ECONOMIC CAPITAL FOR OPERATIONAL RISK: APPLYING THE LOSS DISTRIBUTION APPROACH (LDA)*

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SUMMARY

In the last few years, bank industry has suffered from important losses due to operational failures. Being aware of that, in 2004, the Basel Committee published a New Capital Accord in which financial institutions were encouraged to measure, control and manage operational risk. In this context, Value at Risk (VaR) becomes essential for operational risk measurement and, what is more important, for estimating capital requirements (Capital at Risk). In this paper, we focussed on the Operational Value at Risk (OpVaR) as well as the methodological process for its estimation from the LDA perspective. In particular, we conduct a stress analysis on CaR with regard to the shape and scale parameters which characterize the operational loss distribution. Likewise, we develop a sensitivity analysis on CaR considering different levels of correlation between operational risk events faced by credit institutions.

KEY-WORDS: OPERATIONAL RISK, ADVANCED MEASUREMENT APPROACHES (AMA), AGGREGATE LOSS DISTRIBUTION (LDA), OPERATIONAL VALUE AT RISK (OpVaR), DIVERSIFICATION RATIO.

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1. INTRODUCTION.

Although operational risk is inherent in banking and cannot be completely eliminated, it can be managed and controlled and, in certain cases, insurance against such risk can be obtained. Clearly the management of this type of risk is not a new practice; it has always been essential for banks to prevent fraud, maintain the integrity of their internal controls and reduce errors in the processing of transactions. To do this, the support functions of banks, such as the Organisation department, devise work procedures for particular processes undertaken in the entity, which, among others aspects, include the necessary controls and verifications that enable the bank to provide its products and services to customers with the least degree of risk possible. However, a recent new development is the conception of operational risk as a specific discipline integrated in the global management of the risks faced by the entity, in harmony with credit and market controls, in accordance with the specific recommendations of the Basel Committee on Banking Supervision, in the New Accord of Adequacy of Capital [Basel, 2004]. One of the principal novelties in the text of this Accord is the inclusion of regulatory capital requirements† for this risk; thus, the new coefficient of solvency of 8% includes in its denominator the operational risk (20% of the 8%), and is defined as follows:

$$\frac{\text{Regulatory Capital}}{\text{Credit Risk (weighted assets)} + 12.5 * (\text{Market Risk} + \text{Operational Risk})} \geq 8\% \qquad [1]$$

Until the publication of the new proposal of capital requirements, there was no complete consensus on the definition of operational risk. The term was understood as: *"all those risks that were neither credit nor market risks"* [Hoffman, 1998: 29]. In consequence, as the point of departure for the management and control of this concept, the Committee [2004: 128] proposed to standardise it by defining it explicitly as *"the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events"*. This definition includes legal risk, but excludes strategic and reputational risk. Furthermore, the incorporation of operational risk in the coefficient of solvency has aimed those banks with less sophisticated control systems to manage this risk more effectively, and has encouraged those other entities that were already applying advanced models to improve their risk measurement methodologies. However, the development of these techniques continues to be a dynamic process, and the financial sector continues to devote substantial efforts to achieve these goals. This dynamism is based on two main aspects: firstly, the immaturity of the methodologies themselves, for which there are still no robust reports available on their reliability and efficacy; and, secondly, the flexibility allowed by the Committee regarding the techniques for the calculation of the regulatory

† Regulatory capital represents the minimum amount of equity that larger credit-issuing entities should allocate to cover the possible losses deriving from the different types of risk to which they are exposed. The difference with respect to the concept of economic capital is that, in the case of the former, it is the regulator that sets the minimum level of solvency, while in the latter case, the level of equity required is related to an objective rating established by the entity itself or by its shareholders. However, it should be noted that, in the present work, we shall utilize both terms, in a broad sense, as synonyms

capital and, in particular, for the internal methodologies of measurement. To this end, the recommendation of the Committee consists in following a sequential advance through the range of methods available, as the entity develops increasingly sophisticated systems and practices of measurement. Nevertheless, it should be stated that the development and utilization of more advanced techniques will, to a large extent, depend on the availability of internal data on operational losses. In this context, the main aim of this paper consists on illustrating the Capital at Risk (CaR) estimation process by using an Advanced Measurement Approach (AMA). We also pretend to evaluate the potential impact of the parameters which characterized the operational loss distribution on the capital charge.

For this purpose, we start defining the theoretical framework in which operational risk measurement is settled within the New Capital Accord (Basel II). In the third section, we describe the methodological process for obtaining the economic capital, taking into account the Loss Distribution Approach (LDA), since this model, based on the concept of Operational Value at Risk (OpVaR), seems to be the most appropriate for calculating the Capital at Risk. Once we have completed this step, in the following section, we conduct some different trials in order to reach the mentioned goals.

First of all, to show how LDA approach functions in practice, we developed the trial 1. In particular, we estimate the CaR at different confidence intervals, simulating two operational risks with opposite features in terms of both frequency and severity of losses. Secondly, in trials 2 and 3 we develop a stress-testing on CaR with respect to the shape and scale parameters corresponding to the respective frequency and severity probability distributions, used in the convolution processes. As a result, we have found, on one hand, a direct relationship between the capital consumption and the level of asymmetry and kurtosis, as well as the dispersion of operational losses, on the other. Finally, we made a fourth trial in order to examine the mitigating effect of diversification on the CaR estimate due to the imperfect correlation existing between the different categories of operational risk. This issue is explicitly included by the Committee [2006a: 152] in its New Accord. As a summary, in the last section, the main conclusions are highlighted.

