Company Valuation, Risk Sharing and the Government's Cost of Capital

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Abstract

Assuming a no arbitrage environment, this article analyzes the role of the government in the context of company valuation when firms follow different financial policies. For the two analyzed financial policies, the tax authority's required returns and the value of tax payments are derived. Based on these results, we study the risk sharing effects between equity owners and the government. It is shown that the government's cost of capital is greater than (equals) the equity cost of capital for a fixed debt policy in a (no) growth setting. In contrast to the fixed debt case, the government's cost of capital is smaller than the equity owner's cost of capital in case of a no-growth constant leverage policy and usually smaller in the growth case. Most importantly, these results allow us to analyze the capital budgeting conflicts between the government and equity owners. We show that firms in some situations invest more than socially desirable. Moreover, the possibility of corporate overinvestment depends on the financial policy. Firms that follow a constant leverage policy never overinvest, but always underinvest. In contrast, firms with fixed future levels of debt overinvest if the gain in tax-shields is big enough to outweigh the loss in the unlevered firm value.

Finally, our results illustrate that various policies available to the government to encourage investments to their socially optimal level should depend on the financing policy pursued by the firm.

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1 Introduction

The approaches employed for company valuation discount projected future cash flows with risk-adjusted costs of capital. Using this valuation methodology the role of the tax authority is only implicitly included in the models in terms of tax claims. The government's tax claims are either corrected in the particular numerator (cash flows) or denominator (costs of capital). Concerning the costs of capital, it is commonly recurred to the work of Modigliani/Miller[1] in conjunction with the CAPM.[2]

The aim of this article is to develop an alternative approach, in which tax payments are discounted by the risk-adjusted required return of the tax authority. The value of an enterprise including the present value of tax payments then arises from discounting the before tax cash flow, that is the cash flows to equity holders, debt owners and the government. The resulting gross present value of the firm then builds the basis for distributions to all three parties. Thus, deducting the market value of debt as well as the present value of tax payments from the firm's gross present value leads to the market value of equity. The addition or deduction of whatever tax advantages attributable to debt financing and the consistent treatment in costs of capital is omitted and eases the calculation by explicitly calculating the present value of the government claim.

The performed explicit analysis of the tax authority is of special interest for three reasons. First, the risk sharing between the government and the equity owners in conjunction with the debt holders becomes visible for the first time from a company valuation perspective, where growing perpetuities of cash flows are assumed. Second, the economic interpretation of the costs of capital in case of a fixed debt financial policy and in case of a constant leverage financial policy is eased by the complementary presentation of the tax authority's risk position. Finally, investment distortions introduced by the existence of the government can be analyzed in this framework and potential remedies, such as investment tax credits and subsidy rates may be quantified in further research.

The analysis is important since in many countries, governments provide vast amounts of loans, endow firms with loan guarantees and many other incentives, in order to encourage new investments. Beside the resulting benefits (e.g. job creation, infrastructure improvements etc.), the distortion effect of taxation involved is well documented in the literature (see e.g. Jorgenson and Landau (1993) among others).

[1]See Modigliani and Miller (1958), Modigliani and Miller (1963) and Modigliani and Miller (1969).

[2]These are special cases of the time state preference approach (see Myers (1968)), generalized in the concept of the stochastic discount factor (see Cochrane (2005)) and applied in the context of company valuation by Arzac and Glosten (2005). For the coherence of these modern theories of finance see Hsia (1981).

Especially, the first and the last objective (risk sharing and distortion effects) are closely related since the risk analysis is a prerequisite to calculate the government's share of a company by discounting the expected tax stream at the government's required rate of return. In addition to Modigliani/Miller's (1958, 1963) fixed debt policy with predetermined debt levels for all future periods, the framework is extended by considering Miles/Ezzell's (1980) constant leverage policy, where the future debt outstanding is a function of the realized company value and thus has to be adjusted every period based on the stochastic properties of the cash flow process. These two financial policies yield completely different and interesting results for the government's cost of capital.

It is shown that for a *fixed debt policy* the government's cost of capital is greater than (equals) the equity cost of capital under (no) growth. The growth case differs from the no-growth case since the cash flow stream available for the government and the flow to equity are not proportional anymore, which drives a wedge between the government's cost of capital and the equity owner's cost of capital. In addition, it is shown that the government's cost of capital is, with the exception of extreme case, smaller than equity owner's cost of capital if the firm follows a *constant leverage policy*.

It has to be noted that the derived results differ in important ways from former conclusions in the literature (see Galai (1998) and Rao and Stevens (2006)), since previous studies worked only in an one period setting, where both financing policies can not be distinguished. Most importantly, the proposed valuation approach provides important insights concerning the conflict of interests arising from capital budgeting, namely corporate under- and over-investment problems. We show that these conflicts depend on the financing policy pursued by the firm. Thus, our main finding is that corporate overinvestment, i.e. investing more than is socially desirable, is possible and depends on the financial policy. Firms that follow a constant leverage policy never overinvest, but always underinvest. In contrast, firms with fixed future levels of debt overinvest if the gain in tax-shields is big enough to outweigh the loss in the value of the unlevered firm. These results illustrate that various policies available to the government to encourage investments to their socially optimal level should depend on the financing policy pursued by the firm.

The remainder of this study is organized as follows. Section 2 reviews the equity valuation approaches and provides the important theoretical extension by explicitly modelling the government as a separate claimholder to the company's cash flow stream. Section 3 comprises the derivation of the firm's before-tax cost of capital, which is a necessary prerequisite in order to derive the government's cost of capital. The following section 4 contains the derivation of the governments's cost of capital and value of taxes for the two financing policies in the case of infinite living firms. Furthermore, the implications for company valuation are discussed. Section 5 comprises the analysis of the risk sharing between the government and the stockholders. Section 6 analyzes the conflicts of interest between the equity owners and the government arising from capital budgeting considerations. Section 7 summarizes the results and discusses implications for future research.

2 Company Valuation incorporating the Government Claim

Three equivalent approaches for company valuation are well known: 1. WACC- (weighted average cost of capital), 2. APV- (adjusted present value)[3] and 3. FTE- (flow to equity) method, which, under consistent assumptions, all yield identical equity values (see Booth (2007)).

These methods differ with respect to the valuation relevant cash flows and discount rates. Within the WACC- and the APV-method the corporation's expected free cash flows are discounted, whereas the FTE-method uses the equity owner's free cash flow. We define the corporation's free cash flow (FCF) and equity owner's free cash flow (FTE) in the usual way:

$$FCF_t = X_t \cdot (1-\tau) - NI_t \tag{1}$$

$$FTE_t = (X_t - r \cdot D_{t-1}) \cdot (1-\tau) - NI_t - PP_t \tag{2}$$

where X denotes $EBIT$, NI is net investment (sum of change in property, plant and equipment and net working capital minus depreciation), r denotes the cost of debt, D is the value of debt, and PP denotes the principal payments. Consistent with the literature, we assume that new investments are financed proportional to $NOPAT = X_t \cdot (1-\tau)$, thus $NI_t = b \cdot X_t \cdot (1-\tau)$, where b is the (expected) retention ratio and τ is the corporate tax rate.[4]

Furthermore we denote by k the unlevered cost of equity, k^E is the levered cost of equity, and TS is the present value of tax shields. Hence, if g is the (expected) growth rate of the

[3] The term Adjusted Present Value was introduced by Myers (1974).

[4] See Copeland, Koller, and Murrin (2000) or Fama and Miller (1972).

company's free cash flow, the following three equivalent equations can be used to calculate the equity value (E) in the perpetuity case with growth:[5]

$$E_t^{WACC} = \frac{E_t\left[FCF_{t+1}\right]}{WACC-g} - D_t \tag{3}$$

$$E_t^{APV} = \frac{E_t\left[FCF_{t+1}\right]}{k-g} + TS_t - D_t \tag{4}$$

$$E_t^{FTE} = \frac{E_t\left[FTE_{t+1}\right]}{k^E-g} \tag{5}$$

These formulas hold for both the fixed debt financial policy, which is characterized by a predetermination of planned future amounts of debt, and the constant leverage policy, for which the debt level is tied to the firm value by a constant relation.[6] Due to the definition of the financial policies, it is reasonable to assume for a firm following a fixed debt policy, that the *level* level of debt is known ex-ante, whereas for a firm following a constant leverage policy, the *leverage ratio* is known ex-ante.[7] This implies that the principal payments are assumed to be known for a fixed debt policy but unknown in a constant leverage setting.

It is also important to discuss briefly the implications of growth. It is reasonable to assume that a company can only grow if $NI > 0$ and these investments are only pursued if they are positive net present value projects. Projects add value if the *marginal* rate of return (after taxes) irr' is greater than the hurdle rate, where irr is the *average* return on assets (after tax) the unlevered company is expected to earn in every future period. From this definition follows that we assume a steady-state condition where the balance sheet and income statement items as well as the present value of debt and equity all are expected to grow geometrically with the same rate for all t. It is important to note that this setting is consistent with the well known Gordon-growth framework and therefore guarantees $g = irr \cdot b$. The no-growth case is easily obtained if depreciation equals gross investment ($b = 0$, there are no positive NPV-projects and all income is distributed to the equity holders) or $irr = WACC$ (growth is neutral with regard to the company value V^L).[8]

[5] In order to ease the notation, time subscripts are later on omitted for values that are known at the valuation date t. In general, stock variables (e.g., D) are associated with their actual value in t and flow variables (e.g. FCF) are associated with their expected value in t for $t+1$.

[6] Modigliani/Miller (1960 and 1963) implicitly assumed a fixed debt policy, the constant leverage policy was introduced by Miles and Ezzell (1980) and Miles and Ezzell (1985). In addition, they all derived their formulas in the no growth setting. For the growth formulas in the fixed debt policy case see Stapleton (1972), Kumar (1975), Bar-Yosef (1977) and the reply by Myers (1977), for the growth case and constant debt policy see Arzac and Glosten (2005).

[7] In general, we assume that k, τ, and the cost of debt function are known to the equity owners ex-ante.

