Commodities and Equities: A "Market of One"?

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January 12, 2008

Abstract

Amidst a sharp rise in commodity investing, many have asked whether commodities nowadays move in sync with traditional financial assets. We provide evidence that challenges this idea. Using dynamic correlation and recursive cointegration techniques, we find that the relation between the prices of, and the returns on, investable commodity and equity indices has not changed significantly in the last fifteen years. First, correlations between daily, weekly or monthly returns on equity and commodity investments have remained low. Second, with the exception of the late 1990's, we find little statistical evidence of cointegration between equity and commodity prices – and none in the last eight years. Strikingly, the only period when a common long term factor drives both equity and commodity prices starts with the Asian crisis and ends shortly after Russia's default on its foreign debt. Finally, we find no evidence of a secular increase in co-movement between commodities and equities during periods of extreme daily or weekly returns. Our results have important implications for investors' ability to diversify portfolios and for risk-sharing amidst increased integration of asset markets.

JEL Classification: G10, G13, L89
Keywords: Commodities, Equities, Dynamic conditional correlations, DCC, Cointegration, Extreme-event returns.

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We thank Celso Brunetti, Francesca Carrieri, Vihang Errunza, Pat Fishe, Andrei Kirilenko, Delphine Lautier, Jim Moser, David Reiffen, and Andy Smith for helpful comments and suggestions. All remaining errors and omissions, if any, are the authors' sole responsibility. This paper reflects the opinions of its authors only, and not those of the CFTC, the Commissioners, or any other staff upon the Commission.

``As more money has chased (...) risky assets, correlations have risen. By the same logic, at moments when investors become risk-averse and want to cut their positions, these asset classes tend to fall together. The effect can be particularly dramatic if the asset classes are small -- as in commodities. (...) This marching-in-step has been described (...) as a 'market of one'." The Economist, March 8, 2007.

1. Introduction

In the past decade, investors have sought an ever greater exposure to commodity prices – by directly purchasing commodities, by taking outright positions in commodity futures, or by acquiring stakes in exchange-traded commodity funds (ETFs) and in commodity index funds. This pattern has accelerated in the last few years. To wit, Standard and Poor's GSCI index was created by Goldman Sachs in 1991. This world-production-weighted index tracks the prices of major physical commodities for which there are active, liquid futures markets. As recently as 1999, the sums invested in investment vehicles tracking this index were estimated at less than 5 billion dollars. Nowadays, however, investments linked to the GSCI or to one of the other prominent commodity indices exceed 130 billion dollars. In a similar vein, the first-ever commodity exchange traded fund (the streetTRACKS Gold Shares ETF) was started in November 2004. As of the end of 2007, its market capitalization exceeds 15 billion dollars, and it has been joined by numerous commodity ETF competitors.

One naturally wonders whether this sharp increase in investor appetite for commodities has had a significant impact on the pricing of commodity-related financial instruments. One reason why it could have had an impact is if the large-scale arrival of financial institutions in commodity markets has led to a reduced scope for cross-market arbitrage opportunities (as in Basak and Croitoru, 2006) and, in the process, has more closely linked commodity and equity markets. Another channel for tighter links between commodity and equity markets is if financial institutions respond differently from traditional commercial traders to extreme stock market movements – in particular, if sharp downward movements in one market force financial investors to liquidate positions in commodity markets so as to raise cash for margin calls.[2] In this paper, we investigate the relation between ordinary as well as between extreme returns on passive investments in commodity and equity markets.

Because much of the new commodity exposure has been achieved through direct or indirect participation in futures markets, it should be reflected in the magnitude and composition of commodity futures trading. Haigh, Harris, Overdahl, and Robe (2007; henceforth, HHOR) confirm this intuition, using proprietary data on trader positions in the world's largest-volume futures contract on a physical commodity – the New York Mercantile Exchange's WTI sweet crude oil futures. HHOR show that greater market participation by commodity swap dealers and hedge funds has been accompanied by a change in the relation between crude futures prices at different maturities and by greater price efficiency. Specifically, the prices of one-year and two-year futures have become cointegrated with the price of near-month futures, for the first time ever, since mid-2004.

[2] For an early formal discussion of the link between margin calls and financial contagion, see Calvo (1998).

Whereas this extant research has documented that the prices of different-maturity commodity futures have recently become much more closely linked, we use dynamic correlation and recursive cointegration techniques to show that the degree of co-movement between benchmark commodity- and equity-investment returns has not changed materially over the course of the last fifteen years. In particular, notwithstanding the surge in commodity investment, the already very low correlation between the rates of return on passive investments in these two asset classes has become negative in the last five years. Our results are similar in spirit to the finding that, despite increased capital flows to emerging markets in the years following their financial liberalization and despite greater integration with world equity markets, cross-market return correlations did not increase enough to diminish the benefit, to U.S. investors, of diversifying into emerging-market stocks (Bekaert and Harvey, 2000; Carrieri, Errunza and Hogan, 2007).

We use Standard and Poor's S&P 500 return and GSCI total return data to proxy for the rates of return on representative unlevered investments in, respectively, U.S. equities and commodities. We obtain qualitatively similar results with two other widely-used indices: Dow Jones' DJIA equity and DJ-AIGTR commodity indices[3].

Because much of the commodity investment boom is still quite new, any change in pricing relationships is likely to be a recent phenomenon. HHOR, for example, do not find pricing efficiency changes across crude oil futures maturities until late 2003 (for one-year contracts) or mid-2004 (for two-year contracts). It is therefore important to utilize recent data. Accordingly, we use daily, weekly and monthly returns from January 15th, 1991 (when GSCI products first became available) to July 2nd, 2007.

To identify possible changes in the co-movements between the asset return series, we run all of our analyses on the entire sample period and then focus in particular on three successive five-year sub-periods: June 1992 through May 1997; June 1997 through May 2002; and, June 2002 through June 2007. The first subperiod predates the commodity investment boom, while the third subperiod overlaps with that boom. These two subperiods, however, correspond to times of economic expansion. The second subperiod allows us to assess the relation between commodities and equities during the stock-market bubble and its immediate aftermath -- including an economic contraction, as defined by the National Bureau of Economic Research (NBER).

We find statistically significant differences in the means and standard deviations of the rates of returns across the two asset classes and, for each asset class, across the three sub-periods. By contrast, we find only small differences in cross-asset correlations for the three sub-periods. The simple correlation between equities and commodities, which was slightly positive between 1992 and 1997, becomes slightly negative between 2002 and 2007. We obtain qualitatively similar results at all return frequencies.[4]

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[3] Unlike the GSCI, which uses weights that reflect world-production figures and is consequently heavily tilted toward energy commodities, the DJ-AIG commodity index is specifically designed to provide a "diversified benchmark for the commodity futures market." In particular, it assigns a weight of only about 30% to energy commodities, including about 13% to crude oil. By comparison, as of mid-July, 2007, the GSCI assigned a weight of more than 70% to energy commodities, including 36% to crude oil (WTI nearby contract). Other GSCI competitors include the Deutsche Bank Liquid Commodity Index, Rogers International Commodity Index, and Reuters-CRB.

[4] In the case of monthly returns, the correlation drops from 0.27 in 1992-1997 (statistically significantly different from zero at the 5% confidence level) down to -0.24 in 2002-2007 (10% significance level). In the

Notwithstanding the relative constancy of the simple cross-correlations across our three sub-periods, we find that rolling measures of the correlation between the equity and commodity return series fluctuate substantially throughout the sample period. The *pattern* of fluctuations, however, does not appear to change during the entire sample period. We confirm these findings using the dynamic conditional correlation (DCC) methodology proposed by Engle (2002). On the one hand, the range of values taken by DCC estimates is quite wide; weekly values, for example, can be as low as -0.5 or as high as $+0.5$. On the other hand, most of the time, the DCC estimates are close to 0. What is more important, we find no evidence of a secular increase in correlations in the last few years.

Correlation estimates are relevant for short-term investors. For long-term investors, however, the key issue is whether there exist long-term common trends between the prices of commodity and equity investment even though these prices may diverge in the short term (Kasa, 1992). To answer this question, we apply recursive cointegration techniques (Johansen, 1998,1991; Johansen and Juselius,1990) to examine the stability and the possible strengthening over time of the relation between equity- and commodity-investment price series.

This analysis complements our other results: with the exception of a period in the late 1990's, we find little statistical evidence of cointegration – and none in the last eight years. That is, equity and commodity investment vehicles do not appear to share a common driving factor over long horizons and, hence, passive investors can still achieve substantial gains by diversifying portfolios across the two asset classes. Strikingly, the only time when a common long-term factor drives both equity and commodity prices starts with Thailand's devaluation of the baht and ends shortly after Russia's devaluation of the ruble and moratorium on its foreign debt repayments.

Even though there is little evidence of any structural shift in correlation and cointegration levels, a logical follow-up question is whether financial and commodity markets mights have become a "market of one" *during extreme events*. Hartmann, Straetmans and de Vries (2004), for example, find evidence of cross-asset extreme linkages in the case of bond and equity returns from the G-5 countries. Using a different approach, Longin and Solnik (2001) provide evidence that international equity-market correlations do not jump during periods of high volatility but do increase during bear markets.

Here, we first identify the days and weeks during which returns on equity indices were at least one or two standard deviations away from their means, and then analyze the contemporaneous returns on investable commodity indices. Contrary to extant findings on linkages between other asset markets, this first analysis finds little relation between exceptionally large returns on equities and those on commodities. This is true for the whole sample period as well as for all three of the five-year sub-periods; for positive as well as for negative exceptional returns; and, for periods of stock market upturns as well as downturns. We then use a technique similar to Longin and Solnik's (2001) to analyze equity-commodity linkages when the returns on equity and commodity indices both take values in the tails of their respective distributions. Again, we find that extreme-event correlations are weak. Interestingly, whereas the equity-commodity correlation is mildly positive in the upper tail, it is negative in the lower tail. That is, conditional on both

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case of daily and weekly returns, the simple cross-correlation levels also fall from one sub-period to the next, but they are never statistically significantly different from zero.

equity and commodity returns being very poor, the two return series are negatively correlated.

In sum, the lack of greater return co-movement across equities and commodities suggests that commodities should retain their role as a portfolio diversification tool. The import of this conclusion cannot be overstated, since academics and practitioners have long called for substantial allocations to commodities as an asset class for the purposes of return generation and portfolio diversification.[5]

The remainder of the paper proceeds as follows. Section 2 describes the data. Section 3 presents the correlation analyses. Section 4 shows the robustness of our results to alternative methodological choices. Section 5 concludes.

2. Data and descriptive statistics

This section discusses the data and gives summary statistics for the return series.

2.1 Returns data

We take the perspective of a passive investor on the relation between commodities and traditional financial investments. To assess short-term correlations, we use daily, weekly, and monthly returns on four widely used commodity and equity indices. We focus on results for weekly (Tuesday to Tuesday) holding-period returns, and provide a brief discussion of our (similar) findings for daily and monthly returns. To analyze long-term cointegration, we use the Tuesday close prices for the same four indices.

For equities, we use Standard and Poor's S&P 500 index; in robustness checks, we use Dow-Jones's DJIA index.[6] For commodities, we focus on the unlevered total return on Standard and Poor's S&P GSCI (formerly, the Goldman Sachs Commodity Index), i.e., the return on a "fully collateralized commodity futures investment that is rolled forward from the fifth to the ninth business day of each month." While the GSCI includes twenty-four nearby commodity futures contracts, it is heavily weighted toward energy. For robustness checks, we use total (unlevered) returns on the second most widely used investable benchmark, Dow-Jones's DJ-AIG commodity index (henceforth, DJ-AIG).

[5] See, e.g., Ankrim and Hensel (1993), Froot (1995), Huberman (1995), and Satyanarayan and Varangis (1996) for early work on how commodities help reduce an investor's unconditional portfolio risk. See also Erb and Harvey (2006), Gorton and Rouwenhorst (2006), and Miffre and Rallis (2007) for evidence on the strategic and tactical values of commodity investments. The data series in these newer papers end in 2004 and, hence, do not cover the period during which took place much of the growth in financial traders' positions in commodity futures markets.

[6] We use returns on both of these equity indices that are exclusive of dividend yields. This approach leads to an underestimation of the expected returns on equity investments (Shoven and Sialm, 2000). However, insofar as large U.S. corporations smooth dividend payments over time (Allen and Michaely, 2002), the correlation estimates that are the focus of our paper should be essentially unaffected.

This rolling index, which is composed of futures contracts on nineteen physical commodities, was designed to provide a "diversified benchmark for the commodity futures market."

We also analyze potential changes in the relation between the rates of returns on various types of commodities. For this purpose, we use daily, weekly, and monthly total returns on several investable sub-indices representing key components of the GSCI: Energy, Non-Energy, Industrial Metals, Precious Metals, Agriculture, and Livestock.

We obtain the return series from Bridge-CRB (GSCI, DJ-AIG, S&P 500 and DJIA) or Bloomberg (GSCI sub-indices). Our data cover more than sixteen years from January 15, 1991 to July 2, 2007. We also provide results for three successive five-year subperiods: June 2, 1992 to May 27, 1997; June 3, 1997 to May 28, 2002; and, May 28, 2002 to July 2, 2007.

2.2 Returns on Equity and Commodity Indices: Summary Statistics

Table 1 presents some descriptive statistics for the two equity- and the two commodity-return series. For weekly returns, Table 1A presents statistics for the entire sample period, while Tables 1B, 1C and 1D present the corresponding statistics for each of the three successive five-year subperiods. Panels E and F present statistics for the entire sample period for, respectively, daily and monthly returns.

From January 1991 through mid-2007, the mean weekly total rate of return on the GSCI was 0.14% (or 7.55% in annualized terms), with a minimum of -13.57% and a maximum of 8.09%. The typical rate of return varies sharply across the sample period: it averaged 0.14% in 1992-1997 (7.58 % annualized); 0.0038% in 1997-2002 (or a mere 0.20% annualized); and, 0.28% in 2002-2007 (15.63% annualized). The corresponding figures are very similar for the DJ-AIG total return index. The one exception is the first subperiod (1992-1997), when the average return was 0.21% for the DJ-AIG *versus* 0.14% for the GSCI; Figure 1, which plots the levels of the four indices, indeed shows that the GSCI did not start appreciating until the end of 1996 whereas the DJ-AIG started appreciating in 1994.

