Strategic Role of Debt in Duopoly with Asymmetric Information

Baran Siyahhan *

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Abstract

This paper investigates the strategic role of debt in a duopoly in which firms eventually have to leave the market. The market is characterized by asymmetric information. Comparing the benchmark model without debt to the one with debt, we argue that debt can be used as a signal to induce the competitor to leave the market earlier than it otherwise would. We specify the conditions under which debt has a signaling value and the debt contract that achieves it. We find that debt has signaling value when (lack of) exercise of options does not convey any information.

1 Introduction

Product markets are often characterized by competition and strategic interactions. Although it may be useful to analyze the firms in isolation, ignoring the market structure could fundamentally affect the analysis. Consider, for instance, a firm that contemplates an investment. Real options analysis suggests that the firm should not simply consider the net present value (NPV) of the investment but also take into account the opportunity cost of immediate investment. By delaying the investment, for instance, the firm could benefit from resolving uncertainty concerning the parameters of the investment (see Dixit and Pindyck, 1994). Real options approach, therefore, suggests that it may be worthwhile to invest at a later time than the conventional NPV rule would advise. The presence of competition, however, could potentially change the above analysis. For instance, given that there are preemptive benefits, the firm may not find it optimal to delay investment as much as the real options analysis would advise. Just as decisions to invest, exit decisions from the industry are prone to strategic interactions. The fundamental decision in this case could be when to reduce capacity or

*Vienna Graduate School of Finance, Heligenstädter Strasse 46-48, Room DG1.22/Floor 4, 1190, Vienna, Austria

leave the market altogether. By driving the competitor earlier out of the market, the remaining firm may be entitled to monopoly benefits. In short, analysis of monopolistic or perfectly competitive markets is often insufficient to understand the oligopolistic markets.

A second dimension that may affect competition is the extent of information firms have about each other. As Lambrecht and Perraudin (2003) state, it is usually unrealistic to assume that firms have full information about their competitors. The lack of information about the competitor is particularly important in oligopolistic markets because a particular firm's decision is a function of the rival firm's actions in the market. These actions are likely to be a noisy signal of the competitor's private information.

This article develops a model in which financial structure can be used strategically to resolve asymmetric information in a duopoly. The model considers two firms with an option to leave the market. The remaining firm in the industry becomes the monopolist until its cost becomes sufficiently high to justify exit. We assume that there are benefits to becoming a monopolist. Therefore, firms would be better off if they could induce the exit of their competitor. The exit decision in the model is motivated by the fixed costs from operations. Firms with higher fixed costs leave the market earlier, *ceteris paribus*. Hence, a lower fixed cost in our model implies a competitive advantage over the competitor. The model assumes that one of the firms has private information about its fixed cost. Given this information structure, we first determine the Bayesian-Nash equilibrium for unlevered firms. The analysis of the unlevered firms allows us to identify under what conditions the capital structure might be useful to resolve the information asymmetry. Based on the results of this section, we then extend the model to explore the signaling role of debt. We characterize the optimal debt contract and the perfect Bayesian equilibrium.

The main features of and the intuition behind the model are as follow. First, we find that not only the industry structure but also the firm characteristics play a role in whether firms use capital structure as a strategic tool. In particular, the relative strengths of the firms determine the competitive environment and are a crucial factor in determining which firm leaves the market first. We find that when one of the firms has a significant competitive advantage over the competitor, there is no signaling role for debt. Second, depending on the relative strengths of the firms, the uninformed firm can resolve the information asymmetry by simply observing the actions of the competitor. In other words, the exercise (or the lack thereof) of the option can fully reveal information. In this case as well, the informed firm has no incentive to design its capital structure to reveal its private information.

Such a policy, on the contrary, makes the informed firm strictly worse off by inducing early exit. Is there then really a signaling value for debt? Since the use of debt induces earlier exit, it may seem counterintuitive that the informed firm levers itself up and thereby shortens its stay in the market[1]. However, we argue that the informed low-cost firm would still be willing to reveal information by issuing debt under certain circumstances in which the uninformed firm cannot otherwise learn. When issuing debt, the low-cost firm trades off the cost of debt against the benefit of debt. The benefit of debt in our model entails becoming a monopolist at an earlier date. By designing a contract that separates itself from other types, the signaling firm induces the uninformed firm to exit earlier and thus becomes the monopolist. On the other hand, once the signaling firm has become a monopolist, it enjoys a shorter monopoly tenure than it otherwise would as an unlevered firm. This cost makes the signaling credible. The debt contract that fully reveals private information is also socially optimal. That is, given that the uninformed firm faces a low-cost competitor, it is also in the best interest of the uninformed firm to exit earlier.

Our paper fits naturally into the game theoretic real options literature. Although this strand of literature focuses relatively more on investment decisions, there are papers that model exit decisions. Ghemawat and Nalebuff (1985) develop the equilibrium concept in a Cournot setting in which firms have different market shares and show that the firm with longer potential monopoly tenure outlasts its rival(s). The equilibrium concept of this paper closely resembles Ghemawat and Nalebuff's idea. Closely related to our paper is the study of Fudenberg and Tirole (1986). As in this article, they model exit decisions with incomplete information. However, they do not consider signaling possibilities to resolve the asymmetric information. A model that does consider the effect of capital structure choice on entry and exit decisions is that of Lambrecht (2001). He explores the strategic impact of debt in a duopoly and shows that debt renegotiation can provide competitive advantage. The main difference between his model and ours is that Lambrecht analyzes a setting with complete information. More recently, Murto (2004) and Miltersen and Schwartz (2007) develop models of exit decisions. While the former incorporates a richer set of exit strategies, the latter analyzes not only exit decisions but also switching options. In this paper, we consider simple strategies and instead focus more on the use of debt as a strategic tool. Although models of investment decisions, our paper also relates to those of Grenadier (1999) and Lambrecht and Perraudin (2003). Both papers deal with incomplete information and demonstrate that the state variable that is the ultimate source of uncertainty in the market

[1]Of course, this argument holds if one abstracts from other benefits of debt such as tax shields and mitigation of the free cash problem as discussed by Jensen (1986).

reveals information on which firms can base their strategies. We also recognize, in this paper, that the state variable can reveal information but argue that under certain circumstances debt can still be used as a signal.

Another strand of literature considers signaling through capital structure. By introducing a richer set of financing choices, Brennan and Kraus (1987) show that the firm can signal costlessly its quality to avoid the underinvestment problem created by information asymmetry. Gertner *et al.* (1988) develop a two-audience signaling model in which the choice of capital structure is a signal to both the capital market as well as the product market. They argue the type of equilibrium (i.e. pooling or separating) depends on the particular strategic interaction in the market. In Poitevin (1989), an entrant must signal its quality by issuing debt. However, signaling with debt induces the competitor to engage in predatory actions. The paper shows that high incentives to prey and the low belief that a potential entrant is a high quality firm act as barriers to entry. The intuition behind our model can also be contrasted with the signal-jamming studies. In these papers, firms are willing to distort information to investors (Kanatas and Qi, 2001) or to potential entrants (Fudenberg and Tirole, 1986). In our model, on the other hand, the informed firm is willing to reveal information to reap off monopoly benefits earlier.

The rest of the paper is organized as follows. Section 2 outlines the assumptions and the main features of the model. In Section 3, we focus on the case in which firms are unlevered. Section 4 introduces debt and specifies the the debt contract that credibly signals firm type. Finally, we conclude in Section 5.

2 Assumptions of the Model

Consider a duopolistic industry. We assume that the industry is a declining one in the sense that both firms have no growth options such as capacity expansion and/or new investment opportunities. Upon exit, stockholders are entitled to the scrapping value of the firm assets. The model assumes that the firms employ similar technologies and therefore, the salvage value of assets is fixed at S. The opportunity cost of the assets designate how valuable the assets of the firm are to the shareholders, should they employ these in an alternative use. This redeployment value (henceforth, the salvage value) accrues exclusively to the shareholders of the firm when the firm is unlevered. Stockholders choose the optimal time to leave the market so as to maximize their value. We assume that this is the only option they have. If one of the firms decides to exit, the remaining firm operates as a monopolist until its own optimal exit threshold.

Each firm generates a multiplicative net operating cash flow of $\pi x_t - f_i$ in duopoly and $\Pi x_t - f_i$ in monopoly, $i = 1, 2$. We assume that $\pi < \Pi$ are both constants and the same across firms. The difference $\Pi - \pi$ measures the incremental benefit from becoming a monopolist. Economically, π and Π can be perceived as measures of firm efficiency in the market. Although it would be interesting to model firm efficiency more explicitly by introducing firm-specific parameters, we refrain from doing so in order to simplify the analysis. The shock process x_t is the ultimate source of uncertainty in the model. One can think of this state variable as shocks to industry demand. The shock process is assumed to follow a geometric Brownian motion:

$$dx_t = \mu x_t dt + \sigma x_t dw_t \tag{1}$$

where μ, σ are constant drift and volatility terms and dw_t is the increments of a standard Wiener process. We assume that the drift term, μ, is strictly less than the riskless rate, r, to guarantee finite firm values.

Returning to the information structure, we assume that the information asymmetry relates to the fixed costs. The value of the assets derives from the current operations and firms do not own any other assets. To simplify the subsequent analysis, information asymmetry is not taken as reciprocal. That is, we assume, without loss of generality, that the fixed cost of firm 1, f_1, is common knowledge. On the other hand, only the distribution of firm 2 fixed cost is known at the outset of the game. This could be, for instance, due to the fact that firm 2 is comparatively new in the industry. The fixed cost of firm 2 can be either low or high[2]:

$$\tilde{f}_2 = \begin{cases} f_L, & with \quad 0.5 \\ f_H, & with \quad 0.5 \end{cases}$$

Murto (2004) develops a theory of exit in an oligopoly under the assumption that the strategy space need not be connected sets as in Lambrecht (2001). In such a setting, firms can also exit the market with an *upward* movement of the state variable. In this paper, we restrict our attention to strategies in which exit is triggered by a single threshold. This allows us to keep the model as simple as possible since, as Murto (2004) demonstrates, relaxing the connectedness assumption may under certain conditions result in multiple equilibria.

