The Cost of Equity Capital Implied by Option Market Prices

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Abstract

The estimation of the cost of equity capital (COE) is one of the most important tasks in financial management. Existing approaches compute the COE using historical data, i.e. they are backward-looking methods. This paper derives a method to calculate forward-looking estimates of the COE using the current market prices of stocks and stock options. Our estimates of the COE reflect the expectation of the market investors about the COE during the life of the investment project. We test empirically our method and compare it with the Fama/French (1993) three-factor model for the S&P 100 firms. The empirical results indicate that our COE estimates (1) change with the investment horizon of the project reflecting the fact that different degrees of risk are associated with projects of different maturities, (2) are plausible and stable over the years as required by appropriate discount rates for capital budgeting, (3) yield an equity risk premium close to the market equity risk premium reported by Fama and French (2002), and (4) generate strong return-risk relationships.

1. Introduction

The estimation of the cost of equity capital (COE) is an important issue for both practitioners and academics. The COE is widely used in applications such as the valuation of an investment project of a firm and the estimation of equity risk premiums. In particular, the COE often affects how the services of a firm in the public sector are regulated by its supervising commission. Therefore, the estimation precision of the COE has a significant impact on a firm's value. According to the survey of Bruner, Eades, Harris, and Higgins (1998) and Graham and Harvey (2001), the most popular market-based methods for estimating the COE in practice are the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), the average historical returns, and a multibeta CAPM (with extra risk factors in addition to the market beta).[1] Although these methods are simple to apply, they all rely exclusively on historical data, i.e. they are backward-looking methods. Since the COE estimates are usually aimed to serve as the discount rate for future cash flows of an investment project, a backward-looking method may not perform well unless the patterns of COE are known and stable over the years in the future. As a result, these estimates of COE are usually imprecise, especially when they are applied to estimate the COE of an industry. For example, Fama and French (1997) pointed out that the standard errors of the COE estimates are typically above 3.0% per year.

In contrast to the above backward-looking methods, this paper provides a forward-looking method to estimate the COE using the current market prices of equity and equity options.[2] The option market prices are widely used to estimate implied volatility, which is commonly found to be the best predictor for future volatility (see e.g. Poon and Granger (2003) for a detailed survey on this issue).[3] Many studies indicate that option market prices

[1] The risk factors include the fundamental factors (Fama and French, 1993), the momentum (Jegadeesh and Titman, 1993), and the macroeconomic factors (Chen, Roll, and Ross, 1986; Ferson and Harvey, 1993).

[2] Some accounting-based models also utilize forward information such as analyst forecasts. But we focus on the COE estimation using market-based models only.

[3] About the information content and forecasting performance of implied volatility based on option market prices see also Day and Lewis (1992), Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Christensen and Prabhala (1998), Blair, Poon, and Taylor (2001), Pong, Shackleton, Taylor, and Xu (2004),

contain incremental or superior information in addition to the information provided by historical data because they reflect market expectations. Inspired by the implied volatility literature, one can expect that the estimates of the COE based on option market prices may contain incremental or superior information in addition to the information contained in the traditional estimates of the COE obtained with historical data.

To obtain the COE implied by option market prices, we first develop an option pricing model in which the expected return of the underlying asset is a tractable parameter. To the best of our knowledge there are only two papers in the literature that discuss the estimation of the expected return of assets using option market prices. Heston (1993) presented an option pricing formula based on the log-gamma distribution under which the expected return of the stock is determined by both the location and the volatility parameters. Unfortunately, his pricing formula depends on the location parameter μ but is independent of the volatility parameter, σ. Hence this option pricing model alone can not be used to estimate the COE. McNulty, Yeh, Schulze, and Lubatkin (2002) also developed a forward-looking approach to calculate the COE based on option market prices. Although their approach is interesting, the method is ad hoc and lacks theoretical support. In contrast to Heston (1993) and McNulty et al. (2002), our option pricing formula not only depends on the expected return of the underlying asset or COE but is also derived in an equilibrium representative agent economy. Hence, our COE estimates are obtained in a general equilibrium model. Moreover, our option pricing formula is analytically tractable. Thus our option pricing model can be easily applied to estimate the COE of a firm or industry.

We compare our estimates of the COE for the market and industry portfolios composed by the component firms of the S&P 100 index with the estimates obtained with the Fama/French three-factor model from January 1996 to December 2005.[4] There are at least four interesting findings from our empirical results. First, our estimates of the COE depend on the

Jiang and Tian (2005).

[4]Fama and French (1993, 1996) indicate that the three-factor model can describe the expected returns of financial assets more appropriately than the CAPM. Therefore, in this paper the model is selected to compare with.

investment horizon of the project of investment while the Fama/French's COE estimates do not.[5] When the investment period increases from 30-days to 1-year, our estimates of the averaged COE decrease from around 22% to around 10%. Second, our option-implied COE estimates are more reasonable and stable over the years than those obtained with the Fama/French method, and thus are more reliable discount rates for capital budgeting. The mean and volatility of our estimates of averaged annual COE over the sample period is about 11% and 3%, respectively. In contrast, the mean level of the Fama/French's estimates is too high (14%), and their values sometimes are extremely high (e.g. 63%) or even negative. Third, the equity risk premium of the market and industry portfolios calculated from our COE estimates is consistent with the existing literature on equity risk premium. For example, the equity premium of the market portfolio from our COE estimates is 6.96 percent which is close to the average equity premium reported by Fama and French (2002) of 7.43 percent. Finally, the return-risk relationship for various industry portfolios is stronger using the option-implied COE estimates than using the Fama/French estimates.

With forward-looking information, option prices provide a reliable source for estimating the COEs for both the market and industry portfolios. Therefore, this study contributes to the literature not only by developing an option pricing model in which the expected return is tractable, but also by providing a plausible and reliable alternative for the COE estimation.

The theoretical set up of our paper is closer to Brennan (1979), Stapleton and Subrahmanyam (1984) and Camara (2003, 2005).[6] In ours, like in these papers, there is a single-period economy. It is assumed that the stock price has a continuous distribution at the end of the period, and that dynamic trading does not exist. In such situation a riskless hedge is not possible to construct and to maintain, and markets are incomplete. In order to price options in this single-period economy, it is assumed that there is a nonsatiated, risk-averse

[5] As discussed by McNulty et al. (2002), one problem associated with the COE estimates of CAPM or any other multifactor model is that a company usually calculates just one estimate of its discount rate and applies it to all future projects regardless of the investment horizon. In contrast, the term structures of COE estimates are taken into account in our method.

[6] See also the important papers by Rubinstein (1979) and Schroder (2004).

representative agent who maximizes his expected end-of-period utility of wealth when he selects his optimal portfolio. While in that research the authors looked for utility functions and distributions of wealth that could be linked and produce preference-free option pricing formulas, we search for utility functions and distributions of wealth that can be linked and produce an option pricing formula dependent of the expected rate of return of the stock or cost of equity capital (COE).

The remainder of this paper is organized as follows. Section 2 derives an equilibrium option pricing model whose pricing formulae depends on the expected (mean) return or COE. Section 3 discusses the empirical implementation procedures and describes the data. Section 4 presents the empirical results for the component firms of the S&P 100 index. We provide some concluding remarks in Section 5.

2. The Option Valuation Model

This section starts by presenting our assumptions on the preferences of the representative agent and the stochastic behavior of aggregate wealth. Then we derive a pricing kernel that avoids arbitrage opportunities to arise in the economy. Assuming that stock prices under the actual probability measure are lognormally distributed, we obtain the equilibrium probability density function that is used to price all the assets in the economy. We derive closed-form solutions for call and put prices in this representative agent economy.

We assume that there is a representative agent with the following marginal utility function of aggregate wealth:

$$U^{'}(W_T) = W_T^{\alpha} + \beta, \tag{1}$$

where aggregate wealth, W_T, is positive, and $\alpha < 0$ and $\beta \geq 0$ are preference parameters. The representative agent is nonsatiated and risk-averse since $U^{'}(W_T) > 0$ and $U^{''}(W_T) < 0$ respectively. It can easily be verified that the preferences of the investor are also characterized by decreasing absolute risk aversion (DARA) and decreasing proportional risk aversion

(DPRA).[7]

Elton and Gruber (1995, p. 218) argue that "while there is general agreement that most investors exhibit decreasing absolute risk aversion (DARA), there is much less agreement concerning relative risk aversion". Later, Zhou (1998, p. 1730) provides empirical evidence that "justifies the assumption that consumer's utility function exhibits DPRA".

We assume that aggregate wealth, W_T, has a lognormal distribution:

$$W_T \sim \Lambda\left(\mu_w T, \sigma_w^2 T\right). \tag{2}$$

This assumption precludes negative wealth, and allow us to obtain tractable results. We start by obtaining the pricing kernel of the economy.

