Stale Prices and the Performance Evaluation of Mutual Funds

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Abstracts

Staleness in measured prices imparts a positive statistical bias and a negative dilution effect on mutual fund performance. First, evaluating performance with nonsynchronous data generates a spurious component of alpha. Second, stale prices create arbitrage opportunities for high-frequency traders whose trades dilute the portfolio returns and hence fund performance. Thus, this paper introduces a model that directly estimates these biases and evaluates fund performance net of these effects. Empirical tests of the model show that the statistical bias is small but the dilution effect is large and widespread in the fund industry. Overall, during the sample period, funds lose about 40 basis points in annual performance due to price staleness.

JEL Classifications: G12, G14.

Key Words: Performance evaluation, stale pricing, timing arbitrage, flows.

Typically, studies of mutual fund performance use monthly return data without controlling for the issue of stale pricing, an inequality between the current price of fund shares and the current value of the underlying assets. This discrepancy occurs when the share price set by funds at the end of each day fails to reflect the most current market information on these assets, because the underlying securities are thinly traded or traded in a different time zone. In funds with stale pricing, observed returns differ from true returns. Scholes and William (1977) show that when data are nonsynchronous, estimates of beta and alpha are biased and inconsistent. In addition, stale prices in mutual fund shares create arbitrage opportunities for short-term traders who purchase shares when the fund's net asset value (NAV) is lower than the value of the underlying securities and sell shares after the true value is incorporated into the NAV. The round trip transactions can be as quick as overnight. Chalmers, Edelen, and Kadlec (2001) call such opportunity a "wild card" option. Greene and Hodges (2002) and Bhargava, Bose, and Dubofsky (1998) provide evidence that arbitrageurs take advantage of such opportunities and their trades can dilute fund returns up to 0.5% annually.

This paper develops a performance evaluation model that not only controls for nonsynchronicity in fund data but also considers the arbitrage dilution due to stale pricing. By directly measuring the statistical bias and dilution effect, the model evaluates managerial ability net of these effects. In contrast, performance evaluated using various traditional models is negatively influenced by stale pricing. As shown in this paper, for each one standard deviation increase in stale pricing, traditionally evaluated performance decreases by about 40 basis points. Compared to an average expense ratio of 0.78% for the sample funds, this magnitude is economically important.

This study contributes to two strands of literature: nonsynchronous data in the application of pricing models and performance dilution due to stale pricing arbitrage. As regards the former, Scholes and William (1976) introduce a model that estimates the beta of nonsynchronously traded stocks. Getmansky, Lo, and Makarov (2004) introduce an evaluation model of return smoothing in hedge funds. This paper contributes to such research in two ways. First, it develops a model that estimates mutual fund performance with serially correlated returns. Second, it controls for arbitrage dilution of fund performance. As regards to the second strand of literature, whereas the extant studies on stale pricing and arbitrage dilution draws inferences from the cross-sectional relation between flow and performance, analysis in this paper endogenizes the flow process by modeling the arbitrage timer's response to stale pricing. This endogenization allows estimation of the dilution effect and managerial ability simultaneously and directly from the performance evaluation model.

The model also delivers a method of fund performance decomposition, which is especially interesting for performance attribution studies and applications in the fund industry. The difference between the observed alpha and the true alpha is the sum of a statistical bias and the arbitrage dilution. According to the empirical evidence, the statistical bias is positive but has little economic significance, while the dilution effect is negative and significant at the 1% level for most fund style groups. Furthermore, this dilution effect increases with the fund stale pricing, with funds in the top quintile having an average dilution of 34 basis points annually and those in the lowest quintile, a dilution of only 7 basis points.

The rest of the paper proceeds as follows. Section I introduces the model, Section II describes the data, and Section III presents the empirical results. Section IV then compares the alpha from the proposed model with those from various traditional models, and Section V presents robustness checks and discusses possible extensions of the model. Section VI concludes the paper.

I. The model

The model is defined in terms of stale pricing, the impact of fund flows, and the crucial element of market timing. The literature refers the term of market timing to two types of activity: in one, fund managers increase fund beta by changing portfolio holdings when they expect the market to go up (hereafter, market timing); in the other, daily timers trade in and out of funds frequently (arbitrage timing). This paper focuses only on arbitrage timing and refers to the arbitragers as daily timers.

A. Stale pricing

To address the return process with zero flows, the analysis assumes that the true return of the underlying assets follows a market model. However, nonsynchronous trading (Sholes and William 1977, Lo and MacKinlay 1990) and naïve methods for determining the fair market value or "marks" for underlying assets (Chalmers, Edelen, and Kadlec 2001, Getmansky, Lo, and Makarov 2004) yield serially correlated observed returns of the fund. Thus, assuming that information generated in time t is not fully incorporated into prices until one period later,[1] the observed return becomes a weighted average of true returns in the current and last periods:

$$r_t = \alpha + \beta\, r_{mt} + \varepsilon_t \tag{1a}$$

$$r_t^* = \eta\, r_{t-1} + (1-\eta)\, r_t \tag{1b}$$

where r_t denotes the true excess return of the portfolio and r_{mt} the excess market return. Both r_t and r_{mt} are i.i.d, and the error term ε_t is independent of r_{mt}. $r_{i,t}^*$ is the observed excess return of the portfolio with zero flows, while η is the weight on the lagged true return. That is, the higher the η, the more the staleness in the prices.

This form can be extended to a variety of more complicated models; for example, to estimate market timing ability by assuming $cov(r_t, r_{mt}^2)$ is nonzero or to incorporate time-variant stale pricing by specifying the form taken by the parameter η.[2] In this paper, for simplicity, η is assumed to be constant over time.

B. The arbitrage timer's decision

When share prices are stale, the future return or a part of the future NAV of the fund is predictable. This predictability, combined with the option to trade at the NAV, allows active traders to capture price swings without bearing any risk when moving in and out of funds quickly. Nonetheless, the timer faces a choice between fund shares with return R_p and risk-free assets with return R_f, where R_p is the sum of r^* (the excess return of fund shares) and r_f. The timer maximizes a utility function that depends on the expected excess return conditional on observation of past fund returns:

[1] This assumption can be verified empirically. In this paper, I apply Getmanky, Lo, and Makorov (2004) to mutual fund data and show that, on average, fund returns smooth over the current and one previous period (table IV-a).

[2] Chen, Ferson, and Peters (2005) show that systematic stale pricing can cause a spurious component in timing measurement and the mean of observed fund returns.

$$U(x) = E\{U([xR_p+(1-x)R_f] - [hR_p+(1-h)R_f]) \,|I_{t-1}^o)\tag{2a}$$

where x is the weight on fund shares in period t, and h is an exogenous benchmark preference parameter. I_{t-1}^o includes all the information in past returns $\{r_{t-1}^o, r_{t-2}^o, --- r_1^o\}$, where r_{t-1}^o is the observed return in period $t-1$.

The optimal weights on fund shares are given in equation (2b):

$$x = h + \gamma^{-1} E(r_t^*|\, I_{t-1}^o) / Var(r_t^*|\, I_{t-1}^o)\tag{2b}$$

where γ is the Rubinstein (1973) measure of risk aversion, which is assumed here to be constant. The details of obtaining the weights are presented in appendix A1.

From equations (1a) and (1b), it can be shown that $E(r_t^*|\, I_{t-1}^o) = \eta\, r_{t-1} + (1-\eta)\,\mu$ *and* $Var(r_t^*|\, I_{t-1}^o) = (1-\eta)^2\sigma^2$, where r_t^* is the fund excess return with zero flows. The extent of stale pricing, η, can be inferred by observing all past returns. Therefore, at the end of $t-1$, the timer adjusts the position in fund shares by

$$\Delta x_{t-1} = x_t - x_{t-1} = \eta\,(r_{t-1}-r_{t-2}) / \gamma(1-\eta)^2\sigma^2\tag{2c}$$

To assess the fund's long-term flows, the model assumes that C_{t-1} is long-term flow occurring at the end of period $t-1$. This flow responds to past long-run returns and is uncorrelated with stale pricing or short-term return r_{t-1}. For simplicity, it follows an N(c, σ_c), i.i.d distribution:

$$C_{t-1} \sim N(c, \sigma_c), \text{ i.i.d.}\tag{2e}$$

Therefore, the fund flow at the end of period $t-1$ is

$$f_{t-1} = C_{t-1} + \eta\,(r_{t-1}-r_{t-2}) / \lambda(1-\eta)^2\sigma^2\tag{2d}$$

$$\lambda = \gamma^* \, TNA_{fund,\,t} / Assets_{timer,\,t}\tag{2e}$$

The difference between λ and γ results from the measurement of flows. Δx_{t-1} represents a percentage of the timer's total assets, while f_{t-1} represents a percentage of the fund's TNA.

C. The impact of fund flows

As Greene and Hodge (2002) show, daily flows by active mutual fund traders cause a dilution of returns, which results from the lag between the time that money comes in and the time that fund managers purchase risky assets. Edelen and Warner (2001) note that fund managers typically do not receive a report of the day's fund flow until the morning of the next trading day, by which time the prices of underlying risky assets have changed. For simplicity, this model assumes that the fund manager reacts immediately on seeing the flows so that the response lags by only one day.

The flow occurring at the end of the previous trading day is $f_{t-1}= CF_{t-1} /(N_{t-1} P_{t-1})$, where CF_{t-1} is the dollar amount of flows, N_{t-1} is the number of shares, and P_{t-1} is the observed share price, $P_{t-1} = \text{TNA}_{t-1}/N_{t-1}$. As noted earlier, r_t^* is the observed excess portfolio return with zero flows. Because of the lag in new money flow investment, the new money flow overnight leads only to a risk-free rate return:

$$TNA_t = TNA_{t-1}(1+ r_t^*+R_f) + CF_{t-1} (1+R_f) \tag{3a}$$

and $$N_t = N_{t-1}+CF_{t-1}/P_{t-1}. \tag{3b}$$

The observed excess return with flows is denoted by r_t^0:

$$r_t^0 = (P_t - P_{t-1})/P_{t-1} - R_f \tag{3c}$$

Through the detailed derivation given in appendix A2, it can be shown that r_t^0 differs from r_t^*:

$$r_t^0 = r_t^* / (1 + f_{t-1}) \quad (3d)$$

D. The system

The system is formulated using the following equations derived from equations (1a), (1b), (2d), (2e), and (3d), which define the model. The details are given in appendix A3.

$$E[\, r_t^o\, (1+f_{t-1})] = \mu_p \quad (4a)$$

$$Cov[r_t^o\, (1+f_{t-1}),\ r_{m,t}\,] = (1-\eta)\ Cov(r,r_m) \quad (4b)$$

$$Cov[r_t^o\, (1+f_{t-1}),\ r_{m,t-1}\,] = \eta\ Cov(r,r_m) \quad (4c)$$

$$Var[r_t^o\ (1+f_{t-1]})] = [\eta^2 + (1-\eta)^2]\ \sigma_p^2 \quad (4d)$$

$$E(f_t) = c \quad (4e)$$

$$\mathrm{Cov}(f_{t-1,}\ f_t\,) = -\ \eta^2 / \lambda^2 (1-\eta)^4 \sigma_p^2 \quad (4f)$$

Combining these equations with

$$E(r_m) = \mu_m \quad (4g)$$

$$Var(r_m) = \sigma_m \quad (4h)$$

produces a system with eight parameters $[\mu_m, \sigma_m^2, \mu_p, Cov(r,r_m), \sigma_p^2, \eta, c, \lambda]$, by which fund performance can be estimated.

E. Components of performance and biases

In this paper, the true alpha or true performance is defined as the alpha estimated with true returns. The *true alpha* from the above model is

$$\alpha = E(r_t) - Cov(\, r_t, r_{m,t})\ \mu_m/\sigma_m \quad (5a)$$

which measures the “true” picking ability of fund managers without influences of stale pricing and arbitrage timings.

