Measuring Time-Varying Economic Fears with Consumption-Based Stochastic Discount Factors

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Abstract

This paper shows that the volatility of sensible consumption-based stochastic discount factors predicts future economic and stock market cycles. More specifically, we report a striking change in this forecasting ability which coincides with the structural change in the macroeconomic conditions of the U.S. economy in 1984. Significant forecasting is found from 1985 to 2006, when the economy is characterized by a period of great macroeconomic moderation and increasing risk aversion in stock market investors simultaneously. In particular, the volatility of contemporaneous consumption growth specifications of recursive and durable preferences predict cycles at horizons of one, two and four quarters, while long run specifications better forecast at eight and twelve quarters. Moreover, the explanatory forecasting ability is stronger at recessions than at expansions.

1. Introduction

It is well known that risk-neutral probabilities are adjusted upward (downward) relative to objective or physical probabilities if they are associated with states with high (low) marginal utility of consumption. The stochastic discount factor (SDF hereafter) plays a key role in such adjustment. The interaction of these two probability distributions with the volatility of stochastic discount factors, as a way of measuring time-varying aggregate economic fears, is the focus of this work.

Several procedures have been proposed to obtain comparable risk-adjusted densities. Jackwerth (2000) allows for a changing risk-neutral probability density function while imposing a stationary objective density function. This is problematic and leads to the well known pricing kernel puzzle. To avoid this debatable assumption, Bliss and Panigirtzoglou (2004) assume risk-aversion function stationarity and estimate implied preference parameters from power and exponential utility functions. Finally, Benzoni, Dufresne and Goldstein (2005) argue that the pricing kernel puzzle and the volatility smirk can be rationalized if the agent has recursive preferences and if the aggregate dividend and consumption processes are driven by a persistent stochastic growth variable subject to jumps. In any case, this literature is still characterized by an active debate without a clear-cut solution.

Contrary to this literature, this paper explores empirically the theoretical results underlying objective and risk-neutral probabilities without relying on option data. In a recent theoretical paper, Bakshi, Chen and Hjalmarsson (2004) define a distance between the risk-neutral and the objective probability measures, which can be related to the volatility of the defining stochastic discount factor (SDF). We argue that the distance between both probability measures captures economic fears. Hence, given the association between the distance and the volatility of the defining SDF, our paper analyzes empirically the ability of the volatility of sensible consumption-based SDFs to forecast macroeconomic and stock market cycles, and to measure investors´ implicit recession fears. In other words, we analyze the empirical link between the ex-ante economic fears about macroeconomic and stock market fundamentals and the ex-post economic cycle in the financial market and the economy.

The empirical analysis is performed under five consumption-based SDF specifications: recursive preferences with non-durable consumption, recursive non-separable preferences with non-durable and durable consumption, the long run versions of the two previous cases, and preferences with habit persistence. It should be noted that we do not pursue to compare SDFs from the traditional asset pricing point of view. We just want to study whether the volatility of reasonable SDFs is able to predict future economic cycles. Of course, the forecasting performance of the alternative specifications employed in the paper may be different. This is how the comparison of the proposed consumption-based SDFs should be understood.

The paper shows that recursive and durable consumption-based SDFs with contemporaneous marginal rate of substitution are able to significantly predict macroeconomic and stock market cycles at relatively short horizons. On the other hand, their long run versions present a higher forecasting power at relatively longer horizons. Moreover, their predicting ability is stronger for recessions than for expansions. This suggests that we are actually capturing time-varying economic fears. Surprisingly, these results are only observed after 1984, when a quite dramatic structural change in the macroeconomic conditions of the U.S. economy occurs. Since 1985, there has been a well known decline in the volatility of key macroeconomic time series. The significant forecasting capacity of the volatility of SDFs is precisely found from 1985 to 2006. Interestingly enough, we also report a significant increase in risk aversion in this period relative to the previous years.[1] Therefore, the forecasting ability of the volatility of the SDFs is found when we have simultaneously a period of great macroeconomic moderation and increasing risk aversion in stock market investors.

This paper is organized as follows. Section 2 discusses the theoretical framework that relates risk-neutral and objective probability distributions with the volatility of SDFs. Section 3 presents the stochastic discount factor specifications analyzed in the paper, while Section 4 contains a description of data and some initial empirical results using the Hansen-Jagannathan (1991) volatility bound. Section 5 selects the appropriate parameters of the consumption-based stochastic discount factors, and Section 6 discusses how well these specifications capture future macroeconomic

[1] Note that this sample period practically coincides with the period through which financial research has shown a significant negative slope of the volatility smile for equity options on stock market indices. See Jackwerth and Rubinstein (1996) for the first discussion on how the crash of October 1987 changes the slope of the smile.

and stock market cycles and recessions. Section 7 concludes with a summary of our findings.

2. Risk-Neutral and Objective Probability Densities, and the Volatility of the Stochastic Discount Factor

There are well known economic episodes, like the stock market crash in 1987, the Asian currency crisis during the summer of 1997, the Russian default in the summer of 1998, or the terrorist attack on September 11th, 2001, in which the left-tail of the risk-neutral density becomes considerably fatter than the corresponding left-tail of the risk-adjusted counterpart. This points out that the former distribution overstates poor states of nature, especially during stress economic periods. Marginal utility is higher in those scenarios and this is precisely what is introduced into the estimated risk-neutral densities. Hence, and independently of the estimation method employed, if we calculate the difference between the probabilistic mass assigned to a given left tail percent from the risk-neutral and the power risk-adjusted density, we would observe how these differences are time-varying with a clear increasing pattern for every potentially damaging economic episode.

Given this discussion, we may also argue that, in absolute values, these crash fears may cause a positive overall gap or distance between the risk-neutral and objective probability measures. For a given percentage of the tails of the density functions, the potential economic downturn increases more the probabilistic mass assigned to the left tail of the risk-neutral density over the risk-adjusted density than the probabilistic mass assigned to the right tail of the risk-adjusted density over and above the risk-neutral counterpart. This suggest that the overall distance taken in absolute value between the risk-neutral and objective probability measures may be well suited to proxy for economic fears of investors. Interestingly, Bakshi, Chen and Hjalmarsson (2004) propose a formal overall distance measure between the two probability sets.

We consider an economy endowed with a probability space $(\Omega, \Im, P)$ where Ω denotes the state space and $\Im$ is the tribe of subsets of Ω that are events and can therefore be assigned a probability. We denote P and Q as the objective and risk-neutral probability measures respectively.

Under no arbitrage opportunities, there exists a strictly positive SDF, *M*, such that the price of any financial asset between any two time periods *t* and *t+1* is given by

$$p_{jt} = E_t^P \left(X_{jt+1} M_{t+1} \right), \tag{1}$$

where p_{jt} is the price of asset *j* at time *t*, X_{jt+1} is the future payoff of asset *j*, and E_t^P is the conditional expectation with respect to the objective probability measure P.

Alternatively, the price of the financial asset with respect to the risk-neutral probability *Q* is

$$p_{jt} = \frac{1}{R_f} E_t^Q \left(X_{jt+1} \right), \tag{2}$$

where E_t^Q is the conditional expectation with respect to *Q* and R_f is the gross riskless-rate of interest between *t* and *t+1*.

Bakshi, Chen and Hjalmarsson (2004) define the distance between P and *Q* as

$$D_0 \left(P, Q \right) \equiv \int_{\Omega} \left| \, dQ\left(X \right) - dP\left(X \right) \, \right|. \tag{3}$$

This distance will be zero if and only if P and *Q* assigns the same probability mass to every given event belonging to $\Im$ in the state space Ω. Otherwise, D_0 must be positive and, given our discussion above, D_0 may be time-varying and it may anticipate changes in the economic cycle because it might reasonable be higher just before economic recessions.

We can always choose a particular equivalent probability measure *Q* such that the Radon-Nikodym derivative is $\frac{dQ}{dP} = R_f M_{t+1}$.[2] Given that $E_t^P \left(M_{t+1} \right) = 1 / R_f$, it must be true that

$$dQ = \frac{M_{t+1}}{E_t^P \left(M_{t+1} \right)} dP. \tag{4}$$

[2] One can check that, under no arbitrage, this random variable is strictly positive and has expectation 1.

Substituting the expression (4) into (3) we obtain

$$D_0\left(P,Q\right)=E_t^P\left|\frac{M_{t+1}}{E_t^P\left(M_{t+1}\right)}-1\right|=R_f E_t^P\left|M_{t+1}-\frac{1}{R_f}\right|. \tag{5}$$

In words, the absolute distance between both probability measures is completely determined by the expectation under the objective probability of the absolute difference between *M* and $1/R_f$.[3]

The idea is to obtain an approximation for the distance between both sets of probability measures without relying on option data. By applying Hölder´s inequality to the distance probability measures we obtain

$$D_0\left(P,Q\right)\frac{1}{R_f}\leq\left\|M_{t+1}-\frac{1}{R_f}\right\|=\left\|M_{t+1}-E_t^P\left(M_{t+1}\right)\right\|=\sigma\left(M\right), \tag{6}$$

where $\sigma\left(M\right)$ is the standard deviation of the stochastic discount factor *M*. Hence, the volatility of the stochastic discount factor provides an upper bound for the distance between the risk-neutral and objective probability distribution, up to a constant of proportionality which is equal to the risk-free rate. This does not imply that a higher volatility of the defining SDF will be necessarily accompanied by a larger distance between the probability measures and, therefore, by increasing economic fears from investors. Then, the specific relationship between the volatility of any sensible SDF and the distance between probabilities becomes an empirical issue. However, we expect that, at the beginning of stressed economic periods, the volatility of reasonable SDFs should increase to reflect the overall larger absolute gap between the risk-neutral and objective probability measures. In fact, by employing the Hansen and Jagannathan (1991) volatility bound or the maximal Sharpe ratio, and assuming a mean-reverting process for the ratio, Brennan, Wang and Xia (2004) show a strong counter-cyclical behavior of the maximal ratio under the ICAPM framework of Merton (1973). Note that we are interested in studying whether the volatility of sensible SDFs predicts future economic and stock market cycles, rather than analyzing contemporaneous pricing relationships as in Brennan, Wang and Xia (2004).