[8] The implicit assumptions of the Gordon-growth framework and the resulting effect of growth on corporate investment is discussed in section 6.

Additionally, our analysis builds on the corresponding text-book formulas for cost of equity, WACC, and tax shield. In case of a fixed debt financial policy these are:[9]

$$k_d^E = k + (k - r) \cdot \left(1 - \frac{r}{r-g} \cdot \tau\right) \cdot L \tag{6}$$

$$WACC_d = k \cdot \left[1 + \left(\frac{g}{k} - 1\right) \cdot \left(\tau \cdot \frac{D}{V^L} \cdot \frac{r}{r-g}\right)\right] \tag{7}$$

$$TS_d = \frac{\tau \cdot r \cdot D}{r - g} \tag{8}$$

In case of a constant leverage financial policy they have to be specified as:

$$k_r^E = k + (k - r) \cdot \left(1 - \frac{r}{1+r} \cdot \tau\right) \cdot L \tag{9}$$

$$WACC_r = k - r \cdot \tau \cdot L^* \cdot \frac{1+k}{1+r} \tag{10}$$

$$TS_r = \frac{r \cdot \tau \cdot L^* \cdot V^L}{k - g} \cdot \frac{1+k}{1+r} \tag{11}$$

where $L = D/E$ denotes the debt-equity (or leverage) ratio, and $L^* = D/V^L$ is defined as the (target-) leverage ratio, which could be aspired by the management or influenced by rating agencies.[10]

Furthermore, it is well known that the market value of the firm can always be separated as

$$V^L = E + D = V^U + TS \tag{12}$$

where V^L is the value of the levered company and V^U is the value of the unlevered company.

Based on this summarized presentation of company valuation, the framework is now enhanced by explicitly incorporating the present value of the government's tax claim.[11] The additional cash flows that have to be considered are the company's *before-tax* cash flow

[9] Subscript d denotes the fixed debt policy, whereas subscript r denotes the constant leverage policy.

[10] See Graham and Harvey (2001), p. 211 and 234. 19% of the companies state that they do not have an optimal L, whereas 10% have an optimal L, the rest of the sample firms state, that they have a more or less flexible L.

[11] The following analysis is -if at all- only implicitly carried out by other authors. See for example Fernandez (2004), Cooper and Nyborg (2006), Fieten, Kruschwitz, Laitenberger, Löffler, Tham, Velez-Pareja, and Wonder (2005) or Arzac and Glosten (2005). An exception is Galai (1998), who however works only in a one period setting assuming no taxation of the liquidation proceeds.

(CFBT) and the government's cash flow (FTG). Following from the cash flow definitions above, we get:

$$
\begin{aligned}
CFBT_t &= X_t - NI_t = X_t \cdot (1 - b \cdot (1 - \tau)) & (13)\\
FTG_t &= (X_t - r \cdot D_{t-1}) \cdot \tau & (14)
\end{aligned}
$$

The corresponding valuation equations are:

$$
\begin{aligned}
C &= \frac{CFTB}{k^C - g} & (15)\\
G &= \frac{FTG}{k^G - g} & (16)
\end{aligned}
$$

where C denotes the present value of *before-tax* cash flows, k^C denotes the before tax cost of capital, G is the present value of tax payments, and k^G is the government's cost of capital.[12] An essential part of this paper is to derive tractable representations for these two cost of capital terms. Consistent with the notation above, we furthermore define for the (hypothetical) unlevered case :

$$
G^U = \frac{FTG^U}{k^{GU} - g} = \frac{X \cdot \tau}{k^{GU} - g} \qquad (17)
$$

The present value of all three claimants must add up in the following way:

$$
C = E + D + G = V^L + G = (V^U + TS) + (G^U - TS) = V^U + G^U
$$

The before-tax cash flow's present value comprises from the financing perspective the sum of the market values of equity and debt as well as the discounted tax payments. From the consumption perspective the separation of the assets requires the insight, that the tax shield from financing is attributable to the equity owners and reduces the present value of the government's tax claim. Hence, the tax shield adds to the value of the unlevered firm and must be subtracted from the taxes' present value in an unlevered firm in order to arrive at the respective present values for the levered company.

[12] $CFBT$ grows with the rate g since it is proportional to FCF. FTG grows with the rate g since FTE, FTD, and $CFBT$ grow with the rate g and $FTE + FTD + FTG = CFBT$, where FTD is the flow to debt holders (see (18) below).

The separation of the three present value components is related to a corresponding treatment of the required returns and cash flow measures.[13] Per definition the following relation must hold for the generation and distribution of cash flows:[14]

$$\begin{aligned} CFBT &= \underbrace{(k-g)\cdot V^U + \left(k^{TS}-g\right)\cdot TS - \left(k^{TS}-g\right)\cdot TS + \left(k^{GU}-g\right)\cdot G^U}_{\text{cash flow generation from assets}} \\ &= \underbrace{\left(k^E-g\right)\cdot E + (r-g)\cdot D + \left(k^G-g\right)\cdot G}_{\text{cash flow distribution to claimants}} \qquad (18) \\ &= FCF + FTG^U = FTE + FTD + FTG \end{aligned}$$

where FTD is the flow to debt holders (interest and principal payments).

Correspondingly, the total weighted required rate of return (k^C) can be derived by dividing (18) by C:

$$k^C = k\cdot\frac{V^U}{C} + k^{GU}\cdot\frac{G^U}{C} = k^E\cdot\frac{E}{C} + r\cdot\frac{D}{C} + k^G\cdot\frac{G}{C} \qquad (19)$$

This framework enables us in the subsequent sections to derive the value of the government's tax claim and to analyze what risk positions and risk sharing exists between the government and the equity owners. Moreover, it will be shown that multiple conflicts of interest exist between the government and the equity owners.

3 The Firm's Before-Tax Cost of Capital

The firm's before-tax cost of capital (k^C) is defined as the appropriate discount factor for the firm's before-tax cash flow stream in order to arrive at the gross value of the firm (see 15). However, from identity (19) it is clear that the firm's before-tax cost of capital cannot be directly inferred from the known identities and text-book-formulas since the government's cost of capital is not known, either. Consequently, one of the two unknowns has to be economically derived in order to be able to determine the other. But even though the firm's before-tax cost of capital cannot be directly inferred, one important property is known ex-ante: k^C has to be independent of the firm's financial policy and capital structure, since the value of the before-tax cash flow must be independent of its distribution to the various claim holders.[15]

[13] See Galai (1998).

[14] See Inselbag and Kaufhold (1997).

[15] Otherwise arbitrage opportunities would exist. This proposition is consistent with the Modigliani/Miller Proposition I without taxes, which states that the value of a firm with fixed investments is independent of the distribution of the firm's cash flows (see also Galai (1998)).

The firm's before-tax cost of capital can be derived by considering an unlevered firm. An unlevered firm makes no principal payments, which implies that taxes paid by the unlevered company (FTG^U) are proportional to FCF:

$$FTG_t^U = \tau \cdot FCF_t / \left((1-\tau) \cdot (1-b)\right)$$

Thus, the tax claim and the equity claim have the same risk in the unlevered firm and it follows:[16]

$$k^{GU} = k \tag{20}$$

The argument of proportional cash flows can be similarly carried over to the before-tax cash flow, since $CFBT_t = (1 - b \cdot (1-\tau)) \cdot FCF_t / \left((1-\tau) \cdot (1-b)\right)$. This is consistent with the formal derivation of the firm's before-tax cost of capital by substituting (20) into (19):

$$k^C = k \tag{21}$$

Hence, since FTG^U, FCF, and CFBT are proportional to each other, they all share the same risk and have to be discounted with the unlevered cost of equity.

It follows immediately:

$$C = \frac{X \cdot (1 - b \cdot (1-\tau))}{k - g} \tag{22}$$

as well as

$$G^U = \frac{X \cdot \tau}{k - g} \tag{23}$$

Because the value of the before-tax cash flow must be independent of the financial policy, equations (21) and (22) hold for every debt ratio and every financial policy. This result is central and will be combined in the following analysis with the known identities and textbook formulas in order to derive the cost of capital and valuation representations for the government's claim in a firm.

It has to be noted that (21) and (22) and therefore the following results differ in important ways from former conclusions drawn in the one-period-framework (see Galai (1998) and Rao and Stevens (2006)). This is due to the fact that taxes paid by the unlevered firm are not proportional to FCF in the one-period-framework, since net investment is not proportional

[16] Fernandez reaches the same conclusion (see equation (10) in Fernandez (2004), p. 148). However, his further analysis of a levered company is flawed, since he does not recognize the principal payment's role. He arrives for the non-growing perpetuity case at his equation (12) by assuming that, independent from the financial policy, no principal payments have to be made. But this is true only for a fixed debt policy (see also Cooper and Nyborg (2006), p. 220).

to EBIT.[17] In this case, it can be shown for an unlevered firm that the government's cost of capital is typically greater than the equity cost of capital. This implies the before-tax cost of capital to be greater than the unlevered cost of equity.[18] Thus, some conclusions drawn from earlier research cannot be carried over to the arguably more interesting case of a long-term investment. Moreover, the analysis of the growing perpetual case additionally includes the possibility to investigate the effects of the firm's financial policy and growth on the government's claim.

4 The Government's Cost of Capital and the Value of Taxes

It has been established that the government's cost of capital in the unlevered company (k^{GU}) equals the unlevered cost of equity (k). In the next step, the government's cost of capital (k^G) can be generally derived from equation (19), since the firm's before-tax cost of capital (k^C) is known. Independent of the firm's financial policy, it follows from substituting (21) into (19) after rearranging terms:

$$k^G = k + (k - k^E) \cdot \frac{E}{G} + (k - r) \cdot \frac{D}{G} \tag{24}$$

Since equation (24) generally holds, the corresponding representations for the two financial policies can be derived by substituting the respective text-book-formula for k^E into the general formula.

