Modeling Short-Term and Long-Term Correlations Between Asset Returns

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Abstract
This paper proposes a new approach for modeling the dynamics of correlation between asset returns. The correlation matrix is decomposed into a high frequency and a low frequency component combining the univariate spline GARCH model and a simple DCC specification. Conditional correlations are allowed to mean-revert to slowly time-varying unconditional correlations. The proposed methodology is used for modeling correlation between stock and bond returns. Unconditional correlations are found to increase/decrease over time as a result of increases/decreases in the uncertainty of macroeconomic fundamentals.

1. Introduction
The correlation of financial asset returns is an important theme in financial economics. Correlation estimation and forecasting has important implications in price formation, asset allocation and risk management. Over the last years,
a significant amount of literature has explored the dynamics of asset return comovements although the primary focus has been on univariate modeling. This paper introduces a new approach for modeling both short-run and long-run correlations that allows for a direct link between the time-variation in correlation and economic factors.

The time-varying nature of asset returns correlation has been highlighted in a series of studies (e.g., Von Fustenberg and Jeon, 1989, Koch and Koch, 1991, Erb, Harvey and Viskanta, 1994, Longin and Solnik, 1995). Following the success of univariate GARCH models in volatility modelling, novel and promising multivariate GARCH volatility models have been proposed (e.g., Bollerslev, Engle and Wooldridge, 1988, Bollerslev, 1990, Engle and Kroner, 1995, Alexander, 2001, Engle, 2002, and Tse and Tsui, 2002). Most of these studies estimate and forecast correlation using purely time series information and usually suffer from over-parameterization or/and estimation difficulties. A limited number of studies have developed multivariate specifications that use structural models to capture asset return comovements based, however, on restrictive assumptions (e.g. Ang and Bekaert, 2002, Ang and Chen, 2002, Bekaert, Hodrick and Zhang, 2005). More recently, Engle and Rangel (2006) have proposed the univariate spline GARCH model that decompose the dynamic volatility structure into a short-term and a long-term component. In a succeeding paper, Engle and Rangel (2007) extended their spline GASRCH model to a multivariate context. The factor spline GARCH model proposed captures the short-run and long-run features of stock correlations and allows for a link between high-frequency correlation and low-frequency economic fundamentals. However, their approach is based on the assumption of a one factor pricing model and becomes over-complicated for more than one factor models.

This study proposes a simple model of asset return correlations that allows conditional correlation to mean revert toward a time-varying function. The proposed econometric methodology incorporates the univariate spline GARCH model of Engle and Rangel (2007a) into a Dynamic Conditional
Correlation specification keeping the simplicity and parsimony of the DCC model. The dynamics of the short-term component of both volatilities and correlations are driven exclusively by GARCH models while the long-term component of correlation is a slowly changing deterministic function of time. The estimated low frequency correlations can be examined in relation with macroeconomic fundamentals. The proposed methodology is applied to the measurement of the correlation between stock and bond returns in the G7 countries.

The remainder of this paper is organized as follows. In Section 2, we describe the econometric methodology proposed for modeling conditional and unconditional correlation. Section 3 introduces the data used in the empirical analysis and reports some preliminary statistics. Section 4 introduces the macroeconomic fundamentals used as potential determinants unconditional correlations between stock and bond returns and presents the empirical results of the study. Finally, concluding remarks are made in Section 5.

