Apples and Pears? The relationship between risk capital and required returns in financial institutions*

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Abstract

We use asset pricing theory to investigate the relationship between return on risk capital (RAROC) and required shareholder returns. Both skewness and correlation substantially affect zero-NPV hurdle rates. As a result RAROC is a poor guide to value creation. We propose an alternative measure of return on capital, pricing separately for systematic and balance sheet risks that avoids these difficulties.[61 words]

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1 Introduction

Most of the world's large and internationally active financial institutions now use their risk-management systems for assessing risk-return tradeoffs. This practice is variously described as 'economic capital management' or 'capital allocation'.(1) Capital allocation is used for a wide variety of business applications including performance measurement, product pricing, and the determination of employee remuneration. It is encouraged by regulators as part of the more risk-sensitive approach to regulation developed for banks in Basel II and for European insurers in Solvency II. A major appeal of capital allocation to financial institutions is that it offers a business payoff for the substantial resources that they are already obliged to devote to the measurement of risk capital, the balance sheet net worth needed for protection against the risk of default and financial distress.(2)

The best known capital allocation tool is RAROC, the ratio of expected revenues on a particular exposure to its contribution to institution wide risk capital.(3) The numerator is expected returns over some time horizon (usually one year) net of all operational and funding costs. The denominator – risk capital – is a measure of tail risk quantified using a combination of models, including VaR for market risk, Credit-VaR models for credit risk and other models for operational risk.

RAROC is encouraged by regulators and nowadays very actively pro-

(1)The major consultancy companies are a good source of information on how financial institutions apply these methods, see for example KPMG (2004) and PWC-EIU (2005) on their use in business management and Ernst and Young (2005) on their role in investor disclosure. For a recent published collection of practitioner writing see Dav (ed) (2006).

(2)We use the phrase 'risk capital' rather than 'economic capital' in order to avoid prejudging the issue of when it is appropriate to use a target return on risk capital as an operational goal in a financial institution.

(3)Matten (2000, pp 146-166) describes RAROC alongside several related performance measures. The various acronyms (RAROC, RORAC, RARORAC, etc.) are not applied by practitioners in a consistent manner. While RAROC is the most common acronym for the the most commonly used measure, the one that we discuss in this paper, this same measure is frequently referred to by other names and acronyms, and the term RAROC is also applied to other related performance measures.

moted by the consultancy industry. As a result, it has become the dominant tool used by financial services firms for supporting decisions on business mix, product pricing, and employee remuneration.(4) The following argument is commonly cited by practioners to justify this industry trend. Shareholders provide the equity capital that protects financial institutions from default. RAROC measures the return achieved on this equity capital and hence, using RAROC with some appropriate risk-adusted hurdle rate, ensures that equity capital is being used efficiently across an organisation. Regrettably this argument is incorrect. As we will show, using RAROC with a single institution wide hurdle rate is inconsistent with standard theory of financial valuation.

Table 1 in Section 3.2 below illustrates one of the main sources of this inconsistency. Column (7) shows the return on risk capital required to compensate shareholders for the risk of each exposure, based on standard asset pricing theory. These required returns are the appropriate RAROC hurdle rates for business decision making (what we refer to in this paper as 'zero-NPV' hurdles). An exposure with expected return on risk capital equal to this required return has a net present value of zero so a financial services firm should be accepting all exposures and only those exposures achieving RAROC that exceeds these hurdles. Table 1 reveals that the RAROC hurdle rates for credit exposures are dramatically lower than that for equity.

The difference in RAROC hurdle for equtiy and debt assets is because of the different skewness of their respective return distributions. Returns on equities are right skewed, implying that the volatility of returns (the standard deviation in column (2)) is large relative to the risk capital (the 99.97% left tail of the distribution shown in column (6)). Defaultable debt,

(4)Smithson (2002), page 266, reports that 78% of the respondents to his 2002 Rutter Associates survey of credit portfolio managers, used RAROC to evaluate the performance of their portfolio of credit assets. PWC-EIU (2005), covering more than 200 medium sized and large banks and insurance companies worldwide, finds that more than half now conduct such capital allocation and most use the resulting return measures for various purposes, including business decision making, product pricing, and the determination of bonuses. They write that "economic capital is fast gaining critical mass within the industry". A more recent 2006 update of this survey (not yet available on the web) shows even greater adoption. Asset managers also make widespread use of RAROC as a performance measure when acting on behalf of both retail and institutional investors.

in contrast, has a low volatility of returns relative to risk capital. Correlation with market returns (shown in column (3)) are broadly similar for equity and debt. As a result the RAROC hurdle appropriate for debt instruments are approximately two-thirds lower than that for equity investment.

This implies that the usual practice in financial institutions of using a single institution wide RAROC hurdle rate to determine business decisions results in considerable loss of shareholder value. With the assumptions underlying Table 1, adopting the equity hurdle rate of 21% would lead to rejection of debt opportunities offering a premium over required returns (as a percentage of risk capital) as high as 14% or as more than 30 basis points of their balance sheet asset value. Adopting the AA debt hurdle rate of 7% would be even worse, resulting in in acceptance of equity exposures that destroyed shareholder value by as much as 14% of risk capital or around 3% of the cost of acquisition.

The contribution of our paper is as follows. Section 2 analyses RAROC from the perspective of asset pricing theory, the theory that supports all the standard techniques of financial valuation such as net present value, the capital asset pricing model, and the Black-Scholes pricing equation. We show that the ratio of risk capital and required returns varies from one exposure to another, depending not just upon the shape of the respective return distributions but also the relative degree of correlation with systematic and institution portfolio specific market risk factors. Section 3 then explores the practical implications of our analysis for balance sheet management, in particualr proposing an alternative measure of return on capital that corrects separately for both systematic and institution specific risk and hence can be legitimately applied with a single institution wide hurdle rate. Section 4 is a brief conclusion. Appendix A provides some further illustrative calculations of zero-NPV RAROC hurdle rates.

The remainder of this introduction discusses the relationship between our findings and previous research literature.(5) The use of capital modelling for the twin applications of default protection and for business management has been fairly widely documented by academic researchers.(6) Academic research addresses the question of why financial institutions have developed their own distinct capital based performance measures, rather than using the same approaches used for example in project appraisal by non-financial

(5) A fuller literature survey is provided by Schroeck (2002).

(6) See for example Zaik et. al. (1996) who note that models of risk capital are used for both for finding the proportion of equity to assets that minimizes the cost of funding and for risk-return assessment.

corporates. One reason is the particular importance of credit standing to financial institutions.(7) A further reason for the use of capital based performance measures – not much emphasised in earlier literature – may be that capital allocation can be easily extended to the important non-funded off-balance sheet exposures of financial institutions, where for example the familiar method of internal rate of return cannot be applied.

Only one previous paper, Crouhy et. al. (1999), has discussed the relationship between RAROC and NPV. Like them we focus on the expected return on risk capital achieved by an exposure with zero net present value (the 'zero-NPV' RAROC hurdle rate). We extend their analysis (they consider only the special case of CAPM pricing together with log-normal or arithmetic return distributions) providing a general discussion without any specific assumptions about the pricing of assets or the distribution of their returns, other than the usual investor ratioanlity and and frictionless capital market assumptions of standard asset pricing theory.

A key issue addressed in the research literature, although little discussed by practitioners, is the circumstances under which risk should be priced relative to a financial institution's own portfolio as well as to the market as a whole. Froot, Scharfstein and Stein (1993) point out that, faced with an increasing cost of raising external funds financial institutions will behave in a risk-averse fashion towards risks that are diversifiable at a market level. Specifically, a business unit's contribution to aggregate earnings volatility will be an important factor in the capital allocation and capital structure decisions and also in the decision to hedge earnings risk. Capital structure, hedging and capital budgeting are therefore inextricably linked together.(8)

Froot and Stein (1998) further develop this point, demonstrating in a two period model that the hurdle rate for investments can be calculated from a two factor pricing model, namely the covariance of the return with the market R_m and with the risks of the existing portfolio R_P so $\mu_i = \gamma \mathrm{cov}\,(\mu_i, R_m) + \lambda \mathrm{cov}\,(\mu_i, R_P)$ where γ is the market unit price of risk for the (market) priced factor R_m and λ is the unit cost for volatility of the portfolio. Our proposed measure of return on capital can be seen as an extension of their

(7) Merton and Perold (1993) emphasize this point, arguing that performance measurement in financial institutions is different from industrial companies because their customers are their largest liability holders and as a consequence, a high credit rating is generally essential to maintain their business activities, e.g. as dealers or customers in OTC markets, to underwrite securities or to compete effectively in the corporate banking and deposit markets.