In the case of a *fixed debt policy*, substituting (6) into (24) yields for k_d^G:

$$k_d^G = k + (k - r) \cdot \tau \cdot \left(\frac{r}{r - g} \right) \cdot \frac{D}{G_d} \tag{25}$$

If the firm follows instead of a fixed debt policy a *constant leverage policy*, the government's cost of capital changes, since the firm's tax shield becomes risky. This has important implications for the government's claim because the value of taxes naturally depends on the

[17] Net investment is not proportional to EBIT in the one-period-framework since no cash is spent to finance the capital equipment that wears out. Consequently, net investments solely consists of depreciation, which is assumed to be riskless.

[18] Galai (1988) and Galai (1998) show this under special assumptions. Specifically, Galai assumes that gains and losses from the liquidation proceeds are not taxed. This realistic extension can be found in Rao and Stevens (2006). However, they make no statement regarding the relationship between the government's cost of capital and the equity cost of capital. But given a negative correlation of the before-tax cash flow with the stochastic discount factor, it can be shown that the government's cost of capital is greater than the equity cost of capital, even in an unlevered firm.

value of tax shields. The government's cost of capital in a firm with a constant leverage policy can be derived by substituting (9) into (24):

$$k_r^G = k + (k - r) \cdot \left(\frac{r}{1+r}\right) \cdot \tau \cdot \frac{D}{G_r} \tag{26}$$

An obvious outcome of this analysis is the increasing risk of the government's claim when leverage increases. This follows from the fact that the government is a residual claim holder, no matter which financial policy the firm pursues. Very interesting is the effect of growth on the government's cost of capital. While the growth rate plays an explicit role for the fixed debt policy (see (25)), the growth rate does not show up in (26). This stems from the fact, that the cost of equity in a firm with a constant leverage policy is independent of g.[19] But this does not necessarily imply that the government's cost of capital is independent of g, too. The government's cost of capital in a firm with a constant leverage policy would be independent of g if D/G_r is independent of g. But as will be argued in section 5, D/G_r and therefore (26) are dependent on g. Thus, the government's cost of capital is -in contrast to cost of equity- for both financial policies a function of the growth rate.[20]

Now, the formulas for the government's cost capital can be used to calculate the present value of taxes paid by the firm. The value of the tax claim equals the discounted Flow-to-Government:

$$G = \frac{FTG}{k^G - g} = \frac{(X - r \cdot D) \cdot \tau}{k^G - g} \tag{27}$$

While (27) generally holds, the value of the government's tax claim, again, depends on the firm's financial policy. In case of a *fixed debt policy*, substituting (25) into (27) yields after rearrangements:

$$G_d = \frac{X \cdot \tau}{k - g} - \frac{r \cdot \tau \cdot D}{r - g} \tag{28}$$

Note that (28) is equivalent to the identity that the value of the tax claim equals the difference between the value of the tax claim in the unlevered firm and the value of tax shields ($G = G^U - TS$). A closer inspection of (28) shows that the marginal effect of growth on the value of the tax claim is ambiguous. Although higher growth leads to a higher value of the tax claim in the unlevered firm, it also leads to higher tax-shields. At some point the negative effect of growing tax shields outweighs the positive effect at the margin and increasing growth lowers the value of the tax claim.

[19] It is assumed that the risk class of the company is independent of growth.

[20] Also note that in the unlevered firm for both financial policies $k^G = k$, which is consistent with the conclusions drawn in the last section from the analysis of the government's claim in the unlevered firm (see (20)).

The calculation of the tax value for a *constant leverage policy* is less obvious. This stems from the fact that the level of debt and thus the Flow-to-Government are unknown ex-ante. The critical step is to use the relationship $V_r^L = (1 - b \cdot (1 - \tau)) \cdot X/(k - g) - G_r$ in order to eliminate D and V_r^L from the (27) (note that $L^* \cdot V^L = D$). Taking these relations into account and substituting (26) into (27) yields after solving for G_r:

$$G_r = \frac{X \cdot \tau}{k - g} \cdot \frac{(k - g) \cdot (1 + r) - r \cdot L^* \cdot (1 + k) \cdot (1 - b \cdot (1 - \tau))}{(k - g) \cdot (1 + r) - r \cdot L^* \cdot (1 + k) \cdot \tau} \tag{29}$$

The value of the tax claim equals the value of the tax claim in the unlevered company times a scalar factor. The marginal effect of leverage on the scalar factor is negative[21], which naturally follows from the fact that increasing leverage leads to higher tax shields and therefore to lower tax payments.[22] In addition, the effect of growth on the value of government's claim is, while holding b fixed, ambiguous. Growing cash flows lead ceteris paribus to growing taxes (FTG). However, higher growth implies higher after tax value of the firm (V_r^L), resulting in a higher level of debt since the leverage ratio is held constant. This, in turn, leads to higher tax shields, which lowers the value of the tax claim.[23]

An interesting consequence of knowing how to calculate the value of the government's tax claim is the ability to derive alternative methods of calculating the after-tax value of the company (V^L) and the value of tax shields (TS).

By using the identity that the after-tax value of the firm equals the difference between the gross value of the firm and the value of taxes paid by the firm ($V^L = C - G$), one obtains for a *fixed debt policy*:

$$V_d^L = \frac{X \cdot (1 - b \cdot (1 - \tau))}{k - g} - \left(\frac{X \cdot \tau}{k - g} - \frac{r \cdot \tau \cdot D}{r - g} \right) \tag{30}$$

Obviously, combining terms leads to the corresponding APV-formula. Hence, it is straightforward to interpret the APV-formula as a representation of the difference between the gross value of the firm and the value of the government's tax claim.

In the case of a *constant leverage policy*, one obtains:

$$V_r^L = \frac{X \cdot (1 - b \cdot (1 - \tau))}{k - g} - \frac{X \cdot \tau}{k - g} \cdot \frac{(k - g) \cdot (1 + r) - r \cdot L^* \cdot (1 + k) \cdot (1 - b \cdot (1 - \tau))}{(k - g) \cdot (1 + r) - r \cdot L^* \cdot \tau \cdot (1 + k)} \tag{31}$$

[21] This is true as long as $0 < \tau < 1$ and $b \neq 1$.

[22] Note also that for an unlevered firm ($L^* = 0$), (29) leads to $G_r = X \cdot \tau / (k - g) = G^U$, which corresponds to the fact that G^U is defined as the value of the government's claim in an unlevered firm.

[23] For a discussion of this issue with respect to its consequence for the risk sharing between government and equity holders, see section 5.

By rearranging and combining terms it can be shown that (31) is consistent with the corresponding Miles-Ezzell valuation formula (equation (3) in conjunction with equation (10)).

Next, we use the above results to derive the value of tax shields in a firm that follows a constant leverage policy. Following our prior argumentation, and consistent with Fernandez (2004) and Cooper and Nyborg (2006), we calculate the value of tax shields as the difference between the value of taxes for the unlevered company and the value of taxes for the levered company ($TS_r = G^U - G_r$).

Taking the difference between (23) and (29) yields:

$$TS_r = G^U - G_r = \frac{X \cdot \tau}{k-g} \cdot \frac{r \cdot L^* \cdot (1+k) \cdot (1-\tau) \cdot (1-b)}{(k-g) \cdot (1+r) - r \cdot L^* \cdot \tau \cdot (1+k)} \qquad (32)$$

(32) is especially useful if the leverage ratio instead of the level of debt is known. In that case, the APV-Method ($V_r^L = V^U + TS_r$) and the WACC-Method ($V_r^L = FCF/(WACC - g)$) are equally applicable.[24] Again, it can be easily verified that (32) is consistent with the corresponding text-book formula (11).

Taken together, we derived the government's cost of capital and the value of the government's tax claim in the growing perpetuity case for the first time. We then used this result in order to calculate the after-tax value of the firm and the value of tax shields. Of course, these alternative valuation methods are consistent with the respective text-book formulas. However, the pedagogical advantage of this method is that its derivation and representation is based on the extension of the well understood Modigliani-Miller Proposition I to the tax case by incorporating the government as an additional claimholder. In this extension the gross value of the firm is independent of its capital structure and adds up as the sum of the values of the three claim holders.

5 Risk Sharing between Government and Stockholders

From the above analysis it is clear that the risk of the government and the equity owners increases with increasing leverage since both parties hold a residual claim. But even though the formulas for the government's cost of capital are already derived for the fixed debt and constant leverage financial policy, it is still unclear how much risk the government has to take relative to the stockholders and how this relative risk position depends on the firm's financial strategy and growth.

Some work has been done in the literature on this issue. Fernandez (2004) claims that the government's cost of capital in non-growing firms equals the equity cost of capital. We

[24] Actually, substituting (32) into the APV-formula directly leads to the WACC^{ME}-formula.

show that this is true only for a fixed debt policy. This has also been recognized by several other authors, e.g. Cooper and Nyborg (2006) and Fieten, Kruschwitz, Laitenberger, Löffler, Tham, Velez-Pareja, and Wonder (2005). Fieten, Kruschwitz, Laitenberger, Löffler, Tham, Velez-Pareja, and Wonder (2005) argue, and we show below, that the government's cost of capital in a non-growing firm that follows a constant leverage policy is smaller than the equity cost of capital as long as $k > r_f$, where r_f is the riskless rate of return While these findings pertain to the no-growth case, we furthermore extend the analysis to growing firms and show that the former mentioned results cannot be carried over to the growth case.

In order to simplify the discussion below, we assume that $k > r$.[25] In addition, since we already know from (20) that in an unlevered company $k^{GU} = k^E$, we restrict the following analysis to levered companies.