Lemma 1. (The pricing kernel) *Assume that the marginal utility function of the representative agent is given by equation (1) and that aggregate wealth has a lognormal distribution as in equation (2). Then the pricing kernel is given by:*

$$\phi(W_T) = \frac{W_T^{\alpha} + \beta}{\beta + exp\left(\alpha\mu_w T + \alpha^2 \frac{\sigma_w^2}{2} T\right)}. \tag{3}$$

Proof: By definition (see e.g. Camara (2003)), the pricing kernel is given by:

$$\phi(W_T) = \frac{U^{'}(W_T)}{E^P[U^{'}(W_T)]}. \tag{4}$$

Since $U^{'}(W_T) = \beta + W_T^{\alpha}$, we have $E^P[U^{'}(W_T)] = \beta + E^P[W_T^{\alpha}]$, where P is the actual probability measure. Also, since $W_T \sim \Lambda(\mu_w T, \sigma_w^2 T)$ we have $W_T^{\alpha} \sim \Lambda(\alpha\mu_w T, \alpha^2\sigma_w^2 T)$ by the properties of the standard lognormal distribution. Hence, using the formula of the expected value of a lognormal random variable we write $E^P[W_T^{\alpha}] = exp\left(\alpha\mu_w T + \frac{1}{2}\alpha^2\sigma_w^2 T\right)$. Making the apropriate substitutions in equation (4) yields equation (3). □

It is important to make some observations about the pricing kernel given by equation (3) since this is the stochastic discount factor that adjusts all assets for risk, and rules out

[7] The utility function $U(W_T) = \frac{1}{\alpha+1} W_T^{\alpha+1} + \beta W_T$ is a monotonic transformation of the power utility function. See e.g. Varian (1992) on obtaning utility functions as monotonic transformations of existing utility functions.

arbitrage opportunities to arise in the economy. The pricing kernel is positive since both the numerator and the denominator are positive, it has a displaced lognormal distribution, and has expectation $E[\phi(W_T)] = 1$. The novelty here is that the pricing kernel has a displaced lognormal distribution. This contrasts with the pricing kernel implicit in the Black-Scholes model obtained by Rubinstein (1976), Brennan (1979), Schroder (2004), and others. These authors show that a necessary and sufficient condition for the Black-Scholes model to hold in a representative agent economy is that the pricing kernel $\phi(W_T)$ has a standard lognormal distribution. Hence, the Black-Scholes model does not hold in our economy even if the stock price has a standard lognormal distribution as in Black-Scholes (1973), unless $\beta = 0$ which is the special case studied by those authors.

In a representative agent economy, the price of the stock is given by the following standard valuation equation (see e.g. Cochrane (2001) and Camara (2003)):

$$S_0 = e^{-rT} E^P \left[\phi(W_T) S_T\right]. \tag{5}$$

In our economy, the stock price has a lognormal distribution under the actual probability measure P, as in the Black-Scholes model. The next proposition derives the distribution of the stock price under the equivalent probability measure R. This distribution differs from the lognormal distribution under the risk-neutral probability measure Q implicit in the Black-Scholes model.

Proposition 2. **(The R measure)** *Assume that the marginal utility function of the representative agent is given by equation (1) and that aggregate wealth has a lognormal distribution as in equation (2). Assume that the stock price has a lognormal distribution under the actual probability measure P, i.e. $S_T \sim \Lambda(ln(S_0) + (\mu - \frac{1}{2}\sigma^2)T, \sigma^2 T)$* under P. Then:

$$S_0 = e^{-rT} E^P \left[\phi(W_T) S_T\right] = e^{-rT} E^R \left[S_T\right], \tag{6}$$

where the stock price has a mixture of lognormal distributions under the equivalent probability measure R, i.e. $S_T \sim x \cdot \Lambda(ln(S_0) + (\mu - \frac{1}{2}\sigma^2)T, \sigma^2 T) + (1 - x) \cdot \Lambda(ln(S_0) + (\mu + \alpha\rho\sigma_w\sigma - \frac{1}{2}\sigma^2)T, \sigma^2 T)$ under R, the weight x, with $0 \leq x < 1$, is a preference function (defined in the *proof of the Proposition), and ρ is the correlation between aggregate wealth and the stock price.*

Proof: See Appendix.

The stock price follows a standard lognormal distribution under P (as in the Black-Scholes model), but it follows a mixture of standard lognormal distributions under R. The expected return of the asset under P is the cost of equity capital, μ. In our economy, asset prices are also given by the expectation of the asset payoffs under the equivalent measure R, and then discounted at the riskless rate of return. Therefore, the expected rate of return of any asset under R is the riskless rate of return. There is only one difference between the measure R and the risk-neutral measure Q implicit in the Black-Scholes model. While the risk-neutral measure Q is independent of preference parameters the measure R depends on a preference parameter x. The density function of the stock price at time T under the equivalent measure R is (as we show in the proof of Proposition 2) given by:

$$\begin{aligned} f^R(S_T) &= xf(S_T; ln(S_0) + \mu T - \frac{1}{2}\sigma^2 T, \sigma^2 T) \\ &\quad +(1-x)f(S_T; ln(S_0) + (\mu + \alpha\rho\sigma_w\sigma)T - \frac{1}{2}\sigma^2 T, \sigma^2 T), \end{aligned} \tag{7}$$

which is a mixture of two lognormal densities. This density depends on preference parameters. Option prices are uniquely determined by the evaluation of the expectation of their payoffs under R, and then discounted at the riskless return.

Proposition 3. (Asset prices) *The evaluation of the current prices of the stock, S_0, call, P_c, and put, P_p, yields the following equations:*

$$1 = e^{-rT}\left[xe^{\mu T} + (1-x)e^{(\mu+\alpha\rho\sigma_w\sigma)T}\right], \tag{8}$$

$$\begin{aligned} P_c &= e^{-rT}x\left[S_0e^{\mu T}N(d_1) - KN(d_2)\right] \\ &\quad +e^{-rT}(1-x)\left[S_0e^{(\mu+\alpha\rho\sigma_w\sigma)T}N(d_3) - KN(d_4)\right], \end{aligned} \tag{9}$$

$$\begin{aligned} P_p &= e^{-rT}x\left[KN(-d_2) - S_0e^{\mu T}N(-d_1)\right] \\ &\quad +e^{-rT}(1-x)\left[KN(-d_4) - S_0e^{(\mu+\alpha\rho\sigma_w\sigma)T}N(-d_3)\right], \end{aligned} \tag{10}$$

where:

$$\begin{aligned}
d_1 &= \frac{ln\left(\frac{S_0}{K}\right) + (\mu + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \\
d_2 &= \frac{ln\left(\frac{S_0}{K}\right) + (\mu - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \\
d_3 &= \frac{ln\left(\frac{S_0}{K}\right) + (\mu + \alpha\rho\sigma_w\sigma + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \\
d_4 &= \frac{ln\left(\frac{S_0}{K}\right) + (\mu + \alpha\rho\sigma_w\sigma - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}},
\end{aligned}$$

$N(.)$ is the cumulative distribution function of the standard normal, K is the strike price, and T is the maturity date of the options.

Proof: Proposition 2 shows that, in our economy, the prices of the stock, call, and put are given by:

$$\begin{aligned}
S_0 &= e^{-rT}E^P\left[\phi(W_T)S_T\right] = e^{-rT}E^R\left[S_T\right], \\
P_c &= e^{-rT}E^P\left[\phi(W_T)(S_T - K)^+\right] = e^{-rT}E^R\left[(S_T - K)^+\right], \\
P_p &= e^{-rT}E^P\left[\phi(W_T)(K - S_T)^+\right] = e^{-rT}E^R\left[(K - S_T)^+\right],
\end{aligned}$$

where the stock price has a mixture of lognormal distributions under the equivalent probability measure R, i.e. $S_T \sim x \cdot \Lambda(ln(S_0) + (\mu - \frac{1}{2}\sigma^2)T, \sigma^2 T) + (1 - x) \cdot \Lambda(ln(S_0) + (\mu + \alpha\rho\sigma_w\sigma - \frac{1}{2}\sigma^2)T, \sigma^2 T)$ under R. Hence, evaluating the expectations under R, yields the desired results.□

We obtain the next result when we use the equilibrium relation given by equation (8) into options prices to eliminate a set of preference parameters from option prices.

Proposition 4. (Call and put option prices) *The current prices of the call and put are given by:*

$$\begin{aligned}
P_c &= e^{-rT}x\left[S_0 e^{\mu T}N(d_1) - KN(d_2)\right] \\
&\quad + e^{-rT}(1 - x)\left[S_0\left(\frac{e^{rT} - xe^{\mu T}}{1 - x}\right)N(d_3) - KN(d_4)\right], \qquad (11)
\end{aligned}$$

$$P_p = e^{-rT}x\left[KN(-d_2) - S_0 e^{\mu T}N(-d_1)\right] \\ +e^{-rT}(1-x)\left[KN(-d_4) - S_0\left(\frac{e^{rT} - xe^{\mu T}}{1-x}\right)N(-d_3)\right], \quad (12)$$

where:

$$d_1 = \frac{ln\left(\frac{S_0}{K}\right) + (\mu + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}},$$
$$d_2 = \frac{ln\left(\frac{S_0}{K}\right) + (\mu - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}},$$
$$d_3 = \frac{ln\left(\frac{S_0}{K}\left(\frac{e^{rT}-xe^{\mu T}}{1-x}\right)\right) + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}},$$
$$d_4 = \frac{ln\left(\frac{S_0}{K}\left(\frac{e^{rT}-xe^{\mu T}}{1-x}\right)\right) - \frac{\sigma^2}{2}T}{\sigma\sqrt{T}},$$

$N(.)$ is the cumulative distribution function of the standard normal, K is the strike price, and T is the maturity date of the options.