In contrast, the observed alpha or observed performance is defined as the alpha estimated by treating the observed returns as the true returns: $\alpha^o = E(r_t^o) - Cov(r_t^o, r_{m,t}) \mu_m/\sigma_m^2$. Using equations (1a), (1b), (2d), and (3d), it can then be shown that $E(r_t^o) = [\mu_p - cov(f_{t-1}, r_t^o)]/(1+c)$ and $Cov(r_t^o, r_{m,t}) = [(1-\eta) Cov(r, r_m) - Cov(r_t^o\ r_{m,t}, f_{t-1}) + cov(f_{t-1}, r_t^o) \mu_m]/(1+c)$. Again, the detailed derivation is included in appendix A4. Therefore, the *observed alpha* equals

$$\alpha^o = [\mu_p - cov(f_{t-1}, r_t^o)]/(1+c)$$
$$-\{[(1-\eta)Cov(r,r_m) - Cov(r_t^o\ r_{m,t}, f_{t-1}) + cov(f_{t-1}, r_t^o)\ \mu_m]/(1+c)\}\ \mu_m/\sigma_m^2 \qquad (5b)$$

In an ideal situation, one in which there is neither stale pricing nor flows—that is, $\eta=0$, $c=0$, and $cov(f_{t-1}, r_t^o)=0$, $Cov(r_t^o\ r_{m,t}, f_{t-1})=0$—the observed performance is a consistent estimator of the true performance; that is, $\alpha^o = \alpha$.

When there is stale pricing but zero flows—that is, $c=0$ and $f_{t-1}=0$— the observed alpha becomes $\alpha' = \mu_p - (1-\eta)Cov(r,r_m)\ \mu_m/\sigma_m^2$. The difference between α' and α is the *spurious bias* due to stale pricing.

$$\Delta\alpha_1 = \eta\ Cov(r,r_m)\ \mu_m/\sigma_m^2 \qquad (6a)$$

When there are long-term flows but no arbitrage flows—that is, c is nonzero but $cov(f_{t-1}, r_t^o)=0$ and $Cov(r_t^o\ r_{m,t}, f_{t-1})=0$—the observed alpha becomes $\alpha'' = (1/1+c)\ [\mu_p - (1-\eta)Cov(r, r_m)\ \mu_m/\sigma_m^2]$. The difference between α" and α' is the *dilution effect of the long-term flows*.

$$\Delta\alpha_2 = (-c/1+c)\ [\mu_p - (1-\eta)\ Cov(r,r_m)\ \ \mu_m/\sigma_m^2] \qquad (6b)$$

If c is completely predictable, fund managers can avoid this dilution through certain cash budget arrangements; however, as shown by Edelen (1999), the unexpected flows will dilute fund performance. Specifically, when c is zero, this bias is zero.

Finally, if there are arbitrage flows responding to the expected short-lived returns and correlated with fund returns—that is, $cov(f_{t-1}, r_t^o)$ and $cov(r_t^o\ r_{m,t},\ f_{t-1})$ are nonzero—the observed alpha now becomes (5b). The difference between α^o and α" is the *dilution effect of the arbitrage flows*.

$$\Delta\alpha_3 = - [1/(1+c)] \{ cov(f_{t-1}, r_t^o) - [Cov(r_t^o\ r_{m,t},\ f_{t-1}) - cov(f_{t-1}, r_t^o\ )\ \mu_m]\ \ \mu_m/_m{}^2\} \qquad (6c)$$

The higher the correlation between arbitrage flows and fund returns, the larger the bias (downward). Flows also reduce performance through a decrease in the portfolio beta—that is, $Cov(r_t^o\ r_{m,t},\ f_{t-1}) < 0$.

The relation among observed performance, true performance, and these biases is $\alpha^o = \alpha + \Delta\alpha_1 + \Delta\alpha_2 + \Delta\alpha_3$, where α^o is the observed performance; α, the true performance; $\Delta\alpha_1$, the spurious bias; $\Delta\alpha_2$, the dilution of long-term flows; and $\Delta\alpha_3$, the dilution of arbitrage flows.

II. Data

Both Lipper and Trimtab currently have datasets that claim to record daily fund flows. However, it is widely recognized that, whereas daily flow data should give a time series of daily mutual fund TNAs from which daily flows or the exact flows on a certain day can be computed; in fact, mutual funds have no accurate day-end TNA figure because they do not know how much flow has been received on the current day. Therefore, some funds report TNA on day *t* including the current day's flows, some report it excluding day *t* flows, and some report a mixture that includes part but not all of day *t* flows. In addition, funds may report one way one day and another, the next. In other words, as argued by Edelen and Warner (2001) and Greene and Hodges (2002), true daily flows may sometimes be lagged by one day, without it being clear when or for which funds. Furthermore, there is no way to check the daily data against another source because all fund databases suffer from this problem. The SEC's NSARs are required to include the current day's flow in day *t* TNA, but because they aggregate all share classes, this also cannot be merged with the other data. As a result, it is currently unfeasible to calculate the true day-by-day flows.

A second possibility is monthly return and flow data. In practice, all flows could be invested the next day, meaning that the actual amount that dilutes the monthly return is the aggregation of the dilution of days within the month. Modifying the monthly flow to a daily average flow for the month enables application of equation (3d) and gives a dilution measure of the monthly returns (see appendix A5 for details of this aggregation). Furthermore, monthly returns are favorable in the application of performance evaluation models.

Funds without monthly return and TNA are therefore excluded from the dataset, producing a final sample of 7,246 funds (including different classes) from the CRSP Mutual Fund Database from January 1961 to December 2004. These funds are then sorted based on their objective codes and claimed investment strategy into the following eight style groups: small company growth, other aggressive growth, maximum capital gain, growth funds, income funds, growth and income, sector funds, and timing funds.[3]

(Insert table I here.)

[3] Denoting the objective codes from Wiesenberger as OBJ, those from ICDI as ICDI, and those from Strategic Insight as SI, the styles are classified as following: small company growth funds = OBJ SCG or SI SCG; other aggressive growth funds = OBJ AGG, ICDI AG or AGG, or SI AGG; growth funds = OBJ G, G-S, S-G, GRO or LTG or ICDI LG or GRO; income funds = OBJ I, I-S, IEQ, or ING or ICDI IN or ING; growth and income funds = OBJ GCI, G-I, G-I-S, G-S-I, I-G, I-G-S, I-S-G, S-G-I, S-I-G, or GRI or ICDI GI or GRI; maximum capital gains funds = OBJ MCG; sector funds = OBJ ENR, FIN, HLT, TCH, or UTL or ICDI SF, UT, ENV, FIN, HLT, TEC, UTI, RLE, NTR, or SEC; and timing funds = OBJ BAL, ICDI BL, or SI BAL.

Panel A of table I summarizes the fund returns by style group. During the 1961–2004 period, the maximum capital gain funds show the highest mean in returns (1.28% per month), while the timing funds show the lowest mean in returns (0.76% per month). Fund flow, computed using fund TNA and returns, is the percentage change of total net assets after fund returns are controlled for:

$$Flow_{i,t} = (TNA_{i,t} - TNA_{i,t-1} * (1+R_{i,t}))/TNA_{i,t-1} \quad (7)$$

Panel B of table I summarizes fund flow observation. The flow observations start in year 1991 because the database does not report monthly TNA until 1991. In addition, because of investment code changes, the maximum capital gain funds (MCG) category ceases to exist after 1992, hence the MCGs have only one year of flow observations. Among the other styles, the small company growth group (SCG) exhibits more flows than any other group (1.17% of monthly TNA), while the growth and income group exhibits the least (0.16% of monthly TNA). Fund flows are persistent with a first-order autocorrelation around 0.5.

The analysis in Section IV compares the proposed performance evaluation model with traditional models using the following 11 economy instruments as conditional variables (Ferson and Schadt 1996): short-term interest rate, term structure slope, term structure concavity, interest rate volatility, stick market volatility, credit spread, dividend yield, inflation, industrial output growth, short-term corporate illiquidity, and stock market liquidity.[4] The returns of the following eight asset classes are also used to

[4] These variables take the same definitions as in Ferson and Qian (2004). The short-term interest rate = the bid yield to maturity on a 90-day treasury bill; term structure slope = the difference between and five-year and a three-month discount treasury yield; term structure concavity = y_3 - (y_1 + y_5)/2, where y_j is the j-year fixed maturity yield from the Federal Reserve; interest rate volatility = the monthly standard deviation of three-month treasury rates, computed from the days within the month. Stock market volatility = the monthly standard deviation of daily returns for the Standard and Poor's 500 index within the month;

construct style index returns (Sharpe 1988, 1992):[5] a 90-day treasury bill, a one-year treasury bond, a 10-year treasury bond, a BAA corporate bond, a broad equity index, value stocks, growth stocks, and small-cap stocks. Added to these are the four factors by of Carhart (1997) and the liquidity factor of Pastor Stambaugh (2003). All data on these variables are obtained from the WRDS.

III. Empirical results

Overall, the empirical results indicate that the dilution effect is large, but the statistical bias due to stale pricing is small, and the true performance is free of biases due to stale pricing.

A. Moment conditions and estimations

The system (4a)–(4h) can be written into the following moment conditions for model estimation:

$$g_{1t} = r_t^o \ (1+f_{t-1}) - \mu_p \tag{8a}$$

$$g_{2t} = [r_t^o \ (1+f_{t-1}) - \mu_p]' \ (r_{m,t} - \mu_m) - (1- \eta) \ Cov(r,r_m) \tag{8b}$$

$$g_{3t} = [r_t^o \ (1+f_{t-1}) - \mu_p]' \ (r_{m,t-1} - \mu_m) - \eta \ Cov(r,r_m) \tag{8c}$$

$$g_{4t} = [r_t^o \ (1+f_{t-1}) - \mu_p]' \ \ [r_t^o * (1+f_{t-1}) - \mu_p] - [\eta^2 + (1- \eta)^2] \ \sigma_p^2 \tag{8d}$$

$$g_{5t} = f_{t-1} - c \tag{8e}$$

$$g_{6t} = (f_{t-1}-c)' \ (f_t-c) - (-\eta^2)/ \lambda^2(1-\eta)^4\sigma_p^2 \tag{8f}$$

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dividend yield = the annual dividend yield of the CRSP value-weighted stock index; inflation = the percentage change in the consumer price index, CPI-U; industrial production growth = the monthly growth rate of the seasonally adjusted industrial production index; short-term corporate illiquidity = the percentage spread of three-month high-grade commercial paper rates over three-month treasury rates; and stock market liquidity = the liquidity measure from Lubos and Stambaugh (2003), based on price reversals.

[5] The style-matched benchmark portfolio is a weighted average return of the eight asset classes obtained by minimizing the tracking error between the fund returns (for all the funds of the same style) and the style benchmark.

$$g_{7t} = r_{m,t} - \mu_m \quad (8g)$$

$$g_{8t} = (r_{m,t} - \mu_m)' (r_{m,t} - \mu_m) - \sigma_m^2 \quad (8h)$$

There are eight moment conditions and eight parameters $[\mu_m, \sigma_m^2, \mu_p, Cov(r,r_m), \sigma_p^2, \eta, c, \lambda]$ and the system is exactly identified. Estimation is conducted not only for style groups with average monthly style group flows and returns but also for individual funds with monthly fund flows and returns. The true alpha in equation (5a), the observed alpha in equation (5b), the statistical bias in equation (6a), and the dilution effects of long-term and short-term flows in equations (6b) and (6c) are all estimated.

B. Results for the style groups and individual funds

(Insert table II here.)

Table II presents the estimation results by style groups for the style-group portfolios. Panel A presents the estimated parameters and their *t*-statistics; panel B presents the estimated alpha and beta and the biases. Here, the true alpha is on average positive but not significant. The spurious bias is small for the style portfolios, but the dilution effects of flows are significant. For example, the dilution effect for short-term flows is 38 basis points for small company growth funds, 18 for other aggressive growth, and 40 for sector funds.

(Insert figure A here.)

The model is also estimated for each individual fund. Figure A plots the empirical distributions of the *t*-statistics of the true alphas and the bias components for individual funds by style group.[6] According to these distributions, fund managers generally have no

[6] The maximum capital gain (MCG) group is not presented for the individual fund-level results because the number of funds with a sufficient monthly flow observation is small for this group.

picking ability, and there is little spurious bias or dilution effects of long-term flows; however, the dilution effect due to short-term flows is significant and widespread. Moreover, not surprisingly, the dilutions, while mostly negative, are positive for some funds. This positivity may result, at least partly, from a net outflow causing the cash balance to shift downward or, in a downturn market, from the higher cash balance making the return look better.

C. Relations between stale pricing and biases, dilutions, and true alpha

(Insert table III here.)