2. THEORETICAL FRAMEWORK.

2.1. MEASUREMENT OF THE OPERATIONAL RISK.

Measurement –in terms of economic capital– has become the most complex and, at the same time, the most important aspect when dealing with operational risk. For this purpose, the Basel Committee [2001b] proposes three main approaches for calculating the capital requirements; ranked from lower to higher degree of sophistication and sensitivity to such risk, these are: (1) the Basic Indicator Approach (BIA); (2) the Standardised Approach (SA); and (3) the Advanced Measurement Approach (AMA). The AMA models, in turn, comprise three alternative methodologies: the Internal Measurement

Approach (IMA); Scorecards; and the Loss Distribution Approach (LDA). The Basic and Standardised approaches are considered as top-down methodologies [Basel, 2001b: 3]; both cover the risk with a capital amount equivalent to a fixed percentage of the gross income; this latter variable is used as a proxy of the magnitude or level of the exposure to operational risk of a financial entity. The principal difference between these two methods is that, in the Standardised Approach, the total amount of capital required is calculated as the sum of the regulatory capital requirements for each of the eight business lines described by the Committee [2006: annex 8]. In contrast, the AMA approaches are based on so-called bottom-up methodologies, since the economic capital is calculated from internal loss data classified by loss event type and business line; from these specific calculations the capital required for the bank as a whole is computed. In deciding whether an entity should apply either the Standardised Approach or one of the AMA methodologies, the Committee [2006a: 148-155] proposes that the entity should meet certain specific admission criteria that will be validated by the corresponding national supervisor. However, it is intended that the Basic Indicator Approach should be applicable to any bank, independently of the complexity of its activities, provided that it follows the directives of the document "*Sound Practices for the Management and Supervision of Operational Risk*" [Basel, 2003]. In that sense, the Basic Indicator Approach and the conditions described in the document should be understood as a point of departure in the process of capital calculation.

2.2. THE OPERATIONAL LOSS.

The design and maintenance of an internal database of operational loss events is essential for measuring and controlling operational risk because it will provide the best risk profile of the entity in the future. Consequently, banks should define the mechanisms for the effective monitoring and recording of the events that give rise to operational losses. At the same time, banks should take into account the minimum requirements of quality for the database established by the Committee [2006a: 152-153].

Expected and Unexpected Losses

In order to ensure a homogeneous categorization of losses in the banking sector, it is recommended to use the classification proposed by the Committee [2006a: annex 9], which identifies seven categories of operational risks. But more generally, operational losses can also be broken down into expected loss and unexpected loss. Thus, the set of expected operational losses will cover all those foreseeable losses that are intrinsic to the ordinary activity of the entity. Therefore, if they are seen as representing just one more cost for the business, they should be included implicitly in the final price of the products and services, or even provisioned. One example, fairly typical of this type of loss, would be the “cash differences” recorded almost every day, in banking offices, but generally of trivial amounts. On the other hand, unexpected losses refer to events not initially foreseen by the entity that may, however, give rise to disastrous situations for the institution due to the magnitude of the potential damage. In the first instance, the Committee suggests that such losses should be covered by equity, thus operational risk should be included as one more element in the denominator of the entity's solvency coefficient. However, there are particular dangers of catastrophic dimension, for which additional measures will have to be articulated, such as the risk transfer through insurance contracts.

Severity and Frequency

Independently of whether or not the loss is foreseen, it is essential to define two parameters at the time of identifying it: firstly, the *severity***,** or monetary amount of the loss; and, secondly, the *frequency* with which the event is repeated during a specified period of time, that is, the probability that the event may occurs. In so far as both variables are assumed to be statistically independent, they are modelled separately. In general terms, in the historical set collection of operational losses of a financial entity, a large number of events will be recorded that cause losses of small magnitude, such as the "cash differences" already mentioned, for example. But, given the still limited length of the database, for those events of low or moderate frequency but high severity, the information that a single entity possesses is, at best, insufficient for modelling the distribution of operational losses with statistical robustness. For this reason, the Committee [2006a: 153-154] allows the internal data to be complemented with the utilization of external databases of operational loss events that aggregate available information on these events, which the entity may possibly not have experienced previously but to which it is exposed. In this respect, the bank must implement a systematic procedure that determines under what circumstances the utilization of external data is justified, and what methodologies will be applied for the calibration of these with its internal data [see Baud *et al.*, 2002]. Further, as underlined by Guillen *et al.* [2007], we can consider the phenomenon known as *under-reporting*; this consists in ignoring or not identifying particular losses generated by operational failures, such that, for example, small losses with high frequency that are not computed when the entity calculates the capital charge, although their aggregation could embody a serious threat to the solvency of the entity.

Table 1
External databases of operational loss events.

DATABASE	MANAGER
ORX (Operational Riskdata eXchange Association)	*PricewaterhouseCoopers.*
CERO (Consorcio Español de Riesgo Operacional)	*Spanish Banks in the ORX.*
GOLD (Global Operational Loss Database)	*British Bankers' Association (BBA).*
MORE (Multinational Operational Risk Exchange)	*Netrisk.*
DIPO (The Database Italiano Perdite Operative)	*Bank of Italy.*
Algo OpVantage FIRST	*Fictch Ratings.*

3. THE LOSS DISTRIBUTION APPROACH (LDA).

3.1. THE OPERATIONAL VALUE AT RISK (OPVAR).

The Loss Distribution Approach is a statistical technique, inherited from the actuarial field [see Bühlmann, 1970], the objective of which is to obtain a probability distribution of aggregated losses. The model is constructed from the information of historical losses, recorded in the form of a matrix comprised by the eight business lines and the seven types of risk standardised by the Committee. In total there are 56 boxes or cells and, for each loss category, we must estimate the distribution of both the frequency and the severity. Once these have been defined, the next step is to obtain the distribution of aggregate losses due to operational risk attributed to each cell. For the calculation of the regulatory capital associated with each loss category or cell, the concept of Value at Risk (*VaR*) is applied to the context of the operational risk, adopting the nomenclature of OpVaR (Operational Value at Risk). Since OpVaR is a statistical measure which represents a percentile of the distribution of losses, it requires the establishment of certain parameters:

- *A confidence interval associated with the calculation*. In terms of capital charge, the Committee [2006a: 151] is explicit in setting 99.9% for this.