Proof: Write equation (8) as $\frac{1}{T}ln\left(\frac{e^{rT}-xe^{\mu T}}{1-x}\right) = \mu + \alpha\rho\sigma_w\sigma$. Then use this expression in equations (9) and (10) to eliminate the term $\mu + \alpha\rho\sigma_w\sigma$. □

Corollary 5. (The Black-Scholes model) *If $\beta = 0$ then the Black-Scholes (1973) valuation equations obtain.*

Proof: If $\beta = 0$ then, by equation (24) of the Appendix, we obtain that $x = 0$. If $x = 0$ in equations (11) and (12) then we have the Black-Scholes call and put prices. □

Equations (11) and (12) show that, in our economy, option prices depend on the stock price S_0, the strike price K, the time to maturity T, the interest rate r, the stock volatility σ, the preference function x, and the rate of return required by stockholders or cost of equity capital (COE), μ. The parameters S_0, K, T, and r are observable. Then equations (11) and (12) can be solved for the three unknowns x, σ, and μ by minimizing the sum of squared differences between market prices and theoretical prices of options. This tell us what is the cost of equity capital (COE), μ, implied by market prices including option prices.

3. Implementation Procedures and Data

3.1 Implementation Procedures

Assume that we are in an n-firm economy. As shown in equation (11) or (12), the unknown parameters include x, μ_i, and σ_i (for i=1,2,...,n). Several loss functions can be considered when estimating the parameters with option prices. As is common in the literature, we minimize the sum of squared differences between the market and theoretical prices of options with the same time-to-maturity. In theory, for a time point the risk preference parameter x is unique across n assets, and all parameters (x, μ_i, and σ_i) should be estimated simultaneously by minimizing the following loss function:

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i}(C_i(K_j) - c_i(K_j|x, \mu_i, \sigma_i))^2, \tag{13}$$

where $C_i(.)$ and $c_i(.)$ denote the market and theoretical call prices, respectively, and m_i is the number of option contracts with different strike prices K_j for firm i. However, the problem of the dimension curse will occur for a multi-asset estimation. For example, we need to estimate 201 parameters in an optimization procedure when having 100 assets. Therefore, to make the estimation plausible, we adjust the above procedure to a two-step procedure.

In the first step, given a fixed x, x_0, we can easily estimate μ_i and σ_i for all firms by minimizing their individual loss functions:

$$L_i(x_0; \mu_i, \sigma_i) = \sum_{j=1}^{m_i}(C_i(K_j) - c_i(K_j|\mu_i, \sigma_i, x = x_0))^2, \tag{14}$$

where $L_i(x_0; \mu_i, \sigma_i)$ is the loss function of firm i given that $x = x_0$, where $i = 1, 2, \ldots, n$. For each x_0, we obtain a set of estimates of $\hat{\mu}_i(x_0)$ and $\hat{\sigma}_i(x_0)$ for n firms. By changing x_0 recursively from 0 to 0.99 with the interval of 0.01, we have 100 sets of estimates of $\hat{\mu}_i(x_0)$ and $\hat{\sigma}_i(x_0)$ ($i = 1, 2, \ldots, n$), respectively. We then choose x_0 with which we have the least sum of all individual loss functions and use the corresponding $\hat{\mu}_i(x_0)$ and $\hat{\sigma}_i(x_0)$ as the optimal estimates, i.e. choosing x_0 that minimizes the following function:

$$\sum_{i=1}^{n} L_i(x_0; \hat{\mu}_i(x_0), \hat{\sigma}_i(x_0)). \tag{15}$$

3.2 Data

An empirical implementation is conducted for the component firms (on December 31, 2005) of the S&P 100 index.Therefore, the primary data are the market prices of options written on the stocks of these firms. As both option pricing theories and option trading experiences indicate that the marginal risk of an investment declines as a function of the square root of time and the falling marginal risk reduces the annual discount rate, the cost of equity capital that serves as the discount rate for capital budgeting depends on the investment duration.[8] In order to estimate the cost of equity capital for a fixed horizon and make an appropriate comparison with the conventional estimates generated from an asset pricing model, we have to use the market prices of options with a fixed time-to-maturity at a regular frequency (e.g. monthly). Therefore, in this study we use the month-end market volatility surfaces of options on the stocks of the component firms of the S&P 100 index for the period from January 1996 to December 2005.

The volatility surfaces are collected from the database of OptionMetrics. For every month-end trading day, we have the Black-Scholes implied volatility surfaces made up of 13 strike prices reported as deltas for both call and put options with 4 different time-to-maturities (30, 91, 182, and 365 days).[9] The calculation of volatility surfaces is based on a kernel smoothing algorithm and an interpolation technique. The database also provides the month-end closing prices of the underlying stocks.

The risk-free interest rates are calculated from the OptionMetrics zero curves formed by a collection of continuously-compounded zero-coupon interest rates with various maturities. We use the linear interpolation method to generate the interest rates whose horizons exactly

[8]This point is also emphasized by McNulty et al. (2002).

[9]The delta ranges between 0.2 (-0.2) and 0.8 (-0.8) with the interval of 0.05 for call (put) options.

match the time-to-maturities of options.

As the underlying stocks pay discrete dividends, we use the OptionMetrics projected dividend amounts and ex-dividend dates that are based on the securities' usual payments and frequencies in order to compute the present values of the projected dividend payments prior to the maturity dates. We then deduct the present values of projected dividend payments from the market prices of underlying stocks and then value the options as though the stocks pay no dividends.

With the adjusted prices of underlying assets and the matched risk-free rates, all volatility surfaces are converted to their Black-Scholes (European) option prices of non-dividend-paying stocks. As out-of-money options are usually traded more heavily than in-the-money ones, in-the-money options are excluded, and all put prices are converted to call prices using the put-call parity for the computation of the loss functions.[10]

To compare our estimates of the costs of equity capital with those estimated by a conventional method, the Fama/French three-factor model, we also collect the monthly time series of the three factors - the market portfolio return minus the risk-free interest rate (RmRf), a small-size portfolio return minus a big-size portfolio return (SMB), and a high-book-to-market-equity portfolio return minus a low-book-to-market-equity portfolio return (HML) - from the website of Kenneth R. French for the sample period from January 1993 to December 2005.[11]

4. Empirical Results

First, the general properties of the estimates of the risk-preference, expected-return or COE, and volatility parameters (x, μ and σ, respectively) in our option pricing formulae

[10] This procedure for data selection has been employed by many studies such as Bliss and Panigirtzoglou (2004) and Jiang and Tian (2005).

[11] As a long period of historical prices is necessary for the COE estimation using the Fama/French factor model, the sample period for the historical data is three year longer than that for the option data.

are presented for the market portfolio with the values averaged across the component firms of the S&P 100 index. We follow up to compare the costs of equity capital (COEs) implied in our option pricing model with those estimated from the Fama/Fench three-factor model for both the market and six industrial portfolios composed by the component firms of the S&P 100 index.

4.1 General Properties of Option-implied Estimates

We use the end-of-the-month prices of options written on the component stocks of the S&P 100 index with four different time-to-maturities - 30, 91, 182, and 365 days - and follow the two-step procedure detailed in Section 3.1 to estimate the parameters, x, μ, and σ, for the period from January 1996 to December 2005 (120 months). This research reports both value-weighted and equally-weighted results of the market portfolio composed by the 100 component stocks.

Table 1 and Figure 1 respectively show the summary statistics and processes of x estimates for various time-to-maturities. For any time point, this parameter is fixed across firms and thus no average is required. Both the processes and distributions of x estimates are very similar across time-to-maturities. The estimates of x range from 0.70 to 0.99 and their mean level is about 0.85. As mentioned in Section 2, our option pricing model converges to the Black-Scholes model when x approaches 0. The estimates clearly indicate that the market prices of equity options are very different from the Black-Scholes prices. Moreover, the estimates of x do not change very much with time and their volatility is only about 0.05. The first-order autocorrelation coefficients range from 0.4 to 0.69.

Table 2 and Figure 2 respectively present the summary statistics and processes of the COE estimates μ for the market portfolio. As the properties of the valued-weighted and equally-weighted estimates are almost the same, the following discussion applies to both. The most remarkable finding is that the estimate of μ decreases as the time-to-maturity increases. For example, the mean level for the 30-day maturity is about 22%, while it is down to about 10%

for the 365-day maturity. This finding is consistent with the general argument in option pricing theories and option trading experiences. As the marginal risk of an investment declines as a function of the square root of time and the falling marginal risk reduces the annual discount rate, the cost of equity capital that serves as the discount rate for capital budgeting should depend on the horizon of investment. In addition, the estimates of μ become less volatile as the time-to-maturity increases. For example, volatility decreases from 0.12 to 0.03 when the time-to-maturity increases from 30 to 365 days. In other words, the option-implied COE estimates are less volatile for longer investment horizons. In contrast, the skewness and kurtosis of μ estimates do not exhibit obvious differences across time-to-maturities. Moreover, the estimates of μ have a high first-order autocorrelation that ranges from 0.75 to 0.87, which is consistent with the general sense that the COE for a firm should not change much with time.

Table 3 and Figure 3 respectively show the summary statistics and processes of σ estimates for the market portfolio. Again, the properties of the valued-weighted and equally-weighted estimates are almost identical. Different from the findings for x and μ estimates, all properties of σ estimates do not depend on the time-to-maturity, which can be clearly seen in Figure 3 as all lines being very close - that is, the distributions and processes of σ estimates are almost the same across time-to-maturities. This could be driven by the stylized fact that equity prices follow a random walk, because the similar annualized volatilities for different horizons indicate that the sum of short-term volatilities equals the long-term volatility. The mean level of σ estimates is about 0.25 and the estimates are spread between 0.14 and 0.42. Similar to the finding for μ estimates, the volatility of σ estimates is also very small, at about 0.06. Moreover, σ is highly persistent as the first-order autocorrelation is even as high as 0.96, which is consistent with the stylized fact, volatility clustering, observed in the market prices of financial assets.