(8) For related discussion see also Stulz (1998)

results, like them we propose pricing separately for systematic and balance sheet risk but we do so with a formulation that allows in addition for the possibility of binding prudential or regulatory capital constraints.

Another literature considers the role of performance measures, in both non-financial and financial companies, as a means of overcoming principal-agent costs, by rewarding managers for acting in the interests of shareholders. Zechner and Stoughton (2007) discuss this use of RAROC, in context of delegated decision making within a large financial institution where managers responsible for investment decisions have privileged information. They develop a model, drawing on the literature on capital decisions in non-financial companies, in which the use of return on capital as a performance measure overcomes the information asymmetries between divisional managers responsible for portfolio decisions and central management.

The valuation model adopted by Zechner and Stoughton (2007) assumes that investors impose a cost on the use of equity capital, but are risk-neutral with regard to systematic risk. This is a reasonable simplifying assumption allowing them to focus on the agency cost issues that are central to their analysis. While not providing a formal treatement, we believe that our proposed measure of return on capital, which adjusts separately for systematic and balance sheet risk, can similarly overcome information assymetries in the case where investors are averse to systematic risk; and thus that our our analysis is entirely consistent with that of Zechner and Stoughton (2007).[9]

Another related branch of literature is that on coherent measures of risk, initiated by Artzner et. al. (1999). They demonstrate that the RAROC denominator VaR fails to satisfy their axiom of sub-additivity. This implies that it is possible, when combining portfolios, that the overall VaR of the combined portfolio can be greater than the sum of the individual VaRs. This potential absence of diversification benefits has been interpreted to mean that VaR is an unsatisfactory risk measure and that alternative measures which do not violate the axiom of sub-additivity, e.g. expected tail shortfall, should be preferred instead.

Our analysis points to a different interpretation of the Artzner et. al.

[9]Stoughton and Zechner (2007) also develop a the separate point that the appropriate measure of risk capital for performance measure should depend on each unit's incremental contribution to total portfolio Value at Risk (its "IVaR"). This suggests defining risk capital in such a way that the sum of the incremental contributions (IVaRs) is equal to the institution's overall VaR, a point well understood by practitioners conducting economic capital allocation.

(1999) results. We suggest that VaR remains an acceptable risk measure for assessing the probability of default (where there is no compelling reason to impose the axiom of sub-additivity) but a poor measure of risk for assessing risk-return trade-offs (when the axiom of sub-additivity has a strong appeal). Coherent measures such as expected shortfall are not not required for measuring risk capital but may still be useful for performance measurement, for example in those situations not covered by own analysis where market prices cannot be used for the risk-adjusted valuation of financial institution exposures.

The literature also addresses the business impact of required regulatory capital. Misalignment of economic and regulatory capital is thought to have distorted business decisions and encouraged the use of securitization to reduce regulatory capital requirements.(10) A stated goal of the new Basel II accord on bank capital has been to achieve a closer alignment of regulatory capital with economic capital and so reduce these effects.(11) Capital allocation is itself promoted by a key principle of the new capital regulations, the so called 'use test'.(12)

Some recent research examines these issues. For example Jokivuolle (2006) reviews the approach of the new accord from the perspective of capital allocation; while Elizalde and Repullo (2006) compare regulatory capital computed by the Basel II IRB risk curves with the capital chosen by banks

(10)Jones (2000) provides illustration of this practice of "Regulatory Capital Arbitrage".

(11)The Basel committee writes (Basel Committee (1999), page 11) "...during the 1990s the [1988 Basel] Accord became an accepted world standard, with well over 100 countries applying the Basel framework to their banking system. However, there also have been some less positive features. The regulatory capital requirement has been in conflict with increasingly sophisticated internal measures of economic capital....In addition the accord does not sufficiently recognise credit mitigation techniques such as collateral and guarantees. These are the principal reasons why the Basel committee decided to propose a more risk-sensitive framework in June, 1999."

(12)In order to qualify for the IRB method for credit risk and the AMA method for operational risk under Pillar 1 of the new Basel accord, the underlying systems must be applied by banks to their business decision making, not just used for regulatory compliance. A similar use test will apply for the recognition of advanced modelling methods in the forthcoming European Solvency II insurance regulations.

(both with and without capital regulations) in the context of a simple dynamic model of banking risks.[13]

In our benchmark setting – that of standard frictionless discounted valuation – capital regulation has no impact on risk-return tradeoffs. We argue that when allowing for frictions such as taxation and balance sheet constraints the business impact of capital regulation can still be expected to be quantitatively fairly small.[14]. If this is correct then alignment of regulatory and economic capital is much less important than many parctitioners and policy makers have supposed.

2 Zero-NPV RAROC hurdle rates

This section considers the appropriate hurdle rates applied to the following RAROC (return on risk capital) performance measure:

$$r^{rc} = \frac{\text{Expected Net Revenues}}{\text{Risk Capital}} \qquad (\mathbf{1})$$

Here risk capital represents the amount of losses (measured relative to expected net revenues) that must be absorbed by shareholders in order to maintain the probability that debt is fully repaid at some target level. For simplicity we restrict our analysis to one-period exposures.

Risk-return comparisons of this kind fall within the domain of standard asset pricing theory. With the assumptions of this theory market prices can be used to value any risky investment prospect and hence establish appropriate hurdle rates for any risk-adjusted performance measure.[15] In the case of return on risk capital this hurdle rate is the value of r^{rc} achieved by exposures with a risk-adjusted net present value of zero. We will refer to this return on risk capital as the zero-NPV RAROC hurdle rate ($\hat{r}^{rc}$). The RAROC decision rule consistent with standard valuation theory is then to accept any exposure with a return on risk capital $r^{rc} > \hat{r}^{rc}$ while $(r^{rc} - \hat{r}^{rc}) \times$ risk capital ('EVA') can be used as a measure of the economic value created from acquiring the exposure.

[13] Elizalde and Repullo use the same Vasicek model of credit risks as we use in our Table 1 and Appendix A.

[14] For a formal elaboration of this point see Dimou, Lawrence, and Milne (2005)

[15] This section thus assumes that all exposures can be 'marked-to-market'. The following Section 3 considers the alternative situation where exposures must instead be 'marked-to-model'.

The key assumptiond of asset pricing theory are investor rationality and the absence of capital market frictions implying that all assets are freely traded on liquid markets at prices that reflect investor preferences. We establish conditions under which the resulting $\hat{r}^{rc}$ hurdle rate is then the same for different exposures. Subsection 2.1 sets out our notation and assumptions. Subsection 2.2 considers the determinants of zero-NPV RAROC hurdles. Required returns are not in general proportional to risk capital (Proposition 1). An increase in skewness increases risk capital relative to required returns (Proposition 2). The RAROC hurdle is increased by correlation between asset returns and the systematic market risk; and decreased by correlation between asset returns and returns on the financial institution's own portfolio (Proposition 3).

2.1 Notation and assumptions

2.1.1 The investment opportunity and the definition of RAROC

A financial institution considers an investment in an exposure indexed by i held for a single period with an initial funding cost of $L_i(0)$.[16] The exposure can be one of many different kinds, including a loan, a trading position, an off balance sheet commitment, or an insurance contract. $L_i(0)$ can be positive or negative. At the end of the period this exposure realises a payoff of $R_i(1) + A_i(1)$ with an expected value of $R_i(1)$ i.e. $A_i(1)$ measures the distribution of end-period payoffs about their mean value with $E[A_i(1)] = 0$).

In order to discuss balance sheet diversification we must also pay attention to the distribution of returns on the remainder of the institution's portfolio. Payoffs on the remainder of the portfolio are denoted by $\bar{R}_i(1) + \bar{A}_i(1)$, and on the total portfolio including A_i by $R(1) + A(1) = \bar{R}_i(1) + R_i(1) + \bar{A}_i(1) + A_i(1)$ The upper case for the expected payoffs $R_i(0)$, $\bar{R}_i(0)$, and $R(0)$ indicates that these are all absolute nominal monetary payoffs, not rates of return which we will distinguish using lower case e.g. r.

