American GARCH Employee Stock Option Valuation*

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Abstract

We implement a new and flexible simulation based approach for the fair value of Employee Stock Option (ESO) plans to value American options in order to consider the common ESO characteristics of vesting period, departure risk and voluntary suboptimal early exercise. We introduce GARCH effects on the underlying asset and we analyze the price bias with respect to the constant volatility case. We also find a significant bias if we value the ESO using a short-memory GARCH model instead of a long-memory one. We also develop a sensitivity analysis with respect to changes in several ESO characteristics. We compare this valuation with FAS 123 method using the implicit expected life of the ESO revealing a FAS overvaluation. Finally, we value a real ESO plan providing the confidence intervals for the estimated ESO prices.

JEL: C15, C22, G13

Keywords: Employee Stock Option; GARCH; Least-Squares Monte Carlo; fair value; FAS 123.

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1 Introduction

Recent accounting standards developments require firms to account employee stock option (ESO) grants using a fair value method and recognize this value as a cost. If we compare ESOs with conventional traded options, it is shown that ESOs exhibit several different features[1]. They usually have an initial vesting period for which the exercise is not allowed. The ESO holder is also subject to a departure risk. If he leaves the firm, either voluntarily or not, he must exercise immediately the ESO though it may be suboptimal. Nevertheless, if the departure occurs during the vesting period, the employee looses the ESO. In contrast with conventional options, ESOs are not transferable. Furthermore, the holder is not allowed to hedge his ESOs taking short positions in the company's stock. Because of either diversification or liquidity reasons, the employee could exercise the ESO suboptimally in contrast to being a tradable option. The related empirical evidence of this fact can be found, among others, in Huddart and Lang (1996), Carpenter (1998) and Bettis et al. (2005). In short, as accounting standards establish, the fair-value based method should incorporate, at least, the stylized facts of vesting period, departure risk and suboptimal early exercise.

This paper aims to value ESOs solely from the firm's perspective. We really obtain two different alternative values. The first is the market value, that is, the option value when the ESO holder is unconstrained. The second is the objective value that recognizes the suboptimal exercise. Finally, but not studied here, there exists a subjective value corresponding to the employee's valuation point of view. The objective option value, which lies between the market and subjective ones, will be the cost (fair value) to the firm of issuing ESOs. This suggests why the Black-Scholes (1973) formula overestimates the ESO cost[2].

Many models have been developed in order to estimate the fair value of the ESO plans. Jennergren and Näslund (1993) use the Black-Scholes (1973) framework to get the extended partial differential equation (PDE) for an option that includes the early exercise as an exogenous stopping time measured by the first jump time of a Poisson process with a constant intensity or hazard rate. This Poisson process is a proxy of the executive's early exercise or forfeiture comprising the wish of either portfolio

[1] For a detailed discussion about the differences between standard traded options and ESO grants, see Rubinstein (1995).

[2] For more details about these alternative valuations, Ingersoll (2006) derives a model for both the subjective and objective values of such options.

diversification or consumption and also, the voluntary or unvoluntary employment termination. Carr and Linetsky (2000) develop an analytical specification based on the stochastic intensity framework in which the intensity or exit rate can be decomposed in two parts: the unvoluntary and voluntary exercises measured by a constant and a function of the stock price evolution respectively. Under the same framework but providing closed-form formulae, Cvitanic et al. (2006) also derive the ESO price.

Huddart (1994) and Kulatilaka and Markus (1994) develop a binomial tree model to value the ESO firm cost using a utility-based framework for determining the agent's exercise policy. Nevertheless, the authors recognize that their model cannot be used in practice since it requires some input variables that are difficult to estimate, such as employee risk aversion, non-option wealth of the employee and so forth. Carpenter (1998) compares a utility-based model with an extended American binomial option version of Jennergren and Näslund (1993). Her results demonstrate that the first model shows no improvement over the second one, which is more parsimonious, in a sample of 40 firms ranging the period from 1979 to 1984. Bettis et al. (2005) find similiar results in a larger and more recent ESO database. More recently, Hull and White (2004) have proposed a binomial approach to calculate the ESO fair value where the early exercise behavior is modeled as a barrier. After the vesting period, if the stock price reaches the barrier, the ESO is exercised voluntarily[3]. A similar approach is presented by Ammann and Seiz (2004) such that the early exercise behavior is here modeled adjusting the strike price of the option. Finally, the continuous version with barrier can be found in Ingersoll (2006).

Our main proposal is valuing typical long-dated American style ESOs when the volatility of the underlying asset is assumed to be time-varying under the GARCH framework. We use the methodology of Duan (1995) to get the risk neutral measure for the ESO valuation. We will concentrate here on pricing American GARCH options[4] and consider the main ESO features mentioned before. Our work consists of an extension of Stentoft (2005), that is based on the least-squares simulation approach of Longstaff and Schwartz (2001), for the case of ESO valuation.

For the last decades, empirical evidence supports that conditional volatilities of many daily stock-index returns exhibit long-memory[5], that is, shocks to the conditonal volatility die away at a

[3]The barrier is settled with a multiple (M) of the exercise price. Carpenter (1998) finds in her sample that, in mean, ESOs are exercised when the stock price is 2.8 times the exercise price. This value may be considered as a proxy of M.

[4]See, for instance, Ritchken and Trevor (1999), Duan and Simonato (2001), Duan et al. (2003) and Stentoft (2005). For European GARCH options, see Christoffersen and Jacobs (2004) among others.

[5]See Hyung et al. (2006) and all the references inside.

hyperbolic rate instead of the exponential rate (short-memory) under the popular GARCH structure. As expected, this feature affects the option valuation leading to significant price biases when modeling the conditional volatility of the underlying daily return under a short-memory structure instead of a long-memory one. Because of it, we consider alternative volatility dynamics for ESO pricing to capture the mispricing effects with respect to both the benchmark model based on constant volatility and the approximated valuation proposed by the Financial Accounting Standard 123 in 1995 (FAS 123). It is worth emphasizing that our suggested valuation model is in line with the International Financial Reporting Standard 2 (IFRS 2) and the 2004 revised FAS 123 (FAS 123R).

The rest of the paper is organized as follows. Our valuation method is explained in Section 2 under alternative GARCH specifications. Section 3 shows some numerical results based on a deep simulation analysis. The implications for the accounting standards are explained in Section 4. Section 5 is about pricing a real case of an ESO plan providing GARCH estimates for the underlying asset and the confidence intervals for the estimated ESO fair value. Finally, Section 6 concludes.

2 ESO valuation with GARCH type volatility

In this section, we first introduce some specifications for the volatility under the GARCH context for the ESO pricing and second, we implement the algorithm to obtain this fair value based on an American option under several restrictions.

2.1 GARCH framework

The famous Black-Scholes (1973) formula for European options assumes that the underlying stock price S_t follows a lognormal distribution and hence, the continuously compounded daily return, $R_t \equiv \ln(S_t/S_{t-1})$, is Normal distributed. Nevertheless, empirical evidence suggests that daily returns show some stylized facts like fat tails and asymmetries which lead to a clear rejection of the normality assumption of stock returns. Moreover, it does also hold the clustering phenomenon for the estimated volatility series, that is, a high (low) volatility period is followed by a high (low) volatility one. This pattern is well captured by the ARCH and GARCH models introduced by Engle (1982) and Bollerslev

(1986) respectively. Under the GARCH framework, the conditional variance is a function of lagged variances and lagged residuals. The simple GARCH model for R_t depending only on the past residual and the variance lagged once is formulated under the real measure $\mathbb{P}$ as

$$\begin{aligned} R_t &= \bar{r} - \bar{d} + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \xi_t; \qquad \xi_t = \sigma_t z_t & (1) \\ \sigma_t^2 &= \omega + \alpha\xi_{t-1}^2 + \beta\sigma_{t-1}^2 & (2) \end{aligned}$$

where z_t is assumed to be i.i.d. $N(0,1)$ and hence, $\xi_t|\mathcal{F}_{t-1} \sim N(0,\sigma_t^2)$ such that $\mathcal{F}_{t-1}$ is the time $t-1$ information set, $\bar{r}$ is the daily constant continuously compounded risk-free rate, $\bar{d}$ is the daily continuous compounded dividend yield [6] and λ is the price of risk. Equation (2) is the so-called GARCH(1,1) specification with the three parameters restricted to be positive and a persistence level of $\alpha + \beta < 1$ for covariance stationarity. The unconditional variance for the daily return in (1) given equation (2) is obtained as $\mathbb{E}^{\mathbb{P}}\left[\sigma_t^2\right] = \omega/(1-\alpha-\beta)$. This model becomes a good candidate to capture the time-varying volatility of financial series under a very parsimonious way. More general versions could also be considered and denoted as GARCH(p,q). We concentrate on $p = q = 1$ not only for this group but also for alternative specifications of σ_t^2 in future.

Under the real mesure $\mathbb{P}$, the conditional expectation of S_t from the return equation (1) is obtained as

$$\mathbb{E}^{\mathbb{P}}\left[S_t|\mathcal{F}_{t-1}\right] = S_{t-1}\exp(\bar{r} - \bar{d} + \lambda\sigma_t)$$

and the conditional variance of R_t is

$$Var^{\mathbb{P}}\left[R_t|\mathcal{F}_{t-1}\right] = \sigma_t^2.$$

Note that the conditional distribution of the one-period return is Normal distributed (both skewness and excess of kurtosis equal zero) but the unconditional one exhibits a kurtosis higher than three, that is, it allows for fat tails copying with the empirical evidence.

Later extensions of equation (2) are the asymmetric GARCH models. These models introduce an asymmetric component in the variance equation in order to consider the leverage effect, that is,

[6] Both $\bar{r}$ and $\bar{d}$ are expressed in terms of the yearly parameters r and d respectively, i.e. $\bar{r} = r\delta t$ and $\bar{d} = d\delta t$ where δt is the length of one day in years (see Section 3.1).

stock returns are negatively correlated with variance. This fact is according to the empirical evidence. In this paper, we only work on the asymmetric GARCH model introduced by Glosten et al. (1993), henceforth GJR, and it is defined as

$$\sigma_t^2 = \omega + \alpha\xi_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma\xi_{t-1}^2 \mathbb{1}_{\{\xi_{t-1}<0\}} \tag{3}$$

where $\mathbb{1}_{\{B\}}$ denotes an indicator function that takes the value of one if the logical expression B is true and zero otherwise. The persistence of GJR is obtained as $\alpha + \beta + 0.5\gamma$ that must be again lower than one for covariance stationarity. The daily unconditional variance of return in equation (1) given equation (3) is now $\mathbb{E}^{\mathbb{P}}\left[\sigma_t^2\right] = \omega/(1 - \alpha - \beta - 0.5\gamma)$. Note that equation (3) nests equation (2) when $\gamma = 0$. Moreover, if $\gamma > 0$ then the leverage effect verifies.

Ding et al. (1993) show that the absolute value of S&P 500 returns has the long-memory property, that is, the sample autocorrelation function (ACF) of either $|R_t|$ or R_t^2 —as a proxy of the volatility and variance respectively— remains significant even at long lags. Note that both GARCH and GJR models imply an exponential (fast) decay rate instead of a hyperbolic (slow) decay rate agreed with the long-memory feature for the ACF of ξ_t^2 in equation (1). Some examples of volatility models capable of producing long-memory characteristics are: (i) the fractionally integrated GARCH (FIGARCH) introduced by Baillie et al. (1996) and Bollerslev and Mikkelsen (1996) and (ii) the Component-GARCH (C-GARCH) introduced by Ding and Granger (1996) and Engle and Lee (1999).

As expected, this feature affects the option valuation leading to significant price biases when modeling the conditional volatility of the underlying daily return under a short-memory structure instead of a long-memory one[7]. We also get the ESO valuation under the long-memory volatility specification though we limit attention to the C-GARCH of Engle and Lee (1999). We select this model because it is easier to implement and it belongs to the GARCH family, in concrete, it implies a restricted GARCH(2,2) model.

The C-GARCH structure σ_t^2 of Engle and Lee (1999) can be decomposed into a permanent or long-run component, denoted as q_t in equation (5), and a transitory or short-run component, that is

[7]Some empirical studies on European options under this new context are: (i) Bollerslev and Mikkelsen (1996) for the long-maturity LEAPS on S&P 500 index under the FIGARCH framework, meanwhile (ii) Christoffersen et al.(2006) applies the C-GARCH. Finally, for the case of American options, Stentoft (2005) applies the FIGARCH model on both individual stocks and the S&P 100 index.

defined as $\sigma_t^2 - q_t$ according to equation (4), which is mean-reverting towards the trend component q_t. Hence,

$$\sigma_t^2 = q_t + \alpha(\xi_{t-1}^2 - \sigma_{t-1}^2) + \beta(\sigma_{t-1}^2 - q_{t-1}) \tag{4}$$

$$q_t = \omega + \rho q_{t-1} + \phi\left(\xi_{t-1}^2 - \sigma_{t-1}^2\right) \tag{5}$$

with all five parameters restricted to be positive, $\alpha + \beta < \rho < 1$ and $\phi < \beta$. These parameter restrictions lead to several properties: (i) $\sigma_t^2 - q_t$ reverts to zero at an exponential decay rate of $\alpha + \beta$, (ii) the component q_t evolves following an AR(1) structure and converges to a constant level or daily unconditional variance, $\mathbb{E}^{\mathbb{P}}\left[\sigma_t^2\right]$, to be equal to $\omega/(1-\rho)$ in case of $\rho < 1$ and (iii) the long-run component is more persistent since $\alpha + \beta < \rho$. Rewritting both equations (4) and (5) as functions of the innovation η_t defined as $\xi_t^2 - \sigma_t^2$, the conditional variance σ_t^2 equals $\theta_t + q_t$ where θ_t is the short-term component, defined as $\beta\theta_{t-1} + \alpha\eta_{t-1}$. The C-GARCH specification provides a flexible structure to capture a slowly decaying ACF of η_t by combining the two exponential decaying ACFs for both q_t and θ_t. If we analyze the effect of a past innovation η_{t-k} on the actual conditional variance, then $\partial\sigma_t^2/\partial\eta_{t-k} = \phi\rho^{k-1} + \alpha\beta^{k-1}$, while it becomes $\alpha(\alpha+\beta)^{k-1}$ for the GARCH model in equation (2).