In summary, our empirical properties of the estimates for x, μ, and σ are in line with the theoretical assumptions. In particular, with different time-to-maturities of option prices, we can estimate the COE that properly matches the required investment duration. Even when the options with the time-to-maturity that perfectly matches a particular investment horizon are not traded in the market, we still can utilize an interpolation technique with the COEs estimated from other maturities of option prices to appropriately generate the COE with the desirable maturity. By contrast, the COEs estimated from many conventional methods with historical equity prices do not consider the investment duration.

4.2 Estimating Costs of Equity Capital with Alternative Methods

The conventionally standard approach for estimating the COE is the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965). As an alternative, Fama and French (1993) propose another two pricing factors, SMB and HML.[12] However, the COEs estimated by both models rely on historical data and are indifferent to the holding period. In contrast, our COE estimates from option market prices are forward-looking and depend on the investment horizon. As recent evidence (Fama and French, 1993 & 1996) suggests that the Fama/Fench three-factor model is better than the CAPM in describing expected returns, in this study we compare our COE estimates with those estimated from the Fama/Fench three-factor model for the most common investment duration of capital budgeting, which is one year. First of all, we look at the market portfolio COE by comparing the COE estimates averaged across all component firms of the S&P 100 index. We then investigate the industrial COEs by comparing the COE estimates of various industrial portfolios composed by the component stocks of the S&P 100 index. To avoid forming an industrial portfolio with too few firms, only those industries including at least 10 component firms are selected. As our previous findings indicate that the general properties of value-weighted and equally-weighted COE estimates are almost the same, we only take the value-weighted estimates to compare with in this section.

As it is necessary to use a long period of historical data to obtain a smooth time series of COE estimates for the Fama/Fench three-factor model, we use a three-year sample period

[12]Some studies, such as Blanchard (1993), Claus and Thomas (2001), Gebhardt, Lee, and Swaminathan (2001), and Fama and French (2002), use valuation models with fundamentals to estimate expected returns. In this study we focus on the comparisons with the market-based estimates only.

for the estimation. We first estimate the factor loadings with the three-year historical data and then use the averages of the historical factor values with the estimated factor loadings to generate the COE estimate for the time point. By rolling over the sample period month by month, we form the time series of COE estimates for the period from January 1996 to December 2005.

The first column of Table 4 and Figure 4 displays the summary statistics and processes of alternative COE estimates for the market portfolio, respectively. It clearly presents that our option-implied COE estimates are much more stable, while the Fama/French estimates are very volatile and roughly range from 0.4 to -0.15. Moreover, the level of our option-implied estimates for the market is much more reasonable for serving as the discount rate for capital budgeting. The mean value is about 11% with a small volatility (0.03), which is similar to the previous evidence of Fama and French that the average return of the component stocks of the S&P 500 index is 9.62% for 1951 to 2000. In contrast, the mean level of the Fama/French COE estimates is about 14% and the estimates sometimes are extremely high or even negative. Basically, our findings are consistent with the argument of Fama and French (1997) and Pástor and Stambaugh (1999) in that the COE estimates from both the CAPM and the three-factor models are surely imprecise due to the uncertainty about true factor risk premiums and imprecise estimates of the factor loadings.

To further investigate the differences between the option-implied and the historical-data-generating COE estimates, we follow the ICB industry classifications to construct six industrial portfolios from the component stocks of the S&P 100 index. Only those industries including at least 10 firms are selected. Six industries are selected and they are Industrials, Consumer Services, Consumer Goods, Health Care, Financials, and Information Technology. Table 4 and Figure 5 offer the summary statistics and processes of the portfolio COEs for various industries from alternative approaches. The results show the same pattern as we have found in the COE estimates of the market portfolio. The common findings include that the option-implied COE estimates are much more stable and reasonable, while the Fama/French estimates are very volatile and sometimes unreasonable. For example, the range for the option-implied COEs of Information Technology is 5.63% to 22.9%, while that for the Fama/French COEs is -36.23% to 63.02%. On average, the volatilities of the Fama/French estimates are more than five times of those of the option-implied ones.

In terms of the equity premium (defined as the difference between the portfolio expected return and the risk-free interest rate), the average equity premium for the market portfolio calculated from our option-implied COEs is 6.96 percent, which is close to the estimate of Fama and French (2002) for 1951 to 2000, 7.43 percent. In contrast, the average equity premium of Fama/French model for the market portfolio is 10.71 percent, which is about 3 percent more than its actual value. Moreover, as shown in Table 5, although the premiums estimated from both approaches indicate that the COEs for Financials and Information Technology are higher than the market averages, the levels of equity premiums estimated by the Fama/French model are too high. For example, the equity premiums for Financials and Information Technology are 11.44% and 16.69%, respectively.

According to the literature in volatility forecasting, implied volatility is the best predictor for future volatility. Using the implied volatilities of various industrial portfolios as their total risk proxies and assuming that the systematic risk is proportional to the total risk with the same ranking across industries, we look at the cross-section trade-off relationships between alternative COEs and their risk proxies. The rank correlation coefficient between option-implied COEs and risk (0.89) is much higher than that between Fama/French's COEs and risk (0.43). This means the return-risk relationship is stronger using the option-implied estimates although both approaches show that the Information Technology industry is the most risky industry with the highest COEs among all six industries.

In summary, with forward-looking information, option prices provide a reliable source for estimating the COEs for both the market and industrial portfolios. Compared with the COEs estimated from historical data with a conventional asset pricing model, the option-implied estimates are much more stable and reasonable. Moreover, with option prices we can obtain the COE estimates corresponding to any desirable investment duration, which is particularly valuable for the practical implementation of capital budgeting. Therefore, in terms of both plausibility and reasonability, our option pricing model provides a reliable alternative for estimating COEs.

5. Conclusions

The expected return of the stock or cost of equity capital (COE) does not affect most existing modern option pricing models. Then it is in general impossible to estimate the COE using such models. This paper contributes to the literature by deriving equilibrium option pricing formulae which explicitly depend on the expected return of the stock or COE. Thus, we are able to provide a forward-looking equilibrium estimate of the COE using our option pricing model and current market prices. Our empirical tests for the component firms of the S&P 100 index for the period 1996-2005 indicate that our COE estimates are superior to those obtained with the Fama/French (1993) three-factor model. For example, our COE estimates depend on the investment horizon of the projects of investment. We also found that our COE estimates are reasonable and stable over the years, and thus can be used as discount rates for capital budgeting. They also generate an equity risk premium close to the average equity premium reported by Fama and French (2002). Moreover, our option-implied COEs show that a strong return-risk relationship is observed with our estimates.

Future research can apply our estimates of expected returns to test the validity of asset pricing models such as the CAPM. Many issues can be reinvestigated with our method. For example, one can ask if the expected returns of the assets are linearly related to their betas? In contrast to all existing literature which uses historical returns to test CAPM, empirical tests based on our forward-looking estimates of expected returns and betas[13] would be ex-ante tests. Therefore it should be possible to derive new results and insights using our option-implied expected return.

[13]Forward-looking betas estimated from option prices are now possible, see Christoffersen, Jacobs, and Vainberg (2006)

Appendix

Proof of Proposition 2: The proof is in two steps. In the first step, we derive the asset-specific pricing kernel, $\psi(S_T)$, by conditioning the pricing kernel, $\phi(W_T)$, on the asset payoff S_T. The asset specific pricing kernel is a positive random variable with $E^P[\psi(S_T)] = 1$. The asset-specific pricing kernel precludes arbitrage opportunities to arise between a specific underlying asset and derivatives written on that asset. In the second step, we derive the density function of the asset payoff S_T under the equivalent measure R, i.e. $f^R(S_T)$, by multiplying the asset-specific pricing kernel, $\psi(S_T)$, and the actual density of the stock, $f^P(S_T) = f(S_T; ln(S_0) + (\mu - \frac{1}{2}\sigma^2)T, \sigma^2 T)$, under the actual measure P. We verify that $f^R(S_T)$ is a true density since $f^R(S_T) > 0$ and $\int_{-\infty}^{+\infty} f^R(S_T)dS_T = 1$. Then we conclude that current prices (including option prices) in this economy are given by the expectation of asset payoffs with respect to the density $f^R(S_T)$, and then discounted at the riskless rate of return.

First step: The asset-specific pricing kernel is given by:

$$\begin{aligned}\psi(S_T) &= E^P[\phi(W_T) \mid S_T] \\ &= \frac{E^P[U'(W_T) \mid S_T]}{\beta + exp\left(\alpha\mu_w T + \alpha^2 \frac{\sigma_w^2}{2} T\right)} \\ &= \frac{\beta + E^P[W_T^\alpha \mid S_T]}{\beta + exp\left(\alpha\mu_w T + \alpha^2 \frac{\sigma_w^2}{2} T\right)},\end{aligned} \tag{16}$$

where we have used the pricing kernel given by equation (3).