The distribution of total portfolio payoffs are described by the joint cumulative density function (CDF) denoted by a double HH to indicate this is a joint distribution. $HH(X_i, \bar{X}_i) = p(A_i(1) \leq X_i, \bar{A}_i(1) \leq \bar{X}_i)$. From HH we can derive the stand alone CDF $H_i(X_i) = p(A_i(1) \leq X_i)$, the total portfolio CDF $H(X) = p(A(1) = A_i(1) + \bar{A}_i(1) \leq X)$, and the total portfolio CDF without the investment i $\bar{H}_i(X) = p(\bar{A}_i(1) \leq X)$. All these distributions

[16] Throughout this section we distinguish the timing of cash flows and payoffs, using (0) to indicate the beginning of the period and (1) the end of the period.

have zero expectations, since they describe payoffs relative to their expected values. Otherwise we place no restrictions on these distributions (although we will consider the special case of joint normality in order to analyse the impact of correlation on the RAROC hurdle rates.)

Risk capital is measured using the quantiles of the CDF at a chosen probability threshold p^*. Thus the total portfolio risk capital is given by $-H^{-1}(p^*)$ while the stand alone risk capital for investment i is given by $-H_i^{-1}(p^*)$. Typically, since p^* is small, $H^{-1}(p^*)$ is negative and much greater in absolute magnitude than expected net return, so risk capital is positive. We also allow for diversification at portfolio level by measuring the marginal contribution to risk capital as $\bar{H}_i^{-1}(p^*) - H^{-1}(p^*)$. Note that under this definition risk capital a VaR type quantile risk measure associated with period 1 return distribution and therefore differs from equity capital which is a period 0 source of funding. Sub-section 3.1 discusses the relationship between equity capital and risk capital.

Return on risk capital RAROC for exposure i is then defined either as:(17)

$$r_i^{\text{src}} = \frac{R_i(1) - r_f L_i(0)}{-H_i^{-1}(p^*)} \tag{2}$$

or as:

$$r_i^{\text{mrc}} = \frac{R_i(1) - r_f L_i(0)}{\bar{H}_i^{-1}(p^*) - H^{-1}(p^*)} \tag{3}$$

depending upon whether we are considering return on stand alone risk capital (src) or on marginal risk capital (mrc).

(17) Here we follow industry practice by defining expected returns to risk capital as the expected asset return $R_i(1)$ net of the costs of debt finance $r_f(L_i(0) - \text{RC})$ i.e. risk capital RC (either standalone or marginal) is a source of funding. (**2**) and (**3**) are then obtained from $r_i^{rc} = (R_i(1) - r_f(L_i(0) - \text{RC}))/\text{RC}$.

2.1.2 Asset pricing theory and market valuation

We denote the time $t = 0$ market value of the exposure i by $\hat{A}_i(0)$.[18]. In this subsection we outline the standard asset pricing theory that we use to model this market value $\hat{A}_i(0)$.[19]

Under the assumption that all risks are tradeable in liquid markets the market value of exposure i can be expressed as:[20]

$$\hat{A}_i(0) = E[z(R_i(1) + A_i(1))] = E[z]R_i(1) + E[zA_i(1)] \qquad \mathbf{(4)}$$

where z is a pricing (or stochastic discount) factor.

We will further assume that markets are complete.[21] z is then unique. It weights all the different possible outcomes for $A_i(1)$ according to the marginal valuations of investors.[22] The expected value $E(z) = r_f^{-1}$ is the inter-temporal discount rate for investors. r_f can be described as the risk-free rate of return, since an exposure with no risk $A_i(1) = 0$ has a present value of $E(zR_i) = E(z)R_i = r_f^{-1}R_i$ and therefore offers a rate of return of r_f.[23]

The content of this theory comes from the fact that z is the same for all assets, it does not depend on i or any portfolio characteristics. This

[18] We use a 'hat' to distinguish market measures (e.g. $\hat{A}_i$) from the corresponding accounting measure (the accounting valuation of the exposure measured at cost would $A_i = L_i$). The use of the same 'hat' for the zero-NPV RAROC hurdle is appropriate, since this is the return on capital achieved by an exposure with a market value $\hat{A}_i(0)$ that equals its accounting value measured on a cost of acquisition basis $L_i(0)$

[19] This theory is described in many textbooks. Our presentation follows that in part I of Cochrane (2005).

[20] This is Cochrane (2005), equation 1.4, with our slightly amended notation.

[21] Our main results still obtain under the weaker assumption that all assets are traded in liquid markets (absence of arbitrage opportunities). In this case, while there is no unique discount factor z, market prices are still uniquely determined. It is also then possible that no risk-free portfolio is achievable in which case $1/E(z)$ is the expected return on the minimum risk or zero-beta portfolio rather than the risk free rate of return.

[22] The ratio of z^a and z^b for two different outcomes A_i^a and A_i^b represents the willingness of investors to exchange a small increase in return in the event of outcome A_i^a for a small decrease in return in the event of outcome A_i^b.

[23] We follow the usual convention in the asset pricing literature in which rates

theory supports a number of widely accepted insights about asset pricing. Investors do not need to be compensated for risks that can be diversified through trading of risky investment instruments. Compensation is required only for risks that are correlated with the investor valuation of returns (those that co-vary with z). Portfolio or exposure specific characteristics will not affect valuations.

This theory can be represented in a variety of equivalent ways:

1. Equation (**4**) can be re-expressed as:[(24)]

$$\hat{A}_i(0) = r_f^{-1} R_i(1) + \mathrm{cov}(A_i(1), z) \tag{\textbf{5}}$$

   We will use an amended version of this equation, where the covariance is restated in terms of the correlation between $A_i(1)$ and z, ρ_{iz}, and the standard deviations of $A_i(1)$ and z, σ_i and σ_z:

$$\hat{A}_i(0) = r_f^{-1} R_i(1) + \rho_{i,z}\sigma_i\sigma_z \tag{\textbf{6}}$$

2. The relationship with standard asset return equations can be obtained by projecting (regressing without a constant) $A_i(1)$ on z to yield:

$$A_i(1) = \frac{\mathrm{cov}(A_i(1), z)}{\mathrm{var}(z)} z + \epsilon \tag{\textbf{7}}$$

   where ϵ is the unpriced diversifiable or specific risk of the exposure. This then yields a beta representation in which the expected return is given by:[(25)]

$$r_i = \frac{R_i(1)}{\hat{A}_i(0)} = r_f + \beta_i^z \lambda \tag{\textbf{8}}$$

   where $\beta_i^z = -\mathrm{cov}(z, \frac{A_i(1)}{\hat{A}_i(0)})/\mathrm{var}(z)$ is the beta of exposure i (the negative of the coefficient from projecting returns $\frac{A_i(1)}{\hat{A}_i(0)}$ on z) and $\lambda = r_f \mathrm{var}(z)$ is the market price of risk.[(26)]

of return are expressed in terms of total payoffs rather than incremental payoffs. Thus period 1 risk free payoffs are discounted at a rate of r_f^{-1} not $(1 + r_f)^{-1}$.

[(24)]This corresponds to Cochrane (2005) equations (1.9). It is derived using the identity $E(zx) = E(z)E(x) + \mathrm{cov}(z, x)$.

[(25)]This is Cochrane (2005) equation 1.15.

[(26)]Note that this beta depends on the covariance of returns with the stochastic discount factor; it can be expressed in terms of the covariance with market returns only in the special case of the standard CAPM.

The z are not directly observed. They are a set of relative valuations inferred from the prices of investment assets. However given further assumptions about either investor preferences, or the determinants and distribution of investor returns, z can be modelled using observable economic or market factors. These special cases include the standard Sharpe-Lintner-Mosser CAPM; the log-utility CAPM of Rubinstein (1976); the APT model; and the inter-termporal CAPM of Merton (1973).

2.1.3 Net present value and the RAROC hurdle rate

We now define the zero-NPV hurdle rate for RAROC. Net present value or NPV is the difference between the market value of an exposure $\hat{A}_i(0)$ and its cost of acquisition $L_i(0)$ and can therefore be written as:[27]

$$\text{NPV} = \hat{A}_i(0) - L_i(0) \tag{\textbf{9}}$$

This is a net present value because it is the present discounted value of future returns less the current cost of acquisition of the exposure. Exposures are value creating and should be acquired if and only if NPV > 0.

