GDP Linked Bonds: Contract Design and Pricing

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Abstract

In this paper we analyse the comparative pricing of vanilla and GDP linked sovereign debt. The key feature of GDP linked bonds is that their cashflows-coupons, principal or both-are linked to the evolution of the country's national income. While it has long been argued that indexing debt to national output could be beneficial for emerging market countries, few models for pricing and analysing such debt have been proposed. We present a simple structural model of sovereign default that models the evolution of the sovereign's capacity to pay through observable macroeconomic variables. The model allows us to obtain prices and default profiles for vanilla bonds and various GDP linked structures that could be issued by emerging market sovereigns. We study four types of growth linked bonds: their default term structures, cash flow profiles, pricing under different assumptions about investor risk aversion and behaviour against an assortment of macroeconomic environments. The form chosen for indexation appears to be of paramount importance. Our results suggest that indexation to real growth, in the forms proposed by previous literature, may not provide a significant reduction in the likelihood of default relative to vanilla debt. Indexation to the growth of nominal GDP, expressed in US dollars, provides a considerable reduction in default likelihood. However, the characteristics of this indexation mean that investors would receive a bond linked to both inflation and growth, as well as significant exposure to the nominal exchange rate. A similar risk profile may be achievable by the sovereign from issuing a mixture of inflation linked, growth linked, foreign and local currency instruments.

Keywords: Sovereign debt, GDP linked bonds.

JEL classification: F34, G15.

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Introduction

In this paper, we develop a simple structural model for analysing sovereign GDP linked external (US dollar) bonds. The key feature of GDP linked bonds is that their cashflows—coupons, principal or both—are linked to the evolution of the country's national income. The International Monetary Fund and the United Nations have been actively campaigning in recent years for emerging market countries to make greater use of these instruments. It is argued that by introducing greater state contingency in sovereign debt contracts, GDP linked bonds will help to reduce the likelihood of formal default, which is known to be very costly for issuers, bondholders and the global economy. Issuing GDP linked debt can also help to stabilize government spending and limit pro-cyclicality of fiscal pressures, while investors get the chance to take a direct position on countries' future growth prospects (Borenzstein and Mauro (2004), Griffith-Jones and Sharma (2006)).

To date there have been relatively few attempts to develop a pricing framework for GDP linked bonds. We present a simple structural model of sovereign default that models the evolution of the sovereign's capacity to pay through observable macroeconomic variables, namely the real GDP, potential output of the economy and the real exchange rate. Using Monte-Carlo simulations, the model allows us to obtain prices and default profiles for vanilla bonds and various GDP linked structures that could be issued by an emerging market sovereign. It also allows us to explore the effects of macroeconomic fundamentals and investor risk preferences on the prices of bonds with different cashflow patterns.

We use our structural model to analyse the cash flow patterns and default characteristics of four growth indexed bonds and compare their characteristics to those of plain vanilla bonds. The main conclusion of our analysis is that the form chosen for GDP indexation is of paramount importance. Sovereign borrowers looking to issue these instruments should pay careful attention to their design. In particular, indexation to real growth, as frequently proposed in academic literature (e.g. Borenzstein and Mauro (2004), Miyajima (2005), Chamon and Mauro (2006), Griffith-Jones and Sharma (2006)) and recently implemented by Argentina, does not appear to be effective in reducing the likelihood of default on hard currency debt. Indexation to the growth of nominal GDP, denominated in US dollars, provides a significant reduction in default probability. However, the characteristics of the resulting instrument would effectively make it a hybrid between external and local currency debt and between growth and inflation indexed debt.

We proceed as follows: Section I presents the rationale for GDP linked bonds and the experience of those countries who issued them. Section II reviews the basic methods for pricing sovereign debt and briefly describes some previous work on pricing growth indexed debt. Next we present our theoretical model in Section III. Section IV describes the indexation structures that we examine using our model. Section V describes our pricing approach, while Section VI discusses calibration and simulation methodology. Section VII presents our results and Section VIII concludes.

I Why GDP linked bonds?

There are two key differences between the way sovereigns and corporates raise money in financial markets. First, sovereigns have a more limited set of financial instruments available to them. In particular, unlike corporates, sovereigns are unable to issue equity and equity hybrids such as convertible bonds. By taking a bigger stake, equity is more effective in absorbing bad shocks and encouraging risk sharing. These considerations suggest that the use of equity-like instruments in sovereign debt markets may significantly reduce debt crises and help to promote global financial stability.

Second, compared to corporate debt, the rights of creditors are less clearly defined with sovereign debt. The estimation of the repayment capacity of the country, the length and even the nature of the sovereign restructuring process are all subject to uncertainty. The result is that failure to comply with debt contracts is likely to be more costly in the case of sovereign debts than corporate debts. In some financial systems, corporate bankruptcy results in relatively little disruption in the operation of the company. For instance, Chapter 11 in the US provides the company with a fresh start free from excessive debt burden. In contrast, sovereign crises can have devastating effects on the domestic economy, including capital flight, serious stress on the banking sector and a collapsing domestic currency.

At the time of writing, the predominant way for sovereigns to raise money through the financial markets is by issuing plain vanilla bonds. The structure of sovereign debt repayment has important economic implications; in particular it could influence the susceptibility to crises, the degree of pro-cyclicality in fiscal policies, as well as broad economic performance (Chamon and Mauro (2006)). It is typical for emerging market countries to rely on short term and foreign currency debt, because the cost of financing is prohibitive otherwise. This being the case, investors can easily become concerned about the borrowing country's ability to meet its external obligations when economic growth weakens. As a result, its currency tends to depreciate and any new debt can only be issued at a higher interest rate. Both of these factors increase the local currency cost of servicing the debt while the country is experiencing an economic downturn. In an effort to persuade the creditors of its creditworthiness, the borrowing country may attempt to cut non-interest expenditures or raise taxes at the point in the economic cycle when it is least desirable. These policy responses may further constrain growth, leading to a vicious cycle. The phenomenon has been documented in the literature on "sudden stops" (e.g. Calvo 2003), where international borrowing often fails to serve as a device for smoothing the impact of slowdowns through time.

Considerations of this type have led to the claim that the current widespread use of plain vanilla sovereign bonds may be suboptimal for emerging market countries. This liability structure can make countries prone to financial crises that are seen as exceedingly costly, negatively affecting the citizens of the borrowing country, the international financial markets and other borrowers. Instead it is often postulated that borrowing instruments closer to state contingent contracts (i.e. equity-like instruments) would lead to a less crisis-prone liability structure for emerging market economies. This, in turn, would benefit both borrowers and lenders. Indeed, several authors, such as Caballero (2001, 2003) and Haldane (1999) argued that emerging market countries that rely heavily on commodity exports for revenue would benefit from indexing debt to commodity prices. Froot, Scharfstein and Stein (1989) maintained that linking debt payments to the growth performance of the issuing country would cushion the impact of negative growth shocks on the ability to service debt. Obstfeld and Peri (1989) suggested that the government may be able to reduce their idiosyncratic risks by issuing securities linked to nominal per capita GDP. Shiller (1993, 2003) proposed creation of macro markets for perpetual claims on a fraction of country's GDP, arguing that this would allow the sovereign to buy insurance against growth uncertainty and help smooth the revenue loss from adverse economic performance. Barro (1995), in a model of optimal debt management, where government seeks to smooth tax rates over time, shows that bonds need to be indexed to consumption and government expenditure.

The use of GDP linked bonds, as proposed in this paper, introduces a degree of state contingency into sovereign debt contracts that could potentially benefit both issuers and investors. For emerging economy borrowers it can help stabilize government spending and limit pro-cyclicality of fiscal pressures. GDP linked repayments result in smaller debt service in times of slower growth, providing the government breathing space for a fiscal stimulus of the economy. The need to pay higher interest rates in times of high growth can also curb excessively expansionary fiscal policy.[1] Moreover, by allowing debt service to fall in times of slow or negative growth, GDP linked bonds reduce the likelihood of costly formal default. Investors also benefit from the reduced default frequency, since in sovereign markets default often results in costly litigation and re-negotiations. Moreover, GDP linked bonds would provide market participants with ability to effectively leverage their macroeconomic expertise and take direct positions on the future economic prospects of a country.

Despite the strong incentives outlined above, only a small number of countries have issued GDP linked securities so far. Bulgaria, Bosnia and Herzegovina and Costa Rica issued growth linked bonds as part of their Brady restructurings in the 1990s. The most liquid of these was Bulgaria's issue, where the bonds provided for a GDP kicker, such that once GDP exceeds 125% of its 1993 level, creditors would be entitled for an additional 0.5% in interest for every 1% of GDP growth in the year prior to interest payment. These bonds were callable by the issuer, meaning that Bulgaria could repay the principal and re-finance it should the GDP linked payments appear likely to be triggered. As Miyajima (2006) notes, this feature appears inconsistent with the objective of issuing GDP linked debt, as it offers the opportunity to reduce the share of debt that is resilient to GDP shocks.

More recently, Argentina issued GDP linked bonds in the debt-restructuring exchange completed in June 2005. The GDP link is specified as follows: it will pay 5% of excess cash flows, defined as the difference between actual GDP and threshold GDP, when three trigger conditions are satisfied: real GDP exceeds threshold GDP, the annual growth rate of real GDP exceeds 3% and the total payment cap has not been reached. The GDP "warrants" (i.e. the additional growth linked cash flows) are detachable from the plain vanilla bonds and, as Miyajima (2006) reports, have been traded separately since November 2005. Press reports suggest that after initial scepticism, Argentina's GDP warrants have recently become very popular with emerging market investors.[2] This is despite the fact that, as noted by Fernandez, Pernice, Streb, Alegre, Bedoya and Gonzales (2006, Chapter 5), there is some confusion among practitioners on how to price these warrants, in particular which discount rate to apply.

II Previous studies of sovereign debt

A Comparative pricing considerations

A natural question that comes to mind is why have there been relatively few instances when GDP linked bonds were issued, given their potential advantages over vanilla debt. There may be a number of reasons for this, most of which were outlined in a survey article by Borenzstein and Mauro (2004). According to surveys of market participants, one of the main obstacles in the way of wider acceptance of these instruments is model risk—the lack of a clear conceptual framework for pricing growth linked bonds.[3]

Although there is no generally accepted principle for pricing sovereign bonds, the investor base that normally participates in sovereign debt markets is very familiar with trading (and pricing) plain vanilla bonds. A natural question for us, therefore, is that of comparative pricing: how does the price of a GDP linked bond differ from a plain vanilla one issued by the same issuer? Clearly, the effect would depend on the exact details of the structure of the GDP link. However, for illustration purposes, let us consider a simple case, where a bond that pays a "base case" coupon δ_t. This coupon can be reduced to zero if certain unfavourable growth events occur or increased to an arbitrary value contingent on the occurrence of favourable growth events. The price of this bond can be decomposed into elements shown in Figure 1.

