Portfolio Optimization with Alternative Investments

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First Draft: January 2008

ABSTRACT

Most monthly return distributions of alternative assets are in general not normally distributed. Further, some have biases (e.g. survivor ship bias) that distort the risk-return profile. For that reason every portfolio optimization in the mean-variance framework which includes alternative assets with not normally distributed return distributions and/or biases will most likely be sub-optimal since the risk-return is not covered adequately. As a result the biases and higher moments have to be taken into account. For that reason the return series are corrected for biases in a first step. In the next step the empirical return distributions are replaced with two normal distributions to approximate a best-fit distribution to cover the impact of the higher moments. This procedure is known as the mixture of normal method and is widely used in financial applications. In order to build a strategic asset allocation for a mixed asset portfolio traditional investments (stocks and bonds) and the vast majority of alternative investments (asset backed securities, hedge funds, venture capital, private equity (buy out), commodities, and REITs) are considered. Furthermore real investor's preferences are considered in optimization procedure. In order to test the results for stability robustness tests which allow for the time-varying correlation structures of the strategies are applied.

JEL Classification: G2, G12, G31

Keywords: Alternative Investments, Asset Allocation, Asset Backed Securities, Buy Out, Commodities, Hedge Fund, Higher Moments, Private Equity, REITs, Venture Capital

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Acknowledgements: I would like to thank Dieter G. Kaiser, Lutz Johanning, Rainer Lauterbach, Mark Mietzner and Juliane Proelss for helpful comments and suggestions. Further, I would like to thank Daniel Schmidt, from CEPRES, for providing access to their databases. All remaining errors are my own.

1 Introduction

Alternative investments gain more and more importance in the context of portfolio compositions of institutional investors, like endowments, family offices, pension funds as well as high net worth individuals and reached a volume of 3 trillion US$ at the end of 2006 [Loeys and Panigirtzoglou (2006)]. These investors have the investment capital to raise the required minimum investment capital for alternative investments like private equity or hedge funds, have long investment horizons to hold illiquid investment. The share of alternative investments in the portfolios of high net worth individuals for example rose from 10% in 2002 to 20% in 2005 [for more details see World Wealth Report 2006 of Capgemini and Merrill Lynch], whereas the average share of alternative investments in an endowment portfolio went up from 3% in 1996 to 12% in 2005. However, endowments greater then 1 billion US$ even have an allocation of about 36% [for more details see 2006 NABUCO Endowment Study]. Therefore the question arises why we observe this rush in alternative investments? I argue that there are two main reasons for this observation. Firstly, investors try to diversify their portfolios to avoid substantial losses in large downturns in the equity and bond markets (e.g. Asian crisis 1997, Russian crisis 1998, new economy bubble in 2000 and the attack of the World Trade Centre in 2001). During those phases alternative investments can help diversifying the portfolio, because their return drivers differ from the drivers that affect the equity and bond markets [Schneeweis, Kazemi and Martin (2001)]. Secondly, their positive diversification properties do not reduce the expected portfolio returns and enhance the risk adjusted performance. For example the top US university endowments (e.g. Harvard, Princeton and Yale) reported media-effective realized annual returns in the range of 10-25% in the last three years which highlights the fact that alternative investments can enhance the expected portfolio returns too. Lerner, Schoar and Wang (2007) attribute parts of this success to the willingness to rely on alternative investments.

Admittedly, if the investor wants to build up an alternative investment exposure she or he has to decide which alternative investments to include. Thereafter, the investor is left with the problem of the strategic asset allocation, meaning the determination of the long term allocations to the considered asset classes. This selection is the most important decision in the investment process and explains most of the portfolio return variability and therefore is the major determinant of investment performance [Hoernemann, Junkans and Zarate (2005)[1]]. Precisely, the investor has to analyze the investment opportunities (traditional and alternative) and decide which alternative investments to include and in what proportions. In order to do so she or he has to take into account in the first step the risk-return characteristics as well as other factors unique to alternative investments adequately. This step is necessary because of the fact that the obtained risk-return characteristics are the influencing variables for the strategic asset allocation models. In the next step the models have to be flexible enough to incorporate the risk-return characteristics. When the risk-return characteristics of alternative investments are not captured appropriately or the strategic asset allocation model lacks in flexibility, then the obtained optimal portfolio after the portfolio optimization in general includes only alternative investments [Terhaar, Staub and Singer (2003)].

In the literature the majority of empirical studies concentrate on analyzing the effects of including one alternative investment in a mixed asset portfolio only. When more then one alternative investment is considered than the risk-return profiles are captured inadequately or the chosen model is not flexible enough [e.g. Schneeweis, Karavas and Georgiev (2002) and Conner (2003)]. In the case that the risk-return profiles are captured adequately and the chosen model is flexible enough then the considered alternative investments do not represent the vast majority of

[1] They present an alternative study to the often cited studies of Brinson, Hood and Beebower (1986) and Brinson, Hood and Beebower (1991), which uses a slightly different framework and covers a longer time horizon than the earlier work, includes alternative assets, and utilizes synthetic portfolios.

the alternative investments. In the paper of Huang and Zhong (2006) commodities, REITs and TIPs are considered only whereas Hoecht, Ng, Wolf and Zagst (2006) integrate in their model Asian hedge funds and Asian REITs solely. Therefore, these papers do not answer the question of a strategic asset allocation for broad sample of alternative investments.

This paper is the first attempt, best to authors knowledge, that 1) incorporates a variety of alternative investments (asset backed securities, hedge funds, venture capital, private equity (buy out), commodities, and REITs) as well as traditional investments (stocks and government bonds), 2) adjusts risk-return profiles for biases, 3) uses a model for the strategic asset allocation which is flexible enough to capture the risk-return profile adequately, and 4) and incorporates real investor preferences. Before the optimization the return time series of some alternative investments (hedge funds, venture capital, buy out) are corrected for biases like appraisal smoothing, stale pricing and/or survivor ship bias. The thereupon used optimization model for the strategic asset allocation is flexible to incorporate risk which arises from the higher moments (skewness and kurtosis) which is not covered by the standard deviation. This is due to the fact, that the empirical return distributions of some alternative investments are in general not normally distributed. For that reason every portfolio optimization in the mean-variance will most likely be sub-optimal since many alternative investments have return distributions for which the risk measure variance does not cover all risks adequately and therefore must be sub-optimal. As a result higher moments have to be taken into account. Consequently, the mixture of normal method is used to replace the empirical return distributions which often exhibit skewness and positive excess-kurtosis with two normal distributions to approximate a best fit distribution. These best fit distributions are used in the optimization procedure for the strategic asset allocation. In order to obtain the strategic asset allocation a goal function is applied where real investor’s preferences for different levels of risk aversion can be examined. As a robustness check the obtained results are tested for time-varying correlations.

The rest of the paper proceeds as follows. In section II is evaluated which alternative investments should be included in mixed asset portfolios. Further, the current literature is reviewed where a single alternative investment is integrated in mixed asset portfolios. In the subsequent paragraph (section III) the considered proxy indices for the traditional and alternative investments and the adjustment for several biases are presented. Section IV presents the methodology of the research design. Then, section V, provides discusses the obtained results. The paper concludes with a summary, discussion and implications for further research (section VI).

2. Evaluation of Alternative Investments in Mixed Asset Portfolios

We have known since Markowitz's (1952) seminal paper on portfolio theory that diversification can increase portfolio expected returns while reducing volatility. However, investors should not blindly add another asset class to their portfolios without careful consideration of its properties in the context of the portfolio. Otherwise, the naively chosen allocations to the newly added asset class may not improve the risk-return profile of the portfolio, and may even worsen it. This raises the question of whether alternative investments really improve the (risk adjusted) performance of a (mixed asset) portfolio and therefore should be included in the strategic asset allocation.

In this section the asset classes: commodities, hedge funds, private equity (buy out, venture capital) and REITs are analyzed in the manner of Kat (2007) to their ability enhance the risk-return profile of an existing portfolio of traditional investments (stocks and bonds).[2] Further,

[2] Asset backed securities are not discussed in detail, because a few studies only have analyzed their risk-return characteristics. However, the studies of Funke, Johanning and Gaston (2005) and Benk and Johanning (2008) find asset backed securities a valuable completion for institutional investors' portfolios.

potential biases in the return time series which affect the risk-return profile are discussed. Thereafter, a proxy index which represents the characteristics of each asset class best is selected.

2.1 Commodities

In order to study the risk-return characteristics of commodities one has to determine which exposure to commodities is appropriate. In general, there are several ways to participate in commodity markets via a number of different kinds of financial instruments. The most important are: 1) direct investment in the physical good, 2) indirect investment in stocks of natural resource companies, 3) commodity funds, or 4) an investment in commodity futures indices.

The direct physical investment in commodities is in general not practicable, since most commodities are perishable and or cannot be stored for long periods in time. According to Geman (2005), precious metals like gold, silver, or platinum are an exception, as they do not have high current costs and do not difficult to store. However, a portfolio consisting solely of precious metals would not be a sufficiently diversified portfolio for investors to hold and Till and Eagleeye (2005) find that commodities which are more difficult to store have higher expected returns then commodities which are not difficult to store.

An indirect investment in commodities via commodity stocks is only an insufficient substitute for a direct investment. By investing in such stocks, investors do not receive direct exposure to commodities because listed commodity stocks all have their own characteristics and inherent risks. Fabozzi, Fuess and Kaiser (2007) point out the major sources of varying movements between commodity stocks and the underlying commodity are: operational risk caused by human or technical failure, internal regulations, or external events, the strategic position of the company, management quality, capital structure (the debt/equity ratio), the expectations and ratings of company and profit growth, risk sensitivity, the risk of a total loss if prices decrease below total production costs, as well as information transparency, information credibility and temporary miss pricing due to market disequilibriums. Furthermore, Georgiev (2006) shows that these sector-specific stocks are only slightly correlated with commodity prices.