The return on risk capital on a zero-NPV exposure (one where $\hat{A}_i(0) = L_i(0)$) can then be written as the following ratio of marginal risk capital:

$$\hat{r}_i^{mrc} = \frac{R_i(1) - r_f\hat{A}_i(0)}{\bar{H}_i^{-1}(p^*) - H^{-1}(p^*)} \tag{\textbf{10}}$$

with an equivalent definition for a stand alone measure of risk capital:

$$\hat{r}_i^{src} = \frac{R_i(1) - r_f\hat{A}_i(0)}{-H_i^{-1}(p^*)} \tag{\textbf{11}}$$

An alternative and equivalent decision criteria to $NPV > 0$ is then to accept all exposures for which $r_i^{rc} > \hat{r}_i^{rc}$ i.e. $\hat{r}_i^{rc}$ is the required rate of return on risk capital (either standalone or marginal).

Note that dividing the numerator and denominator of (**10**) and (**11**) by $\hat{A}_i(0)$ and then using $r_i = \frac{R_i(1)}{\hat{A}_i(0)}$, we obtain the relationship $r_i^{rc} = r_f + \frac{r_i - r_f}{RC/\hat{A}_i(0)}$. This is the relationship used to compute column (7) from columns (5) and (6) of Table 1 in our introduction.

[27] This is the NPV formula in a single-period setting. Multi-period NPV formulations are obtained from the valuation of an asset traded at period 0 offering period t future expected payoffs $R_i(t) + A_i(t)$ where $R_i(t)$ is the known expected return and $A_i(t)$ is the distribution around that return for $t = 1, 2, \ldots, T$.

2.2 The impact of skewness and correlation

We now examine the determinants of the the zero-NPV RAROC hurdle rates, basing our discussion on the following proposition:

1. **Proposition 1** *The required rate of return on stand alone risk capital is*

$$\hat{r}_i^{src} = \frac{\rho_{i,z}\sigma_i\sigma_z}{-H_i^{-1}(p^*)} \tag{12}$$

while that on marginal risk capital is:

$$\hat{r}_i^{mrc} = \frac{\rho_{i,z}\sigma_i\sigma_z}{\bar{H}_i^{-1}(p^*) - H^{-1}(p^*))(1)} \tag{13}$$

Proof. Direct substitution of (**5**) into (**10**) and (**11**) QED □

Proposition 1 shows that it is legitimate to use RAROC with single institution wide hurdle rate for all exposures $i = 1, 2, \ldots, I$ only if the required return premium for each exposure (the numerator for equations (**12**) and (**13**)) is proportional to the risk capital (the denominator of these equations. We will now show that the conditions for this to be true are extremely demanding and, in particular, that differences in either skewness or exposure correlation will affect the zero-NPV hurdle rate.

The following proposition applied to return on standalone risk capital:

1. **Proposition 2** *A sufficient condition for the return on standalone risk capital to be the same for all exposures is that the distribution of asset returns* $A^i(1)$ *all have the same correlation with the stochastic discount factor and that for any given* i*, the distribution of* i *can be expressed as a mean-preserving spread of a single underlying asset return distribution* $A^+(1)$*.*

Proof. Let the standard deviation of $A^i(1) = \sigma_i\sigma_+$. Then $H(A^i(1)) = H(\sigma_i A^+(1))$. Since $\rho_i z$ is the same for all exposures, this rescaling increases both the RAROC numerator $\rho_i z\sigma_i\sigma_z$ and the the RAROC denominator $H_i^{-1}(p^*)$ in proportion to σ_i and hence leaves the RAROC hurdle unchanged unchanged. QED □

Proposition 2 states that RAROC can be used for stand alone comparisons provided that they are all equally correlated with the stochastic discount factor, and their skewness and other higher moments are all the same. In this case their risk – both downside risk and risk-return tradeoffs – can be quantified by single parameter such as the standard deviation and required returns are proportional to exposure to downside risk.

If these sufficient conditions do not apply then an institution wide hurdle rate for standalone RAROC can be used only in the unlikely circumstance that differences in skewness, or in other higher moments, affecting $-H_i^{-1}(p^*)$ just happen to be exactly offset by offsetting differences in correlation with the stochastic discount factor. Otherwise RAROC hurdle rates need to be corrected on an exposure specific basis.

We also obtain the following further proposition for return on marginal risk capital:

1. **Proposition 3** *Suppose (i) the return distributions A_i and $\bar{A}_i$ are jointly normally distributed:* $\begin{pmatrix} A_i \\ \bar{A}_i \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_i^2 & \rho_i\sigma_i\bar{\sigma}_i \\ \rho_{ii}\sigma_i\bar{\sigma}_i & \bar{\sigma}_i^2 \end{pmatrix}\right)$; *and (ii)the volatility of the new exposure is small relative to the existing portfolio ($\sigma_i \ll \bar{\sigma}_i$); then the zero-NPV RAROC hurdle $\hat{r}^r c - r_f$ is given by:*

$$\hat{r}_i^{rc} = \frac{-\rho_{iz}\sigma_z}{N^{-1}(p^*)\rho_{ii}} \qquad \mathbf{(14)}$$

Proof. From joint normality:

$$\begin{aligned} H^{-1}(p^*) - \bar{H}_i^{-1}(p^*) &= -N^{-1}(p^*)\left(\sqrt{\bar{\sigma}_i^2 + \sigma_i^2 + 2\rho_{ii}\bar{\sigma}_i\sigma_i} - \bar{\sigma}_i\right) \\ &= -N^{-1}(p^*)\bar{\sigma}_i\left(\sqrt{1 + \tfrac{\sigma_i^2}{\bar{\sigma}_i^2} + 2\rho_{ii}\tfrac{\sigma_i}{\bar{\sigma}_i}} - 1\right) \\ &\approx -N^{-1}(p^*)\bar{\sigma}_i\rho_{ii}\tfrac{\sigma_i}{\bar{\sigma}_i} \\ &= -N^{-1}(p^*)\rho_{ii}\sigma_i \end{aligned}$$

where $N^{-1}(p^*)$ is the standard cumulative normal density function and the approximation uses $\sigma_i \ll \bar{\sigma}_i$. Then, substituting into (**13**), we obtain (**14**). QED □

Proposition 3 shows that there is a common institution wide zero-NPV hurdle rate for marginal return on risk capital in the special case of joint normally distributed return distributions, provided that the ratio $\frac{\rho_{iz}}{\rho_{ii}}$ is similar for all exposures. In many situations this will not be the case and so the RAROC hurdle must be corrected for differences in correlation, for example:

1.
   - A financial institution operating within a particular market and considering expansion into new markets. This new exposure will have a relatively low correlation with its existing portfolio and a relatively high correlation with the market pricing factor z. For these the appropriate the zero-NPV RAROC hurdle is relatively high. This applies, for example, to a mortgage bank that seeks to extend its business into say corporate lending, or an insurance company acquiring a banking subsidiary; it also applies to a financial institution seeking to expand outside its own domestic market base.
   - Exposures with relatively large proportion of operational risks. Since operational risks can be almost entirely diversified at market level (this is why they can often be insured) but not so well diversified, such exposures should subject to a relatively low RAROC hurdle rate.
   - Large exposures with volatility that is large relative to the existing portfolio, in which case the zero-NPV RAROC hurdle needs to be reduced compared to small exposures. Correlation with the portfolio increases as more of an exposure is acquired and therefore marginal risk capital is larger than suggested by the linear approximation to marginal risk capital $-N^{-1}(p^*)\rho_{ii}\sigma_i$ used in the proof of Proposition 3.

Note also that even when a single RAROC hurdle for return on marginal risk capital is appropriate across an entire institution, this hurdle should still differ substantially from one institution to another. Large diversified institutions will have a relatively low value of ρ_{ii} and therefore should impose relatively RAROC hurdle rates. Small specialised institutions will have a relatively high value of ρ_{ii} and should impose a relatively low RAROC hurdle rate.

Summarising the results of this section, we have shown that zero-NPV RAROC hurdles vary considerably, depending both upon the shape of the return distribution and its correlation with both the institution's portfolio returns and with the market pricing factor z. RAROC hurdle rates, rather than being the same for different exposures and different institutions, must be determined on an exposure and institution specific basis.