Since $ln(S_T) \sim N(ln(S_0) + \mu T - \frac{1}{2}\sigma^2 T, \sigma^2 T)$ and $\alpha ln(W_T) \sim N(\alpha\mu_w T, \alpha^2\sigma_w^2 T)$, we have:

$$\alpha ln(W_T) \mid ln(S_T) \sim N\left(\alpha\mu_w T + \rho\frac{\alpha\sigma_w}{\sigma}\left(ln(S_T) - \left(ln(S_0) + \mu T - \frac{1}{2}\sigma^2 T\right)\right), \alpha^2\sigma_w^2 T(1-\rho^2)\right)$$

due to the properties of the bivariate and conditional normal distributions. Then:

$$W_T^\alpha \mid S_T \sim \Lambda\left(\alpha\mu_w T + \rho\frac{\alpha\sigma_w}{\sigma}\left(ln(S_T) - \left(ln(S_0) + \mu T - \frac{1}{2}\sigma^2 T\right)\right), \alpha^2\sigma_w^2 T(1-\rho^2)\right).$$

Using the definition of expectation of a lognormal random variable, the asset-specific pricing kernel given by equation (16) can be written as:

$$\psi(S_T) = \frac{\beta + exp\left(\alpha\mu_w T + \rho\frac{\alpha\sigma_w}{\sigma}\left(ln(S_T) - \left(ln(S_0) + \mu T - \frac{1}{2}\sigma^2 T\right)\right) + \frac{1}{2}\alpha^2\sigma_w^2 T(1-\rho^2)\right)}{\beta + exp\left(\alpha\mu_w T + \alpha^2\frac{\sigma_w^2}{2}T\right)}, \tag{17}$$

which is positive since both the numerator and denominator are positive. We also conclude that:

$$E^P[\psi(S_T)] = \frac{\beta + exp\left(\alpha\mu_w T + \alpha^2\frac{\sigma_w^2}{2}T\right)}{\beta + exp\left(\alpha\mu_w T + \alpha^2\frac{\sigma_w^2}{2}T\right)} = 1. \tag{18}$$

Equation (18) can be obtained by noting that we have implicitly in equation (17) the following relation:

$$E^P[W_T^\alpha \mid S_T] = exp\left(\alpha\mu_w T + \rho\frac{\alpha\sigma_w}{\sigma}\left(ln(S_T) - \left(ln(S_0) + \mu T - \frac{1}{2}\sigma^2 T\right)\right) + \frac{1}{2}\alpha^2\sigma_w^2 T(1-\rho^2)\right), \tag{19}$$

which is a lognormal random variable. Hence:

$$ln\left\{E^P[W_T^\alpha \mid S_T]\right\} = \alpha\mu_w T + \rho\frac{\alpha\sigma_w}{\sigma}\left(ln(S_T) - \left(ln(S_0) + \mu T - \frac{1}{2}\sigma^2 T\right)\right) + \frac{1}{2}\alpha^2\sigma_w^2 T(1-\rho^2)$$

is a normal variate. Evaluating the mean and variance of this normal random variable yields:

$$E^P\left[ln\left\{E^P[W_T^\alpha \mid S_T]\right\}\right] = \alpha\mu_w T + \frac{1}{2}\alpha^2\sigma_w^2 T(1-\rho^2), \tag{20}$$

$$Var^P\left[ln\left\{E^P[W_T^\alpha \mid S_T]\right\}\right] = \rho^2\alpha^2\sigma_w^2 T. \tag{21}$$

Using the relation between the normal and the lognormal random variables yields:

$$E^P[\{E^P[W_T^\alpha \mid S_T]\}] = exp\left(\alpha\mu_w T + \alpha^2\frac{\sigma_w^2}{2}T\right). \tag{22}$$

Summing up these previous remarks yields equation (18).

Second step: We set $f^R(S_T) = \psi(S_T)\cdot f^P(S_T)$. Then $E^P[\phi(W_T)S_T] = E^P[E^P[\phi(W_T)S_T \mid S_T]] = E^P[\psi(S_T)S_T]] = E^R[S_T]$. We write equation (17) as:

$$\psi(S_T) = x + (1-x)exp\left(\rho\frac{\alpha\sigma_w}{\sigma}\left(ln(S_T) - \left(ln(S_0) + \mu T - \frac{1}{2}\sigma^2 T\right)\right) - \frac{1}{2}\alpha^2\sigma_w^2 T\rho^2\right), \tag{23}$$

where:

$$x = \frac{\beta exp\left(-\alpha\mu_w T - \frac{1}{2}\alpha^2\sigma_w^2 T\right)}{1 + \beta exp\left(-\alpha\mu_w T - \frac{1}{2}\alpha^2\sigma_w^2 T\right)}, \tag{24}$$

with $0 \leq x < 1$. Multiplying the asset specific pricing kernel given by equation (23) and the actual density of the stock, $f^P(S_T)$ yields:

$$\begin{aligned} f^R(S_T) &= xf(S_T; ln(S_0) + \mu T - \frac{1}{2}\sigma^2 T, \sigma^2 T) \\ &+(1-x)f(S_T; ln(S_0) + (\mu + \alpha\rho\sigma_w\sigma)T - \frac{1}{2}\sigma^2 T, \sigma^2 T). \end{aligned} \tag{25}$$

Since $f(S_T; ln(S_0 + \mu T - \frac{1}{2}\sigma^2 T, \sigma^2 T)$ and $f(S_T; ln(S_0) + (\mu + \alpha\rho\sigma_w\sigma)T - \frac{1}{2}\sigma^2 T, \sigma^2 T)$ are two lognormal densities, we see that $\int_{-\infty}^{+\infty} f^R(S_T)dS_T = 1$. Also, since that $f^R(S_T)$ is a mixture of lognormal densities and $f^P(S_T)$ is a lognormal density then P and R are equivalent measures since both give the same probabiliy to the set $(0, +\infty)$ and therefore to its complement $(-\infty, 0]$ This completes the proof of proposition 1. □

References

Blair, B., S.H. Poon and S. Taylor, 2001, Forecasting S&P 100 Volatility: The Incremental Information Content of Implied Volatility and High Frequency Index Returns, *Journal of Econometrics*, 105, 5-26.

Black, F. and M. Scholes, 1973, The Pricing of Options and Corporate Liabilities, *Journal of Political Economy*, 81, 637-659.

Blanchard, O.J., 1993, Movements in the Equity Premium, *Brooking Papers on Economic Activity*, 2, 75-138.

Bliss, R. and N. Panigirtzoglou, 2004, Option-Implied Risk Aversion Estimates, *journal of Finance*, 59, 407-446.

Brennan, M.J., 1979, The Pricing of Contingent Claims in Discrete Time Models, *Journal of Finance*, 34, 53-68.

Bruner, R.F., K. Eades, R. Harris, and R. Higgins, 1998, Best Practices in Estimating the Cost of Capital: Survey and Synthesis, *Financial Practice and Education*, 8, 13-28.

Camara, A., 2003, A Generalization of the Brennan-Rubinstein Approach for the Pricing of Derivatives, *Journal of Finance*, 58, 805-819.

Camara, A., 2005, Option Prices Sustained by Risk-Preferences, *Journal of Business*, 78, 1683-1708.

Canina, L. and S. Figlewski, 1993, The Informational Content of Implied Volatility, *Review of Financial Studies*, 6, 659-681.

Chen, N.F., R. Roll, and S.A. Ross, 1986, Economic Forces and the Stock Market, *Journal of Business*, 59, 383-404.

Christensen, B. and N. Prabhala, 1998, The Relation Between Implied and Realized Volatility, *Journal of Financial Economics*, 50, 125-150.

Christoffersen, P., K. Jacobs and G. Vainberg, 2006, Forward-Looking Betas, Working Paper, McGill University.

Claus, J. and J. Thomas, 2001, Equity Premia as Low as Three Percent? Evidence from Analysts'Earnings Forecasts for Domestic and International Stock Markets, *Journal of Finance*, 56, 1629-1666.

Cochrane, J., 2001, *Asset Pricing*, Princeton: Princeton University Press.

Day, T. and C. Lewis, 1992, Stock Market Volatility and the Information Content of Stock Index Options, *Journal of Econometrics*, 52, 267-287.

Elton, E. and M. Gruber, 1995, "Modern Portfolio Theory and Investment Analysis", Wiley.

Fama, E.F. and K.R. French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics*, 33, 3-56.

Fama, E.F. and K.R. French, 1996, Multifactor Explanations of Asset Pricing Anomalies, *Journal of Finance*, 51, 1121-1152.

Fama, E.F. and K.R. French, 1997, Industry Costs of Equity, *Journal of Financial Economics*, 43, 153-193.

Fama, E.F. and K.R. French, 2002, The Equity Premium, *Journal of Finance*, 57, 637-659.

Ferson, W.E. and C.R. Harvey, 1993, The Risk and Predictability of International Equity Returns, *Review of Financial Studies*, 6, 527-566.

Gebharde, W.R., C.M.C. Lee, and B. Swaminathan, 2001, Toward an Implied Cost of

Capital, *Journal of Accounting Research*, 39, 135-176.

Graham, J.R. and C.P. Harvey, 2001, The Theory and Practice of Corporate Finance: Evidence from the Field, *Journal of Financial Economics*, 60, 187-243.

Heston, S.L., 1993, Invisible Parameters in Option Prices, *Journal of Finance*, 48, 933-947.

Jegadeesh, N. and S. Titman, 1993, Returns on Buying Winners and Selling Losers: Implications for Stock Market Efficiency, *Journal of Finance*, 48, 65-91.

Jiang, G. and Y. Tian, 2005, The Model-Free Implied Volatility and Its Information Content, *Review of Financial Studies*, 18, 1305-1342.

Lamoureux, G. and W. D. Lastrapes, 1993, Forecasting Stock-Return Variance: Toward an Understanding of Stochastic Implied Volatilities, *Review of Financial Studies*, 6, 293-326.

Lintner, J., 1965, The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *Review of Economics and Statistics*, 47, 13-37.