Starting from the bottom, the first component is the plain vanilla bond with coupon δ,

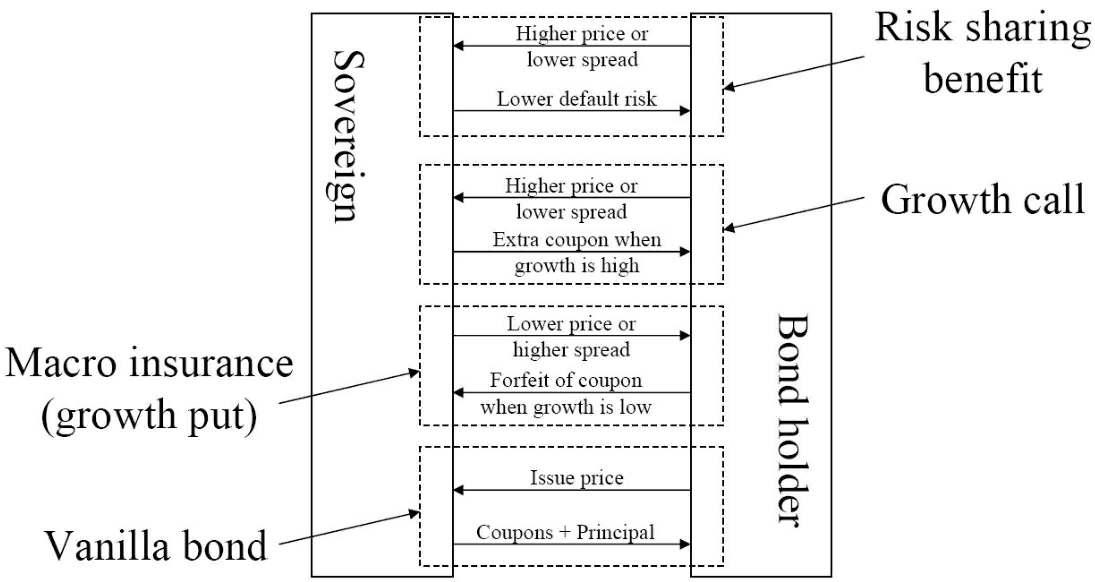

Figure 1: Elements of GDP linked bonds

which the sovereign sells to the bondholder. In the case of GDP linked bond, however, the sovereign also buys insurance from the bondholder that reduces the sovereign's coupon payments contingent on unfavourable GDP developments. The sovereign has to pay a fee for this insurance, which is factored into the price of the GDP linked bond. On the other hand, when growth is higher than expected, the sovereign makes higher payments to the bondholder compared to a plain vanilla bond. Thus the bondholder purchases the possibility of these higher payments, contingent on certain GDP developments, and has to pay a fee to the sovereign for this "growth call". The final element of the GDP linked bond is due to the possibility of more favourable macroeconomic and default dynamics that the link to growth may facilitate. In particular, the possibility of lower likelihood of formal default may also result in greater total payments to the bondholder, which would translate into higher issuing price and lower spread.

A "first best" pricing model for GDP linked debt should therefore have two features. Firstly, we must be able to use the framework of the model to price plain vanilla sovereign bonds in order to enable comparative pricing. Second, the framework should adequately take into account all four components of the price of GDP linked bond outlined above.

B Structural and reduced form models of sovereign default

As noted by Duffie, Pedersen and Singleton (2003), although sovereign debt is notably different from corporate debt, sovereign bonds have typically been priced using similar structural or reduced-form models of a single "default" credit event. In a structural model, default occurs when incentives suggest that it is optimal for the issuer to default or when payment on debt is impossible. For example, in Merton (1974), default occurs at maturity when there are insufficient assets to pay debt. A reduced form model, on the other hand, models default as a random occurrence. Default has an exogenously specified intensity process and arrives as a "surprise" that may not be endogenously linked to (observed) decision variables of the debtor (Jarrow and Turnbull (1995), Duffie and Singleton (1999), Duffie, Pedersen and Singleton (2003)).

The last thirty years saw the development of a substantial body of structural models of corporate default (for example Black and Cox (1976), Leland (1994), Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Fan and Sundaresan (2000)), however this approach has not been applied widely to sovereign borrowers. Conventional wisdom is that structural models of sovereign default can be problematic to develop and implement numerically. It is typically necessary to model the value of the sovereign's assets in terms of observable variables. Theoretically, this is no easy task and is one of the main sticking points in applying the structural approach to sovereign default. A general problem of structural models of sovereign debt is that most of the variables needed for their construction are unobservable, since the value of the sovereign is not traded, unlike the value of the corporation. Moreover, as noted by Duffie et. al. (2003), there may also be measurement issues and the incentives of the sovereign to default may be rather complex. In light of these pragmatic considerations, studies of sovereign debt pricing have focused primarily on models based on an exogenously specified intensity process.[4]

The most notable contribution to the reduced form pricing literature for sovereign debt is Duffie et. al (2003). They develop a model of the term structure of sovereign spreads that accommodates a variety of credit events as well providing for "implicit seniority" of some instruments. In particular, the model accounts for the possibility of outright default, whereby the sovereign announces that it will stop making payments on its debt;

restructuring, where the sovereign and the lenders agree to reduce or postpone remaining payments; and a "regime switch", such as the change in government or the default on another sovereign bond that changes the perceived risk of future defaults. It also encompasses the possibility that similar bonds issued by the same sovereign may be priced in the market using different discount factors. Reasons for this may include bond covenants, differences in collective action clauses or political events that cause the sovereign to differentiate between bond issues (for example Soviet and post-Soviet era debt in Russia)—all of which lead to the so-called "implicit seniority" of certain obligations (Bolton and Jeanne (2005)).

As noted by Anderson and Sundaresan (2000), however, there are several limitations of the reduced form approach. Firstly, an abundance of possible functional forms can be calibrated to a given set of benchmarks, but may imply significantly different values when pricing some other issue. This can result in difficulties in selecting the most appropriate model. Secondly, for many pricing problems there are no reliable benchmarks, making it desirable to establish values from first principles. Our concern is to evaluate the impact of linking debt payments to growth on the payoff distribution and default probability of the bond. The nature of the problem makes it naturally suited to a structural model of default.

C Previous pricing exercises

At the time of writing, there have been relatively few attempts to develop a pricing framework for GDP linked bonds. Prior to the completion of Argentina's restructuring, a number of sell-side institutions published Monte-Carlo exercises that attempted to quantify the value of GDP linked cash flows for that specific case. These early valuations typically relied on simulating only the dynamics of GDP. Looking at Figure 1, they were concerned mainly with the valuation of the growth put and growth call components of the bond, taking as given the price of the vanilla bond and implicitly assuming that the price of the risk sharing benefit was equal to zero. Others taking a similar approach include Kruse Meitner and Schröder (2005), who attempt to apply the Black-Scholes formula to value options on GDP, and Miyajima (2006), whose concern is on designing bonds that match incremental tax revenues received by the sovereign to the extra payments on the GDP linked bonds.

In our view, the major drawback of this strand of research is that it views GDP linked payments and vanilla bonds as additive. That is, the price of the GDP linked bond is derived simply as the price of a vanilla bond in the absence of the link to growth plus the price of GDP linked payments. The underlying implicit assumption is that introducing the GDP-link will not change the default likelihood. Yet one of the main theoretical arguments in favour of GDP linked bonds is that their use may reduce the probability of costly formal default.

A more comprehensive pricing approach was developed by Chamon and Mauro (2006), who concentrate on distress that is due to an increase in the debt burden instead of the deterioration in the sovereign's ability to pay. The paper considers a hypothetical case of a country that has both local currency and dollar debt outstanding and also has issued both vanilla and GDP linked debt. Chamon and Mauro present a framework where an increase in debt burden (relative to GDP) beyond a certain level triggers default. They do not explicitly model how the country's capacity to pay evolves with growth. To value the debt, they estimate, based on past data, the joint distribution of three macroeconomic variables—the growth of real GDP, the real exchange rate and the primary balance (fiscal balance excluding interest payments on consolidated government liabilities)—for a range of emerging market countries. Monte-Carlo simulations are then used to model a range of paths for these variables and derive the evolution of the ratio of total debt to GDP that corresponds to each path. The debt/GDP ratio is specified to be the variable determining the time of default. Specifically, the market prices of plain vanilla bonds are used to extract the trigger level for the debt to GDP ratio. If this ratio rises higher than the trigger level, default occurs. This trigger level is chosen so that (together with an assumed recovery rate) it would yield expected repayments consistent with market spreads. In other words, the resulting probability of default implies that some vanilla bonds would be traded at par (or current market price if available). Their results seem to support a strong case for GDP indexation. With the introduction of GDP indexation, the average price of the country's debt increases, while the likelihood of default falls.

We believe that there are several disadvantages of their approach. The use of debt to GDP ratio as a default trigger can be problematic for a variety of reasons. First, as noted by Borenzstein, Levy Yeyati and Panizza (2007), the debt to GDP ratio at the time of the events of default of countries that have defaulted since the 1980s has had a wide range of values, from around 0.4 to 15. Many countries have had debt levels within the same range and have not fallen into default. This echoes Reinhart, Rogoff and Savastano (2003), who note that emerging countries frequently have far lower levels of debt to GDP than developed countries, yet they default much more frequently. Second, in our view, the debt to GDP ratio does not represent the evolution of the sovereign's capacity to pay through time. Instead it gives the evolution of debt relative to sovereign earnings. As a result, the default term structure for a 10-year plain vanilla bond in Chamon and Mauro's model is arguably unrealistic, with most defaults taking place in the second year of the bond's life. There are also several other drawbacks. By working in a risk-neutral world, they assume that investors take no consideration about the cash flow distribution characteristics of different bonds other than the mean value. By considering local currency and external debt together they need to calibrate the model to some sort of average price and arrive at an average default probability. This ignores the fact that the factors affecting the probability of default may well be very different for local and external debt.

III Modeling the GDP linked cash flows

In this section we develop a simple structural model for sovereign debt that relates debt cash flows to the dynamics of macroeconomic variables. Typically, a structural model of default postulates that default occurs when the variable representing the borrowers' ability to pay (usually the value of the borrower's assets) crosses some barrier (for example the face value of the debt). The first steps, therefore, are to model the value of the sovereign's assets, or sovereign "wealth" that can be used for making debt payments, and the boundary at which default or restructuring of debt occurs. We should note at this stage that the default barrier in this model is determined exogenously—we do not employ a game theoretic approach, whereby the sovereign can make a strategic default decision. We recognise that an endogenous determination of the default barrier may be theoretically desirable, however since our main focus is on the joint determination of national income and sovereign assets we do not use this approach here.

For simplicity, we will assume that all the market debt of the sovereign is represented by one bond, denominated in foreign currency, and that we are concerned with valuing this bond.[5] After initial time $t = 0$, there is no further issuance of debt. Unlike Chamon and Mauro (2006), we are concerned with modelling default due to a deterioration in capacity to pay, rather than default as a result of an increase in debt to an unsustainable level.

Our next assumption is that the sovereign's assets that can be devoted to debt service at time t are a function of the potential output (or output trend) at that time. Potential output is typically defined as a measure of sustainable output in the economy, in which the intensity of resource use is not adding to or reducing inflationary pressure. It is also a measure of real GDP trend, where the influence of short term shocks and the business cycle has been removed. The motivation behind this assumption is that higher sustainable growth makes it possible to accumulate greater resources to service debt. It can also be related to the additional borrowing capacity available to the sovereign, should new debt be used to service old debt in order to avoid default. We denote potential real output (real GDP trend) at time t in domestic currency by $\overline{Y_t}$ and assume that it follows geometric Brownian motion, so that

$$d\overline{Y_t} = \mu \overline{Y_t} dt + \sigma \overline{Y_t} dZ. \tag{1}$$

Intuitively, μ determines the expected long run sustainable growth rate and σ represents the amount of fluctuations in $\overline{Y_t}$.