An investment in a portfolio consisting of commodity stocks via a commodity fund can either be active or passive. In the case of a passive managed fund the same above mentioned discrepancies in the risk-return characteristics of the underlying commodity and the commodity stock still apply. When in investing in an active managed commodity fund (e.g. Commodity Trading Advisor (CTA)) the fund managers' skill additionally distorts the risk-return profile [Gregoriou and Rouah (2004), Akey (2006) and Idzorek (2006)].

Concluding, none of the mentioned forms of obtaining exposure to commodities measures the risk-return characteristics of commodities adequately. Most studies on the risk-return profiles use commodity-futures-indices as appropriate benchmarks for the development of the commodity markets. The most widely used commodity-futures-indices are the CRB/Reuters Commodity Index, S&P GSCI Commodity Index, and Dow Jones-AIG Commodity Index.[3]. Before adding a commodity-futures index to the strategic asset allocation, the potential for diversification benefits and the existence of a risk premium has to be investigated.

In order to answer the question if investors in commodities earn a risk premium is still an ongoing discussion. Early studies of Bodie and Rosansky (1980), Kaplan and Lummer (1998), Greer (2000) and a recent study by Gorton and Rouwenhorst (2005) find historical returns of un-leveraged commodity-futures-indices equal to the stock market. In contrast, Erb and Harvey (2006) find a decreasing returns over time and less evidence of a significant return persistence for

[3] The S&P GSCI Commodity Index, for example, has quintupled in size since 2002 to march 2007 to US$ 60 billion. It is estimated that in march 2007 about US$ 90 billion was invested in commodity-futures-indices, almost seven times the amount invested in 2002 [Doyle, Hill and Jack (2007)]. At the beginning of 2007, Standard & Poor's acquired the GSCI Commodity Index, which was subsequently renamed the S&P GSCI Commodity Index.

the commodity sectors.[4] Kat and Oomen (2006) also found no evidence of a consistent risk premium, except for energy commodities.[5] To summarize these arguments, the existence of a risk premium for commodities is still a contentious issue. Nevertheless, structuring a commodity portfolio will gain in importance for investors, for a simple reason: Even if there is no risk premium for single commodities, a well diversified portfolio of commodities still offers a reliable source of returns, which Erb and Harvey (2006) and Scherer and He (2008) call a diversification return.

In contrast to the controversial discussion on the existence of the risk premium a consensus is found in the literature that investable commodity-futures-indices have positive properties in diversifying mixed asset portfolios [see, for instance, Bodie and Rosansky (1980), Kaplan and Lummer (1998), Anson (1999), Jensen, Johnson and Mercer (2000), Gorton and Rouwenhorst (2005), Georgiev (2006), Gordon (2006), Idzorek (2006), Fabozzi, Fuess and Kaiser (2008), Scherer and He (2008) among others].

Recapitulatory, the most appropriate way to build an exposure to commodities which displays the risk-return characteristics of the commodity market are commodity-futures-indices. Whereas it is not clear that the underlying commodities of the commodity-futures-index generate a risk premium, there is strong evidence that an index can earn a diversification return and have positive properties in diversifying mixed asset portfolios. Therefore, the commodity-futures-index with the highest invested capital, namely the S&P GSCI Commodity Total Return Index[6] is included in the investigation.

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[4] They argue that a commodity futures index is not necessarily a good measure of the aggregate commodity market performance because part of the excess returns is due to a rebalancing effect. The resulting rebalancing bonus (also called volatility pumping) is described by Fernholz and Shay (1982).

[5] Energy is considered a subgroup of commodities, which normally include only natural gas, crude oils, unleaded gasoline, and heating oil.

[6] Fabozzi, Fuess and Kaiser (2008) discuss in detail the different calculation methods spot, excess and total return of the S&P GSCI Commodity Index.

2.2 Hedge Funds

Fung and Hsieh (2000), Amenc, Curtis and Martellini (2003) as well as Fuess, Kaiser and Adams (2007) summarize the most important reasons for investing in hedge funds. The first reason addresses the hedge fund managers' flexibility due to their less regulated investment vehicles to exert their skills and expertise in the attempt to generate positive "alpha returns". While the second reason concentrates on the low correlations of hedge fund returns with traditional asset classes, because the risk premiums of hedge fund strategies are in general low correlated with equity or fixed income risk premiums.[7] Therefore, investors try to diversify away their traditional long-only portfolio which is called a diversification motive.

Empirical investigations regarding the performance persistence of hedge fund managers reached great attention in academic literature and showed mixed results. While, Agarwal and Naik (2000a), Agarwal and Naik (2000b), and Jagannathan, Malakhov and Novikov (2006) find evidence for performance persistence the study of Brown, Goetzmann and Ibbotson (1999) show a contrary picture. The differences in the conclusion can be addressed to different databases, investigation periods, performance measures, and statistical methodologies.[8]

Even if hedge fund managers are not able to generate a persistent alpha return they still have the opportunities to create returns that are low correlated with those of traditional asset classes. Before staring to assess the diversification potential of hedge fund investing the higher moments of the return distribution of hedge fund returns are examined. Researchers like Fung and Hsieh (1997), Fung and Hsieh (2000), Amin and Kat (2003a), Brooks and Kat (2002), among many other have pointed that the return distributions of hedge funds and hedge fund indices returns are not normal distributed and exhibit skewness and excess-kurtosis. Further, the return distributions

[7] Hedge fund managers are not constrained by short-selling and are allowed to invest in other non-traditional asset classes like credit derivatives, mortgaged backed securities, etc. in a way that is not allowed to mutual funds.
[8] See the study of Eling (2007) for a survey.

show positive first order autocorrelation. This autocorrelation causes estimates of the standard deviation of hedge fund returns to exhibit a systematic downward bias [Kat (2003)].[9]

For that reason the mean-variance framework does not seem to cover the characteristics of hedge fund returns appropriate [see, for instance, Fung and Hsieh (1997) and Fung and Hsieh (1999)]. However, if the man-variance framework is applied, the integration of hedge funds in mixed asset portfolios improves the mean-variance properties, but leads to significant lower skewness and higher kurtosis in the portfolios on the efficient frontier [Amin and Kat (2002), Amin and Kat (2003b), and Lhabitant and Learned (2002)]. Therefore they suggest that these risks occurrence with the integration of hedge funds in mixed asset portfolios have to be considered. For that reason Kat (2005) emphasis the use of multi-moment optimizations which accounts for skewness and kurtosis. Kooli, Amvella and Gueyie (2005), Favre and Galeano (2002), Favre and Signer (2002), as well as Lamm (2003) try to incorporate this issue in their optimizations and replace the risk measure variance for example with Cornish Fisher Value at Risk,[10] mean-target semi- deviation and other risk measures that incorporate the downside risk of hedge funds. In contrast Agarwal and Naik (2004) favours the risk measure Conditional Value at Risk, which has several advantages over the Value at Risk and is a coherent risk measure in the sense of Artzner, Delbaen, Eber and Heath (1999).

Other researches did not change the risk measure and investigated hedge fund allocations in different frameworks. Papers that reached attention are the works of Amenc and Martellini (2003) who investigate several models allowing investors to get a quantitative estimate of the

[9] The major reason for the positive autocorrelation is originated by illiquid trading strategies [Lo and McKinlay (1990)].

[10] The Cornish Fisher Value at Risk developed by Cornish and Fisher (1937) is an extension to the Value at Risk concept to adjust for higher moments in the return distribution (e.g. skewness and kurtosis). The Value at Risk concept is criticised by Kaplanski and Kroll (2002) to be an inadequate measure within the expected utility framework. Further drawbacks are if the returns are not normally distributed, the estimation of mean-Value at Risk-efficient portfolios may be very difficult, especially if the return distribution is discrete. In this case, the frontier estimated by the Value at Risk dependent on the portfolio weights is non-convex, non-smooth, and has multiple local extrema [Gaivoronski and Pflug (2005) and Rockafellar and Uryasev (2002)].

optimal portfolio weight for hedge funds. Jurczenko, Maille and Merlin (2005) employ a shortage function as well as Davies, Kat and Lu (2006) who use the Polynomial Goal Programming technique. All models include the moments of the return distribution until the fourth moment. Popova, Morton, Popova and Yau (2007) apply a target function and incorporate the higher moments of the hedge fund return distribution with the mixture of two normal distributions. Summarizing, the more the inherent characteristics of hedge fund return distributions are considered in the models the lower are in general the resulting weights. But the weights in mixed asset portfolios are still substantial and the risk-return profile of the resulting portfolios are enhanced.[11]

Recapitulatory, hedge fund return distributions often exhibit positive autocorrelation and contain additional sources of risk not considered with the normal distribution (e.g. negative skewness and positive excess-kurtosis). When these inherent characteristics are considered in the models and/or in the risk measure the consequential portfolio weights in mixed asset portfolios are in general lower compared with neglecting them. However, the resulting weights in the mixed asset portfolio for hedge funds can still help diversifying the portfolio despite of the mixed evidence on the performance persistence. For that reason, the Credit Suisse/Tremont Hedge Fund Index is considered the further investigation.

2.3 Private Equity

In the literature studies which attempt to quantify the risk-return characteristics of private equity are rare. The rationale behind this is the low transparency of private equity markets and the resulting insufficient available data which hinders a comparison to other asset classes on an aggregate level [Schmidt (2004)]. Further, the target companies of the private equity funds are in

[11] Gueyie and Amvella (2006) and Kooli (2007) also show that the investment in funds of hedge funds, which are the typical start investments in this asset class, can help improving the risk-return profile of a portfolio and Belvedere (2001) as well as Favre and Galeano (2001) suggest that hedge fund investing is favourable for pension funds.

general not traded in a permanent market place with quoted prices. Therefore, most of the investments exhibit a low liquidity and are not divisible. Moreover, the investments in the target companies are accompanied with high transaction costs with less public available information [Kaserer and Diller (2004)]. For that reason, empirical research in private equity is generally based on the reported cash flows and the appraised values of unrealized investments[12] in order to calculate returns.