[3] This is the case since R_f is just a scaling factor.

3. Consumption-based Stochastic Discount Factors

The well known SDF under power utility is given by

$$M_{t+1} = \beta \frac{U'\left(C_{t+1}\right)}{U'\left(C_t\right)} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}, \tag{7}$$

where C_t is aggregate *per-capita* non-durable consumption as calculated at time *t*, $U'\left(C_t\right)$ is marginal utility, β is the subjective discount factor or impatience parameter, and γ is the coefficient of relative risk aversion.

Despite the fact that nondurable consumption growth betas have repeatedly failed to explain the cross-sectional variation of average returns, the recent U.S. empirical evidence has shown that SDFs based on consumption data are able to explain reasonably well mean returns. For example, Parker (2001) and Parker and Julliard (2005) argue that consumption growth rates and stock returns do not covary contemporaneously as preferences in (7) indicate because agents' consumption takes time to respond to changes in wealth. The cost of adjusting consumption to current circumstances is greater than the cost of adjusting investment in financial assets. Furthermore, marginal utility of consumption is related to other slow-adjusting factors such as changes in labour earnings or property investments. Hence, they suggest measuring asset risk as the covariance between returns and consumption growth rate not only in the period to which returns refer, but also in several periods forward.[4] In particular, Parker and Julliard (2005) refer to this as *ultimate consumption risk*. They propose the following SDF

$$M_{t+1}^{S} = \beta^{S+1} \frac{R_{ft+1,t+1+S} U'\left(C_{t+1+S}\right)}{U'\left(C_t\right)}. \tag{8}$$

Under the power specification, the SDF takes the form

[4] Similarly, Bansal and Yaron (2004), Bansal, Dittmar and Lundblad (2005), and Hansen, Heaton and Li (2006) show that the covariance between long run cash flows and long run consumption growth can explain the cross-sectional variation of expected returns. For an excellent review of the main issues involved in this approach, see Bansal (2008).

$$M_{t+1}^{S} = \beta^{S+1} R_{ft+1,t+1+S} \left(\frac{C_{t+1+S}}{C_t} \right)^{-\gamma}, \tag{9}$$

where C_{t+1+S}/C_t is the consumption growth rate between t and $t+1+S$, and $R_{ft+1,t+1+S}$ is the risk-free gross rate corresponding to the same horizon.

At the same time, in a completely different setting, Yogo (2006) has shown that small and value firms are more pro-cyclical than large and growth firms with respect to the growth rate of durable consumption. This suggests that the ratio between durable and nondurable consumption growth rates is a pro-cyclical state variable that accentuates the counter-cyclical behaviour of marginal utility. Moreover, the inclusion of durable consumption can be done under recursive utility where the return of market equity wealth is part of the stochastic discount factor. Once again, this allows for a higher volatility of the stochastic discount factor relative to specifications where only consumption growth is employed.[5]

Finally, the habit persistence model of Campbell and Cochrane (1999) has widely been used as a preference representation in asset pricing modelling. The reason is the extra volatility in marginal utility of consumption obtained throughout the behaviour of the so called surplus consumption ratio which is the percentage difference between consumption and the level of habits.

With this ideas in mind, we now briefly discuss the five alternative SDFs employed in this paper.

Assuming recursive preferences, the SDF has the advantage of separating relative risk aversion and the elasticity of intertemporal substitution. Moreover, this SDF not only incorporates consumption growth but also the return on the market portfolio. In particular, under recursive utility, the contemporaneous SDF is given by

$$M_{t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\eta} \right]^{\kappa} R_{mt+1}^{\kappa-1}, \tag{10}$$

[5] See Campbell (1996).

where η is the elasticity of intertemporal substitution, $\kappa \equiv \frac{1-\gamma}{1-1/\eta}$, and R_{mt} is the gross return on the market portfolio at any time t. The set of parameters to be estimated is given by $\theta = \{\beta, \gamma, \eta\}$.

Similarly, the specification under ultimate consumption risk and recursive utility becomes

$$M_{t+1}^{S} = \left[\beta^{S+1} \left(\frac{C_{t+1+S}}{C_t} \right)^{-1/\eta} \right]^{\kappa} R_{mt+1+S}^{\kappa-1} R_{ft+1+S} . \tag{11}$$

Yogo (2006) incorporates durable consumption to the marginal utility using a recursive preferences specification in which both types of consumption are not separable. The idea is, as usual, to increase the volatility of marginal consumption. The contemporaneous SDF is given by

$$M_{t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\eta} \left(\frac{1-\alpha+\alpha\left(D_{t+1}/C_{t+1}\right)^{(\rho-1)/\rho}}{1-\alpha+\alpha\left(D_t/C_t\right)^{(\rho-1)/\rho}} \right)^{(\eta-\rho)/(\eta(\rho-1))} R_{mt+1}^{1-1/\kappa} \right]^{\kappa}, \tag{12}$$

where D_t is aggregate *per-capita* stock of durable consumption as calculated in time t, α is the expenditure share of the durable consumption good, and ρ is the elasticity of substitution between durable and non-durable consumption. Hence, the set of parameters is given by $\theta = \{\beta, \gamma, \eta, \alpha, \rho\}$.

As before, this paper analyzes the durable consumption-based asset pricing model under the perspective of ultimate consumption risk. In this case, the SDF becomes[6]

[6] In the empirical specification of expressions (11) and (12), we employ a three-year frame (S is 11 quarters) as the time lag for conciliating the consumption growth rate with current returns on equity assets, for both nondurable and durable consumption goods. Julliard and Parker (2005) also employs 11 quarters, while Malloy, Moskowitz and Vissing-Jorgensen (2006) use 16 quarters, although their results are robust to other specifications of long run consumption.

$$M_{t+1}^{S}=\left[\beta^{S+1}\left(\frac{C_{t+1+S}}{C_t}\right)^{-1/\eta}\left(\frac{1-\alpha+\alpha\left(D_{t+1+S}/C_{t+1+S}\right)^{(\rho-1)/\rho}}{1-\alpha+\alpha\left(D_t/C_t\right)^{(\rho-1)/\rho}}\right)^{(\eta-\rho)/(\eta(\rho-1))}\right]^{\kappa} \times R_{mt+1+S}^{\kappa-1}R_{ft+1+S} \tag{13}$$

Finally, the SDF under the external habit persistence model of Campbell and Cochrane (1999) is given by

$$M_{t+1}=\beta\left(\frac{SC_{t+1}}{SC_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}, \tag{14}$$

where H_t is the level of habits and $SC_t=\dfrac{C_t-H_t}{C_t}$ is a state variable known as "surplus consumption ratio" that allows to capture dependencies among states of nature. It is important to point out that SC_t is a recession indicator; it is low after several quarters of consumption declines and high in booms. It should be noted that the recognition of habits eliminates the need of including long-run consumption growth rates in the SDF; the nature of habits should be playing the equivalent role of ultimate consumption risk. Finally, under this specification, relative risk aversion presents a counter-cyclical behaviour. Hence, with recessions, as consumption falls toward habit, people become less willing to tolerate further falls in consumption and they become more risk averse.

Next, we define the habit formation process. Level of habits can be written as a function of past consumption. We use consumption growth rates to ensure that the function is stationary[7]

$$H_t=C_t g\left(\frac{C_{t-1}}{C_t},\ldots,\frac{C_{t-L}}{C_t}\right). \tag{15}$$

A reasonable function that guarantees $H_t<C_t$ is the following:

$$g(x)=h\left(1+e^{-x}\right)^{-1}, \tag{16}$$

[7] See Chen and Ludvigson (2004).

where h is the global habit persistence parameter, $x=\left(\delta\frac{C_{t-1}}{C_t}+\delta^2\frac{C_{t-2}}{C_t}+....+\delta^L\frac{C_{t-L}}{C_t}\right)$, with $0\le h\le 1$, and $0\le\delta\le 1$.

Therefore, the habit specification is given by

$$H_t=C_t h\left(1+e^{-\left(\delta\frac{C_{t-1}}{C_t}+\delta^2\frac{C_{t-2}}{C_t}+....+\delta^L\frac{C_{t-L}}{C_t}\right)}\right)^{-1}. \quad (17)$$

In the actual estimation of this model, and to be consistent with ultimate consumption risk, L will be 12 quarters. The set of parameters to be estimated is $\theta=\{\beta,\gamma,h,\delta\}$.

4. Data and Some Preliminary Results

4.1 Macroeconomic and Stock Market Data

We employ quarterly seasonally adjusted aggregate real per capita consumption expenditure of nondurables and services (*C*) for the period 1947:I to 2006:IV from National Income and Product Accounts (NIPA) Table 7.1. The same source is used to obtain the quarterly real per capita durable consumption expenditures (*E*), and the real per capita Gross Domestic Product (*GDP*). All these figures use year 2000 prices. Durable consumption consists of items such as clothing and shoes, motor vehicles, furniture, appliances, jewelry and watches. Following Yogo (2006), the investor´s stock of the durable good (*D*) is related to its expenditure by the law of motion

$$D_t=(1-dep)D_{t-1}+E_t, \quad (18)$$

where *dep* is the depreciation rate of 6 percent per quarter. We employ data from the previous 44 quarters to calculate the first data point of the stock of durable consumption. This implies that our estimations finally use quarterly real per capita data from nondurables and services, durables and *GDP* from 1958:I to 2006:IV. Quarterly data of the Industrial Production Index (IPI) in real terms from 1958:I to 2006:IV is obtained from the Federal Reserve Data Base.[8]

We also obtain monthly aggregate nominal consumption expenditures for the period 1959:1 to 2006:12 from NIPA Table 2.8.5. Moreover, population numbers are taken from NIPA Table 2.6, and price deflator series from NIPA Table 2.8.4. All this information is used to construct monthly seasonally adjusted real per capita consumption expenditures on nondurable goods and services, and durable expenditures. Each type of consumption is deflated by its corresponding price deflator. As before, we employ year 2000 prices. The law of motion to calculate the stock of durable consumption is also given by expression (18) with a depreciation rate of 2 percent per month, and the previous 60 months are employed to obtain the first data point. Hence, our final monthly data of alternative consumption categories goes from 1964:1 to 2006:12.

