HIGHER ORDER SYSTEMATIC CO-MOMENTS AND ASSET-PRICING: NEW EVIDENCE

Duong Nguyen*
Tribhuvan N. Puri*

<u>Address for correspondence:</u>
Tribhuvan N. Puri, Professor of Finance
Chair, Department of Accounting and Finance
Charlton College of Business
University of Massachusetts Dartmouth
285 Old Westport Road
North Dartmouth, MA 02747
Voice: 508-999-8759
tpuri@umassd.edu

*University of Massachusetts, Dartmouth

HIGHER ORDER SYSTEMATIC CO-MOMENTS AND ASSET-PRICING: NEW EVIDENCE

Abstract:

In this paper, we provide evidence supporting Rubinstein (1973)'s theoretical model that if returns do not follow normal distribution, measuring risk requires more than just measuring covariance, higher order systematic co-moments should be important to risk averse investors who are concerned about the extreme outcomes of their investments. Our paper provides a contribution to the existing literature that not only Fama-French factors (SMB, HML) but also the momentum factor can be explained by higher order systematic co-moments, hence lends a support to the traditional covariance risk-based framework without having to resort to behavior assumptions. We also find that higher order co-moments might subsume the effect of Pastor and Stambaugh liquidity risk. Our results are consistent in both cross sectional and time series framework as well as in several robustness checks.

I. Introduction

In utility maximizing general equilibrium asset pricing models, returns on risky assets are determined by their covariance with the state variables that represent the fundamental sources of risks in the economy. When changes in state variables adversely affect security returns and investors' wealth, marginal utility of wealth increases for risk-averse investors. Hence in equilibrium investors must be compensated by additional reward commensurate with the risk that they undertake. In the standard capital asset pricing model (CAPM), which is a single state variable asset pricing model, the market summarizes presumably all the sources of risk. Cross-sectional returns within CAPM depend upon their sensitivity to a well diversified market portfolio.

Recent researches show that aggregate market liquidity is yet another fundamental source of risk driving financial markets. Studies by Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001), and Huberman and Halka (2001) provide evidence of the existence of commonality across stocks in liquidity fluctuations. Their findings have initiated a new research hypothesis that if liquidity shocks are non-diversifiable and have a varying impact across individual securities, the more sensitive a stock's return to such shocks, the greater its expected return should be. This hypothesis has been supported by Pastor and Stambaugh (2003). They develop a measure of aggregate liquidity, based on daily price reversals, and show that stock whose returns are more sensitive to market liquidity factor command higher required rate of return than stocks whose returns are less sensitive to market liquidity factor. Acharya and Pedersen (2005), Sadka (2006) also provide evidence of premium of systematic liquidity risk (measured as return covariation with particular measures of aggregate liquidity shocks). Thus it is generally accepted that market liquidity is a priced state variable, however, to what extent the liquidity factor has important bearing on asset pricing is still in debate. One of our contributions in this paper is to try to resolve this debate. We argue that liquidity risk might be captured by certain market risk not captured by the CAPM.

Empirical finance research has also documented that some non-market factors such as size, book-to-market ratios, and momentum can explain cross-sectional variations in returns. For example, Fama and French (1993, 1995) show that the size factor, SMB (return on small stocks less the return on big stocks), and the book-to-market factor, HML (return on high book-to-market stocks less the return on low book-to-market stocks), are significantly important in explaining cross sectional of stock returns. Jagadeesh and Titman (1993, 2001) document price momentum of individual stocks. They show that the stocks that do well relative to the market over the last three to twelve months tend to do well in the next few months and stocks that do poorly continue to do poorly. Gundy and Martin (2001), Fama and French (1996) assert that the momentum effect is one of the most serious problems to asset pricing. This effect is distinct from the value effect captured by stock characteristics and is explained neither by the Fama-French three-factor model nor by the CAPM. Carhart (1997) adds a momentum factor MOM (the difference in returns of diversified portfolios of short-term winner and losers) to the Fama-French three-factor model and finds that the momentum factor is significantly important in explaining stock returns.

While there is a consensus that these non-market factors, i.e., SMB, HML, and MOM are significant in explaining stock returns, there is an ongoing debate about what economic mechanisms drive these factors. The debate focuses on two competing categories: risk-based explanations and non-risk based explanations. The proponents of the risk-based explanations suggest that although SMB, HML, and MOM factors are themselves not state variables in the conventional senses, they reflect unidentified state variables that produce non-diversifiable risks (covariances) not captured by market returns and are priced separately from market betas. For example, Fama and French (1993, 1996) view SMB, HML proxy for certain distress factors that are not captured in the CAPM. Lettau and Ludvigson (2001) show that SMB and HML capture common variation in returns because they seem to be related to variation in a consumption-based risk premium that changes over time. Chung, Johnson and Schill (2006) argue that the Fama-French factors simply proxy for the pricing of higher order co-moments. Conrad and Kaul (1998) argue that momentum profit is a result of cross sectional variability in expected stock returns. Chordia and Shivakumar (2002) suggest that profits to momentum strategies can be explained by a set of lagged maroeconomic variables that are related to business cycle. According to this risk-based explanation, small cap stocks, value stocks, and winner stocks have high average returns because they are risky – they have high sensitivity to the fundamental risk factors that are being measured by SMB, HML, and MOM.

Proponents of the non-risk based explanations, on the other hand, suggest that the human behaviors that deviate from rational expectation theory affect stock prices when market frictions limit arbitrage drives these factors. For example, Lakonishok, Shleifer and Vishny (1994) suggest that the book-to-market factor proxies for investor bias in earnings-growth extrapolation. Daniel and Titman (1997) contend that it is "characteristics, not covariances," that produce return dispersion. They argue that high book-to-market stocks have high returns due to some other reason (possibly overreaction), so that the high returns have nothing to do with systematic risk. In their opinion, it is the characteristics (size and book-to-market) rather than the covariances (sensitivities to SMB and HML) that are associated with high returns. Jegadeesh and Titman (1993) suggested that individual stock momentum might be driven by investor underreaction to firm-specific information. Daniel, Hirshleifer, and Subrahmanyam (1998), Hong and Stein (1999) attribute the momentum anomaly to investor cognitive biases.

In this paper, we attempt to provide another explanation for the non-market factors, SMB, HML, MOM within the traditional co-variance risk-based framework, without having to resort to behavior assumptions. We argue that when the returns are not normal, not only SMB and HML but also MOM proxy for certain market risk factors, not captured by CAPM, measured by a set of higher order co-moments. Our motivation comes from the theoretical model of Rubinstein (1973) and empirical evidence in Chung et al (2006). Rubinstein (1973) demonstrates that a risk measure when distributions are non-normal, requires not only measuring covariance with the market but higher order co-moments (co-skewness, co-kurtosis, and so on). His model suggests that only market risk factors (co-moment factors) should matter to investor. Chung *et al*. (2006) tests Rubinstein's assertion and find that, while each co-moment individually is unable to explain returns, a set of co-moments taken together can do so. They find that a set of systematic co-moments of the order 3rd through 10th substantially reduces the level of significance of Fama-French factors (SMB and HML). One might argue that any set of variables would be able to reduce the significance levels of Fama-French factors if enough of them are included. To rule out such a possibility, Chung *et al* employ a set of standard moments and find that the significance levels of SMB and HML remain the same in almost all the cases. Therefore, they conclude that the SMB and HML factors are simply proxies for higher order systematic co-moments.

Chung et al. (2006) provide a useful starting point. We know that price momentum is one of the most serious challenges to asset pricing as most co-variance risk-based models fail to account for it (Fama and French 1996 and Jagadeesh and Titman 2001). Much research, accordingly, has attributed the momentum strategy profit to behavior biases. We argue that most co-variance risk-based models fail to account for momentum since they do not consider the fact that return is not normal, therefore, higher-order co-moments which measure extreme outcomes of an investment should be considered.

If the return is normal, the first two moments (i.e., mean and variance) alone are sufficient to explain the distribution. As a result, investors should not care about higher order moments. However, there is ample evidence that suggest otherwise (see for example, Fama 1965, Arditti 1971, Singleton and Wingender 1986, and more recently, Chung, Johnson, and Schill 2006). This implies investors may care a great deal about the extreme outcomes of their investment.

There is also a large literature examining at the role of higher order co-moments such as co-skewness and co-kurtosis in explaining stock returns. The basic idea is that if the returns are not normal (skewed or leptokurtic), investors are also concerned about portfolio skewness and kurtosis. If investors' preferences contain skewness and kurtosis, each stock's contribution to systematic skewness (coskewness) and kurtosis (cokurtosis) may determine stock's attractiveness and hence require risk premiums. Starting from Kraus and Litzenberger (1976), many studies provide evidence to support the importance of coskewness and cokurtosis (see, for example, Friend and Westerfield 1980, Sears and Wei 1985, Harvey and Siddique 2000, Chen, Hong and Stein 2001, Hung, Shackleton, and Xu 2004). More interestingly, Harvey and Siddique (2000) find that co-skewness accounts for part of the explanatory power of Fama-French size and book-to-market factors and that co-skewness can explain part of the return to momentum strategies.

One might argue that most return distributions can be reasonably characterized by the first four moments, hence resorting to further higher moments does not seem to be appealing.

However, as with Rubinstein (1973) and Chung et al (2006), we argue that there is no reason to stop with the third or fourth moment. Risk-averse investors have a great concern about the extreme outcomes of their investment or the tail of the return distribution. Variance, skewness, and kurtosis might give some information about the tail, but a set of higher order co-moments is capable of characterizing the likelihood of extreme outcomes.

Lotteries and out-of-money options are examples of why investors care about higher moments. For example, the price of a lottery in the US is $1, the expected value is $0.45, hence the expected return is -55 percent. However, lottery is still popular since the downside risk is small and investors have chance to win lottery, which is an example of extreme outcomes. Similarly, the value of out-of-money options should be zero, but the options still have value since options limit the downside risk and give the chance to earn upside return. These examples imply that investors care about the right tail of return distribution, not only skewness and kurtosis, but also higher order moments.

To justify further the importance of higher moments, consider the following lottery example: an investor attempts to optimize a portfolio made of two independent assets: “Buy” and “Sell”. The two assets have the following payoffs:

“Buy” payoff:

- A $1 loss (999 times in 1000)
- A $999 gain (1 time in 1000)

“Sell” Payoff:

- A $1 gain (999 times in 1000)
- A $999 loss (1 time in 1000)

From the above distribution, any rational investor would choose “Buy” because of small downside risk and large upside gain. Therefore, if asset returns are independent, the optimal portfolio should consist of 100 percent “Buy”. However, it might not be the case when we consider this lottery in the mean-variance-skewness-kurtosis framework:

We compute the “Buy” and “Sell” central moments. The results are as follows:[1]

- “Buy” and “Sell” have the same mean (zero) and variance (999),
- Skewness is 31.57 for “Buy” and -31.57 for “Sell”,
- Kurtosis is 998 for both “Buy” and “Sell”.