Actual real GDP in domestic currency at time t, which we denote by Y_t, is related to potential GDP through the output gap. The output gap at time t, G_t, is defined as the ratio of actual to potential output, so that

$$Y_t = G_t \overline{Y_t}. \tag{2}$$

The logarithm of the output gap is assumed to follow an Ornstein-Uhlenbeck process

$$dg_t = -\kappa g_t dt + V dW, \tag{3}$$

where $g_t = \ln G_t$, and κ is the rate of reversion toward zero. In economic terms, V drives the severity of the business cycle, or temporary booms and contractions, while κ determines its length.

By our assumptions and footnote 5, the real value of the sovereign assets in domestic currency at any time t and before debt service can be expressed as a simple linear function of real potential output. Denoting this real value of the sovereign assets, or wealth, in local currency terms by W_t^R, we can write

$$W_t^R = \varsigma \overline{Y_t}. \tag{4}$$

Bond payments, however, will be made in nominal terms and in foreign currency. Denoting the local price level at time t by p_t and the nominal exchange rate (foreign currency price of domestic currency) by s_t, we can express the sovereign assets in nominal foreign currency terms as

$$W_t^{\$} = \varsigma p_t s_t \overline{Y_t}. \tag{5}$$

Denoting the foreign price level at time t by p_t^*,[6] we can also define the (inverse of the) real exchange rate as

$$q_t = \frac{p_t s_t}{p_t^*},$$

so that

$$W_t^{\$} = \varsigma q_t p_t^* \overline{Y_t}. \tag{6}$$

To proceed further, we make one more simplifying assumption: the foreign price level is constant, that is $p_t^* = p^*$.[7] In this case, denoting $\theta = \varsigma p^*$, we can then rewrite (6) as

$$W_t^{\$} = \theta q_t \overline{Y_t}. \tag{7}$$

The evolution of the real exchange rate in our model is given by

$$dq_t = \nu \left(\frac{d\overline{Y_t}}{\overline{Y_t}} - c dt \right) q_t + \Omega q_t dZ^*, \tag{8}$$

where c and ν are constants and Ω is the real exchange rate volatility. This process allows potential output developments to play a role in exchange rate changes, with the magnitude of impact given by parameter ν. Setting $\nu > 0$ can be interpreted in terms of the Balassa-Samuelson effect—the well known observation that a country with faster trend growth relative to its trading partners is likely to experience real exchange rate appreciation. We can think of parameter c as a proxy for the growth of the country's trading partners. High growth of potential output at time t, combined with a low value of c implies that the real exchange rate will tend to appreciate. We could also view c as denoting the market's confidence in government policies. In this case, the lower the value of c the higher the market's confidence—the market requires lower real income growth in order for nominal (and hence real) appreciation to occur. On the other hand, setting $\nu = 0$, leads to

$$dq_t = \Omega q_t dZ^*,$$

which is a Brownian motion with zero mean and volatility Ω. In this case, the evolution of potential output plays no role in real exchange rate movements.[8] Since the real exchange rate is an index, we can initialise it to 1 at the start of the simulation (making real and nominal output the same at $t = 0$).

Let us now define $R_t^{\$}$ to be the nominal foreign currency value of the sovereign's assets after debt service. How should $R_t^{\$}$ evolve over time? $R_t^{\$}$ represents the total dollar "wealth" available for debt service at time t, given that all coupons in periods $t-1, t-2, ..., 0$ have been paid. Denoting the coupon payment at time t by δ_t, we can then write

$$\Delta R_t^{\$} = R_t^{\$} - R_{t-1}^{\$} = W_t^{\$} - W_{t-1}^{\$} - \delta_t = \Delta W_t^{\$} - \delta_t, \tag{9}$$

and hence

$$R_t^{\$} = R_{t-1}^{\$} + \Delta W_t^{\$} - \delta_t, \tag{10}$$

where $\Delta W_t^{\$}$ denotes the growth in sovereign wealth between the ends of periods $t-1$ and t that is due to macroeconomic developments, and δ_t denotes the debt service paid in period t. Additionally, we assume that $R_0^{\$} = W_0^{\$}$. The impact of the coupons is permanent, that is once a coupon is paid it reduces the resources available to the sovereign in all future periods. The lesser the sovereign has paid for servicing debt today, *ceteris paribus*, the more it has available to service debt in the future.[9] In the case of zero coupon bond, $\delta_t = 0$ for all t and $R_t^{\$} = W_t^{\$}$. It is important to note that in our model the size of the debt service does not affect future growth, which is exogenous. However, lower payments to service debt result in greater resources available to the sovereign, thus *ceteris paribus* making default less likely.

Default occurs when the foreign currency value of sovereign assets after debt service $R_t^{\$}$ falls below some critical level R^*. An important assumption in our model is that restructuring does not occur. Rather we assume that at the time of default the bondholders receive immediately a certain percentage of the face value of the bond. This percentage is called a recovery rate. In the implementation, we assume the recovery rate to be 25% and R^* to be equal 1.0675 times the face value of the bond.[10] To allow direct comparison of our results to those obtained by Chamon and Mauro (2006), we set the face value of the bond to 60% of nominal output at the start of the simulation.

The main building blocks of our model are now in place. To summarise, the sovereign's ability to service its debt (measured in real terms and in domestic currency) is a function of its real potential output. Actual real GDP is obtained by combining the dynamics for potential output with the output gap. The real exchange rate is then used to convert the real value of the sovereign's resources into foreign currency terms. Equations (1), (2), (3), (8) and (10) thus describe the dynamics and interactions between different variables. These equations are the basis of the Monte-Carlo simulations described in Section VI. The joint evolution through time of $(\overline{Y_t}, g_t, q_t)'$ corresponds to one path in our simulation.

IV Contract Design

The proponents of GDP indexation for external debt claim that it will be beneficial for both the issuer and the investor. GDP indexation will result in a lower probability of formal default, which is costly for both parties, by providing insurance for the sovereign in case of unfavourable macroeconomic developments. It will also provide the opportunity for investors to take a direct position on countries' future growth prospects. In this study, we consider four different ways of structuring the cash flows of the GDP linked bond. The first is the simplest, whereby the coupon is a shifted linear function of real growth.

The second example resembles Argentina's growth linked debt, whereby the coupon is equal to a base rate plus a variable rate that is a function of excess output, with payments conditioned on both real GDP growth and level exceeding certain targets. In the third and fourth examples, the bondholder is committed to the greatest risk sharing, whereby the coupon is indexed against the growth rate of nominal GDP measured in foreign currency. All of our bonds are non-amortising and have a maturity of 10 years with coupons paid annually.

A Bond 1: Real GDP growth bull spread

It is typically suggested that indexation should take a relatively simple form, so that investors can easily assess the characteristics of the instrument (Borenzstein and Mauro (2004), Chamon and Mauro (2006), Griffith-Jones and Sharma (2006)). Hence, Chamon and Mauro propose the following GDP linked coupon for a 10-year bond:

$$\delta_t^{GDP} = \max\left[0, 3.75\% + g_t\right], \tag{11}$$

where g_t is the annual growth rate of real GDP. Investors receive a zero coupon in year t if real GDP declines by more than 3.75% in year t. The unlimited upward potential of the coupon in (11) is equivalent to a bull spread on growth.

B Bond 2: Conditioned on real GDP growth trend

In the second method of indexing, several conditions have to be met in order for growth linked repayments to be triggered. The aim is to provide greater insurance for the sovereign, while at the same time ensuring adequate payments to bondholders. For example, the growth linked bonds offered to investors during the recent restructuring of Argentina's sovereign debt have coupons similar to the following indexation structure:

$$\delta_t^{GDP} = \begin{cases} \overline{\delta} + \gamma q_t(Y_t - Y_0 e^{\overline{g}t}) & \text{for } Y_t > Y_0 e^{\overline{g}t},\ Y_t > Y_{t-1}e^{g^*}; \\ \overline{\delta} & \text{otherwise.} \end{cases}$$

A minimum flat coupon $\overline{\delta}$ is paid to the bondholders regardless of growth. An extra GDP linked coupon is paid in year t if (i) the real GDP growth with respect to the base year exceeds a preset level $\overline{g}$ per annum, and (ii) the real GDP annual growth is great than g^*. This indexation structure ensures that the country recovering from a severe contraction does not have to make growth linked payments until output has recovered above the pre-contracted trend level. Furthermore, the payments can be linked to the difference between actual output and the trend.

In our implementation the additional bond structure parameters are set as follows: the pre-contracted trend growth, $\overline{g} = 2\%$; the level of annual growth that has to be exceeded in order for growth linked payments to occur, $g^* = 2\%$; the minimum flat coupon received by bondholders, $\overline{\delta} = 5.75\%$; the proportion of excess output paid to bondholders as growth linked payment, $\gamma = 10\%$. While we omit the detailed results here to conserve space, our analysis shows that as γ increases, bond cash flows increase and so does the probability of default. Increasing g^* and $\overline{g}$ decreases the bond cash flows as well as the probability of default. However, it's worth noting that both bond cash flows and the default probability are more sensitive to variations in $\overline{g}$ than those in g^*.

C Bond 3: Conditioned on nominal dollar growth

Real growth is only one of the factors that affects the sovereign's ability to service foreign currency denominated debt. The other, considerably more critical factor, is the real exchange rate q, which converts the real assets of the sovereign into foreign currency terms. Hence, we may envisage an indexation structure that links coupon payments to nominal GDP, measured in US dollars, as follows:

$$\delta_t^{GDP} = \max\left[2\%, g_t^{\$} - 9\%\right],$$

where $g_t^{\$}$ is the growth rate of the nominal GDP in US dollars, that is

$$g_t^{\$} = \frac{Y_t^{\$} - Y_{t-1}^{\$}}{Y_{t-1}^{\$}}, \tag{12}$$

and $Y_t^{\$} = Y_t p_t s_t = Y_t q_t$ is the dollar nominal GDP at time t. This indexation structure guarantees the bondholder a minimum coupon of 2% in all time periods, with an unlimited upward potential should the growth rate of nominal dollar GDP exceed 11%.

D Bond 4: Nominal growth collar

All of the growth linked bonds presented above give the bondholder the potential to receive unlimited payouts. However, the sovereign may be concerned about avoiding high debt service even when its ability to pay rises. To address this concern, the coupon payments can take the following form:

$$\delta_t^{GDP} = \min\left\{\max\left[2\%, g_t^{\$} - 3\%\right], 15\%\right\},$$

where $g_t^{\$}$ is defined in (12). This structure gives the bondholder a minimum coupon of 2%, with potential for greater coupon payments if nominal dollar GDP growth exceeds 5%. However, the maximum coupon paid out in any year is capped at 15%.