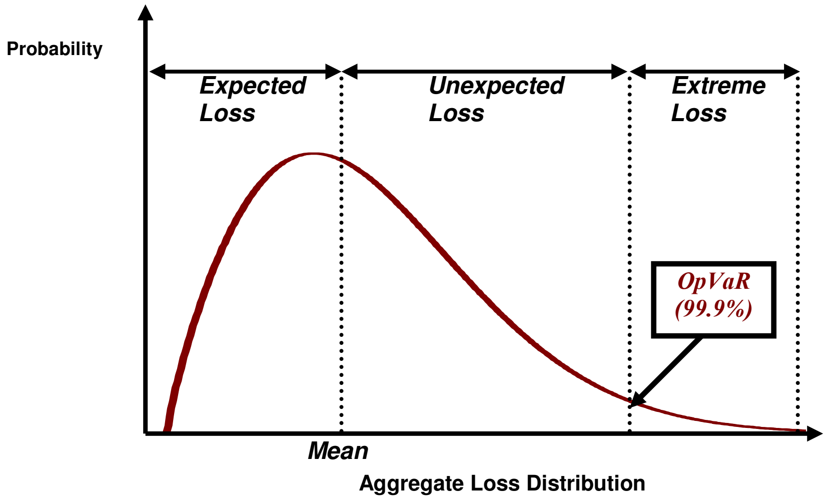
- *A period of time to which the estimation will refer*. In relation to the market risk, the determination of this parameter is not arbitrary but is linked to the nature of the position, and to the period of time necessary for its liquidation or coverage. The Committee [2006a: 151] states that, in the case of operational risk, the estimation must refer to one year time horizon.

- *A currency of reference*. The OpVaR of a business line is expressed in monetary units. In consequence, this variable becomes an intuitive and easily-understandable magnitude for its potential users (regulators, supervisors, risk managers, etc.) who will then be able to take their corresponding decisions.

- *A hypothesis on the distribution of the variable analysed*. In documents prior to the publication of the New Accord [2001b: 34], the Committee proposed the *Lognormal* distribution for dealing with the severity, and the *Poisson* distribution for the frequency. However, the distributions ultimately selected should be those that best fit the historical pattern of losses observed in an entity, and the nature of these losses can obviously be very different from that of other entities.

In short, we can interpret the OpVaR as a figure, expressed in monetary units, that informs us about the minimum potential loss that an entity could incur within a particular business line, *i,* due to operational risk event type, *j*, during a time horizon of one year, and with a level of statistical confidence of 99.9%. See figure 1 in this respect:

Figure 1
Illustration of the OpVaR concept for 99.9% statistical confidence.



3.2. METHODOLOGICAL PROCEDURE OF THE LDA.

As stated before, the LDA methodologies lean on an entity's internal database of losses, complemented with external data, and broken down in the matrix between "business lines" and "event types". Under the assumption that the severities are independent of each other, and that these, in turn, are independent of the frequencies, in each category, the next step consists of modelling separately these two variables.

Fit of the Frequency Distribution.

The random variable *N(i,j)* will symbolise the number of events occurring in a business line *i* due to a type of risk *j*; in a time horizon (ô) of one year; with a probability function $p_{i,j}$. This discrete variable represents the frequency of losses, whose distribution function, $P_{i,j}$, is expressed as:

$$P_{i,j}(n) = \sum_{k=0}^{n} p_{i,j}(k) \qquad [2]$$

According to authors such as Frachot *et al.* [2003] and Mignola and Ugoccioni [2005], the *Poisson distribution* –utilised successfully in actuarial techniques for insurance– is an option offering many advantages for the modelling of frequency. This function is characterised by one single parameter, *lambda (λ),* which represents, on average, the number of events occurring in one year. At the same time, it is worth considering other alternatives such as the *Binomial* or the *Negative Binomial* distribution (see appendix A).

Fit of the Severity Distribution.

Having defined the frequency, we then specify the random variable that represents the amount of loss, henceforth, severity, as *X*(*i,j*), with $\mathbf{F}_{i,j}$ being its probability function. Thus, the parameters of this probabilistic distribution that best match the data observed will have to be determined. For this task, as already mentioned above, the Committee [2001b: 34] first proposed the *Lognormal* distribution; although there are several parametric distributions that may be valid for such an approach, see appendix B in this respect. Thus, Fontnouvelle *et al.* [2004] include the *Pareto*; Böcker and Klüppelberg [2005] propose the *Weibull*; and Mignola and Ugoccioni [2006] add to these the *Burr* function of distribution as an alternative for modelling the severity, in addition to those cited.

The specific values of the parameters of each distribution are estimated by *Maximum Likelihood* (ML)*,* a method proposed by Fischer‡ (1890-1962). Once the parameters have been fixed, we must evaluate which distribution best fits the empirical data. For this purpose, we apply various statistical tests to calibrate the Goodness of Fit (GOF). Moscadelli [2004] proposed performing this test in function of the degree of *kurtosis* of the distribution. Hence he proposes trying with smooth tail distributions such as that of *Weibull*; next to be tested are the distributions of moderate or average tail, including the *Lognormal* or the *Gumbel*, among others; and, lastly, distributions such as the *Pareto,* characterised by presenting fat tails. Following Chernobai *et al.* [2006], the statistical test is symbolised by a null hypothesis, H_0: the observed distribution of operational losses, $F(x)$, is fitted to the theoretical distribution, $\widehat{F}(x)$; and an alternative hypothesis, H_A, which rejects the former:

$$H_0 : F_n(x) = \widehat{F}(x) \qquad\qquad H_A : F_n(x) \neq \widehat{F}(x) \qquad [3]$$

To perform this inferential analysis, we can take support from the following statistical *tests*: *Kolmogorov–Smirnov (K-S)*, *Anderson–Darling (A-D), Smirnov–Cramér–Von Mises* and *Kuiper;* analysed and detailed in Chernobai *et al.* [2005], D'Agostino and Stephens [1986] and Schwarz [1978]. Although most of the functions proposed for modelling the risk severity are usually a good fit for the central data (the body of the distribution), they do, however, tend to underestimate the tail. In addition, since the empirical distribution is asymmetric, this means that the estimation of the parameters is strongly influenced by the computing of observations situated in the middle and central zone of the real distribution; this reduces the informative contribution of those data situated at the end and, in consequence, gives rise to an underestimation of the percentiles. Having arrived at this point, it must be emphasised that the principal difficulty in the modelling of operational risk lies in the extreme behaviour of these distribution tails. Carrillo [2006] asserts that, in general, more than 90% of the capital is due to a very small number of events; moreover, the event accounting for the greatest loss