McNulty, J.J., T.D. Yeh, W.S. Schulze and M.H. Lubatkin, 2002, What's Your Real Cost of Capital? *Harvard Business Review*, 114-121.

Pastor, L. and R.F. Stambaugh, 1999, Costs of Equity Capital and Model Mispricing, *Journal of Finance*, 54, 67-121.

Pong, S., M. Shackleton, S. Taylor and X. Xu, 2004, Forecasting Sterling/Dollar Volatility: A Comparison of Implied Volatility and AR(FI)MA Models, *Journal of Banking and Finance*, 28, 2541-2563.

Poon, S.H. and C. Granger, 2003, Forecasting Volatility in Financial Markets: A Review, *Journal of Economic Literature*, 26, 478-539.

Rubinstein, M., 1976, The Valuation of Uncertain Income Streams and the Pricing of Options. *Bell Journal of Economics and Management Science*, 7, 407-425.

Schroder, M., 2004, Risk-neutral parameter shifts and derivatives pricing in discrete time, *Journal of Finance*, 59, 2375-2402.

Stapeton, R. and Subrahmanyam, M., 1984, The Valuation of Multivariate Contingent Claims in Discrete Time Models, *Journal of Finance*, 39, 707-719.

Sharpe, W. F., 1964, Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, *Journal of Finance*, 19, 425-442.

Varian, H., 1992, *Microeconomic Analysis*, 3rd edition, New York: W. W. Norton & Company.

Zhou, Z., 1998, An Equilibrium Analysis of Hedging with Liquidity Constraints, Speculation, and Government Price Subsidy in Commodity Market, *Journal of Finance*, 53, 1705-1736.

Table 1: Summary Statistics of *x* Estimates

This table presents the summary statistics of the estimates of *x*: the risk preference parameter in the option pricing formula. The estimates are generated from the month-end prices of options written on the component stocks of the S&P 100 index with alternative time-to-maturities. The sample period is from 1996 to 2005.

Time-to-maturity	30 days	91 days	182 days	365 days
Mean	0.9024	0.8580	0.8486	0.8500
Median	0.9000	0.8550	0.8500	0.8500
Maximum	0.9900	0.9800	0.9600	0.9800
Minimum	0.7700	0.7500	0.7000	0.7100
Std. Dev.	0.0531	0.0472	0.0527	0.0624
Skewness	-0.2372	0.4491	0.0423	0.0697
Kurtosis	2.0960	2.9768	2.4913	2.3425
Jarque-Bera Prob.	0.0738	0.1329	0.5143	0.3221
AR(1)	0.4300	0.4000	0.6900	0.6000

Table 2: Summary Statistics of μ Estimates

This table presents the summary statistics of the estimates of μ: the expected-return parameter in the option pricing formula for the market portfolio composed by the component firms of the S&P 100 index. The estimates are generated from the month-end prices of options written on the component stocks of the S&P 100 index with alternative time-to-maturities. The averages are either value-weighted or equally-weighted. The sample period is from 1996 to 2005.

Panel 1: Value-weighted Average

Time-to-maturity	30 days	91 days	182 days	365 days
Mean	0.2295	0.1781	0.1398	0.1055
Median	0.2020	0.1642	0.1292	0.0955
Maximum	0.6252	0.3515	0.2511	0.1979
Minimum	0.0571	0.0952	0.0697	0.0606
Std. Dev.	0.1265	0.0600	0.0409	0.0292
Skewness	0.7170	0.6292	0.5486	0.7034
Kurtosis	2.7041	2.4490	2.4287	2.7340
Jarque-Bera Prob.	0.0047	0.0089	0.0218	0.0059
AR(1)	0.7500	0.8400	0.8700	0.8400

Panel 2: Equally-weighted Average

Time-to-maturity	30 days	91 days	182 days	365 days
Mean	0.2111	0.1699	0.1342	0.1016
Median	0.1805	0.1515	0.1270	0.0939
Maximum	0.5578	0.3261	0.2229	0.1712
Minimum	0.0657	0.0907	0.0630	0.0468
Std. Dev.	0.1116	0.0556	0.0370	0.0266
Skewness	0.8266	0.6769	0.5451	0.4097
Kurtosis	2.8442	2.4538	2.4313	2.2992
Jarque-Bera Prob.	0.0010	0.0049	0.0228	0.0547
AR(1)	0.7600	0.8500	0.8500	0.8400

Table 3: Summary Statistics of σ Estimates

This table presents the summary statistics of the estimates of σ: the volatility parameter in the option pricing formula for the market portfolio composed by the component firms of the S&P 100 index. The estimates are generated from the month-end prices of options written on the component stocks of the S&P 100 index with alternative time-to-maturities. The averages are either value-weighted or equally-weighted. The sample period is from 1996 to 2005.

Panel 1: Value-weighted Average

Time-to-maturity	30 days	91 days	182 days	365 days
Mean	0.2588	0.2489	0.2456	0.2474
Median	0.2547	0.2452	0.2420	0.2423
Maximum	0.4192	0.4121	0.3913	0.3867
Minimum	0.1459	0.1382	0.1370	0.1415
Std. Dev.	0.0664	0.0664	0.0653	0.0635
Skewness	0.4687	0.4431	0.3500	0.3462
Kurtosis	2.5322	2.4048	2.1789	2.1861
Jarque-Bera Prob.	0.0643	0.0579	0.0545	0.0576
AR(1)	0.8900	0.9400	0.9600	0.9600

Panel 2: Equally-weighted Average

Time-to-maturity	30 days	91 days	182 days	365 days
Mean	0.2766	0.2669	0.2602	0.2625
Median	0.2680	0.2568	0.2537	0.2564
Maximum	0.4292	0.4145	0.3922	0.3924
Minimum	0.1711	0.1597	0.1572	0.1616
Std. Dev.	0.0649	0.0658	0.0633	0.0615
Skewness	0.3841	0.3658	0.3259	0.3520
Kurtosis	2.2499	2.1328	2.0736	2.1426
Jarque-Bera Prob.	0.0560	0.0400	0.0405	0.0461
AR(1)	0.9000	0.9400	0.9600	0.9600

Table 4: Summary Statistics of Costs of Equity Capital for Various Industries

This table presents the summary statistics of the costs of equity capital (COE) for the market and various industrial portfolios composed by the component stocks of the S&P 100 index. Only those industries including at least 10 firms are selected. The COEs are estimated with either our option pricing formula or the Fama/French 3-factor model. The option-implied COEs are estimated from the prices of options with one year to expire. The Fama/French COEs are estimated with three-year historical stock prices. The averages are value-weighted. The sample period is from 1996 to 2005.

Panel 1: Option-implied

Sector	Market	Industrials	Consumer Services	Consumer Goods	Health Care	Financials	Information Technology
Mean	0.1055	0.1060	0.1080	0.1003	0.1012	0.1079	0.1189
Median	0.0955	0.0958	0.1026	0.0947	0.0952	0.0984	0.1077
Maximum	0.1979	0.2064	0.2109	0.1871	0.1717	0.2098	0.2290
Minimum	0.0606	0.0557	0.0433	0.0475	0.0411	0.0486	0.0563
Std. Dev.	0.0292	0.0313	0.0315	0.0264	0.0281	0.0316	0.0367
Skewness	0.7034	0.7194	0.5825	0.7015	0.3715	0.6705	0.7515
Kurtosis	2.7340	2.8223	3.0777	3.0054	2.4767	2.8649	2.7416

Panel 2: Fama/French Model

Sector	Market	Industrials	Consumer Services	Consumer Goods	Health Care	Financials	Information Technology
Mean	0.1430	0.1274	0.1159	0.1195	0.1355	0.1503	0.2027
Median	0.1732	0.1787	0.1107	0.1084	0.1984	0.1536	0.2916
Maximum	0.3991	0.3197	0.5713	0.3017	0.3750	0.3989	0.6302
Minimum	-0.1367	-0.1627	-0.2685	-0.0508	-0.1181	-0.0942	-0.3623
Std. Dev.	0.1636	0.1498	0.2032	0.1033	0.1660	0.1361	0.2834
Skewness	-0.1896	-0.5089	0.2426	0.2113	-0.0868	0.0498	-0.3264
Kurtosis	1.5752	1.8647	2.2762	1.5434	1.3702	1.7859	1.7290

Table 5: Equity Premiums

This table presents the equity premiums of the market and various industrial portfolios composed by the component stocks of the S&P 100 index. Only those sectors including at least 10 firms are selected. The risk premium is defined as the difference between the portfolio's expected return and the risk-free interest rate. The expected returns are estimated with either our option pricing formula or the Fama/French 3-factor model. The option-implied expected returns are estimated from the prices of options with one year to expire. The Fama/French expected returns are estimated with three-year historical stock prices. The averages are value-weighted. The sample period is from 1996 to 2005.

Sector	Market	Industrials	Consumer Services	Consumer Goods	Health Care	Financials	Information Technology
Option-implied	0.0696	0.0702	0.0722	0.0644	0.0654	0.0721	0.0831
Fama/ French	0.1071	0.0915	0.0801	0.0837	0.0997	0.1144	0.1669

Figure1: Processes of x Estimates

This figure presents the processes of the estimates of *x*: the risk preference parameter in the option pricing formula. The estimates are generated from the month-end prices of options written on the component stocks of the S&P 100 index with alternative time-to-maturities. The sample period is from 1996 to 2005.