2. An Econometric Methodology for Unconditional Correlation – the SPIINE DCC Model

In a recent paper, Engle and Rangel (2006) propose a univariate GARCH model that incorporates a time-varying level of unconditional correlation. Consider the univariate spline GARCH(1,1) model for the return of asset $i$ at time $t$:

$$ r_{i,t} = \mu_i + \sqrt{g_{i,t}} \varepsilon_{i,t}, \text{ where } \varepsilon_{i,t} | \Phi_{t-1} \sim N(0,1) \quad (1) $$

$$ g_t = (1 - \beta_1 - \beta_2) + \beta_1 \frac{\varepsilon_{i,t-1}^2}{\tau_{t-1}} + \beta_2 g_{t-1} \quad (2) $$
where \( \Phi_t \) denotes the information set including the history of asset returns up to time \( t \). Equation (2) characterizes the short term volatility dynamics keeping the nice properties of a GARCH model. The unconditional expectation of \( g_{i,t} \) equals 1 implying that the unconditional asset volatility depends exclusively on the \( \tau_{i,t} \) term, i.e.

\[
E[r_{i,t}^2] = E[g_{i,t} \tau_{i,t}] = \tau_{i,t}
\]  

(3)

The volatility component \( \tau_{i,t} \) is a deterministic function of time and can be interpreted as the unconditional volatility of asset \( i \). Engle and Rangel (2007a) model the long run volatility component using an exponential quadratic spline with generates a positive smooth curve based exclusively on data evidence. The quadratic spline is specified as the sum of \( k \) truncated quadratic basis functions:

\[
\tau_i = \gamma_0 \exp\left(\gamma_1 t + \gamma_2 t^2 + \sum_{j=1}^{k} \omega_j \left(t-t_j\right)_+^2\right)
\]

(4)

where the ordered sequence \( \{t_i\}_{i=1}^{k}, t_i > 1 \) and \( t_k \leq T \) represents the division of time into \( k \) equally spaced intervals and \( (t-t_i)_+ = \begin{cases} (t-t_i) & \text{if } t > t_i \\ 0 & \text{otherwise} \end{cases} \).

The proposed econometric methodology extends the above described specification in a multivariate context. The individual asset volatilities are modeled as univariate spline GARCH(1,1) processes based on equations (1) - (4). The conditional correlation dynamics are modeled based on the Dynamic Conditional Correlation developed by Engle (2002) allowing for a time-varying long-term correlation component.

\[
H_i = D_iR_iD_i
\]

(5)

\[
\varepsilon_i = D_i^{-1}r_i
\]

(6)
where $H_t = E_{t-1}(r_t r_t')$, $r_t$ is the $n \times 1$ vector of asset returns at time $t$, $D_t$ is a $n \times n$ diagonal matrix of the time-varying standard deviations from the univariate spline GARCH models with $\sqrt{h_{i,t}}$ as the $i$th diagonal element, $R_t$ is the time varying correlation matrix with ones on its diagonal and the conditional pairwise correlations $\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}$ on its off-diagonal elements, $T_t$ is a $n \times n$ matrix with ones on its diagonal and $\tau_{ij,t}$ as off-diagonal elements, $I_n$ is the $n \times n$ identity matrix and $\Gamma_t$ is a $n \times n$ symmetric matrix with element $\gamma_{ij,t}$.

As shown by Engle and Sheppard (2001), the positive definiteness of $Q_t$ ensures the positive definiteness of $R_t$. A sufficient condition for $Q_t$ to be positive definite is that both $T_t$ and $\Gamma_t$ are positive definite. The positive definiteness of both $T_t$ and $\Gamma_t$ is ensured if $\tau_{ij,t}$ is in the (-1,1) interval and the initial $\Gamma_0$ is positive definite.

The specification of $\Gamma_t$ is based on a GARCH(1,1) model and characterizes the short-term component of conditional correlations. The unconditional expectation of $\Gamma_t$ equals the identity matrix implying that the unconditional expectation of $Q_t$ is governed exclusively by $T_t$, i.e. $E[q_{ij,t}] = E[\rho_{ij,t}] = \tau_{ij,t}$. The term $\tau_{ij,t}$ is a deterministic function of time based on a quadratic spline specified as follows:
$$
\tau_{ij,t} = \frac{\exp\left\{2 \left( w_{0ij} + w_{1ij} t + \sum_{i=1}^{k_i} w_{ij} ((t - t_{i-1})_+) \right)^2 \right\} - 1}{\exp\left\{2 \left( w_{0ij} + w_{1ij} t + \sum_{i=1}^{k_i} w_{ij} ((t - t_{i-1})_+) \right)^2 \right\} + 1}, \quad -1 \leq \tau_{ij,t} \leq 1 \quad (10)
$$

The number of knots, $k_i$ and $k_{ij}$, can be optimally selected using the Schwarz or the Akaike information criterion.