Table III summarizes the fund-level estimated true alpha and biases by fund style and by the extent of stale pricing. Panel A presents the means of the estimated true alphas within each style group, while panel B presents the means of the estimated true alphas with the funds sorted into five quintiles according to stale pricing. The estimated true alpha and the dilution effect of long-term flows do not differ significantly across groups; however, the statistical bias is significantly larger in the top quintile than in the bottom quintile. Additionally, the dilution effect of arbitrage flows is larger (in a negative direction) in the top quintile than in other quintiles. Since the magnitude of the dilution effect is larger than that of the spurious bias, the observed alpha is therefore smaller in the top quintile. These results are consistent with the theoretical derivations that the statistical bias is linearly and positively related with stale pricing, whereas the dilution effect of arbitrage flows is linearly and negatively related with stale pricing. Most importantly, the true alpha is independent of stale pricing and flows.

IV. Comparison: Traditional performance measures and stale pricing

Now it has been shown that the alpha estimated with the proposed model is free of the bias caused by stale pricing, this section compares the proposed model with traditional models. The results suggest that the cross-sectional differences in conventional alphas are largely explained by stale pricing even after various fund characteristics have been controlled for.

A. Measures of performance and stale pricing

Fund performance is compared using six conventional alphas estimated by the following equations:

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \varepsilon_{i,t} \quad (9a)$$

$$r_{i,t} = \alpha_i + \beta_i r_{s,t} + \varepsilon_{i,t} \quad (9b)$$

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + c_i r_{m,t}, Z_{t-1} + \varepsilon_{i,t} \quad (9c)$$

$$r_{i,t} = \alpha_i + \beta_i r_{s,t} + c_i r_{s,t}, Z_{t-1} + \varepsilon_{i,t} \quad (9d)$$

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + c_i HML_t + d_i BMS_t + e_i MOM_t + \varepsilon_{i,t} \quad (9e)$$

$$r_{i,t} = \alpha_i + \beta_i r_{s,t} + c_i PREMLIQ_t + \varepsilon_{i,t} \quad (9f)$$

where $r_{i,t}$ is fund *i*'s return in period *t*, $r_{m,t}$ is the market return, $r_{s,t}$ is the style benchmark return, and Z_{t-1} is the lagged economic instruments. In all, the analysis uses 11 instruments. The first four estimated alphas are Jensen's (1968) alpha and the conditional alpha of Ferson and Schadt (1996) estimated with a market benchmark and style benchmarks.[7] The fifth controls for Carhart's (1997) four factors, and the last for Pastor and Stambaugh's (1999) liquidity factor.

[7] Tables on the style benchmark weights and returns are available from the author upon request.

Four measures of stale pricing are also used. The first, introduced by Lo and MacKinlay (1990), is designed for a nontrading scenario:

$$\pi = -Cov(r_t^o, r_{t+1}^o) / \mu_i^2; \;\; if \;\; Cov(r_t^o, r_{t+1}^o) < 0, \; otherwise \; 0 \qquad (10a)$$

where r_t^o is the observed returns. The true return follows a one factor linear model in which μ_i is the mean of the true return. In each period, security *i* has π_i, the probability of nontrading; the larger the π, the staler the price. The detailed derivations of this measure are presented in appendix A6.

The second measure, the smooth index (Getmansky, Lo, and Makorov 2004), is derived for thin trading scenarios in which trades are not deep enough to absorb all information:

$$\xi = \Sigma\theta_j^2 \; ; \; where \; r_t^o = \Sigma\theta_j r_{t-j}^o; \; \Sigma\theta_j = 1; \; \theta_j \in [0,1]; \; j=1, 2, \text{- -} \; k \qquad (10b)$$

where r_t^o is again the observed returns, and the true return follows a one factor linear model. It is then easy to show $1/(1+k) \leq \xi \leq 1$. The wider the distribution of θ_j, the smaller the ξ; the more concentrated the θ_j, the larger the ξ. Indeed, Getmansky, Lo, and Makorov (2004) show that smoothing increases the Sharpe ratio of the observed returns of hedge funds. The detailed derivations of this measure are given in appendix A7.

In addition to π and ξ, stale pricing can also be measured by the covariance beta of fund returns and lagged market returns or the autocovariance beta of fund returns:

$$Beta(r_t, r_{m,t-1}) = Cov(r_t, r_{m,t-1}) / Var(r_m) \qquad (10c)$$

$$Beta(r_t, r_{t-1}) = Cov(r_t, r_{t-1}) / Var(r) \qquad (10d)$$

where r is fund returns, r_t is fund return in period t, r_m is market return, and $r_{m,t-1}$ is market return in period $t-1$.

(Insert table IV-a and table IV-b here.)

Table IV-a summarizes the estimated stale pricing by fund style. Less than half the funds display nontrading properties. Among the funds with $\pi > 0$, the probability of nontrading is as high as 0.5. The minimum smoothing index ξ is 0.17 for all style groups, implying that the return can smooth up to five periods, K = 5. The average smoothing index ξ is about 0.5, implying that prices on average smooth back only one period, K = 1. Table IV-b presents the joint probability of a fund presenting stale pricing in period *t* and $t + \tau$, $\tau = 1,2, - - 5$ (which is similar to Makorov chains), with each block in the table sums up to one. It is shown that he sum of the diagonal in every block is larger than the sum of the off-diagonal, even when $\tau > 2$. These results imply that stale pricing is widespread and persistent in the mutual fund industry.

B. Explanatory and predictive relations

In a two-step process, the first step estimates each fund's α, π, and ξ with a rolling window of three years (36 months). Each rolling moves observations one year (12 months) forward. The second step then applies both the pooled panel regression and the Fama-MacBeth method to examine the cross-sectional relation between performance, the dependent variable, and stale pricing, an explanatory variable. The controlling variables include log(TNA), expense ratio, age, portfolio turnover, income distributed, capital gains distributed, net flows, total loads, and the fund style. Particularly, since fund loads can impede opportunistic trading by imposing high transaction costs, an interactive variable of total loads and stale pricing is also included in the panel regression. All explanatory variables are studentized, and fund styles are controlled for with dummies.

(Insert table V here.)

Table V summarizes the regression results of all six measures of performance and four measures of stale pricing The three panels in the table present respectively the coefficients on stale pricing (panel A), fund loads (panel B), and their interactive variables (panel C). Coefficients between fund performance and other fund characteristics are not shown here. The coefficients in each column and at corresponding row of each panel are from an independent regression, concerning one measure of performance and one measure of stale pricing. As panel A illustrates, each one standard deviation increase in $Beta(r_t, r_{t-1})$ can decrease alpha by 1.24%. Yet, as panel B illustrates, loads alone explain little cross sectional difference in fund performance. On the other hand, the coefficients on the interaction terms are positive and significant, implying that high loads reduce the dilution of returns in funds with stale pricing.

(Insert table VI here.)

In the Fama-MacBeth approach, the standard errors of the coefficients are adjusted[8] for an MA(2) process, which the estimated coefficient follows because of the rolling window approach. The Fama-MacBeth mean and t-statistics of the coefficients on stale pricing, as well as coefficients on other fund characteristics, are presented in table VI, with each panel concerning one measure of stale pricing. The results indicate that each one standard deviation increase in π is associated with a 0.22%–0.51% decrease in alpha, while each one standard deviation increase in ξ is associated with a decrease of 0.29%–0.39% in alpha. All coefficients on staleness measures are significant at the 1% level.

(Insert table VII here.)

[8] Because the alpha and stale pricing are estimated using a three-year rolling window, the estimation of γ_1 follows an MA(2) process; therefore, Newey and West's (1987) approach is used to estimate $\sigma(\gamma_1)$: $\sigma(\gamma_1) =\{(1/T)*[\Sigma_{t=1:T} g_t g_t + (4/3)\Sigma_{t=2:T} g_t g_{t-1} + (2/3)\Sigma_{t=3:T} g_t g_{t-2}]\}^{1/2}$, where $g_t = \gamma_{1t}$ - Mean(γ_1).

If the persistence property is considered, stale pricing may also predict funds' future performance. Therefore, table VII presents the Fama-Macbeth coefficients and t-statistics between fund performance and stale pricing lagged by 36 months. It is shown that even though the coefficients are positive and significant, compared to those in table V and table VI, the magnitude of the predictive relation in table VII is smaller and of less significance.

V. Robustness

A. Errors in variables

As shown in the previous section, stale pricing explains cross sectional differences in conventionally measured performance. The procedure for estimating the relation consists of two stages: estimation of both performance and stale pricing for each fund with time series data and regression of the estimators from the first stage on each other cross sectionally. As a result, the second stage cross sectional regression is likely to suffer an errors-in-variables problem. This section addresses this issue by calculating the bias caused by errors-in-variables problem and shows that the magnitude of the bias does not dramatically influence the results given in the previous section.

Whereas appendix A.8 provides a full outline of the problem and its solution, the two biases involved can be summarized as follows. The first is the attenuation bias, $A = Var\ (a_2\ ) \ /\ [Var\ (a_2) + (x_2'\ x_2)^{-1} Var(\varepsilon_2)]$, where a_2 is the explanatory variable in the second stage and ε_2 is the observation error of a_2. This is the bias studied in a traditional errors-in-variables problem, in which only the explanatory variable is observed with an error. The second is the bias caused by the correlated errors in the dependent and explanatory variables, $\Phi = (x_1' x_1)^{-1} (x_1' x_2)(x_2' x_2)^{-1} Cov(\varepsilon_1, \varepsilon_2) / [Var(a_2) + (x_2' x_2)^{-1} Var(\varepsilon_2)]$, where x_1 *and* x_2 are explanatory variables in the first stage regressions, in which a_1 and a_2 are estimated. a_1 is the dependent variable in the second stage with ε_1 as its observation error. The problem now becomes the calculation of A and Φ, whose values are determined by $Cov(\varepsilon_1, \varepsilon_2)$ and $Var(\varepsilon_2)$.

To estimate the cross sectional variance and covariance of ε_1 *and* ε_2, the equations in the first stage must be regressed simultaneously for all the funds. However, this procedure is plausible only if the number of funds in the cross section is smaller than the number of periods in the time series. Assuming $Cov(\varepsilon_1, \varepsilon_2)$ and $Var(\varepsilon_2)$ *to* be the same for all funds within the same style group, makes it possible to randomly choose one fund from each style to form a 16 equation system and compute A and Φ. This procedure is repeated 10 times, each time with eight funds randomly redrawn (see table VIII for the 10 pairs A and Φ). As the table shows, the attenuation bias A ranges from 0.888 to 0.997, while the bias caused by correlated errors is smaller than 0.00477. The last column illustrates the implied relation between performance and stale pricing, or say the true coefficient g, if the estimated coefficient $\hat{g}$ is 40 basis points in the second stage, where $\hat{g} = g A + \Phi$. According to this estimation, the overall bias due to the errors-in-variable is minimal in 9 out of 10 cases. Therefore, it is safe to conclude that the relation between performance and stale pricing estimated in the previous section is robust.

(Insert table VIII here.)

B. Two-stage approach with the proposed measure of performance

As shown in section III, the true performance derived from the proposed model is free of biases from stale pricing when the model estimates performance and stale pricing simultaneously. In contrast, as illustrated in section IV, performance measures produced by traditional models are affected by stale pricing, but with performance and stale pricing estimated separately. Therefore, another test of robustness is to check whether or not the true alpha from the proposed model is correlated with stale pricing as estimated in section IV. As is apparent from the results presented in table XI true alpha estimated using the proposed model is also free of the stale pricing bias when the two-stage approach is used.

(Insert table IX here.)

C. Staleness in the indices

The estimation with the proposed model uses the S&P 500 index returns as benchmark returns, a favorable choice given that other benchmark returns, even style indices, can be stale. This assumption can be tested with a simplified version of the model that uses zero flows. The application of the simplified model to 12 portfolios—NYSE, AMEX, NASDAQ equally weighted indices, value, growth, small stock indices and the six Fama-French portfolios—shows that most of these portfolios are stale relative to the S&P 500 index.[9]

VI. Conclusion

Stale pricing is prevalent in the mutual fund industry and impacts fund performance through two channels: a statistical bias and an arbitrage dilution. This paper introduces a model of performance evaluation that accounts for the stale prices and

[9] Results are available from the author on request.

endogenous fund flows. Specifically, it differentiates true performance from observed performance and attributes the difference to these two effects. Compared to the true alpha, the observed alpha has a positive spurious bias that is small, but a negative dilution bias that is large and significant. Such decomposition is particularly interesting for performance attribution studies and applications. The structure of the proposed model also allows extension to time-variant stale pricing or to evaluate fund managers' timing ability.