Gompers and Lerner (1997), Moskowitz and Vissing-Jørgensen (2002), Quigley and Woodward (2002), Ljungqvist and Richardson (2003), Kaplan and Schoar (2005), Gottschalg, Phalippou and Zollo (2005) who update the study of Kaplan and Schoar (2005) with superior information, Cochrane (2005), and Phalippou and Gottschalg (2007) have attempted to quantify the risk-return characteristics of private equity on a fund level or on an individual portfolio company level. These studies have the shortcoming that they that they rely predominantly on data given by data vendors, which is self-reported and thus potentially subject to selection biases; and they are based on unrealized as well as realized investments which introduces noise and potentially biases due to subjective accounting treatment. An exception are the studies of Gompers and Lerner (1997) and Ljungqvist and Richardson (2003) who have exact timing information of the investments and distribution of cash flows to investors, and the types of companies contained in each fund's portfolio, but have a small sample size of 1 and 73 funds only. Weidig, Kemmerer and Born (2005) analyze the risk return profiles of fund of private equity funds.

[12] The resulting estimated return series by VentureXpert from Venture Economics and Cambridge Associates usually show a positive autocorrelation. This can be explained by the composition of the reported return in growth in net asset value (NAV) for unrealized investments and Internal Rates of Return (IRRs) for realized investments. The positive autocorrelation of the appraised values are in general due to the illiquidity of private equity investments and the managed pricing of the general partners of the private equity funds [Anson (2002)].

All the previous mentioned studies have concentrated their efforts on determining the risk-return profiles, but do not pay attention to possible positive diversification attributes of private equity investments in mixed asset portfolios. The first study that considers private equity in the strategic asset allocation is one of Lamm and Ghaleb-Harter (2001). They show that an investor should invest between 19% and 51% in private equity. In their study this recommendation is appropriate under a variety of alternative conditions and conservative assumptions regarding future performance. A later study by Chen, Baierl and Kaplan (2002) point out the a low correlation coefficient of 0.04 between venture capital and public stocks. Because of its relatively low correlation with stocks, an allocation to venture capital of 2% (for the minimum variance portfolio) to 9% (in the maximum Sharpe ratio portfolio) is warranted for mixed asset portfolios. Schmidt (2004) recommends a wide range of optimal portfolio weightings to be between 3% and 65% for minimizing mixed-asset portfolio variance or maximizing performance ratios. The latest study conducted by Ennis and Sebastian (2005) implicates differentiating portfolio allocations for different types of investors. They conclude that for thoughtful investors the right decision is to exclude private equity from their portfolio. For others, private equity holdings of a few percentage points may be appropriate. Only moderate-size, equity-oriented funds with exceptional private equity investment skill, strong board-level support, and adequate staff resources should consider allocations of 10% or more. Admittedly, they use the Venture Economics Post-Venture Capital Index as the proxy for private equity which does not grasp the risk-return profile of private equity for the purpose of the term “private” adequately.

Recapitulatory, all above mentioned studies highlight the relevance of private equity of in diversifying mixed asset portfolios and recommend positive allocations in private equity in the strategic asset allocation. For that reason venture capital and buy outs as the traditional components of private equity are incorporated in the further investigation [Ennis and Sebastian

(2005)].[13] In the further investigation the return time series (CepreX US Venture Capital and CepreX US Buyout) from the CEPRES[14] database are used. This database is based on (partially) audited reports and precise cash flow information from the private equity investment funds, enabling accurate financial calculations and therefore does not have the shortcoming of non audited reported filings of investment firms based on unrealized as well as realized investments which introduces noise and potentially biases due to subjective accounting treatment like the databases from Venture Economics and Cambridge Associates. A further advantage is that the indices are transaction based and are available in a monthly frequency. Admittedly, the return time series can not be reported contemporaneously, due to a sufficient number of transactions per month to report a reliable return. Therefore, the last reported return was in general at least 12 month before.

2.4 Real Estate Investment Trusts (REITs)

Before REITs are added to a mixed asset portfolio as a representative for the real estate market the following questions have to be positively answered. Are REITs as a proxy for the real estate market co-integrated with direct real estate market? Since REITs are publicly traded they might have similar movements to the stock market and if this is the case do REITs have a different risk-return profile to the general stock to be a non-redundant asset class? When both questions are positively answered then REITs should offer diversification benefits to mixed asset portfolios.

Several researches investigated the question whether REITs are integrated with the direct real estate market or not. This is due to the fact that if the REITs and the direct real estate market are co-integrated, then there exist common factors that affect both returns series and so the return

[13] One might argue that that the returns of venture capital and buyouts are generated by the same drivers and therefore one representative for private equity should included only. In unreported results both indices are tested for co-integration, but a co-integration relation could not be found. For that reason venture capital and buy outs are considered.

[14]The papers of Cumming, Schmidt and Walz (2004), Cumming and Walz (2004), Schmidt (2004) and Schmidt, Steffen and Szabo (2007) give detailed insights about the index construction and its composition.

series' will eventually adjust to equilibrium. Gyourko and Keim (1992) find a relation between real estate stock portfolio returns and returns of an appraisal-based direct real estate index after controlling for persistence in the appraisal return series. Therefore, they conclude that the stock market reflects information about direct real estate markets that is later imbedded in infrequent property appraisals. Myer and Webb (1993) find that REIT index returns "Granger" cause direct real estate returns for most of the real estate indices and Barkham and Geltner (1995) show evidence for a lagged price information transmission from the REIT markets to the direct real estate markets. Clayton and MacKinnon (2000) showed that the relationship between REITs and direct real estate increased in the 1990s, which is a valuable insight, because the considered time series in this paper start in the 1990s. They conclude further that, when considering REITs and direct real estate market in the short-run, both may have a place in optimal portfolios, but in the long-run, one is a substitute for the other and so only one may have a place in optimal portfolios.

To answer the question whether REITs behave like stock markets Li and Wang (1995) and Ling and Naranjo (1999) use an integration approach and Oppenheimer and Grissom (1998) a spectral analysis and find that REITs are integrated with the stock market and that stock markets have dominant influence on REIT returns. Interestingly, Clayton and MacKinnon (2000) come to similar conclusion, but find that the sensitivity of REIT returns to stock market has declined significantly during the 1990s which they attribute the growth and maturity of the REIT market. Since much evidence is found that both markets show similar characteristics in the next step it has to shown that REITs have their own risk-return profile and therefore are non-redundant.

Liang and McIntosh (1998) shed light on this topic by using the classical style analysis of Sharpe and find that REITs are a "unique" asset class. Chiang and Lee (2002) extend the work of Liang and McIntosh (1998) and find that the price behaviour of REITs is unique and cannot be satisfactorily replicated by combining stocks, fixed-income securities, and direct real estate.

Stevenson (2001a) finds similar result in the UK Saunders (1997) highlights the fact that weights of the indices which help explaining the REIT returns are not stable over time and Clayton and MacKinnon (2000) point out that the correlation of REIT returns to stock markets are not stable over time. This provides additional evidence suggesting that REITs should be seen as a “unique” asset class.

Before REITs are considered in mixed asset portfolios the ability of diversifying mixed asset portfolios is reviewed. The evidence about the beneficial adding of REITs into mixed asset portfolios is mixed. In the former work of Kuhle (1987) no significant performance benefits of REITs in a stock portfolio are found. Whereas Mueller, Pauley and Morrell (1994) find, dependent on the time period, a valuable addition of REITs in multi asset portfolios. Mull and Soenen (1997) show similar findings regarding the time dependence. More recent studies of NAREIT (2002), Hudson-Wilson, Fabozzi, Gordon and Giliberto (2004), and Lee and Stevenson (2005) find that REITs offer considerable benefits of the portfolio performance when REITs are added to mixed asset portfolios. Whereas Lee and Stevenson (2005) highlight the fact that REITs play a significant role for diversification over different time horizons and holding periods. In the studies of Chen, Ho, Lu and Wu (2005) and Chiang and Ming-Long (2007) spanning tests are applied to test whether the additional inclusion of REITs to an already existing portfolio enhances the efficient-frontier. Both studies draw the conclusion that REITs can enhance the efficient frontier and therefore have a rightful place in a mixed asset portfolio. The mixed results in the early studies might be interpreted in the sense of Clayton and MacKinnon (2000) that REIT markets have grown and matured over time to an asset class of its own.

Recapitulatory, strong evidence is found that REITs are an appropriate proxy for the real estate and offers risk-return characteristics which cannot sufficiently be replicated by other asset classes. These risk-return characteristics were investigated if they can be assembled to enhance the efficient frontier of multi asset portfolios. The resulting positive effects are well documented in the literature. For that reason the FTSE/NAREIT Equity REITS - Total Return Index is considered in the investigation.[15]

3. Data Set Description

In the previous section the characteristics of the considered asset classes were discussed in detail and potential biases were highlighted. In this paper two traditional asset classes, the proxy indices are in parentheses, US equity (S&P 500 Composite - Total Return Index) and US government bonds (JPM United States Govt. Bond - Total Return Index) and five alternative asset classes, namely asset backed securities (IBOXX Coll. ABS - Total Return Index), buy out (CepreX US Buyout), commodities (S&P GSCI Commodity Total Return Index), hedge funds (Credit Suisse/Tremont Hedge Fund Index), Real Estate Investment Trusts (FTSE/NAREIT Equity REITS - Total Return Index), and venture capital (CepreX US Venture Capital) are applied.[16] All time series in the further investigation are on a monthly basis and with an inception date in January 1998, because from this date on all indices reported data. The end date for the time series is the July 2006, due to the fact that the indices for buy out and venture capital are transaction based indices. Table 2 reports the descriptive statistics for the raw time series which are partially biased. In order to obtain an un-biased data set the time series are corrected in three steps:

1) In the literature the survivor ship bias of hedge fund indices was investigated by many researchers. Their research studies differ depending on the investigation period, calculation

[15] One might argue that REITs and direct real estate should be included in the investigation. This is point of view is supported by the studies of Mueller and Mueller (2003) and Feldman (2004) who find positive effects of adding REITs to a portfolio mixed asset portfolio which already consists of direct real estate. Contrary, Stevenson (2001b) does not show consistent results in his study. For the reason of this mixed evidence and the fact that REITs and direct real state markets are integrated one proxy for real estate is used only.