2.2 ESO fair value

To obtain fair ESO prices in a GARCH context, we first apply the locally risk-neutral valuation relationship (LRNVR) in Duan (1995) which is satisfied by a measure $\mathbb{Q}$ if

$$\mathbb{E}^{\mathbb{Q}}[S_t|\mathcal{F}_{t-1}] = S_{t-1}\exp(\bar{r} - \bar{d})$$

and

$$Var^{\mathbb{Q}}\left[R_t\,|\mathcal{F}_{t-1}\right] = Var^{\mathbb{P}}\left[R_t\,|\mathcal{F}_{t-1}\right] = \sigma_t^2. \tag{6}$$

Hence, the LRNVR implies that the return dynamics evolves under $\mathbb{Q}$-measure as

$$R_t = \bar{r} - \bar{d} - \frac{1}{2}\sigma_t^2 + \xi_t^*; \qquad \xi_t^* = \sigma_t z_t^*; \qquad z_t^* \sim i.i.d.\ N(0,1) \tag{7}$$

where $z_t^* = z_t + \lambda$ in order to guarantee equation (6) and λ denotes the price of risk. In short, under $\mathbb{Q}$-measure

$$R_t \left| \mathcal{F}_{t-1} \sim N\left(\bar{r} - \bar{d} - \frac{1}{2}\sigma_t^2, \sigma_t^2\right)\right. .$$

Following Stentoft (2005), we simulate GARCH processes under the LRNVR and we apply the least-squares simulation approach to obtain the exercise rule across the paths as in Longstaff and Schwartz (2001). We implement this method to value a typical ESO characterised by an American call option with a maturity of T years, a vesting period of ν years, and a yearly exit rate or intensity ε referred to a Poisson process. We assume that the jump risk is nonpriced, that is, it can be diversified away.

The early exercise behavior is modeled as in Ammann and Seiz (2004). Their model adjusts the strike price —multiplying it by a factor m lower than one— in order to generate an early exercise behavior[8]. Voluntary exercise occurs when the modified current payoff is larger than the discounted one period expected value of the ESO, that is

$$(S_t - mK)^+ \geq e^{-r\delta t}\mathbb{E}_t^{\mathbb{Q}}\left[C_{t+\delta t}\right] \tag{8}$$

where the function $(y)^+$ is the maximum between y and 0 and $\mathbb{E}_t^{\mathbb{Q}}[\cdot]$ denotes the shortening of the conditional expectation. Note in equation (8) that if $m = 1$ we have the same exercise rule than in a tradable American option. In order to obtain an early exercise behavior, we set $m < 1$. Thus, the left side of equation (8) will be larger and the voluntary exercise will occur earlier. For $m > 1$ the exercise would be delayed. At time to maturity, the ESO is exercised if it is in the money

$$C_T = (S_T - K)^+ . \tag{9}$$

One period before, at $T - \delta t$ where δt is the time step length, on one hand there is a probability equals $1 - e^{-\varepsilon\delta t}$ to abandon the firm —either voluntary or unvoluntary employment termination, that is assumed independent of the current stock price and time to maturity— and the payoff would be $(S_{T-\delta t} - K)^+$. On the other hand, the employee remains in the firm with a probability equals $e^{-\varepsilon\delta t}$

[8] Ammann and Seiz (2004) also show that their model lead to rather the same ESO price as in the Hull and White (2004) tree if their respective early exercise parameters are calibrated so as to achieve the same expected life of the ESO.

and thus, he must decide either to hold or to exercise voluntarily the ESO. According to equation (8) he will exercise if $S_{T-\delta t} - mK > e^{-r\delta t}\mathbb{E}^{\mathbb{Q}}_{T-\delta t}\left[C_T\right]$, then the ESO value will be $S_{T-\delta t} - K$. Otherwise, the payoff will be the discounted expected value of the ESO. Thus, the ESO value at any time t verifying that $T > t \geq \nu$ is computed as

$$C_t = e^{-\varepsilon\delta t}\left[X_t \mathbb{1}_{\{S_t - mK < X_t\}} + \left(S_t - K\right)^+ \mathbb{1}_{\{S_t - mK \geq X_t\}}\right] + \left(1 - e^{-\varepsilon\delta t}\right)\left(S_t - K\right)^+$$

where $X_t = e^{-r\delta t}\mathbb{E}^{\mathbb{Q}}_t\left[C_{t+1}\right]$ is the discounted risk-neutral expectation of the ESO value. The conditional expected value of the ESO will be computed by least-squares, that is, for those paths in the money, the one period ahead ESO value (discounted), C_{t+1}, is regressed over some basis functions of both the current stock price and conditional volatility[9]. We work backwards until the vesting date with this scheme.

During the vesting period, the exercise is not allowed and therefore, if departure occurs the payoff would be zero. Moreover, the employee cannot exercise voluntarily his option. Hence, for any $t < \nu$ the option value is obtained recursively as $C_t = e^{-(r+\varepsilon)\delta t}\mathbb{E}^{\mathbb{Q}}_t\left[C_{t+1}\right]$. Finally, the ESO fair value at current date is

$$C_0 = e^{-(r+\varepsilon)\nu}\mathbb{E}^{\mathbb{Q}}_0\left[C_\nu\right]. \tag{10}$$

Note that equation (10) nests the European style ESO price for $\nu = T$.

3 Numerical study

In this section, we start pricing a hypothetical ESO with constant volatility (CV) so as to compare with our time-varying volatility framework. In a second stage, we value the same option but considering different GARCH models and parameter sets to show the effects of persistence, asymmetry and length of memory in the volatility to ESO valuation. Finally, we also implement a sensitivity analysis with respect to changes in the price of risk, vesting period and time to maturity.

[9]We have used both powers and Laguerre polynomials of current stock price, volatility and their cross product as basis functions. We find that the prices are robust to the basis functions. This evidence is in accordance with Longstaff and Schwartz (2001) and Moreno and Navas (2003).

3.1 ESO pricing under constant volatility

We consider an American call option with a maturity of $T = 10$ years. The starting price is $S_0 = 100$ and the ESO is granted at the money (the usual way). The vesting period is $\nu = 3$ years, the annual volatility of daily returns is $\sigma = 0.30$, and the annual risk-free rate is $r = 0.05$. To show the effects of the exit rate and the early exercise parameter, we implement a grid for both ε and m. Table 1 reports the price of our hypothetical ESO under different valuation methods. We consider two dividend policies: no dividend payments (Panel A) and a yearly continuously compounded dividend yield of $d = 0.025$ (Panel B). The case of $d = 0$ becomes interesting in order to isolate the possible early exercise decision of dividends from the employee's diversification restrictions measured through $m < 1$. Note that standard American call options ($\varepsilon = 0$ and $m = 1$) would never be exercised in the absence of dividends. Columns JN_E and JN_A exhibit the prices for European and American style ESO respectively, using the Jennergren and Näslund (1993) proposal. Finally, columns denoted as C_{cv} show the valuation of American ESOs with constant volatility under the simulation approach.

Both JN_E and JN_A hold the same PDE, that is

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}(r-d)S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 - rC + \lambda_e\left[\Psi(S,t) - C\right] = 0 \tag{11}$$

such that $C\left(S;K,T\right)$ is the ESO price with S as the underlying stock price and K is the strike price, λ_e is the intensity per unit time (in years) of a Poisson process for a jump event in the option —that reflects a mixture of both leaving the company and early exercise for the American style while only the first case for the European one— and $\Psi\left(S,t\right)$ is the payoff to the holder if there is a jump event. For European style ESO, $\Psi(S,t) = 0$ for all t while for American style ESO, $\Psi\left(S,t\right) = \mathbb{1}_{\{T \geq t > \nu\}}(S-K)^+$. The boundary condition at maturity of equation (11) is equation (9). Like ε in Section 2.2, the event risk driven by λ_e is nonpriced.

Suppose that ESO cannot be exercised before expiration (European ESO), the solution to equation (11) for the current date with the boundary condition is

$$C(S;K,T) = e^{-\lambda_e T} \times BS\left(S,K,r,d,\sigma,T\right) \tag{12}$$

where $BS(S, K, r, d, \sigma, T)$ is the Black-Scholes formula with a constant continuous dividend yield of d. Equation (12) suggests that the price of a European style ESO is the ordinary BS price multiplied by the probability that the employee will remain in the firm until maturity. It means that $JN_E < BS$ as Table 1 exhibits where BS price is JN_E for $\lambda_e = 0$ in both panels.

For American style ESO, its value is obtained by solving equation (11) numerically under a fully-implicit finite difference method. Specifically, to make the grid we set $2,520$ steps for t (daily steps) and $2,000$ steps for S. Note that JN_E and JN_A must coincide for $d = 0$ and $\lambda_e = 0$. The negligible difference between these two values exhibited in Table 1 is due to the numerical method to compute JN_A. It is also shown that $JN_A > JN_E$ for $d > 0$ as expected. Otherwise, JN_A is lower than BS for $d > 0$ suggesting the well-known fact of BS overestimation. Another result is the increasing value for the American ESO flexibility (measured as $JN_A - JN_E$) with higher values of λ_e. Remember that λ_e in Table 1 represents an intensity that captures both firm abandonment and early exercise for JN_A, while it does not occur for C_{cv} as we will explain right now.

The last four columns of Table 1, denoted as C_{cv}, display the prices using the simulation approach —where the return, under measure $\mathbb{Q}$, is driven by equation (7) but with constant volatility— for different early exercise parameters. Each price is the average of 100 estimates —obtained with 50 different seeds plus the 50 antithetics— and each estimate has been obtained by running $10,000$ paths. Simulated prices also verify the martingale property according to the empirical martingale simulation (EMS) procedure of Duan and Simonato (1998). The exercise is allowed once per day, that is, a daily frequency of 252 time steps per year. Below each price, in parentheses, we show the standard deviation over the 100 estimates. As expected, C_{cv} prices for $m = 1$ are quite similar to JN_A. The small differences between both prices are due to the numerical valuation methods. The content of λ_e in JN is divided in two different components in C_{cv}. That is, the unvoluntary exercise or cancellation rate is measured by ε while the voluntary early exercise is measured through m. Note that C_{cv} achives the higher value for $m = 1$ which is the case of an unconstrained agent, while lower values are obtained with $m < 1$ (constrained agent).

3.2 ESO pricing under time varying volatility

Tables 2 to 4 exhibit prices under alternative GARCH models with the same yearly unconditional volatility value of 0.30 for comparative reasons. In parentheses we show the standard deviation (std) of the estimated prices. Both price and std are obtained in the same way as C_{cv} reported in Table 1. The three panels (A, B and C) in each table change the values of the GARCH parameter set to analyze the impact on the ESO prices by fixing the same unconditional volatility. Moreover, volatility persistence remains constant in Tables 2 and 3, but it increases across panels in Table 4. We assume that the price of risk, λ, is equal to zero. It means that both $\mathbb{P}$ and $\mathbb{Q}$ measures coincide. In the next section, we study the impact of λ in ESO pricing.

Table 2 displays the ESO price for the GARCH model. We can appreciate that prices from panel A are higher than those in panel B and these are higher than those for panel C. Note that this effect occurs going from higher (lower) to lower (higher) values of β (α). Table 3 shows ESO prices for the GJR model. Across panels we increase the leverage effect: $\gamma = 0.05$, $\gamma = 0.1$ and $\gamma = 0.15$ for panels A, B and C respectively. The C-GARCH ESO prices are shown in Table 4. Across panels we increase the persistence of the long-run component variance, i.e. $\rho = 0.99$, $\rho = 0.995$ and $\rho = 0.999$, and we adjust ω to obtain the desired yearly volatility of 0.30. In most cases, it is exhibited a decrease in the ESO price as ρ increases[10].

It becomes a question of interest the possible misspricing when going from the constant to a time-varying volatility model. In Figure 1, we display ESO price biases for the different GARCH models (C_g) with respect to the constant volatility case (C_{cv}). They are calculted as $\frac{C_g - C_{cv}}{C_{cv}} \times 100$. We must point out that the constant volatility model is nested in any of the GARCH family given some specific values for each GARCH parameter set. Remember that these parameters in Tables 2 to 4 verify that their respective implied unconditional volatilities are the same value and coincide with the one under the constant volatility model, which is equal to 0.30. We can observe several features in Figure 1. First, biases are always negative implying an overvaluation when assuming the constant volatility model. This fact will be analyzed later. Second, there is a negative relationship between the size of

[10]We change both ω and persistence level under C-GARCH while it does not occur for the other two models. The persistence is only captured by one parameter in C-GARCH while more parameters are needed for the others. We are really studying the impact on ESO pricing of a different persistence level under C-GARCH, while for the other two models it is restricted to be constant. This may be the reason why a few C-GARCH ESO prices in Table 4 do not show a decreasing behavior across panels but rather an inverted U-shape.

bias and m while keeping fixed ε. This behavior does not hold for the case of the highest persistence in the C-GARCH model. Now, fixing m the bias size decreases as ε increases. Third, the bias size tends to increase with α (and lower β) in the GARCH model. For instance, if $m = 0.995$ and $\varepsilon = 0.05$, the bias is -13.44% when $\alpha = 0.06$ while it becomes -25.64% when $\alpha = 0.12$. Fourth, for the C-GARCH the bias size increases as the persistence value (ρ) increases. Given the above values of m and ε, the bias goes from -22.5% ($\rho = 0.99$) to -29.31% ($\rho = 0.999$).