Using the simple exit strategies, Section 3.1 establishes a one-to-one correspondence between the fixed costs and the exit triggers of the firms. Since firm 1 knows the possible values of fixed costs for firm 2, it also knows the

[2]This distribution is assumed for illustrative purposes. The basic intuition behind the model remains unchanged if another distribution is assumed.

possible exit thresholds of the competitor. Therefore, the model essentially turns into a game in the exit triggers of the firms. Since each firm has the option to leave the industry first, the strategies for each firm consist of deciding whether to exit at the duopoly or at the monopoly threshold. We restrict our attention only to pure strategies.

Finally, we abstract in this paper from the widely argued benefits and costs of debt, namely, the tax advantage and bankruptcy costs. Furthermore, the model does not consider the use of debt to resolve agency problems. This is so since we focus only on the strategic implications of debt in an asymmetric information setting. Before focusing on the role of debt, the next section develops the model for unlevered firms in order to get a sense of when debt can be used as a strategic tool.

3 The Benchmark Model

We start this section by recalling the exit problem of a monopolist. This serves as a useful benchmark and will later be used in the analysis of the duopoly market. We then consider a duopoly in which firms are unlevered. Questions of interest in the analysis are (*i*) *who* leaves the market first (*ii*) *when* each firm optimally exits and (*iii*) *what* the firm values are. We address these questions in the following way. Depending on the competitive environment (i.e. the relative magnitudes of the fixed costs) and taking exit triggers as exogenous, we first determine the firm values based on some conjectured solution. Optimal exit triggers of the firms are then determined. Finally, we explore whether the conjectured solutions indeed constitute an equilibrium and whether there exist other solutions.

3.1 Value of an Unlevered Monopoly

Before considering the duopoly case, we first derive the value of a firm in isolation without debt. Stockholders of the monopoly essentially have to solve an optimal stopping time problem. Continuation in this case means that the firm is active. The value of the unlevered monopoly firm, $V(x)$, with an option to abandon satisfies the following differential equation in the continuation region:[3]

$$
\begin{aligned}
\frac{1}{2}\sigma^2 x^2 V_{xx} + \mu x V_x - rV + \Pi x_t - f = 0 \quad x_t > x^* \\
s.t. \\
x \overset{lim}{\rightarrow} \infty \; V(x) = \frac{\Pi x}{r-\mu} - \frac{f}{r} \\
V(x^*) = S \\
\frac{\partial V(x^*)}{\partial x} = 0
\end{aligned} \tag{2}
$$

where S denotes the salvage value of the firm and we have dropped the firm subscripts for simplicity. Subscripts in the differential equation denote partial derivatives. The differential equation is solved using the three boundary conditions that follow it. The first is the no-bubble condition that states that as the shock process tends to infinity (and hence the net operating profits), the firm value equals the appropriately discounted expected cash flows. The second boundary condition is the value-matching condition that ensures continuity at the exit trigger, x^*. In particular, it tells that at the exit threshold, stockholders obtain the salvage value of the firm. The last condition is the smooth-pasting condition that ensures differentiability at the exit threshold.

The general solution to system (2) is:

$$
V(x) \quad = \quad A_1 x^{\lambda_1} + A_2 x^{\lambda_2} + \frac{\Pi x}{r-\mu} - \frac{f}{r} \tag{3}
$$

where $\lambda_1 > 1$ and $\lambda_2 < 0$ are the roots of the characteristic equation and A_1 and A_2 are constants to be determined.

Using the boundary conditions, we can derive both the value of the firm and the optimal exit trigger in closed-form:

$$
V(x_t) = \left(\frac{\Pi x_t}{r-\mu} - \frac{f}{r}\right) + \left[S + \frac{f}{r} - \frac{\Pi x^*}{r-\mu}\right]\left(\frac{x_t}{x^*}\right)^{\lambda_2}, \lambda_2 < 0 \tag{4}
$$

$$
x^* = \frac{\lambda_2 (rS + f)(r-\mu)}{(\lambda_2 - 1) r \Pi} \Rightarrow \frac{\partial x^*(r, \mu, \Pi, S, f)}{\partial f} > 0 \tag{5}
$$

In equation (4), the first two terms in the parantheses on the right hand side give the fundamental value of the firm. Without the option to shut down, the firm would have to operate forever in the market and its value is simply the discounted expected cash flows. The term in square brackets

[3]These standard arguments can be found in Dixit and Pindyck (1994).

captures the option value to shut down. When the firm decides optimally to abandon, the stockholders forgo the present value of cash flows and instead get the scrapping value of the firm. The last multiplicative term is simply the discount factor. Note also that in equation (5), the trigger is a strictly increasing function of the fixed cost. That is, a higher fixed cost of assets induces stockholders to leave the market sooner. This intuitive result will play an important role in the subsequent analysis in structuring and solving the model.

In the sequel, the monopoly value of a firm is treated as a termination payoff when the competitor leaves the market in the duopoly market. That is, the game ends when one of the firms exits the market. When the firms operate in a duopoly they earn the duopoly operating profits. When one of the firms exit the market, the remaining firm becomes the monopolist and the analysis reduces to one described in this section. Therefore, the monopoly value can be perceived as a "bequest function". Note that salvage value is in fact the termination payoff when a firm leaves the market. However, when one of the firms leaves in the duopoly, the duopoly problem for the winning firm is stopped and it obtains a payoff which is in turn the solution of another optimal stopping problem.

To simplify the analysis in the following sections, we can also derive the exit triggers of the firms if they were to remain forever as duopolists.[4] Denote by index $i = 1, 2$ the firms in the duopoly market. In the duopoly industry, firm i earns operating profits of $\pi x_t - f_i$. Adjusting for this in the differential equation (3), and carrying out the same analysis, the exit trigger of firm i is given by:

$$x^{'} = \frac{\lambda_2 (rS + f)(r - \mu)}{(\lambda_2 - 1) r\pi} \Rightarrow x^{'} > x^{*} \tag{6}$$

Since $x^{'}$ is the trigger given that the competitor never exits, stockholders of firm i would never exit the market for $x_t > x^{'}$. Furthermore, since x^{*} is the optimal exit trigger when the competitor has already left the market, firm i would never stay in the market for $x_t < x^{*}$.

3.2 Duopoly without Debt

In this section, we model the benchmark duopoly industry without debt. Both firms have only to decide the optimal time to leave the market. Just as with the monopoly case, this is an optimal stopping time problem in which

[4]Fudenberg and Tirole (1986) introduce the idea that firms need not be so inefficient as to exclude the possibility that firms remain as duopolists forever. In their paper, this assumption allows them to obtain a unique equilibrium.

continuation means operating as a duopolist. As long as both firms are in the market, firms earn duopoly profits, $\pi x_t - f_i$. When one of the firms optimally leaves the market, the remaining firm becomes the monopolist with operating income Πx_t and is entitled to the bequest function, $V(x_t)$, derived in the previous section. It operates until it is optimal for stockholders to exit at the monopoly trigger.

We have seen in Section 3.1 that the exit threshold of firm i is a strictly increasing function of its fixed cost. Therefore, there is a one-to-one correspondence between the fixed cost and the exit thresholds of the firm, *ceteris paribus*. This correspondence turns the game into one that is played in exit thresholds. In particular, both firms determine their duopoly and monopoly exit triggers given their own fixed costs. Denote by x_{1m} and x_{1d} the monopoly and duopoly exit triggers of firm 1, respectively, and let x_m^k and x_d^k be the respective monopoly and duopoly exit thresholds of type $k = L, H$ firm 2. Since firm 1 knows only the set $\{f_L, f_H\}$ and because there is a one-to-one correspondence between the fixed costs and the exit triggers, firm 1 knows the set of possible monopoly and duopoly exit triggers of firm 2. On the other hand, x_{1m} and x_{1d} are common knowledge. Given this information structure, stockholders have to decide whether to stay in the market until the monopoly threshold or to leave at the duopoly exit threshold.

Since the game is played in exit triggers, the relative magnitudes of these triggers and hence the fixed costs determine the competitive environment and optimal exit policies of the firms. In the sequel, we say that firm i dominates firm j if $f_i < f_j$. Put differently, firm i dominates its competitor if it can optimally leave the market at a later date, captured by the exit trigger.[5] Furthermore, we say that this domination is strict if the fixed costs of both types of firm 2 are below or above that of firm 1. It is now straightforward to derive the value functions of firm i. In the continuation region, firm value satisfies a differential equation of the same sort as in system (2). Consider first the value of the firms at the exit triggers. When firm i leaves the market, its stockholders receive the salvage value, S. Since we assume that S is the same no matter whether firm i is a monopoly or a duopoly, $V_i(x_{id}) = S = V_i(x_{im})$. If firm j leaves the market before firm i, firm i becomes the monopolist and its value at the rival's duopoly trigger satisfies $V_i^d(x_{jd}) = V_i^m(x_{jd})$, where the superscripts d and m denote the values in duopoly and monopoly, respectively. These equations will show up as boundary conditions repeatedly in each of the cases presented below.

[5] Since we assume in this paper that strategies are connected sets, there is one critical duopoly exit trigger for each firm. Given that there are no jumps in the state variable, this structure necessarily implies that x_t hits x_{jd} before it hits x_{id} if $x_{jd} > x_{id}$.

