Uninsurable Risk and Financial Market Puzzles*

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Abstract

Following Kocherlakota and Pistaferri (2007), we consider two market structures: (i) where agents cannot insure at all their consumption against idiosyncratic skill shocks and (ii) where agents can insure their consumption against idiosyncratic skill shocks using the domestic financial markets, but, due to hidden work effort, financial intermediaries strike incentive compatible constraint, which prevents full risk sharing. For each of these two environments, we derive the associated stochastic discount factor. Empirical evidence is that the pricing kernel associated with (ii) jointly explains the equity premium, risk-free rate, currency premium, and consumption real exchange rate puzzles.

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1 Introduction

If financial markets are complete, then individuals are able to fully insure their consumption against idiosyncratic risks (such as labor income risk, loss of employment, or divorce, for example) and hence equalize state by state their intertemporal marginal rates of substitution (IMRS). Under the assumption that the agents have identical preferences, this implies that in equilibrium all individuals have the same consumption and therefore can be aggregated into one representative agent, whose consumption equals aggregate consumption per capita.

One of the most popular asset-pricing models is the consumption-based capital asset pricing model (consumption CAPM). When financial markets are assumed complete, the stochastic discount factor, or pricing kernel, in this model is given by the representative agent's IMRS. With CRRA preferences, the stochastic discount factor is the discounted aggregate per capita consumption growth rate raised to the power $-\gamma$, where γ is interpreted as the coefficient of relative risk aversion. Empirical evidence is that the representative-agent consumption CAPM performs poorly in explaining asset returns. Perhaps the most well-known problems with the representative-agent model are the consumption real exchange rate, equity premium, risk-free rate, and currency premium puzzles.

Economic theory predicts that the log real exchange rate growth between any two countries is equal to the difference in the logs of the foreign and domestic stochastic discount factors (see Brand *et al.* (2006)). With representative agents within each country, the log real exchange rate growth must be perfectly correlated with the difference in the log growth rates of marginal utilities of aggregate per capita consumption of the two countries. This implies that, under the popular assumption of power utility, the log real exchange rate and log relative consumption should be perfectly correlated. In practice however, it is observed that the correlation between relative consumption and the real exchange rate is zero or negative. This is the consumption real exchange rate puzzle first documented by Backus and Smith (1993).

Another anomaly with the representative-agent model is that this model is unable to reconcile the highly volatile excess return on currency with the smooth aggregate consumption growth rate unless the representative agent is assumed to be willing to entertain an implausibly high level of aversion to risk. This is the currency premium puzzle. Lustig and Verdelhan (2007), e.g., show that the representative-agent consumption CAPM can explain the cross-sectional variation in currency premia only if the representative agent's coefficient of risk aversion is around 100. They find that the estimate of the risk aversion coefficient does not change when the representative agent's Euler equations for the currency portfolios are estimated jointly with the Euler equations for US domestic bond portfolios (sorted by maturity) and stock portfolios (sorted by size and book-to-market ratio).

Apart from the above-mentioned international asset pricing puzzles, two well-known domestic puzzles have also been of great interest over the last two decades. They are the equity-premium puzzle and the risk-free rate puzzle. Empirical evidence is that the covariance of aggregate per capita consumption growth with the excess return on the market portfolio over a risk-free asset is very low so that the representative-agent consumption CAPM can explain the observed market premium only if the typical investor is extremely risk averse. This is the equity premium puzzle discussed by Mehra and Prescott (1985) and Hansen and Jagannathan (1991) among others. Another problem with the representative-agent consumption CAPM is that, given the lack of variability of aggregate consumption growth, the representative agent must have a negative rate of time preference for the model to be able to match the observed mean risk-free rate. This finding is referred to as the risk-free rate puzzle (Weil, 1989).

In the absence of certain contingent-claims markets, agents are not able to completely insure their consumption against idiosyncratic risks they face and hence realized IMRS can differ across individuals. Bewley (1982), Mehra and Prescott (1985), Mankiw (1986), Constantinides and Duffie (1996) and Brav *et al.* (2002), among others, argue that consumers' heterogeneity induced by market incompleteness may be relevant for asset pricing.

To assess the potential of the incomplete market hypothesis in explaining the Backus-Smith (1993) puzzle, Kocherlakota and Pistaferri (2006) assume that markets are complete with respect to country-specific shocks (individuals can fully insure their consumption against cross-country shocks), but domestic markets are incomplete (individuals cannot completely insure themselves against idiosyncratic skill shocks).[1] Constantinides and Duffie (1996), Brav *et al.* (2002), Semenov (2004), Balduzzi and Yao (2007), and Kocherlakota and Pistaferri (2007) argue that the model with heterogeneous consumers can help explain the excess return on the market portfolio over the risk-free rate.[2]

[1]Kocherlakota and Pistaferri (2006) consider two forms of partial insurance against idiosyncratic skill shocks. The first one is domestically incomplete markets (hereafter DIM). Under this formulation, individuals are unable to insure their consumption against idiosyncratic skill shocks. The second form of partial insurance is called Private Information Pareto Optimal (hereafter PIPO). Here, the agents are able to sign insurance contracts, which allow them to insure themselves against idiosyncratic shocks, subject to the incentive constraint that agents reveal truth about their private skill shocks to the financial intermediary. For each of the two models of limited risk-sharing, Kocherlakota and Pistaferri (2006) derive an equation relating the cross-sectional distributions of individual consumption and the real exchange rates. Using household-level consumption data from the US and the UK, they find in their calibration exercise that the PIPO model fits the data with the estimate of the relative risk aversion coefficient of around 5, while the DIM model and the complete risk-sharing model both perform poorly.

[2]Constantinides and Duffie (1996) show that in the equilibrium of an economy with heterogeneity in the form of uninsurable, persistent, and heteroskedastic labor income shocks, the pricing kernel is a function not only of per capita consumption growth, but also of the cross-sectional variance of the logarithmic individual consumption growth rate. Brav *et al.* (2002) test empirically the Constantinides and Duffie (1996) pricing kernel using the CEX database and find that this stochastic discount factor fails to explain the equity premium. However, they find that the pricing kernel calculated as the equally weighted average of the investors' IMRS expanded up to the third moment of the cross-sectional distribution of the individual consumption growth rate can account for the mean equity premium with a coefficient of relative risk aversion between 3 and 4. Semenov (2004) finds that the pricing kernel that captures the first three cross-sectional moments of consumption explains the observed mean equity premium with a low (below three) value of the relative risk aversion coefficient. Balduzzi and Yao (2007) derive a stochastic discount factor, which differs from the Constantinides and Duffie (1996) pricing kernel in that the second pricing factor is the difference of the cross-sectional variance of log consumption and not the cross-sectional variance of the log consumption growth rate. Although this pricing kernel specification allows to explain the equity premium with a value of the relative risk aversion coefficient, which is substantially lower than that obtained using the conventional representative-agent model, the value of risk aversion needed to explain the equity premium remains rather high (larger than 9). Kocherlakota and Pistaferri (2007) derive the stochastic discount factor calculated as the reciprocal of the gross growth of the γth non-centered moment of the consumption

The evidence is that, although some progress in explaining asset returns is made, there is no yet pricing kernel that would allow to jointly explain the equity premium, risk-free rate, currency premium, and exchange rate puzzles. Since, by definition, the same stochastic discount factor should price all assets, we may be interested in deriving a pricing kernel that jointly explain the above-mentioned four puzzles.

To find such a pricing kernel, as in Kocherlakota and Pistaferri (2006), we assume that international markets are complete, while domestic markets are incomplete and consider two market structures: (i) where agents are unable to insure their consumption against idiosyncratic skill shocks and (ii) where idiosyncratic shocks to their skills can be partially insured by striking long term insurance contract with truth revelation constraint (the PIPO form of partial insurance against idiosyncratic shocks). For each of these two market structures, we derive the associated stochastic discount factor. We refer these pricing kernels to as the DIM and PIPO stochastic discount factors.

To assess the empirical performance of these two new stochastic discount factor, we test them empirically using data for the US and the UK. Here, for each of the stochastic discount factors, we jointly estimate the Euler equations for the equity premium, the risk-free rate, and the currency premium as well as the linear equation for the real exchange rate. The GMM estimation and testing results show that, in contrast to the DIM pricing kernel, the PIPO stochastic discount factor allows to jointly explain the observed equity premium, risk-free rate, currency premium, and real exchange rate with economically plausible values of the relative risk aversion coefficient (about 1) and the time discount factor (the estimate of this parameter is close to but lower than 1).

The rest of the paper is organized as follows. In Section 2, we describe the DIM and PIPO environments and derive the associated stochastic discount factors. Section 3 addresses the empirical implementation of the models derived in Section 2. The empirical estimation and testing results for these models are reported in Section 4. Section 5 concludes.

2 The DIM and PIPO Environments

Following Kocherlakota and Pistaferri (2006), we relax the assumption of market completeness and assume that although international markets are complete (individuals can fully insure their consumption against country-specific (aggregate) shocks), domestic markets are incomplete (individuals can only partially insure their consumption against individual-specific (idiosyncratic) skill shocks). To take into account this partial insurance against shocks, as in Kocherlakota and Pistaferri (2006), we consider two market structures: (i) where agents are unable to insure their consumption against idiosyncratic shocks and (ii) where idiosyncratic shocks can be partially

distribution, where γ is the coefficient of relative risk aversion, and show that this pricing kernel can explain the mean equity premium with a value of risk aversion between 5 and 6. However, this stochastic discount factor, like the pricing kernel proposed by Brav *et al.* (2002), fares poorly when used to explain the mean risk-free rate. The both these pricing kernels yield an implausibly low estimate of the time discount factor.

insured by striking long term insurance contract with truth revelation constraint (the PIPO form of partial insurance against idiosyncratic shocks).

In this section, we first describe each of these two environments and then derive the associated stochastic discount factors.

