Reservation bid and ask prices for options and covered warrants: portfolio effects

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Abstract

In this paper we show that marginal reservation option prices for risk-averse investors with an existing option portfolio depend on the difference in portfolio hedging costs with and without the marginal option position. If costs increase as a result of the transaction, reservation ask (sell) prices are higher and reservation bid (buy) prices lower; if they decrease reservation ask prices decrease (bid prices increase), reflecting the cost savings. Optimal portfolio hedging costs reflect the portfolio's risk and increase more than proportionally with portfolio size; thus marginal reservation per-option prices decrease monotonically as the investor's inventory holdings increase. This provides an alternative partial explanation for the consistent empirical observation that prices of covered warrants and other structured products are significantly higher than the prices of corresponding traded options. The model generates additional implications for option prices and bid-ask spreads, some consistent with prior empirical evidence, others requiring further empirical testing.

Keywords: covered warrants, bid ask spreads, reservation prices, optimal hedging, transaction costs.

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1 Introduction

Empirical studies of covered warrants[1] have consistently found they trade at prices higher than those of comparable exchange-traded options. In this paper we show that the costs incurred by a covered warrant issuer/market-maker in order to dynamically hedge their warrant position generate reservation bid and ask warrant prices which are consistent with empirical evidence on the values and bid-ask spreads of covered warrants. In addition, our model has wider implications for characteristics of bid and ask prices for structured products and other traded option-like securities.

Covered warrants are bank-issued vanilla options, generally traded on exchanges, where the issuer commits to making a market in the product it has issued. They represent one of the simplest types of equity-linked *structured product*[2], which became popular in the U.S. in the 1980s and have subsequently spread internationally. Markets for covered warrants developed rapidly in Europe and Asia in the 1990s and there are now active markets in Germany, Amsterdam, Italy, Switzerland, Sweden, Spain, Luxembourg, Australia and London (see Bartram & Fehle (2006)).[3]

Key features which distinguish covered warrants from exchange-traded options include the fact that they cannot be held short by the retail investors to whom they are generally issued. Thus issuers have a net short position at all times. Furthermore, in combination with the issuer's commitment to making a market, this means the issuer effectively sets both bid (redemption) and ask (issue) prices for the warrants, since they take one side of virtually all transactions in the warrants issued by them[4]. In contrast, trades on options exchanges may be made with any of a number of competing market-makers or directly with another investor. Additionally, terms in covered warrants' prospectus[5] suggest there is either an obligation on the issuer to hedge the

[1]Studies of other equity-linked structured products have similarly found evidence of systematic overpricing relative to equivalent traded options.

[2]Structured products are bank-issued securities incorporating potentially complex derivative structures sold to retail investors. For more details see *e.g.* Stoimenov & Wilkens (2005).

[3]The largest market for covered warrants and other equity-linked structured products is EuWaX. For descriptions of covered warrant markets see Bartram & Fehle (2004), Horst & Veld (2003),Chan & Pinder (2000), and Abad & Nieto (2007).

[4]Bartram & Fehle (2004) note that "discussions with market participants indicate ... that orders are filled almost exclusively with the issuer's market maker".

[5]Bartram & Fehle (2004, 2006) note "the issuer is obligated (as stated in the prospectus) to hedge". Some more recent prospectus *e.g.* Goldman Sachs (2007) refer explicitly to the issuer's

covered warrants issued or an expectation that such hedging will occur. Finally, the minimum trade size for covered warrants is generally much smaller and the maturity of covered warrants is generally longer than exchange-traded options on the same underlying asset.

Recently, a number of empirical studies have examined covered warrant markets around the world. Comparison of covered warrant prices with the prices of corresponding exchange-traded options has consistently shown that covered warrant prices are higher than those of the corresponding exchange-traded option (equivalently implied volatilities for covered warrants are higher).[6] Most studies also find that bid-ask spreads are lower in covered warrant markets and that both the bid ask spread and the difference between prices of covered warrants and equivalent traded options increase with time to maturity.

Potential explanations for the relative overpricing of covered warrants include a liquidity premium (Chan & Pinder (2000)), investor clienteles (Bartram & Fehle (2004)) and behavioural explanations (Horst & Veld (2003), Abad & Nieto (2007)). However none of these explanations has found universal acceptance. For example, whilst Australian covered warrant markets are more liquid than the corresponding traded options markets, consistent with warrant prices incorporating a premium for liquidity (or reduction of cost risk (Chan & Pinder (2000))), other covered warrants markets are not unambiguously more liquid than their corresponding traded options market (see Bartram & Fehle (2004), Abad & Nieto (2007) for more detailed discussions). Similarly, whilst the relative characteristics of German covered warrant and traded option markets[7] are consistent with investors in warrants having more speculative motives and being more concerned with round trip transaction costs than initial ask price levels, the evidence is not fully consistent with this for *e.g.* the Spanish covered warrant and option markets (Abad & Nieto (2007)).

hedging transactions but do not commit to the form these will take.

[6]Bartram & Fehle (2004, 2006) show this for covered warrant prices quoted on EuWaX compared to traded option prices on EuReX. Similar results are found for the Australian covered warrant market and options exchange (Chan & Pinder (2000)), the covered warrant market on Euronext Amsterdam and Amsterdam options exchange (Horst & Veld (2003)) and the Spanish covered warrants market and options exchange (Abad & Nieto (2007))

[7]Higher warrant prices combined with lower bid-ask spreads and lower trade sizes in the warrant market.

Moreover, whilst some empirical papers on covered warrants (*e.g.* Bartram & Fehle (2004)) acknowledge that issuers will want to hedge, and that there will be costs associated with this hedging, they do not model these costs or incorporate them explicitly in their empirical tests. In this paper, we use a theoretical model to demonstrate how such dynamic hedging costs will affect the prices above which an issuer is willing to issue more covered warrants, and below which they are willing to redeem warrants they have already issued (the issuer's reservation ask and bid prices respectively) and find results consistent with the empirical evidence described above for covered warrants and other structured products issued to retail investors.

Specifically, we build a general model of reservation bid and ask prices[8] for options for investors with an arbitrary portfolio of vanilla options written on a single underlying asset with a single maturity date who hedges optimally, thereby incurring transaction costs.[9] Reservation prices are particularly relevant to issuers of covered warrants and other structured products because of the nature of the markets in these products. As argued above, covered warrant issuers effectively set both bid and ask prices offered to retail investors, with potential for direct competition from other covered warrant issuers on ask prices only.[10] Hence reservation values for such issuers should have greater relevance for the bid and ask prices they set than for options market-makers, who need to take more account of direct competitive influences. Moreover, hedging of covered warrant positions is either obligatory or expected, and will necessarily incur costs, which should be incorporated in the bid and ask prices offered.

We then apply this model specifically to covered warrants by taking into account their primary feature: that issuers always have a *net short* position in

[8]The reservation ask price per option represents the price at which the investor is indifferent between writing a number of such options and receiving that price per option, or keeping his portfolio unchanged. Similarly the reservation bid price per option represent the price at which the investor is indifferent between buying a number of such options at that price, or keeping his portfolio unchanged.

[9]We summarise the literature on theoretical models valuing options in the presence of transaction costs incorporating optimal hedging strategies in section 2.

[10]Since warrants from different issuers are not exchangeable, retail investors can generally sell their warrants only back to the original issuer. Thus competition on bid prices is only indirect, through the impact on original choice of issuer. Additionally issuers have no obligation to issue covered warrants with a specific type or underlying asset, so there may not be competition from an identical covered warrant issue on ask prices.

the covered warrants. This allows us to show specifically that both reservation bid and ask prices for covered warrants are strictly greater than the Black-Scholes value and to draw other specific implications for covered warrant and structured product prices. Finally, we compare the implications of the model with the findings of prior empirical studies on covered warrants and bid-ask spreads for options in general.

We find we can split reservation values for options into four components: the Black-Scholes value and the certainty equivalent values of the initial, final and lifetime optimal hedging costs. Initial and final costs represent one-off transactions and both depend on the Delta. Lifetime costs represent the sum of costs from many (infinitesimally) small transactions in order to maintain the optimal hedging strategy during the lifetime of the option and depend on a fractional power of the option portfolio's Gamma, integrated over the remaining life of the option.

Mid-reservation prices differ from the Black-Scholes value, and the difference is monotonically related to the magnitude of the Gamma of the investor's (market-maker's) existing portfolio. For portfolios of vanilla options, the difference between mid prices and Black-Scholes prices is principally related to the net size of the portfolio, with higher mid-prices for short positions and lower mid-prices for long positions. Since issuers of covered warrants, who quote prices to sell (ask) and buy back (bid) the warrants, generally have a potentially sizeable net short position, whereas market-makers in options markets will generally try to reduce the net size of their option portfolios, this is consistent with the empirical evidence that covered warrant prices are generally higher than corresponding prices in traded options markets.

For covered warrants and structured products, the issuer always holds a net short position, so the mid-price is always strictly greater than the Black-Scholes price[11] and the difference between them increases with the net absolute size of the issuer's portfolio, though at a decreasing rate. This implies the difference should decrease if traded options are available and are used by the issuer for hedging purposes to decrease his overall net position. To date there is unfortunately no empirical evidence on this issue.

The difference has two components, one relating to initial/final costs and proportional to the asset price multiplied by the Delta, and the other to life-

[11] Equivalently, the implied volatility from covered warrants is always strictly greater than the implied volatility from the Black-Scholes price

time costs of dynamic hedging. Whilst the first is generally independent of the existing portfolio,[12] the second depends nonlinearly on the Gamma of the existing portfolio as well as the marginal change in the Gamma and increments through time. Thus controlling for the initial/final costs, the difference above the Black-Scholes value increases with time to maturity. Covered warrant studies have found significant positive relationships between time to maturity and the ratio of ask prices in the covered warrant market to corresponding traded option prices (Bartram & Fehle (2004)) and to the relative price difference between corresponding covered warrants and traded options (Abad & Nieto (2007)), consistent with this.

The model also has implications for bid-ask spreads for options in general and covered warrants in particular. Firstly, it predicts a bid-ask spread comprised of two strictly positive components: again one relating to initial/final costs and proportional to the marginal Delta, and the other to lifetime costs and a nonlinear function of the Gamma during the life of the option. This second component increases with the lifetime of the option and also increases with the quote depth (the number of options for which the bid or ask quote is valid). Hence controlling for Delta, absolute bid-ask spreads[13] in options markets should increase with the bid-ask spread in the underlying market, the quote depth, time to maturity and the per-option Gamma and should decrease as the net magnitude of the investor's position in options increases. Empirical evidence[14] generally supports the existence of both initial and lifetime or bandwidth cost components and is consistent with comparitive static results, althouth the model's predictions with respect to the quote depth and inventory size have not been addressed specifically inthe existing empirical literature.