A simple trading strategy provides the basis for the analysis of the government's position in a firm. The trading strategy rests on the idea to buy a fraction of the government's claim on the firm's future taxes and finance this transaction by selling shares of the firm. Specifically, this trading strategy is designed in a way that the cash flows of both positions cancel out with the exception of the principal payments and net investment. This is easily done, since the equity cash flow (FTE) and the taxes paid by the company (FTG) are proportional in each period with the exception of principal payments and net investment.[26]

Transaction	Payment in t=0	Future payment in period t
Buy a fraction α of the government's tax claim	$-\alpha \cdot G$	$\alpha \cdot (X_t - r \cdot D_{t-1}) \cdot \tau$
Sell a fraction $\alpha \cdot \frac{\tau}{1-\tau}$ of the shares of the firm	$\alpha \cdot \frac{\tau}{1-\tau} \cdot E$	$-\alpha \cdot \frac{\tau}{1-\tau} \cdot (X_t - r \cdot D_{t-1}) \cdot (1-\tau) +$ $\alpha \cdot \frac{\tau}{1-\tau} \cdot (PP_t + NI_t)$
Total	$\alpha \cdot \frac{\tau}{1-\tau} \cdot E - \alpha \cdot G$	$\alpha \cdot \frac{\tau}{1-\tau} \cdot (PP_t + NI_t)$

Table 1: Basic Trading Strategy

We know from the trading strategy that the following relation must hold:

$$\alpha \cdot G_d - \alpha \cdot \frac{\tau}{1-\tau} \cdot E = \alpha \cdot \frac{\tau}{1-\tau} \cdot \left(PV(PP) + PV(NI)\right), \tag{33}$$

where PV(·) denotes the present value operator. While the present value of net investment is independent from the chosen financial strategy, the present value of principal payments critically hinges on the financial policy. Thus, even before we get into a deeper analysis

[25] If $r > k$, the relations derived below between the stockholders's and government's cost of capital are reversed. If $r = k$, there is no priced risk and it follows directly: $k = k^C = k^E = k^G = r = r_f$.

[26] Note that the principal payments and net investment in a given period directly affect the equity owner's cash flow, while the government's contemporaneous cash flow is independent from these cash outlays.

of the trading strategy, it is clear that the relative risk position of the government differs between the fixed debt and constant leverage policy.

The first part of analyzing the trading strategy, the valuation of net investment, is quickly done. Net investment is independent of the financial policy and proportional to EBIT. Hence, it has to be discounted with the business risk rate k.

While net investment is identical for both financial strategies, the characteristics of the principal payments depend on the financial policy. In general, the principal payments correspond to the adjustments of the debt level as follows:

$$D_t - D_{t-1} = -PP_t \tag{34}$$

Positive principal payments reduce the debt level, while negative principal payments relate to new debt issues and raise the debt level. Since $E_{t-1}\left[D_t\right] = (1+g)\cdot D_{t-1}$:

$$E_{t-1}\left[PP_t\right] = -g \cdot D_{t-1}$$

Note that for a given level of debt, the *expected* principal payments are the same for both financial policies. However, the difference between the two financial policies lies in the risk of the principal payments.

This can most easily be illustrated for a default-free firm: if the firm follows a fixed debt policy, the debt level grows with certainty ($D_t = (1+g)\cdot D_{t-1}$). Consequently, the principal payments generally bear no risk and are negative (zero) each period if $g > 0$ ($g = 0$). On the other hand, if the firm follows a constant leverage policy, the principal payments follow the stochastic progression of the firm value and are therefore risky. Thus, the value of the trading strategy critically hinges on the firm's financial policy.

First, we analyze the government's relative risk position in a firm that follows a *fixed debt policy*. Substituting the respective present value formulas for the LHS and the principal payments into (33) yields:

$$\alpha\cdot\frac{(X-r\cdot D)\cdot\tau}{k_d^G-g}-\alpha\cdot\frac{\tau}{1-\tau}\cdot\frac{(X-r\cdot D)\cdot(1-\tau)-PP-NI}{k_d^E-g}=\alpha\cdot\frac{\tau}{1-\tau}\cdot\left(\frac{PP}{r-g}+\frac{NI}{k-g}\right)$$

which can be rearranged to:

$$(X-r\cdot D)\cdot(1-\tau)\cdot\left(\frac{1}{k_d^G-g}-\frac{1}{k_d^E-g}\right)=\frac{PP}{r-g}+\frac{NI}{k-g}-\frac{PP+NI}{k_d^E-g} \tag{35}$$

In the special case that $g = 0$, $PP = 0$ as well as $NI = 0$ in all periods and it follows directly:

$$k_d^G(g = 0) = k_d^E(g = 0) \tag{36}$$

This result can be explained by standard economic reasoning: if the firm makes no principal payments, FTG and FTE are proportional and therefore must exhibit the same risk.[27] Consequently, if $g = 0$ and the firm follows a fixed debt policy, the government's cost of capital equals the equity cost of capital, no matter whether the firm is unlevered (see (20)) or levered (see (36)).

If $g > 0$, the RHS of (35) is not necessarily zero as it was for $g = 0$. On the other hand, if the RHS is either always positive or always negative, a general relation between k_d^G and k_d^E could be derived for growing firms. Though not obvious, it can be shown that the RHS of (35) is under the standard assumptions necessarily negative (see Appendix A). Hence, it follows from (35) that:

$$k_d^G(g > 0) > k_d^E(g > 0)$$

Due to the growing debt level, FTG and FTE are not proportional anymore. This drives a wedge between the government's and the equity owner's cost of capital. The government's cost of capital is now greater than the equity cost of capital, since growing debt levels imply growing riskless tax shields.

Now, we analyze the *constant leverage policy*. Again, we need to determine the value of the principal payments in order to analyze the relative risk position of the government. However, it is already clear that, if $g = 0$, the equity cost of capital does not equal the government's cost of capital.[28] This follows from the stochastic process of the principal payments, which ensures that FTG and FTE are not proportional.

Arzac and Glosten (2005) derived the present value of principal payments for the case of a constant leverage policy:

$$PV(PP) = D - \frac{r \cdot D \cdot (1 + k)}{(k - g) \cdot (1 + r)}$$

Substituting the respective present value formulas into (33) yields after rearranging:

$$(X - r \cdot D) \cdot (1 - \tau) \cdot \left(\frac{1}{k_r^G - g} - \frac{1}{k_r^E - g} \right) = D - \frac{r \cdot D \cdot (1 + k)}{(k - g) \cdot (1 + r)} + \frac{NI}{k - g} - \frac{PP + NI}{k_r^E - g} \tag{37}$$

[27] Fernandez reaches the same conclusion (see equation (13) in Fernandez (2004), p. 148).

[28] The exception is that $k = r_f$.

In the special case that $g = 0$, the RHS of (37) is positive, since $r \cdot (1+k) < k \cdot (1+r)$. Thus the LHS has to be positive, which is true if:

$$k_r^G(g=0) < k_r^E(g=0) \tag{38}$$

Note that if $g = 0$, the expected principal payments and net investment are zero in each period. Nevertheless, the value of principal payments is positive, since the principal payments are positively correlated with the pricing kernel.[29] Additionally, taking derivatives of k_r^E and k_r^G with respect to D/E shows that the difference between the government's and the stockholders' cost of capital increases with increasing leverage.

Unfortunately, the relative risk position of the government in growing companies ($g > 0$) is not obvious. If k_r^G is independent of g, we would be done since k_r^E does not depend on g. In that case, (38) would hold independently of g. However, it can be easily verified by differentiating (26) with respect to b (and/or irr) that the marginal effect of growth on the government's cost of capital is typically not zero.[30] Thus, at this point, it is unclear whether (38) holds for arbitrary growth rates.

The difference between the government's and the shareholder's cost of capital can be explicitly derived by substituting $k_r^G = k_r^E - \Delta_{E,G}$ into (37) and solving for $\Delta_{E,G}$.[31] $\Delta_{E,G}$ then gives the difference between the government's and the shareholder's cost of equity. The resulting term is quite lengthy and is therefore omitted here, but it can be shown that the government's cost of capital does *not* have to be smaller than the cost of equity. However, for most parameter constellations, the government's cost of equity *is smaller* than the cost of equity, which corresponds to the no-growth case. But with an increasing internal rate of return, holding b fixed, the government's cost of capital (value of tax claim) approaches infinity (zero). Since the cost of equity is independent of g, the government's cost of capital must exceed the cost of equity in that circumstance from some growth rate on.

Summing up, the government's cost of capital in a levered firm that follows a fixed debt financial policy is *greater* than (equals) the equity cost of capital if $g > 0$ ($g = 0$). On the other hand, if the firm follows a target leverage ratio, the government's cost of capital, with the exception of some extreme cases, is *smaller* than the cost of equity. This implies that

[29] This follows from the assumption that $k > r$.

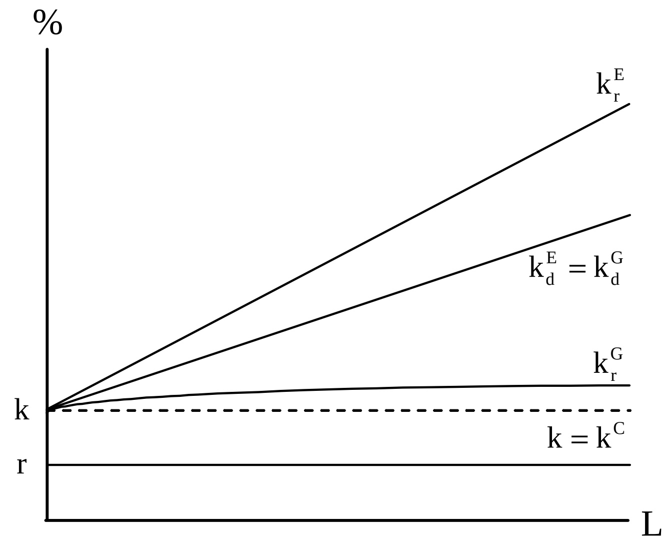
[30] The exact size of the effect is unknown unless a specific functional relationship between irr and b is specified.