Turning to the continuation region in which both firms are active, the value function of firm i is, as in Section 3.1, of the form:

$$V_i^d(x) \quad = \quad A_1 x^{\lambda_1} + A_2 x^{\lambda_2} + \frac{\pi x}{r-\mu} - \frac{f}{r} \tag{7}$$

where λ_1 and λ_2 are, as before, the roots of the characteristic equation. Note that in the continuation region, firms earn the duopoly profits πx_t. The constants A_1 and A_2 are to be determined from the case-specific boundary conditions. The cases (or the competitive environment) depend on how the fixed costs are ordered. Once these values and exit triggers are derived, we can determine which firm leaves the market first. We now turn to each of these cases separately.[6]

Case 1: Firm 1 strictly dominates firm 2 In this case, fixed costs are such that $f_1 < f_L < f_H$. As argued before, this implies that the exit triggers of both types of firm 2 are larger than those of firm 1. Since both firms earn $\Pi x_t - f_i$ and $\pi x_t - f_i$ in monopoly and duopoly, respectively, and $\Pi > \pi$, the triggers have the relation $x_{1m} < x_m^L < x_m^H < x_{1d} < x_d^L < x_d^H$. We conjecture that firm 2 leaves the market first no matter what type it is. Consider first the value of firm 1. Firm 1 knows that it strictly dominates firm 2. Therefore, firm 1 becomes the monopolist when firm 2 leaves at its duopoly exit trigger, $x_d^k, k = L, H$. After firm 2 has left, there is no strategic interaction. The stockholders of firm 1, therefore, determine the optimal time to leave the market as the monopolist firm in Section 3.1. Hence, after x_d^k, the value and the exit trigger of monopoly firm 1 are given by:

$$V_{1m}(x_t) = \left(\frac{\Pi x_t}{r-\mu} - \frac{f_1}{r}\right) + \left[S + \frac{f_1}{r} - \frac{\Pi x_{1m}}{r-\mu}\right]\left(\frac{x_t}{x_{1m}}\right)^{\lambda_2} \tag{8}$$

$$x_{1m} = \frac{\lambda_2(rS + f_1)(r-\mu)}{(\lambda_2 - 1)r\Pi} \tag{9}$$

As argued before, the value of firm 1 in the continuation region is of the form of equation (7). To determine the constants, the following boundary conditions are imposed:

$$lim_{x\to\infty} V_{1d}^k(x) = \frac{\pi x}{r-\mu} - \frac{f_1}{r} \tag{10}$$

$$V_{1d}^k(x_d^k) = V_{1m}(x_d^k) \quad k = L, H \tag{11}$$

[6]Note that the the following arguments essentially demonstrate the conjectured solutions of the game. We show later that the conjectured solutions indeed constitute the equilibrium.

As described in Section 3.1, the first boundary condition captures the event that it may never be optimal for either firm to leave the market. Such will be the case if the state variable, x_t, tends to infinity. Then firm 1 simply earns the discounted duopoly net operating profits. Equation (11) ensures the continuity of firm 1 value function. At the exit trigger of firm 2, the duopoly value of firm 1 equals its monopoly value evaluated at that trigger. Using (10) and (11), firm 1 duopoly value can be derived as:

$$V_{1d}^k(x_t) = \left(\frac{\pi x_t}{r-\mu} - \frac{f_1}{r}\right) + \frac{(\Pi-\pi)x_d^k}{r-\mu}\left(\frac{x_t}{x_d^k}\right)^{\lambda_2} + \left[S + \frac{f_1}{r} - \frac{\Pi x_{1m}}{r-\mu}\right]\left(\frac{x_t}{x_{1m}}\right)^{\lambda_2} \quad (12)$$

In equation (12), the first two terms on the right hand side are the fundamental value described in Section 3.1. The third term captures the strategic effect emanating from the competition. Although firm 1 does not take any action at the exit trigger of its competitor, its value is affected through the rival firm's decision. In particular, at the rival's exit trigger, firm 1 becomes a monopolist. This is exactly what is captured by the third term on the right hand side. Note that this term is absent from the single-firm analysis of Section 3.1. As before, the last term captures the value of the abandonment option.

The information asymmetry embedded in the model can also be seen clearly from equation (12). The stockholders of firm 1 cannot ascertain whether they face a type L or type H competitor. Equation (12); therefore, keeps track of the firm 2 type by the superscript $k = L, H$. The *ex ante* duopoly value of firm 1 is thus a weighted average of these two values. That is, since firm 2 is of type L with probability 0.5, the *ex ante* duopoly value is given by $0.5V_{1d}^L + 0.5V_{1d}^H$. However, the state variable itself is informative about the type of firm 2. To see this, suppose that the state variable has hit x_d^H.[7] If the conjectured solution is indeed an equilibrium and firm 2 exits the market at this point, this will signal the type to firm 1. If firm 2 does not exit the market at x_d^H, firm 1 deduces that its rival is a type L firm and its value is now V_{1d}^L. Thus, the value of firm 1 can be written as:

$$V_1(x_t; x_d^k) = \begin{cases} 0.5V_{1d}^L(x_t; x_d^k) + 0.5V_{1d}^H(x_t; x_d^k) & if x_t \geqslant x_d^H \\ V_{1d}^L(x_t; x_d^k) & if x_d^L \leqslant x_t < x_d^H \\ V_{1m}(x_t) & if x_{1m} \leqslant x_t \leqslant x_d^k \end{cases} \quad (13)$$

where the second line in equation (13) holds if firm 2 is of type L.

As opposed to firm 1, firm 2 earns only duopoly operating profits until its exit trigger, $x_d^k, k = L, H$. The value of type k firm 2 is thus of the form of

[7] We always start at some x_0 greater than the largest exit trigger so that the market is initially a duopoly with private information.

equation (7). To obtain the constants, the following conditions are imposed:

$$lim_{x \to \infty} V_{2d}^k(x) = \frac{\pi x}{r-\mu} - \frac{f_k}{r} \tag{14}$$

$$V_{2d}^k(x_d^k) = S \tag{15}$$

$$\frac{\partial V_{2d}^k(x_d^k)}{\partial x} = 0, \quad k = L, H \tag{16}$$

The first boundary condition reflects the same reasoning as equation (10). The second boundary condition states that at the duopoly exit trigger, the stockholders of the firm retrieve the salvage value. Since the stockholders have to optimally exit, they also determine the exit trigger. The optimality of the trigger is captured by the third boundary condition. Solving equation (7) subject to these boundary conditions yields:

$$V_{2d}^k(x_t) = \left(\frac{\pi x_t}{r-\mu} - \frac{f_k}{r}\right) + \left[S + \frac{f_k}{r} - \frac{\pi x_d^k}{r-\mu}\right]\left(\frac{x_t}{x_d^k}\right)^{\lambda_2} \tag{17}$$

$$x_d^k = \frac{\lambda_2(rS + f_k)(r-\mu)}{(\lambda_2 - 1)r\pi} \tag{18}$$

Case 2: Firm 2 strictly dominates firm 1 Assume that $f_L < f_H < f_1$. By the structure of our model, this implies $x_m^L < x_m^H < x_{1m} < x_d^L < x_d^H < x_{1d}$. In this case, the conjectured solution involves firm 1 always leaving first. Since firm 1 exits first, whether firm 2 is of type L or H is of no consequence for firm 1. Therefore, firm 1 simply earns the net operating profits in the continuation region and leaves the market at x_{1d}. Hence, the value of firm 1 is given by equation (7) subject to boundary conditions (14)-(16) adjusted for firm 1 parameters. The value and the exit trigger of firm 1 is thus:

$$V_{1d}(x_t) = \left(\frac{\pi x_t}{r-\mu} - \frac{f_1}{r}\right) + \left[S + \frac{f_1}{r} - \frac{\pi x_{1d}}{r-\mu}\right]\left(\frac{x_t}{x_{1d}}\right)^{\lambda_2} \tag{19}$$

$$x_{1d} = \frac{\lambda_2(rS + f_1)(r-\mu)}{(\lambda_2 - 1)r\pi} \tag{20}$$

Note that because firm 1 is strictly dominated, it need not make any probability weighting. The *ex ante* value is not affected by whether firm 2 is of a particular type.

Now consider the firm 2 value. In the continuation region, firm 2 value is again of the form of equation (7). By the same arguments as in case 1, the boundary conditions (10) and (11) can be imposed to obtain the constants. On the other hand, when firm 1 leaves the market, firm 2 becomes the monopolist. Firm 2 receives the same monopoly value and exits at the same trigger derived in Section 3.1. Hence, the duopoly, monopoly and exit trigger values are given respectively by:

$$V_{2d}^k(x_t) = \left(\frac{\pi x_t}{r-\mu} - \frac{f_k}{r}\right) + \frac{(\Pi-\pi)x_{1d}}{r-\mu}\left(\frac{x_t}{x_{1d}}\right)^{\lambda_2} + \left[S + \frac{f_k}{r} - \frac{\Pi x_m^k}{r-\mu}\right]\left(\frac{x_t}{x_m^k}\right)^{\lambda_2} \quad (21)$$

$$V_{2m}^k(x_t) = \left(\frac{\Pi x_t}{r-\mu} - \frac{f_k}{r}\right) + \left[S + \frac{f_k}{r} - \frac{\Pi x_m^k}{r-\mu}\right]\left(\frac{x_t}{x_m^k}\right)^{\lambda_2} \quad (22)$$

$$x_m^k = \frac{\lambda_2(rS + f_k)(r-\mu)}{(\lambda_2 - 1)r\Pi}, k = L, H \quad (23)$$

It is interesting to observe that when firm 2 strictly dominates firm 1, the information asymmetry is immaterial and the analysis reduces to one with complete information. In particular, *ex ante* value of firm 1 would again be given by equation (19), had firm 1 known the type of its competitor (cf. Lambrecht, 2001). Given that the conjectured solution of this section indeed constitutes an equilibrium, one can safely argue that debt would have no signaling role when firm 2 strictly dominates firm 1.