A comparative component of the analysis shows that fund performance evaluated with conventional methods is negatively associated with stale pricing, which suggests that reducing stale pricing through fair-value pricing to impede arbitrages may not only protect long-term investors but also benefit portfolio managers directly.

Appendix:

A1: Decision of the timer and flows into the fund

A timer is a utility optimizer with a choice between cash and fund shares:

$$U(x) = E\{U([xR_p+(1-x)R_f] - [hR_p+(1-h)R_f]) \mid I_{t-1}^o)\} \quad \text{(A1.1)}$$

where x is the weight on fund shares in period t, and h is an exogenous benchmark preference parameter, R_p is the return of fund shares, and R_f is the return of cash. I_{t-1}^o includes all the information on fund returns observed in the past $\{r_{t-1}^o, r_{t-2}^o, \text{- - -}\ r_1^o\}$.

By setting the first-order condition of the utility function to zero, it can be simplified using Rubinstein's (1973) lemma:

$$(x\text{-}h) = \gamma^{-1} E(r_p \mid I_{t-1}^o) / Var(r_p \mid I_{t-1}^o) \quad \text{(A1.2)}$$

where γ is the risk aversion parameter and r_p is the excess return R_p-R_f. This calculation then gives the optimal weight:

$$x = \gamma^{-1} E(r_p \mid I_{t-1}^o) / Var(r_p \mid I_{t-1}^o) + h \quad \text{(A1.3)}$$

From equations (1a) and (1b), the following can be derived:

$$E(r_t^* \mid I_{t-1}^o) = E[\eta r_{t-1} + (1-\eta) r_t \mid I_{t-1}^o] = \eta\, r_{t-1} + (1-\eta)\, \mu \quad \text{(A1.4)}$$

$$Var(r_t^* \mid I_{t-1}^o) = var[\eta r_{t-1} + (1-\eta) r_t \mid I_{t-1}^o] = (1-\eta)^2 \sigma^2 \quad \text{(A1.5)}$$

where r_{t-1}^o is the observed return, r_{t-1} is the true return, and r_t^* is the return with zero flows. μ and σ^2 are the mean and variance of the true return, respectively, and η is the stale pricing parameter. The derivation of the conditional mean of r_t^* is based on the assumption that timers will know r_{t-1} once they have observed r_{t-1}^o. This assumption is reasonable because timers can infer the parameter of stale pricing η given I_{t-1}^o. For simplicity, it is also assumed that timers do not consider their own flow's impact on the realized return.

Substituting (A1.4) and (A1.5) into (A1.3), the timer's position in fund shares becomes

$$x = [\eta\, r_{t-1} + (1-\eta)\, \mu\,] \,/\, \gamma\, (1-\eta)^2\, \sigma^2 + h \tag{A1.6}$$

Therefore, the position change at the end of time *t*-1 is

$$\Delta x = \eta\; (r_{t-1} - r_{t-2}) \,/\, \gamma\, (1-\eta)^2\, \sigma^2 \tag{A1.7}$$

Denoting the timer's total assets as M, the arbitrage flow to the fund in time *t* is therefore

$$M\, \Delta x \,/\, TNA = M\, \eta\; (r_{t-1} - r_{t-2}) \,/\, \gamma\, TNA\, (1-\eta)^2 \tag{A1.8}$$

Denoting $\lambda = \gamma\, TNA \,/\, M$ and the long term flow component as a fraction of assets by C then yields the fund flows at the end of period *t*-1:

$$f_{t-1} = C_{t-1} + \eta\, (r_{t-1} - r_{t-2}) \,/\, \lambda\, (1-\eta)^2\, \sigma^2 \tag{A1.9}$$

A2: Dilution impact of fund flows

Denoting the observed excess return of NAV with zero flows by r_t* and the observed excess fund's NAV with flows by r_t^0,

$$r_t^0 = (P_t - P_{t-1})/P_{t-1} - R_f, \tag{A2.1}$$

where P_t is the observed price of the fund shares. Denoting the number of fund shares by N_t,

$$P_t = TNA_t/N_t \tag{A2.2}$$

The essential problem of dilution comes from the investment time lag of the new money flows. That is, trading of shares is transacted at the NAV price at the end of the day, but the new money cannot be invested until the next day. Therefore,

$$N_t = N_{t-1} + CF_{t-1}/\, P_{t-1} \tag{A2.3}$$

$$TNA_t = TNA_{t-1}\, (1 + R_t^*) + CF_{t-1}(1 + R_f) \tag{A2.4}$$

where R_t^* is the total return of the fund, the sum of the risk free rate and the fund's excess return r_t^* . CF_{t-1}, which is the amount of the money flow at the end of t-1, can also be denoted as a percentage of the total assets, f_{t-1}:

$$f_{t-1} = CF_{t-1} / (N_{t-1} P_{t-1}) \qquad \text{(A2.5)}$$

Therefore,

$$\begin{aligned} r_t^0 &= (P_t - P_{t-1})/P_{t-1} - R_f = (TNA_t/N_t)/P_{t-1} - 1 - R_f \\ &= [N_{t-1} P_{t-1} (1 + r_t^* + R_f) + CF_{t-1}(1 + R_f)] / [N_{t-1} P_{t-1} + CF_{t-1}] - 1 - R_f \\ &= [1 + r_t^* + R_f + f_{t-1}(1 + R_f)] / (1 + f_{t-1}) - 1 - R_f \\ &= r_t^* / (1 + f_{t-1}) \qquad \text{(A2.6)} \end{aligned}$$

which is equation (8) in the paper.

A3: System of the model.

The model consists of the following:

$$r_t = \alpha + \beta\, r_{mt} + \varepsilon_t \qquad \text{(1a)}$$

$$r_t^* = \eta\, r_{t-1} + (1 - \eta)\, r_t \qquad \text{(1b)}$$

$$f_{t-1} = C_{t-1} + \eta\, (r_{t-1} - r_{t-2}) / \lambda (1-\eta)^2 \sigma^2 \qquad \text{(2d)}$$

$$C_{t-1} \sim N(c, \sigma_c) \qquad \text{(2e)}$$

$$r_t^0 = r_t^* / (1 + f_{t-1}) \qquad \text{(3d)}$$

Where r_t denotes the true excess return of the portfolio, and r_{mt} the excess market return. Both r_t and r_{mt} are i.i.d, and the error term ε_t is independent of r_{mt}. $r_{i,t}^*$ is the observed excess return of the portfolio with zero flows, η is the weight on the lagged true return, and C_{t-1} is the long-term flows that occur at the end of period t-1. Furthermore, C_{t-1} is independent of r_{t-1} and follows an N(c, σ_c), i.i.d distribution. Finally, λ accounts for the risk aversion γ and the relative size of the timer's assets to the fund's TNA.

Equation (4a) can then be obtained as follows:

Equation (3d) ➔ $r_t^* = r_t^o\,(1+f_{t-1}))$ (A3.1)

Equation (1b) ➔ $E(r_t^*)=E[\,\eta\, r_{t-1} + (1-\eta)\, r_t\,] = \mu_p$ (A3.2)

where μ_p is the mean of the funds' true returns, $\mu_p = E(r_t)$

Therefore, $E(\,r_t^o\,(1+f_{t-1})) = \mu_p$ (A3.3)

Similarly, equation (4b) can be obtained by

$$Cov(r_t^o\,(1+f_{t-1}),\ r_{m,t}\,) = Cov(r_t^*, r_{m,t}) = Cov(\eta\, r_{t-1} + (1-\eta)\, r_t, r_{m,t})$$

$$= (1-\eta)\ Cov(r,r_m) \quad (A3.4)$$

equation (4c) by

$$Cov(r_t^o\,(1+f_{t-1}),\ r_{m,t-1}\,) = Cov(r_t^*, r_{m,t-1}) = Cov(\eta\, r_{t-1} + (1-\eta)\, r_t\,, r_{m,t})$$

$$= \eta\ Cov(r,r_m) \quad (A3.5)$$

and equation (4d) by

$$Var[r_t^o\,(1+f_{t-1]}) - \mu_p] = Var(r_t^*) = Var(\eta\, r_{t-1} + (1-\eta)\, r_t\,)$$

$$= [\eta^2 + (1-\eta)^2]\ \sigma_p^2 \quad (A3.6)$$

where $\sigma_p^2 = Var(r)$.

Equation (4e) is derived directly from equation (2e).

Finally, equation (4f) is obtained as follows:

$$Cov(f_{t-1,}\ f_t\,) \quad =Cov(C_{t-1} + \eta(r_{t-1}-r_{t-2}) / \lambda(1-\eta)^2\sigma^2,\ C_t + \eta\ (r_t-r_{t-1}) / \lambda(1-\eta)^2\sigma^2)$$

$$=Cov(r_{t-1}-r_{t-2,}\ r_{t-1}-r_{t-2}\,)\ [\eta / \lambda(1-\eta)^2\sigma^2]^2$$

$$= -\ \eta^2 / \lambda^2(1-\eta)^4\sigma_p^2 \quad (A3.7)$$

A4: Derivation of the observed alpha and components of the biases

The main elements are $E(r_t^o)$ and $Cov(r_t^o, r_{m,t})$

Since $E(r_t^o (1+f_{t-1})) = \mu_p$ (A3.3)

and $E(r_t^o (1+f_{t-1})) = Cov(r_t^o, f_{t-1}) - E(r_t^o)(1+c)$ (A4.1)

then $E(r_t^o) = [\mu_p - cov(f_{t-1}, r_t^o)]/(1+c)$ (A4.2)

Since $Cov(r_t^o (1+f_{t-1}), r_{m,t}) = (1- \eta) Cov(r,r_m)$ (A3.4)

and $Cov(r_t^o (1+f_{t-1}), r_{m,t}) = E(r_t^o (1+f_{t-1})r_{m,t}) - E(r_t^o (1+f_{t-1})) E(r_{m,t})$

$= Cov(r_t^o r_{m,t}, f_{t-1}) + E(r_t^o, r_{m,t})E(1+f_{t-1}) - E(r_t^o (1+f_{t-1})) E(r_{m,t})$

$= Cov(r_t^o r_{m,t}, f_{t-1})+(1+c)[Cov(r_t^o, r_{m,t})+E(r_t^o) E(r_{m,t})]-E(r_t^o(1+f_{t-1})) E(r_{m,t})$

$= Cov(r_t^o r_{m,t}, f_{t-1})+(1+c)Cov(r_t^o, r_{m,t}) - E(r_{m,t})Cov(f_{t-1}, r_t^o)$ (A4.3)

It should be noted that the last step uses (A4.2) and (A3.3); therefore,

$Cov(r_t^o, r_{m,t}) = [(1- \eta) Cov(r,r_m) – Cov(r_t^o r_{m,t}, f_{t-1}) + cov(f_{t-1}, r_t^o) u_m]/(1+c)$ (A4.5)

As a result, the measured performance is $\alpha^o = E(r_t^o) - Cov(r r_t^o,r_{m,t}) u_m/\sigma_m^2$

$= E(r_t) - cov(f_{t-1}, r_t^o)]/(1+c)$

$–[(1- \eta)Cov(r,r_m) – Cov(r_t^o r_{m,t}, f_{t-1}) +cov(f_{t-1}, r_t^o)u_m]/(1+c)] u_m/\sigma_m^2$

$= E(r_t) - Cov(r_t ,r_{m,t}) u_m/\sigma_m^2$ (α)

$+ \eta Cov(r,r_m) u_m/\sigma_m^2$ ($\Delta\alpha_1$)

$+ (-c/1+c) [\mu_p – (1- \eta) Cov(r,r_m) u_m/\sigma_m^2]$ ($\Delta\alpha_2$)

$+ [-1/(1+c)] \{ cov(f_{t-1}, r_t^o) –[Cov(r_t^o r_{m,t}, f_{t-1})- cov(f_{t-1}, r_t^o) u_m] u_m/_m^2\}$ ($\Delta\alpha_3$)

A5. The aggregation of daily dilution to monthly dilution

Supposing that there are D days in each month and for each day d within the month, the following is derived:

$$r_d^o = r_d^*/(1+f_d) \tag{3d}$$

where r_d^o is the observed excess return of the fund, r_d^* is the would-be excess return of the fund with zero flows, and f_d is the net flow as a percentage of the TNA. The daily dilution in the return is

$$r_d^* - r_d^o = r_d^o * f_d \tag{A5.1}$$

Denoting the observed excess return for the month as r_p^o, the excess return without flows as r_p^*, and the flow of the month f_p, the monthly dilution is then

$$r_p^* - r_p^o = \sum r_d^* - \sum r_d^o$$

$$= \sum r_d^o * f_d$$

$$= (r_p^o * f_p) * \sum (r_d^o / r_p^o) * (f_d / f_p) \tag{A5.2}$$

$$= r_p^o * f_p / D \tag{A5.3}$$

A rigorous approach of the iterating from (A5.2) to (A5.3), requires to expand all the terms at the daily level and it can be shown that the difference between $r_p^* - r_p^o$ *and* $r_p^o * f_p/D$ has an order of magnitude smaller than $r_d^2 * f_d^2$. The details of expansion are available upon request. A sloppy approach is as the follows. Since

$$plim(r_d^* / r_p^*) = 1/D \tag{A5.4}$$

then

$$\sum plim\ (r_d^o / r_p^o) * (f_d / f_p)$$

$$= \sum plim((r_d^* / r_p^*)(1+f_p)/(1+f_d) * (f_d / f_p)$$

$$= 1/D \tag{A5.5}$$

A6: Model of stale pricing—nontrading.