Based on the mean-variance framework, the optimal portfolio should be 50 percent “Buy” and 50 percent “Sell” since this is the minimum variance portfolio (both assets have the same mean and variance). Based on the four moment framework, the optimal portfolio is still 50 percent “Buy” and 50 percent “Sell” since it produces minimum variance, kurtosis. The conclusion based on the mean-variance-skewness-kurtosis only might not reflect the small downside risk and the probability to earn large upside return (extreme event). As a result, we should consider the total return distribution to solve the problem, hence the higher order moments should be considered.

Therefore, we argue that it is worthwhile to examine whether the effect of momentum can be accounted for when higher order co-moments are considered. We also examine whether the liquidity risk can be related to higher order systematic co-moments. The liquidity factor proposed by Pastor and Stambaugh still awaits for attestation. So far few studies incorporate a liquidity risk factor into an asset pricing model, and those that do observe limited success in explaining cross-sectional variation in asset returns. Even less is known about whether liquidity risk can be captured by some forms of market risk not explained by the CAPM.

[1] The proof is available upon request.

Our study is also motivated by a preliminary examination. We regress the factor loadings of size, book-to-market, momentum, and liquidity factors on a set of ten higher order systematic co-moments and find that the systematic comoment can explain these loadings with high R-squares (0.85-0.89 for size factor, 0.73-0.75 for book-to-market factor, 0.61-0.90 for momentum factor, and 0.50-0.67 for liquidity factor).[2] This evidence suggests that there might be a relationship between the common factors and systematic co-moments, hence may provide an explanation for economic mechanism behind these factors within a traditional risk-based covariance framework.

For the cross sectional analyses, using Fama-Macbeth (1973) procedure for portfolios sorted by size, book-to-market, momentum, and liquidity for the period 1970-2005, we find that adding a set of systematic co-moments of order 3 through 10 or higher reduces the explanatory power of SMB, HML, momentum, and Pastor-Stambaugh market liquidity factors to insignificance in almost all cases. Also, consistent with Chung et al (2006), we do not find the similar results when adding a set of standard moments of order 3 through 10 or higher. To check the stability of the results, we divide the sample into two sub-samples: 1970-1987 and 1988-2005, and perform similar analyses. The results still hold in both sub-periods. We also find that the results with a set of 10 systematic co-moments are very similar to those with 15 co-moments or higher. This was done with an objective to verify whether a set of co-moments higher than 10 would more precisely characterize the return distributions. However, the empirical evidence suggests that a set of comments of order 3 through 10 is sufficient to capture the extreme outcomes of the investment.

For the time series analysis, we use multivariate test of Gibbon, Ross, and Shanken (GRS) (1989) and find that the GRS statistics, which test whether the pricing errors from time

[2] The detail results are reported in Appendix Table A.1.

series regression are jointly equal to zero, consistently decrease when more systematic co-moments are added to the model, and become insignificant when enough co-moments are included. This suggests that the pricing error tends to zero when more co-moments are added to the model.

Our findings lend support to Rubinstein (1973) in that higher order systematic co-moments should be considered in asset pricing. We also provide another risk-based explanation for SMB, HML, and MOM that these common factors might proxy for higher order co-moment, which is some certain market risk capturing extreme outcomes of the return distribution and this risk is not captured in the CAPM. Furthermore, we find that Pastor and Stambaugh liquidity risk might be subsumed by the effect of these higher order systematic co-moments. This evidence lends support to the notion that the empirical knowledge of non-market factors in the context of asset pricing models may be of less value because all such factors might be proxied by a set of higher order co-moments with a well diversified market portfolio.

This paper is organized as follows. The next section discusses the data and methodology employed in the paper. Section III presents the empirical findings and section IV concludes the paper

II. Data and Methodology

We examine the issue of high order co-moments and common factors in asset pricing under both cross-sectional and time-series framework. The Fama-MacBeth (1973)'s procedure is employed for cross sectional analysis while the multivariate test of Gibbon-Ross-Shanken (1989) is applied for time series analysis. We form stocks into portfolios based on size, book-to-market, momentum, and liquidity. The detailed procedure is as follows.

1. Portfolio formation

We sort all ordinary common stocks (with the CRSP share code = 10 and 11) traded on three stock exchanges, NYSE, AMEX, and NASDAQ, into 50 portfolios based on size, book-to-market, and momentum and liquidity. In particular, for size portfolios, at the end of each calendar year in the period 1965-2005, all stocks are ranked based on their market capitalization and sorted into 50 portfolios of equal number of stocks. For book-to-market portfolios, all stocks are ranked by their beginning-of-period book-to-market ratios and then divided into 50 portfolios of equal size. The momentum portfolios are constructed along the line with the procedure in Ken French's website.[3] In particular, at the end of each month t, all stocks are sorted into 50 portfolios of equal size based on their prior compound return from month t-2 to t-12. For the liquidity portfolio, we use the liquidity measure of Pastor and Stambaugh (2003). More specifically, each year we perform the following linear regression to obtain the annual liquidity measure for each stock

$$r^e_{i,d+1,t} = \theta_{i,t} + \phi_{i,t} r_{i,d,t} + \gamma_{i,t} sign(r^e_{i,d,t}) \times v_{i,d,t} + \varepsilon_{i,d+1,t} \qquad \mathrm{d} = 1, ..., \mathrm{D} \qquad (1)$$

Where $r_{i,d,t}$ is the return of stock *i* on day d in year *t*; $r^e_{i,d+1,t} = r_{i,d,t} - r_{y,d,t}$ where $r_{y,d,t}$ is the return on the CRSP value-weighted market return on day *d* in year *t*; and $v_{i,d,t}$ is the dollar volume for stock *i* on day *d* in year *t*. The liquidity measure for stock *i* in year *t* is the estimate of $\gamma_{i,t}$ in the above regression (1). We compute the liquidity measure for a stock in a given year only if there are more than 30 observations to estimate (1) (D >30). Then in each year, we form all stocks into 50 liquidity portfolios based on their annual liquidity estimates $\gamma_{i,t}$.

Using portfolios constructed above, we compute equally weighted monthly returns for each of the 50 portfolios. We subtract the 30-day Treasury bill yield to obtain the excess

[3] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french

portfolio return. Once we have constructed the portfolios, we employ both time series and cross-sectional tests to examine the relationship between the common factors and higher order systematic co-moments.

2. Cross-sectional test

We apply the two-step Fama-Macbeth (1973) procedure to our empirical asset pricing tests. In particular, first we test the five-factor model that includes Fama-French, momentum, and Pastor-Stambaugh factors as follows:

$$r\left(j,t\right) = a_0 + a_{rmrf} b_{rm}(j,t) + a_{smb} b_s(j,t) + a_{hml} b_h(j,t) + a_{mom} b_m(j,t) + a_{liq} b_l(j,t) + e(j,t) \qquad (2)$$

Where, $r(j,t)$ is the excess return of portfolio *j* in month *t and* $b_{rm}(j,t)$, $b_s(j,t)$, $b_h(j,t)$, $b_m(j,t)$, $b_l(j,t)$ are factor loadings for excess return of portfolio *j* on factors (*Rm-Rf*), SMB and HML, momentum (MOM), and Pastor-Stambaugh market liquidity (LIQ) respectively, in month *t*.[4]

For each month *t*, the factor loadings are computed by regressing portfolio returns over the last five years on the market factor (*Rm-Rf*), SMB, HML, MOM, and LIQ, respectively. The result is a time series for each factor loading from 1970 to 2005 (we lose the first five-year data in the original sample in order to estimate the factor loadings). Once the factor loadings are computed, for each month *t*, we perform cross-sectional regressions of the period portfolio returns on the factor loadings as in equation (2). Repeating this process for all months in the period 1970-2005, we have 432 sets of coefficient estimates. Following Fama-MacBeth, we average these estimates to get the average coefficients.

[4] We thank the Wharton Research Database Service (WRDS) for proving us the data on these factors.

Next, we examine whether the above factor loadings are still significant when a set of systematic co-moments is added to the model. In particular, for each month t, we perform cross-sectional regressions of excess portfolio returns on the loadings of SMB, HML, MOM, and LIQ, and on the systematic co-moments as follows:

$$r(j,t) = a_0 + a_{smb} b_s(j,t) + a_{hml} b_h(j,t) + a_{mom} b_m(j,t) + a_{liq} b_l(j,t) + \sum_{i=2}^{n} a_i b(i,j,t) + e(j,t), \qquad (3)$$

where $b(i,j,t)$ is the i^{th} systematic co-moment of portfolio j in month t.[5] We compute the systematic co-moments in month t using the past 60 months of portfolio returns as follows:

$$b(i,j,t) = \frac{\sum_{\tau=1}^{60}\left[r(j,t-\tau) - \frac{1}{60}\sum_{k=1}^{60} r(j,t-k)\right]\left[r(m,t-\tau) - \frac{1}{60}\sum_{k=1}^{60} r(m,t-k)\right]^{i-1}}{\sum_{\tau=1}^{60}\left[r(m,t-\tau) - \frac{1}{60}\sum_{k=1}^{60} r(m,t-k)\right]^{i}}, \qquad (4)$$

where, $r(m,t)$ is the return of the CRSP value weighted index. We compute the systematic co-moments up to the 10^{th} order as in Chung et al (2006). We also experiment a set of systematic co-moments up to the 15^{th} order to see whether the findings in Chung *et al* are robust to higher order of co-moments or the findings are only chance results. Our findings which are described in the next section show that the two set of systematic co-moments give similar results.

Since one might argue that including any set of variables would be able to reduce the significant levels of the factors, to address this issue, we also include a set of standard moments (not systematic co-moments) as follows:

$$r(j,t) = a_0 + a_{rmrf} b_{rm}(j,t) + a_{smb} b_s(j,t) + a_{hml} b_h(j,t) + a_{mom} b_m(j,t) + a_{liq} b_l(j,t) + \sum_{i=3}^{n} a_i m(i,j,t)$$
$$+ e(j,t), \qquad (5)$$

[5] We include the loading on the market risk premium factor in the set of systematic co-moment since the loading is the 2nd systematic co-moment.

where, $m(i, j, t)$ is the standard moment order *i* of portfolio *j* in month *t*. We also use the past 60 months of portfolio returns to compute $m(i, j, t)$ as follows

$$m(i, j, t) = \frac{1}{60}\sum_{\tau=1}^{60}\left[r\left(j, t-\tau\right)\right]^{i} \tag{6}$$

The two-step Fama-MacBeth procedure has become a standard for estimation and testing of different asset pricing models. However, this approach has two major drawbacks: (1) error in variable, because betas are estimated in the first-stage regression and then subsequently used as independent variables in the second-stage regression, and (2) serial correlation in return residual. As a check for robustness, we use Shanken (1992)'s correction to adjust for estimation errors in betas. The adjusted covariance matrix is calculated as follows

$$\text{Adj. Var(a)} = V\left[1 + \overline{a}'(Z)^{-1}\overline{a}\right] + Z \tag{7}$$

Where V is the k factor × k factor covariance matrix of mean coefficient estimates and Z is the k × k covariance matrix of monthly risk factors. The respective risk factors are the SMB, HML, MOM, LIQ, and market risk premium (RMRF) for the covariance factor, and RMRF raised to the (*i-1*) power for the higher-order co-moment factors.