[16] Detailed descriptions for the proxy indices are provided in Table 1.

method, and the used data base. The resulting survivor ship bias ranges from 0.16% [Ackermann, McEnally and Ravenscraft (1999)] to 6.22% [Liang (2002)]. Admittedly, most researchers number the survivor ship bias between 2% and 3%.[17] For that reason the mean return for the hedge funds is reduced by 2.5% p.a.

2) The time series of the S&P 500 generated a comparative low mean return of 0.5% per month in the considered time period which is comparable to the mean return of the US government bonds (see Table 2). It seems unreasonable that stocks generate bond like returns in the long run with a standard deviation which is more than three times higher. Further, the mean return of the S&P 500 is the only mean return in this investigation which is more then 100% below the long term mean return. The means of all other time series are in the range of 30% around their long term mean return. Therefore, the mean return of the S&P 500 is raised about 5% p.a. that stocks are not unconsidered systematically.

3) As mentioned in the previous section hedge fund time series often display positive autocorrelation e.g. due to illiquid portfolio positions. In contrast transaction based indices e.g. the index for buy out and venture capital have in general a negative autocorrelation. To test for first order autocorrelation the portmanteau-test of Ljung and Box (1978) is applied. Table 4 reports significant first order autocorrelation for the buy out, CS/T hedge fund index, and venture capital return time series. In order to adjust for the autocorrelation which distorts the standard deviation the method of Geltner (1991) is applied.[18]

After adjusting for the above mentioned biases the resulting descriptive statistics are shown in Table 3. As can be seen from the table, buy out has both the highest mean return return (2.81%) and highest standard deviation (8.79%) followed by venture capital with a mean return of 2.01%

[17] See, for instance, Anson (2006), Brown, Goetzmann and Ibbotson (1999), and Fung and Hsieh (2000).

[18] This method as adapted from the real estate finance literature, where due to smoothing in appraisals and infrequent valuations of properties, the returns of direct property investment indices display positive autocorrelation. See Geltner, MacGregor and Schwann (2003) for an overview of this topic.

and a standard deviation of 6.49% among the eight asset classes considered here. Commodities have a similar standard deviation (6.38%) to venture capital but a lower mean return of 1.01%. US equity and REITs have similar standard deviations of 4.5% and 4.12%, but show differences in the mean return with 0.86% and 1.1%. Asset backed securities, hedge funds and US government bonds have similar return levels (0.57%, 0.55%, and 0.45%), but the corresponding risk for this three asset classes, measured by the standard deviation, is somewhat unexpected. The assets backed securities have the highest mean return and the lowest standard deviation of 1.35% followed by US government bonds with 1.42% and the hedge funds with 2.5%.

The higher moments (skewness and kurtosis) are additional potential sources of risk, which are discussed in the following. The lowest skewness of -0.6505 is found for the REITs, followed by US government bonds (-0.5176) and US equity (-0.4786). A skewness of around zero is found for the hedge funds (-0.0452), asset backed securities (0.0346), and the commodities (0.0545). A positive, high skewness is found for the private equity like asset classes venture capital (0.2269) and buy out (1.8119). The excess-kurtosis for most asset classes is close to zero. Commodities have a negative excess-kurtosis of -0.4204, whereas asset backed securities, US equity, venture capital, and US government bonds have a slightly positive excess-kurtosis (0.3737, 0.4796, 0.5350, and 0.6741). The excess-kurtosis of 1.0580 for REITs is considerable, but not as high as for hedge funds and buy out (6.2968 and 7.9188) which indicates that a substantial probability mass lies on the tails of the return distribution compared to the normal distribution.

The analysis of the higher moments of the return distribution for the asset classes' shows evidence that most considered return distributions do not follow a normal distribution. The Jarque-Bera test denotes that the null hypothesis of a normal distributed return distribution can be rejected for hedge funds, REITs, buy out on a 1% level, for US government bonds on a 5% level and for US equity on a 10% level. Only for asset backed securities, commodities, and venture capital the null hypothesis can not be rejected. For that reason, relying on a mean-variance framework and ignoring the higher moments of the asset classes return distributions will not cover the risk-return profile adequately and therefore must be sub-optimal. As a result higher moments have to be taken into account in the model.

To achieve first insights of the diversification potential of the asset classes the correlation matrix in
Table 6 is discussed. Venture capital has a high diversification potential, because the correlation to all other asset classes is negative except asset backed securities. Buy out has a similar high diversification potential and has a negative correlation to all other asset classes except US equity and commodities. US equity has a negative correlation to the fixed income related asset classes US government bonds and asset backed securities as well as to buy out. The remaining correlations are positive. REITs show a similar behaviour to US equity, admittedly the correlation to US government bonds is higher, but the correlation to private equity related asset classes is lower and even for buy out negative. Hedge funds have a negative correlation to the fixed income related asset classes US government bonds and asset backed securities as well to private related asset classes buy out and venture capital. Positive correlations are found with commodities, US equity and REITs. Commodities have a low correlation to all asset classes and a negative correlation to asset backed securities, venture capital. The fixed income related asset US government bonds and asset backed securities are highly correlated, but do only have the same sign for the correlation coefficient for hedge funds and US equity. For all other asset classes the correlation coefficients have the opposite sign.

Concluding, after reviewing the descriptive statistics of the return distributions for all asset classes which characterize the risk-return profile and their diversification properties it can not determined a priori that one asset class is a substitute for another. Therefore all asset classes should be considered in the portfolio construction. However, the model for the portfolio construction has to consider the characteristics of the presented asset classes adequately to deliver optimal portfolios implications for the investors. The framework for the optimal portfolio construction is presented in the next section.

4. Methodology

The next to last section discussed the characteristics unique to the different alternative asset classes and potential biases whereas the previous section concentrated on correcting these biases from the return series and discussing their statistical properties. Summarizing, the resulting return distributions, after correction for biases, are generally not normally distributed, and they exhibit skewness and excess kurtosis. Before using advanced models for the strategic asset allocation Scott and Horvath (1980) argue that three conditions have to checked before using the mean-variance framework introduced by Markowitz (1952) efficiently. If one of the following conditions 1) the return distributions are asymmetric, 2) the investor's utility function is of higher order then quadratic, or 3) the mean and the variance do not completely determine the distribution holds then the mean-variance framework will fail to deliver efficient results. That is using the mean-variance approach we implicitly assume that either the investor's utility function is quadratic or the return distribution of considered assets is multivariate normal. As can be seen from Table 3 the return distributions of most considered asset classes do follow a normal distribution. Consequently using mean-variance optimization and ignoring higher moments, the obtained portfolios in general over allocate alternative investments.

In order to capture the higher moments of the return distribution a number of alternative distributions to the normal distribution are provided in the literature. The multivariate Student t-distribution is good for fat-tailed data, but does not allow for asymmetry. The non-central multivariate t-distribution also has fat tails and is skewed. However, the skewness is linked directly to the location parameter, making it somewhat inflexible. The log-normal distribution has been used to model asset returns, but its skewness is a function of the mean and variance, not a separate skewness parameter.

To capture higher moments adequately, a distribution that is flexible enough to fit the skewness and the kurtosis is needed. Therefore, a combination of two different geometric Brownian motions to generate a mixture of normal diffusions is used in the further analysis. The normal mixture distribution is an extension of the normal distribution, and has been successfully applied recently in many fields of finance literature. For example, Alaxander and Scourse (2003) and Venkatramanan (2005) have used this distribution to model asset returns and study option pricing problems in this setting. Venkataraman (1997) applied this concept to risk management. And Buckley, Saunders and Seco (2004), Morton, Popova and Popova (2006), and Popova, Morton, Popova and Yau (2007) and Kaiser, Schweizer and Wu (2008) also used the normal mixture distribution in asset allocation problems.

The normal mixture distribution is primarily chosen for its flexibility and its tractability. In particular, let $f_1(x,\mu_1,\sigma_1)$ denote the probability density function of the first normal distribution, with mean μ_1 and standard deviation σ_1, and let $f_2(x,\mu_2,\sigma_2)$ denote the probability density function of the second normal distribution. The empirical distribution of non normally distributed return distributions can then approximated by a new distribution with the following probability density function:

$$\begin{aligned} f(x,\mu_1,\sigma_1,\mu_2,\sigma_2) &= 0.2\cdot f_1(x,\mu_1,\sigma_1)+0.8\cdot f_2(x,\mu_2,\sigma_2) \\ &= 0.2\cdot\frac{1}{\sqrt{2\pi\sigma_1^2}}\exp\left(-\frac{(x-\mu_1)^2}{\sigma_1^2}\right)+0.8\cdot\frac{1}{\sqrt{2\pi\sigma_2^2}}\exp\left(-\frac{(x-\mu_2)^2}{\sigma_2^2}\right) \end{aligned} \tag{1}$$

The economic justification is as follows. Consider a regime-switching model with two economic states: the usual and the unusual. The usual state exists 80% of the time, when a return is achieve with the distribution given by the second normal density; the unusual state exists 20% of the time, when the return is given by the other normal distribution.

Note it is not specified whether the unusual return is better than the usual return in terms of having higher mean and/or lower volatility. Indeed, the unusual return could be better, worse, or even the same. The latter case harks back to the classic assumption that returns are unconditionally normal. In general, this setting allows for conditionally normal returns, but unconditional returns need not be normal.

This specification offers many advantages. First, there are four free parameters, μ_1, σ_1, μ_2, and σ_2, so the first through fourth moments of the empirical distribution can be matched exactly. The skewness and excess kurtosis can also be captured. Second, with the normal density function, the new approximating distribution is still tractable. And third, as noted before, this specification treats the traditional normal approximation as a special case.

Figure 1

gives a visualization of this method.