The underlying intuition for these results relies on two points: firstly that the reservation value of a portfolio of hedged options (or warrants) incorporates the certainty equivalent value of the transaction costs incurred in hedging the portfolio optimally during its lifetime and secondly that the reservation value of a *marginal* option position to an investor with an existing option portfolio is given by the difference between the reservation value of the portfolio

[12]This is true providing the Deltas of the porfolio before and after the trade have the same sign for all S.

[13]Numerical simulations show proportional options bid-ask spreads decrease with moneyness and time to maturity, consistent with empirical findings.

[14]See section 3.2 for more detailed discussion.

including the marginal option position (incorporating the transaction costs involved in hedging this new portfolio optimally) and the reservation value of the optimally hedged existing portfolio.[15] We also need to distinguish between the marginal reservation value, which can be either positive or negative (for long or short marginal option positions respectively) and the marginal reservation price per option,[16] which is always positive.

Transaction costs always reduce the reservation value of a portfolio of options. Thus, for an investor with no existing option holdings, transaction costs decrease the (maximum) price the investor is willing to pay for a long option position and increase the (minimum) price they are willing to accept to take a short position. If the investor already has a portfolio of options on the underlying asset, the effect of transaction costs on the value they place on a change in their portfolio depends on the change in the certainty equivalent value of the transaction costs involved in the optimal hedging. If the costs of hedging the new portfolio are greater than the costs of hedging the existing portfolio (as above for the investor with no existing portfolio), transaction costs reduce the marginal value of the change in the portfolio, decreasing the maximum purchase price per incremental long option bought and increasing the minimum writing price per incremental option sold short. If, however, the costs of hedging the new portfolio are lower than the costs of hedging the existing portfolio (for example, on liquidating the option portfolio, so the new portfolio incurs zero hedging costs), the effect of transaction costs is to increase the marginal value of the change in the portfolio, because of the effective saving of transaction costs. This *increases* the maximum purchase price per incremental long option bought and *decreases* the minimum writing price per incremental option sold short.

Since the incremental future transaction costs saved by liquidating an option portfolio equal the future costs incurred in hedging the portfolio if it is retained, ignoring initial costs[17] the marginal reservation price for purchasing n options for an investor who has an existing holding of $-n$ identical options (n options held short) is the same as the marginal reservation price for writing n such options for an investor with zero initial holdings, *i.e.* greater than

[15] Note reservation values are non-linear so marginal values depend on the investor's existing portfolio.

[16] The marginal reservation price per option is given by the marginal reservation value of the option position divided by the number of options in the marginal portfolio.

[17] The effect of initial costs will be incorporated fully in section 2

the perfect market price. The investor with zero initial holdings requires a minimum price greater than the perfect market price in order to compensate for the future transaction costs involved in hedging the short position they are entering into; the investor with an existing short position is willing to pay up to a maximum price which is greater than the perfect market price because of the saving in future transaction costs the deal will bring. Writing $V(n|N)$ for the marginal reservation price per option for an increment of n options for an investor with an existing portfolio of N identical options and V^{BS} for the Black-Scholes price of the option, we thus have

$$V(-m|0) > V^{BS} > V(m|0) \quad \forall m > 0$$

and

$$V(m|-N) > V^{BS} > V(-m|N) \quad \forall N \geq m > 0$$

The leading order component of the effect of transcation costs on option portfolio values depends on a fractional power of the magnitude of the option portfolio's Gamma. These lifetime costs will increase with a change in the option portfolio if the Gamma of the marginal option position has the same sign as the Gamma of the existing option portfolio. In this case, overall costs will increase, decreasing the reservation value of the marginal option position. On the other hand, if the Gamma of the marginal option position has the opposite sign to the Gamma of the existing option portfolio, then including the marginal option position in the investor's portfolio reduces the magnitude of the Gamma and thus reduces costs, increasing the reservation value of the marginal option position. Thus the sign of the effect of transaction costs on the value of a marginal option position is determined by the relationship between the signs of the Gamma of the existing portfolio Γ_P, and of the marginal option position, Γ_Q:

Issuers of covered warrants always have a net short position, so the Gamma of their existing portfolio is always negative $\Gamma_P < 0$. Thus if the issuer issues additional warrants (*i.e.* increases their short position), the Gamma of the additional position has the same sign. The transaction costs involved in hedging the new portfolio are greater than the costs in hedging the existing portfolio; thus the issuer requires a minimum price strictly greater than the perfect market value in order to compensate them for these additional costs[18].

[18]The effect of transaction costs on the value to the investor of the marginal short transaction in warrants is to decrease the value of the transaction. Since the value of a short position is negative, this increases the price per warrant.

If the issuer buys back some of the warrants they have already issued (*i.e.* decreases the magnitude of their short position), the Gamma of the additional position has the opposite sign to the existing Gamma. Overall transaction costs are reduced by the repurchase; the maximum price the issuer is willing to pay reflects this saving and is thus also *greater* than the perfect market price[19]. Thus both ask reservation prices (the price at which the issuer is willing to write an additional covered warrant) and bid reservation prices (the price at which the issuer is willing to buy back a covered warrant already issued) are strictly greater than the perfect market price due to transaction costs of optimal hedging.

This argument is shown more formally in section 2, which considers the mid reservation price and the reservation bid-ask spread rather than reservation bid and ask prices explicitly. The leading order non-linear component of prices arising from transaction costs, due to costs of lifetime optimal dynamic hedging, affects both the mid prices and bid-ask spreads. Lifetime costs for an option portfolio depend on the magnitude of its Gamma to the power 4/3, *i.e.* as the magnitude of the portfolio increases, lifetime costs also increase at an increasing rate. Hence the value of a marginal change in the portfolio also increases with the magnitude of the portfolio's Gamma (it depends to leading order on the magnitude of the portfolio's Gamma to the power 1/3), whereas the bid-ask spread increases with the magnitude of the marginal Gamma but decreases with the magnitude of the Gamma of the original portfolio. As the quote depth increases, it has a relatively larger effect and the size of the existing portfolio has a relatively smaller effect on the overall size and Gamma of the new portfolio. The mid price for a marginal option position also incorporates the initial costs incurred for any change in the option portfolio, *i.e.* the change in the number of the hedging asset, which to leading order is proportional to the magnitude of the Delta of the marginal option position, whereas the bid-ask spread includes twice the difference in expected final costs, again proportional to the magnitude of the Delta of the marginal option position to leading order.[20] Initial costs are incurred whether the marginal transaction is a purchase or sale and are thus included in the mid price; in contrast

[19]The effect of transaction costs on the value of the marginal long transaction is to increase the value of the position, increasing the reservation price per warrant.

[20]This is becuase expected final costs proportional to $S|\Delta|$ have current certainty equivalent value also proportional to $S|\Delta|$ providing the Deltas before and after the transaction have the same sign for all asset prices.

expected final costs may be greater, if the transaction increases the overall absolute Delta, or there may be a cost saving if the transaction reduces absolute Delta, so expected final costs increase the bid-ask spread.

Section 3 relates the implications of the model for covered warrant prices and for bid-ask spreads for options in general to prior empirical evidence Section 4 concludes.

2 Model

In this section we first find the value of a marginal option position to an investor with an existing portfolio of options on the same underlying asset. The investor is assumed to hedge his option portfolio dynamically using the underlying asset in order to maximise his expected utility (net of the costs the dynamic hedging strategy incurs). Initially in section 2.2 we consider only portfolios of different numbers of options of a single type (European call or put with a single strike price and maturity); later in section 2.4 we extend our analysis to cover more general portfolios of European vanilla options on the same asset. We start in section 2.1 with a recap of the value of a general portfolio of options in the presence of transaction costs.

2.1 Setting: transaction cost models

There is a large and growing literature on the valuation of options incorporating transaction costs. This can be divided in to papers which assume an exogenous hedging strategy and value the option given that hedging strategy and a valuation methodology[21] and papers which derive an endogenous optimal hedging strategy and resulting option value by maximising the investor's utility[22]. Whilst utility-based models are more conceptually appealing, they are also generally more complicated and require computationally intensive

[21]For example Leland (1985), the first paper to recognise that transaction costs necessitated a change from the continuous Black-Scholes hedging strategy, assumed hedging occurs at fixed points in time. Other papers incorporating exogenous hedging strategies include Bensaid, Lesne, Pages & Scheinkman (1992), Boyle & Vorst (1992), Hoggard, Whalley & Wilmott (1992), Edirisinghe, Naik & Uppal (1993), Henrotte (1993), Whalley & Wilmott (1993), Grannan & Swindle (1996) and Avellaneda & Paras (1996).

[22]This strand of the literature was started by Hodges & Neuberger (1989). Other papers using utility-based methods include Davis, Panas & Zariphopoulou (1993), Hodges & Clewlow (1997), Whalley & Wilmott (1997, 1999), Damgaard (2003) and Zakamouline (2006).

numerical solution. However Whalley & Wilmott (1997, 1999) provided an asymptotic expansion solution to the generally-used exponential utility[23] form of the problem which is considerably easier to solve. We thus use this formulation in the remainder of the paper and summarise the set-up of their model and their main results below. Note however that apart from the assumption of small transaction costs, the basic formulation of the problem is the same as that in Hodges & Neuberger (1989) and most subsequent utility-based papers; the particular formulation allows us to derive simple formulae which illustrate explicitly the effects we describe rather than relying exclusively on numerical simulations.

An investor holds a portfolio of options with payoff Λ_P and maturity T written on an underlying asset which follows Geometric Brownian Motion

$$dS = \mu S dt + \sigma S dz$$

The investor hedges optimally using the underlying asset, taking into account the transaction costs associated with the optimal hedging strategy, in order to maximise his expected utility of wealth at some date after maturity of the option portfolio. We consider only transaction costs proportional to the amount traded,[24]

$$k(S, dy) = kS|dy|,$$

where k is the percentage transaction cost fee, including the proportional component of the underlying asset's bid-ask spread, S is the value of one unit of the underlying asset and dy is the change in the number of the underlying asset held in the issuer's portfolio.

Assuming the investor has an exponential utility function with absolute risk aversion γ and that the level of transaction costs is small, $k \ll 1$, Whalley & Wilmott (1997) and Whalley (1998)[25] showed that

[23] Hodges & Neuberger (1989) used exponential utility to help reduce the dimensionality of the problem, since in this case the option value and hedging strategy are independent of the investor's overall wealth. However the resulting problem is still three-dimensional and requires numerical calculation of two free boundaries to determine the hedging strategy and hence option value.