Stock market data is taken from Kenneth French´s web page. We have quarterly and monthly data on real value-weighted stock market portfolio returns, the corresponding real risk-free rate and excess returns of ten size-sorted equally-weighted portfolio returns. The price deflator from NIPA Table 2.8.4 is used to calculate real rates of returns. The final quarterly (monthly) data employed goes from 1958:I to 2006:IV (1964:1 to 2006:12).

4.2 Some Preliminary Results

As discussed before, our hypothesis is that the volatility of the SDFs described in Section 3 is time-varying, and particularly high just before recessions. Before analyzing this issue, we now discuss how the volatility of the model-free SDF behaves along economic cycles.

The Hansen and Jagannathan (1991) bound shows that the volatility of the stochastic discount factor satisfies the following relationship:

$$\sigma(M) \geq \left[\left(1_N - E(M)E(R) \right)' \Sigma^{-1} \left(1_N - E(M)E(R) \right) \right]^{1/2}, \qquad (19)$$

[8] In particular, the Industrial Production Index is taken from the descriptor 6/7 IP Major Industry.

where I_N and $E(R)$ are the N-vector of ones and expected returns respectively, and Σ is the variance-covariance matrix of returns. Any sensible SDF should satisfy this bound.[9]

The Hansen-Jagannathan volatility bound is estimated from 1958 to 2006 with quarterly and monthly data using realized returns of our ten equity portfolios and for a range of different values for $E(M)$. The minimum standard deviation of the SDF associated with the realized risk-free interest rate for the period is about 0.399 for quarterly data (0.263 for monthly data), corresponding to a mean SDF of about 0.987 (0.996).

Figure 1 display the volatility bound of the SDF using expression (19) but now calculated with overlapping sub-periods of 5 years of monthly data from the ten size-sorted portfolios. Each point shown in Figure 1 is the volatility bound for the given average level of the risk-free interest rate for each of the sub-periods. As long as this volatility is associated with the distance between the risk-neutral and objective probabilities, as suggested in Section 2, we may identify these changes in volatility with time-varying economic fears. The shaded regions of Figure 1 are U.S. macroeconomic recessions from peak to trough as defined by the National Bureau of Economic Research (NBER).

Figure 1 shows substantial differences over time in the volatility of the SDF which satisfies the Hansen-Jagannathan bound. The relevant issue is whether these changes are associated with changing recession fears reflecting the distance between the probability measures. Indeed, the time-varying behaviour of the volatility of the SDF seems to reflect increases before or during economic recessions. It is interesting to point out that the volatility of the SDF experienced a continuous increase from 0.276 on December 1986 to a peak of 0.427 on September 1987, just before the crash of October 1987. Moreover, Figure 1 displays a higher unconditional mean of the volatility of the SDF for the second half of our sample period. In other words, since the previous months of the crash of October 1987, the volatility of the model-free SDF does not seem to

[9] Thus, next section employs this expression to select feasible consumption-based SDFs using the vector of our ten size-sorted equity portfolios.

revert to the average levels characterizing the sub-period from the beginning of the sixties to the mid eighties.

5. The Estimation of Parameters for Alternative Consumption-based SDF Specifications

This section selects appropriate parametric SDFs using consumption data and stock returns from the U.S. economy. The idea is to study the behaviour of the volatility of alternative specifications of consumption-based SDFs. The five specifications discussed in Section 3 are analyzed here. In particular, we explore the recursive utility specification of expression (10), the long-run version of recursive utility (*recursive long*) given by (11), the durable SDF suggested by Yogo (2006) as in equation (12), its long-run version (*Yogo long*) given by (13), and the habit persistence model described in equations (14) and (17).

Specifically, for each SDF, we try a large grid of feasible preference parameters. We then select sets of combinations of those parameters that generate a volatility of the SDF which lies above the Hansen-Jagannathan bound of 0.399 with quarterly data and 0.263 with monthly data. Given the selected SDFs, we compute the pricing error at every quarter of each of these SDF in valuating the ten size-sorted portfolios returns as

$$\hat{e}_{jt} = \hat{M}_t \left(R_{jt} - R_{ft} \right), \tag{20}$$

where R_j is the rate of return of each of the ten size-sorted equity portfolios.

Then, we calculate the mean-squared pricing error over time and across portfolios as

$$MSE = \frac{\sum_{j=1}^{10} \sum_{t=1}^{T} \hat{e}_{jt}^2}{NT}. \tag{21}$$

Finally, for each SDF specification, we choose the preference parameters that simultaneously make the SDF to enter inside the feasible mean-volatility space and have the lowest error in pricing the ten portfolios according to expression (21).

Figure 2 displays the location of each of the five SDFs employed in the paper with the lowest mean-squared pricing error on the mean-volatility space for both quarterly and monthly data. Interestingly, they all exactly lie on the frontier. Note that the corresponding locations do not depend on the historical risk-free rate for the period. However, one of the SDF specifications has a mean-volatility pair closer to the historical pair estimated with equity portfolios and expression (19). In particular, this is the case for the recursive utility with contemporaneous growth.

Table 1 contains the actual parameter estimates, the volatility of M, the mean of M, and the mean-squared pricing error for each of the five SDFs chosen throughout the empirical exercise. Panel A contains the results with quarterly data. The lowest mean-squared pricing errors are obtained for the long run specifications. On the other hand, the higher volatilities are associated with habit persistence and the long run version of Yogo´s durable SDF. All estimated elasticities of intertemporal substitution are less than one independently of the specification employed. Moreover, the two long-run versions of recursive and Yogo´s preferences are able to generate very reasonable levels of risk aversion. It seems that the combination of long-run consumption growth and the inclusion of the market portfolio return with either nondurable or durable consumption goods are key properties to obtain low pricing errors and reasonable risk aversion coefficients. Panel B reports similar results with monthly data. In this case, however, reasonable levels of risk aversion are also estimated with the contemporaneous growth versions of recursive and Yogo´s SDF specifications which also display larger estimates of the elasticities of intertemporal substitution relative to Panel A.

Figure 3 represents, over time and across recessions, the volatilities of the five SDFs described in Table 1. They are calculated with quarterly data from the previous five years. In Panel A, the volatility of contemporaneous recursive and Yogo´s SDFs tends to follow a very similar patter, while the volatility of the habit persistence case presents more pronounced peaks. Panel B shows again a similar general pattern for the volatilities of long run SDFs with the exception of a higher peak shown by Yogo´s specification around the crash of October 1987.

6. Macroeconomic and Stock Market Cycles, Recessions, and the Volatility of Consumption-based Stochastic Discount Factors

6.1 Forecasting Macroeconomic and Stock Market Cycles with the Volatility of Consumption-based Stochastic Discount Factors

The volatility of the SDF may reflect the distance between the risk-neutral and objective probability distributions, which contains economic fears implicit in the investment behaviour of investors. If correct, the volatility of the SDF not only should contain information about economic uncertainty, but it may also be able to predict future macroeconomic and stock market cycles. This empirical issue is analyzed in this section.

The analysis consists on determining whether the overlapping standard deviation of our five SDF specifications incorporates information about the future of three selected state variables: the growth rate of the industrial production index, the growth rate of *GDP* and the stock market returns. We therefore perform the following OLS autocorrelation-robust-standard-error regressions for our five alternative SDF specifications:

$$\frac{IPI_{t+\tau} - IPI_t}{IPI_t} = \alpha + \beta\sigma(M_t) + \varepsilon_{t+\tau} \quad ; \tau = 1,2,4,8,12 \qquad (22)$$

$$\frac{GDP_{t+\tau} - GDP_t}{GDP_t} = \alpha + \beta\sigma(M_t) + \varepsilon_{t+\tau} \quad ; \tau = 1,2,4,8,12 \qquad (23)$$

$$R_{mt+\tau} = \alpha + \beta\sigma(M_t) + \varepsilon_{t+\tau} \quad ; \tau = 1,2,4,8,12 \qquad (24)$$

where IPI_t , GDP_t and R_{mt} are the Industrial Production Index, the Gross Domestic Product, and the stock market portfolio return for quarter *t* respectively, and $\sigma(M_t)$ represents the volatility of each SDF specification described in Section 5 estimated with either quarterly or monthly data. The corresponding volatility is always calculated with data from each SDF up to quarter *t* (or last month of quarter *t*). Note that the forecasting regressions always use quarterly data, while the volatility of the SDFs used as the explanatory variable can be estimated with either quarterly or monthly data.

A large number of empirical macroeconomic papers provide evidence of a striking decline in the volatility of U.S. macroeconomic time series after 1984. It is known as the “Great Moderation”. Although the reasons of this extraordinary break are debatable, a recent paper by Galí and Gambetti (2007) shows quite convincingly that this evidence can be attributed to changes in the economy´s structure and/or in the way policy has been conducted rather than to luck. Their paper shows that the Great Moderation can be largely explained by a very significant fall in the contribution of non-technological shocks to the volatility of output (with a simultaneous slightly increase in the contribution of technological shocks), and to the impact of monetary policy given a larger weight to inflation stabilization objectives.