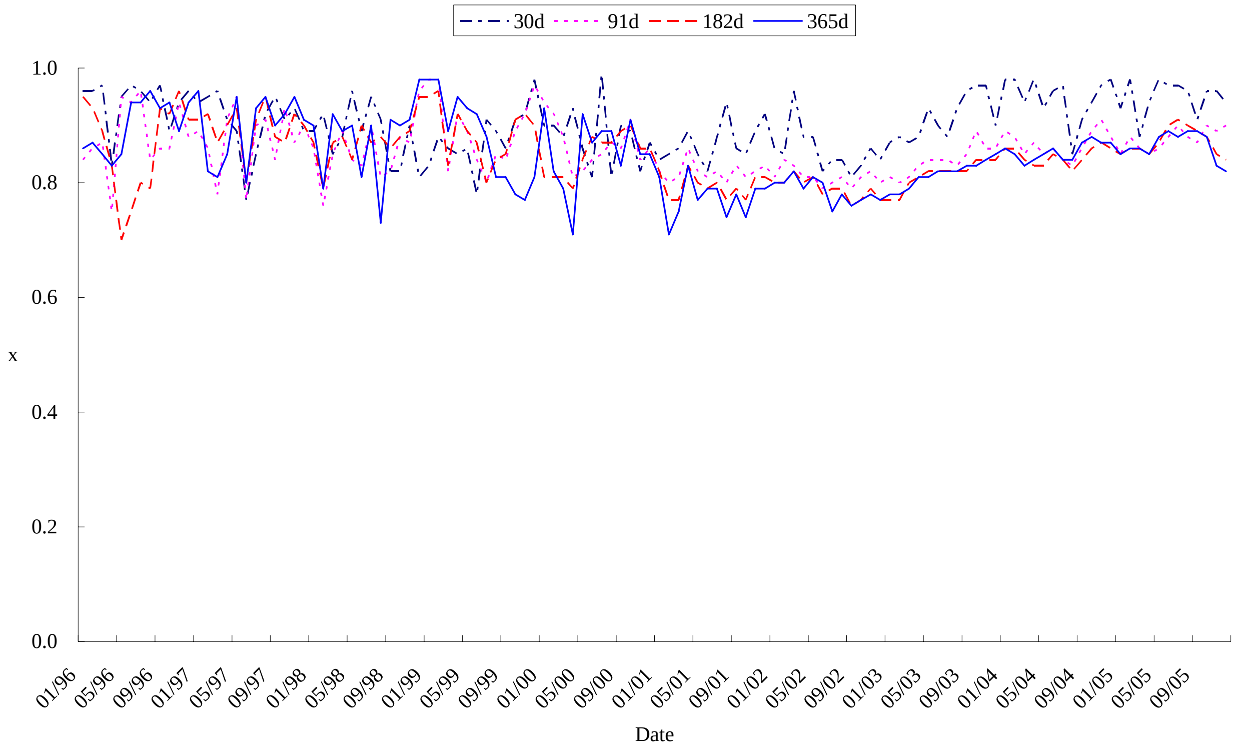
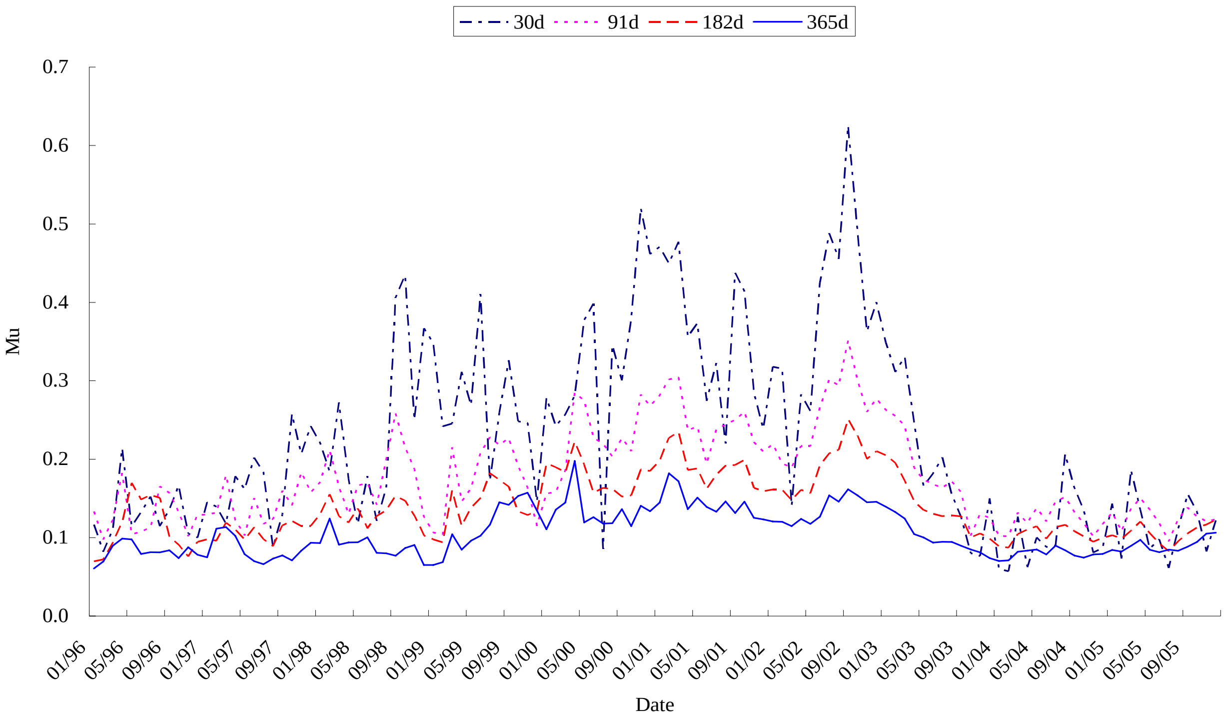


Figure 2: Processes of μ Estimates

This figure presents the processes of the estimates of μ: the expected-return parameter in the option pricing formula for the market portfolio composed by the component firms of the S&P 100 index. The estimates are generated from the month-end prices of options written on the component stocks of the S&P 100 index with alternative time-to-maturities. The averages are either value-weighted or equally-weighted. The sample period is from 1996 to 2005.

Panel 1: Value-weighted Average



Panel 2: Equally-weighted Average

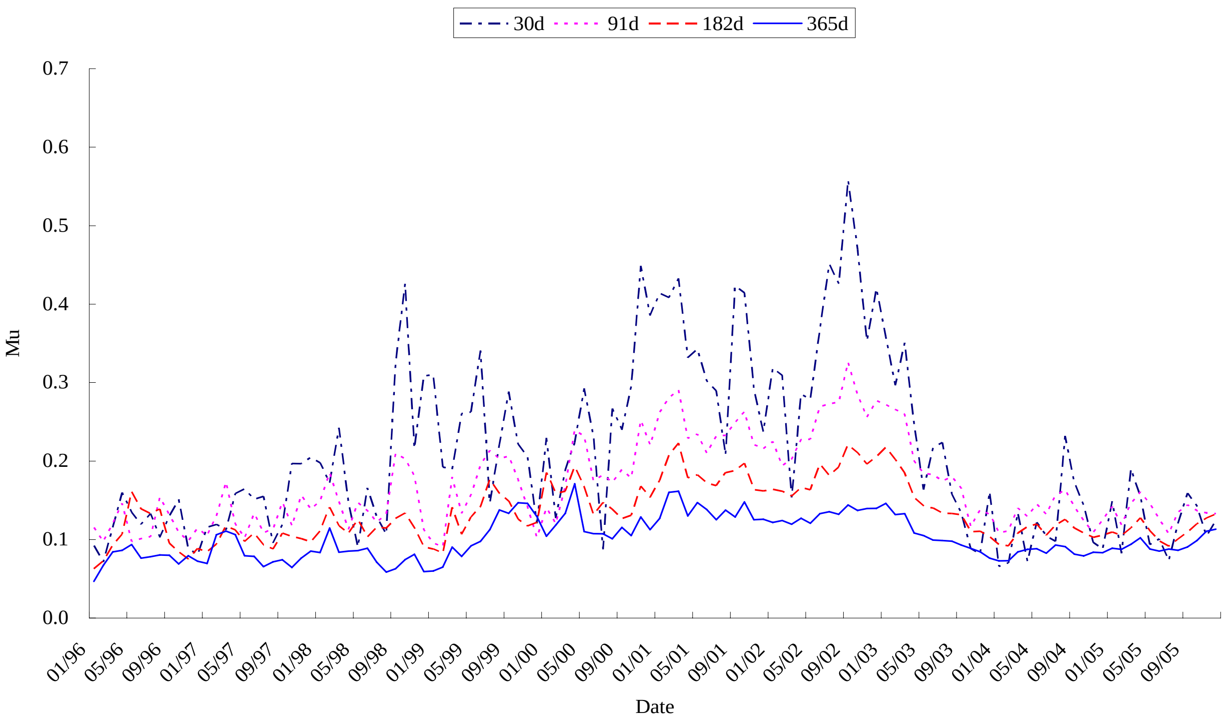
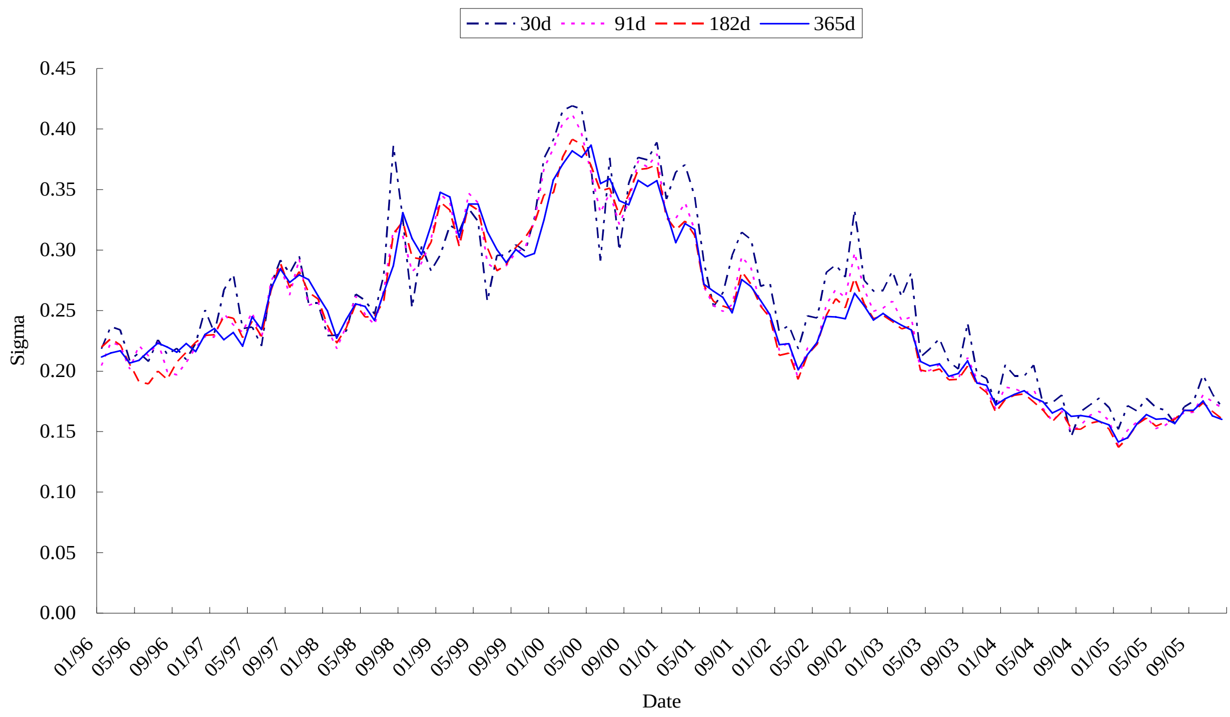