‡ Véase John (1997): "R.A. Fischer and the making of maximum likelihood 1912-1922". *Statistical Science* 12 (3): 162-176.

amount may turn out to be at more than 30 *standard deviations* from the *mean* of the distribution. This appreciation leads us to the application of the *Extreme Value Theory* (*EVT*), which analyzes the extreme behaviour of the random variables [see Gumbel, 1935, and Embrechts *et al.,* 1997]. This methodology, in relation to the computing of the CaR, allocates a greater weight to the tails of the distribution; thus, when modelling the data on past losses, the only data that will be used are those that exceed a high loss threshold, for which the Committee proposes the magnitude of 10,000 euros [2006a: 153]. For many experts, this premise makes the model highly sensitive to the choice of the threshold and to the number of observations of extreme losses. In addition, Mignola and Ugoccioni [2005] and Chernobai *et al.* [2006] warn of the possible inconsistencies on the resulting estimations of capital; these authors remark the risk of overestimation of the CaR, given the percentile utilised (99.9%). Being aware of that, it is not surprising that many authors suggest that it is necessary to use mixtures of distributions, such as, for example, the *Lognormal-Gamma* [Mignola and Ugoccioni, 2006] or the *Lognormal-Pareto* [Carrillo and Suárez, 2006] when modelling operational losses.

Obtaining the Aggregate Loss Distribution (LDA)

As we have already noted, the severity is a continuous variable whereas the frequency only takes discrete values. Consequently, for the purposes of obtaining the distribution of Aggregate Losses from each of these distributions, we first need to convert the severity into discrete values [Panjer, 2006: 411].

Once the distributions of severity and frequency have been characterised and homogenised, the last step of the methodological procedure consists in obtaining the distribution of aggregate losses. Thus, the total loss associated with a business line, *i,* and originated by a type of risk *j,* will be given by:

$$L(i,j) = \sum_{n=0}^{N(i,j)} X_n(i,j) \qquad [4]$$

This amount is therefore what is computed from a random number of loss events, with values that are also random, under the assumption that the severities are independent of each other and, at the same time, independent of the frequency [Frachot *et al*., 2004: 2]. The distribution function of the variable $L(i,j)$ –$\mathbf{G}_{i,j}(x)$– is obtained by:

$$\mathrm{G}_{i,j}(x) = \begin{cases} \sum_{n=1}^{\infty} p_{i,j}(n)\mathrm{F}_{i,j}^{n*}(x) & x > 0 \\ p_{i,j}(0) & x = 0 \end{cases} \qquad [5]$$

The asterisk denotes the convolution[§] in the function **F**, where $\mathbf{F}^{n*}$ is *n-times* the convolution of **F** with itself, that is:

$$\begin{aligned} \mathrm{F}^{1^*} &= \mathrm{F} \\ \mathrm{F}^{n^*} &= \mathrm{F}^{(n-1)^*} * \mathrm{F} \end{aligned} \qquad [6]$$

To obtain the aggregate loss function **G**(x), four possible techniques are proposed:

- Fast Fourier Transforms (FFT) [see Klugman *et al* 2004 : chap. 6].
- The Recursive Algorithm of Panjer [1981].
- The Monte Carlo Simulation Approach [see Klugman *et al* 2004: chap. 17].
- The Single-loss Approximation [see Böcker and Klüppelberg, 2005].

Once the aggregate distribution function has been determined, all that remains to calculate the regulatory capital associated with each cell is to apply the concept of Value at Risk Operational (OpVaR), that is, to calculate the 99.9% percentile of such distribution. In a strict sense, as warned by the Committee [2006a: 151], the economic capital (CaR) should cover, a priori, only the unexpected loss (UL):

$$CaR = UL(i, j; \alpha) \qquad [7]$$

However, if the entity does not demonstrate, in a suitable way, that the expected operational loss has been covered, in a broader sense, the regulatory capital should be considered as being computed to cover both types of loss; in such case, the CaR and OpVaR are identical.

$$\begin{aligned} CaR \equiv OpVaR(i, j; \alpha) &= G_{i,j}^{-1}(\alpha) \\ &= EL(i, j) + UL(i, j; \alpha) \end{aligned} \qquad [8]$$

Mathematically, the expected loss can be defined as:

$$EL(i, j) = E[L(i, j)] = \int_0^\infty x d\mathrm{G}_{i,j}(x) = E[X(i, j)] \text{ x } E[N(i, j)] \qquad [9]$$

Consequently, the unexpected loss would be expressed as follows:

$$\mathrm{UL}(i, j; \alpha) = \mathrm{G}_{i,j}^{-1}(\alpha) - E[L(i, j)] = \inf\{x | \mathrm{G}_{i,j}(x) \geq \alpha\} - \int_0^\infty x d\mathrm{G}_{i,j}(x) \qquad [10]$$

At the level of the entity, the computing of its own capital requirements for operational risk - assuming there is perfect dependence between the risks associated with each cell - does not entail any difficulty, in so far as it consists merely of the aggregation of the capital (CaR) corresponding each of the 56 cells; that is:

$$\mathrm{K}_{\mathrm{LDA}} = CaR(\alpha) = \sum_{i=1}^{8} \sum_{j=1}^{7} CaR_{ij}(\alpha) \qquad [11]$$

[§] The convolution is a mathematical procedure that transforms the distributions of frequency and severity into a third distribution (LDA) by the superposition of the two [see Feller, 1971:143].

3.3. THE DIVERSIFICATION EFFECT.

Up to now, we have assumed the perfect dependence hypothesis; in consequence, we have approximated the economic capital for the whole entity as the simple sum of the CaR of each cell in the matrix. This presumption would lead us, as in the Standardised Approach, to a one factor model characterised by a single source of risk, the random variable *z*. Thus, the loss corresponding to each of the 56 boxes is quantified in function of its sensitivity, β_{ij}, to this variable:

$$L_{ij} = \beta_{ij}(z) \qquad [12]$$

In the strict sense, this scheme is unrealistic since, although the existence of risks with a certain degree of correlation (for example, internal and external fraud) is evident, it is no less evident that an entity also faces other risks for which the correlation coefficient is clearly null or, at least, doubtful[**] –for example, between internal fraud and damage to physical assets–. The hypothesis of perfect dependence increases the degree of conservatism in the calculation of capital by not taking into account the mitigating effect of diversification [see Markowitz, 1952 and 1959]. However, this effect will ultimately depend on the degree of subadditivity of the OpVaR.