The Fama-MacBeth method is designed in such a way that it accounts for cross correlation between return residual. However, it assumes the residuals are not correlated over time, which might not be correct. Therefore, the procedure is not robust to serial correlation (see Cochrane 2001 for more detail). We use Newey-West (1987)'s method to account for serial correlation in the error term.[6]

[6] See Newey-West (1987) for more detail

3. Time Series Test

It is well-known that asset pricing models can also be evaluated by examining the intercepts from the time series regressions of portfolio excess returns on the factors. If the regression intercepts, which are the pricing errors, are jointly equal to zero, the model is valid. Another advantage of time series approach is to avoid the error in variable problem in the Fama-MacBeth approach since the estimation of risk premium is no longer necessary and the implication of the asset pricing theory can be tested by the hypothesis that all the intercepts are jointly equal to zero. To test this hypothesis, we use the test developed by Gibbon, Ross, and Shanken (1989) (GRS). The GRS statistics can be used to examine the relation between common factors and higher order systematic co-moments. We argue that if the common factors proxy for higher order co-moments, the magnitude of GRS statistics should decrease when co-moment factors are added to the model. The procedure is follows

First, we estimate the time-series regression of the excess returns on the 50 portfolios (sorted by size, book-to-market, momentum, and liquidity) on the five-factor model using ordinary least squares:

$$r(i,t) = \alpha_i + \beta_i (R_{mt} - R_{ft}) + \delta_i SMB_t + \gamma_i HML_t + \eta_i MOM_t + \psi_i LIQ_t + e_{it} \tag{8}$$

where $r(i,t)$ is the excess return on portfolio *i* in month *t*, $(R_{mt} - R_{ft})$, SMB_t, HML_t are the Fama and French (1993) three factors related to market premium, firm size, and the book-to-market ratio, MOM_t is the momentum factor, and LIQ_t is the Pastor and Stambaugh (2003) liquidity factor in month *t*. We then compute the GRS statistics to test whether the 50 intercepts from these time series regressions are jointly equal to zero as follows:

Let N be the number of time series observations, L be number of portfolios, K be the number of regression parameters including the constant term, and X be the observation matrix. Then, the GRS test statistic is given by

$$\left(A'\Sigma^{-1}A\right)\frac{N-K-L+1}{L*(N-K)*\omega_{1,1}}$$

where A is the column vector of the regression parameters, Σ is the variance-covariance matrix of the residuals from the regression, and $\omega_{1,1}$ is the diagonal element of $\left(X'X\right)^{-1}$. Under the null hypothesis that the regression constants are zero, this statistic has an F-distribution with L and (N – K – L + 1) degrees of freedom.

The findings are presented in the next section. Briefly, we find that the GRS statistics strongly reject the null hypothesis that the time series intercepts are jointly equal to zero.

The next step is we add higher order co-moment factors to (8) as follows

$$r(i,t)=\alpha_i+\beta_i(R_{mt}-R_{ft})+\delta_i SMB_t+\gamma_i HML_t+\eta_i MOM_t+\psi_i LIQ_t+\sum_{j=3}^{K}\eta_{i,j}\left(R_{mt}-R_{ft}\right)^{j-1}$$

$$+e_{it} \tag{9}$$

Where $\left(R_{mt}-R_{ft}\right)^{j-1}$ is the higher order *j* co-moment factor. For example, $R_{mt}-R_{ft}$ is the covariance factor, $\left(R_{mt}-R_{ft}\right)^2$ is the coskewness factor, $\left(R_{mt}-R_{ft}\right)^3$ is the co-kurtosis factor, and so on.

When adding each co-moment factor, we compute the corresponding GRS statistics. Our argument is that if higher order co-moments are relevant in explaining stock return and if higher order co-moments can subsume the effects of factors SMB, HML, MOM, LIQ, then adding these co-moments reduces the significance level of the GRS statistics since the intercepts (pricing errors) tend to approach zero.

III. Empirical findings

Table 1 provides a summary statistics for the distributions of portfolios returns for size-sorted, book-to-market sorted, and momentum sorted portfolios. We use three statistics, namely Kolmogorov-Smirnov, Cramer-von Mises, Anderson-Darling, to test the normality of the portfolio returns. In all cases, the normality is strongly rejected. Since the returns do not follow normal distribution, the first two moments alone should not be sufficient to characterize the return distribution hence higher order moments should be considered.

Table 2 reports the correlation among factor loadings, SMB, HML, MOM, and LIQ. As can be seen from the table, SMB are generally strongly correlated to HML (0.39, 0.35, -0.05, and 0.59 for size, book-to-market, momentum, and liquidity portfolios, respectively). MOM and LIQ generally have low correlation with SMB and HML. For example, the correlation between LIQ and SMB, HML is -0.02 and 0.02, respectively for size portfolios, -0.01 and 0.07, respectively for book-to-market portfolios, -0.28 and 0.09, respectively for momentum portfolios, and -0.18 and 0.01, respectively for liquidity portfolios. The correlation between MOM and LIQ is also low (-0.01, 0.01, 0.17, 0.07 for size, book-to-market, momentum, and liquidity portfolios, respectively). This indicates that MOM and LIQ are separate effect from each other as well as from SMB and HML and might be priced separately.

A. Cross Sectional Results

We report the two-step Fama-MacBeth (1973)'s procedure applied to the five factor model (2^{nd} systematic co-moment, SMB, HML, MOM, and LIQ) in table 3. For each month, factors loadings: *s*, *h*, *m*, *l*, are computed by regressing portfolio returns on the SMB, HML, MOM, LIQ, respectively. Then, in each month, portfolio returns are regressed on these factor loadings (also including the 2nd systematic co-moment, which is the loading on the market factor) to get the Fama-MacBeth coefficients. As can be seen from the table, at least two out of four factor loadings are significant for all portfolios. The factor loadings on HML and MOM: *h* and *m*, respectively, are significant in all cases, while the factor loading on market liquidity factor LIQ is significant in the case of book-to-market and liquidity portfolios. We compute the F-statistics to examine whether the coefficients on SMB, HML, MOM, and LIQ are jointly different from zero. In all sorting criteria, the F-statistics reject the null hypothesis that all factor loadings are jointly equal to zero, suggesting at least one of the factors is significant in explaining cross sectional stock returns.

The main focus of our analysis is to examine how the significant levels of SMB, HML, MOM, and LIQ change when the systematic co-moments are added to the model. Table 4, Panel A reports the results for size sorted portfolios. We find that the significance levels of factor loadings on SMB, HML, MOM, and LIQ successively diminish as we add more systematic co-moments. Eventually, the factor loadings become insignificant when a set of 10 co-moments is included.[7] We also experiment with a set of 15 co-moments instead of a set of 10 co-moments. We find that the results are not different. More interestingly, the magnitude of the F-statistics that tests the joint significance of coefficients of the factors decreases when more systematic co-moments are added and become insignificant when a set of 10 or 15 co-moments is included. This implies that if a sufficient number of co-moments are considered, the factor loadings become insignificant in explaining cross sectional stock returns. Panel B, C and D report the results for book-to-market, momentum, and liquidity sorted portfolios, respectively. The findings are very similar to those in Panel A.

[7] The significant level of loading on LIQ factor basically remains insignificant throughout all cases.

Since one may argue that any set of variable would be able to reduce the significance levels of the factor loadings if a sufficient number of them are included. Such is not the case when we add standard moments (not systematic co-moments) to the model. Standard moments are computed as in equation (5). The results are reported in Table 5. In all cases (for size, book-to-market, momentum, and liquidity portfolios), whether we add a set of 10 or15 standard moments, the explanatory powers of factor loadings on SMB, HML, MOM, and LIQ remain almost unchanged compared with the original levels when no standard moment is included. The F-statistics are significant and their magnitudes are similar in all cases. The findings imply that standard moments do not reduce the significance of common factors, but the systematic co-moments do. This is consistent with Rubinstein (1973)'s model in that if return is not normal, risk averse investors should be concerned about higher order co-moments.

Another noteworthy observation is that some factor loadings (e.g., LIQ), that are insignificant before adding higher co-moments, remain insignificant even after adding higher systematic co-moments or higher standard moments. This means that a set of systematic co-moments affects only those factors that are significant, and does not affect the ones that are not significant before adding more variables. A set of standard moments does not reduce the significance levels of the factors regardless they are initially significant or insignificant. If adding more variables causes imprecision in estimation, this should not be the case. In some other context, this observation is additional evidence that our results are not driven by simply adding a set of variables.

B. Time Series Results

Our main concern with the two-step Fama-MacBeth procedure is that it might not be robust (see Shanken and Zhou 2007, Lewellen, Nagel, and Shanken 2007, Petersen 2007). Therefore, we conduct time series test as an alternative asset pricing test for robustness check. We report the GRS statistics testing the hypothesis whether the intercepts from time series regressions are jointly equal to zero for portfolios sorted by size, book-to-market, momentum, and liquidity in Table 6. The main idea in the time series test is that if the intercepts (i.e., pricing errors) from time series regressions of portfolio returns on the five-factor model are jointly equal to zero, hence the GRS statistics are not significant, the common factors are sufficient to explain stock returns. On the other hand, if the GRS statistics are significant different from zero, then the model is not able to explain stock returns. In our analysis, we add higher order co-moment factors to the model and compute the corresponding GRS statistics. If the statistics decrease consistently when more co-moment factors are added and become insignificant when enough co-moment factors are included, then higher order co-moments should be considered in asset pricing, common factors may be proxies for these co-moments.

The results in Table 6 support our argument. When the asset pricing model includes only five factors: market risk premium, SMB, HML, MOM, LIQ, the GRS statistics are 12.37, 10.69, 2.75, and 3.45 for size, book-to-market, momentum, and liquidity portfolios, respectively, which are strongly significant different from zero. This suggests these factors are not sufficient to explain stock return. Next, we start adding the higher order co-moment factors, the GRS statistics consistently decrease for all portfolios and become insignificant when a set of 10 or 15 co-moment factors are added. The results here are consistent with those in the cross-sectional analyses in that the factors SMB, HML, MOM, LIQ might proxy for higher order co-moments when explaining stock returns.

C. Other robustness checks

In this section, we present results of other robustness checks for the Fama-MacBeth procedure, namely, (1) test the stability of the results by dividing the sample into two sub-periods, (2) correct for error-in-variable problem using Shanken (1992)'s adjustment, and (3) adjust for serial correlation in return residuals using Newey-West (1987)'s method.