During the sample period, the mean weekly rate of return on the S&P 500 was half again as high as that on commodities: 0.20% for the whole period (or 11.20% in annualized terms), with a minimum of -11.46% and a maximum of 13.17%. Notably, the lowest weekly rate of return on the two equity indices is found in the third sub-period *versus* in the second sub-period for the commodity indices. In the same vein, the median weekly rate of return on the S&P GSCI was negative (-0.24% on the GSCI and -0.13% on the DJ-AIG) between June 1997 and May 2002, whereas the S&P 500 equity index had its highest median weekly rate of return during the same period (+0.37%). These observations suggest that equities and commodities do not move together.

Consistent with the fact that the DJ-AIG is by construction more diversified than is the GSCI, the standard deviation of the weekly rates of return is much lower for the DJ-AIG (1.77% for the whole sample) than for the GSCI (2.63%). This pattern of approximately 45% greater GSCI volatility is observed in all three sub-periods: 1.80% *vs.* 1.26% in 1992-1997; 2.77% *vs.* 1.85% in 1997-2002; and, 3.18% *vs.* 2.18% in 2002-2007. Standard deviations increase throughout the sample for commodities, while they peak in the second sub-period for equities. Interestingly, the standard deviations of the equity returns always fall within those of the two commodity returns, with the DJ-AIG (*GSCI*) volatility playing the role of a lower (*upper*) bound.

Panels E and F show similar patterns for daily and monthly returns that Panels A to D showed for weekly returns, i.e.:

- Between 1991 and 2007, the rates of return on commodity indices were significantly lower than those on equity indices. However, this rank-ordering fluctuates dramatically over the course of that entire period. For example, equity returns trounce commodity returns in 1997-2002, but commodity returns are almost double equity returns in 2002-2007.
- The rates of return on equites are somewhat more volatile than those on a well-diversified basket of commodities (represented by the DJ-AIG), except in the last five years (2002-2007).
- The rates of return on the GSCI are the most volatile throughout the entire sample period. Of note, the GSCI returns are approximately 40-50% more volatile than those on the DJ-AIG.

2.3 Simple Cross-Asset Correlations

Figure 1 gives some preliminary insights into the co-movements between the commodity and equity indices. This graph allows the reader to visualize which sub-periods help determine the co-movements between the index returns that are summarized by the correlations presented in Table 2. In particular, it suggests a high correlation between the two equity indices; a positive, but somewhat weaker, correlation between the two commodity indices; and, a weak or possibly negative correlation between the equity and commodity indices, especially during the second sub-period (June 1997 through May 2002).

Table 2 quantifies these first impressions by providing an overview of the simple correlations between the two four benchmark asset-return series. This summary table is helpful for the interpretation of the empirical results in Section 3. As in Table 1, Panels A to D are for weekly returns; E, daily returns; and F, monthly returns. For weekly returns, Table 2A presents statistics for the entire sample period, while Tables 2B, 2C and 2D present the corresponding statistics for each of the three successive five-year subperiods.

As one would expect, the simple correlation between the returns on the DJIA and S&P 500 equity indices is very high (more than 0.92 from 1991 to 2007), especially in the last five years (0.97). Likewise, the rates of return on the two commodity indices are strongly positively correlated. At all three return frequencies, the simple correlation is 0.89 for the whole sample; it is strongest in the second sub-period (0.94) and is slightly weaker in 1992-1997 (between 0.85 and 0.89, depending on the return frequency).

In sharp contrast, equity-commodity cross-correlations are typically very low or even negative:

- In the case of **daily returns**, Table 2E shows that the rates of return on the commodity indices exhibit very little correlation with either of the equity returns, with the coefficient estimates ranging from -0.08 to 0.01 depending on the index pair and the time period.

- For **weekly returns**, Table 2B and 2C show that the highest weekly correlations, a mere 0.06 to 0.14 depending on the index, were observed in the first (1992-1997) and second (1997-2002) sub-periods.
- For **monthly returns**, equity-commodity correlations are slightly larger in absolute value, yet over the entire sample they are not statistically significantly different from zero. The only statistically significant correlations are observed for the GSCI. However, while the GSCI's correlation with the S&P 500 and the DJIA was 0.27 in 1992-1997 (statistically significantly positive at the 5% level), this correlation became statistically significantly *negative* in 2002-2007 (-0.25 with the DJIA and -0.3 with the S&P 500).

In short, despite a commonly-expressed view that both equity and commodity *prices* have boomed since 2003, the correlation between commodity and equity *returns* is almost nil in our third subperiod – indeed, the total returns on the GSCI are negatively correlated with the returns on both equity indices during that period between June 2002 and July 2007. Figure 1 suggests that, to the extent that the correlations were at all positive prior to 2002, the likely reasons are joint run-ups in commodity and equity prices in 1995-1997 and again in the eighteen month period from late 1998 through Spring 2000.

2.4 Returns on Specific Categories of Commodities

2.4.1 Summary Statistics

Table 3 provides summary statistics for the unlevered (total) rates of return on six investable sub-indices representing key components of the S&P GSCI index: the GSCI Energy, Non-Energy, Industrial Metals, Precious Metals, Agriculture, and Livestock investable indices. Table 3A presents statistics for the entire sample period; Tables 3B, 3C and 3D, for each of our three successive five-year subperiods. Table 3 focuses on weekly returns for the sake of brevity, because the results are similar for daily and monthly return series.

Table 3 shows that, over then entire sample period, individuals who invested in Energy or Metal sub-indices experienced greater average returns (but also more volatility) than investors in other commodity sub-indices. Panels B to D of the same table, however, show that the performance rankings vary significantly from period to period. Industrial as well as Precious Metals, for example, both underperform all other commodity sub-indices between 1992 and 1997, but beat all but Energy between 2002 and 2007. In a similar vein, Agriculture outperforms all other sub-indices in 1992-1997 but is the worst performer in 2002-2007.

2.4.2 Simple Correlations

Table 4 shows the simple correlations between the unlevered rates of return on the S&P 500 equity index, the S&P GSCI, and the six narrow commodity benchmarks introduced in Table 3. Again, Table 4 focuses on weekly returns. Table 4A presents statistics for the entire sample period, while Tables 4B, 4C and 4D present the corresponding statistics for each of our three successive five-year subperiods. Four patterns emerge from Table 4:

- Equity returns exhibit very little correlation with the returns on *any* of the commodity sub-indices. The highest individual correlation is for Industrial Metals, but even that figure is not statistically significantly different from zero. It is a mere 0.13 over the whole sample (Table 4A), peaking at 0.18 in 2002-2007 (Table 4D). All the other cross-correlations are less than 0.12, and quite a few are slightly negative.
- There is no evidence of a material increase, over time, of the correlation between the returns on equities and those on either the Agriculture or the Livestock sub-indices.
- Consistent with the fact that the GSCI is a value-weighted index and is consequently heavily weighted toward energy (as energy contracts make up the world's largest commodity futures markets), the unlevered returns on the GSCI and on the Energy sub-index are very highly positively correlated – between 0.94 and 0.98 depending on the sample period. In contrast, the correlation between the returns on the entire GSCI index and those on the Non-Energy sub-index range from 0.38 to 0.41 depending on the sub-period.
- The returns on the Non-Energy sub-index are strongly positively correlated with the returns on all the other GSCI sub-indices (but not with the Energy sub-index). This finding suggests the possibility of a common economic variable driving the returns on most types of commodities.

3. Short-Term Co-Movements

Tables 1 and 3 show that the unconditional return volatilities vary a lot over time. In particular, the weekly rates of return on equities were 50% more volatile in the third subperiod (2002-2007, Table 1D) than in the first (1992-1997, Table 1B). Even more strikingly, the standard deviation of the returns on commodity investments almost doubled over the course of our entire sample period.

In contrast, although the unconditional correlations between the rates of returns on equity and commodity investments vary somewhat from sub-period to sub-period, Tables 2 and 4 suggest that these fluctuations are quite mild and that the correlations are always close to zero. Put differently, the analysis in Section 2 seemed to suggest that commodity returns exhibit consistently low correlations with their equity counterparts.

Before concluding that commodities provide a good hedge for equity portfolios, however, one should account for possible time variations in these correlation measures. In this Section, we provide estimates of the intensity of co-movements (or the lack thereof) that account for time variations in the various moments of the return series.

3.1 Methodology

Measuring the relationship between variables at various points in time, rather than using a single correlation coefficient over the entire sample period, provides information on the evolution of the relationship over time. For this purpose, simple correlation measures such as rolling historical correlations and exponential smoothing are widely used in the literature.

Rolling historical correlations take into account the time-varying nature of the relationship between variables straightforwardly, by calculating the correlation at any point in time as the estimate for a specified window (say, k observations) that does not overlap with the full sample. The correlation is first estimated over sub-periods 1 to k, then over sub-periods 2 to $k+1$, and so on. The rolling historical correlation estimator is thus:

$$\hat{\rho}_{12,t+1} = \frac{\sum_{s=t-k}^{t} x_{1,s} x_{2,s}}{\sqrt{\left(\sum_{s=t-k}^{t} x_{1,s}^2\right)\left(\sum_{s=t-k}^{t} x_{2,s}^2\right)}} \tag{1}$$

where x_1 and x_2 are the deviations from the means of the two random variables of interest, with mean zero.

Although this simple estimation technique provides some information on the evolution of relationship between two variables, it suffers from assigning an equal weight to all observation in the estimation window and zero weight to older observations. It also raises the issue of window-length determination. On the one hand, if the window is too narrow, one runs the risk of ignoring important observations in the data by giving zero weight to these observations. On the other hand, if the window is too wide, old observations will be given weight even though they may not be relevant to the analysis.

To overcome these problems, exponential smoothing techniques assign declining weights to older observations based on a parameter, λ, without any prior determination on the amount of past data to be used in the analysis. The exponential-smoothing estimator can be written as

$$\hat{\rho}_{12,t+1} = \frac{\sum_{s=1}^{t} \lambda^{t-s} x_{1,s} x_{2,s}}{\sqrt{\left(\sum_{s=1}^{t} \lambda^{t-s} x_{1,s}^2\right)\left(\sum_{s=1}^{t} \lambda^{t-s} x_{2,s}^2\right)}} \tag{2}$$

One drawback of this second approach is that the user must adopt an *ad hoc* approach to choose smoothing parameter λ. Engle (2002) used $\lambda = 0.94$ to analyze daily returns on the major equity indices. We use the same value for monthly and weekly returns, but set $\lambda = 0.97$ for daily returns.

More importantly, like the rolling historical correlation, the exponential-smoothing technique cannot adequately account for changes in volatility. The sensitivity of the estimated correlation to volatility changes restricts inferences about the true nature of the relationship between variables. Since the estimated correlations are subject to volatility shocks, interpreting these correlations becomes more difficult especially during high volatility periods.

The Dynamic Conditional Correlation methodology (DCC) developed by Engle (2002) helps to remedy this problem. The DCC model is based on a two-steps approach to estimating the time-varying correlation between two series. In the first step, time-varying variances are estimated using a GARCH model. In the second step, a time-varying correlation matrix is estimated using the standardized residuals from the first-stage estimation.

More formally, consider a $n \times 1$ vector of normally-distributed with mean zero and covariance matrix H_t returns series r_t of n assets are assumed the have the following structure:

$$r_t \sim N(0, H_t) \tag{3}$$

$$H_t = D_t R_t D_t \tag{4}$$

where, H_t is the conditional covariance matrix; R_t is the time varying correlation matrix; and, D_t is a diagonal matrix of time-varying standard deviations given by $D_t = \text{diag}\sqrt{E_{t-1}(r_{i,t}^2)} = \text{diag}\sqrt{h_{i,t}}$. The $h_{i,t}$ can be thought of as univariate GARCH models, so the standardized disturbance can be expressed as $\varepsilon_{i,t} = r_{i,t} / \sqrt{h_{i,t}} = D_t^{-1} r_{i,t}$, where $\varepsilon_{i,t} \sim N(0, R_t)$. Consider the following conditional correlations:

$$\rho_{ij,t} = \frac{E_{t-1}[r_{i,t} r_{j,t}]}{\sqrt{E_{t-1}[r_{i,t}^2 r_{j,t}^2]}} \tag{5}$$

Re-writing these conditional correlation in terms of standardized residuals from GARCH estimates yields:

$$\rho_{ij,t} = E_{t-1} \varepsilon_{i,t} \varepsilon_{j,t} \tag{6}$$

This implies the equivalence of conditional correlation of returns and conditional covariance between the standardized disturbances. Therefore, the matrix R represent the time-varying conditional correlation matrix of returns as well as the conditional covariance matrix of the standardized residuals (Engle ,2002).

The DCC model of Engle (2002) suggest the following dynamics of the correlation matrix:

$$R_t = Q_{\text{t}}^{*-1} Q_t Q_t^{*-1} \tag{7}$$

$$Q_t = (1 - \alpha - \beta)\overline{Q} + \alpha(\varepsilon_{i,t-1}\varepsilon_{j,t-1}) + \beta Q_{t-1} \tag{8}$$

where $\overline{Q}$ is the unconditional correlation matrix of standardized residuals and Q^*_t is a diagonal matrix composed of square root of the diagonal elements of Q_t. The correlation estimator is given by the typical element of R_t in the form of

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}}$$

This specification ensures the mean reversion as long as $\alpha + \beta < 1$. The resulting estimator is called DCC by loglikelihood with mean reverting model. The log-likelihood of the DCC model outlined above is given by :

$$L = -\frac{1}{2}\sum_{t=1}^{T}(n\log(2\pi) + 2\log(|D_t|) + \log(|R_t|) + \varepsilon' R_T^{-1}\varepsilon)$$

In essence, the log-likelihood function has two components: the volatility part, which contains terms in D_t; and the correlation part, which contains terms in R_t. In the first stage of the estimation, n univariate GARCH(1,1) estimates are obtained, which produces consistent estimates of time-varying variances (D_t). In the second stage, the correlation part of the log-likelihood function is maximized, conditional on the estimated D_t from the first stage.