Following Lo and MaKinlay (1990), the true return of security i is assumed to follow a one-factor model: $R_{i,t} = \mu_i+\beta_i f_t+\varepsilon_{i,t}$; $E(f_t)=E(\varepsilon_{i,t})= E(f_t\, \varepsilon_{i,t}) =0$, where f_t, $\varepsilon_{i,t}$ are both i.i.d. In each period t, security i may not trade with probability π_i. If it does, its observed return for period t is 0, although its true return is $R_{i,t}$. If security i trades at time $t+1$ and $t-k-1$ but does not trade from time $t - k$ to t, then its observed time $t+1$ return is assumed to be the sum of its true returns from $t-k$ to $t+1$. This model captures the essential feature of nontrading as a source of spurious autocorrelation: news affects those stocks that trade more frequently first and influences the returns of thinly traded securities with a lag. An explicit expression for the observed returns process is derived and its time series properties deduced using two related stochastic processes.

Defining δ_{it} and $X_{it}(k)$ as the following Bernoulli random variables,

δ_{it} is an indicator of trading in period t:

$$\delta_{it} = 1 \text{ with probability } 1- \pi_i \qquad \text{(A6.1)}$$

$$= 0 \text{ with probability } \pi_i$$

while $X_{it}(k)$ indicates trading in period t after not having traded in the past K periods:

$$X_{it}(k){:}\ \ \delta_{it}(1- \delta_{it-1})(1-\delta_{it-2}) \text{ - - -}(1- \delta_{it-k}) \qquad k > 0,$$

$$= 1 \text{ with probability } (1- \pi_i)\, \pi_i^k$$

$$= 0 \text{ with probability } 1- (1- \pi_i)\, \pi_i^k$$

$$X_{it}(0) = \delta_{it} \qquad \text{(A6.2)}$$

It should be noted that δ_{it} is implicitly assumed to be an independently and identically distributed random sequence for $i = l, 2, \ldots, N$.

The observed return process R_{it}^o is given by the following stochastic process:

$$R_{it}^{o} = \Sigma_{k=0}^{\infty} X_{it}(k)\, R_{it-k}\,, \qquad i = 1, \ldots, N \qquad \text{(A6.3)}$$

whose moments and comoments can be derived from the definition of $X_{it}(k)$:

$$E\,[X_{it}(k)] = (1- \pi_i)\, \pi_i^{k}, \qquad \text{(A6.4)}$$

For arbitrary i, t, and k,

$$X_{it}(k)\, X_{it+n}(l) \;=\; \delta_{it}(1- \delta_{it-1}) \cdots (1- \delta_{it-k})\, \delta_{it+n}\, (1-\delta_{it+n-1}) \cdots (1- \delta_{it+n-l}) \qquad \text{(A6.5)}$$

If $l > n$, $E\,[X_{it}(k)\, X_{it+n}(l)] = 0$, since both $(1 - \delta_{it})$ and δ_{it} are included in the product, and if $l < n$, it can be shown that the expectation reduces to $(1 - \pi_i)^2\, \pi_i^{k+l}$ to yield

$$E\,[X_{it}(k)\, X_{it+n}(l)] = (1 - \pi_i)^2\, \pi_i^{k+l}\,,\ \text{if } l < n$$

$$= 0,\ \text{if } l >= n \qquad \text{(A6.6)}$$

Therefore, the moments of the observed return can be derived as follows:

$$E\,[R_{it}^{o}] = E\,[\Sigma_{k=0}^{\infty} X_{it}(k)\, R_{it-k}] = \Sigma_{k=0}^{\infty} E\,[X_{it}(k)\, R_{it-k}] = \Sigma_{k=0}^{\infty} E\,[X_{it}(k)]\, E[R_{it-k}]$$

$$= \mu_i\, \Sigma_{k=0}^{\infty}(1- \pi i)\, \pi_i^{k} = \mu_i \qquad \text{(A6.7)}$$

$$E\,[R_{it}^{o}\, R_{it+n}^{o}] = E\,[\Sigma_{k=0}^{\infty} X_{it}(k)\, R_{it-k}\ \ \Sigma_{l=0}^{\infty} X_{it+n}(l)\, R_{it+n-l}]$$

$$= \Sigma_{k=0}^{\infty} \Sigma_{l=0}^{\infty} E\,[X_{it}(k)\, R_{it-k}\, X_{it+n}(l)\, R_{it+n-l}]$$

$$= \Sigma_{k=0}^{\infty} \Sigma_{l=0}^{\infty} E\,[X_{it}(k)\, X_{it+n}(l)]\, E\,[R_{it-k}\, R_{it+n-l}]$$

$$= \Sigma_{k=0}^{\infty} \Sigma_{l=0}^{n-1} (1 - \pi_i)^2\, \pi_i^{k+l}\, E\,[R_{it-k}\, R_{it+n-l}]$$

$$= \Sigma_{k=0}^{\infty} \Sigma_{l=0}^{n-1} (1 - \pi_i)^2\, \pi_i^{k+l}\, \mu_i^{2} = \mu_i^{2}\, (1- \pi_i^{n}) \qquad \text{(A6.8)}$$

$$Cov\,(R_{it}^{o}\, R_{it+n}^{o}) = E\,[R_{it}^{o}\, R_{it+n}^{o}] - E\,[R_{it}^{o}\,]\, E\,[R_{it+n}^{o}]$$

$$= -\, \mu_i^{2}\ \pi_i^{n} \qquad \text{(A6.9)}$$

A7: A model of stale pricing—smoothing index

In the smoothing index introduced by Getmansky, Lo, and Makorov (2004) for a thin trading scenario, the true return is governed by a one-factor linear model:

$$R_t = \mu+\beta f_t+\varepsilon_t; \qquad E(f_t)= E(\varepsilon_t)=0 \qquad \text{(A7.1)}$$

where $f_{t,}\varepsilon_t$ are both i.i.d. However, the measured (observed) return is a weighted average of true returns in the past k periods:

$$R_t^o = \Sigma\theta_j R_{t-j}; \qquad \Sigma\theta_j= 1;\ \theta_j \epsilon[0,1];\ j=1,\ 2,\ \text{- -}\ k \qquad \text{(A7.2)}$$

The smoothing index is defined as the sum of the weights squared:

$$\xi = \Sigma\theta_j^2 \qquad \text{(A7.3)}$$

Smoothed prices vs. smoothed returns

Letting p_t be the natural log of the fund's true net asset value per share (and assuming reinvestment of dividends), then its true return is

$$r_t = p_t - p_{t-1} \qquad \text{(A7.4)}$$

Due to smoothing, the measured price is

$$p_t^o = \delta_t\, p_{t-1} + (1\text{-}\ \delta_t)p_{t-1},\ \ \delta_t\ \varepsilon\ [0,1] \qquad \text{(A7.5)}$$

where the coefficient δt measures the extent of stale pricing during the month *t*.

Therefore, the measured return on a fund is

$$r_t^o = \delta_{t-1}r_{t-1} + (1\text{-}\delta_t)r_t \qquad \text{(A7.6)}$$

In a nondynamic stale pricing setting, the extent of stale pricing is assumed to be constant. Therefore, $r_t^o = \delta r_{t-1} + (1\text{-}\delta)r_t$, which is the same as a smoothed return approach.

A8: The statistical issue of errors-in-variables

Abstract of the issue

The first stage consists of two time series regressions in which estimates of the coefficients are obtained:

$$y_1 = a_1 x_1 + \varepsilon_1 \qquad \text{(A8.1)}$$

$$y_2 = a_2 x_2 + \varepsilon_2 \qquad \text{(A8.2)}$$

where $Cov\ (x_i\ ,\ \varepsilon_j\ ) = 0$, i, j = 1, 2. In the second stage, the estimates of a_1 and a_2 are regressed on each other in cross section, while the true model is the following:

$$a_1 = g\ a_2 + \omega \qquad \text{(A8.3)}$$

That is

$$g = Cov\ (a_1\ ,\ a_2\ )\ /\ Var\ (a_2\ ) \qquad \text{(A8.4)}$$

Assuming x_1 and x_2 are observed without errors, the following can be derived:

$$\hat{g}\ = Cov\ (\hat{a}_1\ ,\ \hat{a}_2)\ /\ Var\ (\hat{a}_2) \qquad \text{(A8.5)}$$

$$= Cov\ (a_1 + (x_1'\ x_1)^{-1}\ x_1\ \varepsilon_1\ ,\ a_2 + (x_2'\ x_2)^{-1}\ x_2\ \varepsilon_2)\ /\ Var\ (a_2 + (x_2'\ x_2)^{-1}\ x_2\ \varepsilon_2\ )$$

$$= [Cov\ (a_1\ ,\ a_2) + Cov\ ((x_1'\ x_1)^{-1}\ x_1\ \varepsilon_1,\ (x_2'\ x_2)^{-1}\ x_2\ \varepsilon_2)]\ /\ [Var\ (a_2) + Var\ (x_2'\ x_2)^{-1}\ x_2\ \varepsilon_2\ )]$$

$$= [Cov\ (a_1\ ,\ a_2) + (x_1'\ x_1)^{-1}\ (x_1'x_2\ )(x_2'\ x_2)^{-1}\ Cov\ (\varepsilon_1\ ,\ \varepsilon_2\ )]\ /\ [Var\ (a_2) + (x_2'\ x_2)^{-1} Var(\varepsilon_2)]$$

$$= g\ A + \Phi \qquad \text{(A8.6)}$$

where $A = Var\ (a_2\ )\ /\ [Var\ (a_2) + (x_2'\ x_2)^{-1} Var(\varepsilon_2)]$ (A8.7)

and $\Phi = (x_1'\ x_1)^{-1}\ (x_1'x_2\ )(x_2'\ x_2)^{-1}\ Cov\ (\varepsilon_1\ ,\ \varepsilon_2\ )\ /\ [Var\ (a_2) + (x_2'\ x_2)^{-1} Var(\varepsilon_2)]$ (A8.8)

A is the attenuation bias in the traditional errors-in-variables problem in which only a_2 is observed with error; Φ is the bias caused by the correlated errors in a_1 and a_2. The issue now becomes one of calculating A and Φ, whose values are determined by $Cov\ (\varepsilon_1\ ,\ \varepsilon_2)$ and $Var(\varepsilon_2)$.