[31] In this case, $\Delta_{E,G}$ depends, among other variables, on D and k_r^E. But D and k_r^E in turn depend on further variables ($D = f(L^*, V^L)$, $k_r^E = f(k, r, \tau, L^*)$). In order to state $\Delta_{E,G}$ as a function of variables that are assumed to be known ex-ante, $\Delta_{E,G}$ can be more easily derived by subtracting (26) from (9) and taking $D = L * V^L$ and (29) into account. This approach shows that $\Delta_{E,G} = f(k, r, \tau, L^*, g)$. Thus, $\Delta_{E,G}$ is independent of X if L^* is fixed but still depends on the growth rate.

if the firm switches from a constant debt financial policy to a constant leverage policy, the government's risk typically decreases significantly while the equity cost of capital increases significantly.

Figure 1 graphs the government's and stockholders' cost of capital as a function of the leverage ratio for a non-growing firm without default risk.[32] Note that the government's cost of capital in a firm that follows a constant leverage policy is, as opposed to the other cost of capital functions, a concave function of the leverage ratio.[33]

Figure 1: Cost of capital in a non-growing default free firm



6 Conflict of Interest arising from Capital Budgeting: Corporate Under- and Overinvestmen

This section studies the conflict of interest between stockholders and the government. The stockholders' objective is to maximize the *after-tax* value of the firm (V^L).[34] This implies that stockholders, who are in charge of the investment decision, will undertake all projects that add to the after-tax value of the firm. The government's objective on the other hand should be to maximize total welfare, which would be achieved by maximizing the *before-tax*

[32] If debt is risky, the cost of debt becomes a function of the leverage ratio. This, in turn, would dampen the marginal effect of leverage on the government's and stockholders' cost of capital, since the debt holders take over some of the business risk.

[33] This can be verified by taking the derivative of k_r^G with respect to L.

[34] We assume that potential conflicts of interest between bond- and stockholders are solved by costless negotiation and side-payments.

value of the firm (C). It is clear that these two objectives are not the same and we will show that the optimal investment plans of the government and stockholders usually differ. Galai (1998) analyzes in this respect a one-period framework and finds that stockholders always invest less than socially desirable. In the perpetuity case we also find that corporate underinvestment is the usual case. However, we are also able to show that in some situations rational stockholders invest more than socially desirable.

Before we can begin to analyze corporate under- and overinvestment, some groundwork has to be done with respect to the stockholders' capital budgeting decision within the Gordon-growth framework. Since we assume that net investment is solely financed by retained NOPAT, the stockholders' capital budgeting problem is to find the retention rate (b) that maximizes V^L.[35] In order to find a unique and economically plausible solution within the Gordon-growth framework, the average internal rate of return (irr) has to be modelled as a function of the retention ratio (b).[36] Following economic reasoning it is reasonable to assume diminishing returns in b ($\partial irr/\partial b < 0$).[37] It is crucial to note that the Gordon-growth framework implicitly assumes that this investment opportunity set (IOS) is expected to be invariant over time.[38]

Additionally, since optimal investment is decided at the margin, it is necessary to define the marginal rate of return (irr'). An investment's marginal return is defined in the usual way as the partial derivative of the total dollar return with respect to the quantity of funds invested. The quantity of funds invested at the valuation date (I_t) in turn establishes the current and expected future retention rate ($b = I_t/NOPAT_t$). The investment's marginal rate of return is therefore defined as:

$$irr' = \frac{\partial(I \cdot irr(I/NOPAT))}{\partial I} = irr + I \cdot \frac{\partial irr}{\partial I}$$

Lintner (1964) shows that the marginal rate of return equals the partial derivative of growth with respect to the retention ratio:[39]

$$irr' = \frac{\partial g}{\partial b} = \frac{\partial(b \cdot irr)}{\partial b} = irr + b \cdot \frac{\partial irr}{\partial b}$$

[35] Outside financing can be introduced but does not change the economic implications of the model (see Gordon and Gould (1978)). Additionally, the *actual* retention rate must not be the same in all future periods; only the *expected* retention rate has to be time invariant.

[36] If irr is independent of b, the company should retain all cash flows or should liquidate, depending on wether $irr \gtreqless WACC$. If $irr = WACC$ retention does not influence V^L.

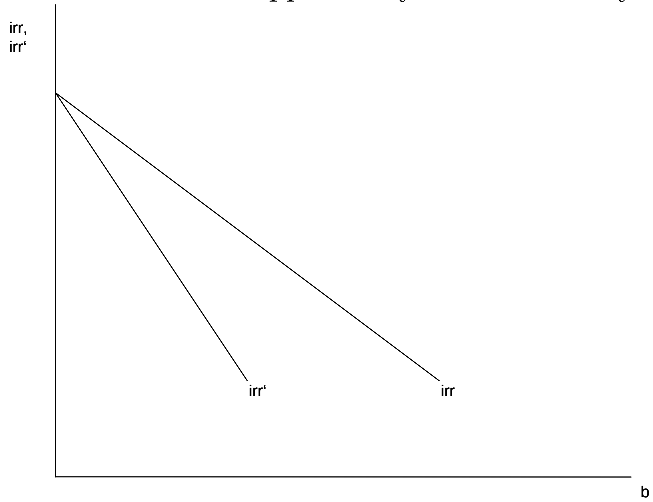
[37] See Williams (1938) and Preinreich (1978).

[38] See Gordon and Shapiro (1956) and Lintner (1963).

[39] See also Bodenhorn (1959) and Elton and Gruber (1976).

Of course, since irr is a decreasing function of b, irr' is a decreasing function of b, too. Moreover, the marginal rate of return is always smaller than the average rate of return ($irr' < irr$). Figure 2 graphs the time invariant IOS, where the average and marginal return are arbitrarily specified as linear functions in b.

Figure 2: Investment Opportunity Set in a Steady State



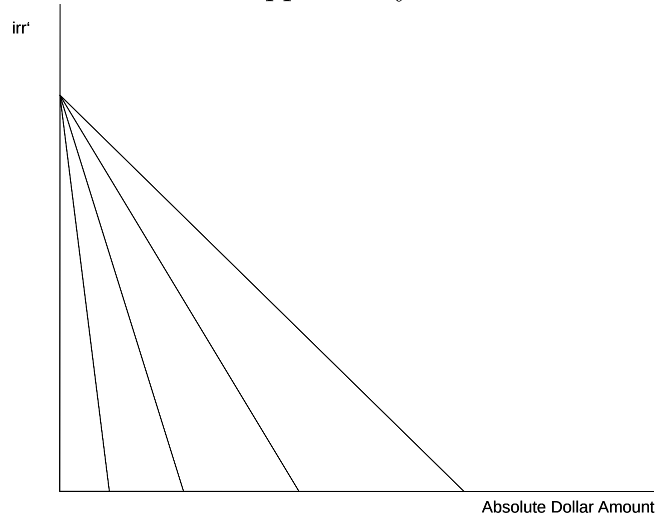
It is essential to note that the definition used here for the *investment's* marginal rate of return is not equal to the *investors'* marginal return (see Lintner (1963)).[40] This is because defining the average rate of return as a time invariant function of b implies that additional investment with positive marginal return increases next period's cash flow, leading to higher dollar amounts of retention that are reinvested at the time invariant *average* rate of return (with $irr > irr'$) and grow at the rate $g = b \cdot irr$. Thus, the *investors'* marginal return definition must include these additional returns generated by future (re)investments.

The pivotal consequence of assuming a time invariant IOS is that the IOS in terms of dollar amounts (IOSD) shifts to the right over time. The shift increases with higher growth and thus with b.[41] Hence, the IOSD is time varying and, more importantly, depends on the stockholders' investment decision at the valuation date. Figure 3 graphs the IOSD for a given b_1. Note that any $b_2 \cdot irr(b_2) > b_1 \cdot irr(b_1)$ would result in an even stronger shift over time to the right.

Due to the specific construction of the IOS in the Gordon-growth framework, it follows that the hurdle rate for the investment's marginal rate of return is typically *not* the weighted

[40] The term "investors" comprises the equity and bond owners.
[41] See Elton and Gruber (1976) and Gordon and Gould (1978).

Figure 3: Investment Opportunity Set in Dollar Amounts



average cost of capital.[42] Put differently: it can pay to undertake investments with $irr' < WACC$ because additional returns can be generated on future investments due to growing retentions that are expected to earn the average rate of return each year. On the other hand, if the IOSD does not depend on the stockholders' investment decision[43], it can be shown that the hurdle rate *is* the weighted average cost of capital.[44]

Unfortunately, the true functional form of the IOSD is unknown and certainly not homogeneous across firms. But since different assumptions on the IOSD yield different capital budgeting solutions, the question remains which specification is appropriate. Assuming that the IOSD is independent of the stockholders decision is problematic. It would imply that today's investment decisions have no impact whatsoever on the company's future investment opportunities. This, of course, contradicts basic economic sense. Thus, some dependency of the IOSD on the stockholders' investment decisions is necessary. Whether the specific dependency implied in the Gordon-growth framework is correct is disputable. We use the Gordon-growth framework because it is analytically tractable and enables us to analyze a setting where future opportunities depend on decisions undertaken in the past.

In this setting a conflict of interest between stockholders and the government arises if the optimal retention rate from the standpoint of the stockholders does not equal the optimal retention rate the government would choose if it were in charge of the investment decision. Economic intuition tells this situation is highly likely since different levels of debt

[42] Note that Vickers (1966) proves that, consistent with traditional finance theory, the hurdle rate for the *investors'* marginal rate of return is the weighted average cost of capital.

[43] Note that this assumption is not consistent with the Gordon-growth framework.

[44] See Elton and Gruber (1976).

and different financial policies most probably lead to different investment decisions by the stockholders whereas the government's optimal retention rate should be independent of the company's financial policy.

We therefore have to analyze for both financial policies which retention rate maximizes V^L (the stockholders' objective) and which retention ratio would maximize C (the government's objective). This can be done by taking the partial derivative of V^L and C with respect to b.