Due to the fact that the approximating parameters μ_1, σ_1, μ_2, and σ_2 cannot be solved analytically, they are solved numerically. The goal is to approximate the empirical distribution, so the considered parameters are of the form x%, where x is an integer. In particular, the numerical solving searches for integer-valued means and standard deviations for the two normal distributions that can approximate the first four moments of the empirical distribution as closely as possible. In general, mean, variance, skewness, and kurtosis have different dimensions, so the chosen objective function has the flexibility to decide to minimize the weighted relative deviation rather than the absolute deviation. Let $w = (w_1,w_2,w_3,w_4)$ be a vector of strictly positive constants, which serve as weights for the four moments we want to match. The objective function is then:

$$\begin{aligned}\min\ & w_1 \times \left|\frac{\text{theoretical mean-empirical mean}}{\text{empirical mean}}\right| + w_2 \times \left|\frac{\text{theoretical variance-empirical variance}}{\text{empirical variance}}\right| \\ & + w_3 \times \left|\frac{\text{theoretical skewness-empirical skewness}}{\text{empirical skewness}}\right| + w_4 \times \left|\frac{\text{theoretical kurtosis-empirical kurtosis}}{\text{empirical kurtosis}}\right|.\end{aligned} \tag{2}$$

The objective function takes the value 0 if all four moments can be matched exactly. Otherwise, it takes positive values. In this investigation, all entries in the w-vector equal 1. The approximating parameters for the return distributions of the asset classes are in Table 7.

Table 8 shows the first four moments of the empirical return distributions, and compares them with the moments obtained from the mixture of normal method. Obviously, the moments are close and therefore the fit is very good.

The next step is to construct a mixed asset portfolio consisting of traditional and alternative asset classes. Since the mean-variance approach does not work, an appropriate objective function has to be determined. By taking a closer look at real-world institutional investors who want to include alternative asset classes in their portfolios, as mentioned in the introduction, are endowments, family offices, high net worth individuals, and pension funds. These investors generally seek to obtain higher expected returns than in a money market, but are risk-averse and therefore pay special attention to downside risk.

Thus an objective function of these investors can be specified as follows: Let r denote the random return of the portfolio, and r_1 and r_2 denote some benchmark returns. Note that the benchmark returns could be constants or random variables. The investors' objective is to maximize the following function:[19]

$$max\ \ Pr(r > r_1) - \lambda \cdot Pr(r < r_2) \tag{3}$$

[19] This objective function is presented and discussed in Morton, Popova and Popova (2006).

In other words, the investor wants to maximize the probability of outperforming some benchmark return, while minimizing the probability of underperforming another benchmark. So the first benchmark could be some constant, e.g., 10% p.a., or a random return of some other indices such as the S&P 500 as the market return. The second benchmark is usually chosen as 0%, or the risk-free rate, or a government bond yield. For the analysis, the first benchmark is defined as a constant 8% p.a., and the second as 0%.

The term λ is a positive constant and represents the trade-off between these two objectives. It is obvious that λ depends on investor risk aversion. The higher λ, the less aggressive the investor, since he or she weights the second objective more highly and is more concerned about losses than gains. Similar to the relative risk aversion coefficient in canonical utility functions, plausible values of λ lie between 1 and 10. The time horizon for achieving the goal is one year. Therefore, the monthly return distributions of the asset classes have to be rescaled to annual return distributions.[20]

For the numerical optimization procedure two constraints are considered: 1) short-selling is not allowed and 2) the maximum portfolio weight for each asset class is restricted to 30%. Using these constraints and the objective function stated above, the optimal portfolio weights for the mixed asset portfolios are numerically calculated for different parameters for λ.[21]

5. Results

After having examined the methodology in the previous section the results for different λ's in the imposed model are discussed now. The resulting portfolio weights for the optimal mixed asset portfolios for λ's ranging from 1 to 6 with a step size of 0.5 are presented in

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[20] Descriptive statistics of the annual return distribution are shown in Table 5. The rescaling technique is described in Appendix A.
[21] For computational details see Morton, Popova and Popova (2006).

Figure **2**. From this figure three regimes of risk aversion can be identified. The first regime of low risk aversion ranges from 1 to 2.5, the second regime of medium risk aversion ranges from 3 to 4, and the third regime of high risk aversion ranges from 3.5 to 6.

In the first regime the asset classes venture capital, buy out, and REITs have the dominant portfolio fraction. Venture capital and buy out are restricted by 30% when the value of λ is 1, but with in creasing risk aversion in this regime their weights are reduced to 27% and 25% respectively. REITs have a constant portfolio weight of 27% in this regime. Asset backed securities, hedge funds, and commodities have minor portfolio weights between 0% and 4% only. US equity and US government bonds have minor portfolio weights of 5% and 4% when the value of λ is 1 too, but when the risk aversion is increased the portfolio weights increase to 8%.

In the second regime four asset classes have a dominant portfolio position. Besides venture capital, buy out, REITs the asset class US government bonds increases importance. All four asset classes have portfolio weights of around 20% in this regime. US equity and asset backed securities have the portfolio weights in this regime of 8% and between 2% and 8%. Commodities have a constant portfolio weight of 3% whereas hedge funds are not considered in the optimal portfolios.

Comparable to the regime of a medium risk aversion in the regime of high risk aversion the same four asset classes have a dominant portfolio weight in the optimal portfolios. However, the portfolio weight of US government bond increase to the restriction of 30%. The portfolio weights for venture capital, buy out and REITs decrease slightly when risk aversion increases. When the risk aversion parameter λ reaches the highest degree of risk aversion the portfolio weights for venture capital, buy out and REITs are 20%, 17% and 21%. The asset classes US equity, hedge funds, and commodities have minor constant portfolios weights regarding the degree of risk aversion in this regime of 7%, 2% and 3%. Asset backed securities are not considered in the optimal portfolios.

General findings of the optimization are that the asset classes US government bonds, venture capital, buy out and REITs have the largest portfolio weights in the optimal portfolios. Whereas the importance of private equity related asset classes and REITs decreases with increasing risk aversion. The weights of venture capital and buy out declines from 30% when λ is equal to 1 to 20% and 17% when the value of λ equals 6. The portfolio weights of REITs are more stable. When the risk aversion is low (λ=1) the portfolio weight for REITs equals 26% and decreases to 21% when the risk aversion is high (λ=6). In contrast to the private equity related asset classes and REITs the US government bonds increase importance when risk aversion increases. US equity and commodities have relative constant portfolio weights regardless the degree of risk aversion of 5%-8% and 3%-4%. Hedge funds are considered in the regimes of high and low risk aversion with a portfolio weight of 1%-2% only. In contrast asset backed securities represented in the portfolios of medium risk aversion with portfolio weights between 1% and 8%.

6. Conclusion

Return distributions of alternative as well as traditional asset classes often exhibit non-normal properties. Further, some return time series of alternative investments suffer from several biases. For that reason the return time series have to be corrected for the biases in a first step to grasp their risk-return profile adequately. In the second step the higher moments of the return distributions have to be considered in the model for the strategic asset allocation, since the variance does not cover all sources of risk. In this paper a model was applied that is flexible enough to integrate the higher moments as well as real investors' preferences. Thereafter, the resulting portfolio weights for three different regimes of risk aversion – low, medium, and high –

are presented. From this follows that alternative investments play an outstanding role in the efficient portfolios, regardless the risk aversion regime. In particular the alternative investment asset classes buy out, REITs, and venture capital denote substantial portfolio weights in all regimes of risk aversion. In contrast US government bonds are underrepresented in efficient portfolios in regimes with low degrees of risk aversion, but when the risk aversion increases the portfolio weight increases until the maximum weight of 30%. This means that US government bonds are most valuable when the focus lies on downside risk protection instead of enhancing expected returns. In contrast US equity is represented in all efficient portfolios with a portfolio weight about 7% regardless the degree of risk aversion. Commodities show a similar behaviour to US equity, but the portfolio weight about 3% is lower. Hedge funds and asset backed securities are not represented in all regimes of risk aversion. Whereas hedge funds are represented in regimes of low and high degrees of risk aversion with minor portfolio weights about 2%, asset backed securities are represented in regimes of a medium risk aversion. Within this regime the portfolio weights are about 4%.

Concluding, alternative investments are important for the strategic asset allocation of institutional investors like endowments, family offices, pension funds as well as high net worth individuals who have the time horizons and the investment capital. However, not all alternative investment classes are of equal importance. Further, alternative investment classes are not substitutes for traditional asset classes they complement the traditional asset classes to better achieving the desired risk-return profiles.

Table 1
Data descriptions

This table reports the proxy indices for the used asset classes. The frequency, inception dates, end date, and sources for additional information are stated for the proxy time series.

Asset Class	Proxy Index	Frequency	Inception Date	End Date	Additional Information
U.S. Equity	S&P 500 Composite - Total Return Index	Monthly	Jan 98	Jul 06	http://www2.standardandpoors.com
U.S. Government Bonds	JPM United States Govt. Bond - Total Return Index	Monthly	Jan 98	Jul 06	http://www.datastream.com
Asset Backed Securities	IBOXX Coll. ABS - Total Return Index	Monthly	Jan 98	Jul 06	http://www.datastream.com
Buy Out	CepreX US Buyout	Monthly	Jan 98	Jul 06	http://www.cepres.de
Commodities	S&P GSCI Commodity Total Return Index	Monthly	Jan 98	Jul 06	http://www.datastream.com
Hedge Funds	Credit Suisse/Tremont Hedge Fund Index	Monthly	Jan 98	Jul 06	http://www.hedgeindex.com
Real Estate Investment Trusts	FTSE/NAREIT Equity REITS - Total Return Index	Monthly	Jan 98	Jul 06	http://www.nareit.com
Venture Capital	CepreX US Venture Capital	Monthly	Jan 98	Jul 06	http://www.cepres.de

Table 2

Descriptive statistics of the (partially biased) monthly return distributions

This table reports the arithmetic mean, median, maximum, minimum, standard deviation, skewness, and kurtosis of the monthly return distributions from January 1998 until July 2006. The reported descriptive statistics are calculated on the raw return time series obtained from the data vendors.