[24] Whalley & Wilmott (1999) showed the hedging strategy can be characterised explicitly when only one type of costs (fixed or proportional) are considered, but must be found as the root of a ninth order polynomial function when both fixed and proportional costs are present. As the size of an option portfolio increases, proportional costs become increasingly important in comparison to fixed costs in determining both the hedging strategy and the reservation price.

[25] Whalley & Wilmott (1997) showed the leading order correction was $P_b(S, t)$ and also included

Proposition 1 *(Whalley & Wilmott (1997), Whalley (1998))*

The reservation value of an optimally hedged option portfolio with final payoff $\Lambda_P(S,T)$ held by such an investor can be approximated by

$$P(\Lambda_P) \approx P^{BS}(S,t) + P_b(S,t) + P_f(S,t) \tag{1}$$

where

1. *$P^{BS}(S,t)$ is the Black-Scholes value associated with final payoff $\Lambda_P(S,T)$,*

2. *P_b satisfies*

$$P_{b_t} + rSP_{b_S} + \frac{\sigma^2 S^2}{2} P_{b_{SS}} - rP_b = \frac{\hat{\gamma}(t)\sigma^2 S^2}{2}\left(H_P^{*^2} - H_0^{*^2}\right) \tag{2}$$

s.t. $P_b(S,T) = 0$ where $\hat{\gamma} \equiv \gamma e^{-r(T-t)}$ and $H_P^*(S,t)$, $H_0^*(S,t)$ represent the optimal 'hedging semi-bandwidths' associated with an option portfolio with final payoff Λ_P and with no option holdings respectively defined below

3. *P_f satisfies*

$$P_{f_t} + rSP_{f_S} + \frac{\sigma^2 S^2}{2} P_{f_{SS}} - rP_f = 0$$

s.t. $P_f(S,T) = -k(|SP_S^{BS}(S,T) + \xi(T)| - |\xi(T)|)$ where $\xi(t) = \frac{\lambda}{\hat{\gamma}(t)\sigma}$.

The optimal hedging strategy is to transact only when the actual number of the underlying asset held differs by more than the 'hedging semi-bandwidth', H_P^*, from the ideal number, $y_P^*(S,t)$, and to trade the minimum required in order to bring the actual number back within this no-transaction band

$$[y_P^* - H_P^*, y_P^* + H_P^*]$$

where

$$\begin{aligned} y_P^*(S,t) &= P_S^{BS} + \frac{\xi(t)}{S} && (3)\\ H_P^*(S,t) &= \left(\frac{3kS}{2\hat{\gamma}}(t)\right)^{\frac{1}{3}} \left|\frac{\partial y_P^*}{\partial S}\right|^{\frac{2}{3}} = \left(\frac{3kS}{2\hat{\gamma}}\right)^{\frac{1}{3}} \left|P_{SS}^{BS} - \frac{\xi}{S^2}\right|^{\frac{2}{3}} && (4)\end{aligned}$$

The optimal hedging semi-bandwidth, H_P^*, is the outcome of the tradeoff between the reduction in risk or hedging error resulting from trading and the transaction costs incurred in doing so[26] and depends on the Gamma of the

inital costs (see later). Whalley (1998) extended the expansion to higher orders; the next term is $P_f(S,t)$.

[26]See Rogers (2000) for a heuristic derivation of the size of the no-transction band

option portfolio being hedged, $\Gamma^P \equiv P^{BS}_{SS}$, to a fractional power.[27] So P_b, which represents the leading order component of portfolio value capturing the effects of 'bandwidth hedging' (transacting in order to remain within the no-transaction band) during the lifetime of the option portfolio reflects the optimal residual risk or hedging error the investor chooses to bear as the outcome of this tradeoff. It thus represents the certainty equivalent cost of bandwidth hedging, incorporating the effects of both transaction costs and residual risk.

The fractional powers in the equations for H^*_P and hence the certainty equivalent cost of bandwidth hedging, P_b, mean the hedging strategy and overall option value is nonlinear. Hence option values are not additive and portfolios must be valued and hedged as a whole. This also means the value of a given option position differs depending on the composition of the investor's existing portfolio, since they value it at its *marginal reservation value*, *i.e.* the difference in the value of their portfolio overall due to the change in the portfolio's composition.[28][29]

Corollary 1 *The marginal reservation value of an option portfolio with final payoff $\Lambda_Q(S,T)$ to an investor with an existing portfolio of options on the same underlying asset with final payoff $\Lambda_P(S,T)$ is given by*

$$Q(\Lambda_Q|\Lambda_P) \approx Q^{BS}(S,t) + Q_b(S,t) + Q_f(S,t) + Q_i(S,t) \tag{5}$$

where

1. *$Q^{BS}(S,t)$ is the Black-Scholes value associated with final payoff $\Lambda_Q(S,T)$,*
2. *Q_b satisfies*
$$Q_{b_t} + rSQ_{b_S} + \frac{\sigma^2 S^2}{2} Q_{b_{SS}} - rQ_b = \frac{\hat{\gamma}(t)\sigma^2 S^2}{2}\left(H^{*2}_{P+Q} - H^{*2}_{P}\right) \tag{6}$$
s.t. $Q_b(S,T) = 0$ where H^*_{P+Q}, H^*_P are the semi-bandwidths associated with portfolios with final payoffs $\Lambda_P + \Lambda_Q$ and Λ_P respectively.
3. *Q_f satisfies*
$$Q_{f_t} + rSQ_{f_S} + \frac{\sigma^2 S^2}{2} Q_{f_{SS}} - rQ_f = 0 \tag{7}$$

[27]Recall the hedging error due to discrete rebalancing is a function of the portfolio's Gamma (Boyle & Emanuel (1980)).

[28]Relevant proofs are in an appendix available from the author on request.

[29]This concept was used by Whalley & Wilmott (1999), who derived the leading order effect on the marginal value given an existing option portfolio in the case of a single marginal option. They did not consider initial or final costs and did not derive reservation bid and ask prices.

$$s.t.\ Q_f(S,T) = -k(|S(P_S^{BS}(S,T) + Q_S^{BS}(S,T)) + \xi| - |SP_S^{BS}(S,T) + \xi|)$$

4. $Q_i = -kS|Q_S^{BS}|$

The marginal reservation value includes the effect of initial as well as lifetime (bandwidth) and final costs, since it represents the cash amount for which the investor is indifferent between doing nothing (keeping their existing portfolio) or paying/receiving the marginal reservation value in cash and taking on the new option position as well as their existing portfolio, thereby incurring one-off costs Q_i associated with changing the number of the underlying asset initially as well as future costs represented by Q_b and Q_f. If the reservation value is negative, the investor requires payment of at least that amount in order to be willing to alter their option portfolio; if it is positive, they are willing to pay up to the marginal reservation value in order to make the change.

To leading order, the initial change in the number of the underlying asset held is given by the (Black-Scholes) Delta of the marginal option portfolio, Q_S^{BS}, so initial costs are simply the proportional transaction cost, k, multiplied by the value of the transaction $S|Q_S^{BS}|$.

If $\mathrm{sgn}[S(P_S^{BS}(S,T)+Q_S^{BS}(S,T))+\xi(T)] = \mathrm{sgn}[SP_S^{BS}(S,T)+\xi(T)]\forall S$ then the final condition for Q_f reduces to $Q_f(S,T) = -\mathrm{sgn}[S(P_S^{BS} + Q_S^{BS}(S,T) + \xi(T)]kSQ_S^{BS}$ and there is an explicit solution for Q_f. Final costs are then also a multiple of kSQ_S^{BS}:

$$Q_f(S,t) = -\mathrm{sgn}[S(P_S^{BS}(S,T) + Q_S^{BS}(S,T))) + \xi(T)]kSQ_S^{BS}(S,t).$$

Thus whereas initial costs are always negative, the effect of final costs can be either positive or negative depending on whether $\mathrm{sgn}[Q_S^{BS}(S,t)(SP_S^{BS}(S,T)+\xi(T))]$ is negative or positive. For example, if P represents N long call options, each with Black-Scholes value $C^{BS} > 0$ and Q is $m < N$ short identical options, then $Q_f(S,t|\Lambda_P) = kmSC_S^{BS} > 0$, representing the saving in final costs as a result of the reduction in the overall magnitude of the position.

The bandwidth cost term for the marginal reservation value, Q_b, reflecting costs incurred over the lifetime of the option portfolio, can also be either positive or negative, as it represents the difference between the certainty equivalent value of the bandwidth costs incurred in hedging the new portfolio and those of bandwidth hedging the existing portfolio. Thus whilst these lifetime costs reduce the value of each portfolio ($P_b \leq 0\ \forall\Lambda_P$), the change in the portfolio's composition can either increase or reduce the magnitude of these costs, depending on the relative magnitude of $\Gamma_P + \Gamma_Q - \xi/S^2$ and $\Gamma_P - \xi/S^2$. Specifically, if $\text{sgn}(\Gamma_Q) \neq \text{sgn}(\Gamma_P - \xi/S^2)\ \forall (S,t)$, (*e.g.* if P represents a holding of N short call options and Q represents a portfolio of $m < N$ long identical options) incorporating the new option position will reduce lifetime bandwidth hedging costs so Q_b will be positive. We shall investigate these effects in more detail for specific cases in sections 2.2 and **??**.

In order to simplify the expositions below, we may make the assumptions hereafter that the existing option portfolio P is large relative to the marginal option portfolio, Q and the optimal holding of the underlying asset in the absence of any option position, ξ/S. This ensures we concentrate on effects due to option hedging rather than optimal investment. Specifically, we may assume

Assumption 1

$$|\Gamma_P + \Gamma_Q| \gg \frac{\xi}{S^2} \quad \textit{and} \quad |\Gamma_P| \gg |\Gamma_Q| \quad \forall S, t$$

where Γ_P, Γ_Q are the Black-Scholes Gammas for the option portfolios with payoffs Λ_P, Λ_Q respectively.

Assumption 2

$$|\Delta_P + \Delta_Q| \gg \frac{\xi}{S} \quad \textit{and} \quad |\Delta_P| \gg |\Delta_Q| \quad \forall S, t$$

where Δ_P, Δ_Q are the Black-Scholes Deltas for the option portfolios with payoffs Λ_P, Λ_Q respectively.

2.2 Special case: portfolios of a single option type

To illustrate the main ideas in the simplest fashion, we initially consider the special case where both P and Q consist of positions of various magnitudes in a single European vanilla option. Specifically, for this section we assume

$$\Lambda_P = N\Lambda_V, \quad \Lambda_Q = n\Lambda_V, \quad |n| \ll |N|$$

where Λ_V represents the payoff to a single long European vanilla option.