Case 3: The "Mixed" Case In the so-called mixed case, we assume that type H firm 2 has the largest fixed cost but that type L firm 2 has a competitive advantage over firm 1. That is, $f_L < f_1 < f_H$. The conjectured solution in this case involves type H firm 2 leaving the market before firm 1 and type L firm 2 exiting after. If the conjectured solution is indeed an equilibrium, firm 1 observes firm 2 action at critical triggers and relies on these actions to design its strategy. In other words, firm 1 can form a strategy in which the shock process fully reveals firm 2 type. Specifically, given that the mentioned profile is indeed an equilibrium, firm 1 observes firm 2 action at x_d^H. If firm 2 has exited, firm 1 then stays in the market until its monopoly trigger. If no action has occurred, firm 1 leaves the market at its duopoly trigger.[8]

[8]The next section argues that this strategy profile constitutes a Bayesian-Nash equilibrium.

Firm 1 value can now be derived by accordingly distinguishing between two regions. First, when $x_t > x_d^H$, firm 1 does not know the type of its competitor. Therefore, its *ex ante* value takes the distribution of firm 2 salvage values into account. Specifically, for $x_t > x_d^H$, firm 1 value is given by:

$$V_1^b(x_t) = 0.5V_{1d}(x_t) + 0.5V_{1d}^H(x_t) \tag{24}$$

where the superscript b captures the event that x_d^H has not yet been hit. In equation (24), the first term on the right hand side captures the likelihood that the competitor is of type L, whereas the second term designates the event that it is of type H.

On the other hand, when $x_\tau = x_d^H$, for some adapted stopping time τ, firm 2 type is fully revealed by the (lack of) exercise of the abandonment option. Once the trigger is hit, firm 1 continues to earn duopoly profits if the competitor is of type L or it becomes the monopolist. Hence, firm 1 value can be written as:

$$V_1^a(x_t) = \begin{cases} V_{1d}(x_t), & x_t \geq x_{1d}, \quad if \quad type \quad L \\ V_{1m}(x_t), & x_t \geq x_{1m}, \quad if \quad type \quad H \end{cases} \tag{25}$$

Returning to firm 2 value, Since type H firm 2 leaves the market before firm 1 does, it will only receive the operating profits in the continuation region. Its value and the duopoly exit trigger are given respectively by equations (17) and (18) by setting $k = H$. The value of type L firm 2, on the other hand, is the same as those derived in case 2. Specifically, its monopoly and duopoly values are given by equations (22) and (21), respectively. Again, in this case, we set $k = L$.

We have now determined the firm values and exit triggers by assuming that the relative magnitudes of the triggers are given exogenously. The next step is to determine whether the above profile is indeed an equilibrium. We also ask the question whether other equilibira are possible within the framework of our model.

3.3 Equilibrium

The previous section has derived the payoffs of the conjectured solutions to the game. In this section, we argue that in all three cases, as long as the relation between the fixed costs is strict, there can only be a unique Bayesian-Nash equilibrium. When the relation is strict, the state variable itself and the action taken by firm 2 are sufficiently informative. The determination of equilibrium also allows us to identify the conditions under which debt could be of signaling value. It turns out that if firm 1 is identical to one of the firm 2 types, debt can be used strategically.[9] We analyze the strategic role of debt only in the mixed case.

Before solving the games, we first introduce some notation and terminology. χ_i denotes the set of strategies available to firm $i = 1, 2$. Since firms can decide either to exit at the duopoly exit trigger or at the monopoly exit threshold, $\chi_i = \{x_{id}, x_{im}\}$. Let $M_i \subset \chi_i$ denote the set of particular moves that is chosen by firm i. Note that the set M_i comprises of a singleton. For instance, if firm i chooses to leave at the duopoly exit trigger, then $M_i = \{x_{id}\}$. Denote by $R_i(M_j)$ the best response of firm i to firm j's move. The equilibrium in case $n = 1, 2, 3$ is a set, Q_n, comprising of a fixed point such that the respective values of the firms are maximized. In particular, we say that $Q_n = \{M_i, M_j\}$ is an equilibrium of game n, if $R_i(M_j) = M_i$ and $R_j(M_i) = M_j$.

Next, we define the concept of reservation trigger. As in Lambrecht (2001), the reservation trigger is the critical threshold of the competitor that makes firm i indifferent between becoming a monopoly at this trigger and leaving first at its own duopoly threshold. Figure 1 maps firm 1 duopoly value function for various levels of duopoly exit trigger of type L firm 2. Note that the value function is increasing in the exit trigger of the competitor. Put differently, the sooner firm 2 leaves, the higher is the value of firm 1 since it has a shorter duopoly but a longer monopoly position. The existence of the reservation trigger is guaranteed by this feature and the fact that the value functions are of the same shape. Specifically, the reservation triggers for firm 1 and type k firm 2, respectively, are the critical value of competitor's duopoly threshold such that

$$V_{1d}^k(x_{1r}) = V_{1d}(x_{1d}) \tag{26}$$

$$V_{2d}^k(x_{2r}) = V_{2d}(x_d^k) \tag{27}$$

In equations (26) and (27), the left-hand-side is the value of the firm that eventually becomes a monopolist at the competitor's duopoly exit trigger. The right-hand-side yields the value when leaving first at the duopoly trigger. Substituting for equations (26) and (27) from the respective analysis in Section 3.2, the reservation triggers can be obtained as:

$$\begin{aligned} x_{1r} &= \left\{ \frac{(rS + f_1)(r-\mu)}{(\Pi - \pi)(1-\lambda_2)r} \left(\frac{1}{x_{1d}^{\lambda_2}} - \frac{1}{x_{1m}^{\lambda_2}} \right) \right\}^{\frac{1}{1-\lambda_2}} \\ x_r^k &= \left\{ \frac{(rS + f_k)(r-\mu)}{(\Pi - \pi)(1-\lambda_2)r} \left(\frac{1}{(x_d^k)^{\lambda_2}} - \frac{1}{(x_m^k)^{\lambda_2}} \right) \right\}^{\frac{1}{1-\lambda_2}} \end{aligned} \tag{28}$$

[9] Firm 1 is said to be identical to type k firm if its fixed cost equals that of type k firm.

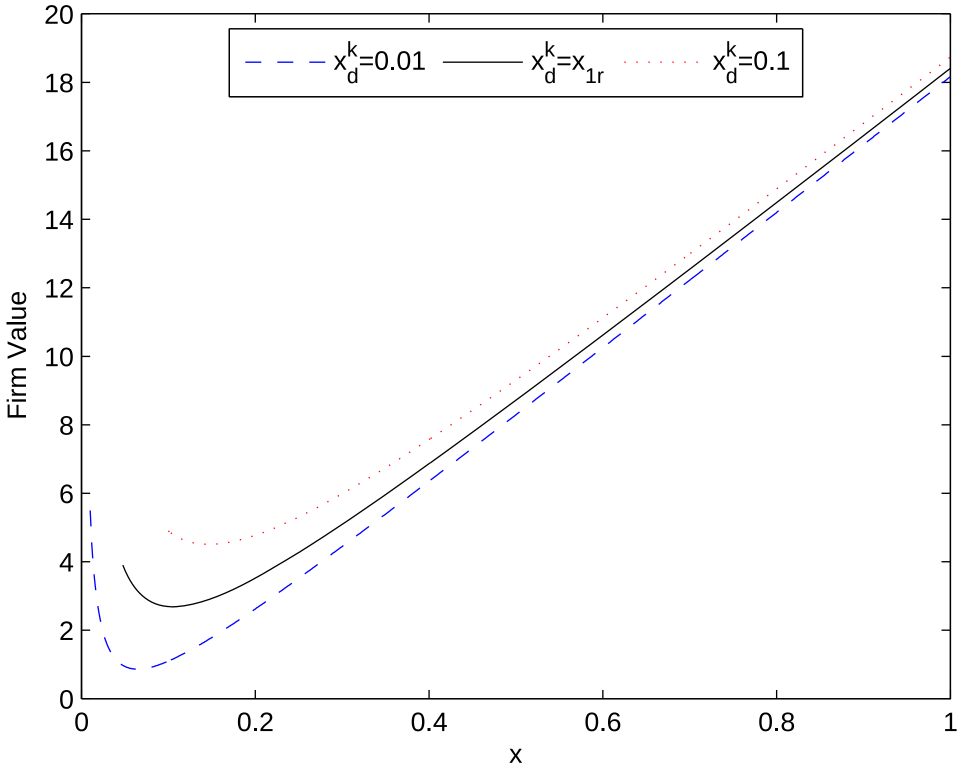

Figure 1: Duopoly Firm 1 Value as a Function of Type L Firm 2 Duopoly Trigger. The parameters are $\mu = 0, \sigma = 0.2, r = 0.05, \Pi = 2, \pi = 1, S = 1, f_1 = 0.1$. The reservation trigger with this parameter set is $x_{1r} = 0.05$

As in Lambrecht (2001), the reservation trigger can be perceived as the point until which firm i is willing to incur losses to reap off monopoly benefits when the competitor leaves the market. Furthermore, substituting for the triggers in equation (28), it turns out that the reservation trigger is linearly increasing in the salvage values. The reservation trigger can be written generically as:

$$x_{ir} = \frac{(r-\mu)}{1-\lambda_2}\left[\frac{\pi^{\lambda_2} - \Pi^{\lambda_2}}{\lambda_2^{\lambda_2}(\Pi - \pi)r}\right]^{\frac{1}{1-\lambda_2}}(rS + f) \tag{29}$$

Equation (29) allows us to make a natural order among reservation triggers. This simplifies the derivation of the equilibrium. In addition, the reservation trigger has the feature that it is between the duopoly and monopoly thresholds, $x_{im} \leqslant x_{ir} \leqslant x_{id}$.

As a final step before determining the equilibrium, it is important observe that the fixed costs are assumed to be given exogenously. Therefore, both firms know the competitive environment (i.e. which of the above cases they are in). The implication is that the equilibrium should be determined for each case separately.