According to Artzner [1999] a consistent measurement of risk must comply with a series of properties, among which is subadditivity. In terms of OpVaR, this translates into:

$$OpVaR(L_1 + L_2) \leq OpVaR(L_1) + OpVaR(L_2) \qquad [13]$$

However, McNeil *et al*. [2005] and Chavez-Demoulin *et al.,* [2004b] warn of several characteristics in the loss distribution functions (L_k), including *extreme asymmetry, heavy tails* and *special dependence*, which could cause the principle of subadditivity to break down; such a phenomenon is known by the name of superadditivity. In this case, precisely the opposite happens, in other words, *the whole is greater than the sum of the parts*; this is:

$$OpVaR(L_1 + L_2) \geq OpVaR(L_1) + OpVaR(L_2) \qquad [14]$$

Having reached this point, it should be stated that, under the assumption of non-subadditivity of the OpVaR, the AMA methodologies could lose some of their appeal, because it may happen that the CaR calculated with this approach turns out to be higher than if it were determined by the Standardised or even the Basic Methods. Aware of this possibility, the Committee [2006a: 152] examines the possibility of including the diversification effect in the calculation of the capital requirements, asserting in the text that: *"(...) the bank will be authorised to use internal estimations in respect of the correlations of losses due to operational risk that exist between the different estimations of the operational risk, provided it can demonstrate to the national supervisor that its systems for*

[**] Frachot *et al.* (2004), based on the statistical analysis of the operational loss data corresponding to *Credit Lyonnais*, estimate the range of values for the correlation coefficient between the different categories of risks, at between 0.05 and 0.1.

determining the correlations are adequate, that they are applied in their totality, and that they take into account the uncertainty surrounding these estimations of correlation (especially in periods of stress). The bank must validate its assumptions of correlation utilising the most appropriate quantitative and qualitative techniques". In this line, the correlation between the aggregate losses of two types of risk is determined, in turn, by the dependence that may exist between the frequencies, or between the severities, or between both the frequencies and the severities of these types. Therefore, following Frachot *et al.* [2004] and using the notation previously employed, we will analyse, in a generic sense, the correlation between two different types of risk. Thus, L_1 and L_2 will represent the aggregate loss of each type of risk and L the aggregate loss at the level of the entity, as follows:

$$L = L_1 + L_2 = \sum_{n=1}^{N_1} X_n + \sum_{m=1}^{N_2} Y_m \qquad [15]$$

where N_i symbolises the annual frequency of events. Hence, with N_1 and N_2 being perfectly correlated and assuming that the frequency follows a *Poisson* of parameter, λ, then: $\lambda_1=\lambda_2=\lambda$. In practice, the correlation between the frequencies of two types of risk will be conditioned by the sensitivity of both to a particular factor (for example, the volume of business or the economic cycle). To quantify this degree of dependence with sufficient reliability, it will have to be supported by a comprehensive historical base of loss data. In respect of the severity, one of the basic principles of the LDA model is the assumption that the amounts of the losses recorded in the cells of the matrix are independent of each other. Taking this premise as the starting point, it is difficult to conceive that, if there is no correlation between the severities of the losses within a single type of risk, there still might be some correlation between the severities of two different categories of risk. Therefore, the correlation of the aggregate loss should be computed, principally, as a function of the degree of dependence of the frequencies observed:

$$cor(L_1, L_2) \neq 0 \begin{cases} cor(N_1, N_2) \neq 0 \\ cor(X, Y) = 0 \end{cases} \qquad [16]$$

In consequence,

$$cor(L_1, L_2) \leq cor(N_1, N_2) \qquad [17]$$

The objective now will be to transfer the correlation of the frequency to the aggregate loss; Frachot *et al.* (2004) and Powojowski *et al.* (2002) approach the calculation of the correlation coefficient by assuming that the frequency is fitted to a *Poisson*. After determining the correlation between the two types of risk, ρ_{12}, we incorporate the effect of diversification into the capital charge by using a new magnitude, *diversified CaR* (CaR^{D}), which is defined as follows:

$$\text{CaR}^{\text{D}} = OpVaR^D = \sqrt{OpVaR_1^2 + OpVaR_2^2 + 2\rho_{12} \cdot OpVaR_1 \cdot OpVaR_2} \qquad [18]$$

It should be observed that, under the assumption of perfect dependence, $\rho_{12} = +1$, the global CaR for the two cells is calculated by simple aggregation of the individual CaR, that is:

$$\mathrm{CaR}^{D} = OpVaR^{D} = \sqrt{OpVaR_1^2 + OpVaR_2^2 + 2 \cdot OpVaR_1 \cdot OpVaR_2} = \\ = \mathrm{CaR}_1 + \mathrm{CaR}_2 \qquad [19]$$

In other words, assuming perfect dependence, there is no benefit from diversification in terms of capital charge reduction because of the coincidence between the *non-diversified or gross CaR* (CaR^{G}) and the *diversified CaR* (CaR^{D}). On the contrary, the advantages from the diversification can be clearly quantified by the following ratio:

$$\text{Diversification Ratio} = \frac{\mathrm{CaR}^{G} - \mathrm{CaR}^{D}}{\mathrm{CaR}^{G}} \qquad [20]$$

4. AN APPROACH TO THE CAR BY MONTE CARLO SIMULATION.

In order to illustrate and test the methodological procedure presented, we have carried out several trials using Monte Carlo Simulation. Thus, in a first step, we estimate CaR from a certain frequency distribution and a severity one, applying the convolution process. On the other hand, we stress the regulatory capital considering the parameters that characterized both probability distributions, with the aim of calibrating how sensitive the CaR is to such parameters. We also compare the raw CaR with the so called diversified CaR for illustrating the diversification effect, considering different correlation scenarios for the simulated operational risks.