V Pricing and the effect of risk aversion

Our approach to pricing GDP linked and vanilla sovereign bonds is similar to the Esscher transform method outlined, for example, in Gerber and Shiu (1994). Let us consider a simple economy with only a stream of risky cashflows x and a risk-free bond. In what follows, we use x_t to denote the risky cashflow that occurs at time t and let $F_{0,t}$ denote the time 0 forward price of this cashflow x_t. The sovereign (GDP linked) bond represents a stream of risky cashflows x_t, $t \in (0, T]$, where T is the bond's term to maturity. These cashflows depend on the structure of the bond, as well as the sovereign's ability to meet its contractual obligations. There is a representative investor who owns a unit of the stream of risky cashflows ($1 face value of the risky bond) and bases his decisions on a risk-averse utility function $u(\cdot)$. Further, let's assume that there exists a derivative security that provides a payment of π_t at time $t > 0$, where π_t is some function of the cashflows $x_0, x_1, ..., x_t$. Let $V_{0,t}$ denote the time 0 forward price for the derivative security. The number of derivative contracts z that the investor would want to buy or sell can be derived by maximising the expected utility function

$$W(z) = E\left\{u\left(x_t + z[\pi_t - V_{0,t}]\right)\right\}.$$

Using the first order condition[11]

$$W'(z) = 0,$$

we can write

$$0 = E\left\{u'\left(x_t + z[\pi_t - V_{0,t}]\right)\left(\pi_t - V_{0,t}\right)\right\}.$$

If we now impose $z = 0$, implying that the derivative is fairly priced and therefore the representative investor would not want to buy or sell any derivative contracts in this equilibrium, we obtain

$$V_{0,t} = \frac{E\left\{\pi_t u'(x_t)\right\}}{E\left\{u'\left(x_t\right)\right\}}. \tag{13}$$

Equation (13) must hold for any derivative security, including the case where $\pi_t = x_t$. For $\pi_t = x_t$, the forward price of the derivative equals the forward price of the cashflow, $F_{0,t} = V_{0,t}$, and we can write

$$F_{0,t} = \frac{E\left\{x_t u'(x_t)\right\}}{E\left\{u'\left(x_t\right)\right\}}.$$

In the case of the exponential utility function $u(x_t) = 1 - e^{-\eta x_t}$, we get

$$F_{0,t} = \frac{E\left\{x_t e^{-\eta x_t}\right\}}{E\left\{e^{-\eta x_t}\right\}}. \tag{14}$$

In other words, the (forward) asset specific pricing kernel[12] is given by

$$\psi\left(x_t\right) = \frac{e^{-\eta x_t}}{E\left\{e^{-\eta x_t}\right\}}, \tag{15}$$

and the forward value of the cashflow can be written in standard form as

$$F_{0,t} = E_t^{\mathbb{Q}}\left\{x_t\right\} = E\left\{\psi\left(x_t\right) x_t\right\}, \tag{16}$$

where $E_t^{\mathbb{Q}}[\cdot]$ denotes the expectation under the risk-adjusted probability measure.

Let us denote the risky cashflow that occurs at time t in state of the world i by

x_{ti}. Here, the risky cashflow stream $x_{1i}, x_{2i}, ..., x_{Ti}$ corresponds to the payments along simulation path i of a vanilla or GDP linked sovereign bond with maturity T. There are three stochastic equations in our model, equation (1) governs the behaviour of potential output $\overline{Y_t}$, equation (3) the output gap g_t and equation (8) the real exchange rate q_t. These equations, together with the contractual designs of the bonds outlined in Section IV, determine the cashflows x_t.[13]

Assuming the simulation is run for N paths, at each point $t \in (0, T]$ there are N states of the world, each with an equal probability of occurring $p_{ti} = 1/N$. In this setting, we can use (15) to define the asset specific pricing kernel in state i as

$$\psi\left(x_{ti}\right) = \frac{e^{-\eta x_{ti}}}{E\left\{e^{-\eta x_{ti}}\right\}} = \frac{N e^{-\eta x_{ti}}}{\sum_{i=1}^{N} e^{-\eta x_{ti}}}, \tag{17}$$

and using (17) we can re-write equation (16) as

$$F_{0,t} = \sum_{i=1}^{N} p_{ti} x_{ti} \psi(x_{ti}) = \sum_{i=1}^{N} x_{ti} \frac{1}{N} \frac{N e^{-\eta x_{ti}}}{\sum_{i=1}^{N} e^{-\eta x_{ti}}} = \sum_{i=1}^{N} \frac{x_{ti} e^{-\eta x_{ti}}}{\sum_{i=1}^{N} e^{-\eta x_{ti}}}. \tag{18}$$

If we assume that the representative investor is risk neutral, then $\eta = 0$, in which case equation (18) becomes

$$F_{0,t} = \sum_{i=1}^{N} \frac{x_{ti}}{N} = \sum_{i=1}^{N} p_{ti} x_{ti} = E\left\{x_t\right\}.$$

This forward price is correct only if investors take no consideration about the cash flow distribution characteristics other than the mean value. However, given the same level of mean return, one would expect a distribution with a higher standard deviation, a negative skewness and a high kurtosis to be a riskier investment. A higher value of η means the investor is more risk averse and will discount large positive cash flows more heavily, since $\frac{\partial \psi}{\partial x} < 0$.

Assuming a constant risk free interest rate r, the spot price of the bond can then be written as the discounted sum of these cashflow streams

$$P_0 = \sum_{t=1}^{T} e^{-rt} F_{0,t} = \sum_{t=1}^{T} e^{-rt} \sum_{i=1}^{N} \frac{x_{ti} e^{-\eta x_{ti}}}{\sum_{i=1}^{N} e^{-\eta x_{ti}}}. \tag{19}$$

VI Simulation and Calibration

A Simulation methodology

In this section we present a detailed discussion of our simulation procedure. We simulate (1) and (3) as follows:[14]

$$\overline{Y}_{t+\Delta t} = \overline{Y}_t \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\varepsilon_t\sqrt{\Delta t}\right],$$

$$g_{t+\Delta t} = g_t \exp\left(-\kappa\Delta t\right) + V\varepsilon_t\sqrt{\frac{1-\exp\left(-2\kappa\Delta t\right)}{2\kappa}}.$$

Equation (8) is slightly more difficult, because the process for q_t at each point in time depends on the contemporaneous actual growth rate of potential output $\frac{d\overline{Y}_t}{\overline{Y}_t}$. To simulate the equation we used the Euler approximation scheme, resulting in (8) being discretised as follows:

$$q_{t+\Delta t} = q_t\left[1 + \nu\left(\frac{\overline{Y}_{t+\Delta t} - \overline{Y}_t}{\overline{Y}_t} - c\Delta t\right) + \Omega\varepsilon_t\sqrt{\Delta t}\right], \quad \varepsilon_t \backsim N(0,1).$$

The accuracy of this scheme is $O(\Delta t)$.

B Convergence rate of the simulation

All simulations were run for 500,000 paths. To give an idea of the convergence of our simulation results, we present some example vanilla bond prices and 95% confidence intervals for alternative numbers of paths and Δt values below. Setting $\Delta t = 0.01$ gave us a price of 99.9987 for the vanilla bond (assuming $\nu = 1$), with a 95% confidence interval of [99.9086, 100.0888]. In the following sections we present all results for 500,000 paths and $\Delta t = 0.01$ (approximately 4 days).

Table I: Convergence rate of simulation results

		Number of paths		
		100,000	500,000	1,000,000
	0.05	99.9333	99.848	99.9564
		[99.7316, 100.1351]	[99.7576, 99.9383]	[99.8926, 100.0202]
Δt	0.01	99.9183	99.9987	99.9633
		[99.7165, 100.1202]	[99.9086, 100.0888]	[99.8995, 100.0271]
	0.005	99.971	100.0089	100.0197
		[99.7695, 100.1725]	[99.9188, 100.0990]	[99.9560, 100.0834]

Note: 95% confidence intervals in brackets. Prices are computed under the baseline macroeconomic scenario (see Section VI.C), assuming ν=0 and the representative investor is risk neutral.

C Selecting macroeconomic parameters

Throughout our analysis we assume that the risk free rate r is constant and equal to 4%. For the baseline scenario in our simulation we set the expected growth rate of potential output $\mu = 3\%$ and the volatility of potential output $\sigma = 2\%$. The mean reversion rate of the output gap κ was set at 50% and the output gap volatility V at 4%.[15] The values were estimated from historical data over the past 100 years of three major emerging market sovereign issuers (Brazil, Mexico and Argentina).[16] Figure 2 shows the evolution of the logarithms of real GDP, estimated potential GDP and the output gap for Argentina since 1900.

The volatility of the real exchange rate was set at 16%, which is again in line with historical data for the major emerging market issuers (see Chamon and Mauro (2006)). Parameter c was set to 3%. The parameters of the baseline macroeconomic scenario are summarised in the table below.

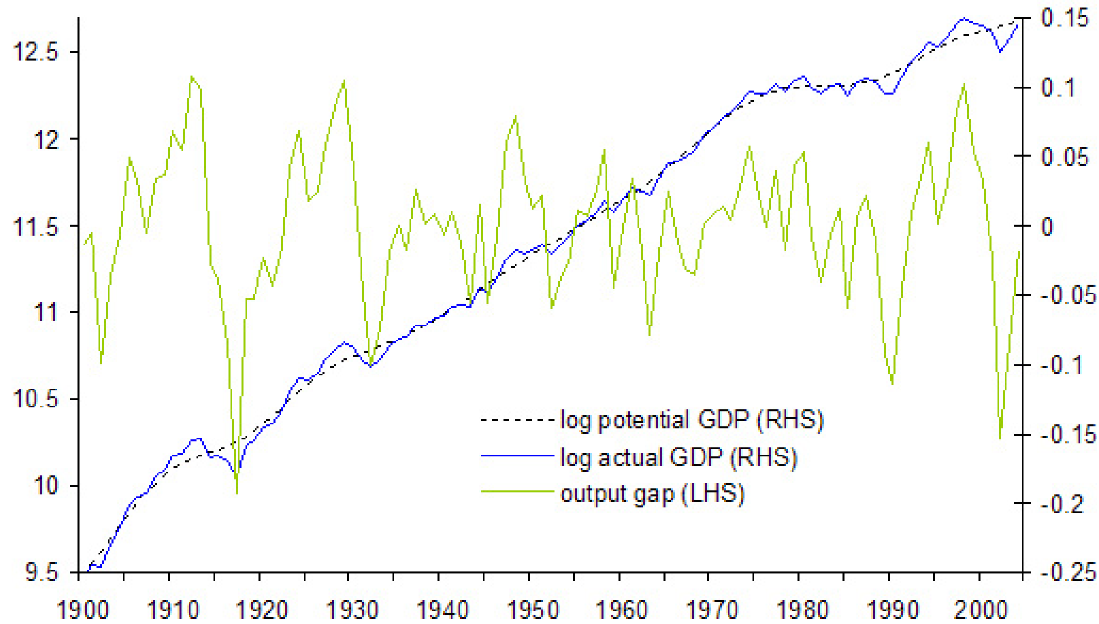

Figure 2: Real GDP, potential output and output gap for Argentina

Table II: Baseline macroeconomic scenario

Variable	Description	Value
μ	mean potential output growth	3%
σ	volatility of potential output	2%
V	volatility of the output gap	4%
κ	output gap mean reversion	50%
c	average GDP growth rate of trading partners	3%
Ω	real exchange rate volatility	16%
r	risk-free interest rate	4%

We have calculated all results for 2 variants of the real exchange rate evolution. First, we set $\nu = 0$, so that the actual growth of potential output does not affect the evolution of the exchange rate. Then we recalculated for $\nu = 1$, which allows for the full impact of the Balassa-Samuelson effect influencing the real exchange rate.

In order to examine the sensitivity of bond prices to changes in the macroeconomic environment, we also considered a range of alternative scenarios. We varied the potential output growth rate μ from 1% to 7% and the volatility of potential output σ from 1% to 10%. We let the mean reversion rate κ in the output gap equation take values from

30% to 80% and the volatility of the output gap V to change from 2% to 7%. Finally, the exchange rate parameters took values of 1-7% for the Balassa-Samuelson constant c, and 10-25% for the volatility of the real exchange rate Ω. We believe that these scenarios set up reasonable ranges for the macroeconomic variables in emerging market countries.