Figure 2 tries to explain why GARCH ESO prices are lower than CV ones under the setting of $\varepsilon = 0.05$ and two alternative values of m: a suboptimal case ($m = 0.995$) and the optimal one ($m = 1$). This figure displays the cumulative probability of exercise, denoted as $F(t)$, per day after the vesting period ranging from 1 to 252 days (one year). That is, $F(t) = Prob(\text{exercise date} \leq t)$. In short, we only concentrate on the probability for the early exercise for the first year just after the vesting period. Each picture exhibits $F(t)$ for all GARCH models and parameter sets from Tables 2 to 4. Moreover, they include $F(t)$ under CV for comparative reasons (solid line). The pictures in the last row show the probability differences between the suboptimal case with respect to the optimal one. We obtain the following results. First, we can always observe a positive difference (bias) suggesting that exercise occurs earlier for $m = 0.995$. This early exercise decision is made by an undiversified employee. This is the reason why ESOs are cheaper than tradable options ($m = 1$) as they can be exhibited in Tables 1 to 4. Second, $F(t)$ under any symmetric GARCH model is higher than under CV. This fact suggests to accelarate the early exercise decisions and hence, more suboptimal situations. This explains why CV overprices (negative biases in Figure 1). It is also verified that $F(t)$ is higher under GARCH as larger (lower) is α (β). This fact explains the findings in Table 2 for the GARCH model: prices (exercise probabilities) decrease (increase) across the panels. For the C-GARCH, $F(t)$ increases with the persistence parameter ρ. Third, for the asymmetric model (GJR) we can appreciate that $F(t)$ decreases with the leverage effect (γ). Note that under the parameter set of panel C, $F(t)$ becomes lower than under CV.

3.3 Sensitivity analysis

This section aims to analyze the impact on the ESO price by changing each time only one of the following parameters: the price of risk (λ), vesting period (ν) or time to maturity (T).

Figure 3 shows GARCH ESO prices for different values of λ under alternative values of m. For $\lambda \neq 0$, the unconditional variance under $\mathbb{Q}$-measure is $\mathbb{E}^{\mathbb{Q}}\left[\sigma_t^2\right] = \omega/(1 - \alpha\left(1 + \lambda^2\right) - \beta)$. This suggests that this variance increases for higher values of λ. We only select the GARCH model under the Panel B parameter set ($\omega = 3.5714 \times 10^{-6}$, $\alpha = 0.09$ and $\beta = 0.90$) with $\varepsilon = 0.05$. The x-axis displays different prices of risk with the corresponding yearly unconditional volatilities (in parentheses). ESO prices for $\lambda = 0$ are obtained from Table 2. As expected, ESO price increases when λ increases for given a fixed value of m.

Figure 4 displays ESO prices (upper graphic) for different vesting periods under alternative volatility specifications. We select the same GARCH parameter sets used in panels B of Tables 2 to 4 with $\varepsilon = 0.05$ and $m = 0.995$. The bottom graphic exhibits the relative biases in percentages, calculated with respect to the constant volatility case. Note that, for $\nu = 10$ ($\nu = 0$) we have a European (fully American) ESO. The main result is that the shorter (longer) is the vesting period, the larger (smaller) is the price bias in absolute value, except for the GJR case that remains constant.

Note also that we would expect a decreasing behavior of the ESO price with the vesting period since as longer is the vesting period, the ESO tends to look like more the European style option. This pattern is observed for both CV and GJR ESO prices. However, both GARCH and C-GARCH cases exhibit an inverted U-shape behavior. It suggests that ESO price increases for short vesting periods and then, it starts decreasing for larger ones. This phenomenon is due to the agent's suboptimal behavior. A long vesting period avoids the agent of accepting a lot of suboptimal situations that might be undertaken in case of either absence or a lower vesting period[11]. This fact is only observed for the symmetric GARCH models since they display the highest exercise probabilities after the vesting period, see Figure 2. Nevertheless, the exercise probabilities become the lowest for the CV and GJR cases, that is why the vesting protection is not observed. In contrast, for very large vesting periods, ESOs become more European, and hence prices decrease, regardless the vesting protection.

[11]Mun (2004) also points out the vesting protection phenomenon under a constant volatility framework and using binomial trees.

Figure 5 shows the behavior of the ESO price as a function of the time to maturity under the volatility specifications with the same parameter sets used in Figure 4. As expected, this relationship is positive, that is, a higher T implies a higher price. If we implement a constant volatility model as our incorrect data generating process (DGP) instead of a GARCH family one, then it always overprices and the bias size increases with time to maturity.

3.4 ESO pricing with a misspecified model

To end, we also study the impact of pricing ESOs when the selected DGP for modeling the underlying stock price is not the right one and compare with the true process. Here, we concentrate on analyzing the sensitivity of ESO prices according to the degree of volatility memory such as, for instance, C-GARCH (long memory) against GARCH (short memory). Since both are symmetric, the possible difference in prices would only come from the difference in the volatility memory.

For this study, we start assuming the C-GARCH structure as the right process (true DGP) for the daily returns of the underlying stock price. The procedure to obtain ESO prices is as follows. First, we simulate $1,000$ sample paths of length $3,000$ daily return observations each under the true DGP driven by the C-GARCH model with the parameters from Panel B in Table 4. Second, for each path, we leave out the first 500 observations to avoid the problem of starting values and undertake the maximum likelihood (ML) estimation assuming by mistake (false DGP) the GARCH specification for the remaining $2,500$ observations. Third, we take the average over the $1,000$ estimates for each GARCH parameter. Finally, we use this mean vector as the GARCH parameter set for the ESO valuation exhibited in Figure 6 for different time to maturities. Specifically, the average GARCH parameters are: $\lambda = 0.0072$, $\omega = 1.748 \times 10^{-5}$, $\alpha = 0.1380$ and $\beta = 0.8090$. This parameter set implies an unconditional volatility (annualized) of 0.2883. We also select $\nu = 3$, $\varepsilon = 0.05$ and $m = 0.995$. We repeat the same experiment but starting at the second step where the incorrect DGP is now driven by the CV model that is also displayed. The average constant volatility (yearly) equals 0.2965. In short, the three ESO prices for the different volatility specifications are exhibited in the upper graphic of this figure. The percentage relative biases displayed in the bottom graphic are obtained with respect the true DGP (C-GARCH).

Some results emerge from Figure 6. First, there is a mispricing when assuming an incorrect volatility specification (either GARCH or CV) with respect to the correct one (C-GARCH). As expected, according to previous results, the size of this price difference becomes larger when assuming the CV model instead the GARCH one. Second, the GARCH process overprices with respect to the true or C-GARCH process except for the shortest maturities. Third, this bias enlarges as the time to maturity increases. The same pattern occurs for the CV model.

4 Accounting standards and ESO valuation

This section aims to relate our valuation approach with the two main accounting standards for ESO plans valuation. In a first stage we justify why our valuation approach is in line with the standards, and secondly we compare our numerical results with respect to older accounting rules.

4.1 FAS 123 and IFRS 2

The Financial Accounting Standard Board (FASB) published the Financial Accounting Standard 123 (FAS 123) in 1995. This statement encourages, but does not oblige, firms to adopt a fair-value based method for accounting ESO expenses. It aims to obtain an approximation of the fair value through the BS formula, but replacing the ESO expiration date (T) by its expected exercise time or expected life (L), henceforth $BS(L)$, and correcting for the probability of departure during the vesting period. Thus, the ESO price may be calculated as

$$C_{FAS,95} = BS(L) \times \exp(-\varepsilon\nu) \tag{13}$$

where the last term is the probability of the employee will remain in the firm until the vesting period.

In February 2004, the International Accounting Standard Board (IASB) published the International Financial Reporting Standard 2, *Share-Based Payments* (IFRS 2). This standard is adopted in the European Union since 2005 and it is obligatory for all traded firms. The IFRS 2 is the first standard that forces firms to recognize all share-based payments, including ESO, as an expense and a fair-value based method is required in the standard. However, how to calculate the fair value has been very discussed and it has become one of the major difficulties in accounting for ESO because IFRS 2 does not specify which pricing models should be used. The IFRS 2 only describes the factors that should be taken in account when estimating the fair value (see IFRS 2 paragraphs B4 to B10). This standard also suggests that formulae like BS may be not suitable because volatility and other parameters are not allowed to be time varying (IFRS 2, paragraph B5).

In December 2004, the FASB revised the FAS 123 in order to be compatible with the IFRS 2 (FAS 123R). Like IFRS 2, the FAS 123R does not specify a preference for a particular valuation technique or model in estimating the fair value, although it enumerates the factors required in the valuation technique at a minimum (FAS 123R, paragraph A18). FAS 123R explicitly says that fair values shall be estimated by applying a valuation technique that would be used in determining an amount at which instruments with the same characteristics would be exchanged. Moreover, the valuation technique should be based on established principles of financial economic theory like time value of money and risk-neutral valuation (FAS 123R, paragraph A8). In a footnote, the FAS 123R recognizes that Monte Carlo simulation technique is a valuation method that may satisfy the requirements of the standard. A method of estimating volatility is not specified in the standard, but it provides a list of factors that should be considered in the estimation procedure. Among others, it should include volatility changes and mean reversions (FAS 123R, paragraph A32).

In short, our proposed valuation method based on simulations is in line with IFRS 2 and FAS 123R because it allows for a time varying volatility, besides the vesting period, the departure risk and the early exercise behavior.

4.2 Consequences of FAS 123 valuation

Both standards recognize that the stochastic life of the ESO is the main characteristic that affects its price. Moreover, in equation (13) the vesting period length is also a relevant feature whose behavior has already been studied in Section 3.3. Using our simulation approach, we can get as an output the average of exercise times, each computed per path, labelled as expected exercise time (L).

The principal advantage of FAS 123 method, or equation (13), is its simplicity since it is a closed-form formula, instead of numerical methods like finite differences or simulations. An interesting question is to compare our simulation-GARCH method with the FAS 123 proposal. To carry out this analysis, we plug the expected life obtained under simulation for the different GARCH models into equation (13). If we consider the GARCH ESO expected life under this setting, we propose to modify equation (13) substituting BS price by the equivalent European GARCH option price in the spirit of FAS 123, that is

$$C_{FAS,g} = C_g^E(L) \times \exp(-\varepsilon\nu) \tag{14}$$

where $C_g^E(L)$ represents the European style ESO price, with L as time to maturity, nested in the American ESO price (C_g) modeled in Section 2.2 when $T = \nu = L$.

The aim of Figure 7 is to compare both approximations, that is, FAS 123 formula (dashed line) and our European GARCH proposal (dotted line) with the correct value C_g (solid line) for different vesting periods under alternative volatility models. The selected parameters for ESO valuation are shown at the bottom of this figure which are the same throughout this section. Each picture in Figure 7 is associated with a different DGP as the true volatility model. It is shown that FAS 123 proposal always overprices C_g. As expected, this bias measured as $C_{FAS,95} - C_g$ (henceforth, bias I) is the lowest under the CV case. Meanwhile, there is a price correction if the bias is now measured as the difference between equation (14) and C_g (henceforth, bias II). It is always lower than bias I except for the CV specification where both coincide. Note also that any bias exhibits a decreasing pattern as the vesting period grows for any volatility structure. Bias II tends to be higher for lower vesting periods and specifically, for the GARCH and C-GARCH models. Meanwhile, bias II is always zero for a vesting period of 10 years since it coincides with the time to maturity of this example and hence, C_g becomes a European ESO. In short, this evidence implies the following results. First, bias II is preferred to bias I and second, if the true DGP is driven by a constant volatility then bias II is not very large while it becomes more significant for the GARCH family, in case of being the true DGP, except for the GJR model that achieves a very low bias II. Also note that the simulation method, C_g, produces the vesting protection phenomenon (see Section 3.3) while $C_{FAS,95}$ and $C_{FAS,g}$ cannot reproduce it although these prices hold the same expected lives.

The values of L used in equations (13) and (14) are displayed in the upper graphic of Figure 8 and they correspond to the ESO prices exhibited in Figure 4. We can see that the expected life across the models is larger the shorter is the vesting period. The GARCH and C-GARCH (GJR) expected lives are always lower (higher) than the constant volatility case. We also quantify the mispricing of FAS 123 formula in terms of the ESO expected life. That is, we obtain the implicit expected life plugged into equation (13) to get an ESO price being equal to the corresponding C_g. This solution is unique from the fact that ESO price increases with time to maturity as Section 3.3 shows. The solution, denoted as L_{imp}, is obtained by implementing a standard nonlinear minimization algorithm. The middle graphic in Figure 8 displays L_{imp} for different vesting period lengths and volatility models. Whereas, the bottom graphic shows the bias (in years) of L_{imp} with respect to the true expected lives from the upper graphic, i.e. $L_{imp} - L$. As longer is the vesting period, the shorter is this bias (in absolute value) except for the GJR model. The excess of L over L_{imp} is the answer for the overpricing evidence of $C_{FAS,95}$ exhibited in Figure 7 since it is verified that $\partial C_{FAS,95}(L)/\partial L > 0$.

Finally, we can see in Figure 9 that both $C_{FAS,95}$ and $C_{FAS,g}$ biases always increase with time to maturity for both GARCH and C-GARCH models. Meanwhile, biases A and B are constant (negligible) for the GJR (CV) DGP. It also holds that $C_{FAS,g} < C_{FAS,95}$, except for CV where both coincide.