We use rolling historical correlation, exponential smoother and dynamic conditional correlation by log-likelihood for mean-reverting model estimation to analyze the dynamic properties of the relevant variables.

3.2 Equities and Commodities

Figures 2 to 5 plot the estimates of the time-varying correlation between the unlevered rates of return on investable equity and commodity indices over the sample period. Figure 2, 3, 4, and 5 provide information on the correlations between, respectively, the S&P 500 and GSCI; S&P 500 and DJ-AIG; DJIA and GSCI; and, DJIA and DJ-AIG. Figure 6 provides similar plots for the correlation between the two equity indices (S&P 500 and DJIA). Note that, in Figures 2 and 3, three panels are provided: Figures 2A and 3A are for weekly returns; Figures 2B and 3B, for daily returns; and Figures 2C and 3C, for monthly returns.

For weekly returns, each panel or Figure contains three plots, one for each of the estimation methods outlined above: rolling historical correlation; exponential smoother with smoothing parameter 0.94; and dynamic conditional correlation (DCC) by log-likelihood for mean-reverting process. For the sake of brevity, only the DCC estimates are presented in the case of daily and monthly returns. The straight line running through each graph shows the relevant simple correlation from Table 2, which is not an average of any of the four time-varying correlation estimates. Several facts are immediately apparent from these graphs.

- The correlation between equity and commodity returns fluctuates notably over time. This finding is robust to the choice of equity or commodity indices – the correlation time-patterns are the same for all four pairs of indices.
- There is little evidence that correlations are any higher after 2002 than they were prior to 2002. If anything, consistent with the results obtained with simple correlations (see Table 2, in particular Table 2D), the time-varying correlation graphs show that correlations are lower since 2002 than before.

- Notwithstanding some amount of fluctuation over time, the correlations between equities and commodities are not often greater than 0.3. In contrast, Figure 6 shows that the correlation between the two equity indices is very high, typically well above 0.9.

In sum, equity-commodity return pairwise correlations fluctuate over the sample period. Quite often, the correlation estimates are even negative. This result underlines the importance of accurate measures of co-movement between asset returns necessary for long-term portfolio investments.

3.3 Commodity Sub-Indices

Figures 7 and 8 complement the analysis of the previous subsection, by plotting estimates of the time-varying correlations between the unlevered rates of return on benchmark equity indices and on specific categories of investable commodity indices. Figure 7 focuses on the difference between "Energy" and "Non-Energy" commodity baskets. Figure 8 refines Figure 7 by breaking down the Non-Energy index further into several investable sub-indices: Precious Metals, Industrial Metals, Agriculture, and Livestock. All of the plots in Figures 7 and 8 are directly comparable, in that they are all drawn using dynamic conditional correlations estimated by log-likelihood for mean-reverting model. Figures 7 and 8 highlight four facts:

- There is a substantial amount of time variation in the correlations between returns on equities and on both the energy and non-energy commodity sub-indices. Depending on the time subperiod, these correlations fluctuate between -0.45 and 0.45. By contrast, the unconditional correlations are close to zero across the entire sample period.
- While the Energy and Non-Energy sub-indices do not move in close sync, they are sufficiently positively correlated that investors do not benefit from a consistently low correlation between equities and commodities.
- Figure 8 suggests, however, that indices based on more narrow categories of commodities exhibit less correlation with equities than the overall non-energy index – raising questions about possible diversification strategies.
- Finally, and importantly for the purpose of the present paper, it is readily apparent from both Figures 7 and 8 that there is no obvious secular pattern toward an increase in correlations in the last few years.

4. Long-term Co-Movements

The foregoing analysis indicates that the correlations between the equity and commodity return series may have fallen amidst the commodity investment boom. The very fact that these correlation estimates fluctuate significantly over time, however, is evidence of their short-term nature. If there is a reason to suspect that equity and commodity return should move together in the long run, however, a complementary technique is required.

4.1 Cointegration Analysis

A large volume of research evaluates the degree of interconnectivity between prices from different markets by employing time-series techniques that are appropriate for non-stationary and co-integrated data. In particular, much work on applied co-integration analysis has relied on Johansen's multivariate approach (Johansen, 1988, 1991; Johansen and Juselius, 1990).

Johansen (1988) proposes and implements a unified vector autoregressive system approach for testing cointegration. Johansen derived the maximum likelihood estimator of the space of cointegration vectors and the likelihood ratio test of the hypothesis that it has a given number of dimensions. The procedure involves the following stages:

- Model checking, determination of lag length;
- Determination of cointegration rank, trace and maximum eigenvalue statistic;
- Estimation of the cointegration space;

The first step of the model building involves the choice of lag order. The most common procedure is to estimate a vector autoregression using the undifferenced data. Then we can use different information criteria to select the number of lag lengths. In our analysis, we use Schwarz (SC) criteria to determine the optimal lag – 2 in our case.

After selecting the lag length, the Johansen procedure estimates a vector error correction model (VECM) to determine the number of cointegrating vectors. According to Johansen (1988), a general polynomial distributed lag process, x_t, involving up to k lags, can be written as:

$$x_t = \Pi_1 x_{t-1} + ... + \Pi_k x_{t-k} + u_t \tag{9}$$

where x_t is a vector of n variables of interest, Π_i is an $(n \times n)$ matrix of parameters, and u is n-dimensional Gaussian independently distributed random variables with zero mean and variance matrix (Λ).This equation can be reformulated into VECM form:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + ... + \Gamma_{k-1} \Delta x_{t-k+1} + \Theta x_{t-k} + u_t \tag{10}$$

where $\Gamma_i = -\Sigma_{j=i+1}^{k} \Pi_j$, ($i = 1,2,...,k-1$), and $\Theta = \Sigma_{i=1}^{k}(\Pi_i - I)$. This way of specifying the system contains information on both the short and long run adjustments to changes in x_t, via the estimates of $\hat{\Gamma}_i$ and $\hat{\Theta}$, respectively. Assuming that x_t is $I(1)$, while r linear combinations of x_t are stationary, we can write

$$\Theta = \alpha\beta', \tag{11}$$

where α is the vector of adjustment coefficients; β is the cointegrating vector; and both are ($n \times r$) matrices. The approach of Johansen is based on the estimation of system (10) by maximum likelihood, while imposing the restriction in (11) for a given value of r.

Johansen (1988) demonstrates that β can be estimated by regressing ΔX_t and X_{t-k} on the lagged differences.

The next step in the Johansen approach involves testing the hypothesis about the rank of the long run matrix Θ, or equivalently the number of columns in β. The likelihood ratio test for the determination of the rank r is discussed in Johansen (1992). In general, tests of the hypothesis that $r \leq q$ use the likelihood ratio test statistics:

$$\lambda_{trace}(q) = -T\Sigma_{j=q+1}^{k} log(1-\hat{\lambda}_j) \tag{12}$$

This test is called the trace test. It checks whether the smallest $k-q$ eigenvalues are significantly different from zero. Furthermore, we can test $H_0 : r \leq q$ versus the more restrictive alternative $H_1 : r = q+1$ using

$$\lambda_{max}(q) = -Tlog(1-\hat{\lambda}_{q+1}) \tag{13}$$

This alternative test is the so-called maximum eigenvalue test, as it is based on the estimated (q+1)th largest eigenvalue.

Most of the existing Monte Carlo studies on the Johansen methodology point out that dimension of the data series for a given sample size may pose particular problems since the number of parameters of the underlying VAR models grows very large as the dimension increases. Likewise, difficulties often arise, when a given lag length of the system is either over or under parameterized. Reimers (1992) argues that for small samples, the Johansen procedure over-rejects when the null is true. To correct this bias, he suggests an adjustment in the degrees of freedom in the trace statistics and the maximum eigenvalue test statistics by replacing T by $T-nk$ for small samples. The corresponding degrees of freedom adjusted trace and maximum eigenvalue test statistics can be written as:

$$\lambda_{trace}^{a}(q) = -(T-nk)\Sigma_{j=q+1}^{k} log(1-\hat{\lambda}_j)$$

$$\lambda_{max}^{a}(q) = -Tlog(1-\hat{\lambda}_{q+1})$$

We first perform a univariate Augmented Dickey Fuller unit root test to determine the order of integration for each variable. Both variables (S&P GSCI total return index and S&P 500 index) appear to be integrated of order one; that is to say, our variables are nonstationary and they only become stationary after we take first differences. This finding suggests that we cannot rely on standard regression procedures, since OLS estimators have sampling distributions that are very different from those derived under the assumption of stationarity. Therefore, we proceed with the Johansen cointegration approach to determine whether there exists a long run relationship between our variables.

Using trace statistics, we fail to observe any cointegrating vector between commodity and equity indices. This result is consistent with the low-correlation that we observe from our dynamic conditional correlation estimation.

4.2 Recursive Cointegration Analysis

To obtain an understanding of the dynamics of the relationship, which the foregoing analysis cannot provide, we examine the dynamics and extent of relationship, if any, between our indices using recursive cointegration method outlined in Hansen and Johansen (1993). Recursive cointegration techniques allow us to test for the level of co-integration among indices during our sample period. The recursive technique allows us to recover two ECM representations. In the "Z-representation," all the parameters of the ECM (β and α) are re-estimated during the recursions, while under the "R-representation" the short-run parameters (α) are kept fixed to their full sample values and only the long run parameters (β) are re-estimated.

The logic behind the recursive cointegration technique is very similar to Johansen (1988) multivariate cointegration approach. Instead of using all observations, we start with an initial sample period from t_0 to t_j to perform Johansen (1988) cointegration approach and calculate the corresponding trace statistics for this sub-sample. Then, we increase the sample size by 1 from t_0 to t_{j+1} and calculate the relevant trace statistics for this sample period. This process continues until we exhaust all the observations and, in the final stage, we perform the cointegration analysis for the full sample and calculate the trace statistics. Of course, the trace statistic calculated in the final stage is equal to standard static trace statistics calculated with the Johansen (1998) method. The recursive method, however, allows us to see the *dynamics* of the trace statistic.

We start with 52 weeks of observations and add one more week in each step until we exhaust all our observations.[7] We re-scale our trace statistics by the 95% quantile of the trace distribution derived for the selected model without exogenous variables or dummies. Re-scaled trace statistics suggest the rejection of null hypothesis of no cointegration if it is above 1. In addition, to see whether there exist a cointegrating vector among our variables, the slope of re-scaled trace statistic determines the direction of co-movements between our variables. An upward slope indicates rising co-movement, while a downward slope for the trace statistics reveals declining co-movement between our variables.

Figure 9 shows the R-1 form of the trace statistic, recursively calculated and scaled by the 5% critical value. The dark blue line gives the estimate calculated using data from the whole sample (i.e., from January 1991 through July 2007).[8] Although there is large variation in trace statistics during our sample period, we can divide our sample period into three distinctive periods.

[7] We also use three years of observation in our initial estimation to see the robustness of our results. Three years of initial estimation did not change our qualitative results.

[8] We utilize weekly price data from the year prior to a given estimation period to start the recursive procedure for that period. Our results are robust to using more weeks for the prior period.

- The first period, from January 1991 to January 1997, is characterized by a relatively stable trace statistics that is generally below the threshold level of 1 (implying no cointegrating relationship).
- The second period, from January 1997 to June 1999, is charaterized by instability in the trace test statistics. The latter is generally above 1, constituting some support for co-movement between our indices. However, the direction of this co-movement is mixed. The period when a common long-term factor most clearly appears to drive both equity and commodity prices coincides broadly with the Asian and Russian crises.
- The last period starts in June 1999 and continues up to the end of our sample. During this period, there is no statistical evidence of any long-run relationship between the benchmark commodity and equity indices.

In sum, there is little evidence of a common long-term trend between investable commodity and equity indices, and no evidence of a possible secular strengthening of any such trend.

5. Extreme Events

Sections 2 and 3 provide evidence that neither the average levels of correlation between equity and commodity returns, nor the pattern of variation of these correlations over time, have been qualitatively very different in the last five years than in the foregoing ten years. The widespread perception that financial markets nowadays move much more in lock-step, however, could be due not as much to changes in average levels and patterns but, instead, to the joint behavior of financial markets on "stressful days." In this Section, we provide evidence that there has been no increase in cross-market co-movements during periods of exceptionally large returns on commodities and equities.

5.1 Summary Statistics

To assess whether cross-asset extreme linkages exist in the case of commodities, we first identify the days and weeks during which the returns on the benchmark S&P 500 equity index was at least one or two standard deviations above or below its sample mean, and then analyze the contemporaneous unlevered returns on the benchmark investable commodity index, the S&P GSCI. Implicit in this approach is the notion that, if changes in extreme linkages have taken place because of commodity investment flows, then the fact that equity markets are much larger than commodity markets suggests that ripple effects are more likely to emanate from the former than from the latter. For the same reason, liquidity problems or panic reactions should be more likely to spread from stock to commodity markets than the reverse.

As in the rest of the paper, we look at joint commodity-equity return behaviors for the whole sample as well as for three successive sub-periods. Tables 6 and 7 summarize our findings for weekly and daily returns, respectively. Table 6A (*resp. 7A*) tallies the episodes when the weekly (*resp. daily*) return on the S&P 500 equity index was "large," i.e., at least one standard deviation away from its mean during a given period. Table 6B (*resp. 7B*) tallies what happens on weeks (*resp. days*) of "extreme" stock returns, i.e., when these returns were at least two standard deviations away from the relevant mean. Tables 6 and 7 show in italics the number of times when the unlevered return on the GSCI index was positive or negative, for a given direction of the large (Tables 6A and 7A) or extreme (Tables 6B and 7B) S&P 500 return. It also shows in **bold** the number of times when the contemporaneous GSCI return itself was also more than one (Tables 6A and 7A) or two (Tables 6B and 7B) standard deviations away from its own sample mean.