D Selecting θ and η from the vanilla bond price

We use our model to find the prices and probabilities of default of five bonds: a 10 year vanilla bond[17] paying an annual coupon of 6.75% and four GDP linked structures. We assume that the vanilla bond is trading at par and the prices of GDP linked bonds are obtained by employing this assumption.

There are two structural parameters in our model that can be calibrated so that the price of the vanilla bond is par—the macroeconomic constant θ in (7) and the risk-aversion parameter η in (19). The macroeconomic constant θ in is a determinant of the resources available to the sovereign for debt service. Increasing θ means that the sovereign has more resources available for debt service. *Ceteris paribus* this decreases the probability of default, increasing the price of the bond. The risk aversion parameter η affects the investor's subjective discounting of cashflows. A higher η means that the investor places more weight on unfavourable outcomes. *Ceteris paribus* this decreases the price that the investor is willing to pay for the bond. The risk aversion parameter in itself does not effect the sovereign's ability to pay or the default rate. It only affects the price through investor preferences. While the two parameters impact the bond price in opposite ways, there are many combinations of θ and η that are consistent with a par price for the vanilla bond.

D.1 The risk neutral case: $\eta = 0$, $\theta = 1.31$

First consider the case of the risk-neutral investor with $\eta = 0$. Setting $\theta = 1.31$ and using the baseline macroeconomic scenario gives the vanilla bond price of par (\$100) and a default probability of around 31% over the 10 year period.[18] The prices of the GDP linked structures range from \$99.74 to \$102.84.[19] The risk neutral payoff distributions of the vanilla bond and the four GDP linked bonds are shown in Figure 3. Under the risk neutral scenario, the cash flows for all bonds are discounted at the assumed risk free rate of 4%

without taking into consideration that these cash flow profiles have different distributional characteristics. The resulting discounted values are the bond prices that investors would pay if they were impartial to the higher moments of the cash flow distributions.

All five discounted cashflow distributions presented in Figure 3 share a common characteristic; they have two modes. The first mode represents the distribution of smaller cash flows when there is a default, and the second node the distribution of the larger cash flows if there is no default. The cumulative default probabilities are, respectively, 31%, 31% and 29% for the vanilla bond, Bond 1 and Bond 2. For bonds linked to nominal growth, Bond 3 and Bond 4, the default probabilities are considerably lower at approximately 20%. We will examine the factors underlying the comparative default probabilities of vanilla and different GDP linked bonds in detail in Section VII.A.

D.2 The risk adjusted case: $\eta = 0.005$, $\theta = 1.70$

Historically, however, a cumulative default probability of 31% within 10 years for a vanilla bond seems high even for issuers with a poor credit rating. Both Moody's and Standard and Poor's compile cumulative corporate default rates over periods of one to twenty years on the basis of historical data. Data given by Moody's (2000) suggests that in recent experience the cumulative 10-year default rates for non-investment grade corporate issuers varied between 7-31%.[20] We believe that the sovereign spread of 250bps over US Treasuries for a 10-year bond, given a risk-free rate of 4%, is consistent with the sovereign rating of between Ba and Baa on the Moody's scale. Using data from 1920 to 1999, the cumulative 10-year default rate for Ba rated corporates was 19%, while for Baa rated corporates it was 8%. We therefore believe that a cumulative 10-year default rate of around 15% could be reasonable for our hypothetical sovereign.

This default rate can be obtained in our model in the baseline macroeconomic scenario by setting $\theta = 1.70$. However, if the risk aversion rate η remains at zero, then the price of the vanilla bond becomes 111.66, which is no longer consistent with our assumption of a par price. To reduce the price of the bond, while maintaining the lower probability of default, we decided to increase the risk aversion parameter. Setting $\theta = 1.70$ and $\eta = 0.005$ is consistent with a vanilla bond price of par and a default rate of 15% under the baseline macroeconomic scenario. These parameters are therefore used in our base case when we analyse the sensitivity to macroeconomic variables in Section VII.B.

VII Results and sensitivity analysis

The analysis in this section is based on $\theta = 1.7$, $\eta = 0.005$, and $\nu = 1$. The first row of Table III presents the prices and the corresponding default probabilities of the vanilla and GDP linked bonds under the baseline macroeconomic scenario. The vanilla bond is priced at 100.05 (approximately par), while the prices of GDP linked structures range from 99.18 to 100.61. The prices of all GDP linked structures are close to par.

Default rates, on the other hand, differ substantially between bonds. The default rates for the vanilla bond and Bond 1 (real growth bull spread) are very similar and in the case of Bond 2 (conditioned on growth trend) there is a slight reduction in the likelihood of formal default. Bonds 3 and 4, indexed to nominal dollar growth, however, have default rates that are almost half those of the vanilla bond. Figure 4 presents the default term structures for all five bonds. The default term structures show that the probability of default in each year increases at a decreasing rate up to maturity for all bonds.

A Analysing the differences in default probability

At the first glance on the default rates for the baseline scenario, it might seem illogical that the GDP linked features of Bond 1 and Bond 2 did not help to bring down the default probability. Indeed, in the case of Bond 1 (the real growth bull spread), the default rate is actually slightly worse than that of the vanilla bond. There are several explanations for the high default probability of Bond 1. The ability of the sovereign to service its debt at time t in our model is determined by two factors: macroeconomic developments, in particular the evolution of potential output and the real exchange rate, and the size of debt service payments up to time t. First, it is possible for GDP to rise through the increase in the output gap, but for potential output (and hence real ability to pay) to remain constant in the course of a normal business cycle. This would, *ceteris paribus*, trigger higher GDP linked coupons without a matching increase in the sovereign's ability to service debt. Second, while real GDP increases, it is possible for the real exchange rate to fall at the same time, potentially reducing the value of $W_t^{\$}$ available for debt service. Third, the indexation structure of Bond 1 does not provide protection for the country recovering from a severe output contraction. As soon as the annual GDP growth exceeds the pre-set threshold, the country is penalised by higher debt service costs even though the level of GDP may be low. Finally, *ceteris paribus*, higher debt service in period t means that the sovereign will have less resources to devote to debt service in periods $t+1$ onwards. Because of the asymmetry of the indexation formula (maximum payments are uncapped) this results in an increase in default probability.

It is possible that all four explanations present different degrees of severity to sovereign's susceptibility to default. In the case of Bond 2, the third explanation does not apply because of the additional conditions set on growth trend. Yet the default probability of Bond 2 is approximately 5% higher than that of Bond 3 and Bond 4, which are indexed to nominal dollar growth, highlighting the critical role the exchange rate and inflation play in determining sovereign's ability to service debt.

B Sensitivity to macroeconomic variables

B.1 Potential output (long run) growth rate, μ

The next row in Table III examines the sensitivity of bond prices and default rates to the mean potential output growth rate. Everything else equal, the higher the underlying trend real GDP growth rate, μ, the higher the sovereign's debt service capacity. Hence, the prices of all bonds rise and the probability of default falls as μ grows. Higher average growth also means that the real exchange rate is more likely to appreciate, increasing the foreign currency debt service capacity further.

The default rate impact of μ is most vivid in the case of vanilla bond, which experiences the largest swing in default rates when μ moves from 1% to 7%. In the case of the GDP linked bonds the effect of μ is two-fold. A lower μ will increase default risk, but the impact is cushioned by the protection of the GDP linked clause that allows the sovereign to (partially) waive coupons. Hence for $\mu = 1\%$, the default rates of all GDP linked bonds are lower than that of the vanilla bond. A higher μ results in an improvement in fundamental determinants of capacity to pay, but also triggers higher coupon payments.

Overall, this results in higher default rates for Bond 1 and Bond 2 (both indexed to real growth), compared to the vanilla bond, as potential output growth increases to 5% and 7%. Bond 2's default rates are slightly smaller than Bond 1 because of the added protection from the extra conditions on growth. Bond 3 and Bond 4, both indexed to nominal dollar growth, have the lowest default rate profiles, largely due to extra protection against real exchange rate movements, an issue which we shall return to later in subsection B.5.

B.2 Potential output volatility, σ

The next row in Table III proceeds to examine the sensitivity to potential output volatility. As σ increases, the probability of default rises for all bonds due to a higher chance of adverse output developments. Since we allow output developments to influence the real exchange rate, higher potential output volatility can also increase the effective volatility of the real exchange rate, making default more likely still. The price of the vanilla bond falls, mirroring the rising default probability. However, in the case of the GDP linked bonds, higher potential output volatility increases the likelihood of higher GDP linked cashflows being paid. The overall result is that the prices of all bonds drop as σ rises, with the vanilla bond being the most sensitive. For Bond 3 (nominal dollar growth bull spread), on the other hand, it is evident that the higher probability of default is partially offset by the possibility of higher nominal growth due to greater real growth and real exchange rate movements (and co-movements). The price of Bond 3 is the least sensitive to increases in σ.

B.3 Output gap parameters κ and V

The parameters of the output gap equation have no effect on the default probability and cash flow profile of the vanilla bond, since they do not influence the sovereign's ability to pay. For the GDP linked bonds, both the mean reversion rate κ and the output gap volatility V seem to have fairly small effect on the default rates and prices.

Changing the output gap mean reversion rate κ from 2% to 10% has no discernible impact on default probability and prices of the four GDP linked bonds. Increasing the output gap volatility V from 2% to 7% slightly increases the default probability, but also produces higher GDP linked cash flows, resulting in a small increase in the price of GDP linked bonds.

B.4 Balassa-Samuelson effect, c

The variable c is our proxy for world growth or, alternatively, the market assessment of the government's credibility. It affects the evolution of the real exchange rate. All other things equal, a high c makes it more difficult for the real exchange rate to appreciate. As c increases, default probability increases and the bond cash flows are substantially reduced for all bonds. The impact is most notable for Bond 3 (nominal dollar growth bull spread) since in this case the sovereign has also passed on some of the real exchange rate risk to the bondholder.

If we set $\nu = 0$ in equation (8), the evolution of the real exchange rate is driven by randomness only, with fundamental factors having no impact. While we do not report detailed results here to conserve space, setting $\nu = 0$ significantly decreases the price of Bond 3 under the default macroeconomic scenario to 96.80. It also lowers the sensitivity of prices to mean potential output growth μ and volatility of potential output σ, since these parameters now have no effect on real exchange rate evolution.

B.5 Real exchange rate volatility, Ω

The real exchange rate volatility Ω has a big impact on the default probability and cashflow profiles of all bonds. The default probability rises substantially for all bonds as Ω is increased from 10 to 25%, as higher real exchange rate volatility makes a deterioration in capacity to pay more likely. However, higher real exchange rate volatility can also make high nominal growth more likely, potentially benefiting the cashflow profiles of Bonds 3 and 4 (both linked to nominal dollar growth).

The net effect is that the prices of the vanilla bond, the bonds linked to real growth and Bond 4 (nominal dollar growth collar) all fall as Ω rises. For Bond 3 (nominal dollar growth bull spread), the price impact is less clear. Setting Ω at 10% reduces the price of Bond 3 in comparison with the default scenario where $\Omega = 16\%$, as the lower volatility decreases the chances of high nominal growth. Increasing Ω to 20% and 25%, however, also reduces the price, as the possibility of higher coupons on the upside becomes offset by higher chances of default on the downside. Therefore, it appears that there may be an optimal value of Ω for which the price of Bond 3 is maximised.