3 Implications for balance sheet management

Section 2 assumes that all exposures can be valued with reference to market prices and that the RAROC denominator is an end-period VaR type measure potential losses on the exposure (what we refer to as 'risk capital'). This section extends the analysis, relaxing these assumptions to take account of several real world features and discussing the practical implications for balance sheet management. Several issues are considered. Section 3.1 addresses the implications of measuring RAROC as a return on beginning of period equity capital rather than return on end-period risk capital. While this makes some difference to the levels of both RAROC and the RAROC hurdle rates, the propositions of Section 2 continue to apply. Section 3.2 considers how RAROC hurdle rates are affected by departures from the frictionless capital markets assumptions made in Section 2, in particular for the presence of balance sheet options such as limited shareholder liability and the ability to mark all exposures to market. Section 3.3 considers the relationship between the RAROC hurdle rates and return on equity, showing that even in the special situation where a single institution wide RAROC hurdle can be applied (when the conditions), this hurdle rate should be the return required by shareholders on the market not the book value equity capital. Section 3.4 and Appendix A, present calculations of return on allocated equity taking account of many of these real world complications. These are the Table 1 figures already described in our introduction. These calculations demonstrate that both skewness and correlation have a very substantial quantitative impact on zero-NPV RAROC hurdles. Finally Section 3.5 presents an alternative measure of risk-adjusted return on capital which can be used to allocate capital with a single institution wide hurdle rate.

3.1 Equity capital versus risk capital

The measures of marginal and standalone risk capital discussed in Section 2 are not the same as the equity capital allocated by financial institution risk-management systems. The difference between the two is that equity capital is a period 0 measure of the funding contribution whereas risk capital is a period 1 measure of risk exposure. This subsection shows that Propositions 1-3 continue to apply to allocated equity capital as well as to allocated risk capsital.

Denote the amount of risk capital allocated to exposure i as $K_i(p^*)$ at an appropriate confidence threshold p^*. This could be either standalone risk capital $K_i(p^*) = -H\ _i(p^*)$ or marginal risk capital $K_i(p^*) = \bar{H}_i(p^*) - H$

$_i\,(p^*)$. The financial institution funds exposure i with a mixture of debt and equity. The promised period 1 debt repayment $D_i\,(1)$ is set at the level which maintains the default probability at p^* i.e. $D_i\,(1) = R_i\,(1) - K\ _i\,(p^*)$.

Suppose, for the moment, that shareholders are subject to unlimited liability. This implies that the expected return to shareholders $E\,[E_i\,(1)] = R_i\,(1) - D_i\,(1) = K_i\,(p^*)$ and that the period zero value of the promised debt repayment is $\hat{D}_i\,(0) = r_f^{-1}\,D_i\,(1) = r_f^{-1}\,(R_i\,(1) - K_i\,(p^*))$. The period 0 book value of equity is then $E_i\,(0) = L_i\,(0) - \hat{D}_i\,(0)$ while the corresponding period 0 market value is $\hat{E}_i\,(0) = \hat{A}_i\,(0) - \hat{D}_i\,(0)$. The expected return on equity capital is then given by:

$$r_i^{\mathrm{ec}} = \frac{\mathbb{E}\,[E_i\,(1)]}{E_i\,(0)} = \frac{K_i\,(p^*)}{L_i\,(0) - \hat{D}_i\,(0)} \qquad \mathbf{(15)}$$

and we can obtain a further proposition:

1. **Proposition 4** *The zero-NPV hurdle rate for return on allocated equity capital (standalone or marginal) $\hat{r}_i^{ec}$is related to the hurdle rate for return on allocated risk capital (standalone or marginal) by $\hat{r}_i^{rc}$by:*

$$\hat{r}_i^{ec} - r_f = r_f \frac{\hat{r}_i^{rc}}{1 - \hat{r}_i^{rc}}$$

and so the earlier Propositions 1-3 also all apply to return on allocated equity capital.

Proof (subject to unlimited liability):

$$\begin{aligned}
\hat{r}_i^{\mathrm{ec}} &= \frac{K_i\,(p^*)}{\hat{A}_i\,(0) - \hat{D}_i\,(0)} = \frac{K_i\,(p^*)}{r_f^{-1}R_i\,(1) + \mathbb{E}\,[zA_i\,(1)] - r_f^{-1}\,(R_i\,(1) - K_i\,(p^*))} \\
&= \frac{K_i\,(p^*)}{\mathbb{E}\,[zA_i\,(1)] + r_f^{-1}K_i\,(p^*)} = \frac{r_f}{1 - \hat{r}_i^{\mathrm{rc}}} = r_f \frac{\hat{r}_i^{\mathrm{rc}}}{1 - \hat{r}_i^{\mathrm{rc}}} + r_f
\end{aligned}$$

where the last step follows from $\hat{r}_i^{\mathrm{rc}} = -E\,[zA_i\,(1)]\,/K_i\,(p^*)$. QED □

This proposition implies that the zero-NPV hurdle rate for return on allocated equity capital must also be both exposure and institution specific and is altered, both by skewness and correlation, in the exactly the same direction as the simpler return on risk-capital measure discussed in Section

2. We prefer to discuss return on risk capital in that section because the proof of propositions and the economic intuition relating to the impact of skewness and diversfiication are then much simpler and hence more easily understood; but the same proofs and intuition apply equally to return on equity capital.

What about if we allow for limited shareholder liability? Suppose that there is no protection of debtors e.g. bank deposit insurance or a financial safety net for institutions that are too big to fail. In this case the period 0 value of equity is augmented, and the period 0 value of debt is correspondingly reduced, by the put option created by limited liability for equity holders. Assuming the change in this put option resulting from accepting exposure i has a period 0 value of V_i^{put} . We then have:

$$\hat{r}_i^{\text{ec}} - r_f = r_f \frac{\hat{r}_i^{\text{rc}}}{1 - \hat{r}_i^{\text{rc}} + \frac{V_i^{\text{put}}}{K_i}}$$

Limited liability lowers the zero-NPV hurdle rate for return on equity capital. This is because debt holders now require a higher rate of return to compensate for the risk of losses in the event of default, in turn implying that the financial institution must find a greater proportion of the period 0 funding $L_i(0)$ out of equity capital, and therefore that the required return on this equity capital is reduced compared to when there is unlimited shareholder liability. It should also be realised that Proposition 1 no longer applies exactly, since now $\frac{V_i^{\text{put}}}{K_i}$ can vary for different exposures i even when the conditions for the proposition hold. But (as demonstrated in Appendix A) for small values of p^* the quantitative impact on $\hat{r}_i^{\text{ec}}$ is very small. Moreover propositions 2 and 3 and hence the conclusions reached in Section 2 about the impact of skewness and correlation on hurdle rates continue to apply even when shareholders benefit from limited liability.

3.2 Capital market frictions

3.3 RAROC and return on equity

3.4 Calculations of zero-NPV RAROC hurdles

Table 1: required returns on risk capital[a]

Exposure	PD %	σ %	ρ	beta	Required return %	Risk capital %	RAROC hurdle %
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Equity		9.00	0.60	0.60	8.60	22.40	21.07
Risk free debt	0	0	0	0	5.00	-	-
AA debt	0.20	0.27	0.55	0.02	5.09	4.49	7.06
A debt	0.50	0.54	0.62	0.04	5.21	7.74	7.73
BBB debt	1.00	0.90	0.68	0.07	5.39	11.18	8.48
BB debt	2.00	1.52	0.75	0.13	5.71	15.46	9.57

[a]This table shows calculations of the required returns for different exposures, measured both as a percentage of market value (column (5)) and as a percentage of risk capital (column (7)). The two are related by $(7) = \frac{(5)-r_f}{(6)} - r_f$ (this is explained in Section 2, it is because the RAROC numerator is column (5) net of funding costs). The capital asset pricing model is used to calculate column (5), assuming a risk free rate of $r_f = 5\%$ and the betas in column (4). These betas are in turn derived using return volatility (column (2)) and correlation with the market (column (3)) and a market price of risk (Sharpe ratio for market returns) of 0.5. Risk capital in column (6) is a VaR type measure calculated to a threshold of 99.7%. Equity is assumed to have a log-normal distribution with a correlation of 0.6 with market. The returns on debt instruments are described by the standard Vasicek credit portfolio model used in Basel Pillar 1 IRB capital calculations, with a asset correlation of 0.4 and an LGD of 40%. The underlying aggregate factor driving portfolio default has a correlation of 0.6 with the market i.e. the same as that of equity. The calculations are detailed in Appendix A.

3.5 An alternative measure of return on capital

One possible response is to abandon the practice of capital allocation altogether and instead base financial institution decision making around the direct application of standard NPV formulae. However this meets with the problem that capital continues to be a very important constraint on the activities of financial institutions. It also does not avoid the necessity of computing different risk-adjusted discount rates for each exposure.

