Forecasting Volatility movements using Markov Switching Regimes

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Abstract

This paper uses Markov switching models to capture volatility dynamics in exchange rates and to evaluate their forecasting ability. We identify that increased volatilities in four euro-based exchange rates are due to underlying structural changes. Also, we find that currencies are closely related to each other, especially in high volatility periods, where cross-correlations increase significantly.Using Markov switching Monte Carlo approach we provide evidence in favour of Markov switching models, rejecting random walk hypothesis. Testing in and out-of-sample Markov trading rules we find that using econometric methodology is able to forecast accurately exchange rate movements. When applied to the Euro / U.S. Dollar and the Euro / British Pound daily returns data, the model provides exceptional out-of-sample returns. However, when applied to the Euro / Brazilian Real and the Euro / Mexican Peso, the model looses power. Higher volatility exercised in the Latin American currencies seems to be a critical factor for this failure.

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1. Introduction

In recent years, researchers and professionals make extensive use of Markov switching models to predict future currency movements and to capture volatility dynamics. This is mainly due to the excellent predictive power of trend following trading rules associated with univariate time series analysis and nonlinearities.

The most popular models for time varying volatility are the integrated GARCH and the Markov switching regimes. GARCH models used in practical applications imply a very high level of persistence in volatility. However, if the data stem from stationary processes that differ in their parameters then, structural breaks can account for a part of the high persistence. Moreover, Markov switching models allow us to identify separate joint normal distributions for the exchange rates of these countries for periods in which the parameters of these distributions are significantly different. Thus, the evaluation of both Markov switching and GARCH models appear to have important econometric and financial implications. Also, we test a Markov switching model with endogenous changes in the parameters. This approach was used in the seminal article by Hamilton (1989).

A primary objective of this research is to generate better forecasts than a naïve random – walk specification. Also, we identify whether asymmetric volatility reported in certain countries are due to an underlying structural change. Finally, this paper provides a solid underpinning of the presence of Markov switching dynamics in exchange rate data.

In this paper we use (i) Markov switching model to capture time – varying volatility dynamics; (ii) Markov switching dynamics in conjunction with Monte Carlo approach to test against random walk in currency movements; (iii) Markov switching model, developed by Dueker and Neely (2007) in order to test ex ante trading rules in the foreign exchange market.

This study contributes to the related literature in that (i) examines the forecasting performance of Markov Switching term structure models of exchange rates; (ii) identifies the dynamics of asymmetric volatilities in four euro based exchange rates; (iii) extends the work of Dueker and Neely (2007) who create and test Markov switching trading model for four dollar based currencies.

The structure of the paper is organized as follows: Section 2 provides a brief literature review. Section 3 analyzes methodological issues. Section 4 presents the data. The empirical results are reported in Section 5. The final section contains the concluding remarks.

2. Literature Review

The vast majority of the empirical literature on forecasting exchange rates has centered on forecasting the level of nominal exchange rates. This literature is highly influenced by the seminal work of Meese and Rogoff (1983a,b), who first reported that empirical exchange rate models, based on conventional macroeconomic fundamentals suggested by international macroeconomics theory, cannot outperform a simple no-change or random walk forecast of exchange rates in terms of standard measures of point forecast accuracy. In the same way, Meese and Rose (1991), Cheung (1993), Chinn and Meese (1995) and Neely and Sarno (2002) demonstrated the inability of structural exchange rate models to generate better forecasts than a naïve random walk specification.

The fact that macroeconomic fundamentals do not offer accurate forecast for exchange rates, random walk specifications gained importance in exchange rate movements. However, recent attempt with Markov switching models appear to yield some encouraging results. Engel and Hamilton (1990), Dewachter (2001), Clarida et al. (2003), and Dueker and Neely (2007) advocated using Markov switching models that allow the exchange rate dynamics to alternate between regimes. They found that Markov switching regimes models perform well for both in-sample and out-of-sample periods.