$Q(n\Lambda_V|\Lambda_P)$ represents the marginal reservation value of an incremental option position and can be positive (if n is positive, so the incremental transaction for the investor is to buy options) or negative (if n is negative, so the marginal transaction for the investor involves writing or selling options). The marginal reservation *prices* quoted are however always positive. We thus define the *marginal bid reservation price* per option for a marginal purchase of

$m > 0$ options to an investor with an existing position of N identical options as

$$V^{bid}(m|N) = \frac{Q(m\Lambda_V|N\Lambda_V)}{m} > 0 \tag{8}$$

and the *marginal ask reservation price* per option for taking a short position in $m > 0$ options (so $\Lambda_Q = -m\Lambda_V$) for an investor with an existing position of N identical options as

$$V^{ask}(m|N) = -\frac{Q(-m\Lambda_V|N\Lambda_V)}{m} > 0 \tag{9}$$

It is easier to work with the reservation mid price, $V^{mid}(m|N)$, and the reservation bid-ask spread, $B(m|N)$ (for a quote depth of $m > 0$ and an existing position of N options) rather than marginal bid and ask reservation prices directly. These are defined as

$$V^{mid}(m|N) = \frac{V^{bid}(m|N) + V^{ask}(m|N)}{2} \tag{10}$$

$$B(m|N) = V^{ask}(m|N) - V^{bid}(m|N) \tag{11}$$

Then expanding (5) in $|m/N| \ll 1$ and taking leading order terms, we find

Proposition 2 *The reservation mid price per option for a quote depth of* m *for an investor with an existing position of* N *identical options is to leading order given by*

$$V^{mid}(m|N) \approx V^{BS} - \frac{4}{3} sgn(N)|N|^{\frac{1}{3}} k^{\frac{2}{3}} L_b(|\Gamma|) - sgn(N)kS|\Delta| \tag{12}$$

where $L_b(|\Gamma|) > 0$ *satisfies*

$$L_{b_t} + rSL_{b_S} + \frac{\sigma^2 S^2}{2} L_{b_{SS}} - rL_b = -\frac{\hat{\gamma}(t)\sigma^2 S^2}{2} \left(\frac{3S}{2\hat{\gamma}(t)}\right)^{\frac{2}{3}} |\Gamma|^{\frac{4}{3}} \tag{13}$$

s.t. $L_b(S,T) = 0$ *and* $\Gamma = V^{BS}_{SS}$ *and* $\Delta = V^{BS}_S$ *are the Black-Scholes Gamma and Delta of a single long option respectively.*

The reservation bid-ask spread per option for a quote depth of m *for an investor with an existing position of* N *identical options is to leading order given by*

$$B(m|N) \approx \frac{4}{9} m|N|^{-\frac{2}{3}} k^{\frac{2}{3}} L_b(|\Gamma|) + 2kS|\Delta| \tag{14}$$

2.3 Discussion - empirical implications

These simple expressions for the reservation mid price and bid-ask spread allow us to draw inferences about the determinants of the signs and magnitudes of each. These are summarised in Corollaries 2 and 3 below:

Corollary 2 1. *The reservation mid price per option for a quote depth of m for an investor with an existing position of N identical options, $V^{mid}(m|N)$, is lower than the Black-Scholes price if the investor already has a long position, and is greater than the Black-Scholes price if the investor's existing position is a net short one*

$$sgn[V^{mid}(m|N) - V^{BS}] = -sgn[N]$$

i.e. $V^{mid}(m|N) > V^{BS}$ *if* $N < 0$ *and* $V^{mid}(m|N) < V^{BS}$ *if* $N > 0$.

2. *The magnitude of the difference between the reservation mid price and the Black-Scholes price increases with the size of the investor's existing portfolio and is independent of the quote depth*

$$\frac{\partial |V^{mid}(m|N) - V^{BS}|}{\partial |N|} > 0, \qquad \frac{\partial |V^{mid}(m|N) - V^{BS}|}{\partial m} = 0$$

3.

$$\frac{\partial L_b(|\Gamma|)}{\partial (T-t)} > 0$$

so the magnitude of the lifetime or bandwidth cost component of the reservation mid price increases with the option's remaining life.

Corollary 3 1. *The reservation bid ask spread per option for a quote depth of m for an investor with an existing position of N identical options, $B(m|N)$ is strictly positive for all m, N:*

$$B(m|N) > 0 \quad \forall (S,t)$$

2. *The magnitude of the bid ask spread increases with the quote depth and decreases with the magnitude of the investor's existing portfolio and is independent of the*

$$\frac{\partial B(m|N)}{\partial |N|} < 0, \qquad \frac{\partial B(m|N)}{\partial m} > 0$$

3.

$$\frac{\partial L_b(|\Gamma|)}{\partial(T-t)} > 0$$

so the magnitude of the lifetime or bandwidth cost component of the bid ask spread increases with the option's remaining life.

Figure 1 shows how the difference between marginal mid reservation price and the Black-Scholes price varies with the number of options the investor already owns. The top graph shows how the at-the-money bandwidth component varies with N: reservation mid prices decrease monotonically with the number of options in the investor's existing portfolio. The bottom graph shows how this, combined with the initial cost component, varies with the moneyness of a call option: the number of options in the investor's existing portfolio affects the bandwidth component only, and thus has greatest impact close to the money.

Figure 2 shows how the the difference between the reservation mid price and the Black-Scholes price per option (left graphs) and the per-option bid-ask spread (right graphs) are affected by time to maturity. The lifetime or bandwidth terms are shown separately in the top graphs. These increase monotonically with time to maturity, confirming the results in corollaries 2 and 3. They are combined with the final or initial cost terms in the lower graphs to give the overall effect. However, since the effect of time to maturity on the option's delta can be either positive or negative depending on the stock price, the overall effect of time to maturity is ambiguous. Note the mid reservation price is greater than the Black-Scholes price in these figures (this is also the case for the bottom graph in Figure 1 and Figure 3), reflecting the assumption of an existing net short position. These are thus appropriate for covered warrants issuers.

Figure 3 shows the effect of asset volatility on the difference between the reservation mid price and Black-Scholes and the per-option bid-ask spread. Again the lifetime or bandwidth terms, in the top graphs, increase monotonically with the volatility of the underlying asset, but no unambiguous statement is possible for the combined terms, as it depends on the relative sizes of the lifetime and initial/final cost terms and also the moneyness level. However, controlling for the initial/final cost level $S|\Delta|$ (*i.e.* isolating the lifetime or bandwidth components) the effect of optimal hedging costs has greater effect on both mid prices and bid-ask spreads, the longer the time to maturity and the higher the asset volatility.

2.4 Generalisations and robustness

In this section we present the more general model for a portfolio of European vanilla options with the same maturity date on the same underlying asset without any assumptions on the relative magnitude of portfolio terms. We then specialise under assumptions 1 and 2) to provide approximations when the marginal portfolio change is relatively small. The latter generalises the formulae from section 2.2 whilst retaining some simplifications.

2.4.1 Results for general option portfolio holdings

The generalised form of the reservation mid-prices and bid-ask spread for a quote to buy or sell m European vanilla options, each with payoff Λ_W at maturity T is given by

Proposition 3 *The reservation mid price per option for a quote depth of m options, each with payoff Λ_W at maturity T for an investor with an existing position of options on the same underlying asset with payoff Λ_P and maturity T is to leading order given by*

$$W^{mid}(m|\Lambda_P) \approx W^{BS} - W_b(m|\Lambda_P) - W_f(m|\Lambda_P) \tag{15}$$

where W^{BS} is the Black-Scholes value of an option with payoff Λ_W, $W_b(m|\Lambda_P)$ satisfies

$$\begin{aligned} W_{b_t} + rSW_{b_S} + \frac{\sigma^2 S^2}{2} W_{b_{SS}} - rW_b = \\ -\frac{\hat{\gamma}(t)\sigma^2 S^2}{4m}\left(\frac{3kS}{2\hat{\gamma}(t)}\right)^{\frac{2}{3}}\left(\left|\Gamma_P + m\Gamma_W - \frac{\xi}{S^2}\right|^{\frac{4}{3}} - \left|\Gamma_P - m\Gamma_W - \frac{\xi}{S^2}\right|^{\frac{4}{3}}\right) \end{aligned} \tag{16}$$

s.t. $W_b(S,T) = 0$
and $W_f(m|\Lambda_P)$ satisfies

$$W_{f_t} + rSW_{f_S} + \frac{\sigma^2 S^2}{2} W_{f_{SS}} - rW_f = 0$$

s.t. $W_f(S,T) = \frac{k}{2m}\left(|S(\Delta_P + m\Delta_W) + \xi| - |S(\Delta_P + m\Delta_W) + \xi|\right)$
where $\Gamma_W = W^{BS}_{SS}$ and $\Delta_W = W^{BS}_S$ are the Black-Scholes Gamma and Delta of a single long marginal option respectively and $\Gamma_P = P^{BS}_{SS}$, $\Delta_P = P^{BS}_S$ are the Black-Scholes Gamma and Delta of the existing option portfolio with Black-Scholes value P^{BS}.

The reservation bid-ask spread per option for a quote depth of m options, each with payoff Λ_W at maturity T for an investor with an existing position *of of options on the same underlying asset with payoff Λ_P and maturity T is to leading order given by*

$$B(m\Lambda_W|\Lambda_P) \approx B_b(m\Lambda_W|\Lambda_P) + 2kS|\Delta_W| \tag{17}$$

where $B_b(m\Lambda_W|\Lambda_P) > 0$ satisfies

$$\begin{aligned} B_{b_t} + rSB_{b_S} + \frac{\sigma^2 S^2}{2} B_{b_{SS}} - rB_b = \\ -\frac{\hat{\gamma}(t)\sigma^2 S^2}{2m}\left(\frac{3kS}{2\hat{\gamma}(t)}\right)^{\frac{2}{3}}\left(\left|\Gamma_P + m\Gamma_W - \frac{\xi}{S^2}\right|^{\frac{4}{3}} + \left|\Gamma_P - m\Gamma_W - \frac{\xi}{S^2}\right|^{\frac{4}{3}}\right. \\ \left. -2\left|\Gamma_P - \frac{\xi}{S^2}\right|^{\frac{4}{3}}\right) \end{aligned} \tag{18}$$

s.t. $B_b(S,T) = 0$.