For the sake of brevity, we focus on weekly results (Table 6) because the daily results are qualitatively similar (Table 7). Between January 15, 1991 and July 2, 2007, there were 116 weeks ($65+51$) when the rate of return on the S&P 500 equity index was below its sample mean by one standard deviation or more, and 20 weeks ($14+6$) when the same return was below its mean by more than two standard deviation. During the 116 weeks of large poor S&P 500 returns, the total return on the GSCI was *positive* (though not necessarily large or extreme) 65 times, and negative only 51 times. Of those 116 times, the GSCI return deviated from its mean by more than one standard deviation a total of 33 times – 15 times below the mean but 18 times *above* the mean. In other words, when the S&P 500 drops a lot, it is not clear which way the GSCI return will go – neither in terms of its sign nor in comparison to its mean. A similar pattern emerges when equities do very well. To wit, in the 87 (38+49) weeks when the S&P 500 return exceeded its sample mean by one standard deviation or more, the GSCI total return was positive only 49 times. Likewise, in the 14 weeks when the S&P 500 return exceeded its mean by more than two standard deviation, the GSCI total return was equally likely to be extremely bad or extremely good (2 in each case).

In sum, contrary to extant findings that there exist extreme linkages between other asset markets (e.g., Hartman et al., 2004; Solnik and Longin, 2001), our evidence is suggestive of little relation between exceptionally large returns on commodities and equities. This negative result holds for the whole sample period as well as for all three of the five-year sub-periods; for positive as well as for negative exceptional returns; and, for periods of stock market upturns as well as for downturns.

5.2 Extreme Correlation Analysis

To further investigate possible links between equity and commodity investments in periods of market stress, we compute the excedence correlation between the two series. We define correlation at an exceedence level q as the correlation between the two series when both of them exceed predefined threshold level of q. We choose empirical distribution of each series to determine threshold levels. This construction allows us to calculate the cross-correlation between the weekly returns on unlevered passive equity and commodity investments for each percentile of the joint return distribution. This technique is similar to that used by Longin and Solnik (2001) to assess pairwise U.S.-international equity market linkages during periods of extreme returns.

Figure 10 summarizes our results. Of particular interest is, naturally, what happens when both equity and commodity returns are either very high or very low. Figure 10 shows that, while equity-commodity correlations are typically very low, they strengthen during extreme events. Figure 10 further shows that, whereas the equity-commodity correlation is mildly positive in the upper tail of the joint distribution, the cross-correlation is *negative* in the lower tail. That is, conditional on both equity and commodity returns being very poor, the two returns series are negatively correlated.

6. Conclusion and Possible Extensions

Amidst a sharp rise in commodity investing, many have asked whether commodities nowadays move in sync with traditional financial assets. We provide evidence that challenges this idea. Using dynamic correlation and recursive cointegration techniques, we find that the relation between the returns on investable commodity and equity indices has not changed significantly in the last fifteen years. We also find no evidence of much co-movement during periods of extreme returns.

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Table 1A: Weekly Rates of Return (%, January 1991 through June 2007)

	DJIA	S&P500	DJAIG	GSCI
Mean	0.2172	0.2043	0.1527	0.1401
Median	0.2832	0.3417	0.1620	0.1869
Maximum	12.6934	13.1729	5.5331	8.0874
Minimum	-9.0967	-11.4591	-7.1159	-13.5768
Std. Dev.	2.1283	2.1465	1.7693	2.6256
Skewness	0.1264	0.0365	-0.1671	-0.4418
Kurtosis	6.6784	6.8592	3.9934	4.8996
Jarque-Bera	486.57	533.25	39.32	157.09
Probability	0.0000	0.0000	0.0000	0.0000
Sum	186.60	175.51	131.21	120.38
Sum Sq. Dev.	3886.29	3953.23	2685.92	5914.67
Observations	859	859	859	859

Table 1B: Weekly Rates of Return (%, June 1992 through May 1997)

	DJIA	S&P500	DJAIG	GSCI
Mean	0.3134	0.2886	0.2059	0.1406
Median	0.3377	0.3507	0.2267	0.1426
Maximum	3.9070	4.2835	3.5734	5.4858
Minimum	-4.2337	-4.0290	-4.1213	-8.7976
Std. Dev.	1.5352	1.4419	1.2607	1.7976
Skewness	-0.2377	-0.2167	-0.2073	-0.2975
Kurtosis	3.2046	3.3696	3.2926	5.1457
Jarque-Bera	2.91	3.53	2.80	53.92
Probability	0.2329	0.1713	0.2466	0.0000
Sum	81.79	75.33	53.73	36.71
Sum Sq. Dev.	612.75	540.59	413.26	840.19
Observations	261	261	261	261

Table 1C: Weekly Rates of Return (%, June 1997 through May 2002)

	DJIA	S & P500	DJAIG	GSCI
Mean	0.1479	0.1237	0.0054	0.0038
Median	0.1553	0.3680	-0.1309	-0.2446
Maximum	11.1719	9.9121	4.8857	7.3270
Minimum	-9.0114	-9.0214	-7.1159	-13.5768
Std. Dev.	2.5491	2.6004	1.8459	2.7737
Skewness	0.0222	-0.0828	0.1018	-0.2096
Kurtosis	4.3983	3.5515	3.3542	4.3768
Jarque-Bera	21.29	3.61	1.82	22.52
Probability	0.0000	0.1649	0.4036	0.0000
Sum	38.61	32.29	1.42	0.98
Sum Sq. Dev.	1689.41	1758.13	885.86	2000.22
Observations	261	261	261	261

Tabe1D: Weekly Rates of Return (%, June 2002 through June 2007)

	DJIA	S & P500	DJAIG	GSCI
Mean	0.1288	0.1477	0.2796	0.2797
Median	0.2808	0.2899	0.4041	0.5931
Maximum	12.6934	13.1729	5.5331	8.0874
Minimum	-9.0967	-11.4591	-6.8533	-11.5571
Std. Dev.	2.2230	2.2814	2.1751	3.1787
Skewness	0.4657	0.3161	-0.2912	-0.4796
Kurtosis	9.6357	10.6177	3.2844	3.4686
Jarque-Bera	497.64	647.59	4.65	12.63
Probability	0.0000	0.0000	0.0976	0.0018
Sum	34.27	39.28	74.38	74.41
Sum Sq. Dev.	1309.59	1379.31	1253.71	2677.64
Observations	266	266	266	266

Notes: Panels A to D of Table 1 provide summary statistics for the **weekly** unlevered rates of return on the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices (excluding dividends), as well as on the Dow Jones DJAIG and S&P GSCI commodity indices (total return). Table 1A uses sample moments computed using weekly rates of returns from January 15, 1991 to July 2, 2007. Tables 1B, 1C and 1D provide the corresponding moments for three successive sub-periods: June 2, 1992 to May 27, 1997; June 3, 1997 to May 28, 2002; and, May 28, 2002 to July 2, 2007.

Table 1E: Daily Rates of Return (%, January 1991 through June 2007)

	DJIA	S & P500	DJAIG	GSCI
Mean	0.0445	0.0420	0.0321	0.0303
Median	0.0493	0.0432	0.0409	0.0255
Maximum	6.3481	5.7327	4.9708	6.7875
Minimum	-7.1838	-6.8657	-8.7461	-16.8332
Std. Dev.	0.9748	0.9908	0.8082	1.2221
Skewness	-0.1375	-0.0229	-0.2205	-0.5839
Kurtosis	7.7874	7.0196	7.8342	13.6844
Jarque-Bera	3949.3880	2775.3450	4047.1260	19840.5100
Probability	0.0000	0.0000	0.0000	0.0000
Observations	4122	4122	4122	4122

Notes: Panel E of Table 1 provide summary statistics for the **daily** unlevered rates of return on the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices (excluding dividends), as well as on the Dow Jones DJAIG and S&P GSCI commodity indices (total return). Table 1E uses sample moments computed using daily rates of return from January 15, 1991 to July 2, 2007.

Table 1F: Monthly Rates of Return (%, January 1991 through June 2007)

	DJIA	S & P500	DJAIG	GSCI
Mean	0.8895	0.8276	0.6919	0.6489
Median	1.1688	1.1096	0.7190	0.7152
Maximum	10.6047	11.1588	10.2253	16.8927
Minimum	-15.1320	-14.5797	-7.5449	-14.4111
Std. Dev.	3.9834	3.8955	3.4936	5.3449
Skewness	-0.5001	-0.4993	0.0874	0.1213
Kurtosis	4.3711	4.0633	3.0579	3.4304
Jarque-Bera	23.6415	17.4662	0.2785	2.0038
Probability	0.0000	0.0002	0.8700	0.3672
Observations	197	197	197	197

Note: Panel F of Table 1 provide summary statistics for the **monthly** unlevered rates of return on the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices (excluding dividends), as well as on the Dow Jones DJAIG and S&P GSCI commodity indices (total return). Table 1F uses sample moments computed with monthly rates of returns from January 15, 1991 to July 2, 2007.

Table 2A: Index-return Correlations (Weekly), January 1991 through June 2007

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000	0.9369	0.0666	0.0077
S & P500		1.0000	0.0807	0.0352
DJAIG			1.0000	0.8973
GSCI				1.0000

Table 2B: Index-return Correlations (Weekly), June 1992 through May 1997

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000	0.9193	0.1029	0.1359
S & P500		1.0000	0.0656	0.1057
DJAIG			1.0000	0.8203
GSCI				1.0000

Table 2C:Index-return Correlations (Weekly), June 1997 through May 2002

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000	0.9117	0.1129	0.0776
S & P500		1.0000	0.1241	0.1026
DJAIG			1.0000	0.9336
GSCI				1.0000

Table 2D: Index-return Correlations (Weekly), June 2002 through June 2007

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000	0.9753	0.0157	-0.0890
S & P500		1.0000	0.0619	-0.0298
DJAIG			1.0000	0.8989
GSCI				1.0000

Note: Panels A through D of Table 2 provide simple cross-correlation tables for the **weekly** unlevered rates of return on four investable indices: the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices, as well as on the Dow Jones DJAIG and S&P GSCI commodity indices. Table 2A uses weekly return data from January 15, 1991 to July 2, 2007. Tables 2B, 2C and 2D provide the corresponding cross-correlations for three successive sub-periods: June 2, 1992 to May 27, 1997; June 3, 1997 to May 28, 2002; and, May 28, 2002 to July 2, 2007.

Table 2E: Index-return Correlations (Daily), January 1991 through June 2007

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000	0.9423	-0.0269	-0.0657
S & P500		1.0000	-0.0081	-0.0412
DJAIG			1.0000	0.8973
GSCI				1.0000

Note: Panel E of Table 2 provide simple cross-correlation tables for the **daily** unlevered rates of return on four investable indices: the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices, as well as on the Dow Jones DJAIG and S&P GSCI commodity indices. Table 2E uses daily return data from January 15, 1991 to July 2, 2007.

Table 2F: Index-return Correlations (Monthly), January 1991 through June 2007

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000	0.9246	0.0974	-0.0245
S & P500		1.0000	0.0860	-0.0064
DJAIG			1.0000	0.8826
GSCI				1.0000

Note: Panel F of Table 2 provide simple cross-correlation tables for the **monthly** unlevered rates of return on four investable indices: the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices, as well as on the Dow Jones DJAIG and S&P GSCI commodity indices. Table 2F uses monthly return data from January 15, 1991 to July 2, 2007. **Bolded** equity-commodity cross-correlations are statistically significant (5% level).

Table 3A: Weekly Rates of Return on Commodity Sub-Indices (%, 1991-2007)

	S&P 500	GSCI	Agri-culture	Energy	Ind. Metals	Live-stock	Non-Energy	Prec. Metals
Mean	0.2043	0.1401	0.0133	0.2070	0.1781	0.0458	0.0633	0.1010
Median	0.3417	0.1869	0.0263	0.2504	0.0690	-0.017	0.0409	0.0701
Maximum	13.1729	8.0874	9.2918	14.6776	9.5520	7.7185	5.4159	16.7883
Minimum	-11.46	-13.58	-6.676	-21.55	-10.04	-11.20	-5.143	-11.54
Std.Dev.	2.1465	2.6256	2.1407	4.0174	2.2845	1.8912	1.4036	1.9902
Skewness	0.0365	-0.442	0.3357	-0.378	0.1062	-0.116	0.1984	0.4851
Kurtosis	6.8592	4.8996	4.0860	4.7313	4.2812	5.2488	3.7764	10.8101
Jarque-Bera	533.25	157.09	58.34	127.71	60.37	182.93	27.21	2216.89
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Sum	175.51	120.38	11.44	177.84	153.01	39.32	54.40	86.78
Sum Sq.Dev.	3953.23	5914.67	3931.90	13847.8	4477.70	3068.72	1690.32	3398.35
Observations	859	859	859	859	859	859	859	859

Table 3B: Weekly Returns on Commodity Sub-Indices: Summary Statistics (1992-1997)

	S&P 500	GSCI	Agri-culture	Energy	Ind. Metals	Live-stock	Non-Energy	Prec. Metals
Mean	0.2886	0.1406	0.2327	0.1164	0.1117	0.1292	0.1612	0.0390
Median	0.3507	0.1426	0.1975	0.0511	0.1546	0.0835	0.1217	0.1098
Maximum	4.2835	5.4858	9.2918	9.0001	5.7449	5.6271	5.0024	5.2064
Minimum	-4.03	-8.80	-5.65	-14.56	-7.57	-3.82	-3.09	-6.76
Std.Dev.	1.4419	1.7976	1.9565	3.1399	2.0259	1.6234	1.2129	1.5065
Skewness	-0.22	-0.30	0.4466	-0.20	-0.26	0.3730	0.3648	-0.06
Kurtosis	3.3696	5.1457	5.3624	4.4955	3.7276	3.3920	4.2227	6.3861
Jarque-Bera	3.53	53.92	69.37	26.07	8.66	7.72	22.05	124.82
Probability	0.1713	0.0000	0.0000	0.0000	0.0132	0.0210	0.0000	0.0000
Sum	75.33	36.71	60.73	30.37	29.16	33.71	42.08	10.17
Sum Sq.Dev.	540.59	840.19	995.29	2563.35	1067.11	685.22	382.48	590.05
Observations	261	261	261	261	261	261	261	261