First, we derive a relation that ensures the maximization of the stockholders' wealth. By noting that g is a function of b and $WACC$ might be a function of b, the after-tax value of the firm can be written as:

$$V^L = \frac{X \cdot (1-b) \cdot (1-\tau)}{WACC(b) - g(b)}$$

The partial derivative with respect to b is:

$$\frac{\partial V^L}{\partial b} = \frac{X \cdot (1-\tau)}{(WACC(b) - g(b))^2} \cdot (g(b) - WACC(b) + (1-b) \cdot (\partial WACC(b)/\partial b - \partial g/\partial b)) \tag{39}$$

Setting (39) equal to zero while noting that $\partial g/\partial b = irr'$, we find that V^L is maximized when b is set so that :

$$irr'(b) = \frac{WACC(b) - b \cdot irr(b)}{1-b} + \frac{\partial WACC(b)}{\partial b} \tag{40}$$

The LHS of (40) is the marginal rate of return on investment and is a decreasing function of b. We call the RHS the hurdle rate (HR) function, since as long as the marginal rate of return is greater than the value of the HR-function, additional profitable investments can be undertaken by increasing the retention rate. The optimal retention is reached where the marginal rate of return function intersects with the HR-function.

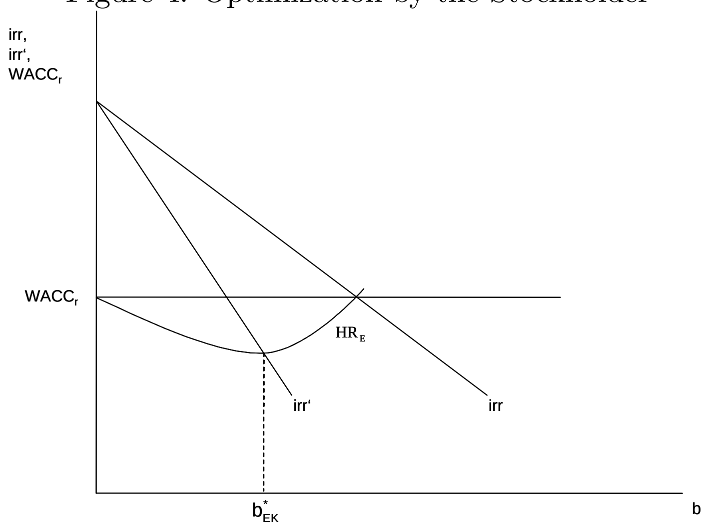
A critical question is how the HR-function depends on b. If $WACC$ is independent of b the HR-function first falls and then rises as b increases. But the $WACC$ might depend in two ways on b. First, the unlevered cost of equity (k) could be a function of growth and therefore of b. Because it is neither theoretically nor empirically clear in which direction the unlevered cost of equity depends on the choice of growth we assume k to be independent of growth.[45] This essentially means that additional investments do not change the risk class of the firm. Second, higher growth implies higher tax-shields and depending on the risk of tax-shields they might change the $WACC$ of the company. We know that growth has no impact on the $WACC$ if the firm follows a constant leverage policy and k is independent of growth (see (10)). On the other hand, the $WACC$ of a firm following a fixed debt policy depends on the growth rate (see (7)).

[45] See Riahi-Belkaoui (2000), p. 41.

Figure 4 graphs the stockholders' optimization problem in a firm that follows a constant leverage policy. This is easily done since $\partial WACC_r/\partial b = 0$. The optimum is reached where[46]

$$irr' = \frac{WACC_r - b \cdot irr}{1-b} \tag{41}$$

Figure 4: Optimization by the Stockholder



At the optimum, it can be easily shown (Lintner (1963), Vickers (1966)) that:[47]

$$irr' < WACC_r \text{ and } irr > WACC_r$$

Unfortunately the HR-function for a firm following a fixed debt policy is less tractable since $WACC$ depends on b. The HR-function is a polynomial of order three in b.[48] The resulting maximization condition is:

$$irr' = \frac{X \cdot (r - b \cdot irr)^2 \cdot (k - b \cdot irr) \cdot (1-\tau)}{A \cdot X + \tau \cdot r \cdot D \cdot (k - b \cdot irr)^2} \tag{42}$$

with

$$A = (\tau - 1) \cdot b^3 \cdot irr^2 + ((1-\tau) \cdot (irr + 2 \cdot r)) \cdot irr \cdot b^2 + ((\tau - 1) \cdot (r + 2 \cdot irr)) \cdot b \cdot r + (1-\tau) \cdot r^2$$

[46] In the following, the notation is simplified by not explicitly stating that irr and irr' are functions of b.

[47] Note that, as argued above, the hurdle rate is smaller than the weighted average cost of capital.

[48] This can be verified by taking the partial derivative of $VF = X \cdot (1-\tau) \cdot (1-b)/(k - g(b)) + \tau \cdot r \cdot D/(r - g(b))$ with respect to b. After solving the resulting first order condition for irr' we get (42).

Comparing the functional form of (41) and (42) it is obvious that the optimal retention rate is not the same for both financial policies, even if the same level of debt (or leverage ratio) is chosen by the stockholders.

Now we derive the equivalent maximization condition for the government. This is done by differentiating C with respect to b and solving the resulting first order condition for $irr\'$. We have

$$C = \frac{X \cdot (1 - b \cdot (1 - \tau))}{k - g(b)}$$

It follows:

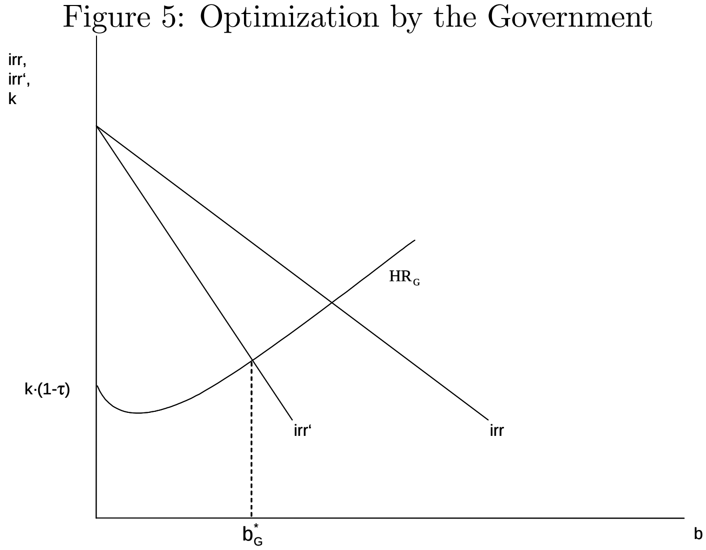
$$\frac{\partial C}{\partial b} = \frac{X}{(k - g(b))^2} \cdot ((1 - \tau) \cdot (g(b) - k) + (1 - b \cdot (1 - \tau)) \cdot \partial g / \partial b) \tag{43}$$

Setting (43) equal to zero and solving for $irr\'$ $(= \partial g / \partial b)$ yields:

$$irr\' = \frac{(1 - \tau) \cdot (k - b \cdot irr)}{1 - b \cdot (1 - \tau)} \tag{44}$$

In the case of no growth ($b = 0$) the value of the government's HR-function is $k \cdot (1 - \tau)$. Note that for any admissible level of debt and for both financial policies this value is smaller than the $WACC$ in the no growth case. This means, with respect to the constant leverage policy, the HR-function of the government starts below the HR-function of the stockholders.[49] Figure 5 graphs the government's optimization problem.

Figure 5: Optimization by the Government



[49]The same does not directly follow for the fixed debt policy because $\partial WACC_d / \partial b \neq 0$ at $b = 0$ (see (40)).

Now we turn to the question whether stockholders invest less than socially desirable. Stockholders underinvest if their optimal retention rate is smaller than the government's optimal retention rate. If the government's HR-function would be below the stockholders' HR-function for all admissible constellations of the valuation relevant parameters, stockholders would even always underinvest.

One obvious case where stockholders underinvest is the case of an unlevered company. This can be seen by comparing the HR-functions of the stockholders and the government:[50]

$$\frac{(1-\tau)\cdot(k-b\cdot irr)}{1-b\cdot(1-\tau)} < \frac{k-b\cdot irr}{1-b} \tag{45}$$

The LHS of (45) is the government's HR-function while the RHS is the stockholders' HR-function for the unlevered company. The statement in (45) is true for all admissible values of k, b, irr, and τ. Hence, we have shown that stockholders always underinvest if the firm is unlevered. We do not dwell on the underinvestment issue for levered companies since not much can be learned by that. It is quite obvious that, for many admissible parameter constellations, stockholders of levered companies invest less than socially desirable.[51]

We have seen so far that a conflict of interest between stockholders and the government regularly arises. This conflict stems from corporate underinvestment, which has also been found in prior research (Galai (1998)). Moreover, the magnitude of underinvestment depends on the financial policy since different financial policies lead to different optimal retention rates chosen by the stockholders. Thus, if the government wants to encourage investment (e.g. by granting investment tax credits) the specific design of these measures should depend on the company's financial policy.

The existence of corporate underinvestment leaves the question whether corporate overinvestment is also possible. Put differently: do admissible constellations of the valuation relevant parameters exist where the stockholders' optimal retention rate is higher than the government's optimal retention rate? In a one-period framework, Galai (1998) shows that overinvestment is impossible. As we will see below, in the growing perpetuity case, the answer to this question depends on the financial policy.

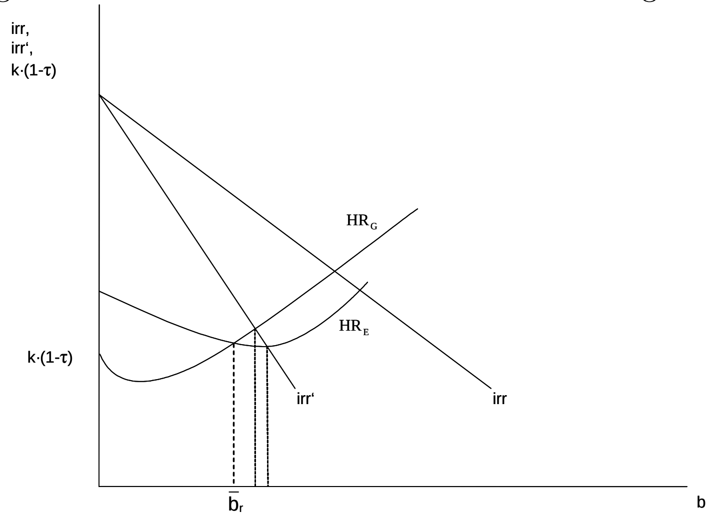
With respect to the two financial policies, this question can be more easily answered for the constant leverage policy. We know that the governments' HR-function starts below the stockholders' HR-Function if the firm follows a constant leverage policy. If it always stays below the stockholders' HR-function in the admissible space of the valuation relevant parameters, then companies always underinvest. Figure 6 gives an example for a firm that

[50] Note that in an unlevered company $WACC = k$.