Corollary 4 *Under assumptions 1 and 2 the reservation mid price simplifies to*

$$W^{mid}(m|\Lambda_P) \approx W^{BS} - \frac{4}{3} sgn(\Gamma_W\Gamma_P) W_b(|\Gamma_W|, |\Gamma_P|) - sgn(\Delta_W\Delta_P) kS|\Delta_W| \tag{19}$$

where $W_b(|\Gamma_W|, |\Gamma_P|) > 0$ satisfies

$$W_{b_t} + rSW_{b_S} + \frac{\sigma^2 S^2}{2} W_{b_{SS}} - rW_b = -\frac{\hat{\gamma}(t)\sigma^2 S^2}{2}\left(\frac{3kS}{2\hat{\gamma}(t)}\right)^{\frac{2}{3}} |\Gamma_W||\Gamma_P|^{\frac{1}{3}}$$

s.t. $W_b(S,T) = 0$.

Under assumptions 1 and 2 the reservation bid-ask spread simplifies to

$$B(m|\Lambda_P) \approx \frac{4}{9} m B_b(|\Gamma_W|, |\Gamma_P|) + 2kS|\Delta_W| \tag{20}$$

where $B_b(|\Gamma_W|, |\Gamma_P|) > 0$ satisfies

$$B_{b_t} + rSB_{b_S} + \frac{\sigma^2 S^2}{2} B_{b_{SS}} - rB_b = -\frac{\hat{\gamma}(t)\sigma^2 S^2}{2}\left(\frac{3kS}{2\hat{\gamma}(t)}\right)^{\frac{2}{3}} |\Gamma_W|^2|\Gamma_P|^{-\frac{2}{3}}$$

s.t. $B_b(S,T) = 0$.

From (19) and (20) the conclusions of corollaries 2 and 3 continue to hold in a generalised form. Specifically, the reservation mid price is greater than the Black-Scholes price if $\text{sgn}(\Gamma_W\Gamma_P) = \text{sgn}(\Delta_W\Delta_P) = -1$, *i.e.* if both the Gamma and Delta of the marginal option position have the opposite signs to the Gamma and Delta of the investor's existing portfolio respectively and is less than the Black-Scholes price if both Gammas and Deltas have the same signs $\text{sgn}(\Gamma_W\Gamma_P) = \text{sgn}(\Delta_W\Delta_P) = +1$. For vanilla options, and recalling $\Gamma_W > 0$ since it is the Black-Scholes Gamma of a single long option, this implies that reservation mid-prices exceed the Black-Scholes price if the investor's existing portfolio has a negative Gamma[30] and are less than the Black-Scholes price if the existing Gamma is positive. Also the magnitude of the difference between the reservation mid-price and the Black-Scholes price is independent of the quote depth to leading order and increases with the Gamma of both the new and existing option positions. Similarly, the bid-ask spread is always positive and increases with the quote depth, and with the magnitude of the new option's Delta, Gamma and time to maturity (holding all else constant) and decreases with the magnitude of the Gamma of the investor's existing portfolio.

3 Applications

3.1 Covered warrants

The model presented in section 2 suggests that if prices of covered warrants[31] are set by issuers to take into account the cost effects of the issuer's optimal hedging strategies,[32] then warrant prices will be greater than the perfect market value of the warrant, by more the larger the issuer's warrant portfolio, the larger the bid-ask spread in the underlying asset market, the larger the initial costs, or equivalently the larger the proportional cost level and the current exposure to stock price risk (measured by $S|\Delta|$) and, holding the initial cost

[30] This ensures the bandwidth component of the difference between reservation mid-prices and Black-Scholes is positive. The condition on the Deltas $\text{sgn}(\Delta_W \Delta_P) = -1$, which ensures the second term is also positive is automatically satisfied if all options are either calls or puts. In fact, this second term arises from initial costs of changing the hedging portfolio and may in practice be smaller if the amount of the underlying asset which needs to be traded in order to move to the edge of the new hedging band is less than the change in the mid-points of the band. If the amount of the underlying asset held is already within the new band, no transaction would be required and this term would be zero. Thus in general this is a less important component of the mid reservation price than the bandwidth component. Thus whilst the initial cost term may in isolated cases lead to exceptions to this result, these are rare and small in magnitude.

[31] Note the term 'covered' would traditionally imply a constant hedge ratio of 1 per covered call warrant, in which case the model in this paper would not be relevant. There may be such obligations in some markets (*e.g.* Australia), but in other markets there is no obligation in exchange rules or in prospectuses *e.g.* Goldman Sachs (2007) to cover in the traditional sense.

[32] Given covered warrant issuers' net short positions, issuers' reservation warrant prices if they optimally hedge dynamically in the underlying market will be higher than the Black-Scholes value.

level constant, the larger the volatility of the underlying asset, the absolute Gamma per option of the issuer's portfolio and the longer the time to maturity. In this section we briefly review the empirical literature on covered warrant prices and relate them to these predictions.

A number of papers have investigated the relationship between prices of covered warrants and those of closely matching or equivalent traded options. Chan & Pinder (2000) investigated the relative pricing of electronically traded equity warrants with exchange traded equity options in Australia over the period January 1997 - June 1998 which spanned the time of a change in the trading method for equity options. Over the whole period they found the warrants to be 'systematically over priced relative to matched options', but they also found evidence of a structural change when the option market began trading electronically which reduced the overpricing significantly. Chan & Pinder interpret their results as evidence of a liquidity premium: investors are willing to pay a higher price for a warrant in the relatively more liquid Australian warrant market than a corresponding exchange traded option.

Horst & Veld (2003)used the implied volatility from comparable long term call options (traded on the Euronext Derivatives Market of Amsterdam) to value covered warrants listed on Euronext Amsterdam, part of the Amsterdam stock exchange during the first five days of trading for issues between January 1999 and December 2001 and found relative overpricing of more than 25% on average, with 99% of the warrants overvalued on issue. They consider various possible explanations and suggest that whilst the relative trading costs and flexibility faced by potential investors in each market can explain some of the overpricing (for low warrant prices), some of the willingness of investors to buy more costly warrants has a behavioural explanation: 'financial institutions have managed to create an image for call warrants that is different from call options.'

Bartram & Fehle (2004) examine relative pricing between the German covered warrants market, EuWax, and the EuRex options exchange during 2000 and found both ask and bid prices on EuWax were consistently and significantly higher than comparable ask (bid) prices on EuRex. Furthermore bid prices differed relatively more than ask prices, so bid-ask spreads on EuWax were significantly smaller than on EuRex. They suggest a clientele effect, where covered warrants are held by more speculative investors, who are more likely to reverse their position before expiry and are thus more concerned with the bid-ask spread than the initial level of the ask price. The lower bid-ask spread in covered warrant markets is thus more importhant than the initially higher ask price. Using the same data, Bartram & Fehle (2006) further find that bid-ask spreads on either market were lowered by competition from the other market.

Abad & Nieto (2007) investigate price differences between the Spanish covered warrants and traded options markets during 2003. They find that Spanish warrants are systematically overpriced, to a greater extent than other studies, and that the data does not fully support either the liquidity or clientele explanations. They also find differences between the level of overpricing between different issuers, which cannot be explained by liquidity, clientele or credit risk arguments.

3.1.1 Relative overpricing of warrants

As indicated above, the empirical stuides of covered warrant markets around the world have consistently found that both ask and bid prices for warrants issued by banks are significantly higher than the prices of comparable traded options. For the largest covered warrant market, EuWaX, Bartram & Fehle (2004) found ask prices were on average 4.7% and bid prices 9.9% higher than prices for comparable options traded on the EuReX options exchange during 2000, a statistically significant difference. For the Australian market, Chan & Pinder found significant median pricing differences of $3-4\%$ and mean pricing differences of $6-7\%$. Horst & Veld's results for the Amsterdam market (relative overpricing of more than 25% on average) are not completely comparable, as they are measured over only the first five days of trading, but are of similar magnitude to the Spanish case (Abad & Nieto) which had a median overpricing of 17% for warrants with similar volumes to those traded on the options market (and between 19 and 25% median overpricing more generally).

Several potential explanations for the overpricing have been put forward. Chan & Pinder's liquidity premium hypothesis is supported for the Australian markets, where the covered warrant market is more liquid than the exchange-traded options market. However, as noted by Bartram & Fehle (2004), the EuWax covered warrant market generally has lower volume and liquidity than the corresponding EuRex traded options market, so a liquidity premium would suggest relative underpricing of warrants, the opposite of what occurs. For the Spanish case Abad & Nieto find that the relative price difference is larger when the bid-ask spread for warrants is relatively smaller, which they interpret as a liquidity effect, but which is also consistent with the model in this paper (see below). Abad & Nieto also test for Bartram & Fehle (2004)'s clientele hypothesis and find that whilst characteristics of the Spanish warrant market suggest investors are predominantly speculators, bid-ask spreads in both markets are similar in size, providing only weak grounds to explain why investors would be willing to pay higher prices to buy warrants rather than corresponding options. Bartram & Fehle (2004) consider transaction costs for investors trading on each market and find that whilst these are lower for warrant trades the difference is very small (less than 1% of the trade value). Similarly, Horst & Veld's examination of relative transaction costs for investors found they were not large enough to explain the relative pricing difference. Bartram & Fehle (2004) considered the issue of hedging costs for warrant issuers. They argue that issuers who are also market-makers on the corresponding options market may face cost advantages in reducing hedging risk but suggest, without developing a formal model, that whilst this would decrease bid-ask spreads it will not necessarily change the location of the mid price.

This paper considers the issuer's perspective and shows that under optimal hedging both reservation bid and ask prices for an issuer with a net short position are both necessarily *above* the perfect market price in order to compensate the issuer for the transaction costs involved in hedging the warrants they have written. Actual prices quoted by covered warrant issuers may, of course, differ from the reservation prices we calculate[33]. However ask prices should be no lower than (and bid prices no higher than) the calculated reservation ask and bid prices respectively, otherwise the issuer would prefer not to transact.Since covered warrants can only be held long by retail investors, holdings (inventory) of the warrant issuer/market-maker will necessarily be short and is thus likely to be much larger in magnitude than the inventory position of a market-maker in the options market,[34] even if the relative size per contract is greater in the options market. Options market-makers will generally prefer to reduce the magnitude of their inventory positions which, in contrast to covered warrant market-makers/issuers, can be either positive

[33]Bid and ask quotes could be calculated by assuming some trading intensity along the lines of Avellaneda & Stoikov (2006). 5[2]. This is beyond the scope of this paper.

[34]The warrant issuer/market-maker's inventory holdings represent the total number of warrants outstanding, in contrast to an options market-maker.

or negative. Overall therefore we would expect covered warrant issuers' positions to be significantly more negative (*i.e.* have larger $|N|$ with $N < 0$) than options market-makers. This drives the prediction that covered warrant prices are rationally set higher than the price of equivalent traded options in order to compensate the issuer for their hedging costs and risk.