Table 3C: Weekly Rates of Return on Commodity Sub-Indices (%, 1997-2002)

	S&P 500	GSCI	Agri-culture	Energy	Ind. Metals	Live-stock	Non-Energy	Prec. Metals
Mean	0.1237	0.0038	-0.313	0.1730	-0.093	-0.146	-0.214	0.0434
Median	0.3680	-0.245	-0.222	0.2061	-0.202	-0.202	-0.191	-0.153
Maximum	9.9121	7.3270	6.4484	14.6776	9.5520	7.7185	3.7125	16.7883
Minimum	-9.021	-13.58	-6.676	-19.36	-4.985	-7.420	-3.626	-5.253
Std.Dev.	2.6004	2.7737	1.9918	4.4310	1.9989	1.9978	1.2922	2.0039
Skewness	-0.083	-0.210	0.1291	-0.103	0.6070	-0.077	0.1167	2.5640
Kurtosis	3.5515	4.3768	3.2067	3.8739	4.5954	5.0011	3.0930	21.3337
Jarque-Bera	3.61	22.52	1.19	8.77	43.70	43.80	0.6864	3941.31
Probability	0.1649	0.0000	0.5516	0.0125	0.0000	0.0000	0.7095	0.0000
Sum	32.29	0.98	-81.72	45.15	-24.20	-38.18	-55.83	11.32
Sum Sq.Dev.	1758.13	2000.22	1031.46	5104.68	1038.84	1037.69	434.13	1044.09
Observations	261	261	261	261	261	261	261	261

Table 3D: Weekly Rates of Return on Commodity Sub-Indices (%, 2002-2007)

	S&P 500	GSCI	Agri-culture	Energy	Ind. Metals	Live-stock	Non-Energy	Prec. Metals
Mean	0.1477	0.2797	0.0811	0.3316	0.5848	0.1406	0.2385	0.2977
Median	0.2899	0.5931	0.0761	0.8012	0.5525	0.1087	0.2511	0.4484
Maximum	13.1729	8.0874	8.6752	10.7600	8.3326	6.4158	5.4159	8.2064
Minimum	-11.46	-11.56	-6.028	-14.90	-10.04	-11.20	-5.143	-11.54
Std.Dev.	2.2814	3.1787	2.4719	4.2669	2.8103	2.1254	1.7109	2.4720
Skewness	0.3161	-0.480	0.3366	-0.439	-0.193	-0.373	0.0232	-0.627
Kurtosis	10.6177	3.4686	3.4485	3.2561	3.8388	5.5486	3.2441	5.0177
Jarque-Bera	647.59	12.63	7.25	9.29	9.44	78.15	0.68	62.53
Probability	0.0000	0.0018	0.0266	0.0096	0.0089	0.0000	0.7103	0.0000
Sum	39.28	74.41	21.59	88.22	155.56	37.40	63.44	79.20
Sum Sq.Dev.	1379.31	2677.64	1619.29	4824.67	2092.88	1197.07	775.66	1619.34
Observations	266	266	266	266	266	266	266	266

Note: Table 3 provides descriptive statistics for the unlevered (``total") weekly rates of return on the S&P 500 equity and GSCI commodity indices, as well as six investable GSCI sub-indices: Agriculture (Wheat, Red Wheat, Corn, Soybeans, Cotton, Sugar, Coffee, and Cocoa); Energy (WTI Crude Oil, Brent Crude Oil, RBOB Gas, Heating Oil, GasOil, and Natural Gas); Industrial Metals (Aluminium, Copper, Lead, Nickel, and Zinc); Precious Metals (Gold and Silver);Livestock (Live Cattle, Feeder Cattle, and Lean Hogs); and, NonEnergy. Table 3A uses sample moments computed from January 15, 1991 to July 2, 2007. Tables 3B, 3C and 3D provide the corresponding moments for three successive sub-periods: June 2, 1992 to May 27, 1997; June 3, 1997 to May 28, 2002; and, May 28, 2002 to July 2, 2007.

Table 4A: Cross-correlations of Weekly Index and Sub-index Returns, 1991-2007

	S&P 500	GSCI	Agri-culture	Energy	Ind. Metals	Live-stock	Non-Energy	Prec. Metals
S&P 500	1.0000	0.0352	0.0059	0.0353	0.1283	0.0430	0.0612	-0.0137
GSCI		1.0000	0.2778	0.9667	0.2410	0.1584	0.3799	0.2251
Agriculture			1.0000	0.1107	0.1344	0.0715	0.8130	0.1887
Energy				1.0000	0.1402	0.0622	0.1775	0.1540
Ind.Metals					1.0000	0.0686	0.5153	0.3396
Livestock						1.0000	0.4295	0.0436
NonEnergy							1.0000	0.3896
Prec.Metals								1.0000

Table 4B: Cross-correlations of Weekly Index and Sub-index Returns, 1992-1997

	S&P 500	GSCI	Agri-culture	Energy	Ind. Metals	Live-stock	Non-Energy	Prec. Metals
S & P500	1.0000	0.1057	-0.038	0.1229	0.0313	0.0377	-0.004	-0.096
GSCI		1.0000	0.3365	0.9409	0.1281	0.1932	0.4153	0.0692
Agriculture			1.0000	0.0610	0.0895	0.0926	0.8353	0.2088
Energy				1.0000	0.0088	0.0374	0.0949	-0.019
Ind.Metals					1.0000	0.0365	0.3515	0.2465
Livestock						1.0000	0.5131	0.0174
NonEnergy							1.0000	0.2653
Prec.Metals								1.0000

Table 4C: Cross-correlations of Weekly Index and Sub-index Returns, 1997-2002

	S&P 500	GSCI	Agri-culture	Energy	Ind. Metals	Live-stock	Non-Energy	Prec. Metals
S & P500	1.0000	0.1026	0.0321	0.0985	0.1532	-0.021	0.0636	0.0016
GSCI		1.0000	0.3210	0.9736	0.2815	0.1733	0.4076	0.1540
Agriculture			1.0000	0.1661	0.1121	0.1477	0.8562	0.0645
Energy				1.0000	0.2040	0.0595	0.2212	0.1245
Ind.Metals					1.0000	0.0972	0.4217	0.1797
Livestock						1.0000	0.5327	0.1150
NonEnergy							1.0000	0.2403
Prec.Metals								1.0000

Table 4D: Cross-correlations of Weekly Index and Sub-index Returns, 2002-2007

	S&P 500	GSCI	Agri-culture	Energy	Ind. Metals	Live-stock	Non-Energy	Prec. Metals
S & P500	1.0000	-0.03	-0.049	-0.039	0.1866	0.1162	0.0733	0.0445
GSCI		1.0000	0.2622	0.9859	0.2825	0.1379	0.3804	0.3040
Agriculture			1.0000	0.1432	0.1674	-0.007	0.7860	0.2905
Energy				1.0000	0.1869	0.0953	0.2315	0.2243
Ind.Metals					1.0000	0.0704	0.6627	0.4896
Livestock						1.0000	0.2852	-0.012
NonEnergy							1.0000	0.5545
Prec.Metals								1.0000

Note: Table 4 provides simple cross-correlation tables for the unlevered weekly rates of return on eight investable indices: the S&P 500 equity and GSCI commodity indices, as well as six GSCI sub-indices: Agriculture (Wheat, Red Wheat, Corn, Soybeans, Cotton, Sugar, Coffee, and Cocoa); Energy (WTI Crude Oil, Brent Crude Oil, RBOB Gas, Heating Oil, GasOil, and Natural Gas); Industrial Metals (Aluminium, Copper, Lead, Nickel, and Zinc); Precious Metals (Gold and Silver);Livestock (Live Cattle, Feeder Cattle, and Lean Hogs); and, Non-Energy. Table 4A uses sample moments computed from January 15, 1991 to July 2, 2007. Tables 4B, 4C and 4D provide the corresponding moments for three successive sub-periods: June 2, 1992 to May 27, 1997; June 3, 1997 to May 28, 2002; and, May 28, 2002 to July 2, 2007.

Table 5: Johansen (1988) Cointegration Analysis
between S&P GSCI Total Return Index and S&P 500 Index, Full Sample (1991-2007)

p-r	H_0=r	Eigenvalue	Trace	Trace*	Trace 95%	P-Value	P**-Value
2	0	0.007	9.224	9.196	20.164	0.717	0.720
1	1	0.003	2.886	1.079	9.142	0.610	0.926

Notes: VAR specification includes unrestricted constant and two lags. * Small sample corrected trace test statistic. ** The approximate *p*-value using the small sample corrected trace statistic.

Table 6A: Large Weekly Co-Movements: S&P 500 *versus* GSCI

	S&P 500 Down		S&P 500 Up	
Sample	GSCI Down	GSCI Up	GSCI Down	GSCI Up
Full Sample	*65* (**15**)	*51* (**18**)	*38* (**10**)	*49* (**17**)
1992-1997	*9* (**0**)	*11* (**1**)	*7* (**0**)	*8* (**1**)
1997-2002	*36* (**9**)	*23* (**10**)	*22* (**6**)	*27* (**12**)
2002-2007	*20* (**6**)	*17* (**7**)	*9* (**4**)	*14* (**4**)

Table 6B: Extreme Weekly Co-Movements: S&P 500 *versus* GSCI

	S&P 500 Down		S&P 500 Up	
Sample	GSCI Down	GSCI Up	GSCI Down	GSCI Up
Full Sample	*14* (**0**)	*6* (**0**)	*5* (**2**)	*9* (**2**)
1992-1997	*0* (**0**)	*0* (**0**)	*0* (**0**)	*0* (**0**)
1997-2002	*10* (**0**)	*1* (**0**)	*4* (**1**)	*5* (**2**)
2002-2007	*4* (**0**)	*5* (**0**)	*1* (**1**)	*4* (**0**)

Note: Table 6 focuses on the episodes when the weekly return on the S&P 500 index was at least 1 standard deviation (``Large" returns, Table 6A) or at least 2 standard deviations (``Extreme" returns, Table 6B) away from its mean during a given period. Table 6 shows in *Italics* the number of times when the unlevered return on the GSCI index was positive or negative, for a given direction of the large (Table 6A) or extreme (Table 6B) S&P return. It also shows in **Bold** the number of times when the contemporaneous GSCI return itself was also more than one (Table 6A) or two (Table 6B) standard deviations away from its own sample mean. For example, the first line in Table 6A shows that, between January 15, 1991 and July 2, 2007, there were 116 weeks ($65+51$) when the rate of return on the S&P 500 equity index was below its sample mean by one standard deviation or more, while the corresponding line in Table 6B shows that there were 20 weeks ($14+6$) when the same return was below its mean by more than two standard deviation. During the 116 weeks listed in Table 6A, the total return on the GSCI was positive (though not necessarily large or extreme) 65 times, and negative the other 51 times. Of those 65 times, the GSCI return deviated from its mean by more than one standard deviation a total of 33 times -- 15 below the mean and 51 above the mean).

Table 7A: Large Daily Co-Movements: S&P 500 *versus* GSCI

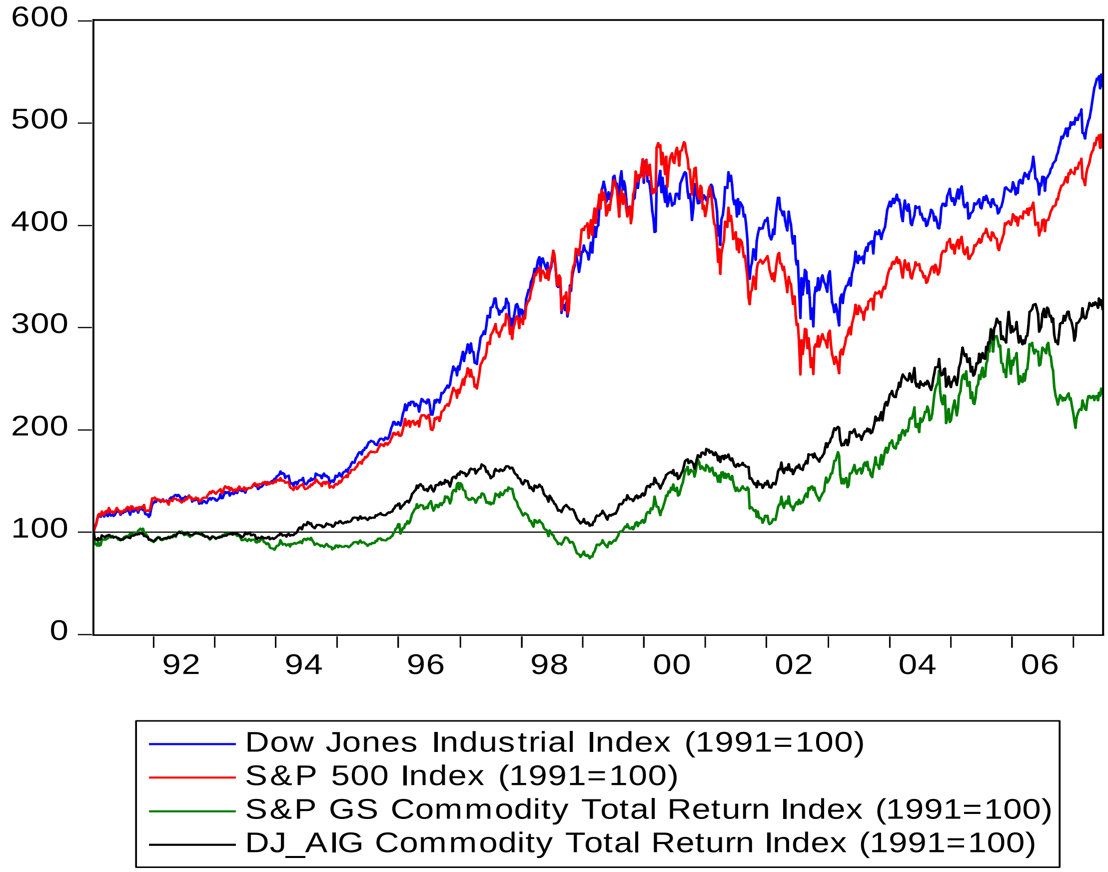
	S&P 500 Down		S&P 500 Up	
Sample	GSCI Down	GSCI Up	GSCI Down	GSCI Up
Full Sample	*187* (**58**)	*162* (**73**)	*163* (**72**)	*176* (**67**)
1992-1997	*31* (**7**)	*31* (**14**)	*36* (**13**)	*34* (**13**)
1997-2002	*98* (**35**)	*80* (**35**)	*84* (**36**)	*90* (**31**)
2002-2007	*58* (**16**)	*51* (**24**)	*43* (**23**)	*52* (**23**)