[51] Further below we even show that stockholders *always* underinvest if the company follows a constant leverage policy.

follows a constant leverage policy and invests more than socially desirable. The question is, whether this hypothetical situation is possible.

Figure 6: Overinvestment in the Constant Leverage Case



It is easy to show that stockholders of firms following a constant leverage policy never decide to invest more than socially desirable. This means, the example given in Figure 5 is impossible. In order to show this, we first identify the retention rate where the two HR-functions intersect ($\bar{b}_r$).

The two HR-functions intersect where

$$\frac{WACC_r - b \cdot irr}{1-b} = \frac{(1-\tau)\cdot(k - b\cdot irr)}{1 - b\cdot(1-\tau)}$$

Solving for b yields:

$$\bar{b}_r = \frac{WACC_r - (1-\tau)\cdot k}{(1-\tau)\cdot(WACC_r - k) + \tau \cdot irr} \tag{46}$$

Substituting $\bar{b}_r$ into the valuation formula for C (15) and V_r^L ((3) in conjunction with (10) and (1)) shows:

$$C(\bar{b}_r) = V_r^L(\bar{b}_r) \tag{47}$$

Thus, $\bar{b}_r$ is the retention rate where the gross value of the firm equals the after-tax value of the firm. Since C and V_r^L both are continuous functions in the admissible parameter space, $C - V_r^L$ is continuous as well. $\bar{b}_r$ is the unique root of $C - V_r^L$ and since for $b = 0$ we know that $C > V_r^L$[52] it follows that for any $b_1 > \bar{b}_r$: $C(b_1) - V_r^L(b_1) < 0$. But by assumption,

[52] This follows from $WACC_r > k\cdot(1-\tau)$.

the gross value of the firm can not be smaller than the after tax value. Otherwise we would have $G_r < 0$, which is the case if $X < r \cdot D$. Since $X < r \cdot D$ does not lie in the admissible parameter space, overinvestment is impossible in the constant leverage case. More general, since $X = r \cdot D$ is not admissible as well, firms that follow a constant leverage policy always underinvest.

In order to show analytically that underinvestment is not necessary in the fixed debt case, we could choose a specific form of the IOS and solve (42) for the optimal retention ratio. Unfortunately, if we choose a simple linear IOS, the resulting first order condition is already a polynomial of order six in b. Thus, in order to avoid unnecessary complexity, we just give a simple example where the stockholders choose an optimal retention rate (b_E^*) that is higher than the government's optimal retention rate (b_G^*).

First, we have to define an explicit form of the IOS. For simplification reasons we assume a linear IOS:[53]

$$irr = a - q \cdot b \tag{48}$$

We demand that $r > a - q$ and $a > 2 \cdot q$. These condition simply imply that the average rate of return falls below the cost of debt with 100% retention while the marginal rate of return is greater zero at that point.[54] The corresponding marginal rate of return function is:

$$irr' = a - 2 \cdot q \cdot b \tag{49}$$

Explicitly modelling the irr'-function enables us to determine the optimal retention rate for the government. Substituting (49) into (44) and solving for b gives the government's optimal retention rate (b_G^*):

$$b_G^* = \frac{1 - \sqrt{1 - \frac{(a - k \cdot (1-\tau)) \cdot (1-\tau)}{q}}}{1 - \tau} \tag{50}$$

Now, after deriving the government's optimal retention rate, we have to choose admissible parameter values that imply corporate overinvestment. We choose: $X = 200$, $\tau = 0.4$, $k = 0.1$, $r = 0.06$, $a = 0.11$, $q = 0.06$, and $D = 500$. Substituting the relevant values into (50) gives the government's optimal retention rate: $b_G^* = 0.6126$. Analog, solving (42) for b gives the stockholders' optimal retention rate: $b_E^* = 0.7165$.[55]

[53] For a similar IOS assumption see Lerner and Carleton (1964).

[54] This ensures that a finite price maximum occurs within the admissible parameter space of b, i.e. within the unit interval.

[55] All other solutions of b lie outside the admissible parameter space (four of them are complex and the fifth is 2.1465).

	$b_G^* = 0.4882$	$b_E^* = 0.7165$
irr	8.07%	6.70%
irr'	5.14%	2.40%
g	3.94%	4.80%
V^U	1013.54	654.39
TS	582.51	1001.06
V_d^L	1596.05	1655.45
C	2333.66	2193.23

Table 2: Overinvestment in case of a fixed debt policy

Table 2 shows that retaining 48.82% of NOPAT maximizes the before-tax value of the firm, yielding a before-tax value of 2333.66 and an after-tax value of 1596.05. By increasing the retention rate to 71.65%, the stockholders decrease the before-tax value to 2193.23, but gain by increasing the after-tax value to 1655.45. Hence, stockholders may invest more than socially desirable since the gain in tax shields (1001.06-582.51) can outweigh the loss in the value of the unlevered company (654.39-1013.54).

Summing up, the main finding of this chapter is that corporate overinvestment is possible and depends on the financial policy. Firms that follow a constant leverage policy never overinvest, but always underinvest. Firms with fixed future levels of debt may overinvest if the gain in tax-shields is big enough to outweigh the loss in V^U.

7 Conclusion

It is shown that treating the government's tax claim explicitly in company valuation provides several advantages over the usual approach of only implicitly correcting the cost of capital or cash flows for the tax burden. First, starting from the Modigliani/Miller framework without taxes, it is clear that after introducing the tax authority the value of the gross cash flow available to all three parties is independent of its distribution among the three claimants. Value additivity is retained. The basic knowledge valid in the no tax case can be transferred to the tax case, since only the distribution but not the creation of value is affected.

Based on arbitrage arguments the cost of capital for the government assuming either a fixed or a proportional debt policy is derived and the present value of the government's claim in the case of a (growing) perpetuity is explicitly calculated. Furthermore, we formulate an alternative approach of firm valuation for both financing policies exhibiting the advantage that all tax effects are explicitly accounted for. This enhances the transparency in the valuation process.

With these new formulas the risk position of the government is analyzed and it is shown that the risk bearing depends on the company's chosen financing policy. In a positive growth (no-growth) setting the government's risk is greater than (equals) the equity cost of capital if the firm follows a fixed debt policy. In turn, if the firm follows a constant leverage policy the government's risk is, with the exception of extreme cases, smaller than the equity cost of capital.

Building on the derivation of the company's before-tax value we show that multiple conflicts of interest exist between the stockholders and the government. Stockholders usually invest less than socially desirable. However, for the first time we show that corporations might even invest more than socially desirable by strongly increasing the value of tax-shields. Moreover, the possibility of corporate overinvestment depends on the financial policy. Firms that follow a constant leverage policy never overinvest, but always underinvest. Firms with fixed future levels of debt may overinvest if the gain in tax-shields is big enough to outweigh the loss in the value of the unlevered company. These conclusions show that the government should take the financial policy of the company into account when encouraging corporate investment.

A Risk Sharing in a Growing Firm with Fixed Debt

Proposition:

If the following relations hold

- $k > r > 0$[56]
- $r > b \cdot irr > 0$[57]
- $irr > WACC_d > r \cdot (1-\tau)$[58]
- $b \in (0, 1)$
- $\tau \in (0, 1)$
- $X > r \cdot D > 0$

and

$$A \equiv \left(\frac{PP}{r-g} + \frac{NI}{k-g} \right) - \left(\frac{PP + NI}{k_d^E - g} \right) \tag{51}$$

then the following relation must hold:[59]

$$A < 0 \tag{52}$$

Proof:

Substituting $PP = -g \cdot D$, $NI = b \cdot X \cdot (1-\tau)$, $k_d^E = k + (k-r) \cdot \left(1 - \frac{r}{r-g} \cdot \tau\right) \cdot D/E$, and $E = X \cdot (1 - b \cdot (1-\tau))/(k - b \cdot irr) + r \cdot \tau \cdot D/(r - b \cdot irr) - D$ (APV-Formula) into (51) yields after simplifications:

$$A \equiv -\frac{b \cdot D \cdot X \cdot (1-\tau) \cdot (irr - r \cdot (1-\tau)) \cdot (k-r)}{(k - b \cdot irr) \cdot (r - b \cdot irr) \cdot [X \cdot (1-b) \cdot (1-\tau) + (b \cdot irr - r \cdot (1-\tau)) \cdot D]} \tag{53}$$

The nominator in (53) is positive. Furthermore, the first and the second bracket in the denominator are positive. Thus, A is negative if and only if the squared bracket in the denominator is positive.

[56] If $k = r$, the firm is either totally debt financed or $k = r = r_f$. We exclude both cases. The first is excluded because the solution is obvious and the second is excluded, since it relates to an all equity financed firm. This case has already been analyzed in section 3.

[57] If $r \leq b \cdot irr$, the value of tax-shields would be infinite.

[58] If $irr \leq WACC_d$, growth would not generate value.

[59] Note that, in contrast to section 5, we do not assume that $r = r_f$. Thus, the proof holds irrelevant of the risk of debt.