Chung et al (2006) consider different return horizons: daily, weekly, monthly, quarterly, and semi-annually. However, since the Pastor and Stambaugh market liquidity factor is constructed using monthly data only, the data on this factor for other higher frequencies is not available. Thus, in order to check robustness of the results, we divide the sample into two sub-periods: (1) 1970-1987 and (2) 1988-2005. The results are reported in table 7 and 8. Since the results with a set of 10 systematic co-moments are very similar to those with a set of 15 systematic co-moments, we report only those with the 10 co-moments.

Table 7 shows that for size portfolios, when the model includes only the 2nd systematic co-moment (covariance), the coefficients of the factor loadings on SMB, HML, MOM, LIQ are all significant. In particular, the t-statistics of *s, h, m, l* are 1.80, -6.55, 5.21, 3.42, respectively. The F-statistics for the joint significance of these loadings is 16.15, which is significant at 1 percent or below. However, when a set of 10 systematic co-moments is included, the t-statistics of *s*, *h*, *m*, *l*, are 0.55, -0.22, -0.53, 1.28, respectively, which are insignificant. The F-statistics reduces to only 1.21, which is also insignificant. The results with book-to-market portfolios are very similar to those with size portfolios. For the momentum portfolios, the loading on momentum factor, m, is significant at the original level (t-stat = 3.20), but becomes insignificant with the addition of a set of co-moments of the order 2nd to 10th. The F-statistics also reduces from 3.32 (significant at 5 percent level) to 0.83 (insignificant). For the liquidity portfolios, the loadings on HML and MOM factors are significant at 5 percent level but all become insignificant when the set of higher order co-moments is included in the model.

The results with standard moments in panel B, Table 7 are also consistent with those obtained for the full sample. The addition of a set of standard moment does not reduce the explanatory power of factor loadings for the size, book-to-market, momentum, and liquidity portfolios. For the momentum portfolios, the coefficients of *s* and *l* are not significant. This, however, is not because of adding standard moments since they are insignificant even before we include the set of standard moments. The magnitudes of F-statistics remain almost unchanged before and after adding the set. This implies the standard moments do not affect the explanatory power of the factors on stock return.

Table 8 presents the results for the second sub-period: 1988-2005. The findings are generally consistent with those in the first sub-period as well as in the whole sample. Only in the case of size portfolio, adding systematic co-moments does not reduce substantially the significance levels of all factor loadings. SMB and HML remain significant and the magnitude of F-stat does not reduce substantially. However, in other cases, the results are in line with previous findings.

The Fama-MacBeth may be biased because of the error-in-variable problem. The right-hand-side variables in the second-pass cross sectional regression are estimates from the first-pass time series regression. We use Shanken (1992)'s adjustment to recalculate all the t-statistics in Table 4. The results are reported in Appendix table A.2. We document consistent results with Chung et al (2006). The adjusted t-statistics are similar when only the 3rd comoment is added, but drop substantially and approach zero when higher order co-moments beyond the 3^{rd} are included in the model. The evidence is consistent for all portfolios: size, book-to-market, momentum, and liquidity. Chung et al suggest that because of the higher-order right-hand-side variables are created, the Shanken adjustment biases the test results too much against the factor loadings causing the t-statistics to approach zero, hence the adjustment appears to be inappropriate for our studies.

We use Newey-West's procedure to recalculate the t-statistics adjusted for serial correlation in the return residuals which is not accounted in the Fama-Macbeth method. The results are reported in Appendix table A.3. We document similar findings as with the Fama-MacBeth method. This implies that the serial correlation problem does not influence the results.

Our computation so far is based on centered systematic co-moments. We also report the results based on non-centered systematic co-moments.[8] The findings are also qualitatively similar (see Appendix Table A.4).

Overall, our robustness checks are consistent with the previous findings that adding a set of systematic co-moments reduces substantially the significance levels of factor loadings: SMB, HML, MOM, and LIQ.

[8] The non-centered systematic co-moments are computed as follows

$$b(i, j, t) = \frac{\sum_{\tau=1}^{60} \left[r(j, t-\tau)\right]\left[r(m, t-\tau)\right]^{i-1}}{\sum_{\tau=1}^{60} \left[r(m, t-\tau)\right]^{i}}$$

where $b(i, j, t)$ is the i^{th} non-centered systematic co-moment of portfolio j in month t, $r(m,t)$ is the return of the CRSP value weighted index.

IV. Conclusion

In this paper, we attempt to provide evidence to support Rubinstein (1973)'s theoretical model that if returns do not follow normal distribution, measuring risk requires more than just measuring covariance, higher order systematic co-moments should be important to risk averse investors who are concerned about the extreme outcomes of their investments. We show that not only the Fama-French factors (SMB and HML), but also momentum and liquidity factors can be explained by higher order co-moments.

In cross-sectional analyses, for all sorting criteria (size, book-to-market, momentum, and liquidity), we find that adding a set of 10 or 15 systematic co-moments reduces substantially the significance levels of the factors: SMB, HML, MOM, and LIQ and causes them to become insignificant in most cases. One might argue that the results are driven by imprecision in estimation due to adding more independent variables, we show that it is not the case. We perform a similar analysis with a set of 10 or 15 standard moments and find that the explanatory powers of the factors remain the same after including the set of standard moments. Also, in some cases, some factors remain consistently insignificant before and after adding the set of variables. Thus, it does not appear that our results are being driven by simply adding more explanatory variables. Our cross sectional results are also consistent in both sub-periods and several other robustness checks.

In time-series analysis, we find that the Gibbon-Ross-Shanken statistics, which test whether the pricing errors from time series regressions are jointly equal to zero, consistently decrease when more systematic co-moments are added to the model, and become insignificant when enough co-moments are included. This suggests that the pricing error tends to zero when more co-moments are added to the model, hence providing evidence that higher order systematic co-moments should be relevant in pricing assets.

We also find that the results with a set of 10 systematic co-moments are very similar to those with 15 co-moments or higher. This suggests that while the more number of higher co-moments are included, the return distributions would be characterized more precisely, but empirically, a set of 10 order co-moments is sufficient to capture the extreme outcomes of the investment.

Our findings provide another explanation for the non-market factors, SMB, HML, MOM within the traditional co-variance risk-based framework, without having to resort to behavior assumptions since these factors are explained by certain market risk not captured in the CAPM and measured by a set of higher order systematic co-moments. We also find that the Pastor and Stambaugh liquidity risk might be captured by these co-moments. The practical implication is that in a well-diversified portfolio, the idiosyncratic moments (e.g., standard deviation, skewness, kurtosis, etc.) are eliminated, investors only earn compensation for exposure to systematic co-moments (e.g., co-variance, co-skewness, co-kurtosis, etc.), and these systematic co-moments should be priced when investors consider their investments.

Reference:

Acharya, V. and L. Pedersen, 2005. Asset Pricing with Liquidity Risk, *Journal of Financial Economics* 77, 375-410.

Arditti, F., 1971. Another Look at Mutual Fund Performance, *Journal of Financial and Quantitative Analysis* 6, 909-912.

Carhart, M., 1997. On Persistence in Mutual Fund Performance**,** Journal of Finance, 52, 57-82.

Chen, J., H. Hong, and J. Stein, 2001. Forecasting crashes: Trading volume, past returns, and conditional skewness in stock prices, *Journal of Financial Economics* 61, 345-381.

Conrad, J. and J. Kaul, 1998. An Anatomy of Trading Strategies, *Review of Financial Studies*, 11, 489-519.

Chordia, T., R. Roll, and A. Subrahmanyam, 2000. Commonality in Liquidity. *Journal of Financial Economics* 56, 3-28.

Chordia, T. and L. Shivakumar, 2002. Momentum, Business Cycle, and Time-Varying Expected Returns. *Journal of Finance*, 57, 985-1019.

Chung, Y., H. Johnson, and M. Schill, 2006. Asset Pricing when Returns are Nonnormal: Fama-French factors vs Higher-Order Systematic Co-Moments, *Journal of Business*, 79, 923-940.

Cochrane, J., 2001. Asset Pricing, Princeton University Press, New Jersey.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam, 1998. Investor Psychology and Security Market Under- and Over-reactions, *Journal of Finance*, 53, 1839-1886.

Daniel, K., and S. Titman, 1997. Evidence on the Characteristics of Cross-Sectional Variation in Stock Returns, *Journal of Finance* 52, 1-33.

Fama, E., 1965, The Behavior of Stock Market Prices, *Journal of Business* 38, 34-105.

Fama, E. and K. French, 1992. Size and Book-to-Market Factors in Earning and Returns, *Journal of Finance* 50, 131–155

Fama, E. and K. French, 1993. Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics* 33, 3–56

Fama, E. and K. French, 1996. Multifactor Explanations of Asset Pricing Anomalies, *Journal of Finance* 51, 55-84.

Fama, E. and J. D. MacBeth, 1973. Risk, Return and Equilibrium: Empirical Tests, *Journal of Political Economy*, 81, 607–636.

Friend, I., and R. Westerfield, 1980. Co-Skewness and Capital Asset Pricing, *Journal of Finance*, 35, 897-914.

Gibbons, M., S. Ross, J. Shanken, 1989. A test of the efficiency of a given portfolio, *Econometrica* 57, 1121–1152.

Gundy, B. and J. Martin, 2001. Understanding the Nature of Risks and the Source of the Rewards to Momentum Investing, *Review of Financial Studies* 14, 29-78.

Hasbrouck, J., and D. J. Seppi, 2001. Common Factors in Prices, Order Flows, and Liquidity, *Journal of Financial Economics* 59, 383-411.

Harvey, C. and A. Siddque, 2000. Conditional Skewness in Asset Pricing Tests, *Journal of Finance* 55, 1263-1295.

Hong, H. and J. Stein, 1999. A Unified Theory of Under-reaction, Momentum Trading and Overreaction in Asset Markets, *Journal of Finance* 54, 2143-2184.

Huberman, G., and D. Halka, 2001. Systematic Liquidity, *Journal of Financial Research* 24, 161-178.

Hung, C., M. Shackleton and X. Xu, 2004. CAPM, Higher Co-Moment and Factor Models of UK Stock Returns, *Journal of Business Finance and Accounting* 31, 87-112.

Jegadeesh, N. and S. Titman, 1993. Return to Buying Winners and Selling Loser: Implications for Stock market Efficiency, *Journal of Finance* 48, 65-91.

Jegadeesh, N. and S. Titman, 2001. Profitability of Momentum Strategies: An Evaluation of Alternative Explanations, *Journal of Finance* 56, 699-720.

Lakonishok, J., A. Shleifer, and R. Vishny, 1994. Contrarian Investment, Extrapolation, and Risk, *Journal of Finance* 49, 1541-1578.