	JPM Gov. Bonds	S&P GSCI	CS/T Hedge Fund Index	IBOXX	NAREIT	S&P 500	CepreX US Buyout	CepreX US Venture Capital
Mean	0.0045	0.0101	0.0072	0.0057	0.0110	0.0050	0.0281	0.0197
Median	0.0050	0.0075	0.0075	0.0064	0.0156	0.0084	0.0167	0.0104
Maximum	0.0356	0.1579	0.0853	0.0498	0.0949	0.0978	0.5694	0.2236
Minimum	-0.0449	-0.1392	-0.0755	-0.0310	-0.1458	-0.1446	-0.2116	-0.2487
Std. Dev.	0.0142	0.0638	0.0202	0.0135	0.0412	0.0450	0.1261	0.0888
Skewness	-0.5176	0.0545	0.0189	0.0346	-0.6505	-0.4786	1.6333	0.1233
Kurtosis	3.6741	2.5796	7.7529	3.3737	4.0580	3.4796	8.2413	3.7849

Table 3

Descriptive statistics of the (un-biased) monthly return distributions

This table reports the arithmetic mean, median, maximum, minimum, standard deviation, skewness, and kurtosis of the monthly return distributions from January 1998 until July 2006. The return time series is of the CS/T Hedge Fund Index is corrected for the survivor ship bias. Therefore, every monthly return is reduced by 2.5% for an annual survivor ship bias. The return time series of the S&P 500 is raised by 5% p.a. Furthermore, the return time series of the CS/T Hedge Fund Index, CepreX US Buyout, and CepreX US Venture Capital are adjusted for first order autocorrelation with the method of Geltner (1991). The Jarque-Bera test [Jarque and Bera (1980)] is used to test for the assumption of normally distributed monthly returns.

	JPM Gov. Bonds	S&P GSCI	CS/T Hedge Fund Index	IBOXX	NAREIT	S&P 500	CepreX US Buyout	CepreX US Venture Capital
Mean	0.0045	0.0101	0.0055	0.0057	0.0110	0.0086	0.0280	0.0201
Median	0.0050	0.0075	0.0039	0.0064	0.0156	0.0084	0.0175	0.0111
Maximum	0.0356	0.1579	0.0916	0.0498	0.0949	0.0978	0.4213	0.1823
Minimum	-0.0449	-0.1392	-0.0985	-0.0310	-0.1458	-0.1446	-0.1530	-0.1528
Std. Dev.	0.0142	0.0638	0.0250	0.0135	0.0412	0.0450	0.0879	0.0649
Skewness	-0.5176	0.0545	-0.0452	0.0346	-0.6505	-0.4786	1.8119	0.2269
Kurtosis	3.6741	2.5796	7.2968	3.3737	4.0580	3.4796	8.9188	3.5350
Jarque-Bera	6.49**	0.80	78.50***	0.61	11.95***	4.87*	204.70***	2.09

***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively, based on monthly returns.

Table 4
First order autocorrelation of the asset classes

This table reports the coefficient of the first order autocorrelation (AR(1)) for monthly returns. The Ljung-Box test [Ljung and Box (1978)] is applied to test for significance.

Asset Class	AR(1)
S&P 500	0.008
ABS	-0.056
CS/T Hedge Fund Index	0.207**
JPM Gov. Bonds	0.008
CepreX US Venture Capital	-0.330***
CepreX US Buyout	-0.411***
S&P GSCI	0.039
NAREIT	-0.064

***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively, based on monthly returns.

Table 5
Descriptive statistics of the rescaled annual return distribution

This table reports the arithmetic mean, median, maximum, minimum, standard deviation, skewness, and kurtosis of the annual return distribution from January 1998 until July 2006. The median, maximum, and minimum are obtained from rolling annual returns. The first four central moments are rescaled from monthly data to annual, because otherwise the model calibration would be based on eight observations.

	JPM Gov. Bonds	S&P GSCI	CS/T Hedge Fund Index	IBOXX	NAREIT	S&P 500	CepreX US Buyout	CepreX US Venture Capital
Mean	0.055	0.122	0.066	0.068	0.132	-0.103	0.336	0.241
Median	0.049	0.195	0.062	0.080	0.174	0.087	0.301	0.089
Maximum	-0.034	-0.357	-0.090	-0.022	-0.211	-0.266	-0.205	-0.343
Minimum	0.154	0.722	0.319	0.136	0.526	0.398	1.499	1.876
Std. Dev.	0.049	0.221	0.087	0.047	0.143	0.156	0.305	0.225
Skewness	-0.149	0.016	-0.013	0.010	-0.188	-0.138	0.523	0.065
Kurtosis	3.056	2.965	3.358	3.031	3.088	3.040	3.493	3.045

***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively, based on monthly returns.

Table 6
Correlation matrix

This table reports the correlation between the asset classes based on monthly returns.

	JPM Gov. Bonds	S&P GSCI	CS/T Hedge Fund Index	ABS	NAREIT	S&P 500	CepreX US Buyout	CepreX US Venture Capital
JPM Gov. Bonds	1.000	0.027	-0.131	0.628	-0.032	-0.269	-0.097	-0.152
S&P GSCI	0.027	1.000	0.025	-0.008	0.145	0.014	0.142	-0.095
CS/T Hedge Fund	-0.131	0.025	1.000	-0.054	0.201	0.106	-0.116	-0.026
ABS	0.628	-0.008	-0.054	1.000	0.028	-0.126	-0.054	0.039
NAREIT	-0.032	0.145	0.201	0.028	1.000	0.106	-0.052	-0.142
S&P 500	-0.269	0.014	0.106	-0.126	0.106	1.000	0.223	-0.003
Buyout	-0.097	0.142	-0.116	-0.054	-0.052	0.223	1.000	-0.002
Venture Capital	-0.152	-0.095	-0.026	0.039	-0.142	-0.003	-0.002	1.000

Table 7
Moments of the normally distributed auxiliary distributions

This table shows the mean and standard deviation of the two auxiliary distributions, as well as the weighting factor for all asset classes. The values in the w-vector are all equal to 1.

Auxiliary Distributions	Distribution 1		Distribution 2	
Weighting Factor	0.2		0.8	
	Mean	Standard Deviation	Mean	Standard Deviation
S&P 500	2%	16%	13%	15%
ABS	8%	4%	6%	4%
CS/T Hedge Fund Index	7%	8%	6%	9%
JPM Gov. Bonds	1%	4%	6%	4%
CepreX US Venture Capital	34%	24%	21%	22%
CepreX US Buyout	48%	39%	30%	27%
S&P GSCI	26%	19%	9%	20%
NAREIT	20%	9%	11%	15%

Table 8
Comparison of the moments of empirical and approximated distributions

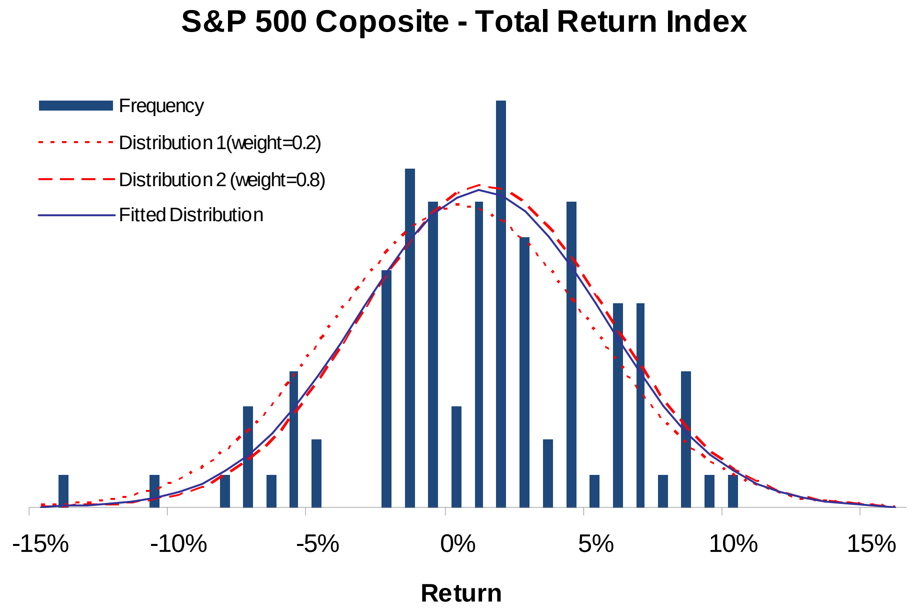
This table shows the first four moments of the empirical and approximated distribution. The number on the left is the theoretical moment in the approximated distribution; the number in parentheses is the empirical moment.

	Mean	Standard Deviation	Skewness	Kurtosis
JPM Gov. Bonds	0.050 (0.055)	0.045 (0.049)	0.020 (-0.149)	3.010 (3.056)
S&P GSCI	0.124 (0.122)	0.209 (0.221)	0.017 (0.016)	2.973 (2.965)
CS/T Hedge Fund Index	0.062 (0.066)	0.088 (0.087)	-0.012 (-0.013)	3.021 (3.358)
ABS	0.064 (0.068)	0.041 (0.047)	0.011 (0.010)	3.000 (3.031)
NAREIT	0.128 (0.132)	0.145 (0.143)	-0.183 (-0.188)	3.075 (3.088)
S&P 500	0.108 (-0.103)	0.158 (0.156)	-0.073 (-0.138)	3.043 (3.040)
CepreX US Buyout	0.336 (0.336)	0.306 (0.305)	0.257 (0.523)	3.510 (3.493)
CepreX US Venture Capital	0.236 (0.241)	0.230 (0.225)	0.064 (0.065)	3.047 (3.045)

Figure 1

Return Histograms and Fitted Return Distributions for the Asset Classes

The figure shows the monthly return histogram of the eight asset classes and the corresponding fitted return distribution for asset classes. The fitted return distribution is composed of two auxiliary distributions – distribution 1 and distribution 2 – that are weighted with factors, 0.2 and 0.8, respectively. The y-axis quotes the frequency in case of the histogram and the probability density in case of the continuous distributions.