5 A real case study

Since ESOs are not tradable derivatives, we can not estimate the vector of unknown true parameters ψ implied in the model for the underlying asset dynamics in the usual way of minimizing the mean squared option valuation error. Here, using the quasi-maximum likelihood (QML) criterion we estimate the parameters, denoted as $\widehat{\psi}$, from a discrete log-return series of an asset price. Let $\sqrt{n}(\widehat{\psi}-\psi) \overset{a}{\sim} N\left(0, V_\psi\right)$ where n is the sample size, V_ψ is the asymptotic variance and consider a nonlinear function of ψ like the ESO price, denoted as $C \equiv C\left(\psi\right)$, then applying the *delta method* —see Lo (1986) for more details— we obtain the asymptotic distribution of the QML ESO price estimator, denoted as $\widehat{C} \equiv C(\widehat{\psi})$, given by

$$\sqrt{n}(\widehat{C} - C) \overset{a}{\sim} N\left(0, V_c\right), \qquad V_c \equiv \frac{\partial C(\psi)^{\prime}}{\partial \psi} V_\psi \frac{\partial C(\psi)}{\partial \psi} \tag{15}$$

where V_c may be estimated in the usual way

$$\widehat{V}_c \equiv V_c(\widehat{\psi}) = \frac{\partial C(\widehat{\psi})'}{\partial \psi} \widehat{V}_\psi \frac{\partial C\ (\widehat{\psi})}{\partial \psi} \tag{16}$$

such that $\widehat{V}_\psi$ is the asymptotic variance estimator. Therefore, for large n the variance of $\widehat{C}$ may be approximated by $Var(\widehat{C}) \approx V_c/n$.

We compute both the fair value and confidence interval of the ESO plan granted by ACS[12] on May 2, 2004. The ACS stock price was trading at 13.91 euros on the grant date. The ESO plan consists on 7,038,000 options issued at the money with a time to maturity of six years. Specifically, a third part of the options has a vesting period of three years, another third part has a vesting period of four years and the last part has a vesting period of five years.

We use a daily time series of ACS returns, denoted as R_t, from January 2, 1998 to April 30, 2004 ($1,580$ observations) to estimate the constant volatility model and the different GARCH models of Section 2.1. Table 5 displays some descriptive statistics for the return series. It shows both excess of kurtosis and positive skewness. As a result, the Jarque-Bera (JB) test rejects the null hypothesis of normality. Moreover, the Ljung-Box test statistic for the squared return series, denoted as $Q^2\,(20)$, clearly rejects the null hypothesis of independence, suggesting the existence of a time-varying variance dynamics and hence, modeling the returns according to a member from the GARCH family may be appropiate. Figure 10 displays the time series and the sample autocorrelation function (ACF), up to lag 100, for the series R_t, R_t^2 and $|R_t|$. For the squared return series, we can observe the clustering phenomenon already described in Section 2.1. Note that most of the significant correlations for R_t^2 occur approximately until lag 30. Meanwhile, the ACF of the absolute returns shows evidence of a long-memory pattern since the correlations remain significant up to lag 80. This evidence also leads to the possibility of modeling under the C-GARCH structure.

Table 6 displays the QML estimates for R_t under the alternative models already mentioned above. Note that the three GARCH models are significant with a high persistence level of size 0.97 for both GARCH and GJR. Meanwhile, the significant persistence measured through ρ is 0.99 for the C-GARCH. There is also evidence for the leverage effect in the volatility, see the significant and positive

[12] ACS is a construction industry Spanish firm and it belongs to the Spanish stock index IBEX-35.

parameter γ for the GJR. If we analyze the goodnesss of fit, we conclude that the C-GARCH makes the best performance according to the Akaike information criterion (AIC). Nevertheless, if we select under either Bayesian (BIC) or Schwarz (SIC) criterion —both penalize the number of the estimated parameters stronger than AIC— then GARCH and C-GARCH score rather the same. Finally, the statistic $Q^2(20)$ for the squared of the standardised residuals clearly accepts the null hypothesis of independence for any GARCH model but not for the constant volatility case. This suggests that such conditional heteroskedastic variance structure is necessary for modeling R_t. We also obtain the same conclusion for the autocorrelation of the standardised residuals, that is, $Q(20)$ always leads to accept the null hypothesis except for the constant volatility case.

Table 7 displays the estimates of the ESO price and the 95% asymptotic confidence intervals for the mean price (C) under different volatility specifications, see panels A to D, according to equations (15) and (16)[13]. For each panel, we also study the sensitivity of the ESO price estimator, $\widehat{V}_c/n$ and the confidence interval ($I_{95\%}$) for alternative values of the early exercise parameter, vesting period and departure rate. For any volatility model or panel, it holds that $\widehat{C}$ decreases with a larger value of ε but increases with m. Of course, these results were already expected according to the analysis made in Sections 3.1 and 3.2. It is always verified that $\widehat{C}$ under C-GARCH always leads to a lower value than GARCH while it occurs the opposite for $\sqrt{\widehat{V}_c/n}$. The estimator for the standard deviation, $\sqrt{\widehat{V}_c/n}$, also decreases in most cases with an increase of either ν or ε and hence, a narrower length for the confidence interval is obtained.

Finally, consider the situation for a lower value of m, for instance, $m = 0.985$ which becomes a strong suboptimal exercise rule. Hence, a larger value of ν gives a larger ESO price since it eliminates a lot of suboptimal situations that would appear under the case of a shorter vesting period. This evidence is already described in Section 3.3. Note also that a higher value for ε leads to a higher probability for leaving the firm before ending the vesting period, then larger values of ν, jointly with a high value of ε, may also lead to a decrease in the ESO price as it is exhibited for $\varepsilon = 0.10$.

[13]Note that the derivative $\partial C(\widehat{\psi})/\partial\psi$ in equation (16) does not show a closed-form solution. It suggests that the derivative must be obtained numerically by perturbating the value of $\widehat{\psi}$ slightly and then, obtaining the approximation of $\Delta C(\widehat{\psi})/\Delta\psi$.

6 Concluding Remarks

This article proposes a simulation based method to value ESOs in line with the IFRS 2 and FAS 123R proposals. It suggests a flexible tool to be suitable for the different ESO characteristics like vesting period, departure risk and the early exercise behavior of the ESO holder. We implement an American ESO valuation with time varying volatility under the GARCH family framework and compare with the constant volatility (CV) model. Specifically, we consider short-memory GARCH models, like GARCH and the asymmetric GJR, and the long-memory model C-GARCH. The first result is the overpricing under CV with respect to GARCH models. Early exercise probabilities also support this evidence, being higher for symmetric GARCH models.

Second, we study how ESO price changes when altering some parameters. If the price of risk increases, a higher ESO price is obtained since it leads to a higher unconditional volatility. We also show that as shorter (larger) is the vesting period the larger (shorter) is the overvaluation under CV. This overvaluation for the same model also increases (decreases) as longer (shorter) is the time to maturity. We also analyze the price effects when a misspecified volatility model is used.

Last but not least, we compare our GARCH ESO prices to those obtained according to the FAS 123 proposal for different vesting periods and time to maturities. For this analysis, we plug the implied expected life of the ESO from the simulation-based method into the FAS 123 formula. A significant bias arises for either shorter vesting periods or longer time to maturities. We try to improve the FAS 123 proposal substituting the BS value in its formula by the equivalent European GARCH option price with the same expected life. The results show a decrease for the initial bias. Finally, we estimate the fair value of a real ESO plan.

Our proposal could be easily adapted to some extensions. First, consider the case of indexed ESOs, see Johnson and Tian (2000) and Duan and Wei (2005). Second, the endogenous departure intensity, see Cuny and Jorion (1995) and Carr and Linetsky (2001). Third, the subjective valuation already mentioned in the introduction, see Hall and Murphy (2002) and Kahl et al. (2003). A fruitful avenue for future research would be to implement these extensions.

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Table 1: ESO price with constant volatility

Panel A: $d = 0$

λ_e/ε	JN_E	JN_A	C_{cv}			
			$m = 0.985$	$m = 0.99$	$m = 0.995$	$m = 1$
0.00	52.567	52.553	43.274	46.312	48.856	50.954
			(5.74)	(4.20)	(3.13)	(2.29)
0.05	31.883	42.112	35.754	38.099	39.708	41.041
			(4.19)	(2.81)	(2.23)	(1.53)
0.10	19.338	34.079	29.677	31.255	32.407	33.388
			(3.05)	(2.27)	(1.44)	(1.03)
0.15	11.729	27.815	24.397	25.833	26.826	27.359
			(2.79)	(1.68)	(1.04)	(0.73)

Panel B: $d = 0.025$

λ_e/ε	JN_E	JN_A	C_{cv}			
			$m = 0.985$	$m = 0.99$	$m = 0.995$	$m = 1$
0.00	34.682	36.300	32.579	33.763	34.927	35.837
			(4.28)	(3.37)	(2.34)	(1.02)
0.05	21.035	29.642	25.853	28.003	28.735	29.348
			(2.89)	(2.28)	(1.51)	(0.49)
0.10	12.759	24.345	22.401	23.240	23.768	24.161
			(2.46)	(1.72)	(1.12)	(0.47)
0.15	7.738	20.152	18.781	19.358	19.731	20.090
			(1.81)	(1.28)	(0.85)	(0.23)

This table shows the price of an American style ESO under constant volatility for the underlying asset. Different valuation methods are reported. We select a time to maturity of $T = 10$ years, an initial vesting period of $\nu = 3$ years, the annualized stock return volatility is $\sigma = 0.30$ and the risk-free interest rate is $r = 0.05$. We consider two dividend policies: no dividend payments (Panel A) and a yearly continuously compounded dividend yield of $d = 0.025$ (Panel B). The column λ_e/ε shows the different exit rates for the JN and simulation models respectively. The columns JN_E and JN_A show the prices for a European and American ESO respectively according to the Jennergren and Naslund (1993) proposals. For European ESO we use the closed form formula while for American ones we employ a fully implicit finite difference scheme given a grid with daily time steps (252 per year) and $2,000$ steps for S. Finally, the last four columns report the ESO prices using simulations (C_{cv}) under different early exercise parameters (m). Each price is a mean of 100 estimations (50 plus 50 antithetic). Each estimate consists of $10,000$ paths. We apply the Duan and Simonato (1998) correction in the simulated paths. Below each price, in parentheses, we display the standard deviation computed over the 100 different estimations. The simulations have been done with daily frequency (252 days per year).

Table 2: ESO price and GARCH model

ε	$m = 0.985$ Price	Std	$m = 0.99$ Price	Std	$m = 0.995$ Price	Std	$m = 1$ Price	Std
Panel A: $\alpha = 0.06$ and $\beta = 0.93$								
0.00	26.215	(3.75)	28.046	(3.93)	29.530	(4.03)	31.402	(3.59)
0.05	21.722	(2.83)	23.590	(2.94)	24.873	(2.93)	26.285	(2.54)
0.10	18.614	(2.18)	19.988	(2.30)	21.098	(2.18)	21.922	(1.86)
0.15	15.912	(1.78)	16.924	(1.79)	17.894	(1.51)	18.518	(1.28)
Panel B: $\alpha = 0.09$ and $\beta = 0.90$								
0.00	24.192	(2.76)	25.548	(3.38)	27.074	(3.59)	28.298	(3.50)
0.05	20.430	(1.98)	21.326	(2.23)	22.859	(2.78)	24.001	(2.67)
0.10	17.352	(1.45)	18.103	(1.75)	19.296	(2.03)	20.342	(1.92)
0.15	14.744	(1.16)	15.332	(1.36)	16.354	(1.56)	17.132	(1.40)
Panel C: $\alpha = 0.12$ and $\beta = 0.87$								
0.00	23.823	(4.05)	24.422	(4.17)	25.484	(3.93)	26.594	(4.54)
0.05	20.162	(2.85)	20.637	(2.99)	21.368	(3.19)	22.384	(3.24)
0.10	17.036	(1.97)	15.535	(2.16)	18.047	(2.30)	18.830	(2.30)
0.15	14.497	(1.37)	14.873	(1.61)	15.331	(1.70)	15.898	(1.75)

This table shows the price of an American style ESO where the underlying asset is simulated under alternative parameter values of the GARCH(1,1) process in equation (2). Each parameter set supports both an annualized unconditional expected volatility of 0.30, and a volatility persistence of $\alpha + \beta = 0.99$. The selected time to maturity of the ESO is $T = 10$ years, the vesting period is $\nu = 3$ years, the risk-free interest rate is $r = 0.05$, the price of risk is $\lambda = 0$ and the continuously compounded dividend yield is $d = 0.025$. The first column, ε, exhibits the different exit rates considered. Different early exercise parameter values are denoted by m. The second column, E, shows the European style ESO price. Each price is a mean of 100 estimations (50 plus 50 antithetic). Each estimate consists of $10,000$ paths. We apply the Duan and Simonato (1998) correction in the simulated paths. In Std columns, we display the standard deviation computed over the 100 different estimations. The simulations have been done with daily frequency (252 days per year).

Table 3: ESO price and GJR model

ε	$m = 0.985$ Price	Std	$m = 0.99$ Price	Std	$m = 0.995$ Price	Std	$m = 1$ Price	Std
			Panel A: $\alpha = 0.065$ and $\gamma = 0.05$					
0.00	25.544	(3.59)	27.695	(3.61)	30.107	(3.11)	31.673	(2.75)
0.05	21.571	(2.80)	23.548	(2.72)	25.216	(2.19)	26.317	(1.95)
0.10	18.084	(2.15)	19.896	(2.09)	20.987	(1.71)	21.973	(1.27)
0.15	15.399	(1.72)	16.805	(1.54)	17.756	(1.27)	18.423	(0.92)
			Panel B: $\alpha = 0.04$ and $\gamma = 0.10$					
0.00	26.888	(4.03)	29.948	(2.74)	31.890	(1.83)	33.175	(1.28)
0.05	22.466	(3.08)	24.757	(2.19)	26.473	(1.14)	27.224	(0.87)
0.10	18.806	(2.46)	20.763	(1.53)	21.955	(0.81)	22.509	(0.62)
0.15	15.861	(1.91)	17.553	(1.01)	18.334	(0.56)	18.687	(0.48)
			Panel C: $\alpha = 0.015$ and $\gamma = 0.15$					
0.00	26.198	(4.10)	29.944	(2.45)	32.083	(1.23)	33.089	(0.83)
0.05	21.809	(3.16)	25.008	(1.70)	26.345	(1.01)	27.078	(0.59)
0.10	18.246	(2.50)	20.806	(1.15)	21.854	(0.61)	22.282	(0.43)
0.15	15.510	(1.95)	17.395	(0.93)	18.180	(0.43)	18.467	(0.33)

This table shows the price of an American style ESO where the underlying asset is simulated under alternative parameter values of the GJR(1,1) process in equation (3). Each parameter set supports both an annualized unconditional expected volatility of 0.30, and a volatility persistence of $\alpha+\beta+0.5\gamma = 0.99$. The leverage parameter γ increases going from panel A to C. The selected time to maturity of the ESO is $T = 10$ years, the vesting period is $\nu = 3$ years, the risk-free interest rate is $r = 0.05$, the price of risk is $\lambda = 0$ and the continuously compounded dividend yield is $d = 0.025$. The first column, ε, exhibits the different exit rates considered. Different early exercise parameter values are denoted by m. The second column, E, shows the European style ESO price. Each price is a mean of 100 estimations (50 plus 50 antithetic). Each estimate consists of $10,000$ paths. We apply the Duan and Simonato (1998) correction in the simulated paths. In Std columns, we display the standard deviation computed over the 100 different estimations. The simulations have been done with daily frequency (252 days per year).