Lettau, M. and S. Ludvigson, 2001. Resurrecting the (C)CAPM: A Cross-sectional Test when Risk Premia are Time-Varying, *Journal of Political Economy* 109, 1238-1287.

Lewellen, J., S. Nagel, and J. Shanken, 2007. A Skeptical Appraisal of Asset Pricing Tests, Working paper.

Newey, W. and K. West, 1987. Hypothesis Testing with Efficient Method of Moments, *International Economic Review* 28, 777-787.

Pástor, L. and R.F. Stambaugh, 2003. Liquidity risk and expected stock returns. *Journal of Political Economy* 111, 642-685.

Petersen, M. A., 2007. Estimating Standard Errors in Finance Panel Datasets: Comparing Approaches. *Review of Financial Studies*, forthcoming.

Rubinstein M., 1973, The Fundamental Theorem of Parameter-Preference Security Valuation, *Journal of Financial and Quantitative Analysis* 8, 61-69.

Sadka, R., 2006, Momentum and Post-earnings-announcement Drift Anomalies: The Role of Liquidity Risk, *Journal of Financial Economics*, 80, 309-349

Sears, R. and K. Wei, 1985. Asset Pricing, Higher Moments, and the Market Risk Premium: A Note, *Journal of Finance*, 40, 1251-1253.

Shanken, J., 1992. On the Estimation of Beta-Pricing Models, *Review of Financial Studies* 5, 1-33.

Shanken, J., and G. Zhou, 2007. Estimating and Testing Beta Pricing Models: Alternative Methods and Their Performance in Simulations, *Journal of Financial Economics* 84, 40-86.

Singleton, C. and J. Wingender 1986. Skewness Persistence in Common Stock Returns, *Journal of Financial and Quantitative Analysis*, 21, 335-341.

Table 1: Summary Statistics of Portfolio Return

The table reports the summary statistics for portfolio returns (size, book-to-market, and momentum) under analysis. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. Liquidity portfolios are constructed by sorting stocks into 50 equal-size portfolios bases on their Pastor and Stambaugh (2003)'s liquidity level estimates. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2005 are used in computation. Kolmogorov-Smirnov, Cramer-von Mises, Anderson-Darling statistics are used to test normality of portfolio returns. ***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

	Size portfolios	Book-to-market portfolios	Momentum portfolios	Liquidity portfolios
Number of portfolio-period observation	21600	21600	21600	21600
Mean	0.0079	0.0083	0.0082	0.0076
Variance	0.0045	0.0046	0.0047	0.0045
Skewness	0.0474	0.0517	1.0992	0.4088
Kurtosis	3.0200	3.1312	16.8590	4.9550
Kolmogorov-Smirnov	0.0345***	0.0371***	0.0639***	0.0405***
Cramer-von Mises	10.9780***	11.1920***	40.6470***	14.5321***
Anderson-Darling	79.9370***	80.8740***	259.6540***	104.4443***

Table 2: Correlation among factor loadings SMB, HML, LIQ, and MOM

The table reports the correlation among factor loadings SMB, HML, LIQ, and MOM for portfolio sorted by size, book-to-market, momentum, and liquidity. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. Liquidity portfolios are constructed by sorting stocks into 50 equal-size portfolios bases on their Pastor and Stambaugh (2003)'s liquidity level estimates. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2005 are used in computation.

Panel A: Size portfolio

	SMB	HML	MOM	LIQ
SMB	1.00			
HML	0.39	1.00		
MOM	-0.17	0.07	1.00	
LIQ	-0.02	0.02	-0.01	1.00

Panel B: book-to-market portfolio

	SMB	HML	MOM	LIQ
SMB	1.00			
HML	0.35	1.00		
MOM	-0.20	0.06	1.00	
LIQ	-0.01	0.07	0.01	1.00

Panel C: Momentum portfolio

	SMB	HML	MOM	LIQ
SMB	1.00			
HML	-0.05	1.00		
MOM	-0.30	-0.09	1.00	
LIQ	-0.28	0.09	0.17	1.00

Panel D: Liquidity portfolio

	SMB	HML	MOM	LIQ
SMB	1.00			
HML	0.59	1.00		
MOM	-0.20	-0.02	1.00	
LIQ	-0.18	-0.01	0.07	1.00

Table 3: Fama-Macbeth Regression Results

This table reports Fama-MacBeth regression estimates for size, book-to-market, and momentum portfolios. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. Liquidity portfolios are constructed by sorting stocks into 50 equal-size portfolios bases on their Pastor and Stambaugh (2003)'s liquidity level estimates. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2005 are used in computation. In each month, portfolio returns are regressed on the factor loadings *b*, *s*, *h*, *m*, *l*. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks (based on their prior compound return from month t-2 to t-12). LIQ is the Pastor-Stambaugh market liquidity factor. The mean coefficient estimates across the sample period are reported with their t-statistics. The F-statistics test the joint significance of the *s*, *h*, *m*, *l* estimates.
***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Portfolios	b	s	h	m	l	Mean adj. R^2 [F-stat]
Size	0.0356 (8.94)***	0.0041 (2.45)**	-0.0127 (-5.60)***	0.0129 (5.54)***	0.0049 (1.29)	0.46 [13.31]***
Book-to-market	0.0355 (9.01)***	0.0026 (1.53)	-0.0076 (-3.82)***	0.0077 (3.39)***	0.0110 (3.14)***	0.44 [7.39]***
Momentum	0.0011 (0.26)	-0.0005 (-0.28)	-0.0045 (-2.18)**	0.0101 (4.13)**	-0.0005 (-0.13)	0.43 [6.03]***
Liquidity	0.01272 (4.10)***	-0.0012 (-0.68)	-0.0046 (-2.74)**	0.0040 (1.96)**	0.0075 (2.04)**	0.342 [3.17]***

Table 4: Systematic Co-moments and Common Factors

This table reports Fama-MacBeth regression estimates for size, book-to-market, momentum, and liquidity portfolios when adding systematic higher order co-moments. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. Liquidity portfolios are constructed by sorting stocks into 50 equal-size portfolios bases on their Pastor and Stambaugh (2003)'s liquidity level estimates. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2005 are used in computation. In each month, portfolio returns are regressed on the factor loadings *b, s, h, m, l*, and the respective number of systematic co-moments. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. The systematic co-moments are estimated using the same rolling five-year portfolio return with the market return. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks (based on their prior compound return from month t-2 to t-12). LIQ is the Pastor-Stambaugh market liquidity factor. The mean coefficient estimates across the sample period are reported with their t-statistics. The F-statistics test the joint significance of the *s, h, m, l* estimates.
***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Panel A: size portfolios

Systematic Co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
2nd to 3rd	0.0030	-0.0123	0.0077	-0.0076	0.46
	(1.50)	(-5.36)***	(3.04)***	(-1.52)	[11.04]***
2nd to 4th	-0.0007	-0.0075	0.0059	-0.0116	0.47
	(-0.31)	(-2.88)**	(2.02)**	(-2.15)**	[5.63]***
2nd to 5th	-0.0018	-0.0089	0.0075	-0.0129	0.48
	(-0.75)	(-2.90)**	(2.42)**	(-2.33)**	[6.46]***
2nd to 6th	0.0020	-0.0121	0.0088	-0.0071	0.48
	(0.73)	(-3.31)***	(2.37)**	(-1.15)	[6.11]***
2nd to 7th	0.0010	-0.0098	0.0095	-0.0072	0.49
	(0.33)	(-2.18)**	(2.39)**	(-1.05)	[4.58]***
2nd to 8th	0.0030	-0.0135	0.0116	-0.0025	0.49
	(0.74)	(-2.66)**	(2.46)**	(-0.31)	[5.39]***
2nd to 9th	0.0070	-0.0156	0.0012	0.0034	0.49
	(1.51)	(-2.50)**	(0.21)	(0.39)	[2.67]**
2nd to 10th	0.0080	-0.0121	-0.0078	0.0045	0.49
	(1.39)	(-1.77)	(-1.16)	(0.47)	[1.38]
2nd to 15th	0.0135	-0.0090	-0.0032	0.0106	0.45
	(1.65)	(-0.98)	(-0.36)	(0.89)	[0.95]

Panel B: book-to-market portfolios

Systematic Co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
2^{nd} to 3^{rd}	0.0015	-0.0077	0.0054	0.0043	0.45
	(0.86)	(-3.58)***	(2.22)**	(0.97)	[4.05]***
2^{nd} to 4^{th}	0.0022	-0.0084	0.0068	0.0059	0.45
	(0.99)	(-3.20)***	(2.30)**	(1.12)	[3.99]***
2^{nd} to 5^{th}	0.0006	-0.0071	0.0080	0.0031	0.46
	(0.26)	(-2.24)**	(2.61)***	(0.57)	[3.21]**
2^{nd} to 6^{th}	0.0027	-0.0074	0.0084	0.0037	0.46
	(0.97)	(-2.04)**	(2.31)**	(0.63)	[2.44]**
2^{nd} to 7^{th}	0.0043	-0.0093	0.0083	0.0095	0.46
	(1.46)	(-2.09)**	(1.98)**	(1.43)	[2.04]*
2^{nd} to 8^{th}	0.0052	-0.0087	0.0074	0.0102	0.47
	(1.39)	(-1.79)*	(1.65)	(1.29)	[1.52]
2^{nd} to 9^{th}	0.0067	-0.0084	-0.0030	0.0120	0.47
	(1.55)	(-1.36)	(-0.55)	(1.41)	[0.77]
2^{nd} to 10^{th}	0.0025	-0.0015	-0.0079	0.0058	0.47
	(0.52)	(-0.22)	(-1.28)	(0.64)	[0.57]
2^{nd} to 15^{th}	0.0031	0.0013	0.0101	0.0020	0.42
	(0.43)	(0.13)	(1.14)	(0.17)	[0.52]

Panel C: momentum portfolios

Systematic Co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
2^{nd} to 3^{rd}	-0.0022	-0.0038	0.0099	-0.0014	0.44
	(-0.93)	(-1.79)*	(3.82)***	(-0.27)	[4.93]***
2^{nd} to 4^{th}	-0.0044	-0.0044	0.0061	-0.0045	0.44
	(-1.65)	(-1.75)*	(1.89)*	(-0.79)	[3.23]**
2^{nd} to 5^{th}	-0.0028	-0.0059	0.0042	-0.0013	0.45
	(-0.95)	(-1.60)	(1.09)	(-0.21)	[1.67]
2^{nd} to 6^{th}	-0.0065	-0.0008	0.0063	-0.0055	0.45
	(-1.92)	(-0.19)	(1.30)	(-0.85)	[1.92]
2^{nd} to 7^{th}	-0.0023	-0.0091	0.0052	0.0036	0.45
	(-0.66)	(-1.54)	(0.94)	(0.47)	[1.76]
2^{nd} to 8^{th}	-0.0001	-0.0079	0.0054	0.0062	0.46
	(-0.01)	(-1.29)	(0.95)	(0.71)	[0.93]
2^{nd} to 9^{th}	-0.0016	-0.0091	0.0054	0.0045	0.46
	(-0.35)	(-1.11)	(0.83)	(0.48)	[0.99]
2^{nd} to 10^{th}	-0.0032	-0.0056	0.0085	0.0013	0.46
	(-0.51)	(-0.63)	(1.08)	(0.12)	[0.81]
2^{nd} to 15^{th}	-0.0033	-0.0063	0.0115	0.0005	0.42
	(-0.42)	(-0.66)	(1.34)	(0.04)	[0.94]