2.1 The DIM Environment

2.1.1 The Problem

Our DIM environment is similar to that in Kocherlakota and Pistaferri (2006) and Golosov and Tsyvinski (2006) except that, within our approach, we have explicit stock, bond, and currency trading. We assume that there are two generic countries in the world: a home country and a foreign country. We further assume that the economy is populated by infinitely many agents with *ex ante* identical preferences. The agents are not country specific in nature and only differ in private history of skill shocks.

At any date t ($t = 0, 1, 2, ..., T$), an agent experiences an idiosyncratic skill shock θ_t, which is drawn from a finite set Θ. In addition, all agents are exposed to the same aggregate shock z_t that is drawn from an uncountable set Z. The date t private skill shock history $\theta^t = (\theta_1, \theta_2, ..., \theta_t)$ and public shock history $z^t = (z_1, z_2, ..., z_t)$ are the tth components of Θ^T and Z^T, respectively, with respective probabilities $\pi(\theta^t)$ and $\psi(z^t)$. We assume that the idiosyncratic skill shock and the aggregate shock are drawn independently, so that by observing the aggregate shock one cannot infer anything about the idiosyncratic skill shock. According to the law of large numbers, at any date t there are exactly $\pi(\theta^t)$ agents with the private history θ^t.

Suppose that the home country produces two goods, tradable (y_t^{TR}) and non-tradable (y_t^{NT}), with the following technologies:

$$y_t^i(z^t, \theta^t) = \phi^i(z^t, \theta^t) l_t^i, \; i = TR, NT, \tag{1}$$

where l_t^i is the labour used in sector i and ϕ^i is the sector i marginal product of labour. Note that the labour productivities depend on the history of the public and idiosyncratic skill shocks, z^t and θ^t.

The aggregate outputs of traded and non-traded goods for the home country are

$$Y_t^i(z^t, \theta^t) = \sum_{\theta^t} y_t^i(z^t, \theta^t)\pi(\theta^t), \; i = TR, NT. \tag{2}$$

Assume that there are the following assets: (i) two home stocks, which are claims to the nominal proceeds from traded and non-traded sectors, (ii) a one-period nominal bond that pays a nominal interest rate of r_t, and (iii) the home country currency, which is traded in the international spot and forward markets. We further assume that only the spot and forward contracts on currency are traded abroad, while stocks and bonds are not internationally traded. The currency plays a twofold role: (i) as a means of exchange (specified as a cash-in-advance constraint) and (ii) as a store of value (the same currency can be invested in the international spot and forward markets). Home and foreign goods are both non-storable.

Consider the following two trading strategies: a spot transaction and a forward transaction. A spot transaction consists in converting the home country currency into the foreign country currency using the spot market and then converting this back into the home country currency using the spot market next period. A forward transaction consists in converting the home country currency into the foreign country currency using the spot market and then converting this back into the home country currency next period using the forward market, which is contracted now.[3]

Financial markets open before the goods market. At the start of the day, agents trade in stocks, bonds, and currency. Once the financial transactions are completed, a household takes the left over cash to transact in goods. Each household has two distinct entities: a shopper and a producer. As a producer, the household produces traded and non-traded goods, while as a shopper it purchases the same goods. Since in the market place there are infinitely many shoppers and producers and a shopper meets a producer randomly, a cash-in-advance constraint is necessitated.

A home country agent faces the following optimization problem:

$$Max\ E_t \sum_{j=t}^{T} \beta^{j-t} \left[\frac{\{u(c_j^{TR}, c_j^{NT})\}^{1-\gamma}}{1-\gamma} - v(l_j^{TR}, l_j^{NT}) \right] \tag{3}$$

s.t.

$$m_j^c + m_j^s + m_j^f + \sum_{i=TR,NT} Q_j^i \xi_j^i + b_j \leqslant$$

$$\sum_{i=TR,NT} (D_j^i \xi_{j-1}^i + Q_j^i \xi_{j-1}^i) + \frac{S_j m_{j-1}^s}{S_{j-1}} + \frac{F_{j-1} m_{j-1}^f}{S_{j-1}} + (1+r_{j-1}) b_{j-1} \tag{4}$$

and

$$\sum_{i=TR,NT} P_j^i c_j^i \leq m_j^c. \tag{5}$$

Here, c_j^i is the date j consumption of sector i goods, P_j^i is the date j nominal price of sector i goods, Q_j^i is the date j sector i nominal price of new equity purchases, $D_j^i = P_j^i Y_j^i$ is the date j dividends from sector i,[4] ξ_j^i is the date j share of sector i, r_j is the date j nominal risk-free rate of interest, m_j^s is the home money invested in the spot market at date j, m_j^f is the home money invested in the forward market at date j, m_j^c is the currency used for a goods purchase at date j, b_j is the risk-free bondholding at date j, F_j stands for the date j forward

[3] In order to keep the equity premium puzzle a purely domestic financial puzzle, we assume that the stocks and the bond are non-traded assets. This is an extreme form of "home bias" documented by Tesar and Werner (1995), e.g. As a result, we rule out the possibility of earning the risk-free interest on the currency held from one period to another.

[4] Note that there is no labour market. Agents supply their own labour and thus the dividends are simply the proceeds from the sale of outputs in the goods market.

exchange rate, S_j represents the date j spot exchange rate. All prices are denominated in home money. The instantaneous utility function $u(c_j^{TR}, c_j^{NT})$ is assumed linearly homogenous as in Backus and Smith (1993) and function $v(l_j^{TR}, l_j^{NT})$ is assumed to monotonically increase in its arguments with usual regularity conditions as in Kocherlakota and Pistaferri (2007). $E_t[\cdot]$ is an expectations operator. Expectation is computed with respect to the probability measures of z^{t+1} and θ^{t+1}. Finally, β is the subjective time discount factor and $\gamma > 0$ is the relative risk aversion coefficient.

Since within the DIM framework agents are assumed unable to insure themselves against idiosyncratic skill shocks, all date t prices, interest rate, and exchange rates are functions of public history of shocks z^t only. The crucial assumption here is that stocks and bonds do not hedge the idiosyncratic skill shocks. In this respect, the markets are domestically incomplete.

2.1.2 First-Order Conditions

The Lagrangian for the above optimization problem is

$$\begin{aligned} L \;=\; & E_t\left[\sum_{j=t}^{T}\beta^{j-t}\frac{u(c_j^{TR}, c_j^{NT})^{1-\gamma}}{1-\gamma} - v(l_j^{TR}, l_j^{NT})\right] + E_t\left[\sum_{j=t}^{T}\mu_j\left(m_j^c - \sum_{i=TR,NT} P_j^i c_j^i\right)\right] \\ & + E_t\sum_{j=t}^{T}\lambda_j\left(\begin{array}{c}\sum_{i=TR,NT}(D_j^i\xi_{j-1} + Q_j^i\xi_{j-1}^i - Q_j^i\xi_j^i) + \frac{S_j m_{j-1}^s}{S_{j-1}} + \frac{F_{j-1}m_{j-1}^f}{S_{j-1}} \\ +(1+r_{j-1})b_{j-1} - b_j - m_j^s - m_j^f - m_j^c\end{array}\right). \end{aligned} \tag{6}$$

The first-order conditions are:

$$c_t^i : u_t^{-\gamma} u_{c_t^i} = \mu_t P_t^i,\ i = TR, NT, \tag{7}$$

$$l_t^i : \beta^t v_{l_t^i}\pi(\theta^t)\psi(z^t) = \mu_t \phi_t^i P_t^i,\ i = TR, NT, \tag{8}$$

$$\xi_t^i : -\lambda_t Q_t^i + E_t\left[\lambda_{t+1}(Q_{t+1}^i + D_{t+1}^i)\right] = 0,\ i = TR, NT, \tag{9}$$

$$b_t : -\lambda_t + E_t\left[(1+r_t)\lambda_{t+1}\right], \tag{10}$$

$$m_t^s : -\lambda_t + E_t\left[\lambda_{t+1}\frac{S_{t+1}}{S_t}\right] = 0, \tag{11}$$

$$m_t^f : -\lambda_t + E_t\left[\lambda_{t+1}\frac{F_t}{S_t}\right] = 0, \tag{12}$$

$$m_t^c : -\lambda_t + \mu_t = 0. \tag{13}$$

Here, the subscripts of u and v state for the partial derivatives of u and v with respect to the relevant argument. The first-order condition (13) means that the agents allocate money for transaction purpose so as to equate the marginal benefit of transaction to the marginal opportunity cost of the foregone earnings from currency trading.

Based on these first-order conditions, it is straightforward to verify the following static efficiency condition for the labour supply decision:

$$\frac{u_{c_t^{TR}}\phi^{TR}}{u_{c_t^{NT}}\phi^{NT}} = \frac{v_{l_t^{TR}}}{v_{l_t^{NT}}}, \tag{14}$$

which shows the equivalence between the ratio of marginal disutlities of labour and the corresponding marginal utilities from consumption in each sector.

2.1.3 Monetary Policy and Initial Distributions of Assets

Monetary policy within this framework represents an initial cross-country distribution of money stocks, namely home money, M_0, and foreign money, M_0^*, to fix the date 0 spot rate such that

$$M_0 = S_0 M_0^*.^5 \tag{15}$$

In other words, central banks in both the home and foreign countries coordinate monetary policies in such a way that the initial spot rate S_0 is pinned down. After this, the central banks let the nominal exchange rate float according to currency trading among countries.

The initial distributions of stocks and bonds are such that

$$\sum_{\theta^0} \xi_0^i(\theta^0, z^0)\pi(\theta^0) = 1, \tag{16}$$

$$\sum_{\theta^0} b_0(\theta^0, z^0)\pi(\theta^0) = 0, \tag{17}$$

and

$$\sum_{\theta^0} m_0(\theta^0, z^0)\pi(\theta^0) = M_0. \tag{18}$$

2.1.4 Composite Good and Price

Following Kocherlakota and Pistaferri (2007), it is convenient to reduce the two good setting to a composite good problem. Exploiting linear homogeneity of the instantaneous utility function and the duality property, we can write:

$$\overline{P}_t c_t = \sum_{i=TR,NT} P_t^i c_t^i, \tag{19}$$

where $\overline{P}_t$ is the minimum expenditure required to attain one unit of utility.