We propose a different approach, the use of the following performance metric:

$$r_i^* = r_f + \frac{R_i(1) - r_f L_i(0) - (-\rho_{iz}\sigma_i\sigma_z)}{\bar{H}_i^{-1}(p^*) - H^{-1}(p^*)} \qquad (\mathbf{16})$$

i.e. applying an adjustment for the cost of systematic risk to the numerator of return on risk capital. This can be thought of as an insurance charge, the minimum amount that the financial institution have to pay on the market in order to fully hedge itself against the uncertainty of returns $A_i(1)$.

What does this mean in practice? It means using standard asset pricing models, such as the CAPM or the APT, in order to value the uncertainty associated with each exposure and netting this cost off from expected returns before calculating return on risk capital. This procedure cannot be entirely consistent with the asset pricing theory applied in Section 2. There every exposure has a market price and can therefore be 'marked to market'. In practice many financial institution exposures are illiquid without definite market prices, so in these cases exposures will have instead to be 'marked to model'.

This leads to the further question: what is the appropriate hurdle rate for r_i^*? With the assumptions of Section 2, where there are no capital market frictions and every exposure can be fully risk-adjusted, the hurdle rate is very simple. It is given by:

$$\hat{r}_i^* = 0 \tag{17}$$

since any exposure with a positive r_i^* will create value for shareholders. This is the appropriate hurdle rate in a Modigliani-Miller world where capital structure does not matter.

In reality there are substantial capital market frictions, exposures are illiquid and cannot be accurately priced using asset pricing tools, financial institutions are opaque with their risks and returns not fully understood by shareholders and so not fully reflected in share prices, and there are major agency costs arising in the contracts both between shareholders and senior management and between senior management and employees. Shareholders need to be compensated for all these frictions and therefore demand a premium return on their capital, suggesting that it is appropriate to ration their capital and that this should be done by setting a non-zero hurdle rate:

$$\hat{r}_i^* = \nu > 0 \tag{18}$$

How large is the appropriate hurdle rate ν? Our analysis provides no definite answer. We can expect that this premium will be larger for more opaque institutions and those with less liquid exposures. It will be worthwhile to undertake quantitative empirical work, in order to recover the value of ν consistent with the observed market pricing of financial institution shares. Nonetheless it is clear that this appropriate hurdle rate ν, the true cost to shareholders of supplying capital, will be very much lower than either return on equity or the returns on capital emerging from RAROC calculations. We anticipate that the appropriate premium will be of the order of 1%-2%, a range that implies for example that loan assets – where risk capital is around 5% of total balance sheet loan value – must earn a minimum premium return of around 5 to 10 basis points per annum after netting out the full costs of management, funding and adjustment for systematic risk.(28)

If it does turn out that ν falls in the range 1%-2%, this in turn implies that practitioners have considerably overstated the costs of balance sheet capital, leading them for example to move assets off balance sheet through securitisation when shareholders would be better served by maintaining the assets on balance sheet. To illlustrate, if a financial institution securitises \$100mn worth of assets saving capital of \$5mn then with this magnitude of ν this will create value for shareholders of about \$0.05mn-\$0.1mn per year. Suppose further that the securitisation vehicle is set up for five years, then the total value created is around \$0.25mn-\$0.5mn, but securitisation fees could easily come to \$1mn or more, in which case the securitisation is value destroying. Securitisation may be an extremely expensive way of releasing capital.

The same arguments apply even more strongly to regulatory capital, since regulatory capital requirements are not associated with agency costs to the same extent as the additional free capital held over and above regulatory capital requirements. This suggests that there should be two values of ν, a lower one applied to regulatory capital and a higher one to any excess of risk capital over regulatory requirements.

Our proposed performance measure results in a much more appropriate assessment of the diversification gains from merger than provided by RAROC. Yes, the merged institution is likely to consume less risk capital than the two institutions operating independently. The benefit to shareholders can be measured by ν times the reduction in risk capital. Since ν is very much smaller than RAROC, the diversification gains from merger – while not entirely ignored – are correspondingly small and must be assessed alongside the revenue and cost synergies and various orginisational and management costs of merger. It will be very unlikely that the business case of a merger to be settled one way or the other by return on risk capital arguments alone.

A potential advantage of (**16**) as a performance measure – surprisingly we have found no research or practitioner discussion of this point – is that the hurdle rate can be adjusted upwards in the situation where a financial institution happens to be constrained in its activities by a lack of balance sheet capital, thus ensuring that its limited balance sheet capital is applied

(28) 5 basis points = 1% × 5

in the most efficient way possible.

This is a situation in which the requriement that total risk capital is less than or equal to actual balance sheet capitalisation:

$$-H^{-1}(p^*) \leq K \tag{19}$$

is a binding contstrain. Here K is available balance sheet capital. If this constraint were not satisfied, then the institution would be running a risk of default of greater than the required level of p^*. The efficient way to then maximise shareholder value, while satisfiying this risk-capital constraint, is to increase the hurdle rate above ν to that level $\nu + \chi$ (a shadow price) at which this balance sheet constraint is first satisfied and choose exposures accordingly. A similar balance sheet rationing could be conducted in relation to equity capital although in that case the performance measure would have to be net present value as a proportion of marginal contribution to equity capital, not marginal contribution to risk capital.

We can summarise our proposed performance measure as follows:

Table 2: Hurdle rates for r^* (net present value over marginal risk capital

hurdle : $\hat{r}^*$	Frictionless markets	Balance sheet constraints
No capital constraint	0 (NPV $\geq$ 0)	ν (NPV $>$ 0)
Capital constrained	n.a.	$\nu + \chi$ (shadow price, NPV $\gg$ 0)

There are three possibilities:[29]

1. (a) In the absence of capital market frictions we would simply apply a positive NPV decision rule i.e. $\hat{r}_i^* = 0$.
   (b) In practice, as a result of capital market frictions, there are balance sheet constraints and shareholders will require a premium return on their capital ($\hat{r}_i^* = \nu > 0$) so all accepted projects have net present values clearly greater than zero.
   (c) If in addition the financial institution is capital constrained then it will ration available exposures so as to meet this binding constraint.

(29) With frictionless capital markets an institution can always recapitalise and hence there is no possibility of a capital constraint.

4 Concluding remarks

Our discussion has a number of implications for the practice of capital management.[30] Practitioners frequently claim a benefit to shareholders from merger between two financial institutions because diversification between their portfolios results in an increase in returns relative to risk capital i.e. in an increase in RAROC. But as standard financial theory indicates, shareholders can achieve the same diversification benefit by purchasing the equity of the two institutions, while avoiding the substantial transactional costs of merger. We also argue that the cost of regulatory capital requirements on the shareholders of financial institutions are generally overstated by practitioners, a misperception that underlies the practice commonly but inaccurately described as 'regulatory capital arbitrage' (securitisation of balance sheet assets in order to reduce regulatory capital requirements).

This paper has examined the relationship between risk capital (the contribution of an exposure to institution wide default risk) and required shareholder returns. We have examined the common industry measure return on risk capital i.e. RAROC (equation (**??**)) and its use as a performance measure with a single institution wide hurdle rate assumes that this relationship is stable. We show in our Propositions 1-3 that this procedure is inconsistent with standard models of financial valuation. In all realistic situations the hurdle rate, the required return on risk capital, must be adjusted on an exposure specific basis.

We employ standard asset pricing theory to show that such adjustment is needed, both in order to correct for differences in skewness and higher moments of return distributions (the reason why the RAROC hurdles differ so much in Table 1) and also for divergence between the correlation of returns with the with the marginal valuations of risk by investors (i.e. with the market pricing factor z) and with the institution's own portfolio returns. The rationale for these results is both standard and once understood fairly obvious. In the frictionless setting of standard asset pricing theory, investors need compensation only for systematic risk (correlation of returns with z). Thus exposures with relative large left-hand tails or relatively high correlation with the institution's own balance sheet should be subject to relatively low RAROC hurdles.

Recognising that balance sheet risks are in reality costly for investors, we propose an alternative measure of return on risk capital, in which there is

[30] There are also substantial practical problems with the use of RAROC, notably the difficulties of modelling extreme tail risk with limited data. Some of these practical difficulties are discussed in Milne (2007).

a separate pricing of systematic risk (a deduction of the market cost of this risk from the numerator) and of balance sheet risk (through the choice of an appropriate hurdle rate). Distinguishing systematic risk and balance sheet risk in this way, further highlights some further shortcomings of the conventional RAROC performance measure. It imposes too great a penalty on the use of balance sheet capital, thus also encouraging excessive securitisation, and overstates the gains from merger between institutions.