Case 1: Firm 1 strictly dominates firm 2 As described above, firm 1 dominates both types of firm 2 since $f_1 < f_L < f_H$. The first proposition shows that the Bayesian-Nash equilibrium in this case involves firm 1 always leaving last.

Proposition 1: Under the assumption $f_1 < f_L < f_H$, the unique Bayesian-Nash equilibrium is given by

$$Q_1 = \left\{x_{1m}, x_d^k\right\}, k = L, H$$

Proof: We derive the equilibrium in three parts depending on whether firm 2 types play separating or pooling strategies.

(1) Suppose that both types play $M_2 = \left\{x_d^k\right\}$, $k = L, H$. Since $x_{1m} \leqslant x_{1r} \leqslant x_{1d} < x_d^k$, $k = L, H$, firm 1 prefers to stay until its monopoly trigger. That is, $R_1(x_d^k) = \{x_{1m}\}$. This is easy to see. Once firm 2 leaves the market at its duopoly trigger, firm 1 becomes the monopolist. Section 3.1 shows that when firm 1 is a monopolist, it maximizes its value if it waits until x_{1m}. Now suppose that firm 1 indeed plays $M_1 = \{x_{1m}\}$. As argued above, firm 2 would be willing to incur losses until x_r^k to become a monopolist. However, by the strict monotonicity of reservation triggers in the fixed cost, firm 1 can make a credible threat to firm 2 by holding on to the market until x_{1r}. Given this, firm 2 would cut its losses and exit at the duopoly threshold. Since $R_1(x_d^k) = \{x_{1m}\}$ and $R_2(x_{1m}) = \left\{x_d^k\right\}$, $Q_1 = \left\{x_{1m}, x_d^k\right\}$ is a fixed point and an equilibrium.

(2) Assume now that both types of firm 2 play $M_2 = \left\{x_m^k\right\}$. It is evident that if $x_{1r} < x_m^k$ and/or $x_{1d} < x_r^k$, firm 2 would never play x_m^k, since firm 1 in either case can hold on to the market longer than its competitor. However, even if $x_{1m} < x_r^k < x_{1d}$, [10] by the strict monotonicity of x_{ir} in f_i, we have $x_{1r} < x_r^k$, $k = L, H$. That is, firm 1 can again hold out longer than firm 2 to reap off monopoly profits. Hence, $R_1(x_m^k) = \{x_{1m}\}$. But since $R_2(x_{1m}) = \left\{x_d^k\right\}$, there cannot be any equilibrium with firm 1 leaving first when both types of firm 2 play x_m^k.

(3) It could also be that type L firm plays x_m^L and type H plays x_d^H and vice versa. However, the arguments in (2) establish that as long as $x_{1r} < x_r^k$, firm 1 can always outlast firm 2 no matter what type it is. Therefore, there

[10] That is, firm 2 could hold out until the duopoly trigger of firm 1 if it could become the monopolist.

cannot be any equilibrium involving firm 1 leaving first when types play separating strategies `Q.E.D.`

Case 2: Firm 2 strictly dominates firm 1 Recall that firm 2 strictly dominates firm 1 if $f_L < f_H < f_1$. Section 3.2 has argued that the conjectured solution presented there is equivalent to a standard duopoly without information asymmetry. Furthermore, by arguing as in the proof of Proposition 1, one can show that there is a unique Bayesian-Nash equilibrium in this case as well. Proposition 2 formalizes this result.

Proposition 2: Under the assumption $f_L < f_H < f_1$, the unique Bayesian-Nash equilibrium is given by

$$Q_2 = \left\{x_{1d}, x_m^k\right\}, k = L, H$$

Proof: It is straightforward to prove the proposition by interchanging the firms in Proposition 1. The proof again rests on the strict monotonicity of the reservation triggers in the fixed costs `Q.E.D.`

Case 3: "Mixed" Case Consider now the more interesting third case in which only type L dominates firm 1. The equilibrium in this case must be derived conditionally. The analysis of the first two cases suggests that firm 1 would leave the market first only if it knew with probability 1 that the competitor is type L. However, as the proof of Proposition 3 shows, firm 1 can design a strategy in which the type of firm 2 is fully revealed. The strategy relies on the strict monotonicity of the reservation triggers in the fixed costs and the informative content of the state variable.

Proposition 3: Under the assumption $f_L < f_1 < f_H$, type L firm 2 leaves the market at its duopoly trigger whereas type H firm 2 exits when x_t hits x_d^H. Firm 1 leaves the market either at its duopoly trigger if firm 2 has not exited at x_d^H or exits at the monopoly trigger if firm 2 has exited at x_d^H.

Proof: Let $\tau = inf\left\{\, t : x_t = x_d^H\right\}$ be an adapted stopping time and denote the value of the state variable at x_d^H by x_τ . Consider the following strategy for firm 1: (i) wait until $x_\tau = x_d^H$, (ii) if at $x_\tau = x_d^H$, firm 2 has not exited, leave the market at x_{1d}, (iii) otherwise exit the market at x_{1m}. We now determine the best responses of the firms to each other under this scheme.

(1) First, consider type H firm 2. Since $f_1 < f_H$, by the strict monotonicity of the reservation triggers in the fixed costs, firm 1 can make a credible threat to type H firm 2 by holding on until $x_{1r} < x_r^H$. Note that the credibility of the threat is not affected by the information asymmetry. In particular, type

H firm 2 cannot mimic type L firm 2 since this requires that type H firm 2 wait credibly until $x_r^L < x_r^H$. Hence, type H firm 2 would always leave the market at its duopoly trigger, x_d^H. Now, consider type L firm 2. Since we have $x_r^L < x_{1r}$ no matter what strategy firm 1 follows, the best response of type L firm 2 would be to leave the market at its monopoly trigger, x_m^L.
(2) We now argue that the above profile outlined for firm 1 is the best response to firm 2 strategies. Observe that for $x_t > x_d^H$, firm 1 does not commit itself to any strategy. Suppose now that $x_\tau = x_d^H$. Since regardless of firm 1 strategy, type H(L) firm 2 will leave the market at its duopoly (monopoly) trigger, firm 1 finds out firm 2 type by observing firm 2 action at x_d^H. If firm 2 has exited, firm 1 finds out that the competitor is of type H and leaves the market at x_{1m} by Proposition 1. If no exit has occurred at this point, firm 1 deduces that the competitor is of type L and thus exits at x_{1d} by Proposition 2. Since the best responses would be the same for any x_t, the proposed profile is indeed a Bayesian-Nash equilibrium `Q.E.D.`

4 Signaling with Debt

This section opens with the discussion and justification of signaling with debt. After having established the role for debt, we specify the debt contract that separates the types. Section 4.2 then carries out a numerical analysis of the sensitivity of equity value with respect to the exogenous parameters. Finally, Section 4.3 derives the conditions under which the separating debt contract constitutes a perfect Bayesian equilibrium.

4.1 The Debt Contract

Section 3 has dealt with a duopoly with asymmetric information in which firms were equity-financed. The analysis shows that as long as one of the firms strictly dominates the other, information asymmetry does not *qualitatively* affect the outcome of the competition. In fact, when the firm with the finer information set dominates its rival, the competition reduces to one in which there is no information asymmetry (case 2). This section explores whether debt can be used as a signal to change the outcome of the competition. Three important questions arise in this context. First, under which conditions can debt be used justifiably? Since the model in this paper abstracts from tax benefits of debt, the main benefit of debt is its signaling value. However, as argued above, the state variable, x_t and the action of the competitor itself have information content. Therefore, debt can only be used as a signal if it adds value beyond the informative characteristic of the state variable. Second, if debt has signaling value, how should the resulting debt contract look like? In this respect, the paper considers the simplest type of debt contract. We assume that the debt contract entails the payment of a coupon, c, forever unless firm leaves the market. Since the main benefit of debt is to distinguish type L firm 2 from type H firm 2, the debt contract should specify a coupon that cannot be mimicked. The proceeds from the debt issuance are assumed to be distributed to the shareholders. Furthermore, there is no call feature and/or renegotiation possibility. These assumptions imply that the capital structure structure decision in the model is essentially a static decision. That is, given the parameters of the game, type L firm 2 decides to initiate a debt issue at the outset. Under the current assumptions, there is no adjustment to the capital structure thereafter. This is admittedly a restrictive assumption. However, a dynamic capital structure would introduce another decision variable in terms of the frequency of adjustment. Therefore, the structure here allows us to isolate the strategic effect of debt in its simplest form. Finally, is the suggested debt contract indeed a perfect Bayesian equilibrium (PBE)? In this paper, we only characterize the conditions under which the suggested use of debt constitutes a PBE. We leave the more rigorous treatment of this issue to further research.

Before moving further, a caveat about the use of debt is in order. The signaling requirement on debt imposes a constraint on the *ex ante* optimal coupon decision. That is, equityholders of the signaling firm must maximize the *ex ante* total firm value when setting the coupon. However, as will be shown in the subsequent analysis, the coupon must be within a certain range if it is to reveal information. In short, capital structure choice becomes a constrained optimization problem.

It is straightforward to see that in the strict domination cases, debt has no signaling value. When firm 2 strictly dominates firm 1 (case 2), in equilibrium, firm 1 leaves the market first no matter what type its competitor is. Because the competition is equivalent to one with complete information, neither type of firm 2 has an incentive to signal. When firm 1 strictly dominates firm 2 (case 1), *ex ante* firm 1 value is a probability distribution across firm 2 types (see equation (13)). However, as argued in the analysis of case 1, the state variable itself is informative as to the type of firm 2. More importantly, firm 1 never exits at its duopoly trigger in equilibrium (Proposition 1). Therefore, it does not pay firm 2 to signal its type when it is strictly dominated.