That is,

$$\overline{P}_t = \underset{c_t^{TR},\, c_t^{NT}}{Min} \sum_{i=TR,NT} P_t^i c_t^i \tag{20}$$

s.t.

$$u(c_t^{TR}, c_t^{NT}) = 1, \tag{21}$$

[5] Here and hereafter, the asterisk denotes the foreign country.

which means that instantaneous utility $u(c_t^{TR}, c_t^{NT})$ is nothing, but the real consumption expenditure or a composite consumption good that we label c_t hereafter.

Based on this composite consumption, equations (7) and (13) can be combined to obtain

$$c_t : \beta^t c_t^{-\gamma} \pi(\theta^t)\psi(z^t) - \lambda_t \overline{P}_t = 0. \tag{22}$$

2.1.5 Equilibrium

In equilibrium, the following market-clearing conditions must hold. Given the assumption that stocks and bonds are not internationally traded, the stock and bond markets must domestically clear meaning

$$\sum_{\theta^t} \xi_t^i(\theta^t, z^t)\pi(\theta^t) = 1 \tag{23}$$

and

$$\sum_{\theta^t} b_t(\theta^t, z^t)\pi(\theta^t) = 0. \tag{24}$$

The other market-clearing conditions are the traded and non-traded market clearing conditions

$$\sum_{\theta^t}(c_t^{TR}(z^t, \theta^t) + c_t^{*TR}(z^t, \theta^t))\pi(\theta^t) = \sum_{\theta^t}(y_t^{TR}(z^t, \theta^t) + y_t^{*TR}(z^t, \theta^t))\pi(\theta^t) \tag{25}$$

and

$$\sum_{\theta^t} c_t^{NT}(z^t, \theta^t)\pi(\theta^t) = \sum_{\theta^t} y_t^{NT}(z^t, \theta^t)\pi(\theta^t), \tag{26}$$

respectively, and the currency market clearing conditions

$$\sum_{\theta^t}(m_t^s(\theta^t, z^t)\pi(\theta^t) = S_t \sum_{\theta^t} m_t^{*s}(\theta^t, z^t)\pi(\theta^t). \tag{27}$$

Some clarification about the spot and forward market clearing conditions is in order here. Within our framework, the spot and forward transactions imply that an agent in the home country first visits the spot market to convert the home currency into the foreign currency regardless of his trading strategy. The spot rate must be such that the supply of the home currency exactly equals the corresponding demand. This explains the currency market clearing conditions (27).[6]

[6] Note that there are two parallel currency markets, the spot market and the forward market. Given the above-described two trading strategies, the spot market clearing conditions are

$$\sum_{\theta^t}(m_t^s(\theta^t, z^t) + m_t^f(\theta^t, z^t))\pi(\theta^t) = S_t \sum_{\theta^t}(m_t^{*s}(\theta^t, z^t) + m_t^{*f}(\theta^t, z^t))\pi(\theta^t).$$

The forward market clearing conditions are

$$\sum_{\theta^t}(m_t^f(\theta^t, z^t) = \sum_{\theta^t}(m_t^{*f}(\theta^t, z^t) = 0 \text{ for all } z^t$$

because the forward rate at each date is contracted in such a way that agents taking long and short positions in the forward market balance each other for all aggregate history z^t. These conditions together imply the currency market clearing conditions (27).

2.1.6 The DIM Pricing Kernel

Assume that the world equilibrium, as laid out in the preceding section, exists. Consider a generic asset n with the real gross return $R_{n,t}$, which is a function of the aggregate shock history z^t only.

The Euler equation for an individual with the shock history (θ^t, z^t) is

$$c_t(\theta^t, z^t)^{-\gamma} = \beta \sum_{z^{t+1}} R_{n,t+1}(z^{t+1})\psi(z^{t+1}|z^t) \sum_{\theta^{t+1}} c_{t+1}(\theta^{t+1}, z^{t+1})^{-\gamma}\pi(\theta^{t+1}|\theta^t). \tag{28}$$

Define

$$E(c_{t+1}^{-\gamma}|z^{t+1}, \theta^t) = \sum_{\theta^{t+1}} c_{t+1}(\theta^{t+1}, z^{t+1})^{-\gamma}\pi(\theta^{t+1}|\theta^t) \tag{29}$$

as the $-\gamma$th non-centred cross-sectional moment of composite good consumption conditional on private history θ^t and public history z^{t+1}.

Thus, equation (28) can be rewritten as

$$c_t(\theta^t, z^t)^{-\gamma} = \beta \sum_{z^{t+1}} R_{n,t+1}(z^{t+1})\psi(z^{t+1}|z^t)E(c_{t+1}^{-\gamma}|z^{t+1}, \theta^t). \tag{30}$$

Integrating the both sides of (30) over θ^t and using the law of iterated expectations, we get

$$E(c_t^{-\gamma}|z^t) = \beta \sum_{z^{t+1}} R_{n,t+1}(z^{t+1})\psi(z^{t+1}|z^t)E(c_{t+1}^{-\gamma}|z^{t+1}), \tag{31}$$

or, equivalently,

$$E_t\left[\beta \frac{E(c_{t+1}^{-\gamma}|z^{t+1})}{E(c_t^{-\gamma}|z^t)} R_{n,t+1}\right] = 1, \tag{32}$$

where

$$SDF_{t+1}^{DIM} = \beta \frac{E(c_{t+1}^{-\gamma}|z^{t+1})}{E(c_t^{-\gamma}|z^t)} \tag{33}$$

is the stochastic discount factor associated with the DIM environment.

Define the real returns on the traded and non-traded stocks as

$$R_{M,t+1}^i = \frac{q_{t+1}^i + d_{t+1}^i}{q_t^i}, \; i = TR, NT, \tag{34}$$

where $q_t^i(z^t) = Q_t^i(z^t)/\overline{P}_t$ is the real ith equity price and $d_{t+1}^i(z^{t+1}) = D_{t+1}^i(z^{t+1})/\overline{P}_{t+1}$ is the real dividend from share i.

Based on these two returns, we can define the market portfolio return as

$$R_{M,t+1} = \sum_{i=TR,NT} \overline{\xi}_{t+1}^i R_{M,t+1}^i \tag{35}$$

where $\overline{\xi}_{t+1}^i = \sum_{\theta^t} \xi_t^i(\theta^t, z^t)\pi(\theta^t)$. According to (23), in equilibrium $\overline{\xi}_{t+1}^i = 1$ and hence the equilibrium market portfolio return is simply the sum of the returns on the traded and non-traded stocks, i.e.

$$R_{M,t+1} = \sum_{i=TR,NT} R_{M,t+1}^i. \tag{36}$$

Define the real risk-free rate as

$$R_{F,t+1} = \frac{(1 + r_{t+1})\overline{P}_t}{\overline{P}_{t+1}}. \tag{37}$$

Based on the first-order conditions (9) and (10), we can thus write the equation for the real equity premium as

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\overline{P}_{t+1}}{\overline{P}_t} (R_{M,t+1} - R_{F,t+1}) \right] = 0 \tag{38}$$

and the equation for the real risk-free rate as

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\overline{P}_{t+1}}{\overline{P}_t} R_{F,t+1} \right] = 1.^7 \tag{39}$$

The first-order conditions (11) and (12) give the spot and forward rate equations as follows:

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{S_{t+1}}{S_t} \right] = 1 \tag{40}$$

and

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{F_t}{S_t} \right] = 1 \tag{41}$$

or, equivalently,

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\overline{P}_{t+1}}{\overline{P}_t} \frac{S_{t+1}\overline{P}_t}{S_t\overline{P}_{t+1}} \right] = 1 \tag{42}$$

and

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\overline{P}_{t+1}}{\overline{P}_t} \frac{F_t\overline{P}_t}{S_t\overline{P}_{t+1}} \right] = 1. \tag{43}$$

Subtracting (42) from (43), we get the real currency premium equation

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\overline{P}_{t+1}}{\overline{P}_t} \left(\frac{F_t - S_{t+1}}{S_t} \right) \frac{\overline{P}_t}{\overline{P}_{t+1}} \right] = 0. \tag{44}$$

From the above equations, it follows that within the DIM framework the stochastic discount factor is

$$SDF_{t+1}^{DIM} = \frac{\lambda_{t+1}}{\lambda_t} \frac{\overline{P}_{t+1}}{\overline{P}_t}. \tag{45}$$

Given that, we can rewrite equations (38), (39), and (44) as

$$E_t \left[SDF_{t+1}^{DIM} (R_{M,t+1} - R_{F,t+1}) \right] = 0, \tag{46}$$

$$E_t \left[SDF_{t+1}^{DIM} R_{F,t+1} \right] = 1, \tag{47}$$

and

$$E_t \left[SDF_{t+1}^{DIM} \left(\frac{F_t - S_{t+1}}{S_t} \right) \frac{\overline{P}_t}{\overline{P}_{t+1}} \right] = 0, \tag{48}$$

[7] It is easy to see that the equation for the equity premium is analogous to the equity premium equation of Kocherlakota and Pistaferri (2007). The only difference is that the consumption is defined in terms of composite consumption units.

respectively, with SDF^{DIM}_{t+1} defined as in (45).