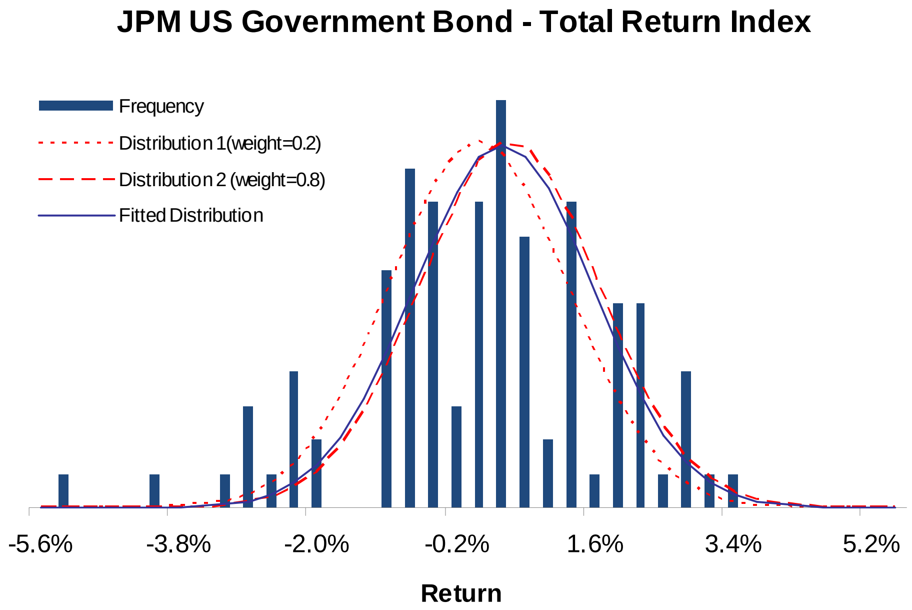
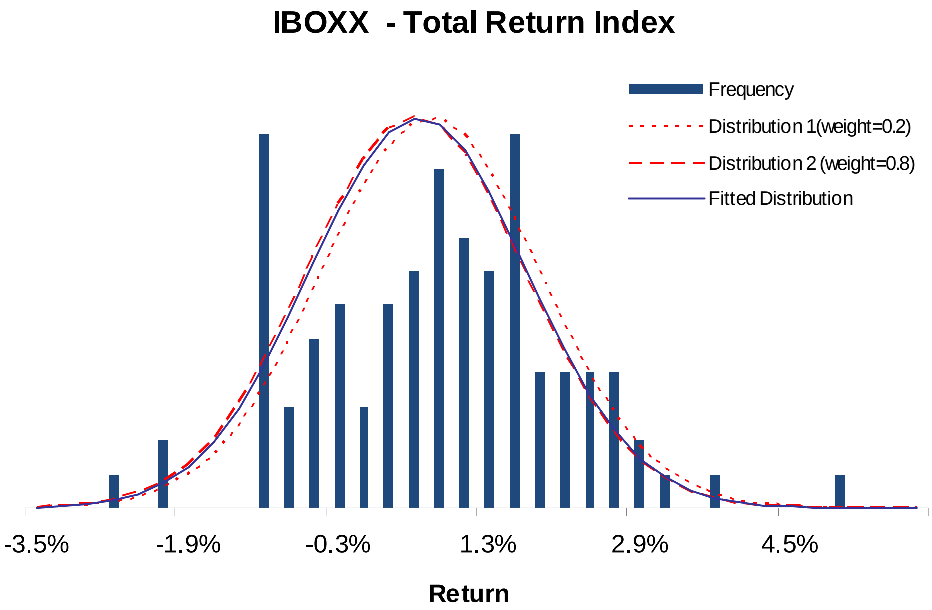


Figure 1—*Continued*



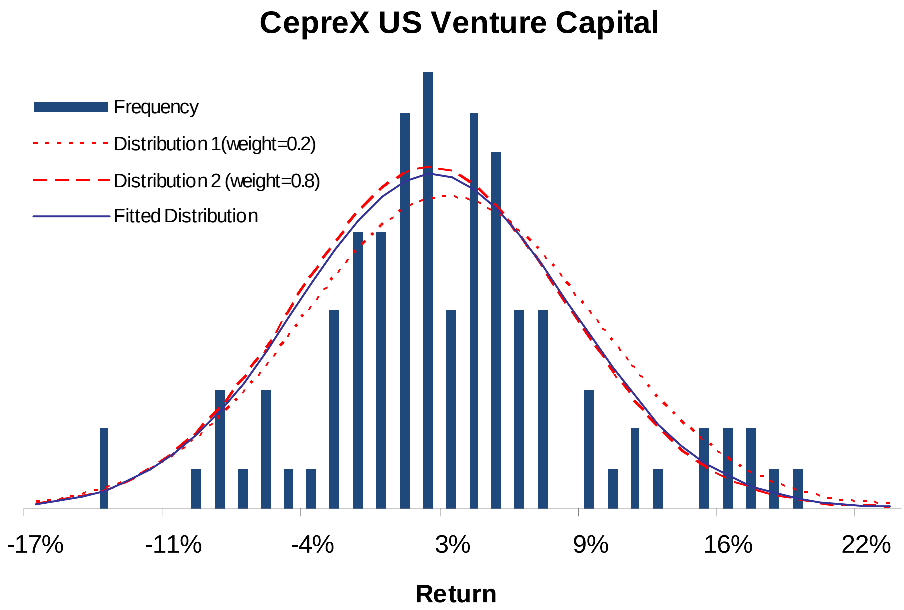
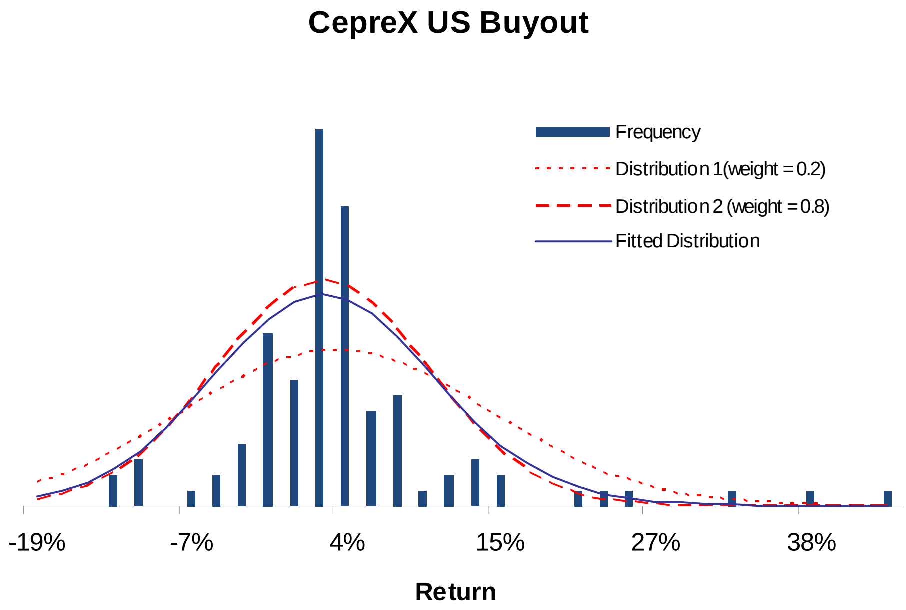


Figure 1—*Continued*



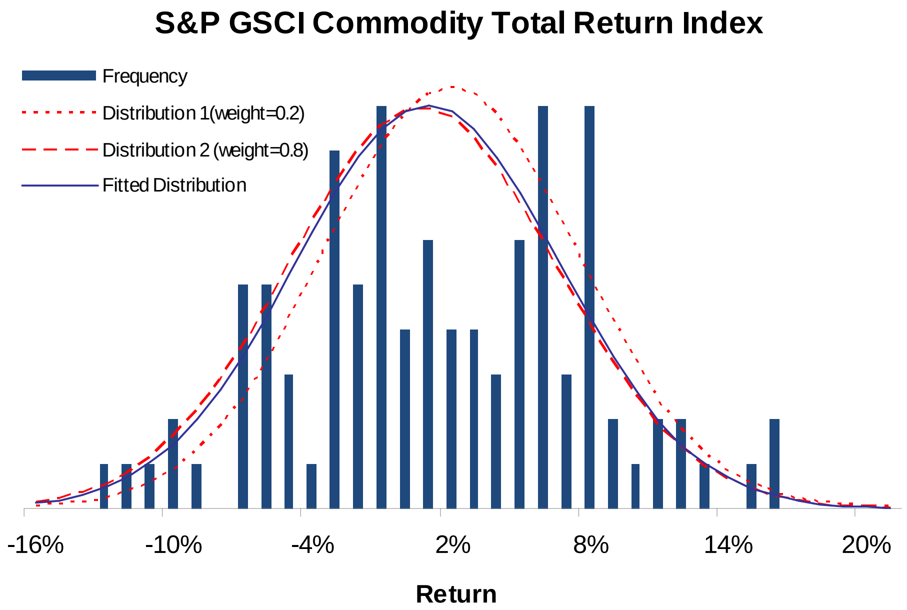
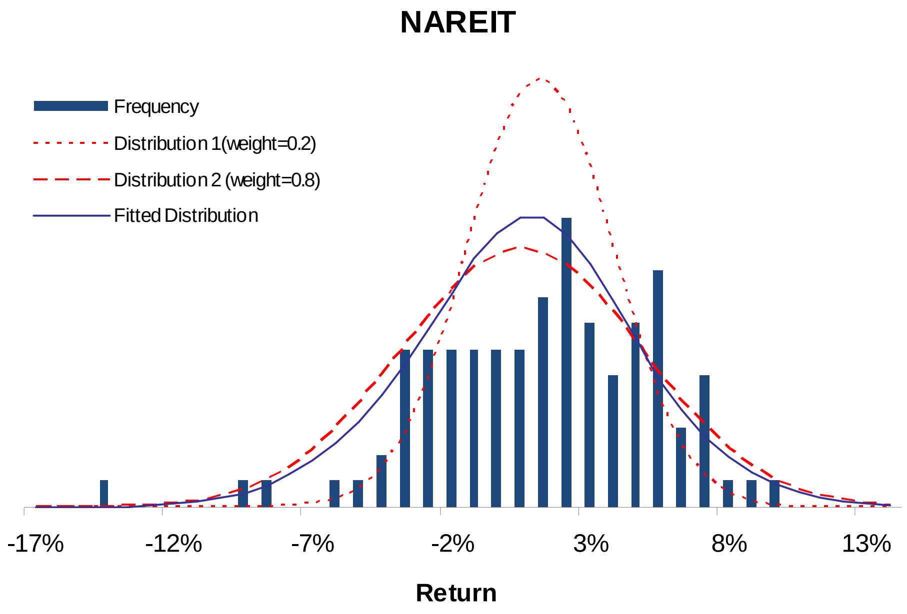