C Varying the model set up

C.1 The effect of risk aversion

Let us now examine the effect on increasing the risk aversion of the representative investor on the prices of the vanilla and GDP linked bonds in different macroeconomic scenarios. Table IV presents prices calculated by setting the risk-aversion rate $\eta = 0.01$ and leaving the macroeconomic constant $\theta = 1.70$. The default rates remain unaffected, as the risk aversion parameter has no effect on the sovereign's ability to pay. In all cases the prices of all bonds drop as η rises, since the investor now places more weight on unfavourable outcomes, which reduces the price he is willing to pay for all bonds. Instead of presenting prices and default probabilities, Table IV presents the prices for $\eta = 0.01$ and the corresponding percentage price changes as η rises from 0.005 to 0.01.

As η doubles from 0.005 to 0.01, the percentage price drops under the default macroeconomic scenario are similar for all bonds, ranging between 14% for Bond 4 (nominal dollar growth collar) and 17% for Bond 1 (real growth bull spread). Typically, those scenarios that lead to a drop in price relative to the baseline macroeconomic set-up when risk-aversion is kept constant are associated with the largest differences in prices as risk aversion rises. For example, if mean potential output growth is 1%, the drops in prices as η rises from 0.005 to 0.01 range from a 18% (Bond 1) to 25% (Bond 3). On the other hand, if mean potential output growth is 7%, the declines in prices vary between 1% (Bond 1) and 6% (Bond 3). This effect is also evident when we vary all other macroeconomic variables. It is due to the greater weight placed on unfavourable outcomes by the more-risk averse representative investor.

Considering differences between various bonds, it appears that Bonds 3 and 1 (nominal and real growth bull spreads) are the most sensitive to greater risk aversion. Looking at Figure 3, the cashflow profiles of these two bonds are more spread out than those of the other instruments, meaning that they derive a larger proportion of their value from the right tail of the cashflow distribution. As investors place less weight on more favourable outcomes, the prices of these bonds experience greater declines.

Looking at individual bonds, increasing risk aversion changes the behaviour of Bond 3 (nominal dollar growth bull spread) in several ways. In particular, Bond 3 becomes considerably more sensitive to a rise in potential output volatility σ. With higher risk aversion, investors care less about the increased possibility of higher payouts and become more concerned with default risk and lower coupons, resulting in a more dramatic fall in price of Bond 3 as σ rises. For the same reason higher risk aversion means that the price of Bond 3 starts to unambiguously decline as the real exchange rate volatility Ω increases.

C.2 The effect of jumps in the exchange rate

Up to this point we have allowed all variables in our model to evolve continuously. However, historically, this has not always been the case. The last decade of the 20th century witnessed a number of currency crises affecting the international financial markets. The economies which suffered financial crises and attacks were quite diverse and in some cases had strong macroeconomic fundamentals. These crises are typically characterised by a sharp depreciation of the nominal and real exchange rate. This sharp change cannot be generated by the dynamics of equation (8). To allow for the possibility of currency crises, we can augment (8) as follows

$$dq_t = \nu \left(\frac{d\overline{Y_t}}{\overline{Y_t}} - cdt \right) q_t + \Omega q_t dZ^* + q_t dJ, \tag{20}$$

where J is a compound Poisson process with intensity λ and the logarithm of jump sizes is distributed lognormally with mean μ_J and volatility σ_J, such that if j is the jump size then $\ln(1+j) \sim N(\ln\,(1+\mu_J), \sigma_J^2)$. Using the modified dynamics for the real exchange rate in our Monte-Carlo simulations, we can assess the impact that sharp depreciations in the exchange rate have on the price of bonds in our model.

To model equation (20), we need to estimate 3 extra parameters: the intensity of the jump process λ, the mean jump size μ_J and the volatility of jumps σ_J. To gauge reasonable values for these parameters, we examined the recent history of the monthly real exchange rate series calculated by the central banks of Argentina, Brazil and Turkey. Any monthly return greater than 3 standard deviations in absolute size was considered a jump. In the 11 years of data available for Argentina there were 4 such moves, in 19 years of data for Brazil there were 3 such moves and in 27 years of data for Turkey there were also 3 such moves. All of these were currency depreciations with approximate average size of 20%. Hence we set $\mu_J = -0.2$. Furthermore if we denote the times of successive jumps by τ_{j+1} and τ_j, then we know that

$$P(\tau_{j+1} - \tau_j \leq \Delta\tau) = 1 - e^{-\lambda\Delta\tau},$$

where $P(\tau_{j+1} - \tau_j \leq \Delta\tau)$ is the probability that the time between two successive jumps is less than $\Delta\tau$. Setting $\lambda = 0.3$ therefore gives a probability of 26% that successive jumps will occur within one year of each other, 78% that they will occur within 5 years of each other and 95% that they will occur within 10 years of each other. We believe that these probabilities are reasonable for our hypothetical emerging market sovereign. Finally, we set the jump volatility $\sigma_J = 0.1$.

The results are presented in Table V. Under our baseline macroeconomic scenario the prices of all bonds—vanilla and GDP linked—drop by approximately 30-40% and the probabilities of default rise by around 40%. For the vanilla bond, and Bonds 1 and 2 (both linked to real growth), the size of the price drop is roughly similar, regardless of whether we assume the representative investor to be more ($\eta = 0.01$) or less ($\eta = 0.005$) risk averse. For the Bonds 3 and 4, linked to nominal growth, however, the price drop is larger when the investor is assumed to be more risk averse. Unsurprisingly, from the investor's perspective, the possibility of a sharp currency depreciation would make holding vanilla bonds or bonds linked to real growth more attractive than bonds linked to nominal growth denominated in foreign currency. It appears, therefore, that bonds linked to nominal dollar growth would be best suited for those issuers judged by the market unlikely to suffer a crisis during the bond's life.

D Summary

Here we summarise how macroeconomic parameters influence the prices of the bonds according to our model. We present a short review of the values of macroeconomic parameters that result in high bond prices in the table below.[21]

Table VI: Optimal macroeconomic parameters to maximise bond prices

	μ	σ	c	Ω
Vanilla	high	low	low	low
Bond 1	high	low	low	low
Bond 2	high	-	low	low
Bond 3	high	low	low	medium (low)
Bond 4	high	low	low	low

Note: values in brackets correspond to $\eta = 0.01$

As we can see from the summary table, high potential GDP growth rate μ, and strong credibility (or weak trading partner growth) c, put the sovereign in a strong position when issuing bonds, whether they are GDP linked or not. If the potential GDP growth is highly volatile (σ is high) and investor risk aversion is low, then issuing Bond 3 could help to bring down the issue costs, as it is the least sensitive to changes σ. If the real exchange rate is volatile, i.e. Ω is high, then a GDP linked structure that ties payoffs to nominal GDP measured in hard currency and does not have a cap on coupons would be the most appropriate.

In all scenarios bonds linked to nominal growth measured in foreign currency have considerably lower default rates than the vanilla bond or bonds linked to real growth. However, when we allow for jumps in the real exchange rate, the differences between default rates become much less pronounced. Thus, from the investor's perspective the possibility of sharp currency depreciation can negate the advantage of bonds linked to nominal foreign currency growth.

VIII Discussion and conclusion

Growth linked sovereign debt has long been proposed as an important device to help stabilize emerging market economies. However, such bond issues are not common, partly because of a lack of understanding of the growth indexation structure and pricing issues. In this paper we present a simple structural framework of sovereign default and use it to analyse the cash flow patterns and default characteristics of GDP linked bonds and compare the characteristics of these instruments to plain vanilla bonds. Specifically, we consider four types of growth indexation structures: Bond 1's coupon is a bull spread on actual real GDP growth, Bond 2 adds additional constraints on growth trend and threshold on annual growth rate similar to the restructured Argentinean debt, while Bond 3 and Bond 4 are conditioned on nominal GDP growth measured in hard currency. Our model is centred on the dynamics governing the growth of potential real GDP, the evolution of the output gap and the real exchange rate. We study the default term structure and cash flow profiles of these GDP linked bonds as well as the defaultable vanilla bond under different macroeconomic environments.

The preceding discussion and results make it clear that the types of GDP indexation seen in Bond 1 and Bond 2, while increasing expected payments to bondholders, do not provide a significant reduction in the likelihood of formal default compared to the vanilla bond under most scenarios. In terms of the basic pricing components, it is the growth call, rather than the risk sharing benefit, that appears to be behind the higher price of these bonds. Moreover, Bond 1 and Bond 2 cannot shield the sovereign against a deterioration in capacity to pay caused by real exchange rate fluctuations. Hence it is important for governments to pursue anti-inflationary policies and potentially also limit excessive nominal exchange rate fluctuations in order to improve their debt service ability (and be able to issue debt at lower cost).

Bond 3 and Bond 4, on the other hand, substantially lower the probability of default of the sovereign, while still allowing investors to benefit from higher cashflows due to macroeconomic developments. However, their risk profile is considerably more complicated than that of the other bonds. Investors' returns increase in three cases: a) growth is higher, b) nominal exchange rate appreciates and c) domestic prices rise. Case (b) suggests that these bonds have similar characteristics to local currency debt, case (c) suggests that they are similar to inflation linked bonds, while case (a) makes them growth linked bonds. They also has a minimum coupon fixed in dollar terms and a principal that is repayable in dollars. This makes Bond 3 and Bond 4 hybrids of many financing instruments—local and foreign currency, inflation linked and growth linked bonds. Alternatively, a similar risk profile may be achievable by the sovereign from issuing a mixture of inflation linked, growth linked, foreign and local currency instruments. Moreover, the analysis of the preceding sections suggests that in this portfolio of instruments managing inflation and exchange rate risks may be more important than managing the risk of lower economic growth.

To make the model tractable, we have not considered in this paper model risk and parameter uncertainties and possible gaming behaviour on the part of the sovereign in adopting strategic default. Also we have focused on modelling only the ability of the sovereign to pay, ignoring the important considerations of the willingness to pay, which considerably complicates bond pricing in emerging markets. Throughout this paper, we set default cost to be 75% of par value, i.e. a recovery value of 25%. In the case of sovereign bonds, the cost of default is rather complex, potentially involving restructuring, litigation, local and global economic instability, and the subsequent increase in borrowing costs and global lending squeeze etc. In the case of sovereign debt the default cost need not be borne by the bondholder only, and could potentially involve parties outside the bond contract. Finally, we model the price in a free market setting and only from the bondholder's perspective, without taking into consideration any third party costs and benefits.

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Notes

[1]This argument is based on the assumption that these sovereign bonds are held by foreigners.

[2]See, for example, Wall Street Journal, 23 April 2007 and Euromoney, 1 January 2007.

[3]See for example Borensztein, Chamon, Jeanne, Mauro and Zettelmeyer (2004) and Schröder, Heinemann, Kruse and Meitner (2004).

[4]Some notable exceptions are Kulatilaka and Marcus (1987); Gray, Merton and Bodie (2006).

[5]Alternatively, our model is consistent with the assumption that a certain proportion of the sovereign's overall debt service capacity is dedicated to servicing the bond we are valuing.

[6]For the ease of exposition, p_0 and p_0^* can both be set equal to one.

[7]Alternatively, we can assume the foreign inflation level to be deterministic.