Given this evidence, we perform the forecasting regression of expressions (22) to (24) separating the sample into two non-overlapping sub-periods from 1965:II to 1984:IV and from 1985:I to 2006:IV.[10]

Table 2 provides descriptive statistics over the two sub-periods which clearly reflect the U.S. structural break due to the Great Moderation. For example, the annualized volatility of the *GDP* growth rate changes from 2.216 percent between 1965 and 1984 to 0.996 percent between 1985 and 2006. Similar results are obtained for all macroeconomic series shown in Table 2, and for the risk-free interest rate. On the other hand, the volatility of the stock market portfolio is practically the same, while the Sharpe ratio (relative to the market return and not relative to the ex-post tangency portfolio) increases from 0.142 to 0.524. Moreover, the volatilities of the recursive-based SDF either with contemporaneous or long run growth also show an increasing pattern from 0.4 to 0.6 approximately.[11] The same result is obtained for the Hansen-Jagannathan volatility bound associated with the average risk-free rate for each sub-period.[12]

Table 3 contains the forecasting results for the first sub-period. Panels A, B and C report the regression results for *IPI*, *GDP* and the stock market return, respectively. We do not find any significant evidence of a negative relationship between the volatility of SDFs and future macroeconomic and stock market cycles. No relevant forecasting ability is found whatsoever. If any, there are few positive and significant

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[10] Although we estimate the parameter preferences from 1958:I to 2006:IV, we need five years of data to estimate the first volatility of the SDFs. Moreover, the maximum forecasting horizon imposed in the regression is twelve quarters. This implies that the first data point to be used in the prediction exercise must be the second quarter of 1965

[11] The estimation procedure for the selected SDFs described in Section 5 is repeated using data separately from each sub-period. Therefore, these are not unconditional means of the volatilities of each SDF for each sub-period calculated under the parameters estimated for the full sample period.

[12] This is consistent with the pattern displayed in Figure 1.

signs in all three cases, and a significant economically sensible predicting capacity of the stock market return at long horizons by Yogo´s long run specification.

Table 4 reports the regression forecasting results for the second sub-period. This table is also organized in three panels following the structure of the previous table. Overall, the empirical results of the second sub-period are strikingly different from the first sub-period. In panel A, and with the exception of habit-based preferences, the volatilities of all SDF specifications systematically predict the future growth rate of *IPI*. All estimated slopes have the expected negative sign; increases in the volatility of SDFs seem to be significantly associated with future declines in the growth rate of *IPI*. The R^2 for the recursive-based preferences with contemporaneous growth goes from approximately 16 percent to 24 percent at horizons of one and four quarters respectively. Interestingly, the explanatory power of the forecasting regressions becomes lower at long run horizons. A similar pattern is reported for Yogo´s contemporaneous case. The long run specifications of both recursive and Yogo´s durable SDFs seem to be able to forecast better at long horizons than at short horizons. This is consistent with theoretical nature of these two SDFs.[13] At horizons of one and twelve quarters, the R^2 goes from approximately 5 percent to 21 percent for the recursive case, and from 6 percent to 46 percent for the Yogo´s specification. Although these long run specifications use forward growth rates of both consumption and stock market returns, it should be pointed out that we never use known information of the forecasting period to construct these SDFs. Panel B contains a very similar although somewhat weaker evidence of the forecasting ability of the volatilities of SDFs relative to *GDP*. Contemporaneous specifications of recursive and durable-based SDFs significantly predict with the correct sign at short horizons, while long run versions significantly forecast at long horizons. Panel C contains the forecasting exercise for the stock market return. The empirical results are very similar. They confirm the significant predicting capacity of the volatilities of SDFs. Again, the contemporaneous cases being strong predictors at short run horizons while long run specifications showing a significant ability at long horizons.[14] Finally, independently of the forecasting horizon,

[13] This result supports the argument of Campbell and Thompson (2007) and Cambpell (2007) who convincingly argue that weak theoretical restrictions on the valuation models and on the signs of the coefficients help predicting stock market returns better than historical average returns.

[14] Cambpell and Thompson (2007) discuss whether the usual *R-squares* reported in the forecasting literature are economically meaningful. For tractability, they assume mean-variance preferences, and

we find higher slope coefficients (in absolute values) when forecasting the stock market than when predicting macroeconomic cycles. On the other hand, the $R^2 s$ are higher for macroeconomic variables than for the stock market return.[15]

Because the volatility of the discount factor is very persistent, we also calculate the bias-corrected estimator and the corresponding bias-corrected *t*-statistic proposed by Amihud and Hurvich (2004).[16] These authors suggest a regression method for hypothesis testing in predictive regressions with either one or multiple autoregressive predictor variables. Their simulations show that their adjustment outperforms other bias-correction methods such as those suggested by Stambaugh (1999) or Lewellen (2004). In particular, we assume the following two equations

$$\Delta V_{t+\tau} = \alpha + \beta\sigma\left(M_t\right) + \varepsilon_{t+\tau} \qquad (25)$$

$$\sigma\left(M_{t+\tau}\right) = \theta + \rho\sigma\left(M_t\right) + \eta_{t+\tau} \ , \qquad (26)$$

where $\Delta V_{t+\tau}$ is the growth rate of *IPI*, *GDP* or directly the market return. The basic idea is to partly eliminate noise by running the following regression

$$\Delta V_{t+\tau} = \alpha' + \beta\sigma\left(M_t\right) + \delta\left[\sigma\left(M_{t+\tau}\right) - \rho\sigma\left(M_t\right)\right] + e_{t+\tau} \, , \qquad (27)$$

where the additional regressor, in brackets, is uncorrelated with the initial regressor $\sigma\left(M_t\right)$ but correlated with $\Delta V_{t+\tau}$. Thus the OLS estimation of (27) still obtains a consistent estimate of the original coefficient β, but it does so with larger efficiency.

We therefore replicate the forecasting regressions with this procedure. Interestingly, the results are qualitative the same, and the coefficients tend to be even higher in absolute values. It should be recalled that the results from Table 4 are already

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show that the gain of observing the predictor variable depends on the observed *R-square*, the Sharpe ratio and risk aversion. If we replicate their exercise for the recursive contemporaneous specification and four quarters as the forecasting horizon, we get that the absolute increase in portfolio return is about 5.8 percent per year for an investor with a risk aversion coefficient of 8.5. As discussed later, this is the risk aversion coefficient obtained for the second sub-period under recursive preferences with long run consumption growth.

[15] As for the case of *IPI*, the habit-based specification does not deliver reasonable results for either *GDP* or the stock market return.

[16] Note that persistence leads to biased coefficients in predictive regressions if innovations in the predictor variable are correlated with the dependent variable.

based on autocorrelation-robust-standard error regressions. Indeed, the regression (27) has a significant impact on coefficients when we run the standard OLS predictive regressions. However, these effects turn out to be negligible once OLS autocorrelation-robust-standard errors regressions are used.

We may therefore conclude that the volatility of consumption-based SDFs seems to be a powerful indicator of both future economic and stock market cycles, at least from 1985 to 2006. This sub-period is characterized by a macroeconomic policy succeeding in stabilizing the economy. At the same time, this sub-period practically coincides with the years in which we know there is a changing behaviour of the slope of the volatility smile in equity options. We next further investigate this coincidence.

6.2 Recessions, Expansions, Risk Aversion and the Volatility of Consumption-based Stochastic Discount Factors

A natural concern motivated by the empirical results reported above is how we can explain the ultimate reasons behind this evidence. We argue at the beginning of this paper that increasing fears, reflected in the higher distance between the risk-neutral and risk-adjusted densities estimated by forward looking (at expiration) assets, may be captured by the volatility of SDFs. We should therefore investigate whether the resulting forecasting capacity during the second sub-period is especially stronger in recessions relative to expansions. If this is actually the case, then fears from the investors´ perspectives would indeed be the key component reflected in the volatility of consumption-based SDFs.

We repeat the regressions of equations (22) to (24) from 1985:I to 2006:IV but now separating recession and expansion periods. The definition of the recession variable is based on the recession dates as identified by NBER. This formal identification of recession dates gives only seven quarters during the second sub-period. We therefore introduce a variant which allows us to have more observations in the regressions without substantially altering the nature of the exercise. Recessions are defined as follows: For a $t + \tau$ forecasting horizon, if at $t + \tau$ there is a recession according to the NBER, then a new recession variable is defined by assigning a value of 1 at any quarter from $t + 1$ to $t + \tau$. [17] The expansion variable is the complementary of the recession variable. We perform the forecasting regressions for the contemporaneous and long run recursive-based SDF specifications.

The results, reported in Table 5, tend to be consistent with the hypothesis that fears strongly characterizes the behaviour of the volatilities of consumption-based SDFs. Panel A contains the evidence regarding *IPI*. Practically in all cases, the R^2s are higher for recessions than for expansions, although the evidence is even more consistent when we employ the contemporaneous version of the recursive-based SDF. A similar pattern is found in the slope coefficients. Panel B presents the evidence for *GDP*. As expected, given the results from Table 4, we now find weaker support for our hypothesis. It is clear that the time-varying behaviour of *GDP* is smoother than *IPI*. This may introduce more difficulties for the forecasting regressions since it may become more complicate to distinguish between alternative states of nature with *GDP* than with *IPI* and, therefore, to deliver significant results. Finally, Panel C reports the results for the stock marker return. Once again, they are stronger for recessions than for expansions. The overall conclusion favours the hypothesis that fears are behind the significant forecasting results observed for the sub-period from 1985:I to 2006:IV. Fears seem to be the key determinant of the negative relationship between the volatility of SDFs and macroeconomic and stock market cycles.

We already know that the period in which we find strong support for the forecasting capacity of the consumption-based SDFs coincides with a remarkable decline in macroeconomic volatility. Then, it may be reasonable to check whether the preference parameters of the SDFs used in the analysis show any sign of a changing investor's behaviour from the first to the second sub-period. We therefore repeat the process of selecting preference parameters for the contemporaneous and long run recursive-based SDF specifications for the two sub-periods separately using quarterly data.[18] Table 6 contains the results for the two selected SDFs with the lowest pricing errors when valuing the ten size-sorted equity portfolios. As expected, the risk aversion coefficients tend to be quite high in both sub-periods when we use the contemporaneous version of the SDF, while they are more reasonable when the long run specification is

[17] Of course, this method delivers exactly the same (small) number of recession data points as the ones provided by the NBER when forecasting one quarter ahead.