The signaling value of debt is a more delicate issue in case 3. It is tempting to think that debt has signaling content in this case. However, it turns out that this is true only if firm 1 is identical to type H firm 2 (i.e. $f_1 = f_H$). The reason is that the informative role of the state variable disappears when the relation between the fixed costs is strict. The proof of Proposition 3 illustrates the informative role of the state variable. The proof rests on the strict monotonicity of the reservation trigger in the fixed cost. When the

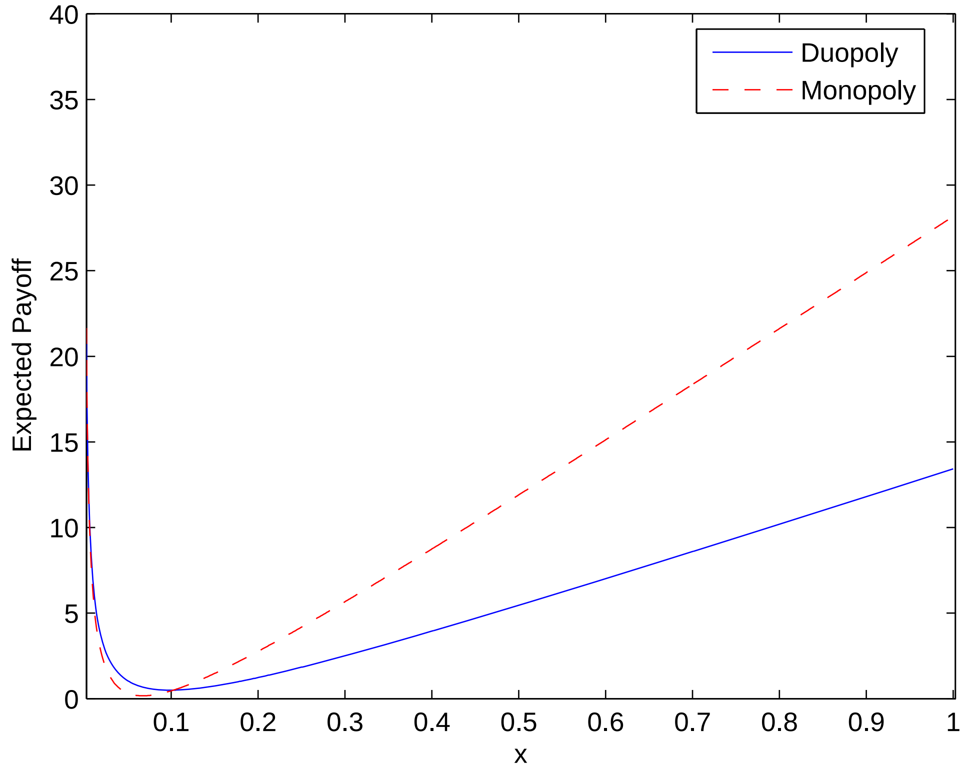

Figure 2: Expected Payoffs from Exiting at Duopoly and Monopoly Triggers When Firm 1 is Identical to Type H Firm 2. The parameters are $\mu = -0.01, \sigma = 0.2, r = 0.05, \Pi = 2, \pi = 1, S = 0.5, f_1 = 0.2$

fixed costs are equal, so are the reservation triggers. Therefore, firm 1 can no longer credibly threaten type H firm 2 by committing itself to waiting until its reservation trigger. If such is the case, game H has two pure strategy equilibria: $\left\{x_{1d}, x_m^H\right\}$ and $\left\{x_{1m}, x_d^H\right\}$. That is, as opposed to Proposition 3, the information content of the state variable disappears. Firm 1 could then commit itself to leaving at its duopoly or monopoly exit trigger depending on the particular parameter set. Figure 2 depicts a case in which the value from exiting at the monopoly trigger is greater than that from leaving at the duopoly trigger. When such is the case, type L firm 2 can benefit from signaling its type to firm 1. If the type were signalled in such a way that type H firm 2 cannot mimic, firm 1 would not commit *ex ante* to exit at the monopoly trigger.

How does the debt contract ensure that type L is distinguished from type H? When the firm issues debt, stockholders maximize the equity value *ex post* rather than the total firm value. The exit triggers, therefore, have to determined anew from equity value optimization. Type L firm 2 would have revealed its type if it sets the coupon, c_L, such that the exit triggers of type H firm 2 are increased while its own competitive position remains unchanged. If type H firm 2 were to mimic type L firm 2, it would place itself at a competitive disadvantage because its exit trigger is now greater than that of firm 1. We have seen in the benchmark model that as long as the monopoly trigger (or equivalently its reservation threshold) of the firm is greater than that of its competitor, the firm finds it optimal to leave the market first.

Let c_k denote the coupon set by the type $k = L, H$ firm and $\omega \in [0, 1]$ be the portion of the salvage value captured by the stockholders when the firm exits the market. It can be shown that the equity value and the exit trigger when the firm becomes the monopolist are given by:

$$E_{2m}^k = \frac{\Pi x_t}{r-\mu} - \frac{f_k + c_k}{r} + \left[\omega S + \frac{f_k + c_k}{r} - \frac{\Pi \bar{x}_m^k}{r-\mu}\right]\left(\frac{x_t}{\bar{x}_m^k}\right)^{\lambda_2} \tag{30}$$

$$\begin{aligned}\bar{x}_m^k &= \frac{\lambda_2(r-\mu)(\omega r S + f_k + c_k)}{(\lambda_2 - 1) r \Pi} \\ &= x_m^k \left(\frac{r\omega S + f_k + c_k}{rS + f_k}\right)\end{aligned} \tag{31}$$

Similarly, if the firm leaves first at its duopoly trigger, its value and the threshold are given by:

$$E_{2d}^k = \frac{\pi x_t}{r-\mu} - \frac{f_k + c_k}{r} + \left[\omega S + \frac{f_k + c_k}{r} - \frac{\pi \bar{x}_d^k}{r-\mu}\right]\left(\frac{x_t}{\bar{x}_d^k}\right)^{\lambda_2} \tag{32}$$

$$\begin{aligned}\bar{x}_d^k &= \frac{\lambda_2(r-\mu)(\omega r S + f_k + c_k)}{(\lambda_2 - 1) r \pi} \\ &= x_d^k \left(\frac{r\omega S + f_k + c_k}{rS + f_k}\right)\end{aligned} \tag{33}$$

Equations (31) and (33) relate the levered firm triggers to the unlevered ones. The former triggers exceed the unlevered triggers if $\frac{r\omega S + f_k + c_k}{rS + f_k} > 1$. In particular, if type L firm 2 sets c_L such that $\frac{r\omega S + f_H + c_L}{rS + f_H} > 1 \Leftrightarrow c_L > (1-\omega) rS$, the exit triggers of type H firm 2 exceed those of firm 1. By Proposition 3, type H firm 2 would then leave at its duopoly trigger if it were to imitate type L firm 2.

Parameter	Value	Parameter	Value
μ	-0.01	S	0.5
σ	0.20	f_1	0.2
r	0.05	f_L	0.1
Π	2	f_H	0.2
π	1	ω	0.6

Table 1: The Baseline Parameter Set

Before moving further, note also that the duopoly value derived above is when firm 2 leaves the market first. It could also be that the use of debt induces firm 1 to exit the market first. This will be the case when type L firm 2 signals its type to firm 1 and firm 1 exits first at its duopoly exit trigger. Making arguments analogous to those in the derivation of boundary condition (10), the equity value of the winning firm can be written as:

$$\begin{aligned}\bar{E}^k_{2d} &= \frac{\pi x_t}{r-\mu} - \frac{f_k + c_k}{r} + \\ &\left\{\frac{(\Pi-\pi)x_{1d}}{r-\mu} + \left[\omega S + \frac{f_k + c_k}{r} - \frac{\Pi \bar{x}^k_m}{r-\mu}\right]\left(\frac{x_{1d}}{\bar{x}^k_m}\right)^{\lambda_2}\right\}\left(\frac{x_t}{x_{1d}}\right)^{\lambda_2} \quad (34)\end{aligned}$$

The use of debt, however, comes at a cost. In fact, this cost is what makes the use of debt a credible signal. The no-mimicking comdition, $\frac{r\omega S + f_H + c_L}{rS + f_H} > 1$ implies that $\frac{r\omega S + f_L + c_L}{rS + f_L} > 1$. Hence, $\bar{x}^L_m > x^L_m$. That is, type L firm 2 reduces its monopoly tenure. It would nevertheless be willing to signal as long as $\bar{x}^L_r \leqslant x_{1r}$. The latter restriction allows type L firm 2 to preserve its competitive advantage. In this case, type L firm 2 could still play x^L_m and leave the market last. Combining this argument with the one presented in the previous page and noting that $f_H = f_1$, type L firm 2 should specify c_L such that

$$(1-\omega)rS < c_L \leqslant (1-\omega)rS + (f_H - f_L) \quad (35)$$

4.2 Comparative Statics

To carry out the comprative statics exercise, a base-case parameter set is determined. The parameter set is given in Table 1. With this choice of the parameter set, the uninformed firm's expected payoff from exiting at the monopoly trigger exceeds that from leaving at the duopoly trigger. Therefore, the baseline parameters ensure that type L firm 2 has incentive to signal its type.