4.1. TRIAL 1

We intend to describe the methodological process for obtaining the CaR at different confidence intervals (95%, 99% and 99.9%), focusing on both expected loss (EL) and unexpected loss (UL) concepts. As a consequence, we propose a relative operational risk measure, named as *hedging ratio*.

According to the results of the *Loss Data Collection Exercise*[††] (Basel, 2002), there are a wide variety of operational risk events in mean of frequency and severity. We have simulated two risks with opposite features regarding those variables, that is: OpRisk1, with high frequency and low severity and, on the contrary, OpRisk2 defined by low frequency and high severity.

In this sense, to ensure the homogeneity of all the trials conducted, we use the same frequency probability functions for all of them, as well as for severity. In a first step, and with the aim of

[††] See appendix F.

modelling the frequency, we have chosen the *Poisson distribution*, whose parameter (λ) symbolises the mean number of events per year. Aware of this, we have tried to reflect this effect by discriminating between a *lambda* equal to 1,000, for the OpRisk1 (high frequency), and one equal to 100, for OpRisk2 (low frequency).

With regard to the severity distribution, we use the *Weibull* model for fitting the operational losses. Since OpRisk2 has been defined as a high severity risk, we have determined a parameter of scale (β) notably higher than in the case of the OpRisk1, by conferring a greater standard deviation. IN particular, the beta for OpRisk2 has been set in 1,000 and 100 for OpRisk1.

In practice, the severity distribution presents a clear positive asymmetry and a high degree of leptokurtosis. In accordance with these assumptions, we have simulated a hypothetical distribution, and have chosen for both OpRisk1 and OpRisk2 a form parameter (α) of less than unity.

However, in order to test the impact of the statistical moments, we have assigned a lower value of *alpha* to OpRisk1precisely to infer a higher degree of asymmetry and kurtosis, compared with that of the OpRisk2. The descriptive analysis conducted for the simulated distributions of severity is given in table 2:

Table 2
Descriptive analysis for the simulated distributions of severity.

	OpRisk1	**OpRisk2**
Mean *(Euro)*	200	1,190
Standard Deviation *(Euro)*	447.21	1,610.8
Skewness	6.62	3.12
Kurtosis	84.72	15.98

Having described the frequency and severity distributions, we then move to the convolution of the distributions using the method of Monte Carlo Simulation, involving the generation of a total of 100,000 iterations. The distributions of the resulting aggregate losses and the CaR associated with these are detailed in table 3.

Table 3
Summary of data of the simulated LDA (thousand Euro).

		OpRisk1		OpRisk2	
Frequency	***Distribution***	***Poisson***		***Poisson***	
	Parameter	λ=1,000		λ=100	
Severity	***Distribution***	***Weibull***		***Weibull***	
	Parameter	α=0.5	β=10	α=0.75	β=80
LDA	***Expected Loss (EL)***	200,000		119,063.93	
	Unexpected Loss (UL)	52,293.12		70,849.73	
	$CaR_{95\%}$	226,420.19		153,879.94	
	$CaR_{99\%}$	238,309.70		170,468.94	
	$CaR_{99.9\%}$	252,293.12		189,913.66	
	$EL/CaR_{99.9\%}$ ***Ratio***	79.27%		37.31%	
	$UL/CaR_{99.9\%}$ ***Ratio***	20.73%		62.69%	

In the light of the above data, the analysis of sensitivity performed on the CaR for different levels of confidence (95%, 99% and 99.9%) demonstrates the powerful impact that this parameter has on the capital consumption. In this respect, although it is the intention of the Basel Accord to cover possible extreme events situated in the tail of the distribution, this high degree of conservatism may, at the same time, become an almost confiscatory element in the capital structure of the entity.

In addition, considering the results obtained for the two risks, we observe a greater CaR in the OpRisk1 than in OpRisk2, independently of the level of confidence established.

Strictly speaking, the capital consumption must take account of both the expected and the unexpected loss. However, as already stated in the preceding lines, in those situations where the entity can prove the adequate provision for its expected loss, the Committee [2006a: 151] accepts a computation of capital based exclusively on the unexpected losses, as can be seen with the formula [7]. This alternative understanding of the CaR gives us the opportunity to re-interpret the data. Since the unexpected loss is higher for OpRisk2, a different hierarchy, in terms of capital charge, is observed between these two risks due to their distinctive nature. Although it is correct that, on average, fewer events occur in OpRisk2, the averaged loss amount is much higher than in OpRisk1, together with the underlying greater deviation of these values with respect to the mean.

On the other hand, the ratios calculated –*"EL/OpVaR"* and *"UL/OpVaR"*– provide very interesting relative measures when financial information on operational risk is reported. These are percentage magnitudes that indicate the relative importance of the expected and unexpected losses in the total regulatory capital. Under the directives of the last assumption, in our example, and for the case of

OpRisk1, 79.37% of the OpVaR would correspond to expected loss (EL) that should be covered, while the remaining 20.73%, the unexpected loss (UL), would represent the percentage associated with the regulatory capital; on this basis, we could call this last figure the hedging ratio for operational risk.

4.2. TRIAL 2

The main aim of the second trial is to calibrate the CaR's sensitiveness with respect to certain parameters: lambda (λ) of the Poisson distribution, which takes five different values (10, 50, 100, 500 and 1,000) and the shape parameter of the Weibull distribution, the so called alpha (α). For the latter, we have used six different values (0.3, 0.5, 0.6, 0.75, 0.9 and 1) while fixing the scale parameter (β). As we the severity distribution is biparametrical and, thus, the sensitivity analysis requires the application of the *ceteris paribus* condition on at least one of such parameters. In this second trial we set the beta value in 100. The table 4 illustrates the descriptive analysis of the simulated distributions of severity under this premise.

Table 4
Descriptive analysis for the simulated distributions of severity in trial 2.