[18] Recall that we employ data since 1958 when estimating the preference parameters.

employed. However, what is more relevant is the increase of risk aversion from the first to the second sub-period. In particular, the relative risk aversion coefficient goes from 21.8 to 29 when we use the contemporaneous recursive-based case, and from 0.35 to 8.5 when the long run specification is employed. Importantly, no other clear patterns are found in other preference parameters. Therefore, the second sub-period is not only characterized by low macroeconomic fluctuations but also for an increasing risk aversion attitude on the investors´ behaviour. This is consistent with increasing fears and with the changing slope of the volatility smile for equity options.

6.3 Out-of-Sample Analysis

The obvious critique to our previous approach is that we are basically performing an in-sample experiment because the preference parameters used in the forecasting regressions are estimated with the full sample period. Hence, we actually use not available data when we carry out the prediction exercise. To check the robustness of our empirical results, we repeat the selection of preference parameters for the contemporaneous recursive-based SDF specifications from 1958:I to 1988:IV using quarterly data.[19] This is the estimation period. These parameters are then employed to construct SDFs for each quarter from 1989:I to 2006:IV. This is the forecasting period. We next repeat the predicting regressions of expressions (22) to (24) for both SDF specifications and for the usual horizons. The results are contained in Table 7. The strong forecasting ability of the volatility of the SDF is again found for both future macroeconomic and stock market variables. This result generates additional confidence in the predicting evidence reported in previous sub-sections.

7. Conclusions

In this paper we present convincing empirical evidence showing that the volatility of appropriated-selected consumption-based SDFs captures implicit recession fears of investors. More specifically, we report a very important change in the forecasting ability of the volatility of the consumption-based SDF specifications which coincides with the structural change in the macroeconomic conditions of the U.S. economy. The predicting capacity is basically observed during the period in which there is a very strong decline in the volatility of key macroeconomic time series. In particular,

[19] The estimated preference parameters, in this case, are 0.9975, 21.8, and 0.22, for the subjective discount factor, the relative risk aversion, and the elasticity of intertemporal substitution, respectively.

from 1985 to 2006, contemporaneous consumption growth specifications of recursive and durable preferences predict economic cycles at horizons of one, two and four quarters, while long run specifications better forecasts at eight and twelve quarters. Even more importantly, the explanatory forecasting ability is stronger at recessions than at expansions. This suggests a strong fear component in the volatility of SFDs. It is also interesting to note that there is an important increase in the risk aversion coefficient from the first sub-period to the second sub-period of our sample, while we do not observe any other significant pattern in the behaviour of other preference parameters. It should be recalled that the second sub-period is also characterized by a significant increase in the slope of the volatility smile of equity options which may also reflect increasing fears from investors in the stock market. Therefore, the forecasting ability of the volatility of consumption-based SDFs is found when simultaneously we have a period of great macroeconomic moderation and increasing fears (higher risk aversion) in stock market participants.

Further research explaining the connection between financial markets and the real economy seems to be crucial for the understanding of this simultaneous phenomenon.

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Table 1
Preference Parameters and Moments for Alternative Consumption-based Stochastic Discount Factors with Lowest Pricing Errors

Panel A. Quarterly Data 1958:I-2006:IV

Preferences	β	γ	η	α	ρ	δ	h	$E(M)$	$\sigma(M)$	*Pricing Errors*
Recursive	0.990	24	0.215	N.A.	N.A.	N.A.	N.A.	0.9563	0.3996	0.0220
Yogo	0.988	25	0.20	0.80	0.42	N.A.	N.A.	0.9545	0.4271	0.0203
Habit	0.997	29	N.A.	N.A.	N.A.	0.76	0.997	0.9441	0.5142	0.0169
Recursive Long	0.991	1.65	0.81	N.A.	N.A.	N.A.	N.A.	0.9528	0.4297	0.0125
Yogo Long	0.980	2.50	0.70	0.85	0.75	N.A.	N.A.	0.9354	0.6052	0.0115

For each SDF specification, we try a large grid of feasible preference parameters. We then select sets of combinations of those parameters that generate a volatility of the SDF, calculated with quarterly data, which lies above the HJ bound for the average level of interest rate for that period. The selected combinations should enter in the HJ space. Given the selected SDFs, we compute the pricing error at every quarter of each of these SDF in valuing the ten-size sorted portfolios. We finally calculate the MSE over time and across portfolios. The parameters reported correspond to selected SDF with the lowest MSE. β is the subjective discount factor for future period utility; γ is the coefficient of relative risk aversion; η is the elasticity of intertemporal substitution; α is the expenditure share of the durable consumption good; ρ is the elasticity of substitution between durable and non-durable consumption; δ is the weight associated with past consumption; *h* is the global habit persistence parameter; and *pricing error* is the mean squared error over ten size-sorted portfolios.

Panel B. Monthly Data 1964:1-2006:4

Preferences	β	γ	η	α	ρ	δ	h	$E(M)$	$\sigma(M)$	*Pricing Errors*
Recursive	0.999	3.8	0.72	N.A.	N.A.	N.A.	N.A.	0.9814	0.2630	0.00390
Yogo	0.999	2.8	0.80	0.96	0.38	N.A.	N.A.	0.9818	0.2660	0.00387
Habit	0.998	43	N.A.	N.A.	N.A.	0.81	0.993	0.9489	1.0318	0.00944
Recursive Long	0.958	0.8	0.02	N.A.	N.A.	N.A.	N.A.	0.9819	0.2725	0.00370
Yogo Long	0.904	0.9	0.11	0.99	0.89	N.A.	N.A.	0.9819	0.2676	0.00369

We repeat the procedure described in Panel A with monthly data.

Table 2
Volatilities of Macroeconomic, Stock Market Variables and Alternative Stochastic Discount Factors by Sub-periods

Variables	1965:II-1984:IV	1985:I-2006:IV
Non-durable Consumption	1.032	0.658
Durable Consumption	1.108	0.918
Gross Domestic Product	2.216	0.996
Industrial Production	4.356	2.068
Market Return	17.488	16.467
Risk-free Rate	1.354	0.976
Sharpe Ratio	0.142	0.524
Recursive-based SDF	0.400	0.596
Recursive-based SDF with Long Run Consumption Growth	0.406	0.598
Hansen-Jagannathan Volatility Bound	0.394	0.595

Volatilities are estimated with quarterly data. Figures reported for the macroeconomic variables, stock market return and risk-free rate are annualized and in percentages. Volatilities for the alternative SDFs are calculated using the preference parameters estimated with data only from the corresponding sub-period.

Table 3
Predicting Future Macroeconomic and Stock Market Cycles with the Volatility of Alternative Consumption-based Stochastic Discount Factors with Quarterly Data 1965:II-1984:IV[1]

Panel A. Future Industrial Production Index and the Volatility of SDFs[2]

$$\frac{IPI_{t+\tau} - IPI_t}{IPI_t} = \alpha + \beta\sigma\left(M_t\right) + \varepsilon_{t+\tau}$$

Horizon	*Estimates*	*Recursive*		*Yogo*		*Habit*		*Recursive Long*		*Yogo Long*	
1 Quarter	*Constant*	-0.002	(-0.17)	-0.015	(-1.05)	0.008	(1.09)	-0.001	(-0.07)	-0.016	(-1.15)
	Slope	0.024	(0.80)	0.055	(1.76)	-0.002	(-0.13)	0.043	(0.48)	0.077	(1.98)
	R^2 (%)	0.90		4.29		0.04		0.55		6.59	
2 Quarters	*Constant*	-0.010	(-0.43)	-0.035	(-1.17)	0.015	(0.95)	-0.001	(-0.02)	-0.028	(-1.07)
	Slope	0.064	(1.11)	0.121	(1.88)	-0.000	(-0.01)	0.077	(0.42)	0.140	(1.94)
	R^2 (%)	2.06		6.76		0.00		0.57		7.18	
4 Quarters	*Constant*	-0.025	(-0.57)	-0.059	(-1.09)	0.031	(1.03)	0.008	(0.10)	-0.033	(-0.76)
	Slope	0.139	(1.32)	0.214	(1.85)	-0.001	(-0.03)	0.107	(0.30)	0.205	(1.66)
	R^2 (%)	3.58		7.83		0.00		0.42		5.80	
8 Quarters	*Constant*	-0.052	(-0.75)	-0.104	(-1.35)	0.039	(0.74)	0.043	(0.34)	0.009	(0.11)
	Slope	0.285	(1.64)	0.401	(2.40)	0.041	(0.46)	0.085	(0.15)	0.166	(0.63)
	R^2 (%)	6.72		12.61		0.86		0.11		1.75	
12 Quarters	*Constant*	0.010	(0.11)	-0.045	(-0.57)	0.044	(0.74)	0.136	(0.94)	0.047	(0.42)
	Slope	0.182	(0.77)	0.306	(1.75)	0.074	(0.73)	-0.284	(-0.41)	0.102	(0.30)
	R^2 (%)	2.40		6.91		1.90		0.87		0.59	

1/ The SDF employed to calculate the volatilities used in the regressions enter in the feasible HJ space and have the lowest pricing errors in valuing the ten size-sorted portfolios. The corresponding preference parameters are estimated with data from the full period from 1958:I to 2006:IV. The volatilities are estimated as overlapping standard deviations of five years of past data.

2/ All panels report OLS autocorrelation-robust standard-error regressions.