Our work has related messages for financial institution regulators. Our analysis makes clear that the contribution of an individual exposure to its regulatory capital requirement is only a business concern to a financial institution if it has – or is danger of having – insufficient capital to meet the overall regulatory requirement. Most banks have a very substantial buffers of capital over and above their regulatory capital requirements. This in turn implies that healthy financial institutions should not be much concerned with the level of capital that regulators require to back a particular exposure. Just as there is no reason for shareholders to require returns based on consumption of risk capital, nor should they require returns based on consumption of regulatory capital. This also implies that divergence between regulatory capital and the financial institution's own measure of risk capital will have only minor business impacts.

We therefore suggest that the goal of 'aligning' economic and regulatory capital in the Basel II accord is misplaced since higher regulatory capital has relatively little impact on the market pricing of risks. We also caution that care is required in the application of the 'use test' in the new risk-sensitive regulations: regulators should not normally expect to see financial institutions take direct account of either risk capital or regulatory capital requirements in business performance measurement. [31] To conclude, risk modelling, financial institution management and prudential regulation will all be substantially improved by recognising that there is no necessary relationship between risk capital and required returns.

1.

[31] This should be done *only* when the bank is under pressing risk or regulatory capital constraints.

A Appendix Illustrations of zero-NPV RAROC hurdles

Section 2 has shown that the use of RAROC as a performance measure is only consistent with standard asset pricing theory under highly restrictive conditions.

This Appendix explores the quantitative magnitude of the variation in zero-NPV RAROC hurdles when these conditions are not satisfied, assuming different return distributions or altering exposure characteristics such as volatility of returns or probability of default.[32] Sub-section A.1 compares two standard cases appropriate to the analysis of market risks, those of arithmetic and lognormal returns. This sub-section also analyses the impact of diversification on the RAROC hurdle, showing that constant RAROC hurdle is biased against specialised institutions whose asset portfolio is not fully diversified against movements in market risk factors. Finallyl we are able to replicate, fairly closely, the findings of Crouhy et. al. (1999).

Sub-section A.2 analyses the determinants of the hurdle RAROC in a standard credit risk model, the asymptotic portfolio loss model of Vasicek underlying the Basel II pillar 1 risk curves and widely used in contexts such as CDO tranche pricing. This suggests that the RAROC hurdle rates applied when using Basel II measures of risk capital for loan credit portfolios should be much lower, much less than one half those applied to investments in marketable securities.

The figures of the zero-NPV RAROC hurdle for return on risk capital reported throughout this section are all based directly on the analysis is Section 2, computed for standalone RAROC using (equation (**??**)). For any given return distribution $A(1)$ and confidence threshold p^* for avoiding default, we compute the current market value $A(0)$ of the prospective investment and thus the market value of the initial equity $E(0) = A(0) - D(0)$ that must be provided by shareholders to reduce the default probability to p^*, yielding the required return on this risk capital.

We assume quadratic investor utility, so that the pricing function $q(z)$ used to compute $A(0)$ is the capital asset pricing model, in which the expected rate of return on the market value of the asset is given by:

$$r_A - r_f = \beta_{A,M} \left(r_M - r_f \right) = s_A \rho_A \Phi_M \tag{\textbf{A20}}$$

and $\beta_{A,M}$ is the beta of the return on asset A with the market M and $r_M(t)$

[32] Mathematica coding for all the Figures reported in the Appendix is available from the authors.

is the market return at time t. This assumption, while convenient, is not especially restrictive. We could instead have adopted one of many other asset pricing models. While the quantitative differences between required returns and risk capital might differ from those we report here, the general conclusions would be unaffected. The calculations presented here in fact make use of the right-hand expression in (**A20**), the reformulation more closely related to the Sharpe ratio, where ρ_A is the correlation of the asset return with the return on the single factor driving market returns, where s_A is the standard deviation of asset returns, and Φ_M is the market price of risk. [33]

All figures in this section assume a target default probability of $p^* = 0.03\%$. Except where otherwise indicated all the portfolios (equity or credit) are fully diversified – an appropriate assumption when risk capital is measured by contribution to the default risk of a very large financial institution where the factors driving its returns may be assumed identical with those for the economy as a whole. We assume that $\Phi_M = 1$, but this is only a scaling factor, assuming a larger value would raise all RAROC hurdles proportionately and not affect the differences in these hurdles which we report.

A.1 Arithmetic versus lognormal returns with full and partial diversification.

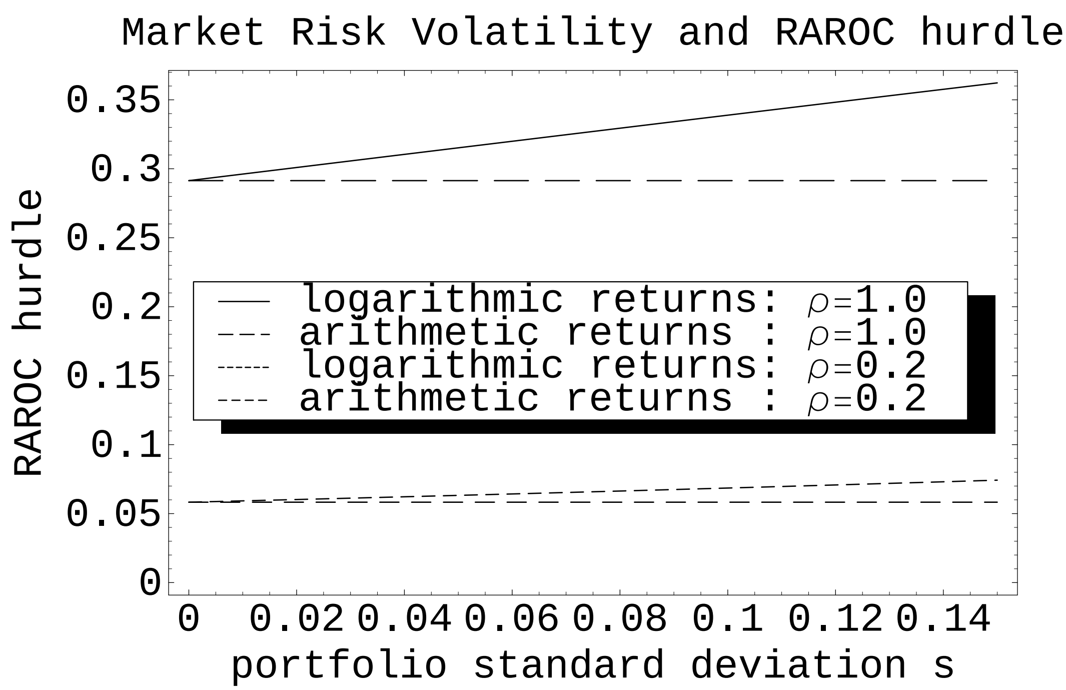
This subsection presents calculations of the RAROC hurdle for a marginal investment opportunity (the $\hat{r}^{rc}$ evaluated on a market value basis as in equation) while varying the standard deviation of returns on a market investment portfolio.

The results are shown in Figure 1. Consider first the upper pair of lines, for a fully diversified portfolio with correlation against the market of $\rho = 1$. The horizontal line is derived assuming an arithmetic normal distribution. This is as predicted by proposition 2, in this case an increase in the standard deviation of returns is an mean-preserving spread in the return distribution, and hence $\hat{r}^{rc}$ remains constant. In this case standalone r^{rc} can be used as a valid performance measure with a constant hurdle rate.

The lines that slope upwards are for the log-normal distribution of returns. This distribution or returns has a right hand skew. An increase in the standard deviation of returns results in a less than proportionate increase in downside tail risk. The denominator of the expression for return on economic capital rises less than proportionately to the increase in asset returns

[33] obtained using $\beta_{A,M} = \rho_A/(s_A s_M)$ and $(r_M - r_f)/s_M = \Phi_M$

Figure 1: RAROC hurdles for market risks†



† Computed assuming $\rho = 1$, $\Phi = 1$. In the case of the log-normal using exact discrete time expected returns and standard deviations.