Necessary assumptions

Estimating the cross-sectional variance and covariance of ε_1 and ε_2 requires that the equations (A8.1) and (A8.1) be regressed simultaneously for all funds. However, this step is plausible only if $N<<T$, where N denotes total number of funds in the cross section and T denotes the number of period in the time series. In the sample used for this paper, N is over 7,000, while the maximum possible T is 360. Therefore, the cross-sectional relation (A8.4)–(A8.6) is assumed to be the same for all funds within the same style group, meaning that one fund can be selected from each group to form a system with 16 equations. This system can be estimated using the seemingly unrelated regression method and calculating A and Φ. Another complication arises from the functional form of the first-stage equations, in which stale pricing does not take a linear form. Therefore, another assumption that stale fund returns follow an AR(1) process is imposed to simplify the form.

Estimation

In this paper, the two time series regressions are as follows:

$$r_{p,t}^{i} = \alpha^{i} + \beta^{i} r_{m,t} + \varepsilon_{1t}^{i} \tag{A8.9}$$

$$r_{p,t}^{i} = \eta^{i} r_{p,t-1}^{i} + \varepsilon_{2t}^{i} \tag{A8.10}$$

Denoting $Y^{i} = [r_p^{i}\ r_p^{i}]'$

$X^{i} = [\ [\underline{1}\ r_m\ \underline{0}\ \ \underline{0}]\ [\underline{0}\ \ \underline{0}\ \underline{0}\ r_{p(-1)}^{i}\ ]\ ]'$

$$\varepsilon^{i} = [\varepsilon_1^{i}\ \ \varepsilon_2^{i}\ ]' \tag{A8.11}$$

$r_{p,t}^{i}$ is fund i's return in period t, $r_{m,t}$ is the market return, and r_p^{i} r_m $r_{p(-1)}^{i}$ ε_1^{i}, and ε_2^{i} are column vectors with all time series observations of fund returns, market returns, lagged fund returns, and residuals. Stacking the eight pairs into one large system gives the following:

$$Y = [Y^1\ Y^2 \text{ - - - } Y^8]'$$

$$X = [\,[X^1\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}]\ \ [\underline{0}\ X^2\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}] \text{ - - - } [\underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ X^8]\,]'$$

$$\theta = [\,\alpha^1\ \beta^1\ \eta^1\ \alpha^2\ \beta^2\ \eta^2 \text{ - - - } \alpha^8\ \beta^8\ \eta^8]'$$

$$\varepsilon = [\varepsilon^1\ \ \varepsilon^2 \text{ - - - } \varepsilon^8\ ]' \qquad \text{(A8.12)}$$

$$Y = X\theta + \varepsilon \qquad \text{(A8.13)}$$

The system (A8.13) is estimated using eight random funds, one from each style. $Cov\ (e_1\ ,\ e_2\ )$ is computed to proxy for $Cov\ (\varepsilon_1\ ,\ \varepsilon_2\ )$ and $Var(e_2)$ for $Var(\varepsilon_2)$. *Var* (a_2) *equals* $Var\ (\hat{a}_2) - (x_2'\ x_2)^{-1} Var(\varepsilon_2)$. Finally, A and Φ are computed. This procedure is repeated ten times, each with eight funds redrawn. The ten pairs of A and Φ are presented in table VIII.

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Table I
Summary Statistics for Monthly Returns and Flows by Fund Style

This table summarizes the fund returns and flows. All funds in each fund style are grouped month by month to form an equally weighted portfolio whose time series of returns and flows are summarized here. Panel A presents the returns; panel B gives the flow summary. Returns are reported in percentage rate per month; fund flows are calculated as a percentage of the fund TNA.

$$Flow_{i,t} = (TNA_{i,t} - TNA_{i,t-1} * (1+R_{i,t}))/TNA_{i,t-1} \qquad (7)$$

Panel A: Summary of fund returns							
Fund Style	Begin	End	Mean	Min	Max	Std.	ρ_1[a]
Growth	1961	2004	0.995	-20.05	18.80	4.93	0.079
Maximum Capital Gains	1968	1992	1.283	-15.37	12.83	4.57	-0.142
Other	1989	2004	0.886	-18.37	18.00	5.67	0.075
Income	1961	2004	0.866	-11.53	9.03	2.41	0.041
Growth and Income	1961	2004	0.954	-15.75	13.52	4.09	0.028
Sector	1988	2004	1.009	-14.35	18.35	5.21	0.007
Small Company Growth	1989	2004	1.248	-19.53	20.19	5.47	0.083
Timing	1961	2004	0.758	-9.01	7.68	2.6	-0.016
Panel B: Summary of fund flows							
Growth	1991	2004	0.52	-4.34	35.23	3.18	-0.003
Maximum Capital Gains	1991	1992	1.20	0.00	3.39	0.88	-0.098
Other	1991	2004	0.89	-2.02	4.64	1.23	0.538
Income	1991	2004	0.96	-13.46	64.07	4.89	0.321
Growth and Income	1991	2004	0.16	-3.92	4.84	0.94	0.563
Sector	1991	2004	0.67	-3.59	8.37	1.67	0.503
Small Company Growth	1991	2004	1.17	-2.60	6.27	1.40	0.617
Timing	1991	2004	0.73	-6.22	5.72	1.32	0.581

Table II
Performance Evaluation Considering Stale Pricing and Endogenous flows—by Style Groups

This table presents the estimation results of the GMM system (equations 8a to 8h) at the style portfolio level for the 1991–2004 sample period. The style portfolio is equally weighted month by month with funds within that style group. Panel A presents the estimated parameters, including η, stale pricing, and λ, the product of Rubinstein risk aversion γ and the relative assets of the fund to the timer. Panel B presents the estimated performance and biases in annual percentage, with α being the true alpha; α^o, the measured alpha; $\Delta\alpha_1$, the statistical bias; $\Delta\alpha_2$, the dilution of long-term flows; and $\Delta\alpha_3$, the dilution of arbitrage flows.

Panel A: Estimated parameters

	u_m	σ_m^2	μ_p	$Cov(r,r_m)$,	σ_p^2	η	c	λ
Growth	1.87	**21.04**	5.89	**15.76**	**43.70**	-0.33	0.01	2264
	(0.03)	**(6.12)**	(1.19)	**(4.56)**	**(2.88)**	(-0.38)	(0.16)	(0.00)
MCG	7.86	**12.14**	**14.54**	**10.03**	**38.34**	-0.41	0.01	1222
	(0.18)	**(6.88)**	**(3.24)**	**(4.95)**	**(5.75)**	(-0.43)	(1.40)	(0.00)
Other	4.99	**17.54**	4.91	**22.44**	**33.74**	0.00	0.01	474
	(0.10)	**(7.43)**	(0.85)	**(7.28)**	**(67.48)**	(0.00)	(0.67)	(0.00)
Income	3.15	**21.19**	3.73	**5.96**	**8.77**	-0.23	0.01	732997
	(0.06)	**(6.05)**	(1.56)	**(3.90)**	**(3.51)**	(-0.35)	(0.19)	(0.00)
G&I	0.75	**22.53**	4.64	**19.47**	**17.70**	-0.00	0.00	145
	(0.01)	**(6.49)**	(1.14)	**(6.69)**	**(7.08)**	(-0.00)	(0.20)	(0.00)
Sector	4.94	**17.42**	6.25	**18.68**	**23.34**	-0.00	0.01	471
	(0.10)	**(7.49)**	(1.21)	**(7.00)**	**(5.60)**	(-0.00)	(0.30)	(0.00)
SCG	7.42	**16.80**	**12.67**	**16.10**	**31.00**	0.00	0.01	340
	(0.15)	**(7.17)**	**(2.29)**	**(5.55)**	**(6.20)**	(0.00)	(0.80)	(0.00)
Timing	6.99	**18.54**	**5.81**	**11.24**	**7.32**	0.00	0.01	99
	(0.14)	**(7.89)**	**(2.24)**	**(7.78)**	**(8.79)**	(0.00)	(0.44)	(0.00)

Panel B: Alpha and components of biases at portfolio level

	α	β	α^o	$\Delta\alpha_1$	$\Delta\alpha_2$	$\Delta\alpha_3$
Growth	4.43	**0.75**	3.79	-0.46	**-0.02**	**-0.14**
	(0.05)	**(21.65)**	(0.03)	(-0.01)	**(-2.27)**	**(-16.44)**
MCG	7.63	**0.83**	4.67	-2.52	-0.06	**-0.20**
	(0.02)	**(6.22)**	(0.01)	(-0.01)	(-1.03)	**(-3.98)**
Other	-1.43	**1.28**	-1.60	0.00	0.01	**-0.18**
	(-0.01)	**(17.88)**	(-0.01)	(0.00)	(0.47)	**(-7.97)**
Income	2.83	**0.28**	4.86	-0.20	**-0.02**	**2.22**
	(0.08)	**(7.15)**	(0.11)	(-0.01)	**(-1.79)**	**(23.14)**
G&I	3.97	**0.86**	3.90	-0.00	**-0.01**	**-0.06**
	(0.04)	**(59.43)**	(0.04)	(-0.00)	**(-3.43)**	**(-16.83)**
Sector	0.90	**1.07**	0.49	-0.00	-0.005	**-0.40**
	(0.01)	**(16.84)**	(0.00)	(-0.00)	(-0.32)	**(-11.91)**
SCG	5.24	**0.96**	4.78	0.00	-0.06	**-0.38**
	(0.04)	**(11.31)**	(0.03)	(0.00)	(-1.37)	**(-12.59)**
Timing	1.56	**0.61**	1.52	0.00	**-0.01**	**-0.03**
	(0.02)	**(43.48)**	(0.02)	(0.00)	**(-2.21)**	**(-4.33)**

Table III
Performance Evaluation Considering Stale Pricing and Endogenous flows—by Individual Fund

This table summarizes the estimated results of the GMM system (equations 8a to 8h) at the individual fund level for the 1991–2004 sample period. The system is first estimated for each fund, then the estimated true alpha, observed alpha, and the biases are summarized in annualized percentage. Panel A summarizes the estimates for each style group, although the maximum capital gain style group is missing because of the limited number of observations. Panel B shows the funds sorted into five quintiles according to their stale pricing, as well as the statistical differences between the highest and lowest quintiles.

	α: True Alpha	α^o : Measured Alpha	$\Delta\alpha_1$: Statistical Bias	$\Delta\alpha_2$: Dilution of Long-Term Flows	$\Delta\alpha_3$: Dilution of Arbitrage Flows
Panel A: By style of the funds					
Growth Funds	0.64	0.47	0.00040	-0.036	-0.135
Other Aggressive	0.86	0.64	0.00026	-0.037	-0.187
Income Funds	0.20	0.06	0.00048	-0.001	-0.132
Growth and Income	1.95	1.62	0.00065	-0.099	-0.233
Sector	5.98	5.55	0.00068	-0.123	-0.314
Small Company	2.55	2.35	0.00057	-0.085	-0.117
Timing Funds	0.93	0.84	0.00030	-0.014	-0.078
Panel B: By fund stale pricing					
Lowest: 1	2.09	1.98	0.00002	-0.05	-0.07
2	2.00	1.86	0.00007	-0.06	-0.09
3	2.34	2.07	0.00020	-0.08	-0.19
4	2.02	1.71	0.00043	-0.07	-0.24
Highest: 5	1.47	1.09	0.00088	-0.04	-0.34
t-Stat of (Highest–Lowest)	-1.32	**-1.87**	**2.23**	0.34	**-1.70**

Table IV-a
Summary of Estimated Stale Pricing of Individual Funds by Style Groups

This table summarizes the stale pricing of funds. For each fund, stale pricing is measured by estimating equations (10a) and (10b) with 36 rolling window returns. Panel A summarizes the estimated stale pricing, π, by fund styles. $\pi = -Cov(r_t^o, r_{t+1}^o) / E(r_{i,t}^o)^2$; *if* $Cov(r_t^o, r_{t+1}^o) < 0$; *otherwise 0; (10a). O*nly the mean of the uncensored observations are given and the percentages of uncensored observations are in parentheses. Panel B summarizes the estimated stale pricing, ξ, by fund styles. $\xi = \Sigma\theta_j^2$; *where* $R_t^o = \Sigma\theta_j R_{t-j}$; *and* $\Sigma\theta_j = 1$; $\theta_j \epsilon [0,1]; j=1, 2, - - k;$ (10b).