Panel D: liquidity portfolios

Systematic Co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
2^{nd} to 3^{rd}	0.0010	-0.0044	0.0058	0.0036	0.369
	(0.53)	(-2.28)**	(2.34)**	(0.73)	[2.34]*
2^{nd} to 4^{th}	0.0010	-0.0052	0.0045	0.0043	0.377
	(0.46)	(-2.28)	(1.67)	(0.80)	[2.10]*
2^{nd} to 5^{th}	-0.0003	-0.0057	0.0054	0.0018	0.384
	(-0.12)	(-2.13)	(1.77)	(0.33)	[2.27]*
2^{nd} to 6^{th}	0.0006	-0.0061	0.0075	0.0035	0.390
	(0.20)	(-1.74)	(1.97)	(0.59)	[1.89]
2^{nd} to 7^{th}	0.0030	-0.0097	0.0080	0.0125	0.396
	(0.97)	(-2.02)	(1.87)	(1.72)	[2.01]*
2^{nd} to 8^{th}	-0.0012	-0.0072	0.0077	0.0040	0.400
	(-0.33)	(-1.38)	(1.67)	(0.52)	[1.64]
2^{nd} to 9^{th}	-0.0017	-0.0066	0.0089	0.0045	0.401
	(-0.41)	(-1.12)	(1.65)	(0.54)	[1.41]
2^{nd} to 10^{th}	-0.0002	-0.0122	0.0072	0.0069	0.402
	(-0.04)	(-1.88)	(1.21)	(0.77)	[1.79]
2^{nd} to 15^{th}	-0.0045	-0.0044	0.0123	-0.0010	0.341
	(-0.67)	(-0.58)	(1.54)	(-0.10)	[0.35]

Table 5: Standard Moments and Common Factors

This table reports Fama-MacBeth regression estimates for size, book-to-market, momentum, and liquidity portfolios when adding a set of standard moment up to order 10 or 15. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. Liquidity portfolios are constructed by sorting stocks into 50 equal-size portfolios bases on their Pastor and Stambaugh (2003)'s liquidity level estimates. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2005 are used in computation. In each month, portfolio returns are regressed on the factor loadings *b, s, h, m, l,* and the respective number of standard moments. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. The standard moments are estimated using the same rolling five-year portfolio return with the market return. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks (based on their prior compound return from month t-2 to t-12). LIQ is the Pastor-Stambaugh market liquidity factor. The mean coefficient estimates across the sample period are reported with their t-statistics. The F-statistics test the joint significance of the *s, h, m, l* estimates.
***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Standard moments	s	h	m	l	Mean adj. R^2 [F-stat]
A. Size portfolios					
3rd to 10th	0.0171 (5.95)***	-0.0170 (-6.63)***	0.0082 (2.57)**	0.0134 (2.50)**	0.55 [13.75]***
3rd to 15th	0.0160 (5.78)***	-0.0156 (-5.89)***	0.0068 (2.14)**	0.0113 (2.11)**	0.54 [13.39]***
B. Book-to-market portfolios					
3rd to 10th	0.0122 (4.76)***	-0.0134 (-5.82)***	0.0078 (2.77)***	0.0113 (2.04)**	0.52 [11.64]***
3rd to 15th	0.0130 (4.70)***	-0.0131 (-5.63)***	0.0062 (2.05)***	0.0130 (2.21)**	0.51 [10.37]***
C. Momentum portfolios					
3rd to 10th	0.0031 (1.03)	-0.0054 (-2.67)***	0.0092 (2.89)***	0.0023 (0.48)	0.53 [3.57]***
3rd to 15th	0.0025 (0.69)	-0.0048 (-2.11)**	0.0084 (2.45)**	-0.0003 (-0.07)	0.50 [2.65]**
D. Liquidity portfolios					
3rd to 10th	0.0092 (2.86)**	-0.0071 (-2.31)***	0.0051 (1.54)	0.0130 (2.29)**	0.453 [2.96]**
3rd to 15th	0.0093 (2.88)	-0.0071 (-2.32)**	0.0069 (2.06)	0.0129 (2.14)	0.446 [3.51]***

Table 6: Gibbon-Ross-Shanken (1989) Test

This table reports Gibbon-Ross-Shanken (1989) statistics examining whether the intercepts (pricing errors) from time series regressions are jointly equal to zero for size, book-to-market, momentum, and liquidity portfolios when adding higher order co-moment factors. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. Liquidity portfolios are constructed by sorting stocks into 50 equal-size portfolios bases on their Pastor and Stambaugh (2003)'s liquidity level estimates. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2005 are used in computation.
***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Systematic Co-moments	Size portfolio	Book-to-market portfolio	Momentum portfolio	Liquidity portfolio
original	12.37***	10.69***	2.75**	3.45***
2^{nd} to 3^{rd}	8.45***	7.10***	2.46**	2.53**
2^{nd} to 4^{th}	8.27***	6.94***	2.68**	2.49**
2^{nd} to 5^{th}	6.61***	5.49***	2.10**	1.98**
2^{nd} to 6^{th}	6.11***	5.19***	1.73*	2.01**
2^{nd} to 7^{th}	5.03***	4.28***	1.46	1.61
2^{nd} to 8^{th}	4.23***	3.77***	1.56	1.51
2^{nd} to 9^{th}	4.00***	2.94***	1.55	1.19
2^{nd} to 10^{th}	3.69***	2.93***	1.55	1.18
2^{nd} to 15^{th}	1.60	1.28	0.87	0.62

Table 7: Sub-period 1970-1987

This table reports Fama-MacBeth regression estimates in the sub-period 1970-1987 for size, book-to-market, momentum, and liquidity portfolios when adding a set of systematic co-moments or a set of standard moments of order 3 through 10. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. Liquidity portfolios are constructed by sorting stocks into 50 equal-size portfolios bases on their Pastor and Stambaugh (2003)'s liquidity level estimates. In each month, portfolio returns are regressed on the factor loadings *b, s, h, m, l*, and the respective number of systematic co-moments or standard moments. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. The systematic co-moments and standard moments are estimated using the same rolling five-year portfolio return with the market return. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks. LIQ is the Pastor-Stambaugh market liquidity factor. The mean coefficient estimates across the sample period are reported with their t-statistics. The F-statistics test the joint significance of the *s, h, m, l* estimates. In each panel A and B, the original indicates the model including only the 2^{nd} systematic co-moment (covariance), and the factor loadings on SMB, HML, MOM, and LIQ.***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Panel A: Regressions with systematic co-moments

Systematic co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
A. Size portfolios					
Original	0.0038	-0.0179	0.0158	0.0172	0.47
	(1.80)*	(-6.55)***	(5.21)***	[3.42]***	[16.15]***
2^{nd} to 10^{th}	0.0057	-0.0018	-0.0055	0.0176	0.49
	(0.55)	(-0.22)	(-0.53)	(1.28)	[1.21]
B. Book-to-market portfolios					
Original	0.0019	-0.0114	0.0071	0.0183	0.44
	(0.90)	(-4.86)***	(2.41)**	(4.39)***	[10.30]***
2^{nd} to 10^{th}	0.0004	0.0048	-0.0048	0.0107	0.47
	(0.04)	(0.70)	(-0.56)	(0.85)	(1.50)
C. Momentum portfolios					
Original	-0.0006	-0.0022	0.0097	-0.0030	0.38
	(-0.24)	(-0.99)	(3.20)***	(-0.77)	[3.32]**
2^{nd} to 10^{th}	-0.0090	0.0062	0.0156	-0.0199	0.41
	(-0.80)	(0.68)	(1.38)	(-1.30)	(0.83)
D. Liquidity portfolios					
Original	-0.0011	-0.0068	0.0065	0.0015	0.381
	(-0.54)	(-3.17)***	(2.11)**	(0.39)	[3.25]**
2^{nd} to 10^{th}	-0.0070	-0.0110	0.0118	-0.0021	0.420
	(-0.76)	(-1.43)	(1.29)	(-0.16)	[1.71]

Panel B: Regressions with standard moments

Standard moments	s	h	m	l	Mean adj. R^2 [F-stat]
A. Size portfolios					
3^{rd} to 10^{th}	0.0134	-0.0157	0.0110	0.0182	0.54
	(3.38)***	(-5.36)***	(2.56)**	(3.36)***	[9.05]***
B. Book-to-market portfolios					
3^{rd} to 10^{th}	0.0131	-0.0124	0.0107	0.0183	0.50
	(3.38)***	(-4.41)***	(2.59)**	(3.41)***	[8.39]***
C. Momentum portfolios					
3^{rd} to 10^{th}	0.0038	-0.0071	0.0118	0.0010	0.46
	(0.56)	(-1.99)**	(2.11)**	(0.13)	[3.32]**
D. Liquidity portfolios					
3^{rd} to 10^{th}	0.0111	-0.0069	0.0107	0.0083	0.461
	(2.38)**	(-2.33)**	(2.41)**	(1.36)	[3.40]***

Table 8: Sub-period 1988-2005

This table reports Fama-MacBeth regression estimates in the sub-period 1988-2005 for size, book-to-market, momentum, and liquidity portfolios portfolios when adding a set of systematic co-moments or a set of standard moments of order 3 through 10. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. Liquidity portfolios are constructed by sorting stocks into 50 equal-size portfolios bases on their Pastor and Stambaugh (2003)'s liquidity level estimates In each month, portfolio returns are regressed on the factor loadings *b, s, h, m, l*, and the respective number of systematic co-moments or standard moments. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. The systematic co-moments and standard moments are estimated using the same rolling five-year portfolio return with the market return. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks. LIQ is the Pastor-Stambaugh market liquidity factor. The mean coefficient estimates across the sample period are reported with their t-statistics. The F-statistics test the joint significance of the *s, h, m, l* estimates. In each panel A and B, the original indicates the model including only the 2nd systematic co-moment (covariance), and the factor loadings on SMB, HML, MOM, and LIQ.***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Panel A: Regressions with systematic co-moments