Particularly, Dewachter (2001) constructs a Markov switching model that is able to produce simulated weekly data on which trend-following trading rules were successful. On the other hand, Clarida et al. (2003) generated an exchange rate forecasting model from a three regime switching vector error correction model (VECM) using weekly term structure data on forward exchange rates. The model outperformed a random walk in terms of out-of-sample forecasting. Also, Dueker and Neely (2007) found that a portfolio of Markov and conventional technical rules has better risk-adjusted performance than either individually.

However, Marsh (2000) showed that Markov switching models for exchange rates are unstable over time and unsuitable for forecasting. Also, Dacco and Satschell (1999) reported that the forecast performance of Markov switching models is very sensitive to misclassification of regimes.

Hence, in this study we firstly examine the time – varying volatility dynamics, then we test a Markov Switching Monte Carlo model which has been tested only once from Cheung and Erlandsson (2005) without finding evidence in favour of Markov switching models. Finally, we re-examine the forecasting power of the Markov switching trading model developed by Dueker and Neely (2007).

3. Methodology

3.1 Random walk vs. Markov Switching

A Markov switching model is a non-linear specification in which different states of the world affect the evolution of a time series. In a Markov switching model the observed change in a variable between period *t* and *t+1* is assumed to be a random draw from one of two distributions. The appropriate distribution is observed by the state variable s_t. Hence, when $s_t = 1$, the observed change, y_t, is a random draw from a $N(\mu_1, \sigma_1^2)$ distribution. On the other hand, when $s_t = 2$, y_t, is a random draw from a $N(\mu_2, \sigma_2^2)$ distribution. Accordingly, the state variable is assumed to evolve based on a Markov chain, so that the probability of being in state 1 at time *t* when state 1 obtained at time *t-1,* is p_{11}. Accordingly, the probability of being in state 2 is p_{22}. So, mathematically we have:

$$\begin{aligned} &p(s_t = 1 \mid s_{t-1} = 1) = p_{11} \\ &p(s_t = 2 \mid s_{t-1} = 1) = 1 - p_{11} \\ &p(s_t = 1 \mid s_{t-1} = 2) = 1 - p_{22} \\ &p(s_t = 2 \mid s_{t-1} = 2) = p_{22} \end{aligned} \tag{1}$$

In a transition matrix, P, we have:

$$P = \begin{pmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{pmatrix} \tag{2}$$

Hamilton (1994) shows that in the case of N=2 (i.e. two regimes) the $p(s_t = j)$ state probabilities can be expressed from the transition probabilities with the following form:

$$\pi_1 = \frac{1 - p_{11}}{2 - p_{11} - p_{22}} \tag{3}$$

and

$$\pi_1 = \frac{1 - p_{22}}{2 - p_{11} - p_{22}} \tag{4}$$

Then, collecting the population parameter into a vector $\theta = (\mu_1, \mu_2, \sigma_1, \sigma_2, \pi_1, \pi_2)'$, we observe the unconditional density of y_t from the following equation:

$$f\left(y_t;\theta\right) = \frac{\pi_1}{\sqrt{2\pi\sigma_1^2}} \exp\left(\frac{-(y_t - \mu_1)^2}{2\sigma_1^2}\right) + \frac{\pi_2}{\sqrt{2\pi\sigma_2^2}} \exp\left(\frac{-(y_t - \mu_2)^2}{2\sigma_2^2}\right) \tag{5}$$

From equation (5) is clear that the Markov switching model is a mixture of normal distribution. We assume that foreign exchange rate changes are not autocorrelated, and the conditional volatilities are time-independent.