Panel B. Future Gross Domestic Product and the Volatility of SDFs[2]

$$\frac{GDP_{t+\tau} - GDP_t}{GDP_t} = \alpha + \beta\sigma\left(M_t\right) + \varepsilon_{t+\tau}$$

Horizon	*Estimates*	*Recursive*		*Yogo*		*Habit*		*Recursive Long*		*Yogo Long*	
1 Quarter	*Constant*	-0.000	(-0.02)	-0.006	(-0.89)	0.005	(1.24)	-0.005	(-0.54)	-0.002	(-0.39)
	Slope	0.014	(0.98)	0.029	(1.79)	0.002	(0.21)	0.051	(1.16)	0.025	(1.53)
	R^2 (%)	1.22		4.55		0.09		3.00		2.74	
2 Quarters	*Constant*	-0.003	(-0.31)	-0.014	(-1.08)	0.008	(0.98)	-0.008	(-0.44)	-0.002	(-0.17)
	Slope	0.036	(1.32)	0.061	(2.05)	0.006	(0.45)	0.092	(1.06)	0.041	(1.29)
	R^2 (%)	3.02		7.92		0.63		3.80		2.89	
4 Quarters	*Constant*	-0.006	(-0.33)	-0.020	(-0.91)	0.013	(0.90)	-0.006	(-0.17)	0.007	(0.34)
	Slope	0.070	(1.43)	0.100	(2.03)	0.016	(0.59)	0.131	(0.80)	0.046	(0.74)
	R^2 (%)	4.52		8.41		1.52		3.10		1.46	
8 Quarters	*Constant*	-0.010	(-0.33)	-0.034	(-1.13)	0.019	(0.83)	0.011	(0.22)	0.034	(0.82)
	Slope	0.132	(1.67)	0.184	(2.76)	0.043	(0.96)	0.151	(0.60)	0.027	(0.22)
	R^2 (%)	7.15		13.08		4.72		1.77		0.23	
12 Quarters	*Constant*	0.035	(0.96)	0.002	(0.08)	0.030	(1.32)	0.077	(1.42)	0.043	(0.83)
	Slope	0.054	(0.52)	0.131	(1.83)	0.052	(1.28)	-0.107	(-0.39)	0.040	(0.27)
	R^2 (%)	1.18		7.03		5.25		0.69		0.52	

Panel C. Future Market Portfolio Returns and the Volatility of SDFs

$$R_{mt+\tau} = \alpha + \beta\sigma\left(M_t\right) + \varepsilon_{t+\tau}$$

Horizon	*Estimates*	*Recursive*		*Yogo*		*Habit*		*Recursive Long*		*Yogo Long*	
1 Quarter	*Constant*	-0.037	(-0.85)	-0.033	(-0.58)	-0.007	(-0.28)	-0.041	(-0.38)	0.050	(1.42)
	Slope	0.123	(1.20)	0.109	(0.84)	0.033	(0.79)	0.253	(0.93)	-0.124	(-1.12)
	R^2 (%)	1.36		0.99		0.68		1.11		0.99	
2 Quarters	*Constant*	-0.059	(-0.73)	-0.063	(-0.60)	-0.014	(-0.32)	-0.072	(-0.64)	0.118	(1.78)
	Slope	0.204	(1.09)	0.208	(0.87)	0.066	(0.84)	0.455	(0.89)	-0.311	(-1.46)
	R^2 (%)	1.62		1.53		1.15		1.56		2.72	
4 Quarters	*Constant*	-0.121	(-0.86)	-0.192	(-1.12)	-0.051	(-0.58)	-0.171	(-0.90)	0.273	(2.46)
	Slope	0.416	(1.26)	0.571	(1.47)	0.174	(1.16)	1.041	(1.21)	-0.744	(-2.00)
	R^2 (%)	3.59		6.16		4.24		4.42		8.53	
8 Quarters	*Constant*	-0.185	(-1.00)	-0.329	(-1.57)	-0.140	(-1.15)	-0.506	(-2.14)	0.517	(3.05)
	Slope	0.664	(1.59)	0.985	(2.19)	0.409	(2.18)	2.850	(2.71)	-1.422	(-2.51)
	R^2 (%)	6.00		12.52		14.09		21.11		21.20	
12 Quarters	*Constant*	-0.167	(-1.13)	-0.183	(-0.90)	-0.253	(-2.22)	-0.769	(-3.42)	0.709	(5.06)
	Slope	0.613	(1.82)	0.617	(1.41)	0.660	(3.72)	4.232	(4.02)	-2.024	(-4.54)
	R^2 (%)	5.26		5.45		29.29		37.35		45.28	

Table 4
Predicting Future Macroeconomic and Stock Market Cycles with the Volatility of Alternative Consumption-based Stochastic Discount Factors with Quarterly Data 1985:I-2006:IV[1]

Panel A. Future Industrial Production Index and the Volatility of SDFs[2]

$$\frac{IPI_{t+\tau}-IPI_t}{IPI_t}=\alpha+\beta\sigma\left(M_t\right)+\varepsilon_{t+\tau}$$

Horizon	*Estimates*	*Recursive*		*Yogo*		*Habit*		*Recursive Long*		*Yogo Long*	
1 Quarter	*Constant*	0.030	(5.88)	0.029	(5.76)	0.008	(2.05)	0.014	(4.62)	0.014	(4.88)
	Slope	-0.052	(-4.05)	-0.055	(-3.96)	-0.000	(-0.00)	-0.017	(-2.06)	-0.010	(-2.01)
	R^2 (%)	15.57		14.72		0.00		5.41		6.35	
2 Quarters	*Constant*	0.059	(6.37)	0.058	(6.18)	0.016	(1.96)	0.028	(4.69)	0.029	(5.25)
	Slope	-0.104	(-4.36)	-0.109	(-4.24)	0.002	(0.10)	-0.034	(-2.15)	-0.022	(-2.35)
	R^2 (%)	19.10		17.90		0.02		6.60		9.42	
4 Quarters	*Constant*	0.117	(6.71)	0.114	(6.37)	0.028	(1.81)	0.056	(4.72)	0.062	(5.70)
	Slope	-0.199	(-4.67)	-0.208	(-4.46)	0.013	(0.42)	-0.066	(-2.27)	-0.050	(-2.95)
	R^2 (%)	23.95		22.02		0.54		8.52		15.89	
8 Quarters	*Constant*	0.208	(7.15)	0.198	(6.51)	0.040	(1.52)	0.122	(5.38)	0.143	(8.14)
	Slope	-0.328	(-4.68)	-0.330	(-4.25)	0.075	(1.48)	-0.146	(-2.61)	-0.127	(-4.74)
	R^2 (%)	21.82		18.80		8.06		13.82		34.37	
12 Quarters	*Constant*	0.236	(4.43)	0.217	(3.76)	0.043	(1.35)	0.193	(6.00)	0.217	(10.76)
	Slope	-0.303	(-2.47)	-0.277	(-1.96)	0.156	(2.75)	-0.247	(-2.80)	-0.192	(-5.50)
	R^2 (%)	10.12		7.23		22.83		21.31		45.69	

1/ The SDF employed to calculate the volatilities used in the regressions enter in the feasible HJ space and have the lowest pricing errors in valuing the ten size-sorted portfolios. The corresponding preference parameters are estimated with data from the full period from 1958:I to 2006:IV. The volatilities are estimated as overlapping standard deviations of five years of past data.

2/ All panels report OLS autocorrelation-robust standard-error regressions.

Panel B. Future Gross Domestic Product and the Volatility of SDFs

$$\frac{GDP_{t+\tau}-GDP_t}{GDP_t}=\alpha+\beta\sigma\left(M_t\right)+\varepsilon_{t+\tau}$$

Horizon	*Estimates*	*Recursive*		*Yogo*		*Habit*		*Recursive Long*		*Yogo Long*	
1 Quarter	*Constant*	0.012	(5.11)	0.012	(5.20)	0.005	(3.77)	0.006	(4.50)	0.006	(4.90)
	Slope	-0.017	(-2.85)	-0.019	(-2.92)	-0.002	(-0.53)	-0.004	(-1.05)	-0.003	(-0.96)
	R^2 (%)	7.18		7.21		0.30		1.35		1.72	
2 Quarters	*Constant*	0.023	(5.68)	0.023	(5.74)	0.009	(3.19)	0.012	(4.50)	0.013	(5.28)
	Slope	-0.031	(-3.01)	-0.034	(-3.05)	0.001	(0.08)	-0.007	(-0.96)	-0.005	(-1.02)
	R^2 (%)	9.59		9.42		0.01		1.70		2.75	
4 Quarters	*Constant*	0.041	(5.44)	0.041	(5.34)	0.016	(2.74)	0.024	(4.83)	0.027	(6.64)
	Slope	-0.051	(-2.69)	-0.054	(-2.64)	0.010	(0.92)	-0.013	(-0.95)	-0.012	(-1.45)
	R^2 (%)	8.98		8.50		1.90		1.84		5.62	
8 Quarters	*Constant*	0.074	(5.35)	0.071	(4.89)	0.023	(2.14)	0.055	(5.66)	0.068	(7.92)
	Slope	-0.075	(-2.20)	-0.075	(-1.96)	0.048	(2.28)	-0.036	(-1.60)	-0.045	(-3.02)
	R^2 (%)	5.84		4.93		16.69		4.27		22.09	
12 Quarters	*Constant*	0.092	(4.43)	0.086	(3.70)	0.028	(2.20)	0.094	(6.07)	0.115	(10.32)
	Slope	-0.064	(-1.25)	-0.053	(-0.87)	0.088	(3.59)	-0.082	(-2.17)	-0.088	(-4.90)
	R^2 (%)	2.17		1.27		34.78		11.30		45.76	