Figure 3 maps the equity value of type L firm for various levels of the monopoly profits factor, Π. The solid line in the figure depicts the base-

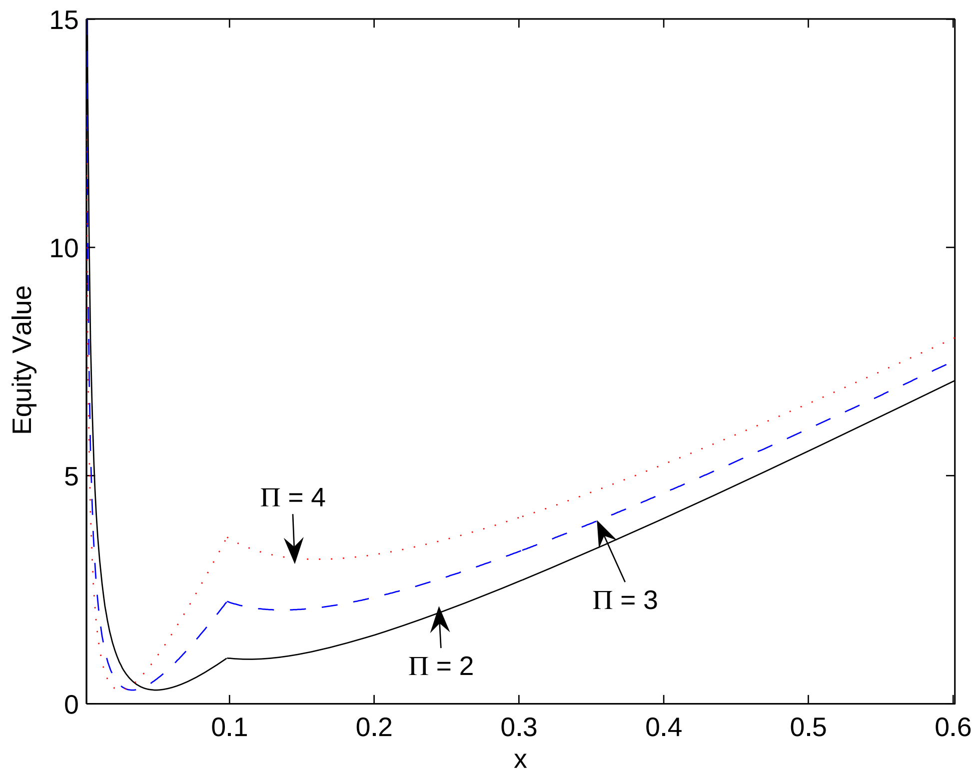

Figure 3: Comparative Statics with respect to Monopoly Benefits. The parameters are $\mu = -0.01, \sigma = 0.2, r = 0.05, \pi = 1, S = 0.5, f_1 = f_H = 0.2, f_L = 0.1, \omega = 0.6$

line case[11] and the kinks in the graphs capture the point at which firm 1 leaves the market. Note that $\frac{\Pi}{\pi}$ measures the incremental benefit from becoming a monopolist firm. Except for small values of the state variable, x_t, larger monopoly benefits translate into higher equity value as Π, and hence this fraction increases. This suggests that information revelation through debt-signaling becomes more essential as the informed firm reaps off larger monopoly profits.

Figure 4 relates the equity value to the changes in the portion of the salvage value retrieved by the equityholders upon exit. As expected, equity value is higher when equityholders obtain a larger fraction of the salvage value upon exit. Figure 5, on the other hand, provides a more interesting insight for the debt policy. In the previous section, we have specified a debt contract that satisfies two conditions. First, it ensures that the type

[11] In all of the subsequent comparative statics graphs, the curves with the solid lines designate the benchmark case.

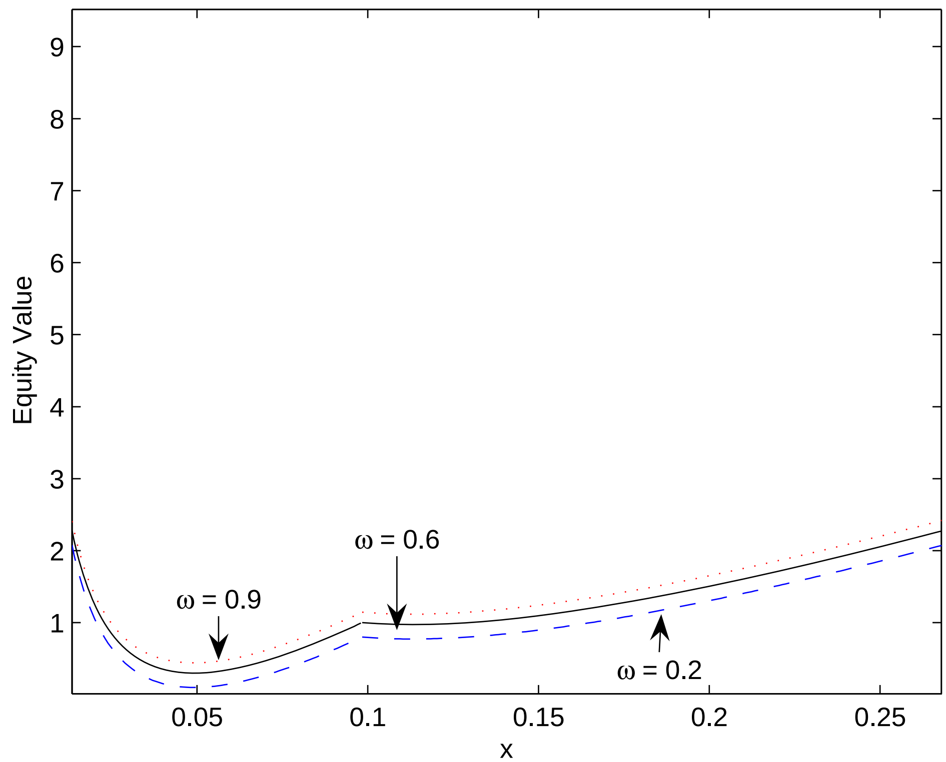

Figure 4: Comparative Statics with respect to ω. The parameters are $\mu = -0.01, \sigma = 0.2, r = 0.05, \Pi = 2, \pi = 1, S = 0.5, f_1 = f_H = 0.2, f_L = 0.1$

H firm cannot mimic and second, the coupon is determined such that the signaling firm does not lose its competitive advantage over the uninformed firm. However, the firm is not indifferent to the choice of coupon even if these two requirements are satisfied. Except for the small values of the state variable, Figure 5 shows that a higher coupon reduces the equity value. The difference is more emphasized as the shock process increases. As the shock process increases, firm 1 is less likely to leave the market soon. On the other hand, the signaling firm faces the obligation to pay the coupon and must wait longer to reap off the monopoly benefits. This analysis suggests that type L firm 2 prefers to set its coupon as low as possible. Recall from inequality (35), however, that the lower bound of the separating coupon contract is a strict inequality. Therefore, the signaling firm chooses a coupon in the $\epsilon > 0$ neighborhood of the lower bound.

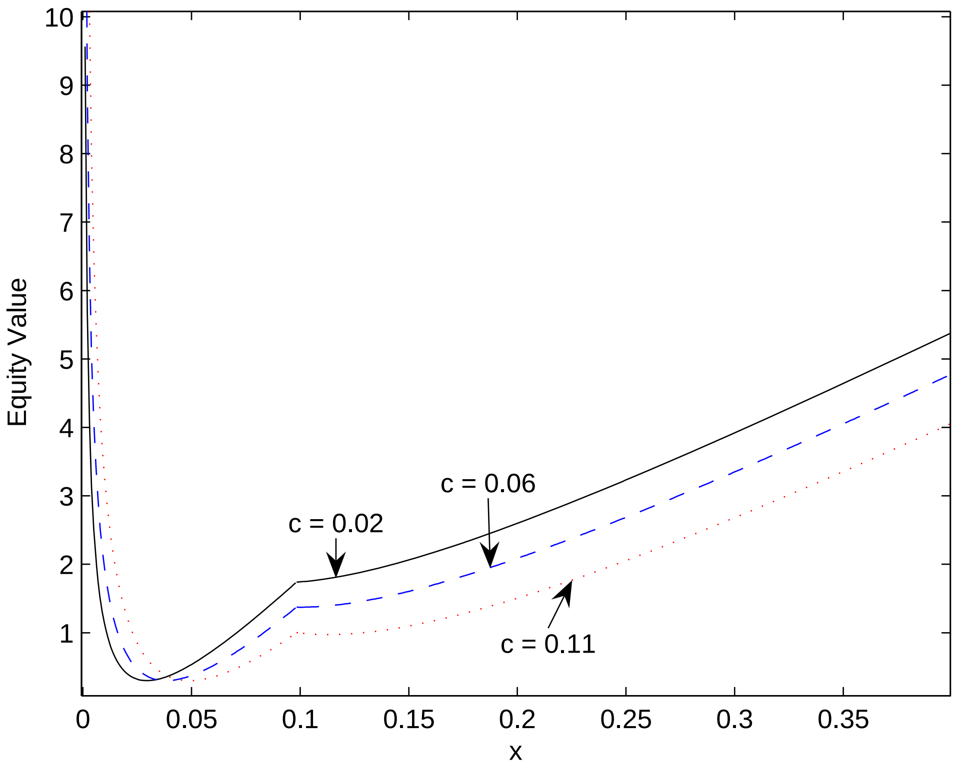

Figure 5: Comparative Statics with respect to Coupon. The parameters are $\mu = -0.01, \sigma = 0.2, r = 0.05, \Pi = 2, \pi = 1, S = 0.5, f_1 = f_H = 0.2, f_L = 0.1, \omega = 0.6$

4.3 Equilibrium

This section characterizes the conditions under which the use of debt as outlined in Section 4.1 constitutes a perfect Bayesian equilibrium (PBE). We first describe the signaling game. By its choice of issuing debt, firm 2 sends a signal to firm 1 that may reveal its type (depending on whether the equilibrium is pooling or separating). Let $\Sigma = \{s_L, s_H\}$ denote the set of signals that type $k = L, H$ firm 2 sends, respectively. Firm 2 may either issue debt (D) or may remain unlevered (ND). Firm 1 observes the signal and then must decide whether to exit at its duopoly or monopoly threshold. The decision is based on firm 1's belief that the signal comes from a particular type. On the other hand, firm 2 also has to decide when to leave the market. In other words, after the signal has been observed, firms play simultaneously a game in exit triggers as in Section 3. For simplicity, we denote the duopoly and monopoly strategies of firms 1 and 2 as $\{d, m\}$, respectively. To further simplify the exposition, we define the following action sets:

$$\begin{aligned} a_1 &= \{d, d\} \\ a_2 &= \{d, m\} \\ a_3 &= \{m, d\} \\ a_4 &= \{m, m\} \end{aligned} \tag{36}$$

where $a_i = \{f1, f2\}, i = 1, ...4$ designate the moves of firm 1 and firm 2, respectively. For instance, $a_1 = \{d, d\}$ states that both firms leave when the shock process hits their respective duopoly triggers.