Table 7B: Extreme Daily Co-Movements: S&P 500 *versus* GSCI

	S&P 500 Down		S&P 500 Up	
Sample	GSCI Down	GSCI Up	GSCI Down	GSCI Up
Full Sample	*35* (**5**)	*20* (**5**)	*24* (**8**)	*33* (**2**)
1992-1997	*2* (**0**)	*2* (**0**)	*2* (**1**)	*4* (**0**)
1997-2002	*20* (**4**)	*12* (**4**)	*16* (**5**)	*19* (**1**)
2002-2007	*13* (**1**)	*6* (**1**)	*6* (**2**)	*10* (**1**)

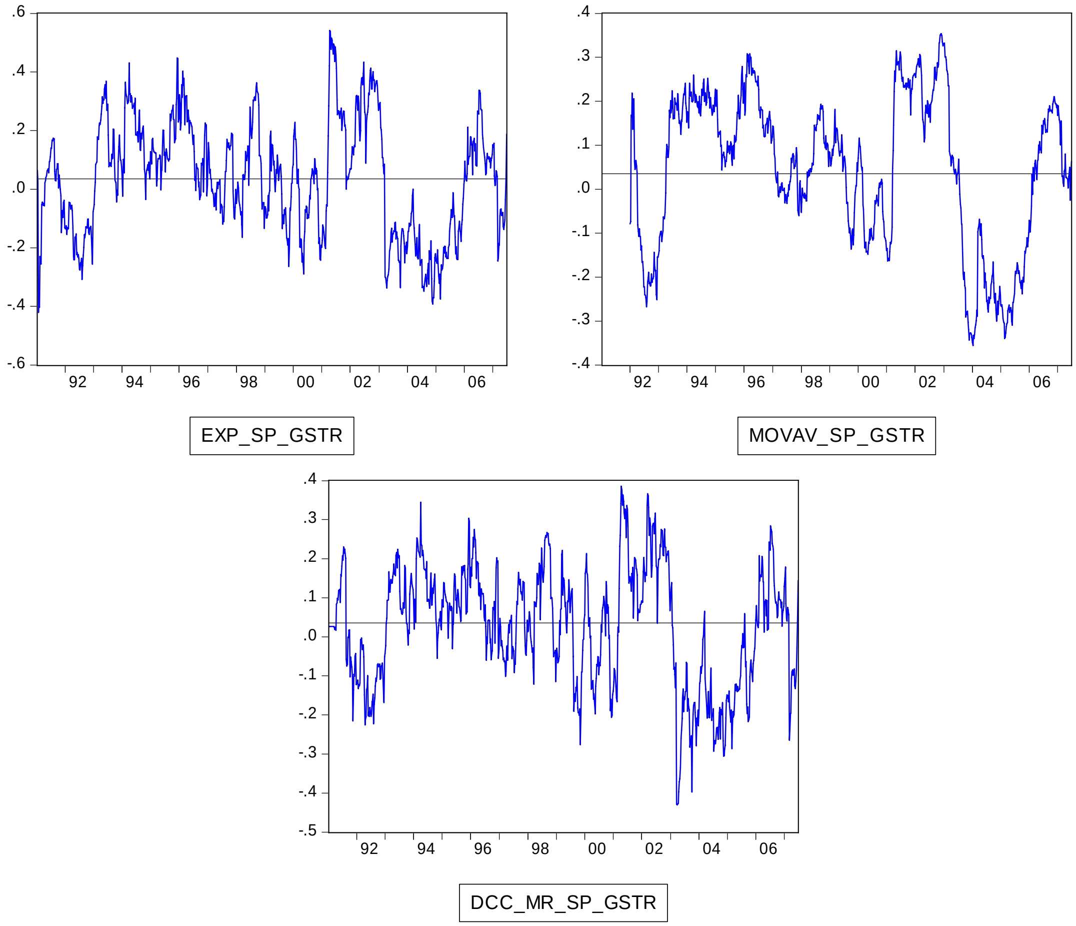
Note: Table 7 focuses on the episodes when the daily return on the S&P 500 index was at least 1 standard deviation (``Large" returns, Table 7A) or at least 2 standard deviations (``Extreme" returns, Table 7B) away from its mean during a given period. Table 7 shows in *Italics* the number of times when the unlevered return on the GSCI index was positive or negative, for a given direction of the large (Table 7A) or extreme (Table 7B) S&P return. It also shows in **Bold** the number of times when the contemporaneous GSCI return itself was also more than one (Table 7A) or two (Table 7B) standard deviations away from its own sample mean. For example, the first line in Table 7A shows that, between January 15, 1991 and July 2, 2007, there were 349 days ($187+162$) when the rate of return on the S&P 500 equity index was below its sample mean by one standard deviation or more, while the corresponding line in Table 7B shows that there were 55 days ($35+20$) when the same return was below its mean by more than two standard deviation. During the 349 days listed in Table 7A, the total return on the GSCI was positive (though not necessarily large or extreme) 187 times, and negative the other 162 times. Of those 187 times, the GSCI return deviated from its mean by more than one standard deviation a total of 131 times -- 58 below the mean and 73 above the mean).

Figure1: Major Commodity and Equity Indices, 1991-2007



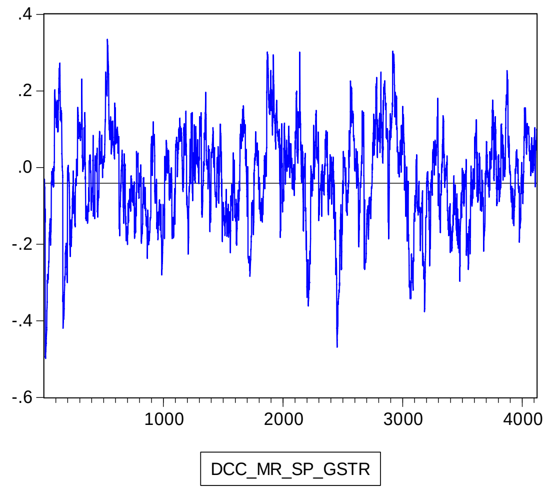
Note: Figure 1 plots the levels of four indices from January 15, 1991 to July 2, 2007. Starting with the index that appreciated the most and ending with the index that appreciated the least, they are: the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices, and the S&P GSCI and Dow Jones DJ-AIG commodity indices. The base level is set for January 15, 1991. The two equity indices (top two trends) appear to move closely together, as do the two commodity indices most of the time. Two exceptions are 1994-1995 and 2006-2007, when the DJ-AIG index rose while the GSCI either stagnated or outright dropped in value.

Figure 2A: Equity and Commodity Weekly Return Correlations: S&P 500 *vs.* GSCI, 1991-2007



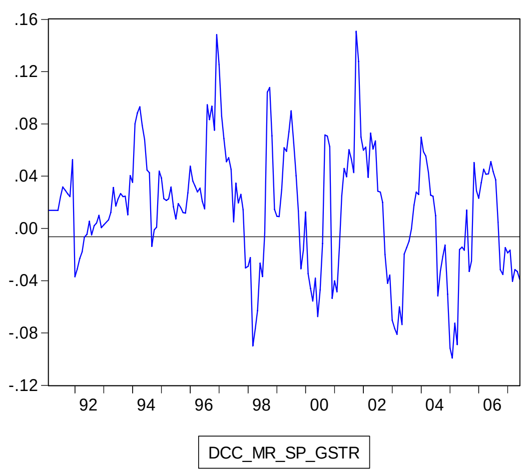
Note: Figure 2A depicts estimates of the time-varying correlation between the **weekly** unlevered rates of return on the S&P 500 (SP) and GSCI total return (GSTR) indices from January 15, 1991 to July 2, 2007. The Figure provides plots for the following three estimation methods: exponential smoother with 0.94 smoothing parameter (top left panel), rolling historical correlation (top right) and dynamic conditional correlation by log-likehood for mean reverting model estimation (bottom panel). The straight line running through each graph is the unconditional correlation from Table 2A.

Figure 2B: Equity and Commodity Daily Return Correlations: S&P 500 *vs.* GSCI, 1991-2007



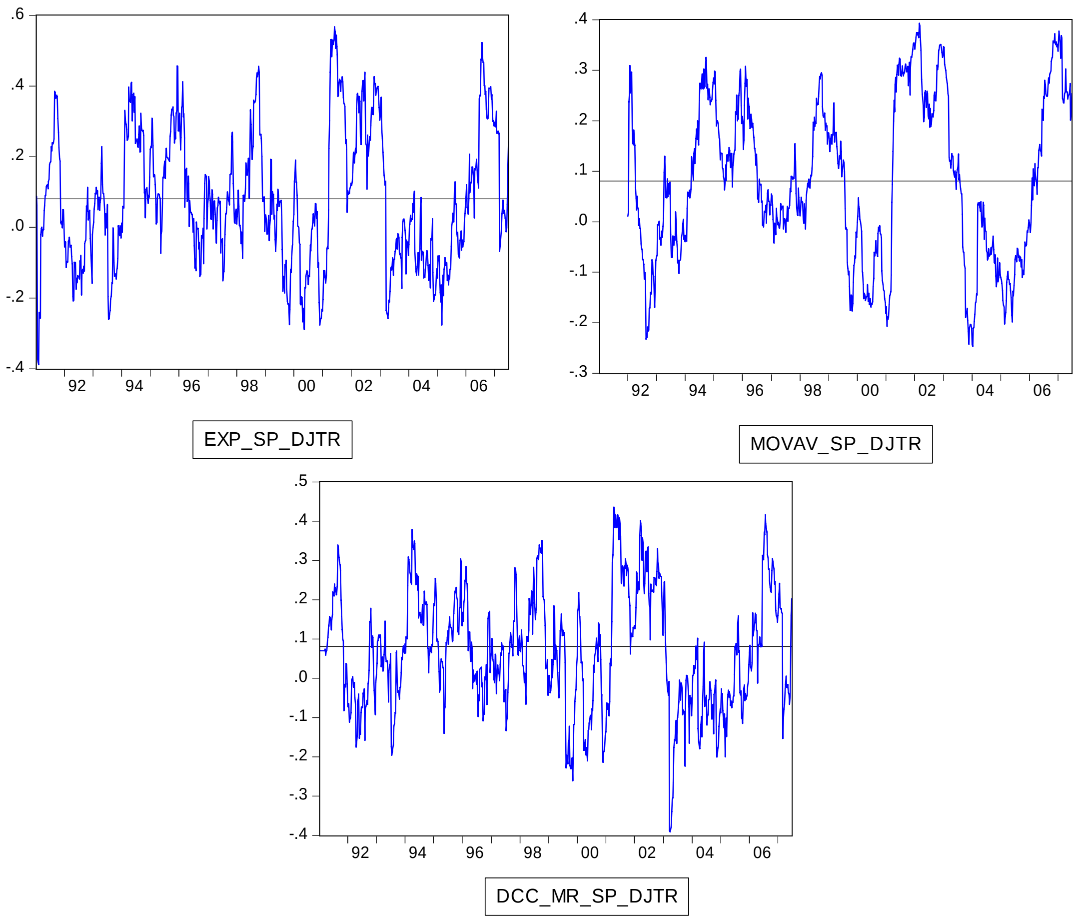
Note: Figure 2B depicts estimates of the time-varying correlation between the **daily** unlevered rates of return on the S&P 500 (SP) and GSCI total return (GSTR) indices from January 15, 1991 to July 2, 2007. The method used to compute estimates is dynamic conditional correlation by log-likehood for mean reverting model estimation. The straight line running through each graph is the unconditional correlation.

Figure 2C: Equity and Commodity Monthly Return Correlations: S&P 500 *vs.* GSCI, 1991-2007



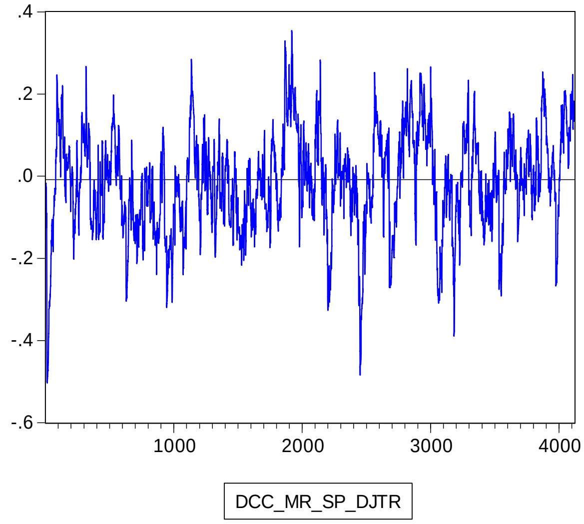
Note: Figure 2C depicts estimates of the time-varying correlation between the **monthly** unlevered rates of return on the S&P 500 (SP) and GSCI total return (GSTR) indices from January 15, 1991 to July 2, 2007. The method used to ompute estimates is dynamic conditional correlation by log-likehood for mean reverting model estimation. The straight line running through each graph is the unconditional correlation.