The random walk with drift specification is given by:

$$\Delta s_t = \mu + \varepsilon_t \tag{6}$$

where Δ is the first – difference operator, s_t is the log exchange rate at time *t*, μ is the drift term, and $\varepsilon_t \sim N(0,\sigma^2)$ is an error term. Then, the two-regime Markov switching model can be written as:

$$\Delta s_t = \sum_{i=1,2} I(S_t = i)[\mu_i + \varepsilon_{it}] \tag{7}$$

where $I(\bullet)$ is the indicator function, $\mu_1 \neq \mu_2$ are the drift terms across regimes 1 and 2, $\varepsilon_{it} \sim N(0,\sigma_i^2)$ is the regime specific error term, and S_t is the regime variable. Equation (7) will be used in order to test the Markov switching model under the alternative (i.e. random walk).We estimate the coefficients of Markov switching model with the fully unconstrained maximum likelihood method, similar to Engel and Hamilton (1990) and Cheung and Erlandsson (2005). Also, we use Monte Carlo simulation to test for the number of regimes. We produce 400 randomized starting values for each interaction in the estimation. Although simulation may not offer general conclusions, it circumvents the issues of nonstandard statistical inferences inherent in regime switching modeling and provides some useful sample-specific results. Also, the use of data-specific distributions helps mitigate finite-sample biases. Using the Monte Carlo approach we derive the empirical distribution of the likelihood ratio statistic. For each exchange rate in our sample we tested H0 (number of regimes is 1, i.e. n=1) against H1 (number of regimes is 2, i.e. n=2). In order to test for those hypotheses, we assume that θ_n and θ_{n+1} are the parameter vectors with $\hat{\theta}_n$ and $\hat{\theta}_{n+1}$ the Maximum Likelihood Estimators (MLEs). Then, following the approach of Cheung and Erlandsson (2005) we search for the number of regimes which will provide us with sufficient evidences if the Markov switching dynamics generate better forecasts from a naïve random walk specification.

3.2 Markov Switching trading model

We propose a regime switching model based on Hamilton (1989) and Dueker and Neely (2007). Following Deuker and Neely, at this stage we introduce some notation. The exchange rate at date *t* (euro per unit of foreign currency) is given by E_t, while r_t is the log of the deviation from uncovered interest parity, and the domestic (foreign) overnight interest rate is $i_t(i^*_t)$.

So:

$$r_{t+} = \ln S_{t+1} - \ln S_t + \ln(1+i^*_t) - \ln(1+i_t). \qquad (8)$$

We allow the conditional mean to be a function of three distinct Markov switching state variables. We assume a student –t error distribution with n_t degrees of freedom in the dependent variable r:

$$r_t = \mu_t + \varepsilon_t,\ \varepsilon_t \sim \text{student} - t(\text{mean} = 0, n_t, h_t),\ n_t > 2 \qquad (9)$$

while the variance of this distribution is:

$$\sigma_i^2 = h_t n_t/(n_t - 2). \qquad (10)$$

The parameter h_t is a scale parameter for the variance such that $(r_t - \mu_t)/h_t^{1/2}$ is a standard student –t variable with n_t degrees of freedom. h_t switches between high and low states, based on the realization of a binary variable, $S1_t$, governed by the following first – order Markov process:

$$h_t = h_0 S1_t + h_1(1-S1_t),$$

$$S1_t \in \{0,1\}, P(S1_t = 0 \mid S1_{t-1}=0) = p_1, P(S1_t = 1 \mid S1_{t-1}=1) = q_1 \qquad (11)$$

where h is the dispersion parameter.

Similarly with Duecker (1997) and Dueker and Neely (2007) we allow for switching in the degrees of freedom, n_t, so that the thickness of the tails of the conditional distribution varies across time.

The second binary variable that follows a Markov process is the kurtosis parameter,$S2_t$. The Markov process is in this case:

$$n_t = n_0 S2_t + n_t\,(1-S2_t)$$

$$S2_t \in \{0,1\}, P(S2_t = 0 \mid S2_{t-1}=0) = p_2, P(S2_t = 1 \mid S2_{t-1}=1) = q_2 \qquad (12)$$

Finally, the third switching binary variable, $S3_t$, provides another independent source of shifts in the expected return:

$$\mu_t = \mu_0 + \mu_1 S1_t + \mu_2\, S2_t + \mu_3 S3_t,$$

$$S3_t \in \{0,1\}, P(S3_t = 0 \mid S3_{t-1}=0) = p_3, P(S3_t = 1 \mid S3_{t-1}=1) = q_3 \qquad (13)$$

where μ_1 and μ_2 reflect how the dispersion and kurtosis affect the mean return.