The next step is to define the set of actions that the firms take after firm 1 observes firm 2's message. Let $\theta = \{a_i, a_j\}, i, j = 1, ...4$ denote a particular move by the firms when firm 2 has $\{D, ND\}$. For example, if $\theta = \{a_1, a_2\}$, firms play a_1 when firm 2 issues debt and a_2 when firm 2 has no debt. Hence, our task in this section is to show that $\theta = \{a_2, a_3\}$ is a separating PBE when $\Sigma = \{D, ND\}$. In words, if firm 2 is type L, it signals by a debt contract as outlined in Section 4.1. Then, firm 1 exits at its duopoly trigger and firm 2 leaves at its monopoly threshold. If, on the other hand, firm 2 is of type H, it does not issue (because mimicking changes its exit trigger and gives firm 1 competitive advantage) and the game is played between two identical firms. As argued in Section 4.1, in this case, there are two equilibria: The first involves firm 1 leaving first at its duopoly threshold and firm 2 leaving at its monopoly trigger. The second equilibrium is the opposite.

Finally, we recall the definition of equilibrium in a signaling game. A PBE must satisfy three requirements:[12]

1. The firm with incomplete information (i.e. firm 1) must have a belief about the type of its competitor after having observed the message. To capture this (conditional) belief, let p denote the probability that firm 2 is of type L if it has debt. On the other hand, q denotes the probability that firm 2 is of type L if it has no debt.

2. Both firms maximize their value. In the current context, if firms have no debt, they maximize the total firm value whereas if firms have debt, they maximize over the equity value.

3. The beliefs of firm 1 must be "reasonable". By reasonableness, we mean that firm 1 updates its beliefs after the firm 2's message in a Bayesian way.

[12]Note that these are only the minimal requirements that refine the Bayesian-Nash equilibrium of static games of incomplete information. Additional refinements can be added to those mentioned here based on the specific context.

We now show that $\theta = \{a_2, a_3\}$ and $\Sigma = \{D, ND\}$ form a PBE. This strategy implies that firm 1 leaves at its duopoly trigger if it observes that firm 2 has debt and at its monopoly threshold otherwise. At the same time, firm 2 leaves at its monopoly trigger if it is type L and has debt and at its duopoly threshold if it is type H and has not issued debt. Suppose that $(1-\omega)rS < c_L \leqslant (1-\omega)rS + (f_H - f_L)$. That is, type L firm 2 sets c_L such that if mimicked, the exit triggers of firm 2 exceed those of firm 1. In terms of the signals, given that $\theta = \{a_2, a_3\}$, $\Sigma = \{D, ND\}$ is the best response for firm 2 if

- for type L firm, payoff from signaling with debt exceeds that from without debt. That is,

$$\bar{E}^L_{2d} + E^L_{2m} > V^L_{2d} \tag{37}$$

  In equation (37), the left-hand-side shows the payoff to type L firm 2 from signaling with debt and the right-hand-side is the payoff if it does not issue debt. If type L firm 2 signals with debt, it will have revealed its type and thus it exits the market last at its monopoly trigger. On the other hand, if it does not signal and $\theta = \{a_2, a_3\}$ is played, firm 2 leaves at its duopoly trigger and firm 1 at its monopoly trigger. Since $x_{1m} < x^L_d$ by assumption, firm 2 would only earn duopoly profits in this case.

- for type H firm, it holds that

$$V^H_{2d} > \bar{E}^H_{2d} + E^H_{2m} \tag{38}$$

  In equation (38), the left-hand-side is the value from issuing no debt. If type H firm 2 issues no debt and exits at its duopoly trigger while firm 1 exits at its monopoly trigger, then it simply earns duopoly profits as given in equation (17) with $k = H$. The right-hand-side shows the payoff from having debt and exiting at the monopoly trigger.

If the above holds, it implies by requirement 3 that $p = 1$ and $q = 0$. That is, if firm 1 observes that firm 2 has debt, it deduces that firm 2 is of type L. It now remains to check whether $p = 1$ and $q = 0$ satisfy the belief requirement of firm 1. That is, we need to determine for which values of p and q firm 1 would be willing to exit at its duopoly trigger when it observes debt and at its monopoly threshold otherwise. In particular,

- the expected payoff from duopoly should be greater than that from monopoly when the competitor has debt:

$$pV_{1d} + (1-p)(V^H_{1d} + \bar{V}_{1m}) > p\bar{V}_{1d} + (1-p)(V^H_{1d} + V_{1m}) \tag{39}$$

In equation (39), the left-hand-side is the value from leaving the market at the duopoly trigger. Note that firm 1 weights its payoff since with probability p, the levered competitor is of type L. With probability $(1-p)$, the levered rival is of type H. Since, in this case, type H will have mimicked type L firm 2, its exit triggers would be greater than those of firm 1 and it would therefore leave the market before firm 1. On the right-hand-side, we have the payoff from leaving at the monopoly trigger. With probability p, the competitor is a type L firm that leaves at its monopoly trigger even if firm 1 leaves at its monopoly threshold (note that this follows from the fact that $\bar{x}_m^L < x_{1m}$ and Proposition 2). Hence, firm 1 remains a duopolist until its exit at the monopoly trigger and $\bar{V}_{1d}$ is obtained by imposing exit at the monopoly threshold on the duopoly value of the firm. With probability $(1-p)$, it is a type H firm that leaves at its duopoly trigger if firm 1 leaves at its monopoly threshold.

- the expected payoff from monopoly should be greater than that from duopoly when the competitor has no debt:

$$q\bar{V}_{1d} + (1-q)(V_{1d}^H + V_{1m}) > qV_{1d} + (1-q)V_{1d} = V_{1d} \tag{40}$$

The left-hand-side in equation (40) shows the payoff from leaving the market at the monopoly trigger. With probability q the competitor is of type L and its rational response would be to exit at the monopoly trigger. Hence, firm 1 would only earn duopoly profits until its monopoly trigger. With probability $(1-q)$ firm 1 faces a type H firm 2 whose best response is to exit at its duopoly trigger. On the right-hand-side, when firm 1 exits at its duopoly trigger, the best response of both types of firm 2 is to exit at their monopoly triggers. Hence, no matter what type the competitor is, firm 1 earns duopoly profits in this case.

Before concluding, it is useful to put the model into context by reviewing the literature. We also specify some of the issues that need further consideration but are not addressed in detail in this paper. Gertner *et al.* (1988) develop a model of financial signaling in a static framework. In their model, the signal sent through the capital structure choice is received both by the capital market and the product market. The type of equilibrium (pooling or separating) depends on the particular type of interaction between the firms. Specifically, an incumbent firm prefers to signal its type only if it faces a potential entrant whereas the reasonable equilibria are pooling when if the competitor is already in the market. Our dynamic model deviates from the Gertner *et al.* approach in several respects. First, we consider an exit game whereas exit is not explicitly modelled in Gertner *et al.*. As Murto (2004) states, strategic interactions have different implications for exit games than for investment and/or entry games. The fundamental question in the latter is preemption whereas in the former the game is a war of attrition. Second, Gertner *et al.* consider signaling with a linear contract that encompasses both debt and equity. In our model, we consider the signaling role of straight debt. In this regard, it is desirable not to confine signaling to a specific contract but rather to determine the characteristics of the signaling contract. This approach allows one to specify a richer set of contracts including hybrid securities that might resolve asymmetric information. Third, Gertner *et al.* provide a more detailed analysis of the equilibrium in their model. They characterize the environments in which separating and pooling equilibria are obtained. In contrast, we focus on separating equilibria in this paper. This stems from the fact that our model assumes that the only strategic decision to take is determining when to leave the market. On the other hand, the equilibrium in Gertner *et al.* depends on whether there is a potential entrant or not. Therefore, future research needs to address in more detail the interaction between market characteristics and equilibria. Another important issue regarding equilibria concerns the existence of multiple equilibria. We have seen that when firm 1 is identical to one of the firm 2 types, there are two (pure strategy) symmetric equilibria. The interesting question is how one chooses between the two. Thijssen *et al.* (2003) address this problem. Instead of making unsatisfactory assumptions as to how one decides which equilibrium is attained, they attempt to solve the coordination problem by redesigning the game. It is also worth investigating whether capital structure choice could remedy this problem. Finally, Poitevin (1989) explores a model of financial signaling with the "deep-pocket argument". In his model, the potential entrant must signal its quality by issuing debt. However, this creates predation incentives for the incumbent firm. A deep-pocket argument can also be important in games of exit. We have argued that type L firm 2 may find it optimal to signal its type given that it does not lose its competitive advantage over firm 1. However, if firm 1 has built up liquidity, it can then engage in predatory actions to force its competitor out of the market. This reduces the signaling value of debt and must be taken into account when determining the optimal debt contract.

5 Conclusion

Exit games in oligopolistic markets and its nexus with capital structure are a relatively neglected area compared to games in which the main research question is the investment problem. In this paper, we have investigated the strategic impact of debt on the exit timing in a duopoly with incomplete information. The model developed shows that issuance of debt does have strategic role to play. In particular, by fully revealing the private information, it can change the exit time of the rival firm and thereby ensure higher profits for the remaining firm. The model has also shown that signaling through debt bears a cost. The main cost is the fact that it reduces the monopoly tenure of the signaling firm. As discussed in the previous section, however, more research needs to be done to better assess the signaling role of debt. In particular, equilibrium issues such as the coordination problem seem to be the demanding research areas. It is also important to address whether the use of debt is *economically* significant. These issues have been left to future research.

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