Systematic co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
A. Size portfolios					
Original	0.0045 (1.70)*	-0.0076 (-2.10)**	0.0100 (2.83)***	-0.0073 (-1.27)	0.45 [3.98]***
2nd to 10th	0.0103 (2.12)**	-0.0224 (-2.06)**	-0.0101 (-1.17)	-0.0085 (-0.64)	0.48 [3.32]***
B. Book-to-market portfolios					
Original	0.0032 (1.23)	-0.0037 (-1.17)	0.0082 (2.39)**	0.0037 (0.66)	0.43 [1.78]
2nd to 10th	0.0048 (0.95)	-0.0078 (-0.68)	-0.0110 (-1.22)	0.0008 (0.06)	0.47 [0.50]
C. Momentum portfolios					
Original	-0.0004 (-0.15)	-0.0069 (-1.95)*	0.0106 (2.73)***	0.0018 (0.23)	0.49 [3.75]***
2nd to 10th	0.0024 (0.40)	-0.0175 (-1.15)	0.0014 (0.12)	0.0226 (1.49)	0.51 [0.98]
D. Liquidity portfolios					
Original	-0.0009 (-0.42)	-0.0032 (-1.10)	0.0025 (0.78)	-0.0054 (-0.98)	0.345 [1.05]
2nd to 10th	0.0066 (1.39)	-0.0133 (-1.28)	0.0026 (0.34)	0.0160 (1.27)	0.382 [0.63]

Panel B: Regressions with standard moments

Standard moments	s	h	m	l	Mean adj. R^2 [F-stat]
A. Size portfolios					
3rd to 10th	0.0129	-0.0151	0.0015	-0.0054	0.55
	(4.11)***	(-3.75)***	(0.37)	(-0.67)	[7.81]***
B. Book-to-market portfolios					
3rd to 10th	0.0113	-0.0144	0.0049	0.0044	0.52
	(3.35)***	(-3.95)***	(1.28)	(0.45)	[5.75]***
C. Momentum portfolios					
3rd to 10th	-0.0026	-0.0044	0.0114	-0.0040	0.60
	(-0.80)	(-1.42)	(2.63)***	(-0.50)	[2.48]**
D. Liquidity portfolios					
3rd to 10th	0.0020	-0.0037	0.0040	0.0094	0.437
	(0.53)	(-0.79)	(0.86)	(1.10)	[0.79]

Appendix

Table A1: R-squares of the regressions of factor loadings (SMB, HML, MOM, LIQ) on a set of systematic co-moments

This table reports the R-squares from the regressions of factor loadings (SMB, HML, MOM, LIQ) on a set of 10 or 15 systematic comoments for size, book-to-market, and momentum portfolios when adding systematic higher order co-moments. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. Liquidity portfolios are constructed by sorting stocks into 50 equal-size portfolios bases on their Pastor and Stambaugh (2003)'s liquidity level estimates. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. The systematic co-moments are estimated using the same rolling five-year portfolio return with the market return. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks. LIQ is the Pastor-Stambaugh market liquidity factor.

Panel A: Common factors and a set of 10 systematic co-moments

Factors	Size portfolio	Book-to-market portfolio	Momentum portfolio	Liquidity portfolio
SMB	0.89	0.88	0.85	0.85
HML	0.75	0.73	0.74	0.73
MOM	0.62	0.63	0.90	0.61
LIQ	0.54	0.50	0.67	0.51

Panel B: Common factors and a set of 15 systematic co-moments

Factors	Size portfolio	Book-to-market portfolio	Momentum portfolio	Liquidity portfolio
SMB	0.91	0.90	0.86	0.87
HML	0.77	0.75	0.76	0.75
MOM	0.66	0.67	0.92	0.65
LIQ	0.58	0.54	0.69	0.55

Table A2: Shanken (1992)'s correction for Fama-MacBeth estimates

This table reports Shanken (1992)'s correction for error-in-variable problem in Fama-MacBeth regression estimates for size, book-to-market, momentum, and liquidity portfolios when adding systematic higher order co-moments. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. Liquidity portfolios are constructed by sorting stocks into 50 equal-size portfolios bases on their Pastor and Stambaugh (2003)'s liquidity level estimates. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2002 are used in computation. In each month, portfolio returns are regressed on the factor loadings *b, s, h, m, l,* and the respective number of systematic co-moments. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. The systematic co-moments are estimated using the same rolling five-year portfolio return with the market return. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks (based on their prior compound return from month t-2 to t-12). LIQ is the Pastor-Stambaugh market liquidity factor.
***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Panel A: size portfolios

Systematic Co-moments	s	h	m	l
Original	0.0041	-0.0127	0.0129	0.0049
	(1.50)**	(-3.77)***	(3.47)***	(0.88)
2nd to 3rd	0.0030	-0.0123	0.0077	-0.0076
	(0.79)	(-2.91)***	(1.60)	(-0.85)
2nd to 4th	-0.0007	-0.0075	0.0059	-0.0116
	(-0.007)	(-0.070)	(0.049)	(-0.052)
2nd to 5th	-0.0018	-0.0089	0.0075	-0.0129
	(-0.008)	(-0.032)	(0.026)	(-0.025)
2nd to 6th	0.0020	-0.0121	0.0088	-0.0071
	(0.000)	(-0.000)	(0.000)	(-0.000)
2nd to 10th	0.0080	-0.0121	-0.0078	0.0045
	(4×10^{-10})	(-5×10^{-10})	(-3×10^{-10})	(1.5×10^{-10})

Panel B: book-to-market portfolios

Systematic Co-moments	s	h	m	l
original	0.0026	-0.0076	0.0077	0.0110
	(0.95)	(-2.54)**	(2.14)	(2.15)*
2nd to 3rd	0.0015	-0.0077	0.0054	0.0043
	(0.48)	(-2.08)**	(1.24)	(0.58)
2nd to 4th	0.0022	-0.0084	0.0068	0.0059
	(0.25)	(-0.82)	(0.59)	(0.29)
2nd to 5th	0.0006	-0.0071	0.0080	0.0031
	(0.030)	(-0.261)	(0.304)	(0.067)
2nd to 6th	0.0027	-0.0074	0.0084	0.0037
	(0.000)	(-0.000)	(0.000)	(0.000)
2nd to 10th	0.0025	-0.0015	-0.0079	0.0058
	(39×10^{-10})	(-17×10^{-10})	(-95×10^{-10})	(43×10^{-10})

Panel C: momentum portfolios

Systematic Co-moments	s	h	m	l
original	-0.0005	-0.0045	0.0101	-0.0005
	(-0.21)	(-1.74)*	(3.12)***	(-0.11)
2nd to 3rd	-0.0022	-0.0038	0.0099	-0.0014
	(-0.72)	(-1.37)	(2.83)**	(-0.23)
2nd to 4th	-0.0044	-0.0044	0.0061	-0.0045
	(-1.08)	(-1.15)	(1.23)	(-0.54)
2nd to 5th	-0.0028	-0.0059	0.0042	-0.0013
	(-0.002)	(-0.004)	(0.002)	(-0.000)
2nd to 6th	-0.0065	-0.0008	0.0063	-0.0055
	(-0.000)	(-0.000)	(0.000)	(-0.000)
2nd to 10th	-0.0032	-0.0056	0.0085	0.0013
	(-0.6×10^{-10})	(-0.8×10^{-10})	(1.3×10^{-10})	(1.6×10^{-10})

Panel D: liquidity portfolios

Systematic Co-moments	s	h	m	l
2nd to 3rd	0.0010	-0.0044	0.0058	0.0036
	(0.32)	(-1.41)	(1.43)	(0.49)
2nd to 4th	0.0010	-0.0052	0.0045	0.0043
	(0.02)	(-0.13)	(0.09)	(0.04)
2nd to 5th	-0.0003	-0.0057	0.0054	0.0018
	(-0.000)	(-0.010)	(0.008)	(0.001)
2nd to 6th	0.0006	-0.0061	0.0075	0.0035
	(0.000)	(-0.002)	(0.002)	(0.000)
2nd to 10th	-0.0002	-0.0122	0.0072	0.0069
	(-1.1×10^{-10})	(-50×10^{-10})	(32×10^{-10})	(20×10^{-10})

Table A3: Newey-West adjustment for Fama-MacBeth estimation

This table reports Newey-West (1987)'s adjustment for serial correlation in return residual problem in Fama-MacBeth regression estimates for size, book-to-market, momentum, and liquidity portfolios when adding systematic higher order co-moments. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. Liquidity portfolios are constructed by sorting stocks into 50 equal-size portfolios bases on their Pastor and Stambaugh (2003)'s liquidity level estimates. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2002 are used in computation. In each month, portfolio returns are regressed on the factor loadings *b, s, h, m, l,* and the respective number of systematic co-moments. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. The systematic co-moments are estimated using the same rolling five-year portfolio return with the market return. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks (based on their prior compound return from month t-2 to t-12). LIQ is the Pastor-Stambaugh market liquidity factor.
***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Panel A: size portfolios

Systematic Co-moments	s	h	m	l
original	0.0041	-0.0127	0.0129	0.0049
	(2.23)**	(-4.35)***	(5.26)***	(1.13)
2^{nd} to 3^{rd}	0.0030	-0.0123	0.0077	-0.0076
	(1.37)	(-4.35)***	(2.70)***	(-1.28)
2^{nd} to 4^{th}	-0.0007	-0.0075	0.0059	-0.0116
	(-0.29)	(-2.56)**	(1.73)*	(-1.83)*
2^{nd} to 5^{th}	-0.0018	-0.0089	0.0075	-0.0129
	(-0.72)	(-2.72)***	(2.13)**	(-2.09)**
2^{nd} to 6^{th}	0.0020	-0.0121	0.0088	-0.0071
	(0.73)	(-3.04)***	(2.08)**	(-1.00)
2^{nd} to 7^{th}	0.0010	-0.0098	0.0095	-0.0072
	(0.34)	(-1.99)**	(2.15)**	(-0.95)
2^{nd} to 8^{th}	0.0030	-0.0135	0.0116	-0.0025
	(0.78)	(-2.59)**	(2.28)**	(-0.30)
2^{nd} to 9^{th}	0.0070	-0.0156	0.0012	0.0034
	(1.45)	(-2.50)**	(0.19)	(0.37)
2^{nd} to 10^{th}	0.0080	-0.0121	-0.0078	0.0045
	(1.38)	(-1.79)	(-1.12)	(0.46)
2^{nd} to 15^{th}	0.0135	-0.0090	-0.0032	0.0106
	(1.73)	(-0.93)	(-0.34)	(0.88)