Because the log real exchange rate growth between any two countries is equal to the difference in the logs of the foreign and domestic stochastic discount factors (see Brand *et al.* (2006)), within the DIM framework

$$ln\left(\frac{e_{t+1}}{e_t}\right) = ln\left(SDF^{DIM*}_{t+1}\right) - ln\left(SDF^{DIM}_{t+1}\right). \tag{49}$$

2.2 The PIPO Environment

2.2.1 The Problem

In this alternative setting, agents are able to partially insure against idiosyncratic skill shocks. The model is a dynamic extension of Mirrlees (1971) type private information setting. Trading convention is similar to that in Golosov and Tsyvinski (2006) and Kocherlakota and Pistaferri (2006). All agents are assumed to have *ex ante* identical preferences. There is a continuum of insurance firms, which act on behalf of the households and play the following roles: (i) produce the traded and non-traded goods by hiring workers, (ii) sell these goods in national and international markets, (iii) trade among themselves in stock, bond, and currency in sequential markets, and, finally, (iv) with the resulting profits from this trade insure the households against idiosyncratic skill shocks. Timing of financial and goods markets is the same as in the DIM setting. The same cash-in-advance constraint applies to the insurance companies when they trade in goods.

The insurance firms are owned equally by all agents. At date 0, before the realization of aggregate and idiosyncratic shocks, the contract market opens only once. In this market, the competitive insurance firms offer contracts to the households about consumption bundles of traded and non-traded goods $\{c^{TR}_t, c^{NT}_t\}$, which provide maximum *ex ante* utility to the households. Since the insurance company does not observe the idiosyncratic shock history and labour supply, it stipulates contract about the observed output sequence of traded and non-traded goods $\left\{y^{TR}_t, y^{NT}_t\right\}$, such that it is incentive compatible for the agents to reveal the truth about the history of idiosyncratic shocks. These contracts are long-term contracts with full commitment on both sides. After the contract market closes, from date 1 onward the insurance firms start trading in goods and financial markets in the same sequential manner as within the DIM framework.

A typical insurance company, located in the home country, maximizes the present value of the nominal payoffs to its owners:

$$\underset{\{c^{TR}_t, c^{NT}_t, y^{TR}_t, y^{NT}_t, \xi_t, b_t, m^s_t, m^f_t\}}{Max} \sum_{t=0}^{T} \prod_{i=1}^{t} (1+\rho_i(z^i))^{-1} \Pi_t(z^t)\psi(z^t) \tag{50}$$

s.t.

$$\Pi_t(z^t) + m_t^s(z^t) + m_t^f(z^t) + m_t^c(z^t) + \sum_{i=TR,NT} Q_t^i(z^t)\xi_t^i(z^t) + b_t(z^t) \leqslant$$
$$\sum_{i=TR,NT} \xi_{t-1}^i(z^{t-1}) \sum_{\theta^t} \pi(\theta^t) D_t^i(\theta^t, z^t) + \sum_{i=TR,NT} \xi_{t-1}^i(z^{t-1}) Q_t^i(z^t) + \frac{S_t(z^t)}{S_{t-1}(z^{t-1})} m_{t-1}^s(z^{t-1})$$
$$+ \frac{F_{t-1}(z^{t-1})}{S_{t-1}(z^{t-1})} m_{t-1}^f(z^{t-1}) + (1 + r_t(z^{t-1}))b_{t-1}(z^{t-1}), \tag{51}$$

the cash-in-advance constraint

$$\sum_{\theta^t} \sum_{i=TR,NT} \pi(\theta^t) P_t^i(z^t) c_t^i(\theta^t, z^t) \leqslant m_t^c(z^t), \tag{52}$$

the participation constraint

$$\sum_{t=0}^{T} \beta^t \sum_{\theta^t, z^t} \left[\frac{\{u(c_t^{TR}(\theta^t, z^t), c_t^{NT}(\theta^t, z^t))\}^{1-\gamma}}{1-\gamma} - v\left(\frac{y_t^{TR}(\theta^t, z^t)}{\phi_t^{TR}(\theta^t, z^t)}, \frac{y_t^{NT}(\theta^t, z^t)}{\phi_t^{NT}(\theta^t, z^t)} \right) \right] \pi(\theta^t)\psi(z^t) \geqslant \underline{u}, \tag{53}$$

and the incentive constraint

$$\sum_{t=0}^{T} \beta^t \sum_{\theta^t, z^t} \left[\frac{u(c_t^{TR}(\theta^t, z^t), c_t^{NT}(\theta^t, z^t))^{1-\gamma}}{1-\gamma} - v\left(\frac{y_t^{TR}(\theta^t, z^t)}{\phi_t^{TR}(\theta^t, z^t)}, \frac{y_t^{NT}(\theta^t, z^t)}{\phi_t^{NT}(\theta^t, z^t)} \right) \right] \pi(\theta^t)\psi(z^t) \geqslant$$
$$\sum_{t=0}^{T} \beta^t \sum_{\theta^t, z^t} \left[\frac{u(c_t^{TR}(\theta_R^t, z^t), c_t^{NT}(\theta_R^t, z^t))^{1-\gamma}}{1-\gamma} - v\left(\frac{y_t^{TR}(\theta_R^t, z^t)}{\phi_t^{TR}(\theta_R^t, z^t)}, \frac{y_t^{NT}(\theta_R^t, z^t)}{\phi_t^{NT}(\theta_R^t, z^t)} \right) \right] \pi(\theta^t)\psi(z^t), \tag{54}$$

where $\Pi_t(z^t)$ is the date t cash flow of the insurance firm contingent on the shock history z^t, $\rho_t(z^t)$ is the z^t contingent discount rate, and θ_R^t is the history of shocks that the household reports to the financial intermediaries. Since the insurance firm does not observe the idiosyncratic shock history, all its relevant choices depend on the aggregate shock history z^t.

2.2.2 First-Order Conditions

Let $\lambda_t(z^t)$, $\mu_t(z^t)$, $\omega_t(z^t)$, and $\eta_t(z^t)$ be the Lagrange multipliers associated with the flow budget constraint (51), the cash-in-advance constraint (52), the participation constraint (53), and the incentive constraint (54), respectively. The first-order conditions for problem (50) through (54) are as follows:

$$\Pi_t(z^t) : \prod_{i=1}^{t} (1 + \rho_i(z^i))^{-1} - \lambda_t(z^t) = 0, \tag{55}$$

$$c_t^i : \beta^t(\omega_t(z^t) + \eta_t(z^t)) u_t^{-\gamma} u_{c_t^i} \pi(\theta^t) = \mu_t(z^t) P_t^i \pi(\theta^t), \; i = TR, NT, \tag{56}$$

$$y_t^i : \beta^t(\omega_t(z^t) + \eta_t(z^t)) v_{l_t^i} \pi(\theta^t) = \lambda_t(\theta^t, z^t) \xi_{t-1}^i(z^{t-1}) \phi^i(z^t, \theta^t) P_t^i \pi(\theta^t), \; i = TR, NT, \tag{57}$$

$$\xi_t^i : -Q_t^i(z^t)\lambda_t(z^t)\psi(z^t) + \sum_{z^{t+1}} (Q_{t+1}(z^{t+1}) + D_{t+1}(z^{t+1}))\lambda_{t+1}(z^{t+1})\psi(z^{t+1}) = 0, \; i = TR, NT, \tag{58}$$

$$b_t : -\lambda_t(z^t)\psi(z^t) + \sum_{z^{t+1}} \lambda_{t+1}(z^{t+1})(1 + r_{t+1}(z^t))\psi(z^{t+1}) = 0, \tag{59}$$

$$m_t^s : -\lambda_t(z^t)\psi(z^t) + \sum_{z^{t+1}} \lambda_{t+1}(z^{t+1}) \frac{S_{t+1}(z^{t+1})}{S_t(z^t)} \psi(z^{t+1}), \tag{60}$$

$$m_t^f : -\lambda_t(z^t)\psi(z^t) + \sum_{z^{t+1}} \lambda_{t+1}(z^{t+1}) \frac{F_t(z^t)}{S_t(z^t)} \psi(z^{t+1}), \tag{61}$$

and

$$m_t^c : -\lambda_t(z^t) + \mu_t(z^t) = 0. \tag{62}$$

A few clarifications are in order. Based on (55), the Lagrange multiplier λ_t represents the date 0 state claims price of a dollar to be delivered at date t contingent on z^t. Because of (62), this state claims price is the same as the marginal transaction benefit of a dollar, $\mu_t(z^t)$.

It can be seen that the other first-order conditions are similar to those in the DIM setting. Note that the use of (56) and (57) yields the same static efficiency condition as (14).

2.2.3 Monetary Policy and Initial Distributions of Assets

The monetary policy and the initial distributions of assets are the same as those described by equations (15) through (18) in Section 2.1.2.

2.2.4 Composite Good and Price

As within the DIM framework, we can reduce the two good setting to a composite good problem described by equations (20) and (21). This means that the two goods can be reduced to a composite good c_t with an associate composite price $\overline{P}_t$ so that the following equality holds:

$$\overline{P}_t c_t = \sum_{i=TR,NT} P_t^i(z^t) c_t^i(\theta^t, z^t). \tag{63}$$

2.2.5 Equilibrium

Following Kocherlakota (2005), we can show that the equilibrium allocation $\{c_t^{TR}, c_t^{NT}, y_t^{TR}, y_t^{NT}\}$ for this decentralized economy solves a constrained social planning problem, where the constraints involve the truth revelation incentive constraint. Because of this optimality, Kocherlakota and Pistaferri (2007) call this allocation Private Information Pareto Optimum. In equilibrium, the market-clearing conditions (23) through (27) hold.