4. Data

Five exchange rates are examined; the Euro (EUR), the American dollar (USD), the pound sterling (GBP), the Brazilian Real (BRL), and the Mexican Peso (MXN), all quoted as foreign currency per unit of Euro. We use daily closing prices from Bloomberg database for the period 3 January 2000 through 2 March 2007. We remove weekends and holiday values in our observations.

5. Empirical Results

5.1 Volatility Dynamics

Our results show that there are two regimes. We interpret the estimated regimes as high and low volatility periods. The two regime switching model provides evidence that the volatilities were not constant over time and they changes simultaneously across the five countries. That is, some of these currencies are closely related to each other, especially in the high volatility periods, where cross-correlations increase significantly. Although these currencies did not necessarily and consistently moved into the same direction in the high – volatility periods, we could detect a consistent and simultaneous increase in their volatilities (Table 1).

Insert Table 1

In addition, from Table 1 we identify that volatilities are significantly different in the two regimes and that certain cross-correlations increase significantly in the high – volatility period. State probability estimates present information for the historical frequency of each regime, whilst the transition probabilities reflect regime persistence (Table 1). The estimated probability of having a high-volatility day was 25.6%.

Insert Figure 1

On the other hand, the transition probabilities in the same table show that the probability of staying in the low volatility regime was much higher at the 88.4%. In other words, the probability of switching to the high volatility regime was 11.6%, while the probability of staying in the high volatility regime, at 61.4%, was lower than for the low-volatility regime.

5.2 Random Walk against Markov Switching

Testing the random walk against the Markov switching model we find that the random walk is rejected for the euro, pound sterling and American dollar exchange rates, in favour of the Markov switching model, while is not rejected for the Brazilian Real and the Mexican Peso. The sample likelihood ratio statistics are given in the first column of Table 2. On the other hand, the Monte Carlo likelihood ratio test provides supportive evidence of Markov switching dynamics. Particularly, the Monte Carlo approach provides evidence that the random walk is rejected for all exchange rates.

Insert Table 2

In Table 2 we report the empirical distributions and the p value. All currencies exercise larger means than medians. Also, all currencies share positively skewed distributions, implying that they have a long tail to the right. According to the p values for the Pound Sterling (GBP) and the American dollar (USD) the random walk null is rejected in favour of the Markov switching model. Indeed, the p values for the three currencies are lower than 5%. However, the p value for the Brazilian Real (BRL) and the Mexican Peso (MXN) are higher than 10% implying that there is no strong evidence to reject the random walk null in favour of the Markov switching model.

The rejection frequency under a 10% test is reported as the empirical power in the last column. The Monte Carlo –test procedure, reported as the empirical power in the last column of Table 2 show that less than 30% of the simulated Markov switching series produced by all currencies in our sample are rejected at the 10% level. This provides strong evidence against the random walk null hypothesis.

5.3 The Markov switching trading model

The Markov switching trading model created by Dueker and Neely (2007) is tested each day with a vector for filter sizes used to map the data to a trading decision. We have used the best filter sizes to maximise the in-sample excess return.

Insert Table 3

In Table 3 we report the trading rule statistics. The top rows show the filter sizes chosen on the basis of in-sample information. The rest of the table shows the annualised return, net of 10 basis point transaction costs in percentage, the *t* statistic for the annual return, mean trades per year, and the percentage of business days the trading rule was long in the foreign currencies (i.e. euro currency is domestic and all other currencies are foreign). The left side of the panel shows the in-sample results, the middle side shows the out of the sample results and the right side shows the three year subsample breakdown.