The two-stage estimation procedures of the standard DCC model can also be applied for the estimation of the proposed spline DCC model. In the first stage, univariate spline GARCH models are estimated for each returns series. In the second stage returns standardized by their standard deviations estimated at the first stage are used to estimate the parameters of the spline dynamic correlation model. The resulting log likelihood function is the following:

$$
L(\theta) = L_v(\theta) + L_c(\theta) \quad (11)
$$

where

$$
L_v(\theta) = -\frac{1}{2} \sum_{t=1}^{T} \left( T \log(2\pi) + \log(|D_t|^2) + r_t^2 r \right) \\
= -\frac{1}{2} \sum_{t=1}^{T} \left( T \log(2\pi) + \log(\tau_{ij,t}) + \frac{r_t^2}{\tau_{ij,t}} \right) \quad (12)
$$

is the volatility component of the likelihood, and

$$
L_c(\theta) = -\frac{1}{2} \sum_{t=1}^{T} \left( \log(|R_t|) + \epsilon_t R_t^{-1} \epsilon_t \right) \quad (13)
$$

is the correlation component of the log likelihood. In the fist stage the volatility component is maximized by maximizing the log likelihoods of the individual spline GARCH equations. The second stage is estimated using the
correctly specified likelihood, conditioning on the parameters estimated in the first stage likelihood.

3. Data Description
The Spline DCC model is applied to stock-bond correlations in G7 countries (US, UK, France, Germany, Japan, Canada and Italy). Our dataset consists of stock market returns and bond market returns at a daily frequency and macroeconomic variables at a quarterly frequency. We use the national total market return stock indices and total return government bond indices for maturities greater than 10 years at a daily frequency for the period 01/01/1990-29/12/2000. Stock and bond indices are collected from Datastream International and expressed in local currency. Moreover, stock price indices are not corrected for dividend payments (as suggested by Berben and Jansen, 2005). Long-term government bonds with more than 10 years of maturity have been used to match their duration with stocks, which are generally viewed as long-term investments. The continuously compounded market returns are computed as the log difference of the closing index levels from one trading day to the other, i.e. \( r_{i,t} = \ln \left( \frac{P_{i,t}}{P_{i,t-1}} \right) \), where \( P_{i,t} \) is the closing index level for country \( i \) at day \( t \).

The sources for our macroeconomic variables are the IMF International Financial Statistics database and the OECD Main Economic Indicators database. The macroeconomic series include the Consumer Price Index, the real Gross Domestic Product, the short-run nominal interest rate and the real personal consumption expenditure.

To provide some perspective of the data, Table 1 presents the sample statistics of stock and bond returns for the G7 countries. Over the last 16 years the mean non-dividend adjusted stock returns vary from -0.009% for Japan to 0.0353% for Canada while the mean bond returns vary from 0.0192% for

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1 As a further step in our empirical analysis we intend to use the dividend adjusted stock index returns.
Japan to 0.0398% for Italy. Stock and bond return correlations are within the range of -0.07 to 0.15. Most of the returns series are highly persistent based on the Ljung-Box(20) Q statistic and exhibit negative asymmetry and fat tails.