2.2.6 The PIPO Pricing Kernel

From (55), we obtain the following useful relationship between the Lagrange multipliers and the stochastic discount factor:

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{1 + \rho_{t+1}(z^{t+1})}. \tag{64}$$

Using (62) and (64), and defining the conditional probability $\psi(z^{t+1}|z^t) \equiv \psi(z^{t+1})/\psi(z^t)$, we get

$$\sum_{z^{t+1}} \frac{Q^i_{t+1}(z^{t+1}) + D^i_{t+1}(z^{t+1})}{Q^i_t(z^t)} \frac{\psi(z^{t+1}|z^t)}{1+\rho_{t+1}(z^{t+1})} = 1, \; i = TR, NT. \tag{65}$$

Likewise, using (59), (60), (61), and (64), we obtain the following equations:

$$\sum_{z^{t+1}} (1 + r_{t+1}(z^t)) \frac{\psi(z^{t+1}|z^t)}{1+\rho_{t+1}(z^{t+1})} = 1, \tag{66}$$

$$\sum_{z^{t+1}} \frac{S_{t+1}(z^{t+1})}{S_t(z^t)} \frac{\psi(z^{t+1}|z^t)}{1+\rho_{t+1}(z^{t+1})} = 1, \tag{67}$$

and

$$\sum_{z^{t+1}} \frac{F_t(z^t)}{S_t(z^t)} \frac{\psi(z^{t+1}|z^t)}{1+\rho_{t+1}(z^{t+1})} = 1. \tag{68}$$

To characterize the discount rates $\rho_t(z^t)$, we follow Kocherlakota (2005) and Golosov *et al.* (2006). Fix the date t history θ^t and z^t. Decrease the composite good at date t for this history group by an infinitesimally small amount $\beta\Delta_t$ and increase across the board the date $t+1$ composite good by Δ_t. This compensating variation leaves the objective function and the incentive and participation constraints unaffected. It only impacts the resource constraints. The insurance company now makes sure to minimize the cost of resources at dates t and $t+1$ for all possible evolutions of the private and public shocks.

To solve this problem, define

$$\frac{\widetilde{c}_t(\theta^t, z^t)^{1-\gamma}}{1-\gamma} \equiv \frac{c_t(\theta^t, z^t)^{1-\gamma}}{1-\gamma} - \beta\Delta_t \tag{69}$$

and

$$\frac{\widetilde{c}_{t+1}(\theta^{t+1}, z^{t+1})^{1-\gamma}}{1-\gamma} \equiv \frac{c_{t+1}(\theta^{t+1}, z^{t+1})^{1-\gamma}}{1-\gamma} + \Delta_t. \tag{70}$$

The insurance company thus chooses Δ_t such that the cost of resources at dates t and $t+1$ evaluated at the respective state claims prices $\lambda_t(z^t)$ and $\lambda_{t+1}(z^{t+1})$ is minimized at $\Delta_t = 0$. Using the flow resource constraint (51) and (63), this cost minimization problem can be rewritten as

$$\begin{aligned} &\underset{\Delta_t}{Min}\; \lambda_t(z^t)\overline{P}_t(z^t)\left(c_t(\theta^t, z^t)^{1-\gamma} - \beta(1-\gamma)\Delta_t\right)^{1/(1-\gamma)} \pi(\theta^t) \\ &+\lambda_{t+1}(z^{t+1})\overline{P}_{t+1}(z^{t+1}) \sum_{\theta^{t+1}} \left(c_{t+1}(\theta^{t+1}, z^{t+1})^{1-\gamma} + (1-\gamma)\Delta_t\right)^{1/(1-\gamma)} \pi(\theta^{t+1}). \end{aligned} \tag{71}$$

The first-order condition with respect to Δ_t evaluated at $\Delta_t = 0$ and the use of (55) and (62) yield the following inverse Euler equation:

$$\beta\overline{P}_t(z^t)c^{\gamma}_t(\theta^t, z^t)\pi(\theta^t) = (1+\rho_{t+1}(z^{t+1}))^{-1}\overline{P}_{t+1}(z^{t+1}) \sum_{\theta^{t+1}} c^{\gamma}_{t+1}(\theta^{t+1}, z^{t+1})\pi(\theta^{t+1}). \tag{72}$$

Next, first integrating the right-hand side of (72) with respect to θ^{t+1} for a given θ^t and then integrating the left-hand side of (72) with respect to θ^t, and applying the law of iterated expectations, we get

$$\beta\overline{P}_t(z^t)E(c_t^\gamma\,|\,z^t) = (1+\rho_{t+1}(z^{t+1}))^{-1}\overline{P}_{t+1}(z^{t+1})E(c_{t+1}^\gamma\,|\,z^{t+1}) \tag{73}$$

or, equivalently,

$$\frac{1}{1+\rho_{t+1}(z^{t+1})} = \beta\frac{E(c_t^\gamma\,|\,z^t)}{E(c_{t+1}^\gamma\,|\,z^{t+1})}\frac{\overline{P}_t(z^t)}{\overline{P}_{t+1}(z^{t+1})}. \tag{74}$$

Plugging (74) into (65) through (68), we obtain that within the PIPO framework the stochastic discount factor is

$$SDF_{t+1}^{PIPO} = \frac{1}{1+\rho_{t+1}(z^{t+1})}\frac{\overline{P}_{t+1}(z^{t+1})}{\overline{P}_t(z^t)} = \beta\frac{E(c_t^\gamma\,|\,z^t)}{E(c_{t+1}^\gamma\,|\,z^{t+1})}. \tag{75}$$

With this pricing kernel, the Euler equations for the equity premium, the risk-free rate, and the currency premium can be rewritten as

$$E_t\left[SDF_{t+1}^{PIPO}\left(R_{M,t+1}-R_{F,t+1}\right)\right] = 0, \tag{76}$$

$$E_t\left[SDF_{t+1}^{PIPO}R_{F,t+1}\right] = 1, \tag{77}$$

and

$$E_t\left[SDF_{t+1}^{PIPO}\left(\frac{F_t-S_{t+1}}{S_t}\right)\frac{\overline{P}_t}{\overline{P}_{t+1}}\right] = 0, \tag{78}$$

respectively, with SDF_{t+1}^{PIPO} defined as in (75).

The log real exchange rate growth between two countries equals the difference in the logs of the foreign and domestic pricing kernels and hence within the PIPO framework

$$ln\left(\frac{e_{t+1}}{e_t}\right) = ln\left(SDF_{t+1}^{PIPO*}\right) - ln\left(SDF_{t+1}^{PIPO}\right). \tag{79}$$

3 Empirical Formulation

3.1 Consumption Process

For the sake of empirical application, consider a specific parameterization of the post-trade world composite consumption process. Define the date t consumption of the hth investor in the kth country as

$$c_{hk,t} = c_{k,t}(\theta^t, z^t).$$

In a similar spirit as in Sarkissian (2003), we represent the post-trade allocation of consumption as

$$c_{hk,t} = \delta_{hk,t}\delta_{k,t}C_t, \tag{80}$$

where $\delta_{hk,t}$ is the hth investor's share in country k's consumption and $\delta_{k,t}$ is the country k's share in world consumption C_t.[8]

We assume the following processes for $\delta_{hk,t}$ and $\delta_{k,t}$:

$$\delta_{hk,t} = exp\left(u_{hk,t}\sqrt{x_{k,t}} - \frac{x_{k,t}}{2}\right) \tag{81}$$

and

$$\delta_{k,t} = exp\left(u_{k,t}\sqrt{x_t} - \frac{x_t}{2}\right), \tag{82}$$

where $u_{hk,t}$ and $u_{k,t}$ are standard normal shocks, which are IID across countries, individuals, and time, $x_{k,t}$ is the within-country variance of country k's log consumption level and x_t is the between-country variance of log consumption level of country k and the rest of the world.

The sth non-centred moment of the cross-sectional distribution of consumption is given by

$$E_h\left(c^s_{hk,t}\right) = C^s_t exp\left(\frac{s^2 - s}{2}(x_{k,t} + x_t)\right). \tag{83}$$

Note that, by construction, the aggregate consumption is the sum of individual consumption, what can be checked by setting $s = 1$. Therefore, this lognormal process satisfies the feasibility condition. The next issue is: Does it satisfy the optimality conditions? We follow a reverse engineering approach here. If we can find a pricing kernel that supports this allocation of world consumption and is also independent of the agent's private history, then it must be satisfying the individual optimality conditions.

3.2 Pricing Kernels

Plugging (83) into (33) and evaluating at $s = -\gamma$, we obtain the following pricing kernel for the DIM environment from the kth country's perspective:

$$SDF^{DIM}_{k,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} exp\left(\frac{\gamma + \gamma^2}{2}\left(\triangle x_{k,t+1} + \triangle x_{t+1}\right)\right), \tag{84}$$

where $\triangle x_{k,t+1} = x_{k,t+1} - x_{k,t}$ and $\triangle x_{t+1} = x_{t+1} - x_t$.

Likewise, plugging (83) into (75) and evaluating at $s = \gamma$, we get the stochastic discount factor associated with the PIPO environment from the kth country's perspective:

$$SDF^{PIPO}_{k,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} exp\left(\frac{\gamma - \gamma^2}{2}\left(\triangle x_{k,t+1} + \triangle x_{t+1}\right)\right). \tag{85}$$

[8] Sarkissian (2003) writes the post-trade allocation in terms of the consumption growth rate, while we write it in terms of the level of consumption. The motivation for doing so is to apply this post-trade allocation to the Kocherlakota and Pistaferri (2007) discounting methodology. The Kocherlakota and Pistaferri (2007) incomplete market stochastic discount factor is based on the growth rates of the cross-sectional moments of consumption level, while Sarkissian (2003) uses the Constantinides and Duffie (1996) pricing kernel that is based on the cross-sectional average of the intertemporal marginal rates of substitution. See Kocherlakota and Pistaferri (2007) for a discussion about the difference in methodology.

3.3 Real Exchange Rates in two Environments

From (49) and (84), we obtain the following relation between the log real exchange rate growth between kth and k'th countries and individual consumption in the DIM environment:

$$ln\left(\frac{e_{t+1}}{e_t}\right) = \frac{\gamma+\gamma^2}{2}\left[\triangle x_{k',t+1} - \triangle x_{k,t+1}\right]. \tag{86}$$

As follows from (79) and (85), in the PIPO environment, the log real exchange rate growth between kth and k'th countries is

$$ln\left(\frac{e_{t+1}}{e_t}\right) = \frac{\gamma-\gamma^2}{2}[\triangle x_{k',t+1} - \triangle x_{k,t+1}]. \tag{87}$$

The immediate implication is that the real exchange rate is independent of the cross-country variance of consumption. Since $\gamma > 0$, for any value of γ the home country currency depreciates (appreciates) in response to the increase in the foreign (home) within-country variance in the DIM environment. In the PIPO model, the implication is the same if $\gamma < 1$ and is exactly reverse as long as $\gamma > 1$.