	Mean		Std	Min	Max
Panel A: Stale pricing π (Lo and MacKinlay 1990)					
Growth Funds	0.515	(29.3%)[a]	0.28	0.01	1
Maximum Capital Gain	0.445	(11.2%)[a]	0.27	0.02	0.94
Other Aggressive Growth	0.419	(9.5%)[a]	0.28	0.03	1
Income Funds	0.537	(18.6%)[a]	0.29	0.01	1
Growth and Income	0.536	(34.1%)[a]	0.27	0.01	1
Sector Funds	0.464	(26.7%)[a]	0.24	0.02	1
Small Company Growth	0.443	(4.6%)[a]	0.29	0.03	0.97
Timing	0.525	(28.6%)[a]	0.29	0.01	1
Panel B: Stale pricing ξ (Getmansky, Lo, and Makorov 2004)					
Growth Funds		0.421	0.17	0.17	1
Maximum Capital Gain		0.451	0.2	0.17	1
Other Aggressive Growth		0.464	0.2	0.17	1
Income Funds		0.495	0.18	0.17	1
Growth and Income		0.438	0.16	0.17	1
Sector Funds		0.447	0.17	0.17	1
Small Company Growth		0.492	0.21	0.17	1
Timing		0.452	0.16	0.17	1

Table IV-b
Persistence in Stale Pricing

For each year, funds are classified into H or L according to their stale pricing. This table presents the joint probability of a fund falling into the high or low category in year t and t + τ. Staleness, π, and ξ are from equations (10a) and (10b). For π, H means π > 0 and L means π = 0. For ξ, H means ξ ≥ 0.5 and L means ξ < 0.5.

t (τ = 0)	τ =1		τ =2		τ =3		τ =4		τ =5	
Panel A: Stale pricing π (Lo and MacKinlay 1990)										
	H	L	H	L	H	L	H	L	H	L
H	0.28	0.12	0.24	0.17	0.21	0.22	0.18	0.22	0.19	0.20
L	0.13	0.47	0.17	0.42	0.2	0.36	0.23	0.37	0.22	0.39
Panel B: Stale pricing ξ (Getmansky, Lo, and Makorov 2004)										
	H	L	H	L	H	L	H	L	H	L
H	0.15	0.18	0.13	0.22	0.14	0.24	0.13	0.25	0.14	0.25
L	0.19	0.48	0.20	0.45	0.21	0.41	0.22	0.40	0.23	0.38

Table V
Traditional Performance Measures, Stale Pricing, and the role of loads—Panel Regressions

This table summarizes all the coefficients and *t*-statistics from regressions in which the traditional alphas are explained by stale pricing. Each pair (in each column and at the corresponding row) is from an independent regression. The dependent variables are the traditional alphas in percentage (equations 9a–9h). The explanatory variables are stale pricing estimated using equation 10a–10d, fund characteristics, and fund styles. Explanatory variables are studentized. Coefficients on stale pricing, total loads, and their interactive variables and t-statistics are presented. The sample period is 1973–2004.

	Alpha from Unconditional CAPM	Alpha from Conditional CAPM	Unconditional Alpha with Style Benchmarks	Conditional Alpha with Style Benchmarks	Alpha Excess of Carhart's Four Factors	Alpha Excess of Market & Liquidity Premium
Panel A: Effects of staleness						
Staleness π	-0.11	**-0.16**	-0.12	**-0.15**	-0.01	**-0.08**
	(-1.63)	**(-2.22)**	(-1.61)	**(-2.17)**	(-0.31)	**(-2.03)**
Staleness ξ	**-0.65**	**-0.52**	**-0.56**	**-0.41**	**-0.30**	**-0.56**
	(-6.64)	**(-5.16)**	**(-6.40)**	**(-3.99)**	**(-3.75)**	**(-5.03)**
Beta (r_t, r_{mt-1})	-0.12	-0.07	-0.12	-0.05	0.03	-0.04
	(-1.48)	(-1.07)	(-1.59)	(-0.89)	(0.65)	(-0.71)
Beta(r_t,r_{t-1})	**-1.14**	**-0.97**	**-1.24**	**-1.20**	-0.12	**-0.93**
	(-9.56)	**(-7.81)**	**(-10.63)**	**(-9.62)**	(-1.19)	**(-7.65)**
Panel B: Effects of load						
Total Load	0.12	0.12	-0.07	-0.10	-0.03	0.14
(π)	(1.16)	(1.05)	(-0.74)	(-0.96)	(-0.40)	(1.40)
Total Load	0.13	0.13	-0.07	-0.10	-0.02	0.14
(ξ)	(1.20)	(1.12)	(-0.71)	(-0.92)	(-0.30)	(1.44)
Total Load	0.13	0.13	-0.06	-0.09	-0.03	0.15
(Beta (r_t, r_{mt-1}))	(1.29)	(1.14)	(-0.61)	(-0.88)	(-0.41)	(1.50)
Total Load	0.07	0.08	-0.12	-0.13	-0.05	0.11
(Beta(r_t,r_{t-1}))	(0.70)	(0.69)	(-1.18)	(-1.22)	(-0.57)	(1.06)
Panel C: Effects of interaction variables						
π*Total Load	0.01	0.05	0.00	0.04	-0.02	-0.01
	(0.29)	(0.78)	(0.08)	(0.77)	(-0.64)	(-0.33)
ξ*Total Load	0.01	-0.05	0.01	-0.01	-0.08	0.00
	(0.11)	(-0.64)	(0.19)	(-0.17)	(-1.21)	(-0.03)
Beta (r_t, r_{mt-1})*	**0.42**	**0.33**	**0.43**	**0.34**	0.05	**0.42**
Total Load	**(4.75)**	**(4.31)**	**(5.20)**	**(4.66)**	(0.79)	**(5.27)**
Beta(r_t,r_{t-1})*	**0.59**	**0.50**	**0.61**	**0.52**	0.10	**0.45**
Total Load	**(5.35)**	**(4.48)**	**(5.70)**	**(4.68)**	(1.11)	**(4.14)**

Table VI

Traditional Performance Measures and Stale Pricing Contemporary—Fama and MacBeth Approach

This table displays the relation between fund stale pricing and traditionally evaluated performance. The dependent variables are alphas in annual percentages. The explanatory variables are stale pricing, lagged fund characteristics, and fund styles. The discretionary turnover is the turnover component that is orthogonal to fund flows. The explanatory variables are studentized. Both alphas and stale pricing are estimated with 36 rolling window returns. The Fama-MacBeth coefficients and *t*-statistics are presented. The standard errors of the coefficients on staleness are adjusted by an MA(2) process.

	Alpha from Unconditional CAPM	Alpha from Conditional CAPM	Unconditional and Style Benchmarks	Conditional and Style Benchmarks	Carhart's Four Factors	Market & Liquidity Premium
Panel A: Staleness, π, measured as in Lo and MacKinlay (1990)						
Intercept	-0.11	0.16	0.29	0.05	-0.90	0.16
	(-0.16)	(0.28)	(1.11)	(0.16)	(-3.28)	(0.26)
Staleness	**-0.51**	**-0.45**	**-0.47**	**-0.41**	**-0.22**	**-0.42**
	(-7.11)	**(-5.88)**	**(-7.70)**	**(-5.57)**	**(-4.72)**	**(-7.00)**
Flow	-0.09	-0.07	-0.07	-0.04	-0.06	-0.11
	(-0.90)	(-0.96)	(-0.84)	(-0.54)	(-1.24)	(-1.07)
Age	**-0.25**	**-0.37**	**-0.29**	**-0.33**	**-0.27**	**-0.39**
	(-3.01)	**(-3.61)**	**(-3.53)**	**(-3.80)**	**(-5.60)**	**(-5.53)**
Logtna	**-0.37**	-0.29	-0.34	-0.29	-0.08	-0.19
	(-2.34)	(-1.65)	(-1.66)	(-1.31)	(-1.07)	(-1.22)
Income	0.03	-0.37	-0.09	-0.35	-0.31	-0.29
	(0.12)	(-1.04)	(-0.37)	(-0.80)	(-1.06)	(-0.79)
CAP_GNS	0.01	0.33	0.18	0.47	0.42	0.32
	(0.08)	(1.00)	(0.75)	(1.15)	(1.56)	(0.98)
Discretionary	0.31	0.35	0.29	0.22	0.06	0.19
Turnover	(1.66)	(1.53)	(1.84)	(1.22)	(0.61)	(0.93)
Total Load	-0.06	**-0.14**	0.00	-0.06	-0.03	-0.01
	(-0.94)	**(-3.55)**	(-0.02)	(-1.21)	(-0.45)	(-0.08)
Expense	**-0.64**	-0.53	**-0.66**	-0.64	**-0.59**	-0.34
	(-2.18)	(-1.65)	**(-2.1)**	(-1.98)	**(-5.04)**	(-1.46)
Styles			Controlled with dummy variables			
Panel B: Staleness, ξ, measured as in Getmansky, Lo, and Makorov (2004).						
Intercept	-0.11	0.16	0.29	0.05	-0.90	0.16
	(-0.16)	(0.28)	(1.11)	(0.16)	(-3.28)	(0.26)
Staleness	**-0.39**	**-0.32**	**-0.36**	**-0.31**	**-0.33**	**-0.43**
	(-2.48)	**(-2.41)**	**(-2.43)**	**(-2.32)**	**(-4.03)**	**(-2.56)**
Flow	-0.06	-0.05	-0.04	-0.02	-0.05	-0.09
	(-0.74)	(-0.78)	(-0.61)	(-0.30)	(-0.93)	(-0.92)
Age	**-0.24**	**-0.37**	**-0.28**	**-0.33**	**-0.28**	**-0.41**
	(-3.01)	**(-3.58)**	**(-3.70)**	**(-3.99)**	**(-5.46)**	**(-5.55)**
Logtna	**-0.38**	-0.30	-0.35	-0.30	-0.07	-0.18
	(-2.38)	(-1.68)	(-1.70)	(-1.34)	(-0.94)	(-1.19)
Income	-0.01	-0.40	-0.13	-0.38	-0.31	-0.29
	(-0.03)	(-1.07)	(-0.46)	(-0.82)	(-1.08)	(-0.80)
CAP_GNS	0.04	0.35	0.20	0.49	0.42	0.31
	(0.22)	(1.03)	(0.81)	(1.17)	(1.54)	(0.97)
Discretionary	0.30	0.34	0.29	0.21	0.05	0.18
Turnover	(1.69)	(1.50)	(1.85)	(1.19)	(0.59)	(0.90)
Total Load	-0.04	**-0.12**	0.01	-0.05	-0.02	0.00
	(-0.69)	**(-3.32)**	(0.16)	(-1.01)	(-0.41)	(-0.03)
Expense	**-0.65**	-0.54	**-0.66**	**-0.64**	**-0.60**	-0.35
	(-2.21)	(-1.68)	**(-2.14)**	**(-2.02)**	**(-5.22)**	(-1.52)
Styles			Controlled with dummy variables			

TableVII

Predictive Relation between Stale Pricing and Traditional Performance Measures

This table presents the relation between the traditional alphas and the predetermined stale pricing. The dependent variables are the traditional alphas in annual percentages. The explanatory variables are stale pricing, fund characteristics, and fund style. Both alphas and stale pricing are estimated with 36 rolling window returns. Stale pricing are lagged by 36 months and studentized. The discretionary turnover is the turnover orthogonal to fund flows. The Fama-MacBeth coefficients and *t*-statistics are presented. Standard errors of the coefficients on staleness are adjusted by an MA(2) process.