Figure 3A: Equity and Commodity Weekly Return Correlations: S&P 500 *vs.* DJ-AIG, 1991-2007



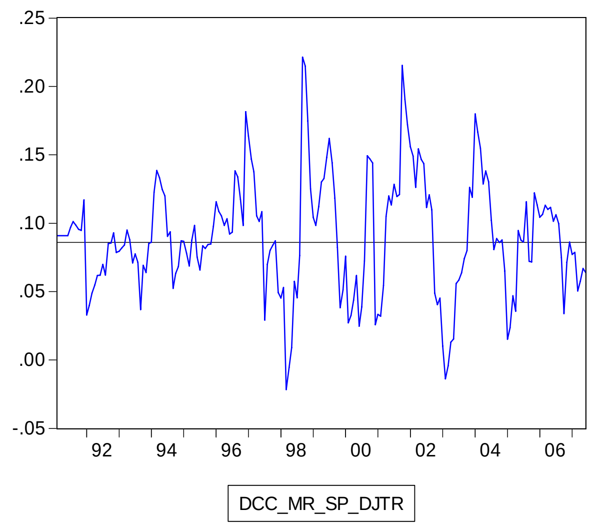
Note: Figure 3A depicts estimates of the time-varying correlation between the **weekly** unlevered rates of return on the S&P 500 (SP) and DJ-AIGCI total return (DJTR) indices from January 15, 1991 to July 2, 2007. The Figure provides plots for the following three estimation methods: exponential smoother with 0.94 smoothing parameter (top left panel), rolling historical correlation (top right) and dynamic conditional correlation by log-likehood for mean reverting model estimation (bottom panel). The straight line running through each graph is the unconditional correlation.

Figure 3B: Equity and Commodity Daily Return Correlations: S&P 500 *vs.* DJ-AIG, 1991-2007



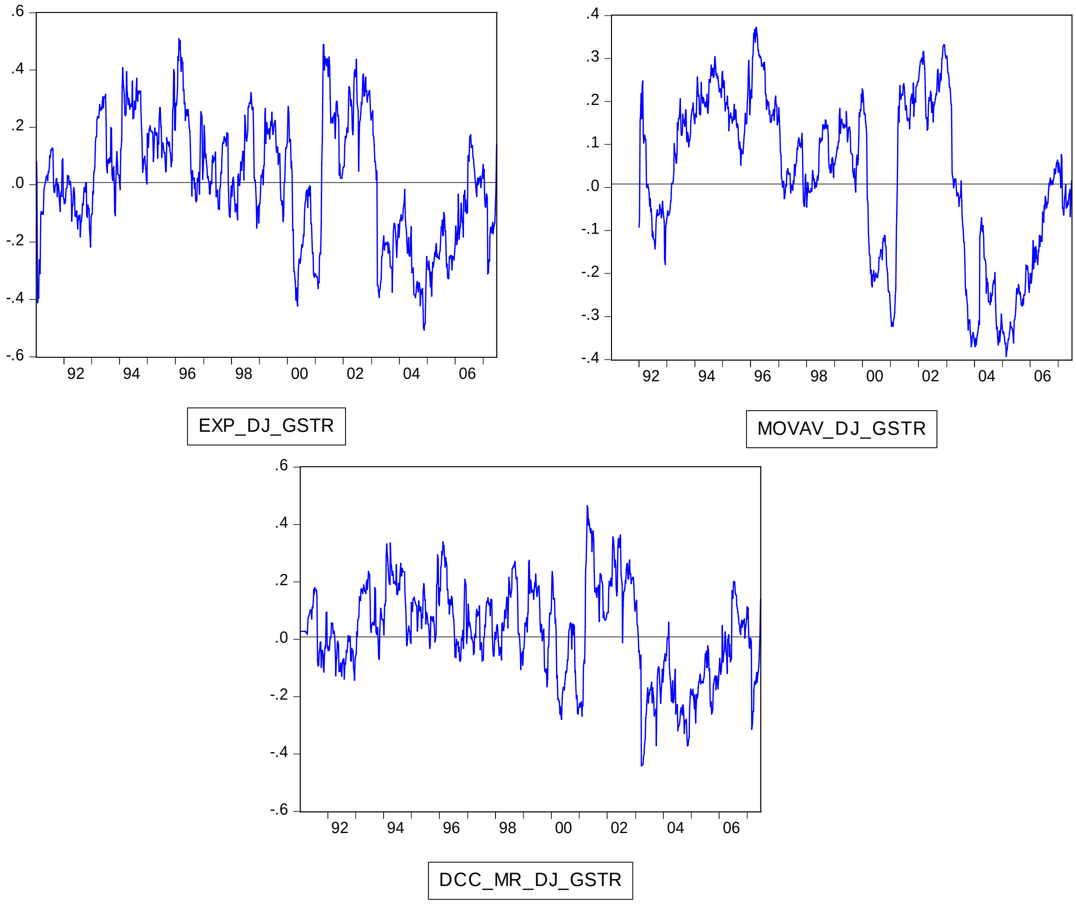
Note: Figure 3B depicts estimates of the time-varying correlation between the **daily** unlevered rates of return on the S&P 500 (SP) and DJ-AIGCI total return (DJTR) indices from January 15, 1991 to July 2, 2007. The method used to compute estimates is dynamic conditional correlation by log-likehood for mean reverting model estimation. The straight line running through each graph is the unconditional correlation.

Figure 3C: Equity and Commodity Monthly Return Correlations: S&P 500 *vs.* DJ-AIG, 1991-2007



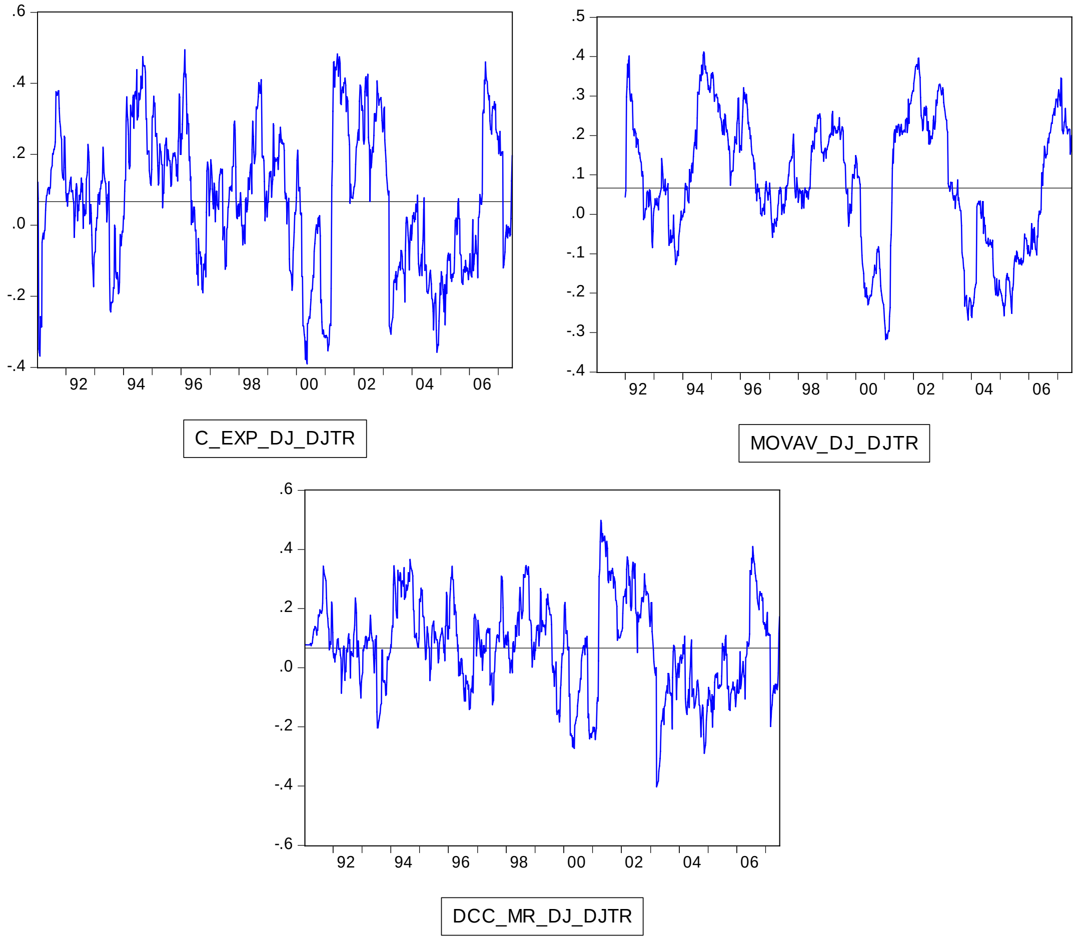
Note: Figure 3C depicts estimates of the time-varying correlation between the **monthly** unlevered rates of return on the S&P 500 (SP) and DJ-AIGCI total return (DJTR) indices from January 15, 1991 to July 2, 2007. The method used to compute estimates is dynamic conditional correlation by log-likehood for mean reverting model estimation. The straight line running through each graph is the unconditional correlation.

Figure 4: Equity and Commodity Weekly Return Correlations: DJIA *vs.* GSCI, 1991-2007



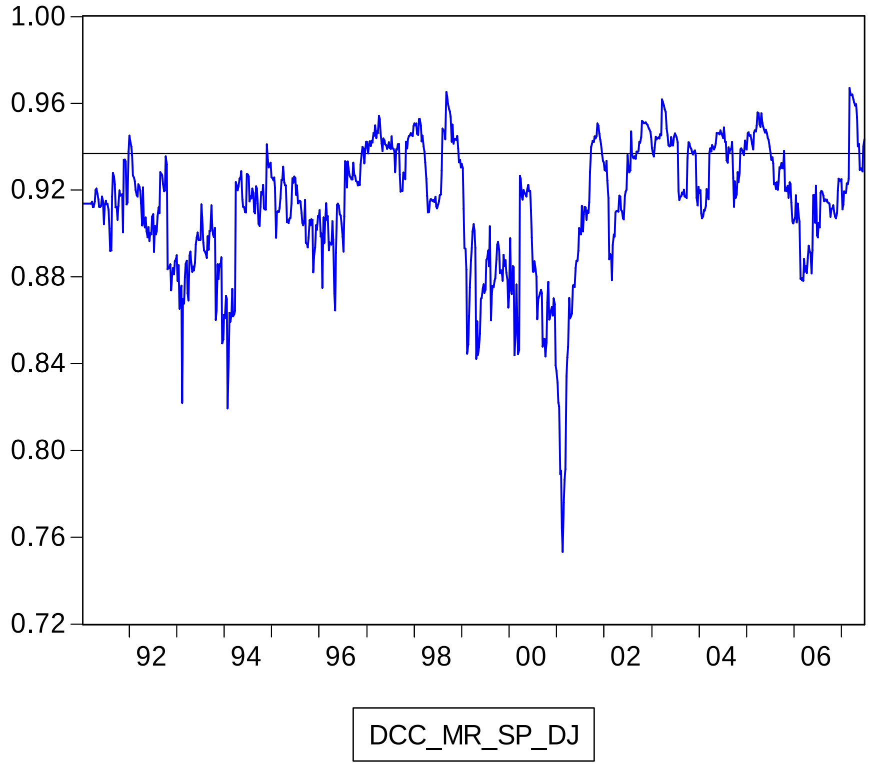
Note: Figure 4 depicts estimates of the time-varying correlation between the **weekly** unlevered rates of return on the Dow Jones Industrial Average (DJ) and GSCI total return (GSTR) indices from January 15, 1991 to July 2, 2007. The Figure provides plots for the following three estimation methods: exponential smoother with 0.94 smoothing parameter (top left panel), rolling historical correlation (top right) and dynamic conditional correlation by log-likehood for mean reverting model estimation (bottom panel). The straight line running through each graph is the unconditional correlation from Table 2A.

Figure 5: Equity and Commodity Weekly Return Correlations: DJIA *vs.* DJ-AIG, 1991-2007



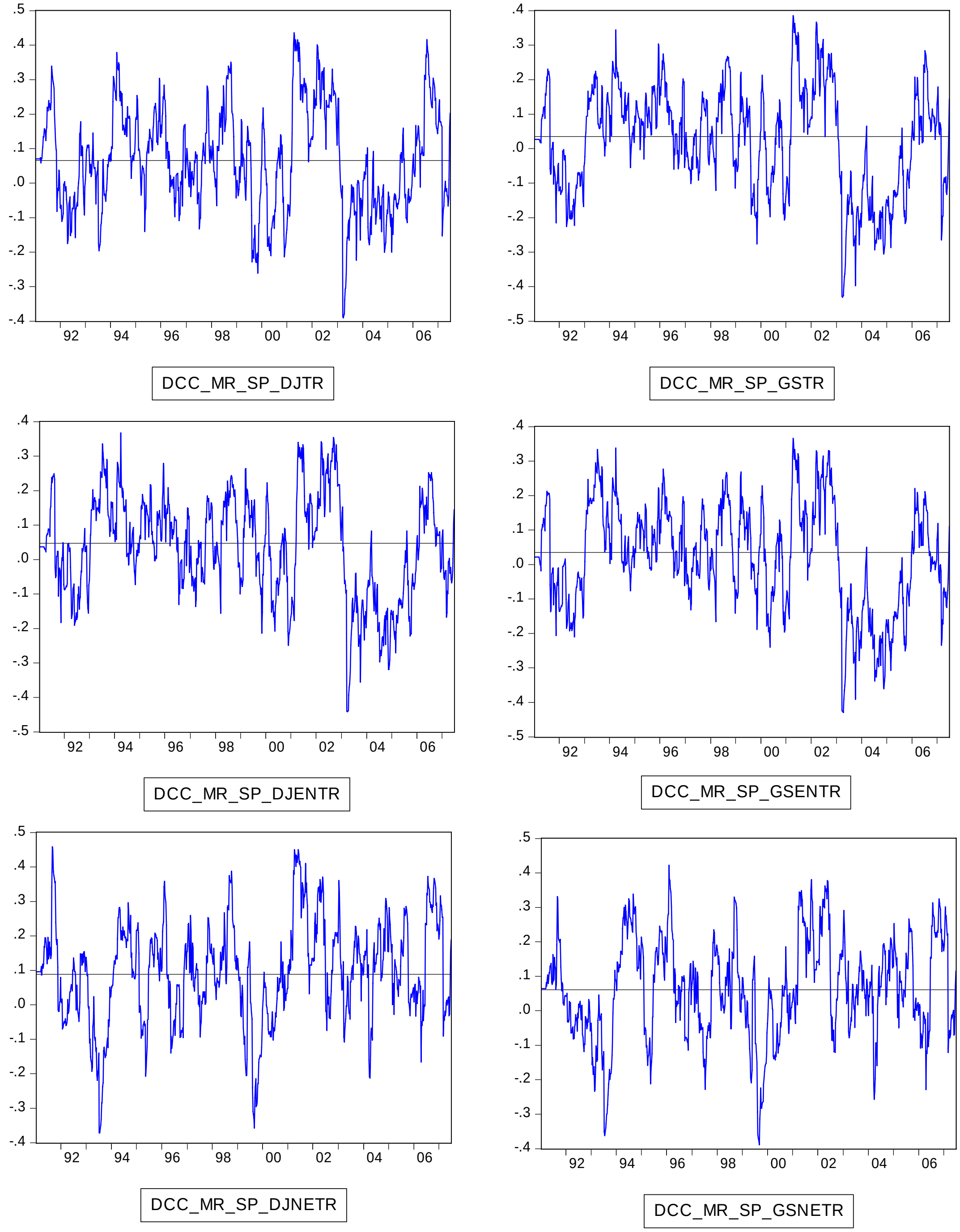
Note: Figure 5 depicts estimates of the time-varying correlation between the **weekly** unlevered rates of return on the Dow Jones Industrial Average (DJ) and DJ-AIGCI total return (DJTR) indices from January 15, 1991 to July 2, 2007. The Figure provides plots for the following three estimation methods: exponential smoother with 0.94 smoothing parameter (top left panel), rolling historical correlation (top right) and dynamic conditional correlation by log-likehood for mean reverting model estimation (bottom panel). The straight line running through each graph is the unconditional correlation from Table 2A.