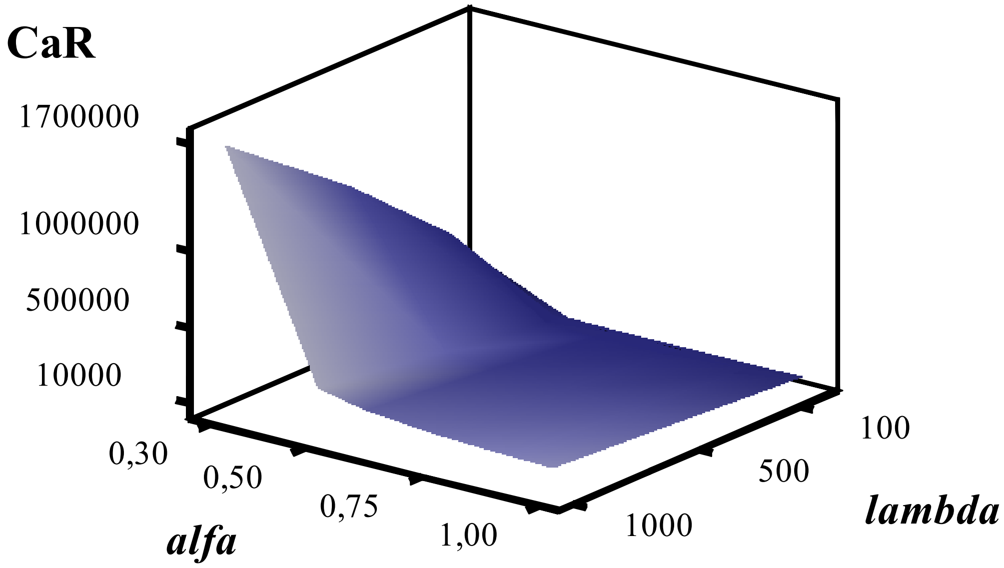
beta = 100	***alpha***					
	0.3	**0.5**	**0.6**	**0.75**	**0.9**	**1**
Mean *(thousand Euro)*	926.05	200	150.46	119.06	105.22	100
Standard Deviation *(thousand Euro)*	5,007.8	447.21	264.51	161.08	117.11	100
Skewness	28.33	6.62	4.59	3.12	2.35	2
Kurtosis	2,343.3	84.72	37.48	15.99	8.53	6

From the observation of the previous table, we can conclude that, remaining beta stable, as long as alpha increases, both the asymmetry and the skewness are reduced. After generating the corresponding frequency and severity distributions, we make the convolution process by using Monte Carlo Simulation. As a result we obtain thirty LDA's from which we infer the CaR with 99.9% of statistical confidence (see appendix D).

In the following chart, we have plotted the three-dimensional CaR surface depending on the joined values of lambda and alpha.

Figure 2

CaR estimates based on the joined alpha and lambda values.



4.3. Trial 3

Having tested the impact of the shape parameter in the severity distribution, in the third trial, we develop a similar analysis to the previous one, but this time stressing the scale parameter, beta, with five different values, 10, 50, 100, 1,000 and 2,000. In order to assess, in isolation, the potential influence of beta on the CaR, the value of the scale parameter is fixed in 0.75. The table 5 presents a descriptive analysis of the simulated distributions of severity under this hypothesis.

Table 5

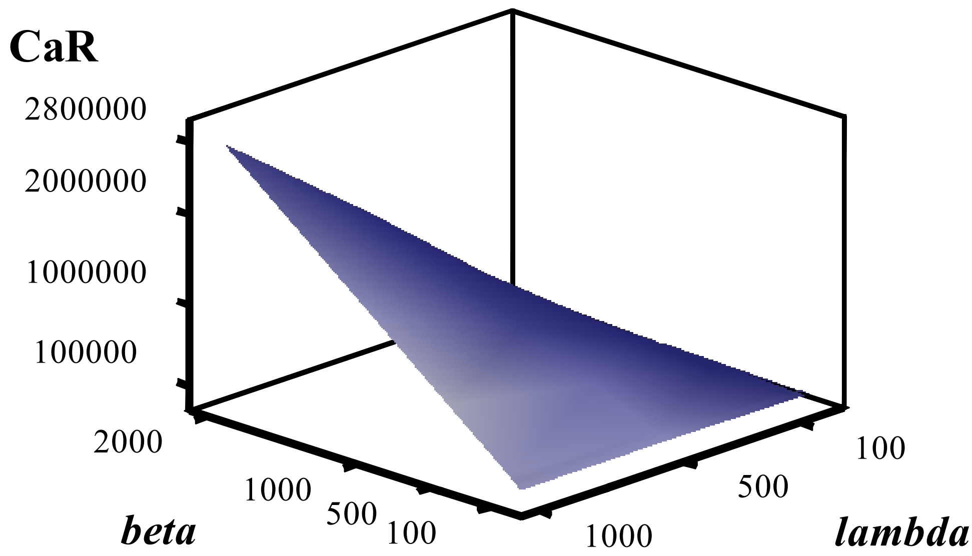
Descriptive analysis for the simulated distributions of severity in trial 3.

alpha = 0.75	*beta*					
	10	**50**	**100**	**500**	**1000**	**2000**
Mean *(thousand Euro)*	11.9	59.53	119.06	595.32	1,190.6	2,381.3
Standard Deviation *(thousand Euro)*	16.11	80.54	161.08	805.38	1,610.8	3,221.5
Skewness	3.12	3.12	3.12	3.12	3.12	3.12
Kurtosis	15.99	15.99	15.99	15.99	15.99	15.99

Looking at the previous table, there is a clear impact of the scale parameter on the dispersion observed in the respective severity distributions, measured by the standard deviation. As alpha remains constant and equal to 0.75, higher values of beta implies much more dispersion with respect to the mean. Reaching this point, once again we conduct the convolution process. The resulting CaR's from the joined values of beta and lambda are summarized in the appendix E and they are also illustrated in the chart 3:

Figure 3

CaR estimates based on the joined beta and lambda values.