The out of sample excess returns are much lower than the in sample returns, ranging from -3.2 percent for the Brazilian Real to 7.4 percent for the American dollar. Using the trading rule for the American dollar and the British pound we achieve a 7.4 percent and 5.56 percent excess return. This is a strong out of sample figure. However, the power of the trading model is in doubt when used for the Brazilian Real (-3.2 percent return) and for the Mexican Peso (-3.07 percent). The model does not produce satisfactory out-of-sample forecasting performance for these currencies. This may be due to (i) the small database used in our tests; (ii) the high volatility exercised in these countries during 2000 – 2006 due to shocks in their economies. All the returns to the rules are statistically different from zero at any reasonable level with Newey – West standard errors calculated with a lag order of 3. Also, the rules trade 2.85 – 15.49 times per year in the out of sample period. The MXN is most often long in the foreign currency. On the other hand, the GBP is most often short in the foreign currency.

6. Conclusion

This paper has presented evidence that increased volatilities in four euro-based exchange rates are due to underlying structural changes. The increased volatility and cross-correlations reported in this study reflect portfolio – allocation decisions.

Moreover, this paper provides evidence that a Markov switching model constitutes a better forecast than a naïve random-walk specification. A Monte Carlo approach was adopted to circumvent the statistical inference problem inherent to the modeling of regime switching. Our simulation results provide sufficient evidence in favour of Markov switching models, rejecting the random walk hypothesis.

Finally, this study used a Markov switching model created by Duecker and Neely to create ex-ante trading rules for four Euro based foreign exchanges. The ability of the Markov trading rule to identify trends is higher for the USD and the GBP and very low for the BRL and the MXN. Thus, the use of econometric techniques enables an investor to earn excess returns in their portfolio.

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Table 1. Regime switching model estimation

Currencies	Regimes	2000 – 2007
	Conditional Daily Volatility Estimations (%)	
EUR/USD	Regime 1	0.63
	Regime 2	0.36
EUR/GBP	Regime 1	0.56
	Regime 2	0.31
EUR/BRL	Regime 1	0.51
	Regime 2	0.28
EUR/MXN	Regime 1	0.43
	Regime 2	0.21
	Conditional Daily Average Return Estimation (%)	
EUR/USD	Regime 1	0.11
	Regime 2	0.05
EUR/GBP	Regime 1	0.05
	Regime 2	-0.02
EUR/BRL	Regime 1	0.17
	Regime 2	0.06
EUR/MXN	Regime 1	0.20
	Regime 2	0.07
	State Probability and Transition Probability Estimations	
P	(Regime 1)	25.6%
P	(Regime 1 |Regime 1)	61.4%
P	(Regime 2 |Regime 2)	88.4%

Table 2. Testing Random Walk against Markov Switching

	p value	Mean	Median	SE	Skew	Max	Power
USD	0.042	4.766	4.585	3.274	0.859	19.002	0.283
GBP	0.048	4.581	4.429	3.890	0.733	16.514	0.259
BRL	0.137	4.202	4.175	3.455	0.882	15.782	0.265
MXN	0.190	4.194	4.126	3.627	0.960	16.225	0.220

Table 3. Markov trading rule statistics

	USD	GBP	BRL	MXN
Filter 1	-0.0286	-0.0131	-0.0189	-0.0196
Filter 2	0.0472	0.0378	0.0253	0.0278

	In-sample 2000-2003				Out-of-sample 2003-2006			
	USD	GBP	BRL	MXN	USD	GBP	BRL	MXN
Return	14.73	11.69	5.52	3.80	7.40	5.56	-3.20	-3.07
t-stat	2.83	2.66	4.79	5.94	2.27	2.18	4.60	4.65
Trades	2.87	1.64	8.61	10.19	4.58	2.85	11.02	15.49
% Long	48.76	51.63	78.49	84.62	54.96	49.05	61.33	64.21