Table 4: ESO price and C-GARCH model

ε	$m = 0.985$ Price	Std	$m = 0.99$ Price	Std	$m = 0.995$ Price	Std	$m = 1$ Price	Std
	Panel A: $\omega = 3.5714 \times 10^{-6}$ and $\rho = 0.99$							
0.00	23.700	(1.86)	24.416	(2.46)	25.983	(3.12)	27.666	(3.78)
0.05	20.345	(1.75)	21.008	(2.11)	22.254	(2.60)	23.900	(2.91)
0.10	17.479	(1.49)	17.968	(1.74)	19.065	(1.99)	20.382	(2.17)
0.15	14.908	(1.14)	15.382	(1.32)	16.343	(1.60)	17.270	(1.58)
	Panel B: $\omega = 1.7857 \times 10^{-6}$ and $\rho = 0.995$							
0.00	23.556	(2.15)	24.488	(2.85)	25.706	(3.45)	27.121	(3.95)
0.05	20.151	(1.83)	20.989	(2.32)	21.976	(2.68)	22.929	(2.90)
0.10	17.362	(1.62)	17.979	(1.92)	18.711	(2.14)	19.573	(2.22)
0.15	14.733	(1.17)	15.293	(1.52)	15.937	(1.64)	16.609	(1.68)
	Panel C: $\omega = 3.5714 \times 10^{-7}$ and $\rho = 0.999$							
0.00	23.750	(2.62)	24.008	(2.70)	24.302	(2.76)	24.522	(2.84)
0.05	19.962	(1.76)	20.102	(1.81)	20.314	(1.86)	20.631	(1.98)
0.10	16.923	(1.25)	17.064	(1.29)	17.195	(1.32)	17.440	(1.38)
0.15	14.429	(0.93)	14.522	(0.95)	14.623	(0.97)	14.831	(1.06)

This table shows the price of an American style ESO where the underlying asset is simulated under alternative parameter values of the C-GARCH process in equations (4) and (5). Each parameter set supports both an annualized unconditional expected volatility of 0.30. The persistence parameter ρ increases going from panel A to C. The selected time to maturity of the ESO is $T = 10$ years, the vesting period is $\nu = 3$ years, the risk-free interest rate is $r = 0.05$, the price of risk is $\lambda = 0$ and the continuously compounded dividend yield is $d = 0.025$. The first column, ε, exhibits the different exit rates considered. Different early exercise parameter values are denoted by m. The second column, E, shows the European style ESO price. Each price is a mean of 100 estimations (50 plus 50 antithetic). Each estimate consists of $10,000$ paths. We apply the Duan and Simonato (1998) correction in the simulated paths. In Std columns, we display the standard deviation computed over the 100 different estimations. The simulations have been done with daily frequency (252 days per year).

Table 5: ACS stock returns descriptive statistics

mean (×100)	0.0429
median (×100)	0.0000
maximum	0.1379
minimum	-0.0841
standard deviation (annualized)	0.3223
skewness	0.2914
kurtosis	6.0676
JB statistic	641.8800
	[0.0000]
$Q(20)$ statistic	36.646
	[0.0129]
$Q^2(20)$ statistic	362.82
	[0.0000]

This table reports some descriptive statistics of the ACS stock return series. The sample contains 1,580 observations for the period January 2, 1998 to April 30, 2004. The data are provided by Sociedad de Bolsas. The JB row shows the Jarque-Bera statistic for testing normality. Under the null hypothesis of normality, the JB statistic is distributed as a χ^2 disribution with 2 degress of freedom, i.e. $JB \sim \chi^2_2$. The row $Q(20)$ is the Ljung-Box portmanteau test for up to 20th order serial correlation in the returns, whereas $Q^2(20)$ denotes the same but for the square return series. A statistic $Q(k)$ is asymptotically distributed as a χ^2 with k degrees of freedom. P-values are in brackets.

Table 6: GARCH model estimates for ACS return series

	CV		GARCH		GJR		C-GARCH	
parameter	estim.	std.error	estim.	std.error	estim.	std.error	estim.	std.error
λ	0.02348	(0.02516)	0.0238	(0.0238)	0.0105	(0.0251)	0.0252	(0.0241)
$\omega(\times 10^{-6})$	412.031**	(14.6887)	13.232**	(3.6166)	12.369**	(3.3781)	0.9172	(1.2754)
α	–	–	0.1354**	(0.0214)	0.1032**	(0.0236)	0.1236**	(0.0227)
β	–	–	0.8371**	(0.0238)	0.8452**	(0.0238)	0.7883**	(0.0459)
γ	–	–	–	–	0.0531*	(0.0269)	–	–
ρ	–	–	–	–	–	–	0.9973**	(0.0027)
ϕ	–	–	–	–	–	–	0.0241	(0.0184)
$\mathcal{L}(\hat{\psi})$	3915.66		4064.01		4065.92		4070.15	

Goodness of Fit Analysis

	stat.	p-value	stat.	p-value	stat.	p-value	stat.	p-value
Akaike	-7827.32	–	-8120.01	–	-8121.84	–	-8128.29†	–
BIC	-7816.59	–	-8098.55†	–	-8095.01	–	-8096.10	–
SIC	-4.9472	–	-5.1257†	–	-5.1234	–	-5.1241	–
JB	641.0669	[0.0000]	122.2564	[0.0000]	140.7006	[0.0000]	138.2324	[0.0000]
$Q(20)$	36.64147	[0.0129]	26.7789	[0.1415]	26.6754	[0.1446]	25.9027	[0.1690]
$Q^2(20)$	364.7673	[0.0000]	15.4429	[0.7505]	15.0966	[0.7708]	16.8783	[0.6608]

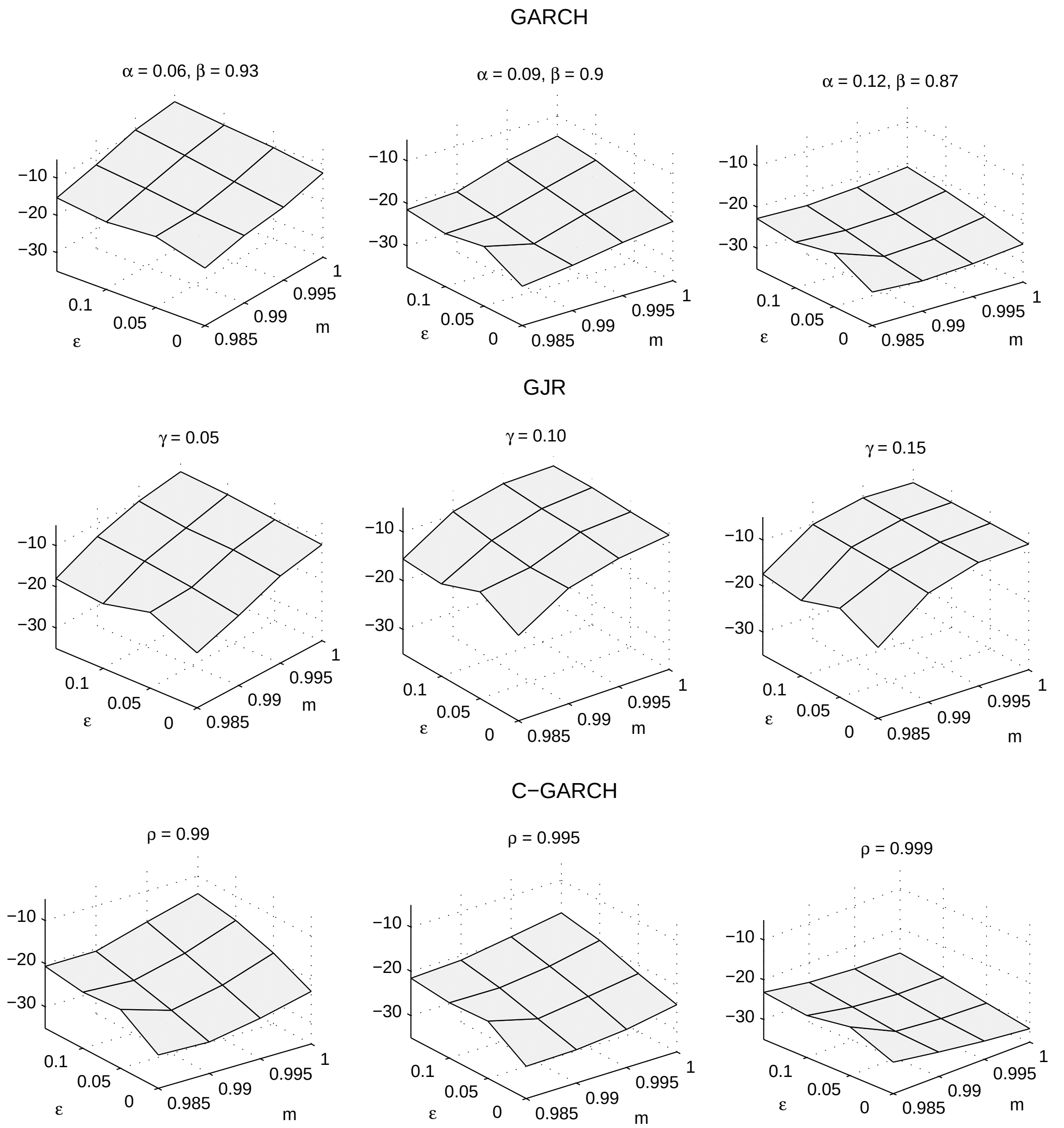
This table shows the QML estimates for ACS daily stock returns for the period January 5, 1998 to April 30, 2004. The GARCH(1,1), the GJR(1,1) and the C-GARCH models are described in equations (2), (3) and (4) to (5) respectively. On the right side of each estimate, we display the standard error of the estimation (in parenthesis). The row $\mathcal{L}(\hat{\psi})$ contains the log-likelihood of the estimation. The three rows named AIC, BIC and SIC report the Akaike, Bayesian and Schwarz information criteria respectively. The JB row exhibits the Jarque-Bera statistics for the estimated residuals and the corresponding p-values. Finally, the last two rows show the Ljung-Box statistic for both the residuals and its squares for the first 20 lags and their p-values. The symbols * and ** denote significance levels at 5 % and 1 % respectively. The symbol † denotes the best model according to the selected statistic criterion.

Table 7: ACS ESO plan: prices and confidence intervals

		$m = 0.985$			$m = 0.99$			$m = 0.995$		
ν	ε	$\widehat{C}$	$\sqrt{\widehat{V}_c/n}$	$I_{95\%}$	$\widehat{C}$	$\sqrt{\widehat{V}_c/n}$	$I_{95\%}$	$\widehat{C}$	$\sqrt{\widehat{V}_c/n}$	$I_{95\%}$
				Panel A: Constant Volatility						
3	0.05	2.706	0.001	(2.709, 2.711)	2.708	0.089	(2.589, 2.936)	3.553	0.004	(3.147, 3.162)
	0.10	2.327	0.001	(2.327, 2.331)	2.645	0.072	(2.432, 2.715)	2.994	0.003	(2.690, 2.701)
4	0.05	2.880	0.001	(2.865, 2.869)	2.883	0.044	(2.784, 2.955)	3.391	0.001	(2.992, 2.998)
	0.10	2.358	0.001	(2.344, 2.348)	2.544	0.036	(2.374, 2.517)	2.741	0.001	(2.445, 2.450)
5	0.05	2.995	0.001	(2.954, 2.959)	3.001	0.001	(2.955, 2.959)	3.190	0.001	(2.961, 2.964)
	0.10	2.333	0.001	(2.298, 2.302)	2.396	0.002	(2.301, 2.308)	2.473	0.001	(2.305, 2.310)
				Panel B: GARCH						
3	0.05	2.992	0.066	(2.842, 3.099)	3.019	0.021	(3.027, 3.110)	3.330	0.026	(3.144, 3.244)
	0.10	2.571	0.053	(2.450, 2.659)	2.600	0.002	(2.684, 2.690)	2.822	0.007	(2.739, 2.765)
4	0.05	3.203	0.025	(3.004, 3.101)	3.254	0.018	(3.050, 3.128)	3.286	0.022	(3.125, 3.210)
	0.10	2.633	0.018	(2.462, 2.532)	2.662	0.003	(2.548, 2.559)	2.680	0.010	(2.587, 2.625)
5	0.05	3.274	0.019	(3.114, 3.190)	3.286	0.018	(3.135, 3.206)	3.305	0.017	(3.167, 3.233)
	0.10	2.548	0.016	(2.418, 2.481)	2.562	0.016	(2.437, 2.498)	2.567	0.016	(2.448, 2.510)
				Panel C: GJR						
3	0.05	3.008	0.085	(2.974, 3.309)	3.032	0.062	(3.064, 3.309)	3.328	0.033	(3.238, 3.369)
	0.10	2.581	0.018	(2.583, 2.652)	2.603	0.071	(2.560, 2.876)	2.849	0.030	(2.762, 2.881)
4	0.05	3.156	0.064	(3.014, 3.263)	3.184	0.038	(3.103, 3.251)	3.306	0.022	(3.235, 3.323)
	0.10	2.579	0.004	(2.558, 2.572)	2.601	0.047	(2.520, 2.705)	2.709	0.017	(2.638, 2.704)
5	0.05	3.238	0.025	(3.157, 3.257)	3.286	0.024	(3.182, 3.277)	3.367	0.021	(3.218, 3.302)
	0.10	2.521	0.021	(2.457, 2.538)	2.542	0.019	(2.474, 2.549)	2.618	0.016	(2.499, 2.563)
				Panel D: C-GARCH						
3	0.05	2.529	0.031	(2.423, 2.543)	2.616	0.030	(2.440, 2.560)	2.640	0.052	(2.445, 2.650)
	0.10	2.169	0.027	(2.082, 2.188)	2.253	0.025	(2.101, 2.200)	2.271	0.070	(2.233, 2.508)
4	0.05	2.706	0.029	(2.567, 2.681)	2.719	0.033	(2.587, 2.716)	2.742	0.039	(2.678, 2.829)
	0.10	2.216	0.028	(2.092, 2.203)	2.224	0.022	(2.119, 2.205)	2.238	0.041	(2.180, 2.339)
5	0.05	2.832	0.032	(2.657, 2.782)	2.843	0.031	(2.687, 2.810)	2.860	0.029	(2.719, 2.833)
	0.10	2.208	0.024	(2.068, 2.164)	2.212	0.025	(2.089, 2.188)	2.228	0.022	(2.118, 2.206)