[8]This would be consistent with some econometric evidence that finds fundamental factors have a limited role in explaining exchange rate movements (see Sarno and Taylor (2002), Chapter 4).

[9]The simple analogy for $R_t^{\$}$ is a well with water; rainfall (macroeconomic developments) determines how much water is added to the well, whereas the amount of water in the well at time t is also determined by how much you have taken out of it (debt service payments).

[10]The final payout for the vanilla bond that we use to calibrate the model; see Section VI.D.

[11]This is a necessary and sufficient condition, since $W''(z) < 0$ if $u''(x) < 0$.

[12]See Poon and Stapleton (2005, p. 43).

[13]Please see the next section for a detailed description of our simulation method and the calibration of parameters.

[14]Derivations of these formulae can be found for example in Hull (4th edition, p 407) and http://www.puc-rio.br/marco.ind/sim_stoc_proc.html#mc-mrd.

[15]Chamon and Mauro (2005) use similar values for mean output growth and volatility in their model.

[16]Potential output was estimated by applying the Hodrick-Prescott filter with $\lambda = 100$ to (the logarithm of) annual real GDP data. The Hodrick-Prescott (HP) filter (for a series x_t) consists of specifying an adjustment rule whereby the trend component of the series x_t moves continuously and adjusts gradually. Formally, the unobserved trend component x_t^* is extracted by solving the following minimization problem

$$\min_{x_t^*} \left\{ \sum\nolimits_{t=1}^{T} (x_t - x_t^*)^2 + \lambda \sum\nolimits_{t=2}^{T} \left[(x_{t+1}^* - x_t^*) - (x_t^* - x_{t-1}^*) \right]^2 \right\}.$$

Thus, the objective is to select the trend component that minimizes the sum of squared deviations from the observed series, subject to the constraint that changes in x_t^* vary gradually over time. The coefficient of λ is a positive number that penalizes changes in the trend component. The larger the value of λ the smoother the resulting trend series. The usual practice is to set λ to 100 with annual data series (see Agenor (2004, p. 361)).

[17]Chamon an Mauro (2005) use the same vanilla bond to calibrate their model.

[18]Chamon and Mauro (2005) report that their model gives a default probability of 25% for the same

bond given similar assumptions.

[19]Given our assumption that the vanilla bond trades at par, the indexation of GDP linked Bonds 1 to 4 was structured such that they price approximately at par.

[20]Although these are corporate default rates (very few rated sovereigns have defaulted since the agencies started collecting their data, making a compilation of sovereign default rates impossible), it is common practice to assume that corporate and sovereign default rates for the same credit rating are similar. As Standard and Poor's (2001) puts it, given that both sovereign and corporate groups use the same rating definitions, S&P expects sovereign credit rating stability and default probability to converge to those of the corporate sector over time as the number of sovereign observations increases.

[21]The output gap parameters κ and V appear to have relatively minor impacts on bond prices and are omitted from this analysis.

Table III: Prices and default probabilities of vanilla and GDP-linked sovereign bonds for θ=1.70
Risk averse representative investor with η=0.005

$$d\overline{Y}_t = \mu \overline{Y}_t dt + \sigma \overline{Y}_t dZ; \qquad dg_t = -\kappa g_t dt + V dW; \qquad dq_t = \left(\frac{d\overline{Y}_t}{\overline{Y}_t} - c dt \right) q_t + \Omega q_t dZ^*; \qquad R_t^{\$} = R_{t-1}^{\$} + \Delta(\theta q_t \overline{Y}_t) - \delta_t \qquad R_0^{\$} = \theta q_0 \overline{Y_0}$$

where $\overline{Y}_t$ is the potential output measured in domestic currency; g_t is the output gap; q_t is the real exchange rate, and $R_t^{\$}$ is the nominal foreign currency value of the sovereign assets after debt service.

			Vanilla		Bond 1		Bond 2		Bond 3		Bond 4	
			Price	Default	Price	Default	Price	Default	Price	Default	Price	Default
Baseline scenario	μ =	3.0%										
	σ =	2.0%										
	V =	4.0%										
	κ =	50%	100.05	15.22%	100.30	15.26%	100.60	13.88%	99.18	7.45%	100.61	8.11%
	Ω =	16.0%										
	θ =	1.70										
μ	=	1%	78.70	38.87%	71.98	31.05%	75.57	34.90%	73.72	23.81%	75.61	25.51%
		5%	115.15	3.89%	128.02	5.96%	130.68	4.17%	119.05	1.50%	118.26	1.64%
		7%	120.42	0.65%	151.46	1.87%	166.44	0.96%	135.95	0.20%	129.92	0.21%
σ	=	1%	101.15	14.32%	100.96	14.64%	101.20	13.02%	99.54	6.55%	101.47	7.28%
		4%	95.58	19.03%	97.66	18.22%	97.88	17.48%	97.51	11.09%	97.01	11.58%
		6%	89.37	24.80%	93.83	23.53%	93.51	23.25%	94.21	17.69%	91.27	17.39%
		10%	76.47	38.70%	85.48	38.87%	84.17	37.81%	86.36	36.97%	78.05	32.98%
κ	=	30%			100.29	15.34%	100.71	13.95%	99.35	7.45%	100.71	8.10%
		60%			100.28	15.25%	100.57	13.86%	99.27	7.37%	100.69	8.06%
		80%			100.37	15.15%	100.60	13.79%	99.14	7.40%	100.64	8.08%
V	=	2%			100.57	14.84%	100.69	13.75%	98.75	7.23%	100.55	7.97%
		5%			100.33	15.68%	100.68	13.96%	99.50	7.61%	100.67	8.23%
		7%			100.90	16.82%	101.04	14.10%	100.38	7.98%	100.84	8.47%
c	=	1%	108.87	8.21%	109.28	8.12%	109.94	7.30%	109.88	3.52%	110.52	3.86%
		5%	89.49	25.50%	89.62	25.70%	89.57	23.68%	86.92	14.02%	88.71	15.14%
		7%	78.77	38.76%	78.78	39.23%	78.26	36.71%	73.81	23.69%	75.65	25.44%
Ω	=	10%	117.68	2.51%	118.32	2.28%	117.48	1.91%	96.47	0.25%	106.19	0.34%
		20%	88.30	25.84%	88.41	26.03%	88.78	24.39%	93.43	19.12%	90.16	18.54%
		25%	76.87	38.29%	76.90	38.59%	77.21	36.98%	86.40	36.29%	78.38	32.49%

Notes: The vanilla bonds pays 6.75% annual coupon. Bond 1 coupon is $\delta_t^{GDP} = \max\left[0, 3.75\% + g_t\right]$, where g_t is the annual growth rate of real GDP. Bond 2 coupon is $\delta_t^{GDP} = \overline{\delta} + \gamma q_t \left(Y_t - Y_0 e^{\overline{g}t}\right)$ for $Y_t > Y_0 e^{\overline{g}t}$ and $Y_t > Y_{t-1} e^{g^*}$. If either of the two growth conditions is not met, $\delta_t^{GDP} = \overline{\delta}$. In the above simulations, $\overline{\delta} = 5.75\%$, $\gamma = 10\%$, and $\overline{g}$ and g^* are both set equal to 2%. In the case of Bond 3, $\delta_t^{GDP} = \max[2\%, g_t^{\$} - 9\%]$, where $g_t^{\$}$ is the annual growth rate of nominal GDP measured in US dollars. For Bond 4, $\delta_t^{GDP} = \min\{\max[2\%, g_t^{\$} - 3\%], 15\%\}$. The risk free rate is set at 4%.

Table IV: Prices of vanilla and GDP-linked sovereign bonds for θ=1.70, Risk averse representative investor with η=0.01

$$d\overline{Y}_t = \mu \overline{Y}_t dt + \sigma \overline{Y}_t dZ; \qquad dg_t = -\kappa g_t dt + V dW; \qquad dq_t = \left(\frac{d\overline{Y}_t}{\overline{Y}_t} - c dt \right) q_t + \Omega q_t dZ^*; \qquad R_t^{\$} = R_{t-1}^{\$} + \Delta(\theta q_t \overline{Y}_t) - \delta_t \qquad R_0^{\$} = \theta q_0 \overline{Y}_0$$

where $\overline{Y}_t$ is the potential output measured in domestic currency; g_t is the output gap; q_t is the real exchange rate, and $R_t^{\$}$ is the nominal foreign currency value of the sovereign assets after debt service.

		Vanilla		Bond 1		Bond 2		Bond 3		Bond 4	
		Price	% chg in price	Price	% chg in price	Price	% chg in price	Price	% chg in price	Price	% chg in price
Baseline scenario	μ = 3.0% σ = 2.0% V = 4.0% κ = 50% Ω = 16.0% θ = 1.70	83.86	-16.18%	83.03	-17.21%	84.46	-16.05%	83.77	-15.55%	86.45	-14.07%
μ =	1%	64.51	-18.03%	55.90	-22.34%	60.78	-19.57%	55.64	-24.52%	58.43	-22.72%
	5%	106.95	-7.12%	115.26	-9.97%	120.00	-8.17%	109.71	-7.85%	112.08	-5.22%
	7%	118.65	-1.47%	144.87	-4.35%	160.18	-3.76%	127.27	-6.38%	126.83	-2.38%
σ =	1%	85.19	-15.78%	84.03	-16.77%	85.40	-15.61%	85.20	-14.40%	88.08	-13.20%
	4%	78.81	-17.54%	79.31	-18.79%	80.67	-17.58%	78.71	-19.28%	80.54	-16.98%
	6%	72.68	-18.68%	74.58	-20.52%	75.76	-18.98%	72.16	-23.41%	73.00	-20.01%
	10%	61.99	-18.94%	66.68	-21.99%	67.64	-19.64%	62.77	-27.32%	60.65	-22.28%
κ =	30%			82.95	-17.29%	84.51	-16.08%	83.93	-15.53%	86.56	-14.04%
	60%			83.07	-17.17%	84.43	-16.05%	83.93	-15.45%	86.60	-14.00%
	80%			83.23	-17.07%	84.48	-16.03%	83.82	-15.45%	86.53	-14.02%
V =	2%			84.00	-16.48%	84.58	-16.00%	83.72	-15.22%	86.56	-13.92%
	5%			82.60	-17.68%	84.49	-16.08%	83.82	-15.76%	86.40	-14.17%
	7%			82.09	-18.64%	84.77	-16.10%	84.03	-16.29%	86.35	-14.37%
c =	1%	95.87	-11.93%	95.27	-12.82%	97.30	-11.50%	97.99	-10.82%	100.69	-8.89%
	5%	73.04	-18.38%	72.17	-19.47%	72.78	-18.75%	68.96	-20.67%	71.67	-19.21%
	7%	64.55	-18.05%	63.71	-19.14%	63.51	-18.84%	55.72	-24.51%	58.46	-22.72%
Ω =	10%	112.40	-4.48%	112.34	-5.06%	113.15	-3.69%	94.62	-1.92%	103.96	-2.09%
	20%	71.69	-18.81%	70.82	-19.90%	71.70	-19.24%	71.02	-23.99%	71.76	-20.41%
	25%	62.31	-18.94%	61.49	-20.04%	62.03	-19.66%	62.86	-27.25%	60.91	-22.29%

Notes: The vanilla bonds pays 6.75% annual coupon. Bond 1 coupon is $\delta_t^{GDP} = \max\left[0, 3.75\% + g_t\right]$, where g_t is the annual growth rate of real GDP. Bond 2 coupon is $\delta_t^{GDP} = \overline{\delta} + \gamma q_t \left(Y_t - Y_0 e^{\overline{g}t}\right)$ for $Y_t > Y_0 e^{\overline{g}t}$ and $Y_t > Y_{t-1} e^{g^*}$. If either of the two growth conditions is not met, $\delta_t^{GDP} = \overline{\delta}$. In the above simulations, $\overline{\delta} = 5.75\%$, γ =10%, and $\overline{g}$ and g^* are both set equal to 2%. In the case of Bond 3, $\delta_t^{GDP} = \max[2\%, g_t^{\$} - 9\%]$, where $g_t^{\$}$ is the annual growth rate of nominal GDP measured in US dollars. For Bond 4, $\delta_t^{GDP} = \min\{\max[2\%, g_t^{\$} - 3\%], 15\%\}$. The risk free rate is set at 4%.