Panel B: book-to-market portfolios

Systematic Co-moments	s	h	m	l
original	0.0026	-0.0076	0.0077	0.0110
	(1.41)	(-3.27)***	(3.24)***	(2.92)***
2nd to 3rd	0.0015	-0.0077	0.0054	0.0043
	(0.79)	(-3.09)***	(2.18)**	(0.93)
2nd to 4th	0.0022	-0.0084	0.0068	0.0059
	(0.92)	(-2.96)***	(2.20)**	(1.05)
2nd to 5th	0.0006	-0.0071	0.0080	0.0031
	(0.26)	(-1.98)**	(2.70)***	(0.55)
2nd to 6th	0.0027	-0.0074	0.0084	0.0037
	(0.97)	(-1.77)*	(2.20)**	(0.60)
2nd to 7th	0.0043	-0.0093	0.0083	0.0095
	(1.56)	(-1.91)*	(1.93)*	(1.56)
2nd to 8th	0.0052	-0.0087	0.0074	0.0102
	(1.37)	(-1.60)	(1.56)	(1.33)
2nd to 9th	0.0067	-0.0084	-0.0030	0.0120
	(1.52)	(-1.28)	(-0.53)	(1.46)
2nd to 10th	0.0025	-0.0015	-0.0079	0.0058
	(0.52)	(-0.21)	(-1.14)	(0.67)
2nd to 15th	0.0031	0.0013	0.0101	0.0020
	(0.43)	(0.13)	(1.12)	(0.19)

Panel C: momentum portfolios

Systematic Co-moments	s	h	m	l
original	-0.0005	-0.0045	0.0101	-0.0005
	(-0.26)	(-2.00)**	(4.53)***	(-0.13)
2nd to 3rd	-0.0022	-0.0038	0.0099	-0.0014
	(-0.83)	(-1.59)	(4.22)***	(-0.26)
2nd to 4th	-0.0044	-0.0044	0.0061	-0.0045
	(-1.44)	(-1.53)	(2.13)**	(-0.79)
2nd to 5th	-0.0028	-0.0059	0.0042	-0.0013
	(-0.84)	(-1.44)	(1.25)	(-0.21)
2nd to 6th	-0.0065	-0.0008	0.0063	-0.0055
	(-1.69)	(-0.17)	(1.33)	(-0.81)
2nd to 7th	-0.0023	-0.0091	0.0052	0.0036
	(-0.57)	(-1.26)	(0.96)	(0.45)
2nd to 8th	-0.0001	-0.0079	0.0054	0.0062
	(-0.01)	(-1.10)	(1.02)	(0.75)
2nd to 9th	-0.0016	-0.0091	0.0054	0.0045
	(-0.33)	(-1.04)	(0.94)	(0.51)
2nd to 10th	-0.0032	-0.0056	0.0085	0.0013
	(-0.45)	(-0.60)	(1.20)	(0.13)
2nd to 15th	-0.0033	-0.0063	0.0115	0.0005
	(-0.39)	(-0.61)	(1.41)	(0.04)

Panel D: liquidity portfolios

Systematic Co-moments	s	h	m	l
2^{nd} to 3^{rd}	0.0010	-0.0044	0.0058	0.0036
	(0.51)	(-2.15)**	(2.34)	(0.77)
2^{nd} to 4^{th}	0.0010	-0.0052	0.0045	0.0043
	(0.43)	(-2.14)	(1.62)	(0.86)
2^{nd} to 5^{th}	-0.0003	-0.0057	0.0054	0.0018
	(-0.12)	(-2.06)	(1.74)	(0.36)
2^{nd} to 6^{th}	0.0006	-0.0061	0.0075	0.0035
	(0.19)	(-1.66)	(1.97)	(0.61)
2^{nd} to 7^{th}	0.0030	-0.0097	0.0080	0.0125
	(0.92)	(-2.00)	(1.84)	(1.76)
2^{nd} to 8^{th}	-0.0012	-0.0072	0.0077	0.0040
	(-0.33)	(-1.46)	(1.67)	(0.54)
2^{nd} to 9^{th}	-0.0017	-0.0066	0.0089	0.0045
	(-0.44)	(-1.19)	(1.73)	(0.58)
2^{nd} to 10^{th}	-0.0002	-0.0122	0.0072	0.0069
	(-0.04)	(-1.96)	(1.23)	(0.83)
2^{nd} to 15^{th}	-0.0045	-0.0044	0.0123	-0.0010
	(-0.72)	(-0.64)	(1.81)	(-0.12)

Table A4: Non-centered Systematic Co-moments and Common Factors

This table reports Fama-MacBeth regression estimates for size, book-to-market, and momentum portfolios when adding **non-centered** systematic higher order co-moments. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. Liquidity portfolios are constructed by sorting stocks into 50 equal-size portfolios bases on their Pastor and Stambaugh (2003)'s liquidity level estimates. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2002 are used in computation. In each month, portfolio returns are regressed on the factor loadings *b, s, h, m, l,* and the respective number of systematic co-moments. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. The non-centered systematic co-moments are estimated using the same rolling five-year portfolio return with the market return without demeaning. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks (based on their prior compound return from month t-2 to t-12). LIQ is the Pastor-Stambaugh market liquidity factor. The mean coefficient estimates across the sample period are reported with their t-statistics. The F-statistics test the joint significance of the *s, h, m, l* estimates.
***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Panel A: size portfolios

Systematic Co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
2^{nd} to 3^{rd}	0.0096	-0.0019	0.0076	0.0186	0.494
	(5.65)***	(-1.09)	(3.05)***	(4.27)***	[11.25]***
2^{nd} to 4^{th}	0.0056	0.0005	0.0057	0.0117	0.499
	(2.78)**	(0.23)	(2.08)**	(2.51)**	[4.08]***
2^{nd} to 5^{th}	0.0019	-0.0013	0.0072	0.0043	0.506
	(0.86)	(-0.53)	(2.51)**	(0.93)	[1.78]
2^{nd} to 6^{th}	0.0034	-0.0008	0.0063	0.0007	0.507
	(1.30)	(-0.27)	(1.82)*	(0.12)	[1.52]
2^{nd} to 7^{th}	0.0003	0.0064	0.0074	-0.0062	0.512
	(0.09)	(1.41)	(1.96)**	(-1.04)	[1.90]
2^{nd} to 8^{th}	-0.0004	0.0054	0.0082	-0.0062	0.515
	(-0.11)	(1.06)	(1.97)**	(-0.83)	[1.55]
2^{nd} to 9^{th}	0.0002	0.0066	0.0058	-0.0053	0.515
	(0.05)	(1.00)	(1.18)	(-0.66)	[0.90]
2^{nd} to 10^{th}	0.0008	0.0031	-0.0012	-0.0028	0.515
	(0.16)	(0.42)	(-0.20)	(-0.32)	[0.18]
2^{nd} to 15^{th}	-0.0006	0.0135	0.0026	-0.0047	0.474
	(-0.07)	(1.51)	(0.28)	(-0.41)	[0.91]

Panel B: book-to-market portfolios

Systematic Co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
2nd to 3rd	0.0071	-0.0018	0.0034	0.0218	0.464
	(4.12)***	(-1.07)	(1.48)	(5.38)***	[9.22]***
2nd to 4th	0.0079	-0.0030	0.0044	0.0243	0.470
	(3.56)***	(-1.31)	(1.55)	(5.04)***	[7.12]***
2nd to 5th	0.0033	-0.0027	0.0069	0.0143	0.477
	(1.44)	(-1.03)	(2.33)**	(3.01)***	[3.38]***
2nd to 6th	0.0035	-0.0011	0.0083	0.0097	0.480
	(1.29)	(-0.33)	(2.37)	(1.75)	[2.13]*
2nd to 7th	0.0038	0.0025	0.0058	0.0100	0.486
	(1.30)	(0.53)	(1.49)	(1.63)	[2.09]*
2nd to 8th	0.0050	0.0034	0.0034	0.0074	0.489
	(1.38)	(0.69)	(0.84)	(1.05)	[1.63]
2nd to 9th	0.0034	0.0083	0.0009	0.0044	0.491
	(0.85)	(1.28)	(0.20)	(0.56)	[1.60]
2nd to 10th	0.0060	0.0061	-0.0057	0.0068	0.492
	(1.11)	(0.86)	(-0.95)	(0.77)	[1.52]
2nd to 15th	0.0068	0.0060	0.0008	0.0051	0.446
	(0.89)	(0.68)	(0.10)	(0.46)	[0.84]

Panel C: momentum portfolios

Systematic Co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
2nd to 3rd	0.0035	-0.0041	0.0109	0.0076	0.446
	(1.70)*	(-2.02)**	(4.38)***	(1.44)	[6.84]***
2nd to 4th	0.0010	-0.0025	0.0077	0.0011	0.451
	(0.43)	(-1.06)	(2.51)**	(0.21)	[1.90]*
2nd to 5th	0.0013	-0.0065	0.0054	0.0024	0.455
	(0.52)	(-2.32)**	(1.57)	(0.43)	[1.94]*
2nd to 6th	-0.0044	0.0003	0.0047	-0.0068	0.460
	(-1.34)	(0.09)	(1.03)	(-1.07)	[0.96]
2nd to 7th	-0.0020	-0.0052	0.0019	-0.0019	0.465
	(-0.57)	(-0.91)	(0.36)	(-0.26)	[0.55]
2nd to 8th	-0.0015	-0.0033	0.0011	-0.0011	0.470
	(-0.32)	(-0.52)	(0.19)	(-0.13)	[0.90]
2nd to 9th	-0.0024	-0.0069	0.0012	0.0002	0.471
	(-0.48)	(-0.78)	(0.20)	(0.02)	[0.62]
2nd to 10th	-0.0044	-0.0022	0.0017	-0.0038	0.473
	(-0.63)	(-0.23)	(0.22)	(-0.33)	[0.88]
2nd to 15th	-0.0017	-0.0036	-0.0004	-0.0005	0.424
	(-0.18)	(-0.33)	(-0.04)	(-0.04)	[0.98]

Panel D: liquidity portfolios

Systematic Co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
2nd to 3rd	0.0018	-0.0023	0.0047	0.0044	0.375
	(1.07)	(-1.31)	(2.04)	(1.03)	[1.50]
2nd to 4th	0.0014	-0.0027	0.0025	0.0046	0.383
	(0.70)	(-1.24)	(0.96)	(0.95)	[0.68]
2nd to 5th	0.0002	-0.0030	0.0026	0.0017	0.388
	(0.10)	(-1.25)	(0.94)	(0.35)	[0.67]
2nd to 6th	-0.0001	-0.0043	0.0045	0.0011	0.393
	(-0.04)	(-1.26)	(1.26)	(0.20)	[0.97]
2nd to 7th	0.0014	-0.0072	0.0041	0.0066	0.399
	(0.48)	(-1.61)	(1.01)	(1.09)	[1.07]
2nd to 8th	-0.0038	-0.0044	0.0049	-0.0034	0.406
	(-1.06)	(-0.88)	(1.10)	(-0.49)	[1.35]
2nd to 9th	-0.0053	-0.0023	0.0070	-0.0051	0.407
	(-1.24)	(-0.39)	(1.35)	(-0.67)	[1.15]
2nd to 10th	-0.0023	-0.0082	0.0052	-0.0016	0.405
	(-0.47)	(-1.28)	(0.92)	(-0.20)	[1.20]
2nd to 15th	-0.0081	-0.0040	0.0052	-0.0071	0.348
	(-1.21)	(-0.53)	(0.64)	(-0.71)	[1.12]