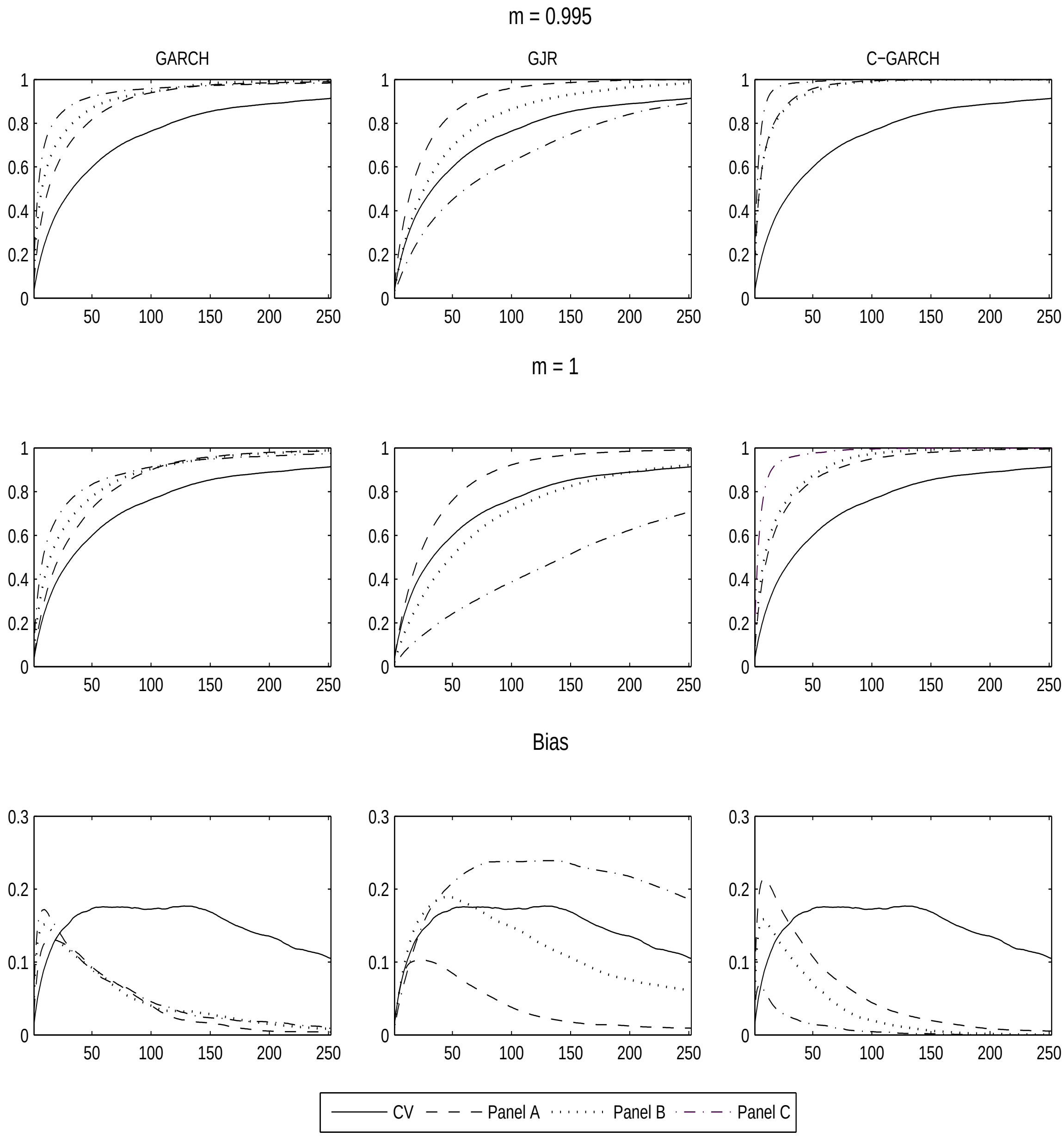
This table shows both the QML estimated prices ($\widehat{C}$) and their corresponding standard deviations ($\sqrt{\widehat{V}_c/n}$) of the three different ESOs, each depending on a different vesting period (ν) in the first column, of the ACS plan granted on May 2, 2004. All ESOs have a time to maturity of six years. The initial price is 13.91 euros and ESOs are issued at the money. Each panel holds a different volatility model and contains the valuation of the same ESO for different early exercise parameters (m) and exit rates (ε).The parameters for the different GARCH models are those from Table 6. $I_{95\%}$ denotes the 95% confidence interval for the population mean of ESO price (C).

Figure 1: Price bias under different volatility models



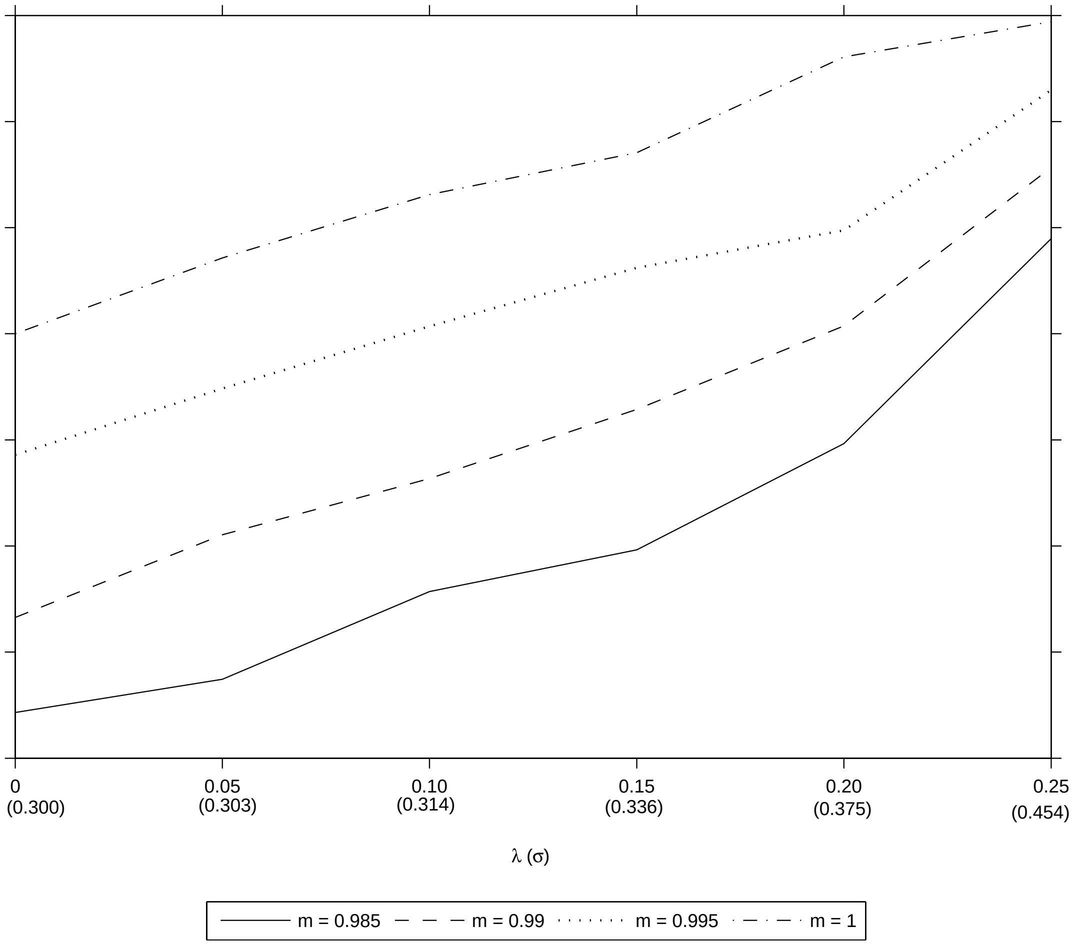
These graphics show percentage relative biases of the ESO prices for the three different GARCH models, compared with the constant volatility price. The bias is computed as $\frac{C_g - C_{cv}}{C_{cv}} \times 100$ where C_g is the ESO price for each GARCH model reported in Tables 2 to 4, and C_{cv} is the ESO price for the constant volatility case reported in the last four columns of Table 1. A negative bias implies an undervaluation of the ESO under the GARCH framework. All cases have the same expected unconditional volatility. The parameters m and ε denote the early exercise rule and exit rate respectively.

Figure 2: ESO exercise probabilities



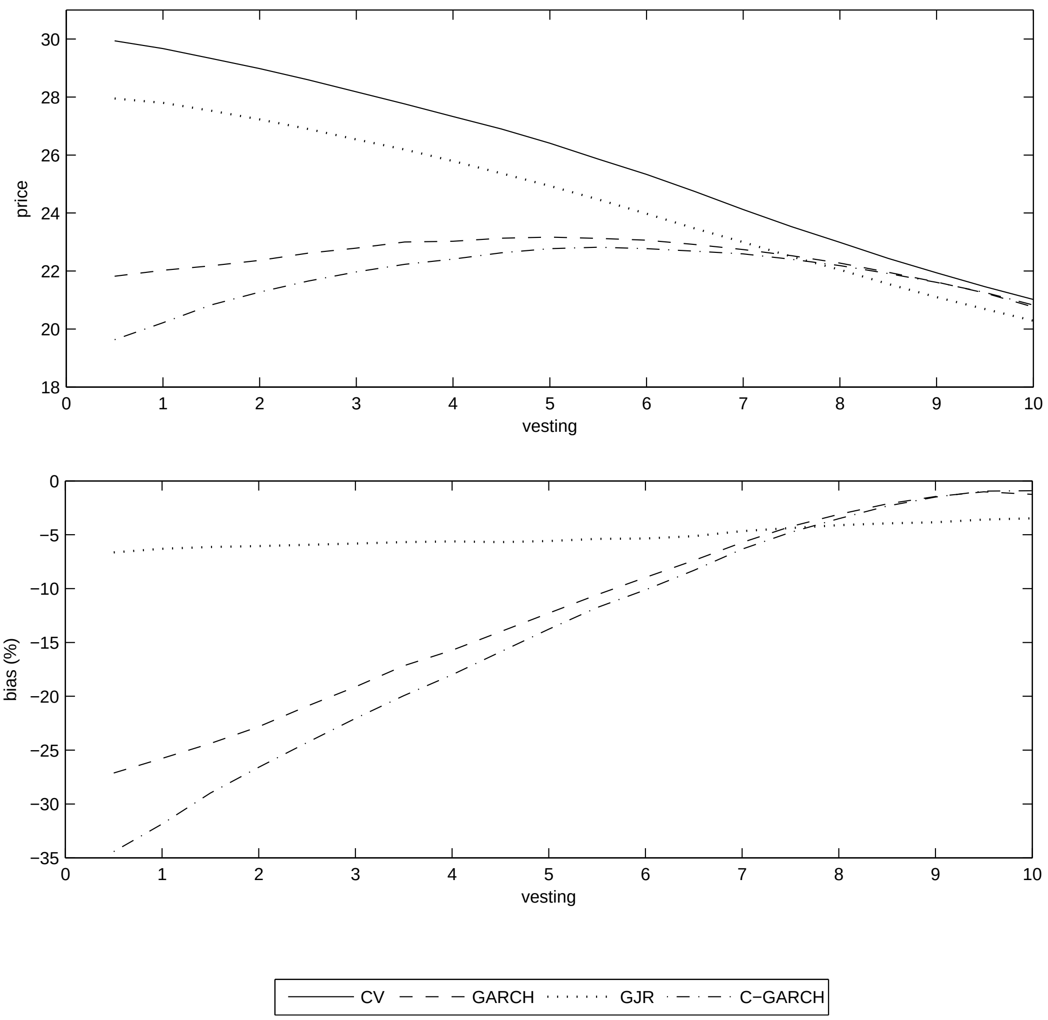
This figure shows the cumulative probability of ESO exercise per day, i.e. $F(t) = Prob(\text{exercise date} < t)$, for the first year (252 days) just after the vesting period. The ESO characteristics are: $T = 10$, $\nu = 3$, $r = 0.05$, λ=0, $d = 0.025$ and $\varepsilon = 0.05$. The upper graphics hold an early exercise parameter of $m = 0.995$, while the middle ones hold the optimal exercise parameter, $m = 1$. The bottom graphics exhibit the probability differences between the suboptimal and the optimal cases. The GARCH parameter sets are those used in Tables 2 to 4.