(the numerator). Hence the RAROC hurdle rises as the standard deviation of returns σ increases. This variation in RAROC hurdle for the lognormal distribution has been previously reported by Crouhy et. al. (1999). The main difference between our results and ours is that they use a balance sheet calculation of risk capital, similar to that we explore in Section 4. We have recomputed the RAROC hurdle using their definition of risk capital and obtained almost exactly the same results as they report.[(34)] Comparing the two cases – arithmetic and log-normal distribution – the figure shows that increasing the volatility of returns from 0% to 14%, the required return on risk capital increases from 29% to about 36% at the 99.97% confidence threshold for the log-normal distribution, whereas for the arithmetic normal it remains constant at 29%.

Figure 1 also reports the zero-NPV or required returns on risk capital for a relatively undiversified portfolio with $\rho = 0.2$. Once again for the arithmetic distribution the required return is constant; while in the case of the log-normal distribution the required return rises as volatility and hence the right skew of the distribution increases. The main point illustrated by these new curves is the required return on the partially diversified portfolio than on the fully diversified portfolio. The intuition here is simple - holding the standard deviation of returns constant, the same amount of equity capital is required to protect an undiversified portfolio as a fully diversified portfolio. However – for any given level of portfolio volatility – shareholders are exposed to much less systematic risk with the partially diversified portfolio than with the fully diversified portfolio, in the former case they are able to remove much of this volatility through diversification within their own holding of the market portfolio. Therefore investors have a very much lower required return on risk capital for the partially diversified institution, the lower lines in Figure 1.

[(34)]Specifically we are able to replicate Table 1 on page 12 of Crouhy et. al. (1999). This replication is not exact for two technical reasons. First we do not include the put option arising from deposit insurance. Secondly we use an exact rather than approximate conversion between continuous time returns and standard deviations (for the log-normal distribution) and discrete time returns and standard deviations.

A.2 An asymptotic credit portfolio distribution

Figure 2 illustrates the RAROC zero-NPV hurdles for the standard credit portfolio model proposed by Vasicek (1987), an asymptotic model of the distribution of returns on a portfolio of defaultable claims and the model underlying the IRB risk-curves in pillar 1 of the Basel II accord.

This Vasicek model of defaultable losses reproduces many basic features of credit risk that cannot captured by either arithmetic or log-normal return distributions. The return distribution is leftward skewed, bounded above at the par value and bounded below at zero. In this model of risky credit portfolio returns, for most plausible parameter choices, the standard deviation of annual returns is relatively small relative to the amount of risk capital required to reduce the probability of default to the required target level. Figure 2 illustrates that because of this left skew, the resulting required RAROC hurdle is very much lower than that reported in Figure 1 appropriate for market investments such as equities.

Figure 2 is derived as follows. In the asymptotic Vasicek model of portfolio returns the end period portfolio return $A0$ (relative to a promised value of 1) is given by:

$$A(0) = 1 - \text{LGD} * N(\frac{N^{-1}(PD) + R^{0.5}X}{\sqrt{1-R}}) \qquad (\mathbf{A21})$$

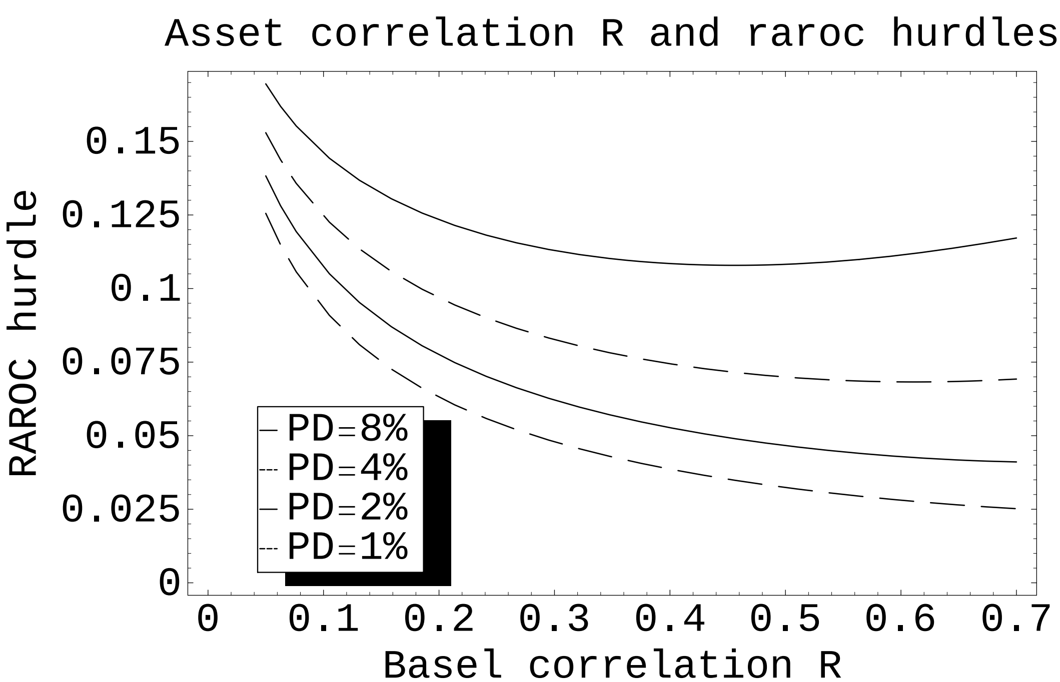
This return is conditional on the underlying normally distributed aggregate factor of X, the constant loss given default LGD, the probability of default PD, and the underlying asset correlation between any two credits R

We assume that returns on the market portfolio are proportional to the same risk factor X (i.e. that both the credit and market portfolios are fully diversified, so the appropriate comparison is with the lines $\rho = 1$ in Figure 1). We then use numerical integration over the range $X \in [-6, +6]$ to compute the correlation of credit portfolio returns with the returns on the market portfolio – and CAPM pricing to obtain the period 0 market value of the credit portfolio $\hat{A}(0)$ – and use the Basel risk curve formula (the right hand part of equation (**A21**) with $X = N^{-1}(p^*)$) to compute the required risk capital $\hat{E}(0)$. The hurdle rates shown in the Figure are then computed directly from (**??**).

The two parameters varied in Figure 2 are the probability of default PD and the underlying asset correlation R between two credit risky assets.[35]

[35] We do not report sensitivity to LGD since this has almost no impact on the zero-NPV RAROC hurdle, the change in the spread of returns and of the correlation with the aggregate factor almost offsetting each other.

Figure 2: RAROC hurdles and credit correlation†



† Computed assuming $\rho = 1$, $\Phi = 1$.

We show the zero-NPV RAROC hurdle for a range of these parameter values covering most bank credit portfolios (PD here ranges from 0.5% to 8%. R for most corporate loan portfolios is fall in the range 0.3-0.5; while for retail credit portfolio exposures it is much lower, typically in the range 0.01 to 0.15.)

Figure 2 indicate that the zero-NPV RAROC hurdle rate for a corporate loan portfolio with returns behaving according to the Vasicek model should be very much lower than the corresponding RAROC hurdle for market exposures shown in Figure 1. For good quality corporate exposures ($PD < 4\%$, $R \approx 0.3$) this hurdle is around 5-6% at a 99.97 % confidence threshold. This compares with thresholds of around 30% for market exposures. This is a very large difference. It is not due to the impact diversification, since the Vasicek model is an asymptotic model which assumes that the credit portfolio is already fully diversified. If the credit portfolio were 'granular' i.e. not fully diversified then the zero-NPV hurdle rates would be even lower.

Figure 2 suggests that RAROC hurdles will be higher for retail credit portfolios (characterised by rather higher PD and much lower R than corporate portfolios) perhaps around 15%, but still very much lower than for market portfolios.

Why are these required RAROC hurdles for credit and market risks so hugely different? This is because of pronounced differences in the shape of the loss distributions. The credit portfolio return distribution computed using the Vasicek model have a very pronounced left skew. This is in contrast to the arithmetic and log-normal distributions used for Figure 1. This substantial difference in the skewness of returns means that the amount of shareholder equity i.e. the risk capital, required to protect a credit portfolio from default can be around five times larger as multiple of portfolio return volatility than is required to protect investment in an equity portfolio. A credit portfolio thus absorbs a much larger amount of risk capital than an equity portfolio, relative to the return required to compensate shareholders for accepting the portfolio risk (which under the CAPM assumption underlying these figures depends only on the volatility of returns and their correlation with market returns.)

1.

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