Figure 6: S&P 500 and DJIA Weekly Equity Returns Correlations, 1991-2007



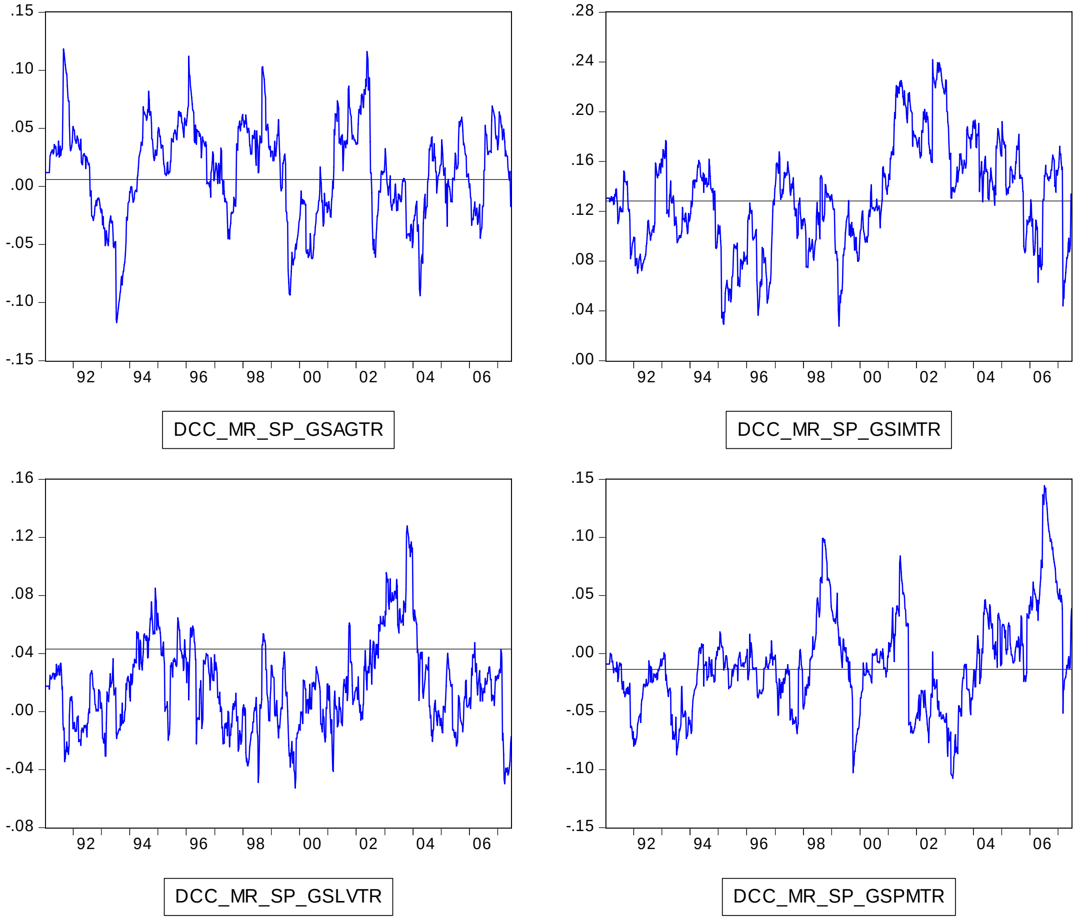
Note: Figure 6 depicts estimates of the time-varying correlation between the **weekly** unlevered rates of return on the Dow Jones Industrial Average (DJ) and S&P 500 (SP) equity indices from January 15, 1991 to July 2, 2007. The method to compute the estimates is the dynamic conditional correlation by log-likehood for mean reverting model estimation. The straight line running through each graph is the unconditional correlation from Table 2A.

Figure 7: S&P 500 vs. Energy and Non-Energy Weekly Return Correlations, 1991-2007



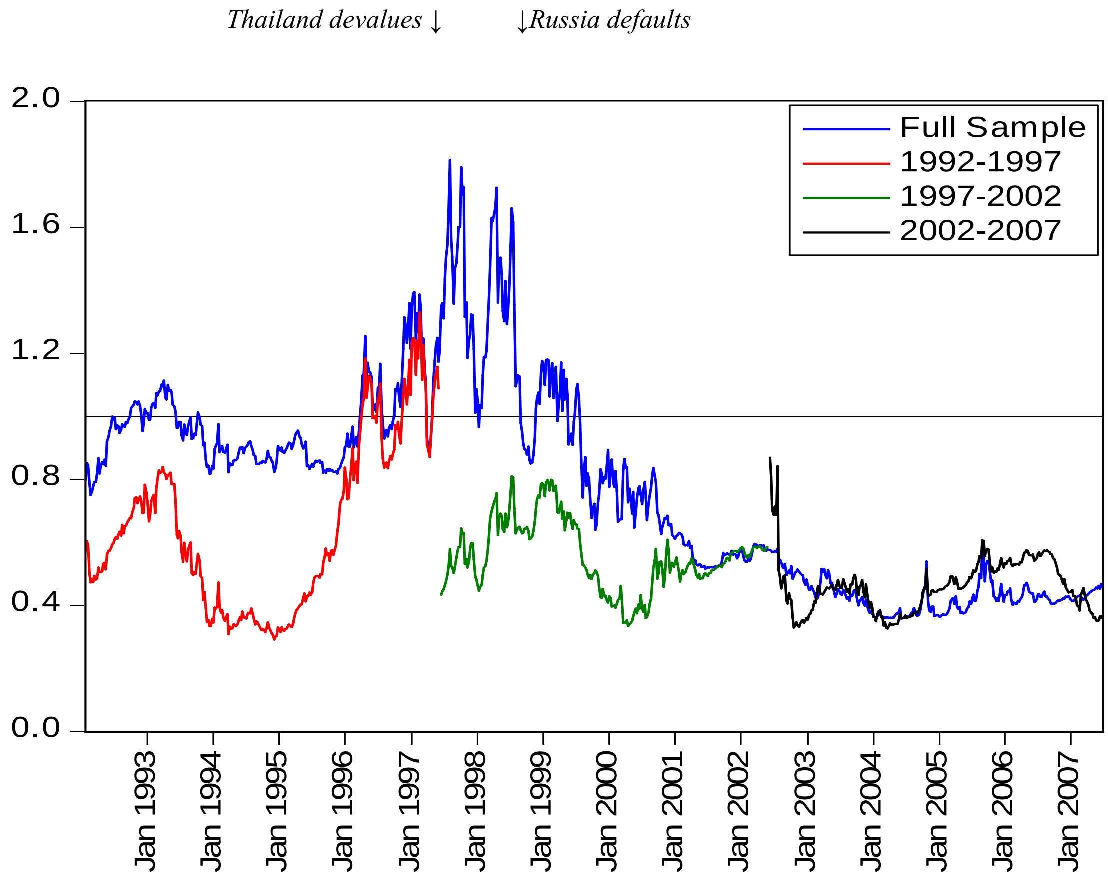
Note: Figure7 plots the time-varying correlations between the unlevered rates of return on the S&P 500 (SP) equity index and six investable commodity products. Dynamic conditional correlation are estimated by log-likelihood for mean-reverting model (Engle,2002). The straight lines through the graphs show the unconditional correlations from January 15, 1991 to July 2, 2007. Clockwise from top right: GSCI total return index (GSTR); GSCI energy total return index (GSENTR); GSCI non-energy total return index (GSNETR); DJ-AIG non-energy total return index (DJNETR); DJ-AIG energy total return index (DJENTR); and, DJ-AIG total return index (DJTR).

Figure 8: S&P 500 and GSCI Weekly Sub-Index Returns Correlations, 1991-2007



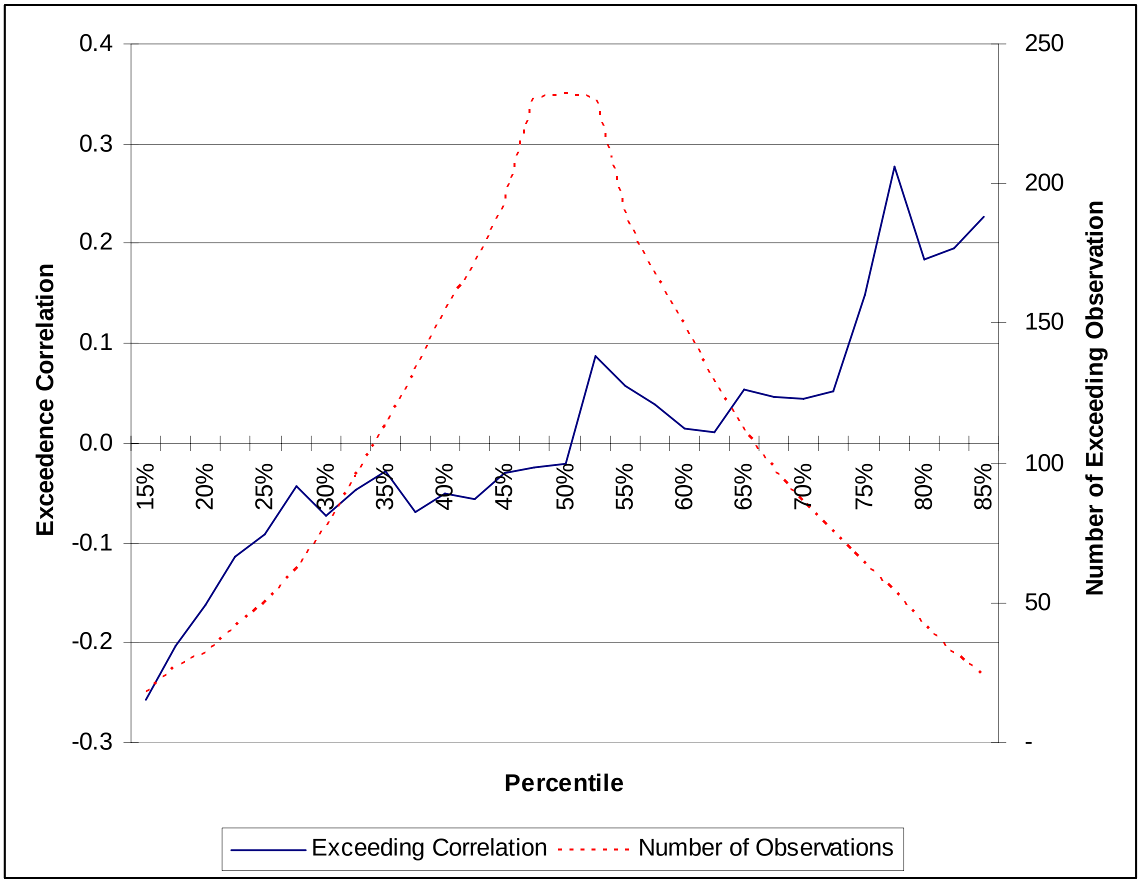
Note: Figure 8 depicts estimates of the time-varying correlation between the unlevered rates of return on the S&P 500 (SP) equity index and various investable GSCI total return commodity sub-indices. Counter-clockwise from the top left corner: GSCI total return (GSTR) index; GSCI Industrial-Metals total return index (GSIMTR); GSCI Precious-Metals total return index (GSPMTR); GSCI Livestock total return index (GSLVTR); and, GSCI Agriculture total return index (GSAGTR). Dynamic conditional correlation are estimated by log-likelihood for mean-reverting model (Engle,2002). The straight lines through the graphs show the unconditional correlations from January 15, 1991 to July 2, 2007.

Figure 9: Recursively Calculated Trace Test Statistic Scaled by the 5% Critical value, 1992-2007



Note: Figure 9 shows the R-1 form of the trace statistic. The 5% critical value is represented by the solid (horizontal) black line. The dark blue graph shows the estimate calculated recursively using data from the whole sample, i.e., from January 1991 through July 2007. Because the first year of observations was used to start the recursive procedure, the plot starts in mid-January, 1992. The red, green and black lines plot the estimate for three successive sub-periods: 1992-1997; 1997-2002; 2002-2007. For the each of the three sub-samples, weekly price data from the year prior to a given estimation period are utilized to start the recursive procedure for that period.

Figure 10: Extreme-Event Cross-Correlations, 1991-2007



Note: Figure 10 plots, for each percentile of the joint return distribution, the cross-correlation between the weekly returns on unlevered passive equity and commodity investments. The dotted line shows the number of observations in each percentile. The absolute value of the correlation is virtually nil (less than 0.1) for the vast majority of stock returns, namely, those between the 25th and the 75 percentiles of the joint return distribution. However, the solid line shows that, between 1991 and 2007, the cross-correlation was negative when equity and commodity returns were both poor, and positive when these returns were both strongly positive.