The model predicts some of the determinants of 'overpricing': the mid price should be higher (relative to Black-Scholes) the larger the magnitude of the issuer's net position in warrants and options, the larger the bid-ask spread in the primary market, (k), the larger the initial costs of setting up the hedge (proportional to $S|\Delta|$) and, controlling for initial costs, the longer the time to maturity, the larger the magnitude of the portfolio's Gamma[35] and the more risk averse the issuer. Many of these predictions have not been tested explicitly in prior empirical studies; however we now examine the correspondence between these and results found in earlier papers.

3.1.2 Determinants of relative overpricing

To our knowledge no study has investigated the effects of the magnitude of the issuer's net position in warrants and options directly.

Abad & Neito regress the relative price difference between warrant and option markets[36] on the ratio of the bid-ask spreads in the option market to those in the warrant market, relative volume in the two markets, time to maturity and moneyness amongst other characteristics and find that warrants are more expensive when the bid-ask spread in the warrants market just before the transaction is smaller. They interpret the bid-ask spread ratio as a proxy for the relative liquidity of the two markets. However, this finding can also be viewed as consistent with the effects of optimal hedging costs.

As argued above, covered warrant markets are characterised by market makers with larger short positions (larger $|N|, N < 0$). The model predicts that for $N < 0$ mid prices increase with N whereas bid-ask spreads decrease with N. Thus smaller bid-ask spreads may correspond to issuers' larger short positions, which in turn implies relatively higher (mid) warrant prices.

Several of the above studies have however included time to maturity in their regressions. Bartram & Fehle (2004) regress the ratio of ask prices in

[35]Strictly, the larger the magnitude of the integrated absolute Gamma over the option's lifetime

[36]Relative price difference is measured as $\frac{V^W - V^O}{V^O}$ where V^W is the warrant price and V^O the price of the corresponding traded option.

the warrant market to the option market (AR) on, amongst other characteristics, the corresponding ratio of bid prices, time to maturity, moneyness, asset volatility and the number of competing EuWax warrants (on the same underlying asset). Once cross-sectional variation has been removed they find a positive relationship between the ask ratio and the time to maturity, which they regard as consistent with the idea that prices on warrant and options markets move closer as the expected holding period decreases. Similarly Abad & Nieto find the relative price difference is significantly positively related to the time to maturity. These positive relationships are consistent with the model of optimal hedging costs for covered warrant issuers: controlling for inital hedging costs, the magnitude of difference between the reservation mid price and the Black-Scholes price increases with time to maturity (see Corollary 2).

Existing studies thus offer some support to the idea that the effects of transaction costs under optimal hedging as modeled in this paper influence the relative pricing of covered warrants by issuers to traded options by market-makers. Empirical studies on the relative pricing of other structured products,[37] which show these are also consistently over-priced relative to corresponding exchange traded options for structured products with both convex *and concave*[38] payoffs, are also consistent with the model. However, the model also has implications for the bid-ask spread of covered warrant markets and options markets more generally.

3.2 Bid-ask spreads

The predictions of the model in section 2 for reservation bid-ask spreads for options are that they increase with the quote size, the bid-ask spread of the underlying asset, the initial costs (or current exposure to stock price risk $S|\Delta|$) and, controlling for initial costs, the market maker's per-option Gamma, the option's time to maturity and the volatility of the underlying asset, and

[37] See, for example Stoimenov & Wilkens (2006).

[38] See *e.g.* Burth, Kraus & Wohlwend (2001) for empirical evidence on a structured product with a concave payoff. The dependence of the reservation mid price on terms which depend on the *absolute value* of the warrant's Gamma or Delta, combined with the sign of the (net) number of options held in the portfolio, means the issuers of structured products (SPs), who also generally have a net short position, will have a reservation mid-price for any SP which is *higher* than the SP's perfect market value (since $\text{sgn}(N) = -1$) irrespective of the sign of the Gamma of a single long SP.

decrease as the size of the market maker's portfolio increases.

Most empirical studies of bid-ask spreads in options markets consider however the *percentage* bid-ask spread rather than the absolute value of the bid-ask spread. In the model this corresponds to B/V^{mid}. To gain some insight into the effect of this, consider the case when the market maker's portfolio consists of a single issue of options. The percentage bid-ask spread can be written as

$$\frac{B(m|N)}{V^{mid}(m|N)} \approx \frac{\frac{4}{9}m|N|^{-\frac{2}{3}}k^{\frac{2}{3}}L_b(|\Gamma|) + 2kS|\Delta|}{V^{BS} - \frac{4}{3}\mathrm{sgn}(N)|N|^{\frac{1}{3}}k^{\frac{2}{3}}L_b(|\Gamma|) - sgn(N)kS|\Delta|} \tag{21}$$

Figure 4 plots this for call options differing times to maturity. In contrast to the absolute reservation bid-ask spread, the proportional reservation bid-ask spread decreases rapidly as moneyness increases and decreases as time to maturity increases. However it still increases with the bid-ask spread of the underlying asset and the quote depth and decreases as the size (absolute value) of the investor's existing portfolio increases.

Empirical studies of bid-ask spreads in options markets include Jameson & Wilhelm (1992), George & Longstaff (1993), Cho & Engle (1999), Kaul, Nimalendran & Zhang (2004) and, for covered warrant markets, Petrella (2004).

Jameson & Wilhelm (1992) recognise that option market-makers face risk because they are only able to rebalance their portfolios at discrete points in time and hence are exposed to a level of stock price risk which varies stochastically as the stock price itself varies. They use the option's Gamma as a proxy for this rebalancing risk and in regressions of the proportional bid-ask spread on proxies for stock price risk exposure, expected holding period, asymmetric information, discrete rebalancing risk and stochastic volatility risk find all are statistically significant, with discrete rebalancing risk contributing around 8% to the overall proportional bid-ask spread. Their results provide evidence that 'the inability to costlessly and continuously rebalance an option portfolio imposes undiversifiable risks on market participants that may influence the theoretical bounds on option prices' and conclude that it is important to account for discrete rebalancing risk (proxied by Gamma) and stochastic volatility risk (proxied by Vega) when studying options bid-ask spreads as they represent an important additional dimension of market-makers risk for option portfolios not present in the risk for market-makers in stocks. George & Longstaff (1993) investgate *inter alia* the relationship between bid-ask spreads across options and find that spreads on call options are positively related to the spreads on corresponding put options and vice versa, consistent with a

Gamma-related component, since the Gammas for European call and put options are identical.[39]

Cho & Engle (1999) propose a 'derivative hedge theory' of options market microstructure. This predicts that 'option market percentage spreads will be inversely related to the option market-makers ability to hedge his position in the underlying market, as measured by the liquidity of the latter.' They suggest initially that options market makers are able to hedge their options positions perfectly in the underlying market. Perfect hedging means neither inventory risk nor the presence of informed traders in the options markets will affect the option spread, whereas informed trading in the underlying asset market will affect the option spread through the illiquidity of the underlying market (asset bid-ask spread). This perfect version of 'derivative hedge theory' is supported by their findings that asset spreads, implied volatility and the option's Delta all have significantly positive coefficients and, crucially, that options volume is insignificant in regressions of proportional bid-ask spreads on these and other characteristics. However, their finding that proportional bid-ask spreads are a U-shaped function of duration between trades, consistent with the effect of asymmetric information at short durations and inventory risk considerations at long durations, provides evidence that options market activity as well as underlying market activity affects option spreads, suggesting that options market-makers may only be able to hedge imperfectly. As cost determinants they include the option's Delta but not the Gamma in their regressions, effectively considering only initial hedging costs and not ongoing rebalancing costs.

Kaul, Nimalendran & Zhang (2004) investigate the impact of hedging costs and adverse selection on the bid-ask spread for individual stock options on CBOE in February 1995. They regress absolute bid-ask spreads on proxies for initial costs ($kS\Delta$), kVega, which they interpret as a proxy for rebalancing costs, trading volume, which they interpret as capturing the effects of a fixed component to order processing costs, volatility skew, to account for the inaccuracy of the volatility estimate, and adverse selection. They find that whilst proportional bid-ask spreads decrease as moneyness increases, absolute bid-ask spreads increase with moneyness. Initial costs have a positive and very significant effect on option spreads, representing up to 64% (for well in-the-

[39] They also find the bid-ask spread for S&P100 index options in 1989 is positively related to the option value and the length of time between trades and negatively related to the time to maturity and the squared Delta.

money options) of the option's bid-ask spread. The rebalancing cost proxy also has a positive and significant effect but represents only 6.9% of option spreads. Kaul *et al* interpret this as 'suggesting that options market-makers hedge their positions but do not have to incur large rebalancing costs, either because they may not hold positions very long or because they can effectively diversify their portfolios.' They also find that adverse selection in the options markets explains $6 - 22\%$ of option spreads, which they interpret as indicating that option market makers are not able to delta hedge perfectly (and thus eliminate the risk arising from trading with informed agents completely).

Petrella (2006) considers the market-making cost determinants of proportional bid-ask spreads on the Italian covered warrants market during December 2000 - January 2001. He finds that initial costs (proxied by $kS\Delta$), rebalancing costs (given by a measure of stock price variability multiplied by the option's Gamma) and a 'reservation proportional bid-ask spread', or minimum spread required to avoid scalping[40] (proportional to the tick-size $\times m\Delta/V$ where m is the number of assets underlying one warrant contract) are all significantly positively related to the warrant proportional bid-ask spread. He interprets his results as showing that options market makers hedge their positions by rebalancing to keep their portfolio delta-neutral and suggests that thus 'representative market maker does not fear to trade with informed traders, because his position is hedged'. However he does not test for adverse selection explicitly, and does not recognise that discrete rebalancing results in imperfect hedging and hence residual risk.

Together, these studies suggest a modified version of Cho & Engle's theory may be more appropriate, allowing for imperfect and costly dynamic hedging. Options market makers trade off the costs and benefits of dynamic hedging and choose to hedge imperfectly, using *e.g.* the hedging strategy underlying the models in section 2. The value of their portfolios is thus affected by the magnitude of the portfolio Gamma.[41] Moreover, informed trading on either market can affect the option spread: in the options market as a result of the options market-maker's inability to hedge perfectly, and in the underlying asset market through the effect on the asset spread, k, which in turn increases

[40]If the warrant's bid price after a one tick upward movement in the underlying asset would be greater than the warrant's current ask price short term unhedged speculators can make a short term profit from small movements in the underlying

[41]Both the rebalancing costs they expect to incur and the hedging error they choose to retain are proportional to a power of the absolute Gamma of their option portfolio

the costs of optimal dynamic hedging for the options market-maker and increases option spreads. Options market-makers' inventory costs are reduced by the dynamic hedging they pursue, but cannot be eliminated and are thus reflected in bid-ask spreads.