Hence, we have to show that

$$X \cdot (1-b) \cdot (1-\tau) + (b \cdot irr - r \cdot (1-\tau)) \cdot D > 0 \tag{54}$$

The first summand in (54) is always positive. Therefore, (54) can only be negative if the bracket term of the second summand is negative. Since D is bounded above by C, we have to show that

$$X \cdot (1-b) \cdot (1-\tau) + (b \cdot irr - r \cdot (1-\tau)) \cdot C > 0 \tag{55}$$

Substituting $C = X \cdot (1 - b \cdot (1-\tau)/(k - b \cdot irr)$ into (55) yields:

$$\frac{X \cdot [b \cdot (\tau \cdot (k + irr - 2 \cdot r) - (k-r) + r \cdot \tau^2) + (k-r) \cdot (1-\tau)]}{(k - b \cdot irr)} > 0 \tag{56}$$

Since X and the denominator are positive we are left to show that the squared bracket in the nominator of (56) is positive. Note that the marginal effect of the internal rate of return on the term in the squared bracket is positive. Hence, by substituting $irr = r \cdot (1-\tau)$ into (56), we get a lower bound for (56) since irr is actually greater than $r \cdot (1-\tau)$. Thus, the proposition is true if:

$$b \cdot (\tau \cdot (k + r \cdot (1-\tau) - 2 \cdot r) - (k-r) + r \cdot \tau^2) + (k-r) \cdot (1-\tau) > 0$$

Collecting terms yields:

$$(1-b) \cdot (1-\tau) \cdot (k-r) > 0 \tag{57}$$

All three factors on the LHS of (57) are by assumption positive. This proves the proposition.

B Example

Table 3: Valuation of a hypothetical firm maintaining a constant leverage ratio

Assumptions:		$(g = 0)$	$(g = 5\%)$
Investment ratio	b	0.00%	52.08%
Internal rate of return	irr	-	9.60%
Risk free rate	r_f	6.00%	6.00%
Unlevered beta	β^U	1	1
Equity premium	p	4.00%	4.00%
Beta of debt	β^D	0.25	0.25
Tax rate	τ	40%	40%
Initial EBIT	X	\$320	\$320
Target leverage ratio	L^*	24.226%	23.498%
Implied Input-Factors:			
Growth	$g = b \cdot irr$	0.00%	5.00%
Leverage ratio	$L = L^*/(1 - L^*)$	31.97%	30.716%
Unl. cost of equity	$k = r_f + p \cdot \beta^U$	10.00%	10.00%
Cost of debt	$r = r_f + p \cdot \beta^D$	7.00%	7.00%
Net investment	$NI = b \cdot X \cdot (1 - \tau)$	\$0	\$100
Initial free cash flow	$FCF = X \cdot (1 - \tau) \cdot (1 - b)$	\$192	\$92
Initial free taxes	$FTG^U = X \cdot \tau$	\$128	\$128
Before-Tax cash flow	$CFBT = X - NI$	\$320	\$220
Valuation and CoC:			
Gross value of firm	$C = CFBT/(k - g)$	\$3200	\$4400
Value of unl. firm	$V^U = FCF/(k - g)$	\$1920	\$1840
Value of unl. taxes	$G^U = FTG^U/(k - g)$	\$1280	\$2560
Value of taxes	$G_r = \frac{X \cdot \tau}{k-g} \cdot \frac{(k-g)\cdot(1+r) - r \cdot L^* \cdot (1+k)(1-b\cdot(1-\tau))}{(k-g)\cdot(1+r) - r\cdot L^* \cdot (1+k)\cdot \tau}$	\$1136.1	\$2272.15
Value of levered firm	$V_r^L = C - G_r$	\$2063.91	\$2127.85
Value of tax-shields	$TS_r = \frac{X \cdot \tau}{k-g} \cdot \frac{r \cdot L^* \cdot (1+k)\cdot(1-\tau)\cdot(1-b)}{(k-g)\cdot(1+r) - r\cdot L^* \cdot \tau \cdot (1+k)}$	\$143.93	\$287.85
Debt value	$D = L^* \cdot V_r^L$	\$500	\$500
Equity value	$E = C - G_r - D$	\$1563.92	\$1627.85
Beta of Government	$\beta_r^G = \beta^U + \left(\beta^U - \beta^D\right) \cdot \left(\frac{r}{1+r}\right) \cdot \tau \cdot \frac{D}{G_r}$	1.0086	1.0043
Beta of Equity	$\beta_r^E = \beta^U + \left(\beta^U - \beta^D\right) \cdot \left(1 - \frac{r}{1+r} \cdot \tau\right) \cdot L$	1.2335	1.2243
Gov. cost of capital	$k_r^G = k + (k - r) \cdot \left(\frac{r}{1+r}\right) \cdot \tau \cdot \frac{D}{G_r}$	10.03%	10.02%
Equity cost of capital	$k_r^E = k + (k - r) \cdot \left(1 - \frac{r}{1+r} \cdot \tau\right) \cdot L$	10.93%	10.90%
WACC	$WACC_r = k - r \cdot \tau \cdot L^* \cdot (1 + k)/(1 + r)$	9.3027%	9.3236%
Crosschecks:			
Initial PP	$PP = -g \cdot D$	\$0	-\$25
Initial Flow-to-Equity	$FTE = (X - r \cdot D) \cdot (1 - \tau) - PP - NI$	\$171	\$96
Initial Flow-to-Gov.	$FTG = (X - r \cdot D) \cdot \tau$	\$114	\$114
Value of taxes	$G_r = G^U - TS$	\$1136.1	\$2272.15
Value of taxes	$G_r = FTG/(k_r^G - g)$	\$1136.1	\$2272.15
Value of tax-shields	$TS_r = r \cdot \tau \cdot D/(k - g) \cdot (1 + k)/(1 + r)$	\$143.93	\$287.85
Value of levered firm	$V_r^L = FCF/(WACC_r - g)$	\$2063.91	\$2127.85
Value of levered firm	$V_r^L = V^U + TS_r$	\$2063.91	\$2127.85
Gov. cost of capital	$k_r^G = r_f + p \cdot \beta_r^G$	10.03%	10.02%
Equity cost of capital	$k_r^E = r_f + p \cdot \beta_r^E$	10.93%	10.90%
Equity value	$E = V_r^L - D$	\$1563.92	\$1627.85
Equity value	$E = FTE/(k_r^E - g)$	\$1563.92	\$1627.85

Table 4: Valuation of a hypothetical firm following a fixed debt policy

Assumptions:		$(g=0)$	$(g=5\%)$
Investment ratio	b	0.00%	52.08%
Internal rate of return	irr	-	9.60%
Risk free rate	r_f	6.00%	6.00%
Unlevered beta	β^U	1	1
Equity premium	p	4.00%	4.00%
Beta of debt	β^D	0.25	0.25
Tax rate	τ	40%	40%
Initial EBIT	X	\$320	\$320
Initial debt level	D	\$500	\$500
Implied Input-Factors:			
Growth	$g = b \cdot irr$	0.00%	5.00%
Unl. cost of equity	$k = r_f + p \cdot \beta^U$	10.00%	10.00%
Cost of debt	$r = r_f + p \cdot \beta^D$	7.00%	7.00%
Net investment	$NI = b \cdot X \cdot (1-\tau)$	\$0	\$100
Initial free cash flow	$FCF = X \cdot (1-\tau) \cdot (1-b)$	\$192	\$92
Initial free taxes	$FTG^U = X \cdot \tau$	\$128	\$128
Before-Tax cash flow	$CFBT = X - NI$	\$320	\$220
Valuation and CoC:			
Gross value of firm	$C = CFBT/(k-g)$	\$3200	\$4400
Value of unl. firm	$V^U = FCF/(k-g)$	\$1920	\$1840
Value of unl. taxes	$G^U = FTG^U/(k-g)$	\$1280	\$2560
Value of tax-shields	$TS_d = r \cdot \tau \cdot D/(r-g)$	\$200	\$700
Value of taxes	$G_d = G^U - TS_d$	\$1080	\$1860
Value of levered firm	$V_d^L = C - G_d$	\$2120	\$2540
Equity value	$E = C - G_r - D$	\$1620	\$2040
Leverage ratio	$L = D/E$	30.86%	24.51%
Beta of government	$\beta_d^G = \beta^U + \left(\beta^U - \beta^D\right) \cdot \tau \cdot \left(\frac{r}{r-g}\right) \cdot \frac{D}{G_d}$	1.1389	1.2823
Beta of equity	$\beta_d^{EK} = \beta^U + \left(\beta^U - \beta^D\right) \cdot \left(1 - \left(\frac{r}{r-g}\right) \cdot \tau\right) \cdot L$	1.1389	0.9265
Gov. cost of capital	$k_d^G = k + (k-r) \cdot \tau \cdot \left(\frac{r}{r-g}\right) \cdot \frac{D}{G_d}$	10.56%	11.13%
Equity cost of capital	$k_d^E = k + (k-r) \cdot \left(1 - \frac{r}{r-g} \cdot \tau\right) \cdot L$	10.56%	9.71%
WACC	$WACC_d = k \cdot \left[1 + \left(\frac{g}{k} - 1\right) \cdot \left(\tau \cdot \frac{D}{V^L} \cdot \frac{r}{r-g}\right)\right]$	9.0566%	8.622%
Crosschecks:			
Initial PP	$PP = -g \cdot D$	\$0	-\$25
Initial Flow-to-Equity	$FTE = (X - r \cdot D) \cdot (1-\tau) - PP - NI$	\$171	\$96
Initial Flow-to-Gov.	$FTG = (X - r \cdot D) \cdot \tau$	\$114	\$114
Value of taxes	$G_d = FTG/(k_d^G - g)$	\$1080	\$1860
Value of levered firm	$V_d^L = FCF/(WACC_d - g)$	\$2120	\$2540
Value of levered firm	$V_d^L = V^U + TS_d$	\$2120	\$2540
Gov. cost of capital	$k_d^G = r_f + p \cdot \beta_d^G$	10.56%	11.13%
Equity cost of capital	$k_d^E = r_f + p \cdot \beta_d^E$	10.56%	9.71%
Equity Value	$E = V_d^L - D$	\$1620	\$2040
Equity Value	$E = FTE/(k_d^E - g)$	\$1620	\$2040

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