Figure 3: ESO price and the price of risk



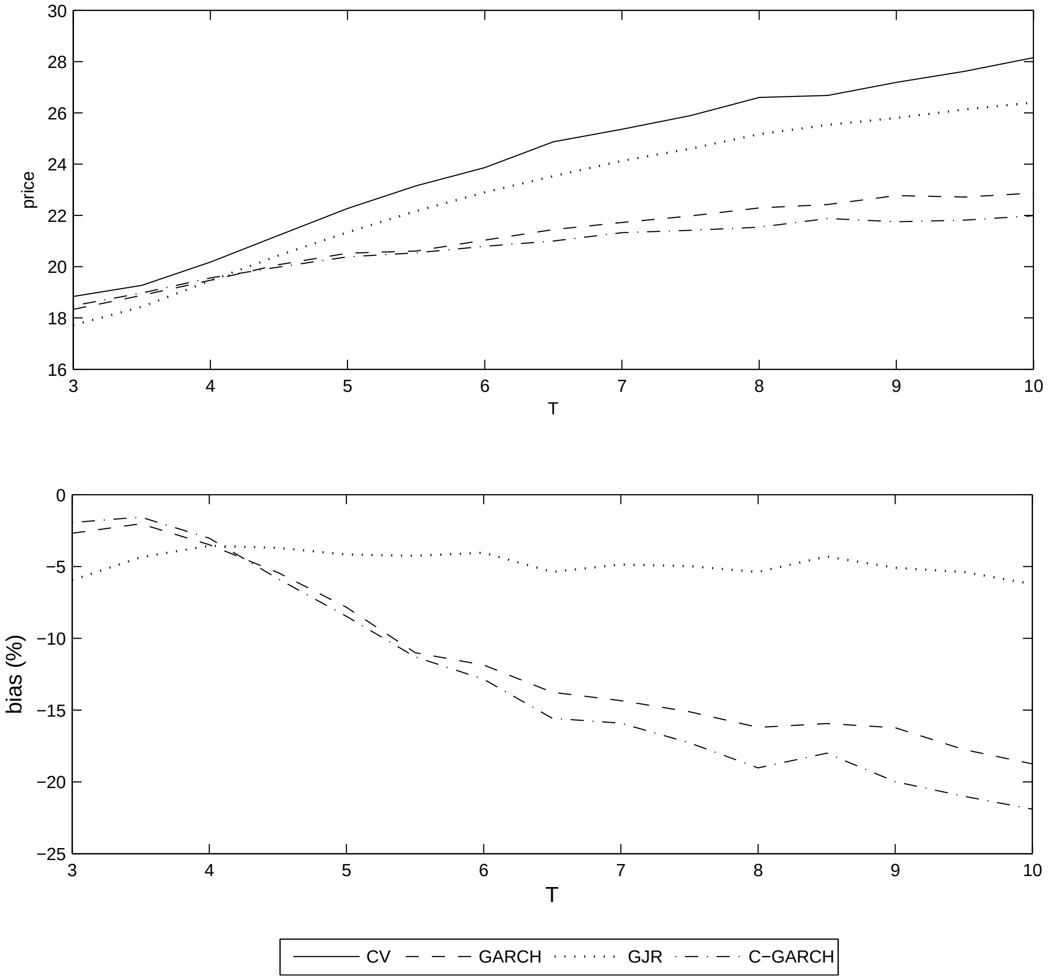
This figure shows the ESO price behavior as a function of the price of risk (λ) for different early exercise parameters (m). The other parameters for ESO valuation are: $T = 10$, $\nu = 3$, $r = 0.05$, $d = 0.025$ and $\varepsilon = 0.05$. The conditional variance of the underlying asset follows a GARCH process with parameters: $\omega = 3.5714 \times 10^{-6}$, $\alpha = 0.09$ and $\beta = 0.90$. Below each value of λ, in parentheses, we display the corresponding yearly unconditional volatility implied in the GARCH model under the $\mathbb{Q}$-measure. Each price is a mean of 100 estimations (50 plus 50 antithetic). Each estimate consists of 10,000 paths. We apply the Duan and Simonato (1998) correction in the simulated paths. The simulations have been done with daily frequency (252 days per year).

Figure 4: ESO price and vesting period



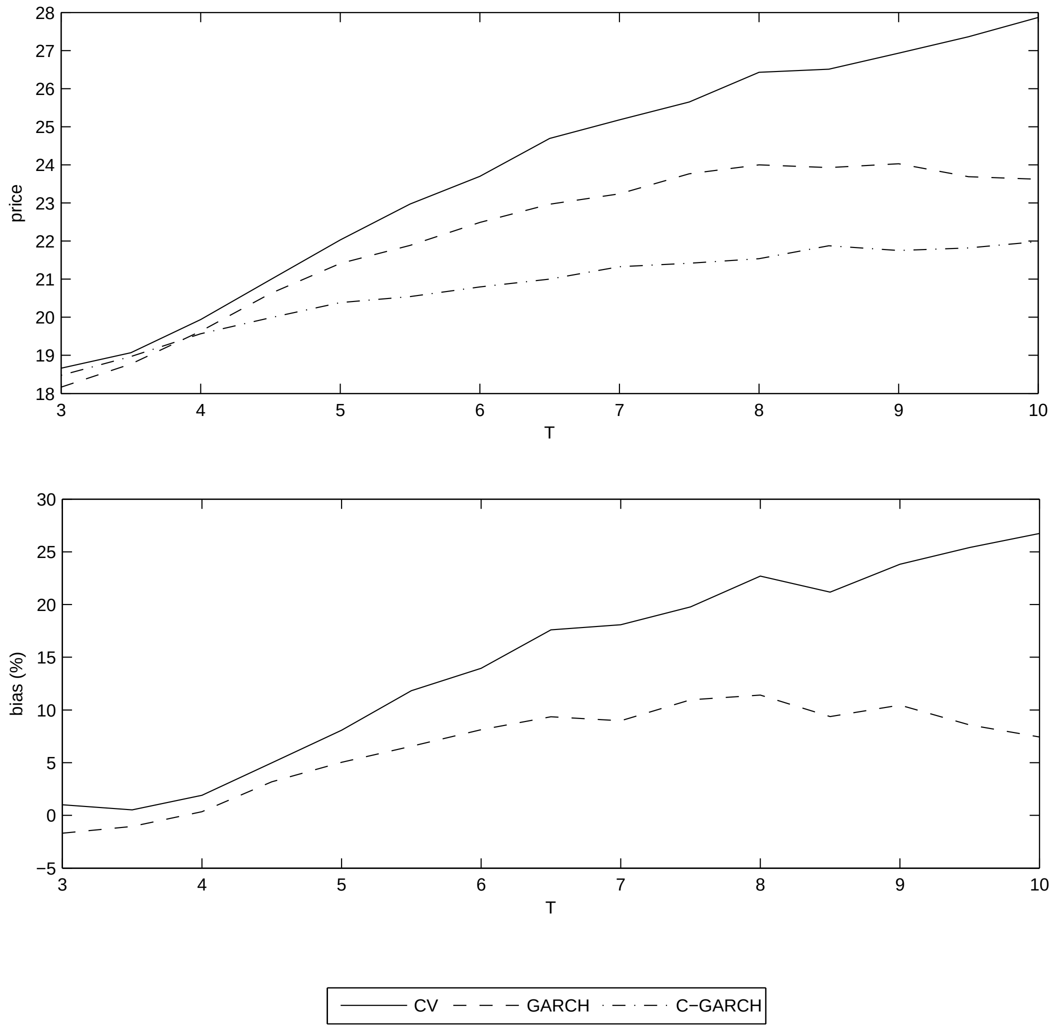
The upper graphic shows ESO prices for different volatility models and vesting periods (in years). The bottom graphic shows the percentage relative bias of the ESO price under alternative GARCH models (C_g), i.e. $\frac{C_g - C_{cv}}{C_{cv}} \times 100$ where C_{cv} is the constant volatility (CV) ESO price. We select the same GARCH parameter sets of Panels B from Tables 2 to 4. The parameters for the ESO valuation are: $T = 10$, $r = 0.05$, λ=0, $d = 0.025$, $\varepsilon = 0.05$ and $m = 0.995$. Each price is a mean of 100 estimations (50 plus 50 antithetic). Each estimate consists of $10,000$ paths. We apply the Duan and Simonato (1998) correction in the simulated paths. The simulations have been done with daily frequency (252 days per year).

Figure 5: ESO price and time to maturity



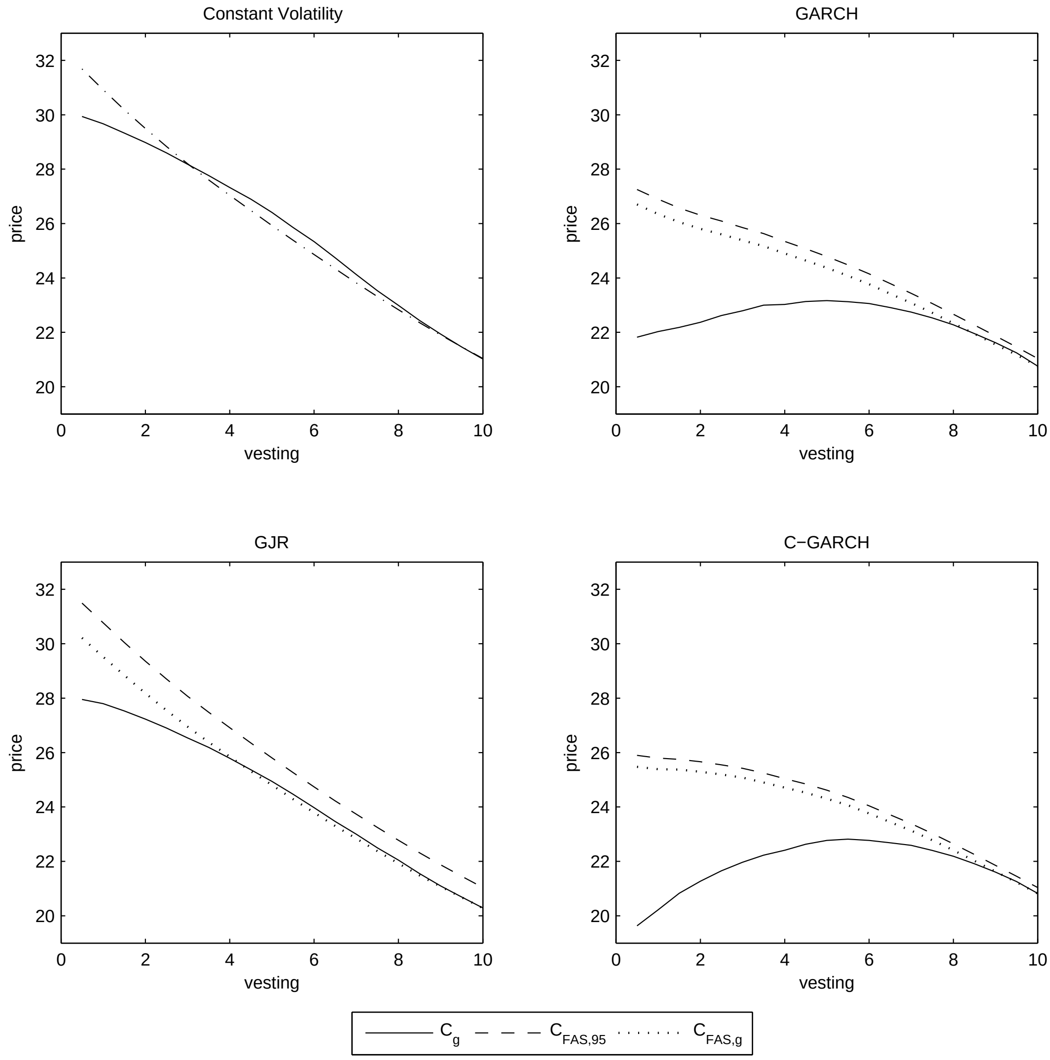
The upper graphic shows ESO prices with $\nu = 3$ for different volatility models and time to maturities (in years). The bottom graphic shows the percentage relative bias of the ESO price under alternative GARCH models like in Figure 4. We select the same GARCH parameter sets of Panels B from Tables 2 to 4. Both the remaining parameters and price estimation method for ESO pricing are the same as in Figure 4.