In this light, the model in section 2 can be viewed in the spirit of an inventory cost model for options market-makers[42]: inventory costs (*i.e.* the effect of the number of options held, N) are reflected in reservation bid and ask prices indirectly through the effect on the optimal hedging bandwidth and hence the residual risk the market-maker optimally incurs. Since the optimal total hedging bandwidth increases less than proportionally with the net number of options in the portfolio, total risk increases more than proportionally with the net size of the market-maker's portfolio. Thus, as in inventory cost models of underlying asset markets (*e.g.* Ho & Stoll (1981)), the mid-price is negatively related to the inventory level (for options this relationship is non-linear) and both bid-ask spread and mid price are positively related to the risk of the marginal (existing portfolio) position (recall the bandwidth component of the effect of hedging costs can be viewed as the integral of the squared bandwidth over the option's life).

We now consider how the detailed predictions of the model are related to the existing empirical evidence.

The general principle that dynamic hedging costs affect bid-ask spreads has relatively wide-spread support. Several studies[43] have found the option proportional bid-ask spread to be significantly positively related to a measure of the initial hedging costs $kS\Delta$, and hence to the bid-ask spread in the underlying market, k and the initial hedging amount $S|\Delta|$ (it is generally assumed Δ is positive). Proxies for rebalancing costs have been considered

[42]The model gives reservation prices, at whcih a market-maekr would be *indifferent* between selling (ask price) or buying (bid price) the quoted number of options and hedging optimally, or not. Hence it does not incorporate expected order flow or maximise the market-maker's profit. We also do not consider adverse selection issues. A fuller model is beyond the scope of this paper; however, since we would expect utility-maximising ask and bid prices to incorporate the same factors which underlie reservation prices based on inventory considerations, these are thus likely to lie outside the reservation bid-ask spread.

[43]Cho & Engle (1999), Kaul *et al* (2004), Petrella (2006)

explicitly by Jameson & Wilhelm (1992)[44], Kaul *et al* (2004),[45] and Petrella (2006). All found the proxies to be positively and statistically significantly related to spreads. Similarly, regressions which have included proxies for option or underlying asset price risk as (a component) of explanatory variables have found results consistent with a positive impact of asset price volatility on option spreads.[46]

The more detailed predictions relating to quote depth and market-maker's inventory have not been tested directly, but some partial support can be gained from related variables. The model predicts that absolute and proportional bid-ask spreads per option should increase with the quote depth. Whilst they do not include quote depth in their regressions, Cho & Engle note that out-of-spread transactions have higher average volume per transaction *i.e. higher transaction depth.* Bartram & Fehle (2004) find that proportional bid ask spreads are significantly smaller than those for corresponding traded options (2.8% *vs* 7.1% on average, with the bid-ask spread differences almost all significant at the 1% level or better). They also note that minimum trade sizes are much smaller for covered warrants than exchange traded options and that trading volume is also lower. If trade sizes and quote sizes (m) are smaller for covered warrants, the model in section 2 implies that bid-ask spreads per option should also be smaller. Similarly, model bid-ask spreads will be smaller if the overall magnitude of the market-maker's inventory position ($|N|$) is larger for warrants than options due to the nature of the warrant market. Bartram & Fehle (2006) also find proportional bid-ask spreads are statistically significantly lower for covered warrants than traded options and suggest the difference could be due to the greater depth offered by EuRex market-makers as compared to EuWax issuers. The theoretical costly hedging model in this paper demonstrates this directly. Finally Petrella finds a

[44] Jameson & Wilhelm (1992) showed the option's Gamma was a significant positive explanatory factor in the proportional bid-ask spread. They interpreted this as a proxy for discrete rebalancing risk; however it is also consistent with the effect of lifetime bandwidth costs.

[45] They proxy this using k times the option's Vega, rather than Gamma as would be more appropriate. Note however that for European vanilla options, Vega $= \sigma S^2 T \Gamma$.

[46] Jameson & Wilhelm include a proxy for stock price risk exposure of $\sigma^2 S \times$ the option's squared elasticity and find this is positive and significant. Cho & Engle find a positive and significant relationship between proportional bid-ask spread and the option's implied volatility. Petrella's rebalancing cost variable incorporates the difference between the maximum and minimum daily asset price, which is related to asset price volatility; the coefficient on rebalancing cost is positive and significant.

significant and positive relationship between the proportional bid ask spread and the 'reservation proportional bid-ask spread', proportional to the tick-size $\times m\Delta/V$ where m is the number of assets underlying one warrant contract. The imperfect hedging model predicts a larger m should be associated with a larger proportional bid-ask spread. Thus whilst these results do not show directly the positive relationship between quote depth and per-option spreads, they provide some support.

Even fewer regressions have included variables linked to the size of the market maker's inventory. As mentioned above the finding by Abad & Nieto that Spanish covered warrants were relatively more 'overpriced' relative to equivalent traded options when the bid-ask spreads before the transactions were smaller is consistent with the predictions of the model: bid-ask spreads decrease with $|N|$ whereas the difference between covered warrants reservation prices and perfect market prices increase with $|N|$. Similarly, Bartram & Fehle (2004)'s results on the relative size of bid-ask spreads on covered warrants and traded options markets are consistent with the predictions of the model for the effects of issuers holding larger (short) positions in warrants than market-makers in options markets.

Finally, consistent with our discussion of the difference in characteristics between absolute and proportional bid-ask spreads (B *vs* B/V^{mid}) above, empirical studies have consistently found that proportional bid-ask spreads decrease with moneyness and time to maturity[47]. Kaul *et al* (2004), the only study to use absolute bid-ask spread, showed empirically that whilst average absolute bid-ask spreads increase across moneyness groupings, proportional bid-ask spreads generally decrease across the same moneyness groups, and that at- and out-of-the-money bid-ask spreads decreased with time to maturity, whereas at-the-money bid-ask spreads increased with time-to-maturity, well out-of-the-money spreads decreased, and the results for in-the-money spreads were ambiguous. These results are broadly consistent with Figures 2 and 4 respectively.

[47] Cho & Engle (1999), Bartram & Fehle (2006) and Petrella (2006) all find significantly negative coefficients in their regressions of proportional bid-ask spreads on moneyness. Jameson & Wilhelm (1992) do not include moneyness but do include elasticity $S\Delta/V$, which is strictly decreasing in moneyness for call options and find a significant negative coefficient. Similarly George & Langstaff (1993), Cho & Engle (1999) and Petrella all find significantly negative coefficients for time-to-maturity.

4 Conclusion and further work

In this paper we have developed a model for the reservation bid and ask prices of options, taking into account hedging costs and an investor's portfolio composition and risk aversion and applied it to covered warrant markets and bid-ask spreads in options markets more generally.

The existing empirical evidence on the relative pricing of covered warrants and exchange traded options and on bid-ask spreads in options markets is broadly supportive of the implications of the model. Further work needs to be done to investigate some novel implications of the model, *e.g.* the effects of the magnitude of a covered warrant issuer's overall position in warrants on the relative overpricing of covered warrants and of a market-maker's inventory on bid and ask prices, as these have not been addressed explicitly in the literature thus far.

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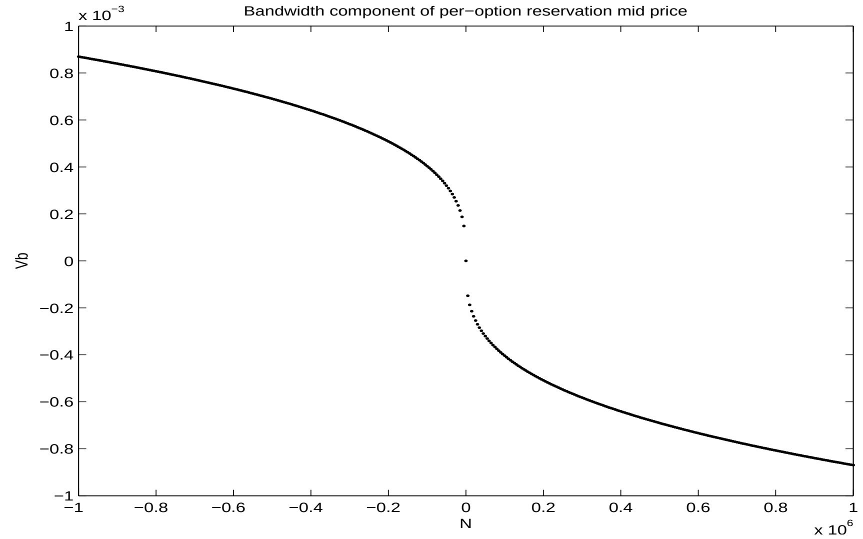

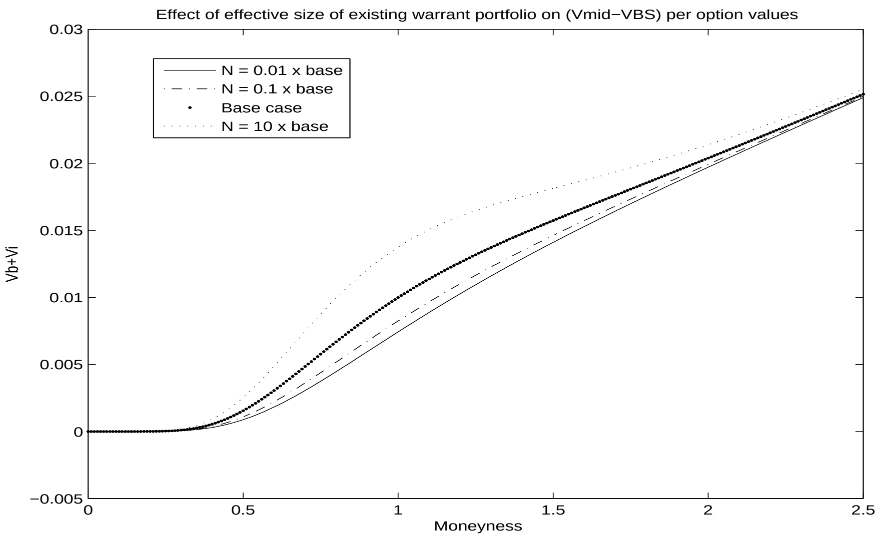

Figure 1: Top graph: Bandwidth cost component of reservation mid price *vs* size of existing portfolio for at-the-money options with 2 years to expiry.
Bottom graph: Difference between reservation mid price and Black-Scholes price *vs* moneyness for different sizes of existing portfolio using (**??**) for options with 2 years to expiry. Base case has an existing short position of $N = -1 \times 10^6$ options. Other parameter values: $\sigma = 0.32$, $r = 0.05$, $\gamma = 1 \times 10^{-3}$ and $k = 0.01$.