Within the DIM setting, higher home country uninsurable risk (that results in a higher value of $\triangle x_{k,t+1}$) raises the precautionary demand for both traded and non-traded goods, which makes the home country currency appreciate. In the PIPO economy, agents buy contracts from insurance firms to insure against individual shocks subject to incentive constraints. Hence, there are two opposite effects: the precautionary saving effect and the incentive effect. When the coefficient of relative risk aversion is lower than 1, the precautionary saving effect dominates and therefore, as in the DIM environment, the home country currency appreciates with the increase in the home country uninsurable risk. The higher the agent's aversion to risk, the greater the incentive effect, so that, when γ becomes greater than 1, the incentive effect dominates the precautionary effect, what results in the depreciation of the home country currency.

4 Empirical Investigation

The DIM and PIPO pricing kernels both incorporate incomplete consumption risk sharing by default. As described earlier, in the DIM environment agents cannot insure consumption at all using the domestic financial market. In the PIPO environment, agents can insure consumption using the domestic financial markets, but, due to hidden work effort, financial intermediaries strike incentive compatible constraint, which prevents full risk sharing. Which of these two environments reconciles the observed fluctuations of the real exchange rate, the equity premium, the risk-free rate of return, and the currency premium better? In this section, we attempt to answer this question.

4.1 Estimation Strategy

For each of the alternative environments DIM and PIPO, we jointly estimate four equations. These equations are the real exchange rate equation, the Euler equations for the equity premium and the risk-free rate of return, and the currency risk premium (the excess real currency return) equation. Hereafter, we treat the US as the home country and the UK as the foreign country. Thus, the pricing kernel refers to the US.

For the DIM environment, we jointly estimate the real exchange rate equation as

$$ln\left(\frac{e_{t+1}}{e_t}\right) = \frac{\gamma+\gamma^2}{2}\left[\triangle x_{t+1}^{UK} - \triangle x_{t+1}^{US}\right] + \eta_{t+1}, \tag{88}$$

the Euler equation for the equity premium as

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} exp\left(\frac{\gamma+\gamma^2}{2}\left(\triangle x_{t+1}^{US} + \triangle x_{t+1}\right)\right)\left(R_{M,t+1} - R_{F,t+1}\right)\right] = 0, \tag{89}$$

the Euler equation for the risk-free rate as

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} exp\left(\frac{\gamma+\gamma^2}{2}\left(\triangle x_{t+1}^{US} + \triangle x_{t+1}\right)\right) R_{F,t+1}\right] = 1, \tag{90}$$

and the currency risk premium equation as

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} exp\left(\frac{\gamma+\gamma^2}{2}\left(\triangle x_{t+1}^{US} + \triangle x_{t+1}\right)\right)\frac{(F_t - S_{t+1})\overline{P}_t}{S_t\overline{P}_{t+1}}\right] = 0. \tag{91}$$

For the PIPO environment, we jointly estimate the real exchange rate equation as

$$ln\left(\frac{e_{t+1}}{e_t}\right) = \frac{\gamma-\gamma^2}{2}[\triangle x_{t+1}^{UK} - \triangle x_{t+1}^{US}] + \xi_{t+1}, \tag{92}$$

the Euler equation for the equity premium as

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} exp\left(\frac{\gamma-\gamma^2}{2}\left(\triangle x_{t+1}^{US} + \triangle x_{t+1}\right)\right)\left(R_{M,t+1} - R_{F,t+1}\right)\right] = 0, \tag{93}$$

the Euler equation for the risk-free rate as

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} exp\left(\frac{\gamma-\gamma^2}{2}\left(\triangle x_{t+1}^{US} + \triangle x_{t+1}\right)\right) R_{F,t+1}\right] = 1, \tag{94}$$

and the currency risk premium equation as

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} exp\left(\frac{\gamma-\gamma^2}{2}\left(\triangle x_{t+1}^{US} + \triangle x_{t+1}\right)\right)\frac{(F_t - S_{t+1})\overline{P}_t}{S_t\overline{P}_{t+1}}\right] = 0. \tag{95}$$

We use the GMM estimation technique to explore the potential of each of the two above-mentioned models to jointly explain the real exchange rate, the equity premium, the risk-free rate, and the currency risk premium.

4.2 Data

Consumption. As is conventional in the literature, the consumption measure used in this paper is consumption of nondurables and services. For the US, data on household quarterly consumption of nondurables and services are from the Consumer Expenditure Survey (CEX), produced by the Bureau of Labor Statistics (BLS). For each household, we calculate quarterly consumption expenditures for all the disaggregate consumption categories offered by the CEX. Then, we deflate obtained values in 2005:Q1 US dollars by the CPI's (not seasonally adjusted, urban consumers) (the CPI series are obtained from the BLS) for appropriate consumption categories. Aggregating the household's quarterly consumption across these categories is made according to the National Income and Product Account definition of consumption of nondurables and services.

Following Brav *et al.* (2002), in each quarter we drop households that do not report or report a zero value of consumption of food, consumption of nondurable and services, or total consumption. We also delete from the sample the nonurban households, the households residing in student housing, the households with incomplete income responses, the households that do not have a fifth interview, and the households whose head is under 19 or over 75 years of age.

To calculate the household's quarterly per capita consumption, we divide the quarterly consumption expenditure of each household by the number of people in the household in that quarter. The within-country consumption variance for each quarter, x_t^{US}, is then calculated as the cross-sectional variance of the log household's quarterly real, per capita consumption.

For the UK, we use the Family Expenditure Survey (FES), a voluntary survey of a random sample of private households in the UK, conducted by the Office for National Statistics (ONS).[9] The data of approximately 6,500 households are collected throughout the year to cover seasonal variations in expenditures, with either the week or month, in which the fieldwork is carried out, being randomly assigned to each individual household. Of the data available in the FES, we use the diary records of daily expenditure, kept for two weeks by each individual aged 16 or over in the household survey.

Using these diary data, the cross-sectional variance of the log household's quarterly real, per capita consumption of nondurables and services for each quarter is computed as follows.[10] First, we calculate the household-wide consumption of nondurable and services by adding the consumption only of nondurables and services (measured in UK pounds) for each individual in the household. The definition of nondurable and services follows that of Attanasio and Weber (1995). Second, given that the household consumption data are for the two week durations only, we multiply them by 6.5, so that the data are converted into quarterly frequency. Third, we divide the quarterly consumption expenditure of each household by the number of people in the household in that quarter to derive quarterly nominal, per capita consumption of nondurables

[9] In April 2001, the FES was replaced by the Expenditure and Food Survey (EFS), which also covered the National Food Survey (NFS).

[10] Our procedure mimics Kocherlakota and Pistaferri (2006).

and services. Fourth, we categorize the household consumption data into four quarterly groups, based on the quarter or month the survey was conducted for the household. By dividing the data by the quarterly CPI for all items (not seasonally adjusted) (the CPI is from the OECD main economic indicators) with the basis of 2005:Q1, the quarterly real, per capita consumptions are calculated. Finally, we take the logarithms of the quarterly real, per capita consumptions calculated in the previous step, followed by the calculation of the cross-sectional variance of the log household's quarterly real, per capita consumption for each quarter, x_t^{UK}.

For each quarter, the between-country consumption variance x_t is calculated as a weighted average of x_t^{US} and x_t^{UK} with weights being the proportions of the populations of the respective countries in the total population of the US and the UK.

The US data on quarterly seasonally adjusted US dollar nominal aggregate consumption of nondurables and services are from the US Bureau of Economic Analysis (BEA). The real aggregate consumption of nondurables and services is calculated by dividing the nominal seasonally adjusted aggregate consumption of nondurables and services by the CPI (2005:Q1=1) for nondurables and services (from the US BEA). The UK data on seasonally adjusted nominal aggregate consumption of nondurables and services (in pounds) are from the ONS and the UK Data Archive (UKDA). The UK real aggregate consumption of nondurables and services is calculated by dividing the nominal seasonally adjusted aggregate consumption of nondurables and services by the CPI (2005:Q1=1) for nondurables and services (from OECD main economic indicators).

The world aggregate consumption C_t is defined as

$$C_t = C_t^{US} + e_t C_t^{UK}, \tag{96}$$

where C_t^{US} is the US real aggregate consumption of nondurables and services (in US dollars), C_t^{UK} is the UK real aggregate consumption of nondurables and services (in UK pounds), and e_t is the real US$ into Sterling spot exchange rate. The real world per capita consumption of nondurables and services is calculated by dividing the real world aggregate consumption of nondurables and services by the sum of the US (from the U.S. Department of the Commerce, BEA) and UK populations.

The Spot and Forward Exchange Rates. The nominal spot (S_t) and 3-month forward (F_t) exchange rates US$ into Sterling are from DATASTREAM (series XUDLUSS and XUDLDS3, respectively). The real US$ into Sterling spot exchange rate (e_t) is calculates as

$$e_t = \frac{S_t CPI_t^{UK}}{CPI_t^{US}}, \tag{97}$$

where CPI_t^{US} and CPI_t^{UK} are respectively the US and UK Consumer Price Indexes (CPI) (2005:Q1=1) for consumption of nondurables and services.

Asset Returns. We use two proxies for the market portfolio return. They are the value- and equal-weighted returns (capital gain plus dividends) on all stocks listed on the NYSE, AMEX, and NASDAQ. The data on the nominal quarterly value- and equal-weighted returns on all stocks listed on the NYSE, AMEX, and NASDAQ for the period from 1982:Q1 to 2004:Q4 are obtained from the Center for Research in Security Prices (CRSP) of the University of Chicago. The nominal quarterly value-weighted returns on five NYSE, AMEX, and NASDAQ industry portfolios ((i) consumer durables, nondurables, wholesale, retail, and some services (laundries, repair shops), (ii) manufacturing, energy, and utilities, (iii) business equipment, telephone and television transmission, (iv) healthcare, medical equipment, and drugs, and (v) other) are from Kenneth R. French's web page.