	Alpha from Unconditional CAPM	Alpha from Conditional CAPM	Unconditional and Style Benchmarks	Conditional and Style Benchmarks	Excess of Carhart's Four Factors	Excess of Market & Liquidity Premium
Panel A: Staleness, π, measured as in Lo and MacKinlay (1990).						
Intercept	-0.57	-0.34	-0.07	-0.33	-1.10	-0.23
	(-0.87)	(-0.60)	(-0.32)	(-1.34)	(-4.02)	(-0.39)
Staleness	**-0.11**	**-0.14**	**-0.11**	**-0.12**	-0.03	-0.06
	(-1.77)	**(-1.98)**	**(-1.66)**	**(-1.65)**	(-0.57)	(-0.94)
Flow	-0.07	-0.08	-0.08	-0.06	-0.03	-0.12
	(-0.75)	(-0.81)	(-0.89)	(-0.63)	(-0.39)	(-1.09)
Age	-0.18	**-0.29**	**-0.24**	**-0.27**	**-0.24**	**-0.33**
	(-1.43)	**(-2.04)**	**(-2.50)**	**(-2.48)**	**(-7.30)**	**(-3.50)**
Logtna	**-0.35**	-0.27	-0.34	-0.29	-0.05	-0.22
	(-2.44)	(-1.62)	(-1.66)	(-1.26)	(-0.64)	(-1.46)
Income	0.08	-0.22	0.00	-0.19	-0.14	-0.16
	(0.29)	(-0.83)	(0.03)	(-0.78)	(-1.30)	(-0.58)
CAP_GNS	-0.06	0.17	0.05	0.26	**0.16**	0.13
	(-0.34)	(0.98)	(0.31)	(1.26)	**(1.91)**	(0.78)
Discretionary	0.24	0.29	0.23	0.21	-0.08	0.10
Turnover	(1.54)	(1.35)	(1.56)	(1.16)	(-0.96)	(0.59)
Total Load	-0.07	**-0.15**	-0.02	-0.08	-0.04	-0.01
	(-0.80)	**(-2.44)**	(-0.26)	(-1.24)	(-0.42)	(-0.08)
Expense	**-0.81**	-0.68	**-0.83**	**-0.79**	**-0.69**	**-0.47**
	(-2.44)	(-1.87)	**(-2.28)**	**(-2.07)**	**(-5.38)**	**(-1.92)**
Styles	Controlled with dummy variables					
Panel B: Staleness, ξ, measured as in Getmansky, Lo, and Makorov (2004).						
Intercept	-0.57	-0.34	-0.07	-0.33	-1.10	-0.23
	(-0.87)	(-0.6)	(-0.32)	(-1.34)	(-4.02)	(-0.39)
Staleness	**-0.15**	-0.16	-0.13	-0.13	**-0.12**	-0.08
	(-1.74)	(-1.64)	(-1.46)	(-1.28)	**(-1.74)**	(-0.88)
Flow	-0.07	-0.07	-0.08	-0.05	-0.02	-0.09
	(-0.70)	(-0.75)	(-0.84)	(-0.58)	(-0.21)	(-0.86)
Age	-0.17	**-0.29**	**-0.24**	**-0.27**	**-0.23**	**-0.32**
	(-1.41)	**(-2.01)**	**(-2.47)**	**(-2.43)**	**(-6.59)**	**(-3.40)**
Logtna	**-0.35**	-0.27	-0.33	-0.28	-0.04	-0.21
	(-2.46)	(-1.63)	(-1.67)	(-1.28)	(-0.61)	(-1.42)
Income	0.06	-0.23	-0.01	-0.18	-0.15	-0.17
	(0.22)	(-0.86)	(-0.05)	(-0.77)	(-1.48)	(-0.62)
CAP_GNS	-0.04	0.18	0.06	0.26	**0.17**	0.14
	(-0.23)	(1.02)	(0.41)	(1.25)	**(2.13)**	(0.84)
Discretionary	0.24	0.29	0.23	0.21	-0.09	0.09
Turnover	(1.52)	(1.34)	(1.55)	(1.16)	(-1.00)	(0.53)
Total Load	-0.07	**-0.16**	-0.02	-0.09	-0.04	-0.02
	(-0.81)	**(-2.59)**	(-0.25)	(-1.34)	(-0.48)	(-0.17)
Expense	**-0.79**	-0.68	**-0.82**	**-0.78**	**-0.68**	**-0.46**
	(-2.41)	(-1.86)	**(-2.26)**	**(-2.07)**	**(-5.42)**	**(-1.86)**
Styles	Controlled with dummy variables					

Table VIII
The Attenuation Bias and the Bias Caused by Correlated Errors

This table presents the estimated attenuation bias and the bias caused by correlated errors in the two-stage regressions. The system (A8.9) to (A8.13) in appendix A8 is estimated using eight funds, each randomly drawn from a style group. Each row presents a random drawn sample. $Cov(\varepsilon_1, \varepsilon_2)$, $Var(\varepsilon_2)$, A, Φ, and implied g are given in the table.

$$\hat{g} = Cov(\hat{a}_1, \hat{a}_2) / Var(\hat{a}_2) = g A + \Phi$$

where g *is the true slope,* $\hat{g}$ *is the estimated slope,*

$$A = Var(a_2) / [Var(a_2) + (x_2' x_2)^{-1} Var(\varepsilon_2)],$$

and

$$\Phi = (x_1' x_1)^{-1} (x_1' x_2)(x_2' x_2)^{-1} Cov(\varepsilon_1, \varepsilon_2) / [Var(a_2) + (x_2' x_2)^{-1} Var(\varepsilon_2)]$$

x_1 *and* x_2 are explanatory variables in the first stage regressions, in which a_1 and a_2 are estimated. a_1 is dependent variable in the second stage and a_2 is the explanatory variable in the second stage regression. ε_1 and ε_2 are observation errors for a_1 and $a_2$in the second stage regression.

$Cov(e_1, e_2)$ is computed to proxy for $Cov(\varepsilon_1, \varepsilon_2)$ and $Var(e_2)$ for $Var(\varepsilon_2)$. $Var(a_2)$ *equals* $Var(\hat{a}_2) - (x_2' x_2)^{-1} Var(\varepsilon_2)$. The attenuation bias A, and the bias caused by correlated errors Φ, are also computed. This procedure is repeated ten times with funds redrawn randomly for each iteration.

		$Cov(\varepsilon_1, \varepsilon_2)$	var(ε_2)	A	Φ	Implied g If $\hat{g} = 40$ Basis Points
Jan.1992–Dec. 2000	1	0.1477	0.3052	0.8998	0.00101	**33**
	2	0.2054	0.3800	0.9067	0.00037	**40**
	3	0.2054	0.3800	0.9067	0.00037	**40**
	4	0.2045	0.3849	0.8885	0.00020	**43**
	5	0.2045	0.3849	0.8885	0.00020	**43**
Jan. 1981–Dec.1989	6	0.2605	0.2759	0.9258	0.00477	**-8**
	7	0.1926	0.2056	0.9950	0.00034	**37**
	8	0.2061	0.2179	0.9973	0.00017	**38**
	9	0.2528	0.2677	0.9961	0.00023	**38**
	10	0.2192	0.2310	0.9946	0.00042	**36**

Table IX
Performance Estimated with the Proposed Model and Stale Pricing using a 2-Stage Approach

This table displays the relation between the proposed performance measure and the stale pricing of the fund. The dependent variable is the true alpha estimated from the proposed model. The explanatory variables are stale pricing measured using equation (9a)–(9d), fund characteristics, and fund styles. The stale pricing measure and the fund characteristics are studentized. The sample period is 1991-2004.

	(1)	(2)	(3)	(4)
Intercept	2.66	2.18	2.67	2.68
	(6.68)	(4.63)	(6.78)	(6.81)
Staleness π	-0.31			
	(-0.47)			
Staleness ξ		0.47		
		(0.80)		
Staleness: Beta (r_t, r_{mt-1})			-0.01	
			(-0.01)	
Staleness: Beta(r_t,r_{t-1})				0.56
				(0.40)
Flow	0.62	0.66	0.63	0.62
	(1.25)	(1.27)	(1.25)	(1.24)
Age	**-0.44**	**-0.40**	**-0.44**	**-0.44**
	(-4.74)	**(-4.21)**	**(-4.71)**	**(-4.74)**
Logtna	**0.78**	**0.82**	**0.79**	**0.79**
	(3.44)	**(3.42)**	**(3.47)**	**(3.49)**
Income	**-0.12**	**-0.13**	**-0.12**	**-0.12**
	(-2.21)	**(-2.35)**	**(-2.26)**	**(-2.24)**
CAP_GNS	0.23	0.19	0.23	0.23
	(1.15)	(0.95)	(1.14)	(1.14)
Discretionary Turnover	-0.11	-0.14	-0.13	-0.13
	(-0.51)	(-0.65)	(-0.62)	(-0.61)
Total Load	**0.70**	**0.69**	**0.70**	**0.70**
	(2.24)	**(2.21)**	**(2.24)**	**(2.24)**
Expense	**-0.52**	**-0.44**	**-0.51**	**-0.52**
	(-2.68)	**(-2.19)**	**(-2.68)**	**(-2.69)**
Growth	**-1.36**	**-1.17**	**-1.37**	**-1.36**
	(-3.08)	**(-2.57)**	**(-3.10)**	**(-3.09)**
Other	0.21	0.33	0.18	0.19
	(0.24)	(0.35)	(0.2)	(0.21)
Income	**-2.13**	**-1.92**	**-2.15**	**-2.13**
	(-4.43)	**(-3.75)**	**(-4.44)**	**(-4.38)**
G & I	**-2.04**	**-1.89**	**-2.05**	**-2.05**
	(-4.84)	**(-4.24)**	**(-4.85)**	**(-4.84)**
Sector	0.43	0.94	0.39	0.40
	(0.54)	(1.20)	(0.50)	(0.51)
SCG	**6.90**	**7.17**	**6.89**	**6.90**
	(11.57)	**(11.19)**	**(11.51)**	**(11.53)**
Timing	**-1.53**	**-1.30**	**-1.52**	**-1.52**
	(-3.69)	**(-3.04)**	**(-3.69)**	**(-3.68)**
# of obs.	**2541**	2318	2541	2541
F-Test	**27.99**	24.39	27.8	27.88
R^2	**0.15**	0.15	0.15	0.15

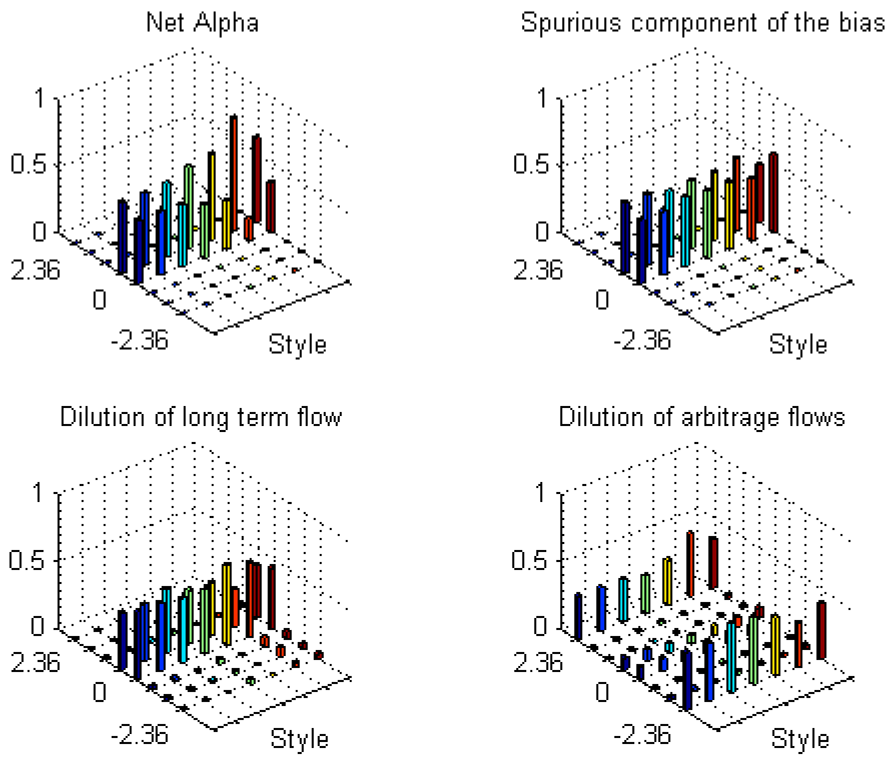

Figure A: Distribution of *t*-statistics of Alpha and Bias Components in the Observed Performance

The GMM system (8a) to (8h) is estimated fund by fund, and their *t*-statistics are grouped and then plotted by style group. The four plots are for the net alpha, the spurious bias, the dilution of the long-term flows, and the dilution of the arbitrage flows.

The X-axis presents the fund style group. From 1–7, they are growth funds, other aggressive growth, income, growth and income, sector, small capital growth, timing.

The Y-axis presents the range of the *t*-values. From 1-8, they are $t>2.36$, $1.96<t<2.36$, $0<t<1.96$, $0<t<-1.96$, $-1.96<t<-2.36$, $t<-2.36$.

The Z-axis presents the percentage of funds that fall into the range of the *t*-values for each style group.