Figure 3: Processes of σ Estimates

This table presents the processes of the estimates of σ: the volatility parameter in the option pricing formula for the market portfolio composed by the component firms of the S&P 100 index. The estimates are generated from the month-end prices of options written on the component stocks of the S&P 100 index with alternative time-to-maturities. The averages are either value-weighted or equally-weighted. The sample period is from 1996 to 2005.

Panel 1: Value-weighted Average



Panel 2: Equally-weighted Average

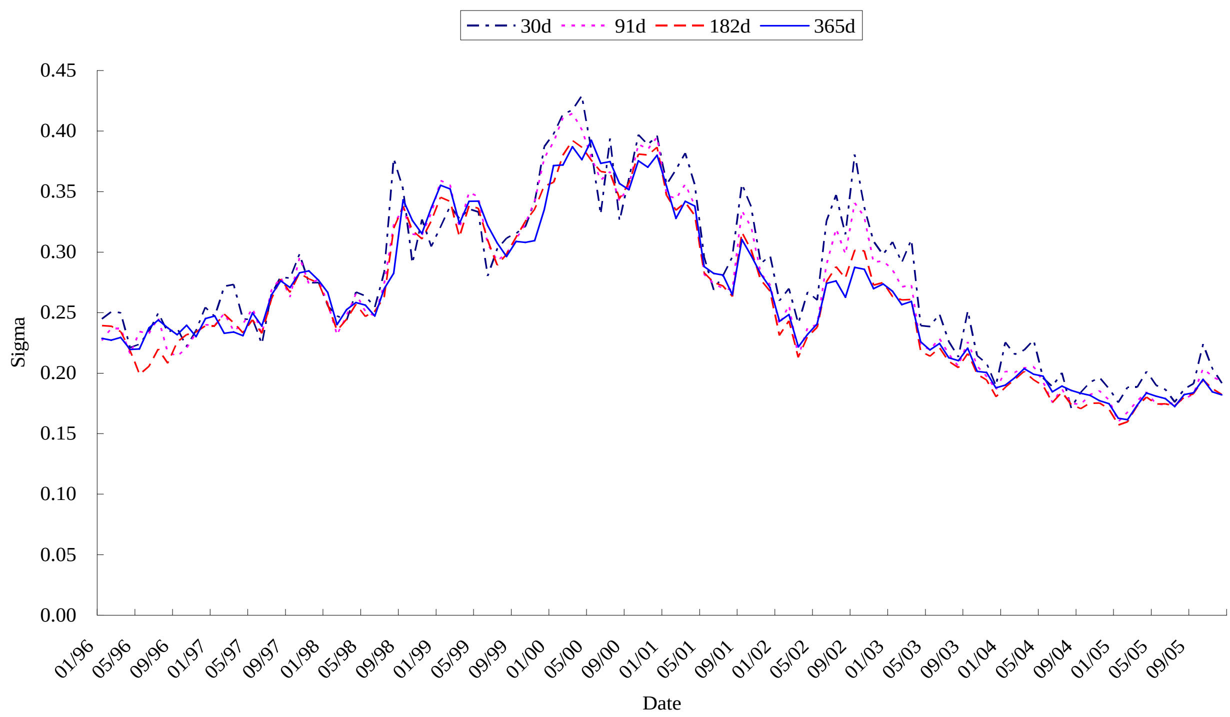


Figure 4: Processes of Costs of Equity Capital

This figure presents the processes of the costs of equity capital (COE) for the market portfolio composed by the component firms of the S&P 100 index. The COEs are estimated with either our option pricing formula or the Fama/French three-factor model. The option-implied COEs are estimated from the prices of options with one year to expire. The Fama/French COEs are estimated with three-year historical stock prices. The averages are value-weighted. The sample period is from 1996 to 2005.

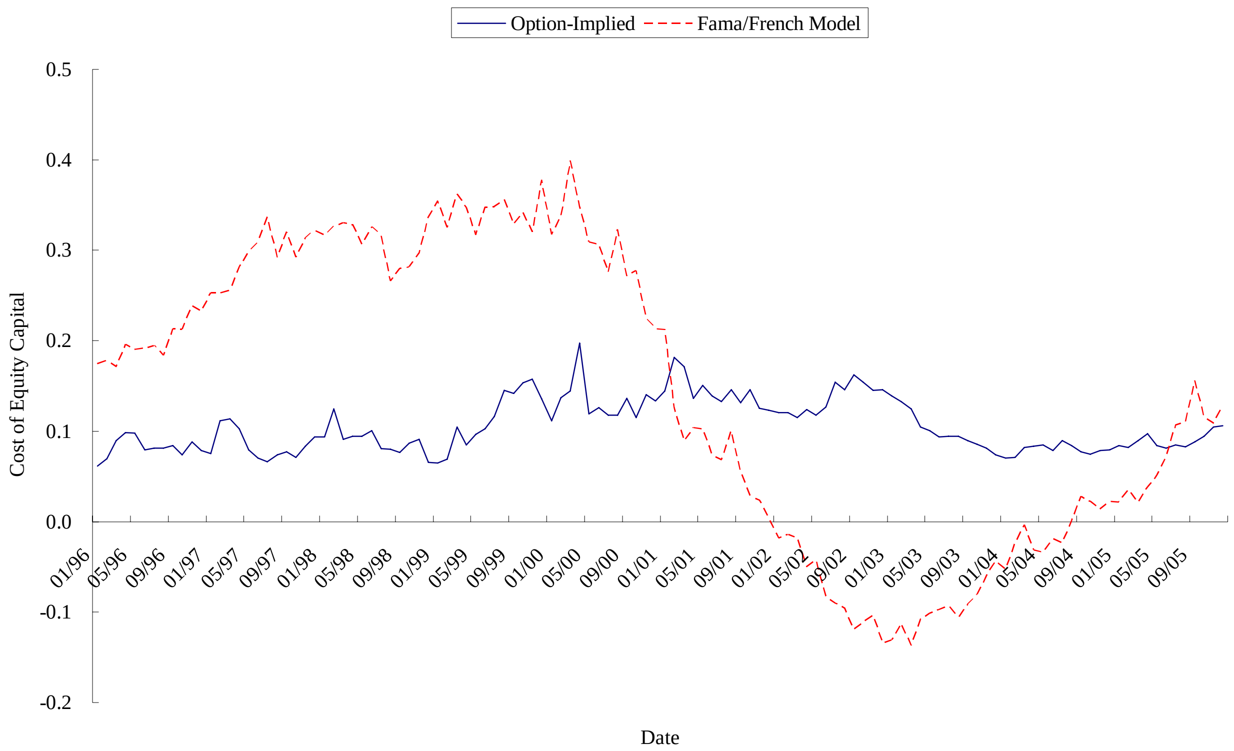


Figure 5: Processes of Costs of Equity Capital for Various Industries

This figure presents the processes of the costs of equity capital (COE) for various industrial portfolios composed by the component stocks of the S&P 100 index. Only the industries including at least 10 firms are selected. The COEs are estimated with either our option pricing formula or the Fama/French three-factor model. The option-implied COEs are estimated from the prices of options with one year to expire. The Fama/French COEs are estimated with three-year historical stock prices. The averages are value-weighted. The sample period is from 1996 to 2005.