Panel C. Future Market Portfolio Returns and the Volatility of SDFs

$$R_{mt+\tau} = \alpha + \beta\sigma\left(M_t\right) + \varepsilon_{t+\tau}$$

Horizon	*Estimates*	*Recursive*		*Yogo*		*Habit*		*Recursive Long*		*Yogo Long*	
1 Quarter	*Constant*	0.098	(2.47)	0.095	(2.41)	0.010	(0.51)	0.053	(2.47)	0.055	(3.50)
	Slope	-0.174	(-1.72)	-0.179	(-1.66)	0.045	(1.05)	-0.077	(-1.15)	-0.051	(-1.75)
	R^2 (%)	2.69		2.42		0.78		1.72		2.54	
2 Quarters	*Constant*	0.220	(2.97)	0.214	(2.93)	0.028	(0.81)	0.103	(2.55)	0.108	(3.65)
	Slope	-0.41	(-2.14)	-0.422	(-2.10)	0.062	(0.91)	-0.148	(-1.19)	-0.099	(-1.85)
	R^2 (%)	7.84		7.19		0.89		3.42		5.03	
4 Quarters	*Constant*	0.403	(3.64)	0.386	(3.52)	0.035	(0.56)	0.192	(2.62)	0.206	(4.06)
	Slope	-0.714	(-2.45)	-0.730	(-2.35)	0.185	(1.61)	-0.250	(-1.14)	-0.177	(-2.01)
	R^2 (%)	12.3		10.87		4.62		4.91		8.05	
8 Quarters	*Constant*	0.701	(4.89)	0.661	(4.56)	0.054	(0.52)	0.400	(3.53)	0.426	(5.01)
	Slope	-1.169	(-3.07)	-1.160	(-2.84)	0.394	(2.04)	-0.535	(-1.68)	-0.373	(-2.94)
	R^2 (%)	14.31		11.95		11.55		9.60		15.24	
12 Quarters	*Constant*	0.757	(2.71)	0.674	(2.30)	0.043	(0.32)	0.723	(4.58)	0.758	(7.57)
	Slope	-0.981	(-1.52)	-0.846	(-1.16)	0.718	(3.22)	-1.118	(-2.30)	-0.732	(-4.48)
	R^2 (%)	5.17		3.29		23.55		21.38		32.52	

Table 5
Predicting Future Macroeconomic and Stock Market Recessions and Expansions with the Volatility of Alternative Recursive-based Stochastic Discount Factors with Quarterly Data[1]
1985:I-2006:IV

Panel A. Future Industrial Production Index and the Volatility of SDFs[2]

$$\frac{IPI_{t+\tau} - IPI_t}{IPI_t} = \alpha + \beta\sigma\left(M_t\right) + \varepsilon_{t+\tau}$$

Forecasting Horizon	*Regresión Estimates*	*Recessions*				*Expansions*			
		Recursive		*Recursive Long*		*Recursive*		*Recursive Long*	
2 Quarters	*Constant*	0.110	(1.67)	0.049	(1.44)	0.053	(7.48)	0.028	(4.89)
	Slope	-0.284	(-2.07)	-0.166	(-2.18)	-0.080	(-5.30)	-0.023	(-1.84)
	R^2 (%)	38.28		38.68		20.79		5.58	
4 Quarters	*Constant*	0.309	(3.17)	-0.063	(-0.79)	0.116	(8.38)	0.063	(5.79)
	Slope	-0.734	(-3.15)	0.159	(0.70)	-0.183	(-5.97)	-0.066	(-2.91)
	R^2 (%)	32.48		7.77		33.52		14.24	
8 Quarters	*Constant*	0.331	(9.90)	0.036	(0.63)	0.219	(6.78)	0.133	(6.05)
	Slope	-0.726	(-6.65)	0.057	(0.36)	-0.335	(-4.64)	-0.169	(-2.91)
	R^2 (%)	59.96		1.06		23.64		21.72	
12 Quarters	*Constant*	0.299	(6.47)	0.249	(6.37)	0.052	(0.36)	0.165	(4.14)
	Slope	-0.474	(-3.56)	-0.396	(-3.42)	0.094	(0.31)	-0.187	(-1.95)
	R^2 (%)	30.38		34.42		0.32		13.82	

1/ The SDF employed to calculate the volatilities used in the regressions enter in the feasible HJ space and have the lowest pricing errors in valuing the ten size-sorted portfolios. The corresponding preference parameters are estimated with data from the full period from 1958:I to 2006:IV. The volatilities are estimated as overlapping standard deviations of five years of past data. Recessions are defined as follows: If at $t + \tau$ there is a recession according to the NBER, then a new recession variable is defined by assigning a value of 1 at any quarter from $t + 1$ to $t + \tau$. The expansion variable is the complementary of the recession variable.
2/ All panels report OLS autocorrelation-robust standard-error regressions.

Panel B. Future Gross Domestic Product and the Volatility of SDFs

$$\frac{GDP_{t+\tau} - GDP_t}{GDP_t} = \alpha + \beta\sigma\left(M_t\right) + \varepsilon_{t+\tau}$$

Forecasting Horizon	*Regresión Estimates*	*Recessions*				*Expansions*			
		Recursive		*Recursive Long*		*Recursive*		*Recursive Long*	
2 Quarters	*Constant*	-0.012	(-0.32)	-0.004	(-0.22)	0.021	(7.33)	0.013	(5.21)
	Slope	0.015	(0.20)	-0.003	(-0.09)	-0.024	(-3.14)	-0.004	(-0.58)
	R^2 (%)	0.65		0.09		11.05		0.98	
4 Quarters	*Constant*	0.079	(1.49)	-0.026	(-0.63)	0.041	(7.07)	0.027	(6.24)
	Slope	-0.172	(-1.43)	0.085	(0.81)	-0.046	(-3.02)	-0.013	(-1.14)
	R^2 (%)	9.75		12.22		11.58		3.11	
8 Quarters	*Constant*	0.127	(7.29)	0.048	(1.95)	0.064	(3.75)	0.055	(5.34)
	Slope	-0.219	(-3.81)	-0.009	(-0.14)	-0.053	(-1.35)	-0.041	(-1.61)
	R^2 (%)	32.87		0.15		2.85		5.95	
12 Quarters	*Constant*	0.115	(7.83)	0.114	(15.02)	0.064	(3.75)	0.079	(3.29)
	Slope	-0.102	(-2.14)	-0.119	(-4.26)	0.254	(2.02)	-0.055	(-1.10)
	R^2 (%)	17.18		38.42		9.03		4.76	

Panel C. Future Market Portfolio Returns and the Volatility of SDFs

$$R_{mt+\tau} = \alpha + \beta\sigma\left(M_t\right) + \varepsilon_{t+\tau}$$

Forecasting Horizon	*Regresión Estimates*	*Recessions*				*Expansions*			
		Recursive		*Recursive Long*		*Recursive*		*Recursive Long*	
2 Quarters	*Constant*	0.315	(0.97)	0.040	(0.36)	0.201	(2.77)	0.106	(2.68)
	Slope	-0.849	(-1.20)	-0.269	(-1.08)	-0.330	(-1.79)	-0.119	(-0.97)
	R^2 (%)	8.03		2.37		6.51		2.81	
4 Quarters	*Constant*	1.469	(2.66)	0.117	(0.34)	0.394	(3.71)	0.206	(2.93)
	Slope	-3.563	(-2.84)	-0.437	(-0.46)	-0.632	(-2.27)	-0.217	(-1.03)
	R^2 (%)	35.92		2.74		13.78		5.25	
8 Quarters	*Constant*	1.113	(5.93)	0.210	(1.02)	0.788	(5.28)	0.429	(3.69)
	Slope	-2.594	(-4.34)	-0.202	(-0.32)	-1.280	(-3.32)	-0.558	(-1.67)
	R^2 (%)	51.52		0.89		16.83		11.56	
12 Quarters	*Constant*	1.005	(3.11)	0.969	(5.35)	0.677	(1.22)	0.684	(3.84)
	Slope	-1.814	(-2.07)	-2.058	(-4.75)	-0.765	(.0.66)	-0.959	(-1.86)
	R^2 (%)	20.62		43.08		0.96		16.61	

Table 6
Preference Parameters and Moments for Alternative Consumption-based Stochastic Discount Factors with Lowest Pricing Errors by Sub-periods
Quarterly Data 1958:I-2006:IV

Preferences	*Periods*	β	γ	η	$E(M)$	$\sigma(M)$	*Pricing Errors*
Recursive-based SDF	*1958:I-1984:IV*	0.9975	21.8	0.23	0.9744	0.3996	0.0255
Recursive-based SDF	*1985:I-2006:IV*	0.97	29	0.25	0.9278	0.5955	0.0282
Recursive Long-based SDF	*1958:I-1984:IV*	0.915	0.35	0.41	0.9719	0.4056	0.0137
Recursive Long-based SDF	*1985:I-2004:IV*	0.92	8.5	0.33	0.9295	0.5984	0.0112

For each recursive-based SDF specification, we try a large grid of feasible preference parameters. We then select sets of combinations of those parameters that generate a volatility of the SDF, calculated with quarterly data, which lies above the HJ bound for the average level of interest rate for that period. The selected combinations should enter in the HJ space. Given the selected SDFs, we compute the pricing error at every quarter of each of these SDF in valuing the ten-size sorted portfolios. We finally calculate the MSE over time and across portfolios. The parameters reported correspond to selected SDF with the lowest MSE. We repeat the procedure for each sub-period. Hence, each parameter reported is estimated using only data from the particular sub-period. β is the subjective discount factor for future period utility; γ is the coefficient of relative risk aversion; η is the elasticity of intertemporal substitution, and *pricing error* is the mean squared error over ten size-sorted portfolios.