Figure 6: ESO price under misspecified volatility models



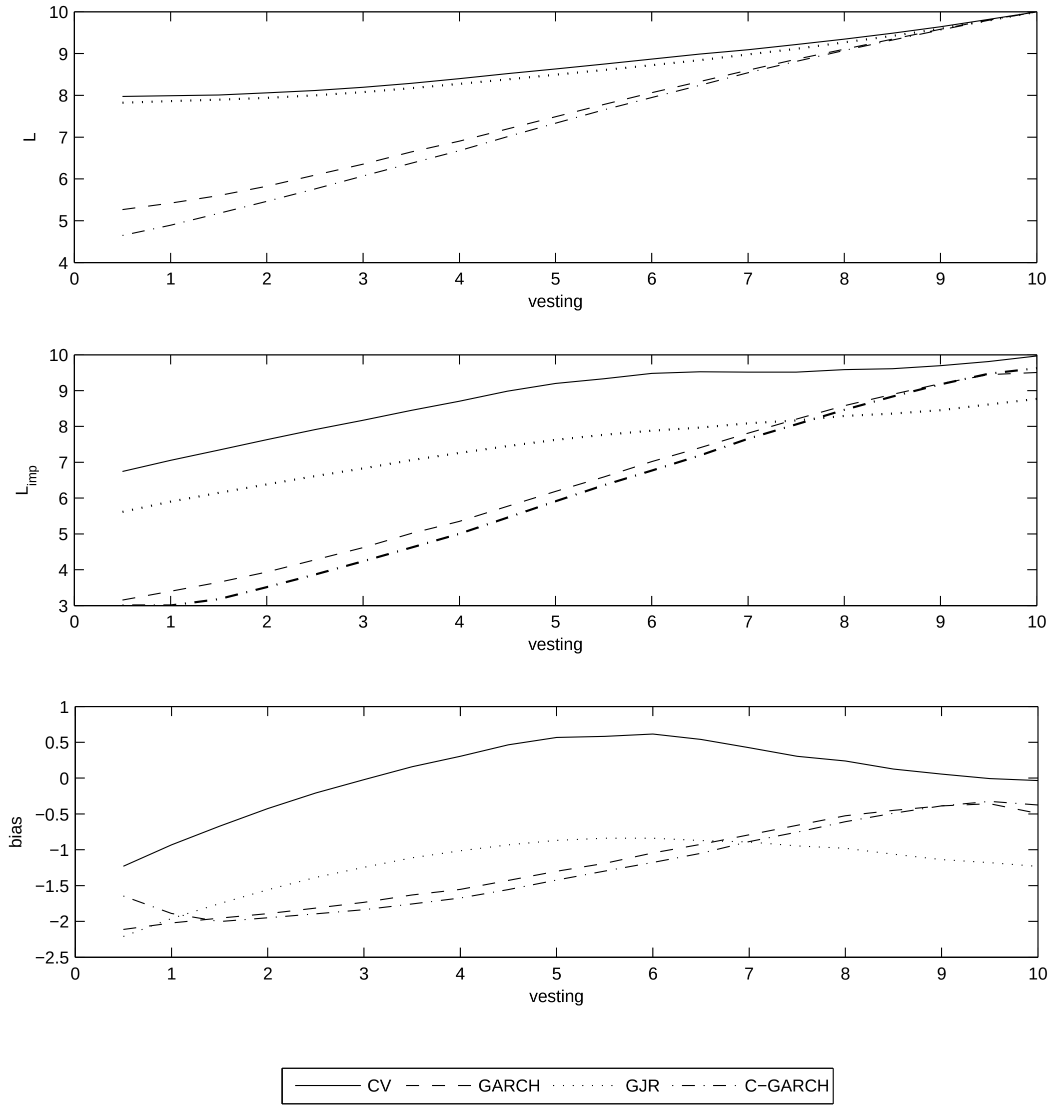
This figure presents ESO prices for different time to maturities in years (T) when either GARCH or CV model is implemented by mistake (false DGP). We assume that C-GARCH model is the right process (true DGP). The procedure to obtain the ESO prices is as follows. First, we simulate $1,000$ sample paths of length $3,000$ daily return observations each under the true DGP with those parameters of Panel B from Table 4. Second, for each path, we leave out the first 500 observations to avoid the problem of starting values and undertake MLE estimation for both GARCH and CV models for the remaining $2,500$ observations. Third, for each model (GARCH, CV) we take the average over the $1,000$ estimated parameter sets. Finally, we use these average parameters as the ones for both GARCH and CV ESO valuation. The average GARCH parameters are $\lambda = 0.0072$, $\omega = 1.748 \times 10^{-5}$, $\alpha = 0.1380$ and $\beta = 0.8090$ while the estimated constant volatility (yearly) equals 0.2965. The other parameters for ESO valuation are: $\nu = 3$, $r = 0.05$, $d = 0.025$, $\varepsilon = 0.05$ and $m = 0.995$. The price estimation method for ESO pricing is the same as in Figure 4.

Figure 7: ESO price and FAS 123 approximation



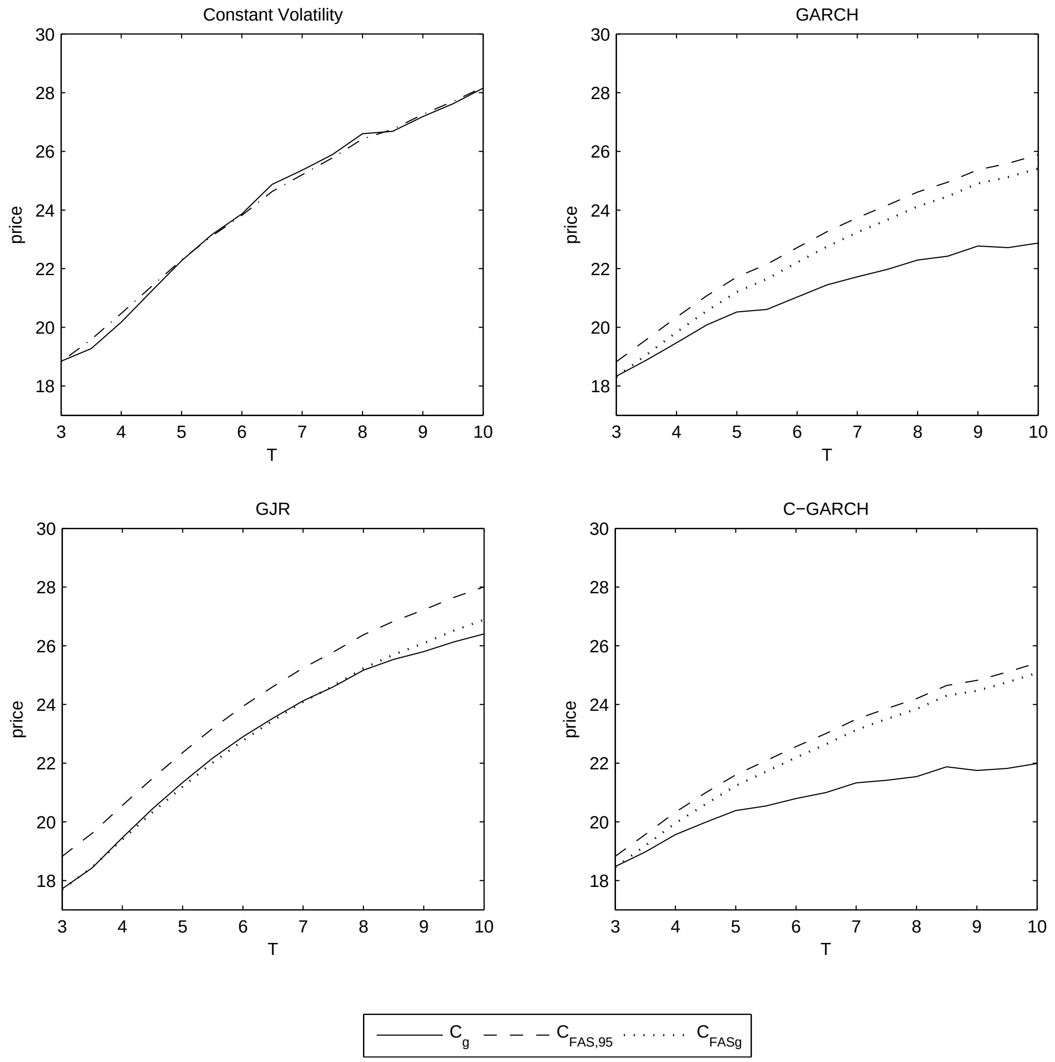
These graphics show the ESO price (C_g) using the simulation method (solid line), the equivalent FAS 123 price (dashed line) and the modified FAS 123 price (dotted line) under different volatility models and vesting periods (in years). The price under FAS 123 is $C_{FAS,95} = BS(L) \times e^{-\varepsilon\nu}$, where $BS(L)$ is the Black-Scholes price where the time to maturity is the ESO expected life obtained with the simulation method. The price under modified FAS 123 is $C_{FAS,g} = C_g^E(L) \times \exp(-\varepsilon\,\nu)$, where C_g^E denotes the European style ESO price nested in C_g when $T = \nu = L$. We select the same parameter sets used in Table C_{cv} and also, Panels B of Tables 2 to 4 for CV and the different GARCH models respectively. The other parameters for ESO valuation are: $T = 10$, $r = 0.05$, λ=0, $d = 0.025$, $\varepsilon = 0.05$ and $m = 0.995$.

Figure 8: Expected exercise time and vesting period



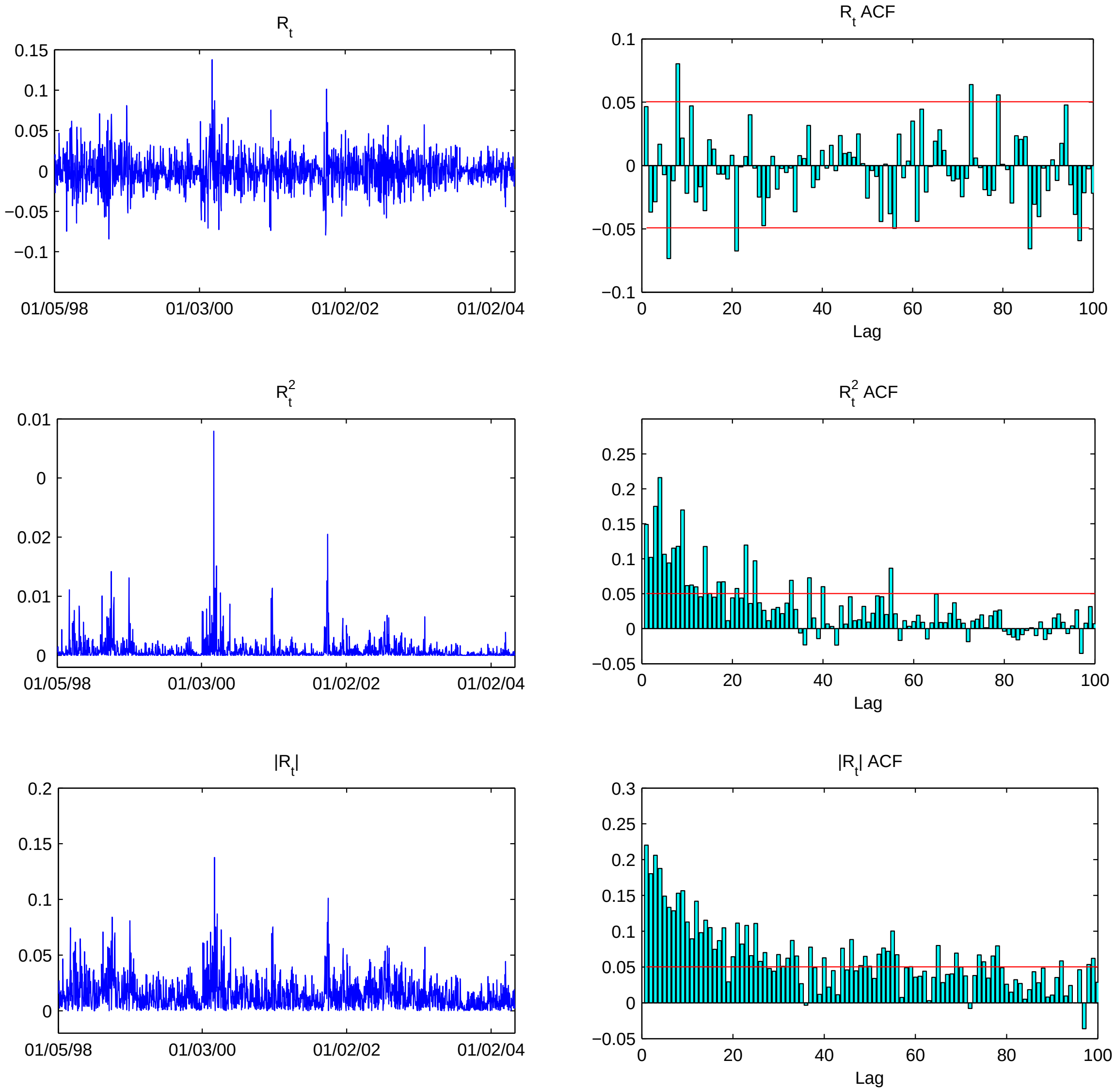
The upper graphic shows the expected exercise time, L, using the simulation method for different volatility models and vesting periods (in years). The middle graphic shows implicit expected life (L_{imp}) under FAS 123 obtained by nonlinear minimization under the alternative models. The bottom graphic displays the yearly expected life bias calculated as $L_{imp} - L$. We select for the volatility the same parameter sets used in Table C_{cv} and also, Panels B of Tables 2 to 4 for CV and the different GARCH models respectively. The other parameters for ESO valuation are: $T = 10$, $r = 0.05$, λ=0, $d = 0.025$, $\varepsilon = 0.05$ and $m = 0.995$.

Figure 9: ESO price, FAS 123 approximations and time to maturity



These graphics show the ESO price (C_g) using the simulation method (solid line), the equivalent FAS 123 price (dashed line) and the modified FAS 123 price (dotted line) under different volatility models and time to maturities (in years). The price under FAS 123 is $C_{FAS,95} = BS(L) \times e^{-\varepsilon\nu}$, where $BS(L)$ is the Black-Scholes price where the time to maturity is the ESO expected life obtained with the simulation method. The price under modified FAS 123 is $C_{FAS,g} = C_g^E(L) \times \exp(-\varepsilon\,\nu)$, where C_g^E denotes the European style ESO price nested in C_g when $T = \nu = L$. We select the same parameter sets used in Table C_{cv} and also, Panels B of Tables 2 to 4 for CV and the different GARCH models respectively. The other parameters for ESO valuation are: $\nu = 3$, $r = 0.05$, λ=0, $d = 0.025$, $\varepsilon = 0.05$ and $m = 0.995$.

Figure 10: Time series and autocorrelation plots of ACS



The left hand side exhibits the time series plots of ACS daily returns (R_t), its squared returns (R_t^2) and its returns in absolute value ($|R_t|$). Meanwhile, the right hand side exhibits the corresponding plots of the autocorrelations (ACF) for the first 100 lags with the confidence intervals ($\pm 1.96 \times n^{-1/2}$ where n is the sample size). The holding period goes from January 5, 1998 to April 30, 2004 ($1,580$ observations).