Figure 1—*Continued*



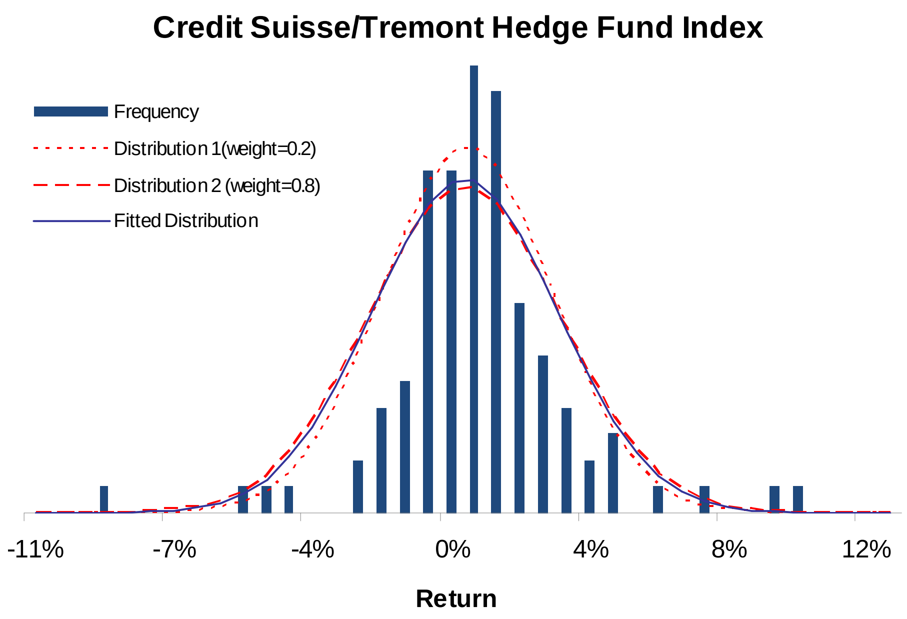
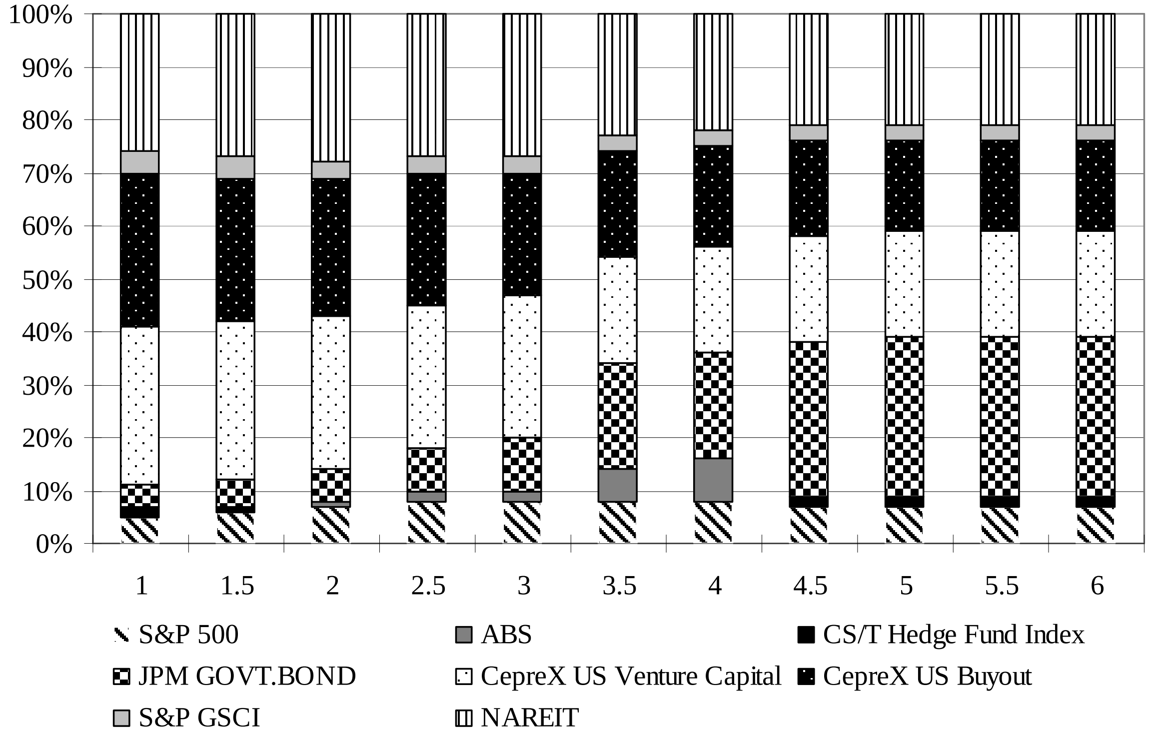


Figure 2
Optimal Portfolio Weights

This figure displays the relationship between the risk aversion factor λ and the corresponding optimal portfolio weights for the asset classes.



Appendix A

The moments of a monthly return distribution can be rescaled to an annual return distribution as follows. Let r_i denote the monthly return i and R denote the annual return.

It is obvious that

$$R = \sum_{i=1}^{12} r_i \ .$$

Assume r_i's are iid. Let $E[r_i] = \overline{r}$, $Var(r_i) = \sigma_r^2$, $E[R_i] = \overline{R}$, $Var(R_i) = \sigma_R^2$. It is well known that

$$\overline{R} = 12\overline{r}$$

$$\sigma_R = \sqrt{12}\sigma_r .$$

The skewness of the annual return is defined as

$$\begin{aligned}
Skew(R) &= \frac{E\left(R-\overline{R}\right)^3}{\sigma_R^3} \\
&= \frac{E\left(\sum_{i=1}^{12} r_i - 12\overline{r}\right)^3}{12\sqrt{12}\sigma_r^3} \\
&= \frac{E\left[\sum_{i=1}^{12} \left(r_i - \overline{r}\right)\right]^3}{12\sqrt{12}\sigma_r^3} \\
&= \frac{E\left[\sum_{i=1}^{12}\sum_{j=1}^{12}\sum_{k=1}^{12} \left(r_i - \overline{r}\right)\left(r_j - \overline{r}\right)\left(r_k - \overline{r}\right)\right]}{12\sqrt{12}\sigma_r^3} \\
&= \frac{\sum_{i=1}^{12}\sum_{j=1}^{12}\sum_{k=1}^{12} E\left[\left(r_i - \overline{r}\right)\left(r_j - \overline{r}\right)\left(r_k - \overline{r}\right)\right]}{12\sqrt{12}\sigma_r^3} .
\end{aligned}$$

Now since

$$E\left[\left(r_i - \overline{r}\right)\left(r_j - \overline{r}\right)\left(r_k - \overline{r}\right)\right] = \begin{cases} E\left[\left(r_i - \overline{r}\right)^3\right] = Skew(r_i)\sigma_r^3, & \text{if } i = j = k; \\ 0 & \text{if } i, j, k \text{ are not the same.} \end{cases}$$

The equation above can be written as

$$
\begin{aligned}
Skew(R) &= \frac{\left(\sum_{i=1}^{12} Skew(r_i)\sigma_r^3\right)}{12\sqrt{12}\sigma_r^3} \\
&= \frac{12Skew(r_i)\sigma_r^3}{12\sqrt{12}\sigma_r^3} \\
&= \frac{Skew(r_i)}{\sqrt{12}}.
\end{aligned}
$$

The kurtosis of the annual return is defined as

$$
\begin{aligned}
Kurt(R) &= \frac{E\left(R-\overline{R}\right)^4}{\sigma_R^4} \\
&= \frac{E\left(\sum_{i=1}^{12} r_i - \overline{r}\right)^4}{144\sigma_r^4} \\
&= \frac{E\left[\sum_{i=1}^{12}(r_i-\overline{r})\right]^4}{144\sigma_r^4} \\
&= \frac{E\left[\sum_{i=1}^{12}\sum_{j=1}^{12}\sum_{k=1}^{12}\sum_{l=1}^{12}(r_i-\overline{r})(r_j-\overline{r})(r_k-\overline{r})(r_l-\overline{r})\right]}{144\sigma_r^4} \\
&= \frac{\sum_{i=1}^{12}\sum_{j=1}^{12}\sum_{k=1}^{12}\sum_{l=1}^{12}E\left[(r_i-\overline{r})(r_j-\overline{r})(r_k-\overline{r})(r_l-\overline{r})\right]}{144\sigma_r^4}.
\end{aligned}
$$

Now since

$$
E\left[(r_i-\overline{r})(r_j-\overline{r})(r_k-\overline{r})(r_l-\overline{r})\right] =
\begin{cases}
E\left[(r_i-\overline{r})^4\right] = Kurt(r_i)\sigma_r^4, & \text{if } i=j=k=l; \\
E\left[(r_i-\overline{r})^2(r_j-\overline{r})^2\right] = \sigma_r^4 & \text{if respective two of } i, j, k, l \text{ are the same;} \\
0, & \text{otherwise.}
\end{cases}
$$

The above equation can be rewritten as

$$\begin{aligned}
Kurt(R) &= \frac{\left(\sum_{i=1}^{12} Kurt(r_i)\sigma_r^4\right) + \frac{12\cdot 11}{2}\cdot\frac{4\cdot 3}{2}\sigma_r^4}{144\sigma_r^4} \\
&= \frac{12Kurt(r_i)\sigma_r^4 + 396\sigma_r^4}{144\sigma_r^4} \\
&= \frac{Kurt(r_i)}{12} + \frac{11}{4}.
\end{aligned}$$

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