The risk-free rate is the 3-month US Treasury Bill secondary market rate on a per annum basis obtained from the Federal Reserve Bank of St. Louis. In order to convert from the annual rate to the quarterly rate, we raise the 3-month Treasury Bill return on a per annum basis to the power of 1/4.

The real quarterly returns are calculated as the quarterly nominal returns divided by the 3-month inflation rate based on the deflator defined for consumption of nondurables and services. We calculate the equity premium as the difference between the real equity return and the real risk-free rate.

Because of the poor quality of the CEX data before 1982, the sample period is from 1982:Q1 to 2004:Q4. Table I reports the descriptive statistics for the data set used in estimation.

4.3 GMM Estimation and Testing Results

To implement the GMM estimation approach, first, we have to identify a set of instruments. When choosing instruments, we use the fact that the error terms associated with Euler equations are uncorrelated with any variables in agents' information sets. From this, it follows that we can use as instruments any variables lagged one or more periods. We identify 5 sets of instruments. The first set of instruments has a constant and consumption growth lagged one and two periods ($INST1 = \{1, C_t/C_{t-1}, C_{t-1}/C_{t-2}\}$). The second set has a constant, consumption growth lagged one period, and the change in the between-country consumption variance lagged one and two periods ($INST2 = \{1, \triangle x_t, \triangle x_{t-1}\}$). The third set has a constant, consumption growth lagged one and two periods, and the change in the US cross-sectional variance of the log real per capita consumptions lagged two periods ($INST3 = \{1, C_t/C_{t-1}, C_{t-1}/C_{t-2}, \triangle x_{t-1}^{US}\}$). The forth set has a constant, the change in the between-country consumption variance lagged one and two periods, and the change in the US cross-sectional variance of the log real per capita consumptions lagged two periods ($INST4 = \{1, \triangle x_t, \triangle x_{t-1}, \triangle x_{t-1}^{US}\}$). Finally, our fifth set of instruments has a constant, consumption growth lagged one and two periods, the change in the between-country consumption variance lagged one and two periods, and the change in the US cross-sectional variance of the log real per capita consumptions lagged two periods ($INST5 = \{1, C_t/C_{t-1}, C_{t-1}/C_{t-2}, \triangle x_t, \triangle x_{t-1}, \triangle x_{t-1}^{US}\}$). Variable $\triangle x_t^{US} = x_t^{US} - x_{t-1}^{US}$ is not included in any of the considered sets of instruments because equations (88) and (92) both include $\triangle x_{t+1}^{US} = x_{t+1}^{US} - x_t^{US}$ and hence the forecast errors η_{t+1} and ξ_{t+1} in these two equations are correlated with $\triangle x_t^{US}$ through x_t^{US}.

As argued by Hall (1988), the second lag in instrumental variables helps in reducing the effect of time aggregation. Furthermore, Ogaki (1988) demonstrates that the use of the second lag is consistent with the information structure of a monetary economy with cash-in-advance constraints.

When assessing the empirical performance of model (88) - (91), we first estimate this model for the sample period from 1982:Q1 to 2004:Q4 with the CRSP quarterly value-weighted return index used as a proxy for the return on the market portfolio. The GMM estimation and testing results are reported in Panel A.1 of Table II. These results show that for any set of instruments the DIM model is not rejected statistically at the 5% level of significance according to Hansen's test of overidentifying conditions and yields positive and statistically significant at the 5% significance level estimates of the relative risk aversion coefficient (except for the set of instruments INST1, for which the hypothesis that the coefficient of relative risk aversion is positive is accepted at the 10% level of significance).[11] The estimate of the time discount factor is less than 1 for all sets of instruments.

To test whether the obtained results are robust to the used proxy for the market portfolio return, we jointly estimate equations (88) - (91) for the same sample period with the return on the market portfolio proxied by the CRSP quarterly equal-weighted return index. Panel B of Table II shows that the estimation results are very similar to those obtained for the CRSP value-weighted return index.

In order to check for sensitivity of the estimation results to the chosen sample period, apart from the entire sample period, we also estimate equations (88) - (91) for two subperiods. The first subperiod is from 1982:Q1 to 1993:Q4 and the second subperiod is from 1994:Q1 to 2004:Q4. The estimation results for these two subperiods are reported in Panels A.2 and A.3 of Table II. If for the first subperiod the hypothesis of positiveness of the relative risk aversion coefficient is accepted at the 5% significance level only for the instrument sets INST2, INST4, and INST5, for the second subperiod this hypothesis is accepted at the 5% level of significance for all sets of instruments. For the both subperiods and all sets of instruments, the DIM model is not rejected statistically by Hansen's test of overidentifying conditions and the estimate of the time discount factor is lower than 1.

Then, we jointly estimate equations (92) - (95). The estimation results for the CRSP value-weighted index are presented in Panel A of Table III. The evidence is that, in contrast to the DIM model, the PIPO model (92) - (95) yields economically realistic estimates of the coefficient of relative risk aversion and the time discount factor for any set of instruments and any sample period. Only for the period from 1994:Q1 to 2004:Q4 and the instrument set INST3 the point estimate of the time discount factor is slightly grater than 1, but the null hypothesis that the

[11] We test $H_0 : \gamma = 0$ against $H_1 : \gamma > 0$.

true value of this parameter is lower than or equal to 1 is not rejected statistically at the 5% significance level. As for the DIM model (88) - (91), the estimation results for the PIPO model (92) - (95) are only slightly sensitive to the chosen proxy for the return on the market portfolio (see Panel B of Table III).

To check whether the DIM model (88) - (91) and the PIPO model (88) - (91) can explain the cross-section of asset returns, we estimate the both these models with the excess value-weighted returns on five NYSE, AMEX, and NASDAQ industry portfolios for the period from 1982:Q1 to 2004:Q4. The estimation results are reported in Table IV. They show that the DIM model is unable to precisely estimate the coefficient of relative risk aversion. The estimate of the relative risk aversion coefficient yielded by the DIM model is not significantly different from zero at the 5% significance level for any set of instruments, except for INST1. In contrast to the DIM model, the PIPO model yields economically plausible and statistically significant estimates of the risk aversion coefficient and the time discount factor for any set of instruments. The PIPO model is never rejected statistically at the 5% level of significance by Hansen's test of overidentifying restrictions.

Our estimation results are consistent with Kocherlakota and Pistaferri (2006) who also find that PIPO model is supported by the data. The redeeming feature of our study is that we have an integrated model, which is capable of reconciling various puzzles on the domestic and international fronts. What is especially noteworthy is that we are able to reconcile the equity premium and currency premium puzzles with a plausible degree of risk aversion within the PIPO framework.

5 Conclusion

This paper addresses a few extant domestic and international financial markets anomalies, namely the equity premium, risk-free rate, consumption real exchange rate, and currency premium puzzles. We investigate the potential of two stochastic discount factors, which allow incomplete risk sharing in economies with consumer heterogeneity, to resolve these anomalies (see Kocherlakota and Pistaferri, 2006). The first stochastic discount factors is the DIM pricing kernel. This stochastic discount factor describes the market structure with domestically incomplete financial markets, where idiosyncratic privately observed shocks are uninsured, while sequential trade in assets enables agents to partially hedge publicly observed shocks. The second stochastic discount factor is the PIPO pricing kernel that describes the market environment, in which both private and public shocks are insured subject to truth revelation constraint by agents.

We test empirically the both these stochastic discount factors using household-level data for the US and UK. The GMM approach is implemented to obtain the estimates of the parameters of interest. Empirical evidence is that the PIPO stochastic discount factor outperforms the DIM pricing kernel. We find that the model with the PIPO pricing kernel is able to jointly explain the consumption real exchange rate, equity premium, risk-free rate, and currency premium puzzles with economically plausible values of risk aversion and the time discount factor. This evidence is found to be robust to the chosen sample period and measure of the return on the market portfolio as well as to the chosen set of risky assets. This suggests that the observed behavior of equity premium, risk-free rate, currency premium, and the real exchange rates are consistent with a world economy whose real allocation mimics a dynamic Mirrlees economy's social planning optimum.

The immediate question arises about the practical implementation of this Mirrlees type allocation in a world economy, where agents trade in assets, while they are exposed to private skill shocks. In our model, this is implemented by fictitious insurance firms striking incentive compatible contracts with full commitment, which is a stretch from the real world. About the issue of practical implementation of the PIPO allocation, one may speculate a bit and leave it for future research. Perhaps a global fiscal policy coordination among countries with nonlinear taxes as in Kocherlakota (2005) could be a way to solve this mechanism design problem for a world economy.