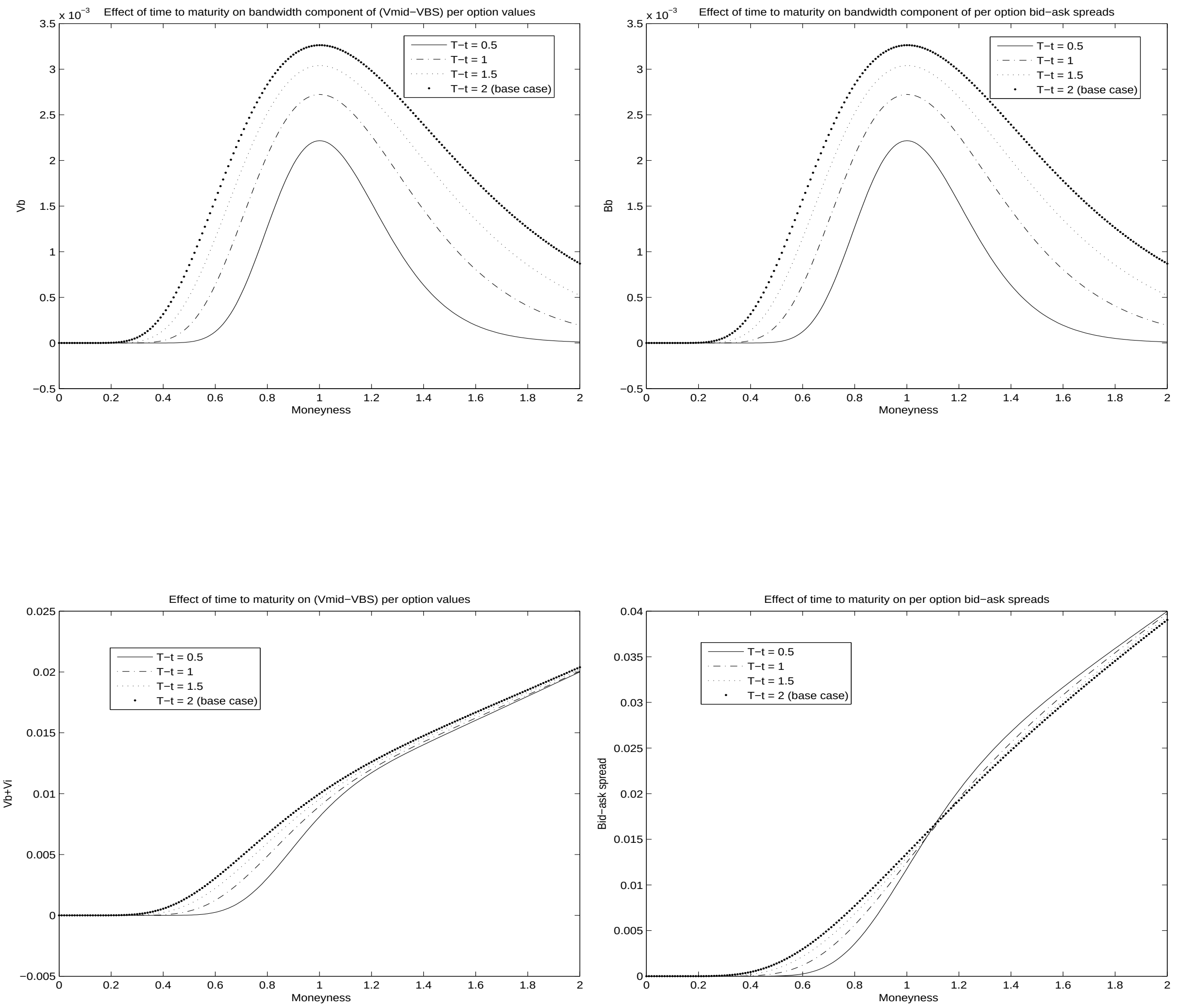

Figure 2: Bandwidth component (top) and total (bottom) of the difference between the per-option reservation mid price and the Black-Scholes price (left) and the bid-ask spread (right) all *vs* moneyness, S/K for a portfolio of European call options for different times to maturity. Parameter values where not stated: $\sigma = 0.32$, $r = 0.05$, $\gamma = 1 \times 10^{-3}$, $|N| = 1 \times 10^{6}$, $m = 1 \times 10^{3}$ and $k = 0.01$.

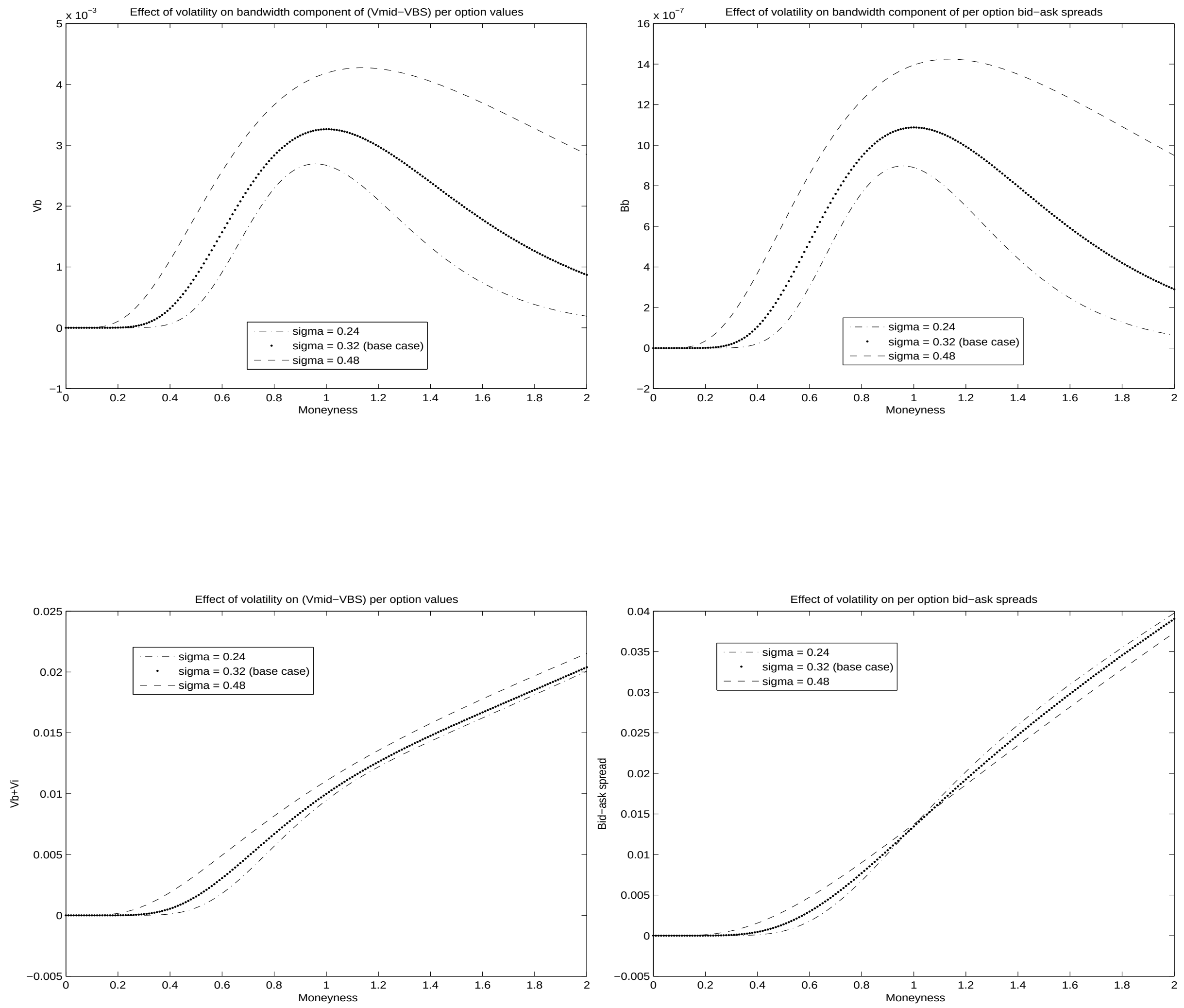

Figure 3: Bandwidth component (top) and total (bottom) of the difference between the per-option reservation mid price and the Black-Scholes price (left) and the bid-ask spread (right) all *vs* moneyness, S/K for a portfolio of European call options for different asset volatilities. Parameter values where not stated: $T = 2$, $r = 0.05$, $\gamma = 1 \times 10^{-3}$, $|N| = 1 \times 10^{6}$, $m = 1 \times 10^{3}$ and $k = 0.01$.

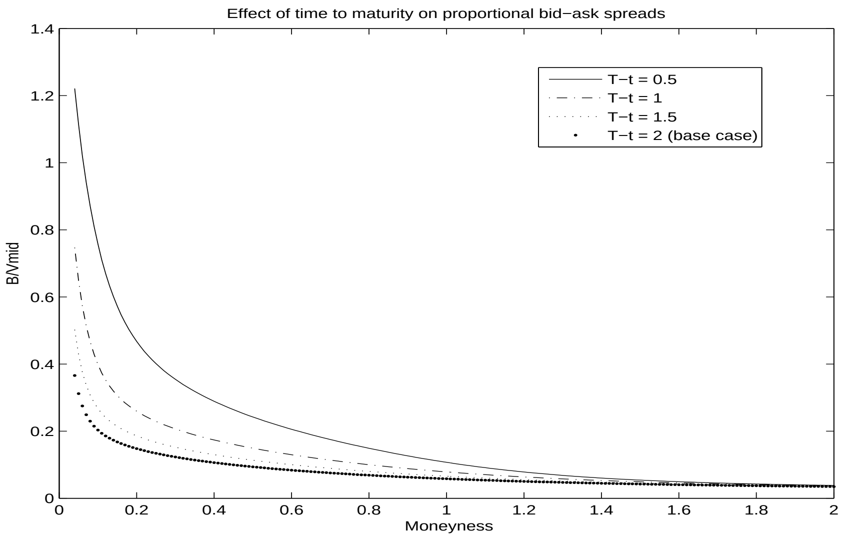

Figure 4: Proportional bid-ask spread B/V^{mid} *vs* moneyness S/K for different times to maturity. Parameter values where not stated: $T = 2$, $\sigma = 0.32$, $r = 0.05$, $\gamma = 1 \times 10^{-3}$ $|N| = 1 \times 10^{6}$, $m = 1 \times 10^{3}$ and $k = 0.01$.