Table V: The impact of jumps in the real exchange rate on bond prices

$$d\overline{Y}_t = \mu \overline{Y}_t dt + \sigma \overline{Y}_t dZ; \qquad dg_t = -\kappa g_t dt + VdW; \qquad dq_t = \left(\frac{d\overline{Y}_t}{\overline{Y}_t} - cdt \right) q_t + \Omega q_t dZ^* + q_t dJ;; \qquad R_t^{\$} = R_{t-1}^{\$} + \Delta(\theta q_t \overline{Y}_t) - \delta_t \qquad R_0^{\$} = \theta q_0 \overline{Y_0}$$

where $\overline{Y}_t$ is the potential output measured in domestic currency; g_t is the output gap; q_t is the real exchange rate, and $R_t^{\$}$ is the nominal foreign currency value of the sovereign assets after debt service. *J* is a compound Poisson process with intensity λ. The logarithm of jump sizes is distributed log-normally with mean μ_J and volatility σ_J, such that if *j* is the jump size, then $\ln(1+j) \sim N(\ln(1+\mu_J), \sigma_J^2)$. For this simulation μ=3.0%, σ=2.0%, *V*=4.0%, κ=50%, Ω=16.0%, μ_J =-0.2, σ_J =0.1.

		Vanilla		Bond 1		Bond 2		Bond 3		Bond 4	
		Price	Default	Price	Default	Price	Default	Price	Default	Price	Default
η = 0.005, θ = 1.70	λ = 0	100.05	15.22%	100.30	15.26%	100.60	13.88%	99.18	7.45%	100.61	8.11%
	λ = 0.3	67.55	53.86%	67.53	54.36%	66.57	52.21%	61.54	43.30%	62.83	44.25%
% change		-32.48%	38.64%	-32.67%	39.10%	-33.83%	38.33%	-37.95%	35.85%	-37.55%	36.14%
η = 0.01, θ = 1.70	λ = 0	83.86		83.03		84.46		83.77		86.45	
	λ = 0.3	56.50		55.72		55.06		45.88		48.56	
% change		-32.62%		-32.89%		-34.81%		-45.23%		-43.83%	

Notes: Percentage changes indicate percentage differences in prices and default probabilities between cases where λ=0 and λ=0.3. If λ=0 then the real exchange rate evolves continuously. Default probabilities do not change as η rises from 0.005 to 0.01. The vanilla bonds pays 6.75% annual coupon. Bond 1 coupon is $\delta_t^{GDP} = \max\left[0, 3.75\% + g_t\right]$, where g_t is the annual growth rate of real GDP. Bond 2 coupon is $\delta_t^{GDP} = \overline{\delta} + \gamma q_t \left(Y_t - Y_0 e^{\overline{g}t}\right)$ for $Y_t > Y_0 e^{\overline{g}t}$ and $Y_t > Y_{t-1} e^{g^*}$. If either of the two growth conditions is not met, $\delta_t^{GDP} = \overline{\delta}$. In the above simulations, $\overline{\delta} = 5.75\%$, γ =10%, and $\overline{g}$ and g^* are both set equal to 2%. In the case of Bond 3, $\delta_t^{GDP} = \max[2\%, g_t^{\$} - 9\%]$, where $g_t^{\$}$ is the annual growth rate of nominal GDP measured in US dollars. For Bond 4, $\delta_t^{GDP} = \min\{\max[2\%, g_t^{\$} - 3\%], 15\%\}$. The risk free rate is set at 4%.

Figure 3: Cashflow distributions of vanilla and GDP-linked sovereign bonds

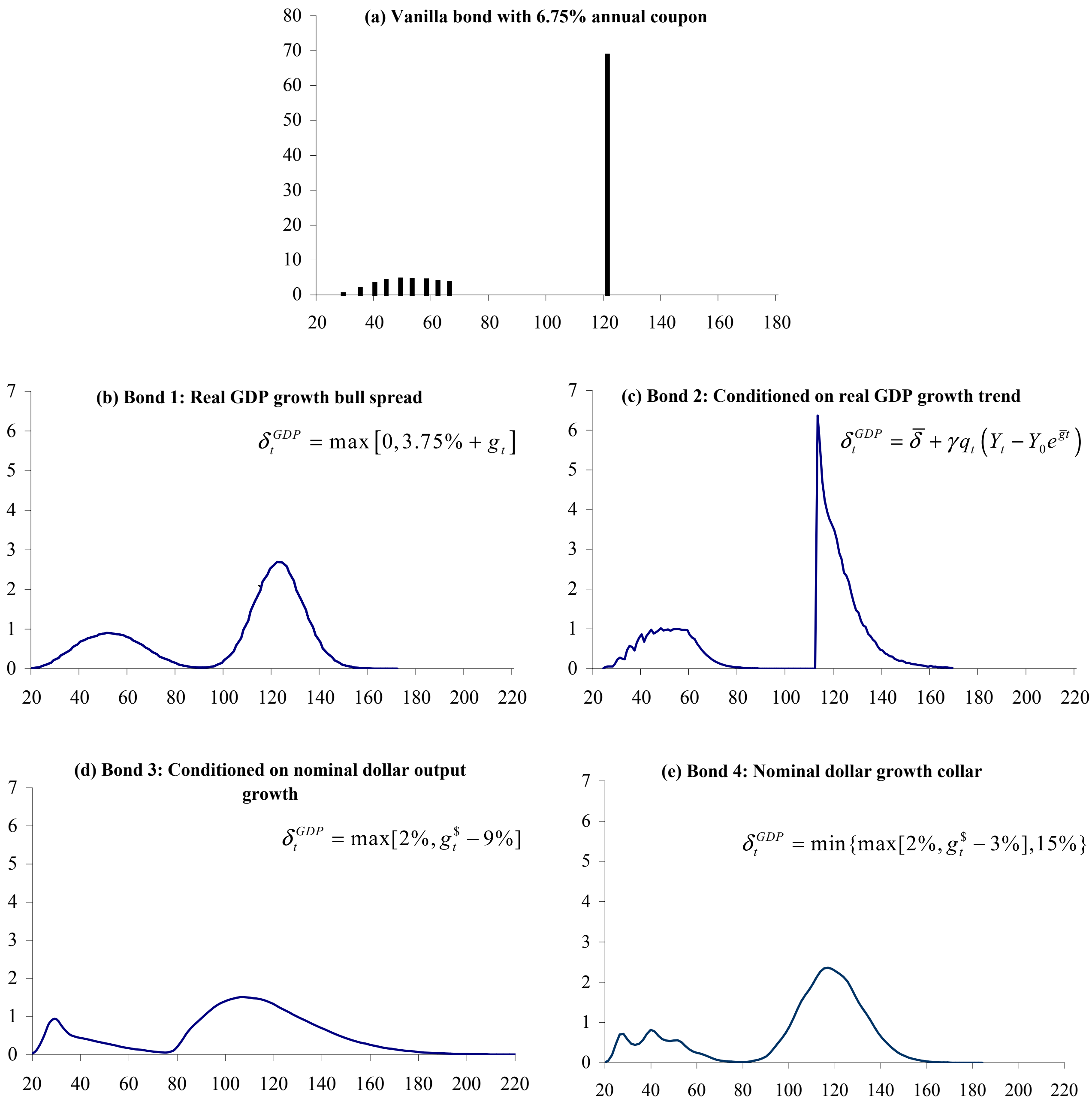
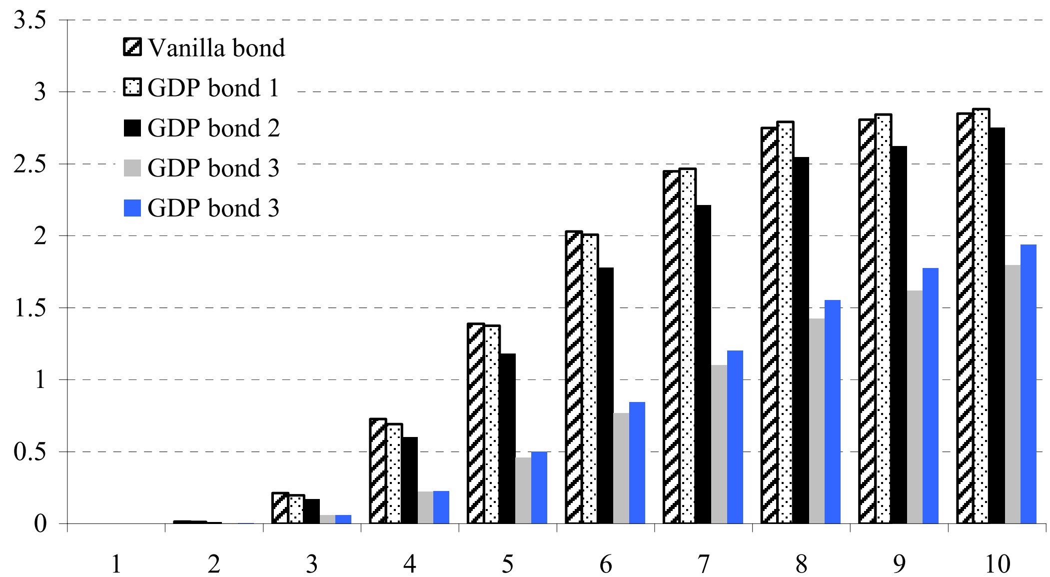


Figure 4
Default term structure of 10-year vanilla and GDP-linked sovereign bonds



Notes: The vanilla bonds pays 6.75% annual coupon. Bond 1 coupon is $\delta_t^{GDP} = \max\left[0, 3.75\% + g_t\right]$, where g_t is the annual growth rate of real GDP. Bond 2 coupon is $\delta_t^{GDP} = \overline{\delta} + \gamma q_t \left(Y_t - Y_0 e^{\overline{g}t}\right)$ for $Y_t > Y_0 e^{\overline{g}t}$ and $Y_t > Y_{t-1} e^{g^*}$. If either of the two growth conditions is not met, $\delta_t^{GDP} = \overline{\delta}$. In the above simulations, $\overline{\delta} = 5.75\%$, γ =10%, and $\overline{g}$ and g^* are both set equal to 2%. In the case of Bond 3, $\delta_t^{GDP} = \max[2\%, g_t^{\$} - 9\%]$, where $g_t^{\$}$ is the annual growth rate of nominal GDP measured in US dollars. For Bond 4, $\delta_t^{GDP} = \min\{\max[2\%, g_t^{\$} - 3\%], 15\%\}$. The risk free rate is set at 4%.