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Table I
Descriptive Statistics

Variable	Mean	St. Dev.	Skewness	Kurtosis	JB
		A. 1982:Q1 - 2004:Q4			
C_{t+1}/C_t	1.0059	0.0064	-0.4269	4.2178	8.20
$\triangle x_{t+1}^{US} = x_{t+1}^{US} - x_t^{US}$	-0.0006	0.0222	0.3099	2.7978	1.58
$\triangle x_{t+1}^{UK} = x_{t+1}^{UK} - x_t^{UK}$	0.0022	0.0436	-0.2116	4.5216	9.25
$R_{VW,t+1}$	1.0278	0.0856	-0.4832	3.5251	4.49
$R_{EW,t+1}$	1.0342	0.1157	-0.0777	3.1646	0.19
$R_{F,t+1}$	1.0050	0.0049	-0.4022	3.3181	2.78
$ln(e_{t+1}/e_t)$	0.0016	0.0529	-0.1290	3.2866	0.55
$(F_t - S_{t+1})\overline{P}_t/S_t\overline{P}_{t+1}$	-0.0081	0.0523	-0.0643	3.0450	0.07
$\triangle x_{t+1} = x_{t+1} - x_t$	-0.0001	0.0208	0.1989	2.7595	0.80
$R_{1,t+1}$	1.0319	0.0910	-0.3438	4.0195	5.61
$R_{2,t+1}$	1.0287	0.0672	-0.9112	4.7373	23.51
$R_{3,t+1}$	1.0282	0.1220	-0.3723	3.9002	5.06
$R_{4,t+1}$	1.0321	0.0972	-0.2595	3.0661	1.02
$R_{5,t+1}$	1.0323	0.0939	-0.6661	3.5549	7.72
		B. 1982:Q1 - 1993:Q4			
C_{t+1}/C_t	1.0060	0.0080	-0.4342	3.1429	1.45
$\triangle x_{t+1}^{US} = x_{t+1}^{US} - x_t^{US}$	-0.0012	0.0237	0.1723	2.8521	0.26
$\triangle x_{t+1}^{UK} = x_{t+1}^{UK} - x_t^{UK}$	0.0038	0.0360	-0.6423	5.2018	12.18
$R_{VW,t+1}$	1.0319	0.0807	-0.7139	4.6793	9.11
$R_{EW,t+1}$	1.0329	0.1178	0.0197	3.5348	0.54
$R_{F,t+1}$	1.0067	0.0042	0.4530	2.3894	2.24
$ln(e_{t+1}/e_t)$	-0.0006	0.0664	-0.1128	2.3883	0.80
$(F_t - S_{t+1})\overline{P}_t/S_t\overline{P}_{t+1}$	-0.0068	0.0656	-0.0376	2.2267	1.13
$\triangle x_{t+1} = x_{t+1} - x_t$	-0.0003	0.0207	0.0242	2.5554	0.38
$R_{1,t+1}$	1.0416	0.0994	-0.6027	4.2479	5.64
$R_{2,t+1}$	1.0312	0.0637	-0.9993	6.2068	26.77
$R_{3,t+1}$	1.0284	0.0907	-0.3121	4.2124	3.49
$R_{4,t+1}$	1.0340	0.1080	-0.2935	2.6126	0.93
$R_{5,t+1}$	1.0357	0.0958	-0.7314	3.9633	5.75

Note: JB is the Jarque-Bera statistic.

Table I (continued)

Variable	Mean	St. Dev.	Skewness	Kurtosis	JB
		C. 1994:Q1 - 2004:Q4			
C_{t+1}/C_t	1.0059	0.0042	0.0292	2.9432	0.01
$\triangle x_{t+1}^{US} = x_{t+1}^{US} - x_t^{US}$	0.0000	0.0208	0.5302	2.3676	2.79
$\triangle x_{t+1}^{UK} = x_{t+1}^{UK} - x_t^{UK}$	0.0006	0.0507	0.0085	3.7020	0.90
$R_{VW,t+1}$	1.0236	0.0911	-0.2746	2.6335	0.80
$R_{EW,t+1}$	1.0355	0.1148	-0.1807	2.5967	0.54
$R_{F,t+1}$	1.0033	0.0051	-0.7075	2.4173	4.29
$ln(e_{t+1}/e_t)$	0.0039	0.0346	0.3929	2.6696	1.33
$(F_t - S_{t+1})\overline{P}_t/S_t\overline{P}_{t+1}$	-0.0094	0.0345	-0.4842	2.6956	1.89
$\triangle x_{t+1} = x_{t+1} - x_t$	0.0001	0.0211	0.3614	2.8187	1.02
$R_{1,t+1}$	1.0220	0.0815	-0.0238	3.3378	0.21
$R_{2,t+1}$	1.0261	0.0713	-0.7948	3.5044	5.10
$R_{3,t+1}$	1.0281	0.1485	-0.3430	2.9965	0.86
$R_{4,t+1}$	1.0301	0.0860	-0.2025	3.5063	0.77
$R_{5,t+1}$	1.0288	0.0928	-0.5784	2.9376	2.46

Table II
GMM Results for the DIM Model

Parameter	INST1	INST2	INST3	INST4	INST5
			A. CRSP Value-Weighted Index		
			A.1 1982:Q1 - 2004:Q4		
γ	0.1390	0.0509	0.0565	0.0582	0.0548
	(1.37)	(2.17)	(2.49)	(2.46)	(2.88)
β	0.9934	0.9925	0.9925	0.9922	0.9927
	(1486.03)	(1696.74)	(1856.02)	(1885.46)	(2330.88)
J	16.13	16.44	16.77	17.06	18.10
	[0.0959]	[0.0877]	[0.2688]	[0.2529]	[0.7003]
			A.2 1982:Q1 - 1993:Q4		
γ	-1.0726	0.0390	-0.0083	0.0604	0.1124
	(-7.88)	(1.68)	(-0.30)	(3.10)	(8.28)
β	0.9814	0.9914	0.9901	0.9921	0.9929
	(944.86)	(1808.64)	(2817.30)	(2277.72)	(3210.13)
J	9.61	9.79	9.97	10.05	11.32
	[0.4750]	[0.4593]	[0.7646]	[0.7589]	[0.9699]
			A.3 1994:Q1 - 2004:Q4		
γ	0.5516	0.0995	0.2595	0.1516	0.1271
	(3.21)	(3.38)	(5.16)	(5.85)	(8.20)
β	0.9981	0.9917	0.9944	0.9930	0.9931
	(591.37)	(1736.78)	(1359.72)	(1845.57)	(2768.64)
J	7.59	9.13	9.75	9.83	10.72
	[0.6690]	[0.5194]	[0.7803]	[0.7746]	[0.9786]
			B. CRSP Equal-Weighted Index		
			1982:Q1 - 2004:Q4		
γ	0.1657	0.0525	0.0731	0.0526	0.0486
	(1.53)	(2.17)	(3.36)	(2.17)	(2.54)
β	0.9930	0.9919	0.9921	0.9918	0.9923
	(1327.99)	(1733.10)	(1861.89)	(1953.12)	(2370.82)
J	15.85	16.73	17.36	17.42	19.07
	[0.1039]	[0.0806]	[0.2376]	[0.2344]	[0.6409]

Note: $INST1 = \{1, C_t/C_{t-1}, C_{t-1}/C_{t-2}\}$, $INST2 = \{1, \triangle x_t, \triangle x_{t-1}\}$, $INST3 = \{1, C_t/C_{t-1}, C_{t-1}/C_{t-2}, \triangle x_{t-1}^{US}\}$, $INST4 = \{1, \triangle x_t, \triangle x_{t-1}, \triangle x_{t-1}^{US}\}$, $INST5 = \{1, C_t/C_{t-1}, C_{t-1}/C_{t-2}, \triangle x_t, \triangle x_{t-1}, \triangle x_{t-1}^{US}\}$. t-statistics are in parentheses. J is Hansen's test of the overidentifying restrictions. Asymptotic p-values are in brackets.

Table III
GMM Results for the PIPO Model

Parameter	INST1	INST2	INST3	INST4	INST5
			A. CRSP Value-Weighted Index		
			A.1 1982:Q1 - 2004:Q4		
γ	3.0679	0.9794	1.0038	0.9720	0.9714
	(3.91)	(25.74)	(24.64)	(26.94)	(29.56)
β	0.9963	0.9982	0.9986	0.9979	0.9980
	(113.23)	(1349.42)	(1595.83)	(1376.60)	(1744.80)
J	9.06	16.21	16.98	16.94	18.57
	[0.5263]	[0.0937]	[0.2573]	[0.2592]	[0.6717]
			A.2 1982:Q1 - 1993:Q4		
γ	0.9262	0.9708	1.0538	0.9301	1.0794
	(7.40)	(30.01)	(24.72)	(32.04)	(54.72)
β	0.9971	0.9982	0.9994	0.9984	0.9985
	(1179.74)	(1546.69)	(1672.47)	(1729.90)	(2917.16)
J	9.45	8.80	10.09	9.21	10.97
	[0.4903]	[0.5512]	[0.7557]	[0.8173]	[0.9752]
			A.3 1994:Q1 - 2004:Q4		
γ	3.8499	0.8862	1.9871	0.7544	0.7857
	(7.37)	(12.23)	(16.05)	(9.19)	(16.72)
β	0.9718	0.9951	1.0017	0.9956	0.9954
	(50.35)	(1601.53)	(387.57)	(1382.57)	(2218.72)
J	6.12	9.09	8.67	9.74	10.72
	[0.8052]	[0.5234]	[0.8518]	[0.7808]	[0.9785]
			B. CRSP Equal-Weighted Index		
			1982:Q1 - 2004:Q4		
γ	2.3681	0.9883	0.9828	0.9908	0.9905
	(3.95)	(27.31)	(25.95)	(29.30)	(32.99)
β	1.0028	0.9977	0.9979	0.9975	0.9976
	(239.26)	(1529.02)	(1841.64)	(1567.87)	(2050.93)
J	9.48	16.57	17.59	17.26	19.20
	[0.4871]	[0.0845]	[0.2259]	[0.2428]	[0.6327]

Note: See Table II.

Table IV
GMM Results for Industry Portfolios

Parameter	INST1	INST2	INST3	INST4	INST5
			A. DIM Model		
γ	2.1832	0.0171	0.0243	0.0175	-0.0001
	(3.58)	(0.84)	(1.18)	(0.83)	(-0.01)
β	1.0063	0.9932	0.9923	0.9916	0.9912
	(97.32)	(2304.75)	(3569.50)	(3802.67)	(5363.36)
J	17.29	19.73	20.64	20.37	21.44
	[0.7473]	[0.5997]	[0.8987]	[0.9066]	[0.9993]
			B. PIPO Model		
γ	3.3753	1.0392	1.0512	1.0170	1.1206
	(4.91)	(38.48)	(28.21)	(40.83)	(63.24)
β	0.9977	0.9998	0.9998	0.9990	0.9999
	(87.04)	(1812.13)	(2594.97)	(2701.81)	(3900.50)
J	16.83	19.08	20.20	20.17	21.39
	[0.7728]	[0.6404]	[0.9113]	[0.9120]	[0.9993]

Note: See Table II.