4. **High and low-frequency correlations between stock and bond returns and macroeconomic factors**

Many researchers have tried to understand the time-varying nature between stock and bond returns including amongst other Campbell and Ammer (1993), Fleming, Kirby and Ostdiek (1998), David and Veronesi (2001), Stivers and Sims (2002), Conolly, Stivers and Sun (2005), Li (2002) and Baele, Bekaert and Inghelbrecht (2007). These studies indicate that economic uncertainty explains some of the variation in the volatilities and covariances of stock and bond returns. The Spline Dynamic Conditional Correlation model is applied for estimating the long-term and short term component of time-varying correlations between the stock and bond market returns of the G7 countries. The model specification allows the linking of long-term correlations with high-frequency economic variables.

Figure 1 shows the graphs of the estimated conditional and unconditional correlations for the stock and bond returns series of the G7 countries over the sample period. There are significant variations in the conditional correlations of the stock and bond returns. Moreover, the assumption that correlations similar to volatility revert towards a constant appears to be rejected for the data series examined. The unconditional correlations exhibit a cyclical behavior that might be attributed to the business cycle and economic fundamentals.

The second part of our empirical analysis examines the macroeconomic variables that might explain the time variation in stock and bond unconditional correlations. Given that correlations are not observable, an estimate of unconditional correlations is needed to construct the dependent variable of our analysis. For each country, we use the Spline DCC model
introduced previously in the paper and derive the unconditional expectation of correlation as described in equation (10). Thus, the dependent variable of our model is the quarterly average of the estimated unconditional correlation for each country.

Concerning the economic factors used in the analysis, it is important to note that it is the uncertainty, rather than the levels, of the macroeconomic variables that affects correlation between stock and bond returns. As suggested by Li (2002) and Baele, Bekaert and Inghelbrecht (2007) expected inflation influences the discounting of expected and unexpected cash flows of stock and bonds. Thus, the inflation uncertainty is an economic factor that might explain the correlation between stock and bond returns. Quarterly inflation rate is constructed as the log-difference of the Consumer Price Index. We proxy the inflation uncertainty with the conditional volatility of inflation generated from a GARCH(1,1) model.

Bekaert, Engstrom, and Xing (2005), Bansal and Yaron (2004) and Wachter point out that economic uncertainty and risk aversion may affect risk stock and bond risk premia. We use the uncertainty on GDP growth as a measure of economic uncertainty and uncertainty on consumption expenditure growth as a measure of risk aversion following Baele, Bekaert and Inghelbrecht (2007). Both measures of uncertainty are constructed on the basis of the conditional GARCH(1,1) volatility of the initial economic fundamentals. Finally, most of the variation in bond returns is driven by the levels of the interest rates. We proxy interest rate uncertainty with the conditional variance of real interest rates from a GARCH(1,1) model.

A panel of the G7 countries data for the period 1990-2006 is constructed in order to explore the economic sources of the cross-sectional variation in stock and bond market unconditional correlations. We estimate the following system:

\[ UC_{i,t} = X_{i,t} \beta + u_{i,t}, \quad i = 1, ..., N, \quad t = 1, ..., T \]  

(14)
where $X_{i,t}$ is the $k \times 1$ vector of explanatory variables and $u_{i,t}$ is the error term with the assumption

$$E(X_{i,t}u_{i,t}) = 0, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T$$

(15)

The model is estimated using the Seemingly Unrelated Regressions (SUR) method developed by Zellner (1962) to take into account possible serial correlation in the unobserved error terms.

In a preliminary analysis, we examine the effect of each of the explanatory variables on unconditional correlations with individual SUR regression for each variable. The cross-sectional results of the estimation are presented in Table 2. A significant positive relationship is observed for all regressors i.e. the uncertainty of inflation rate, consumption growth, GDP growth and real interest rate differential. Next, we estimate the full system of equations for unconditional correlations as dependent variable including the four economic factors as explanatory variables. The results of the SUR regression are presented in Table 3.