Table 7

Out-of-Sample Prediction of Future Macroeconomic and Stock Market Cycles with Volatility of the Recursive-based Stochastic Discount Factor with Quarterly Data 1989:I-2006:IV[1]

$$\frac{V_{t+\tau} - V_t}{V_t} = \alpha + \beta\sigma\left(M_t\right) + \varepsilon_{t+\tau},$$

V is either industrial Production Index, Gross Domestic Product, or Market Portfolio Returns

Forecasting Horizon	*Regression Estimates*	*Industrial Production Index*		*Gross Domestic Product*		*Stock Market Portfolio Returns*	
1 Quarter	*Constant*	0.031	(6.57)	0.011	(4.28)	0.095	(2.40)
	Slope	-0.060	(-4.44)	-0.018	(-2.47)	-0.183	(-1.67)
	R^2 (%)	18.08		7.31		3.28	
2 Quarters	*Constant*	0.062	(7.10)	0.022	(4.80)	0.198	(2.69)
	Slope	-0.117	(-4.69)	-0.035	(-2.75)	-0.390	(-1.89)
	R^2 (%)	21.55		11.10		8.17	
4 Quarters	*Constant*	0.121	(7.72)	0.042	(4.79)	0.408	(3.54)
	Slope	-0.225	(-5.11)	-0.062	(-2.70)	-0.808	(-2.44)
	R^2 (%)	27.47		12.81		15.61	
8 Quarters	*Constant*	0.215	(7.19)	0.078	(4.68)	0.762	(5.30)
	Slope	-0.375	(-4.77)	-0.109	(-2.53)	-1.440	(-3.43)
	R^2 (%)	27.03		15.63		21.62	
12 Quarters	*Constant*	0.262	(4.84)	0.106	(4.72)	0.918	(3.16)
	Slope	-0.395	(-2.93)	-0.129	(-2.27)	-1.540	(-2.19)
	R^2 (%)	16.42		15.05		11.67	

1/ The SDF employed to calculate the volatilities used in the regressions enter in the feasible HJ space and have the lowest pricing errors in valuing the ten size-sorted portfolios. The corresponding preference parameters are estimated with data using data from 1958:I to 1988:IV. The volatilities are estimated as overlapping standard deviations of five years of past data. They are assumed to remain constant over the forecasting period. All regressions only contain information available up to quarter *t*.

2/ All panels report OLS autocorrelation-robust standard-error regressions.

Figure 1
Hansen-Jagannathan Volatility Bound by Overlapping Five-year Sub-periods
1958-2006

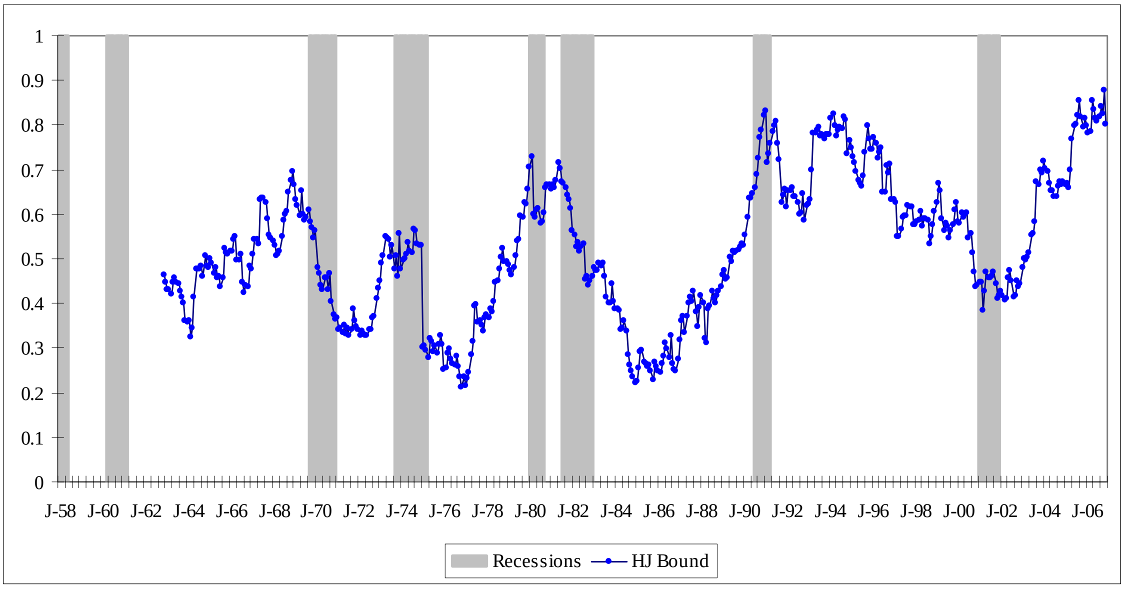
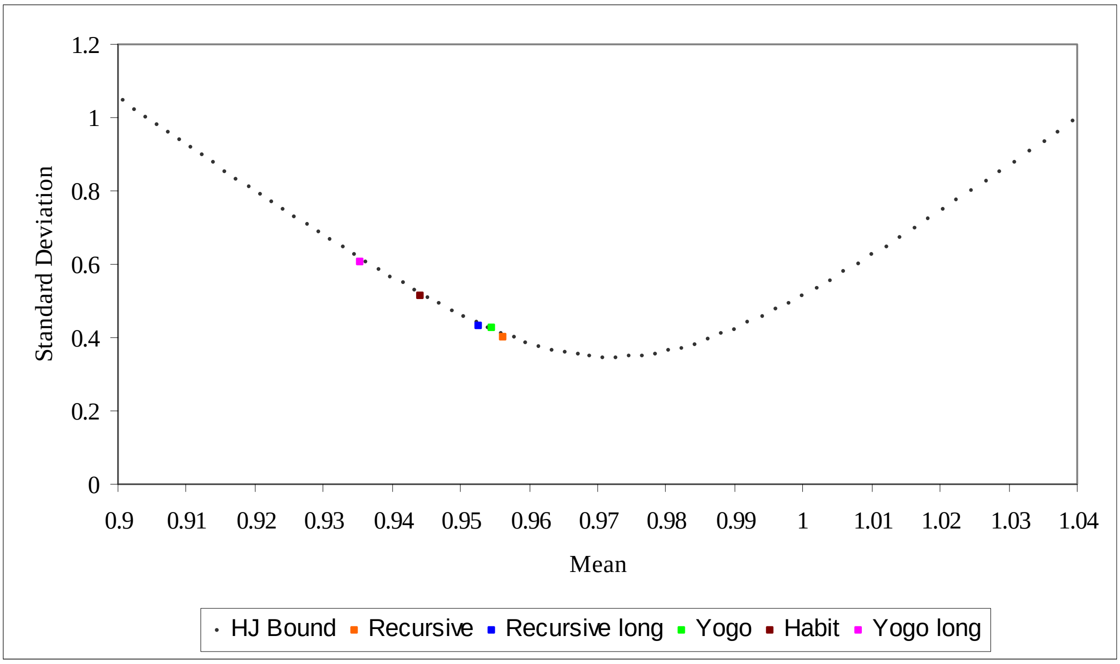


Figure 2

Panel A. Hansen-Jagannathan Volatility Bound and Consumption-based Stochastic Discount Factors with Lowest Pricing Errors
Quarterly Data 1958:I-2006:4



Panel B. Hansen-Jagannathan Volatility Bound and Consumption-based Stochastic Discount Factors with Lowest Pricing Errors
Monthly Data 1964:1-2006:12

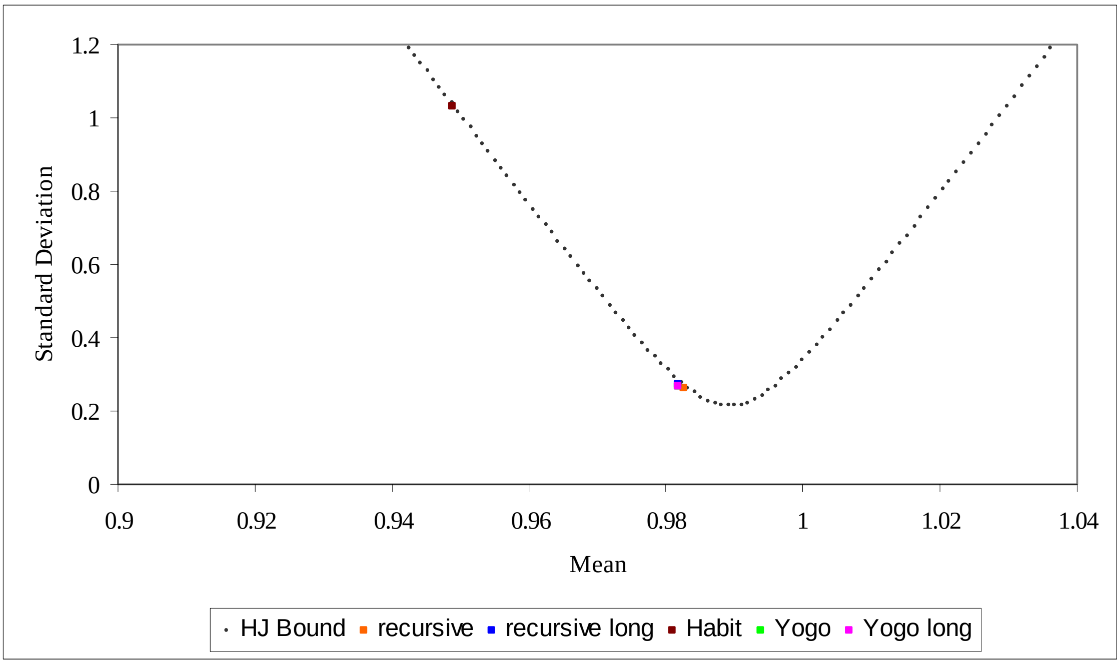


Figure 3

Panel A. Volatilities of Consumption-based Stochastic Discount Factors for Recursive, Yogo and Habit Preference Specifications

Quarterly Data 1965:II-2006:IV

Panel B. Volatilities of Consumption-based Stochastic Discount Factors for Recursive and Yogo Preference Specifications with Long Run Consumption Growth

Quarterly Data 1965:II-2006:IV