The results indicate that uncertainty in GDP growth, consumption growth and inflation have a significant positive effect on stock-bond market correlations. The positive sign of expected inflation uncertainty as an explanatory variable of stock-bond relationship is consistent with the empirical findings of Li (2002). In periods of high economics uncertainty, asset returns tend to be more volatile giving the incentive to investors to diversify more. The consequent increase of stock-bond correlations will reduce the diversification opportunities when they are needed most. Real short-term interest rate uncertainty has a significant negative return on stock-bond correlations. This finding could be attributed to the fact that stock return are not adjusted for dividends and interest rates uncertainty influence the discount factor of cashflows only for bond coupon payments.
5. Concluding Remarks

We develop a multivariate volatility model that generalizes dynamic conditional correlation models by allowing both conditional volatilities and correlations to mean revert toward a time-varying trend. Dynamic correlations are decomposed into a short-term and a long-term component. The short-term component is modeled through a GARCH specification while the long-term component is modeled as a slowly time-varying trend based on a spline function. The proposed generalization does not affect the estimation simplicity of the original DCC model and estimation remains feasible even for a large number of assets.

The proposed methodology is applied in the investigation of the stock and bond correlations across G7 countries over the past 16 years. The empirical evidence suggests that the uncertainty of macroeconomic fundamentals is an important determinant of stock-bond correlations. More specifically, the uncertainty in GDP growth, inflation and consumption expenditure present a significant positive effect in long-term correlations. This result is bad news for investors since during period of high inflation risk and economic uncertainty the diversification benefits decrease. In addition, uncertainty in short-term interest rates show a significant negative effect in stock-bond correlations.
References


Figure 1
Conditional and unconditional correlations estimated from the Splice DCC
Table 1
Descriptive statistics for stock and bond index returns (%) for the G7 counter over the period 01/01/1990-29/12/2006

<table>
<thead>
<tr>
<th>Bond Index Return</th>
<th>Mean Return</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Q(20)</th>
<th>Stock Index Return</th>
<th>Mean Return</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Q(20)</th>
<th>Stock and bond returns correlation</th>
</tr>
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<tbody>
<tr>
<td>Canada</td>
<td>0.0326</td>
<td>0.0042</td>
<td>-0.3046</td>
<td>3.0533</td>
<td>46.478</td>
<td>0.0352</td>
<td>0.8260</td>
<td>0.5966</td>
<td>-0.5966</td>
<td>6.1101</td>
<td>54.673</td>
<td>0.0414</td>
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<td>France</td>
<td>0.0305</td>
<td>0.3500</td>
<td>-0.1863</td>
<td>2.7860</td>
<td>17.628</td>
<td>0.0291</td>
<td>1.1689</td>
<td>-0.2095</td>
<td>-0.5966</td>
<td>3.0334</td>
<td>44.336</td>
<td>0.0643</td>
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<td>Germany</td>
<td>0.0262</td>
<td>0.3139</td>
<td>-0.6539</td>
<td>3.7878</td>
<td>45.048</td>
<td>0.0227</td>
<td>1.1331</td>
<td>-0.3827</td>
<td>-0.1863</td>
<td>2.7860</td>
<td>43.318</td>
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<td>0.0398</td>
<td>0.4066</td>
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<td>7.2930</td>
<td>69.235</td>
<td>0.0287</td>
<td>1.2631</td>
<td>-0.1930</td>
<td>-0.6539</td>
<td>3.7878</td>
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<td>34.672</td>
<td>-0.0090</td>
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### Table 2
SUR individual regressions of unconditional correlations on each explanatory variable

<table>
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<tr>
<th></th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>P-value</th>
<th>Determinant residual covariance</th>
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<td>Consumption growth uncertainty</td>
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<td>0.0262</td>
<td>7.0226</td>
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<td>6.23E-12</td>
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<td>2.3923</td>
<td>8.6023</td>
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### Table 3
SUR regression of unconditional correlations on all explanatory variables

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<th>t-Statistic</th>
<th>P-value</th>
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<td>5.6827</td>
<td>0.0000</td>
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<tr>
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<td>Determinant residual covariance</td>
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