4.3. TRIAL 4

Lastly, to test the diversification impact on the quantification of the aggregate CaR, we have performed an analysis of sensitivity for different values of the correlation coefficient ($\rho_{OpRisk,OpRisk2}$) between the operational risks defined in trial 1(OpRisk1 and OpRisk2). In fact, the results presented in the table 3 rested on the assumption of perfect dependence between such risks. However, if we now consider the existence of imperfect correlations (see Frachot et al., 2004), the effect of diversification becomes evident in a notable saving of capital, as can be observed clearly enough in table 6. As noticed, for low values of the correlation coefficient, the diversified CaR is notably reduced, while the ratio of diversification increases in consequence.

Table 6
Analysis of the Diversification effect on the CaR.

Correlation Coefficient	Diversified CaR		Diversification Ratio	
	$CaR^{D}_{(EL+UL)}$ (thousand Euro)	$CaR^{D}_{(UL)}$ (thousand Euro)	$DR_{(EL+UL)}$	$DR_{(UL)}$
0	315,783	88,058	28.59%	28.49%
0.1	330,608	92,170	25.24%	25.15%
0.2	344,796	96,106	22.03%	21.96%
0.3	358,423	99,886	18.95%	18.89%
0.4	371,551	103,529	15.98%	15.93%
0.5	384,230	107,048	13.11%	13.07%
0.6	396,504	110,455	10.34%	10.30%
0.7	408,410	113,760	7.64%	7.62%
0.8	419,978	116,971	5.03%	5.01%
0.9	431,235	120,097	2.48%	2.47%
1	442,207	123,143	0.00%	0.00%

5. FINAL CONSIDERATIONS.

The Basic and Standardised Methods proposed by the Committee for the calculation of the regulatory capital for operational risk, present certain conceptual deficiencies, particularly with respect to the exposure indicator, that is, the gross income. The fact is that its quantification depends, ultimately, on the accounting framework of each country, and this makes regulatory arbitrage possible. Similarly, we have to ask ourselves whether an entity with higher gross income but better management practices might not incur lower operational risks than one with less income but poorer management. In addition, the Basic Approach offers an entity little incentive to develop better risk control systems, since it does not require the regulator to monitor compliance with any qualitative requirement for its implementation. Given the foregoing, both methods are conceived, a priori, as transitional models in a progression towards superior states that are exemplified by the advanced methodologies; it can be argued that both these simpler methods are currently being used as "escape routes" in the face of the imminent entry into force of the New Accord.

Therefore, the financial entities that aim to manage their exposure to operational risk as effectively as possible should combine efforts in the development and application of the Advanced Measurement Approach (AMA). In this respect, according to the latest directives of the Committee, the AMA approach that appears best placed is the Loss Distribution Approach (LDA), reinforced by the concept of Operational Value at Risk or OpVaR. Hence, to ensure the correct implementation of the LDA, it is considered necessary to have available historical information of operational losses, broken down by type of risk and business line, on which to model the frequency and severity of these losses. However, it is precisely on this point that the banks encounter the main obstacle in applying the advanced methods, because the lack of a sufficiently comprehensive and representative internal database of operational losses significantly weakens this approach. Although the Committee has foreseen that, under certain circumstances, external databases may be used, this does not appear to resolve the problem. Therefore, scenario analysis and simulation of losses have become productive sources of progress, given the lack of information, at least for the moment. On another aspect, for the purposes of computing the regulatory capital, an excessively high percentile, 99.9%, has been established. Thus, the level of confidence proposed by the Basel Committee makes the calculated capital requirement for operational risk a far too conservative measurement. In particular, for those loss distributions with fat tails, a very high figure for OpVaR can be obtained, and this consequently consumes very large amounts of capital. With reference to the modelling of the variables, the *Poisson distribution* is the one most used for fitting the frequency, although it is important to consider other alternatives such as the *Binomial* or the *Negative Binomial distributions.* Regarding the severity, we are faced with a series of parametric distributions *(Lognormal, Weibull, Pareto, etc*.) that could, a priori, be good candidates for such an approach. However, the empirical evidence demonstrates that, in practice, no simple distribution fits exactly; hence it is necessary to apply the so-called mixture of distributions.

The Committee also raises the possibility of incorporating the effect of diversification in the measurement of operational risk. Under the principle of subadditivity of the OpVaR, the resulting economic capital, also known as diversified CaR, is notably sensitive to the correlation coefficient. Nevertheless, to be able to take advantage of the reduction of capital that this should offer, financial entities must articulate the appropriate estimation methods to approximate the correct correlation coefficients. Paradoxically, the empirical studies made of this aspect situate the values of this coefficient very close to zero, far from the conservatism that inspires the New Capital Accord in relation to operational risk.

Finally, it must be emphasised that, although the LDA approach enjoys wide acceptance in the banking industry, especially in those entities that are already utilising advanced approaches, it still lacks sufficient robustness for its consequent practical implementation.

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APPENDIX.

A. Distribution functions proposed for modelling the frequency.

Distribution	*Probability Mass Function (PMF)*	*Parameters*
Poisson	$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$	$\lambda > 0$
Binomial	$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$	$n > 0$ $0 < p < 1$
Negative Binomial	$f(x) = \binom{s+i-1}{x} p^S (1-p)^i$	$s > 0$ $0 < p < 1$

B. Distribution functions proposed for modelling the severity.

Distribution	*Probability Density Function (PDF)*	*Parameters*
Lognormal	$f(x) = \frac{1}{x\sqrt{2\pi\sigma}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$	$\mu, \sigma > 0$
Weibull	$f(x) = \frac{\alpha x^{\alpha-1}}{\beta^\alpha} e^{-(x/\beta)^\alpha}$	$\alpha, \beta > 0$
Gamma	$f(x) = \frac{1}{\beta\Gamma(\alpha)} \left(\frac{x}{\beta}\right)^{\alpha-1} e^{-x/\beta}$	$\alpha, \beta > 0$
Pareto	$f(x) = \frac{x^\alpha \alpha}{(\lambda + x)^{\alpha+1}}$	$\alpha, \lambda > 0$
Gumbel	$f(x) = \frac{1}{\beta} e^{\frac{x-\mu}{\beta}} e^{-e^{\frac{x-\mu}{\beta}}}$	$\mu, \beta > 0$
Burr	$f(x) = \tau\alpha\beta^\alpha x^{\tau-1} (\beta + x^\tau)^{-(\alpha+1)}$	$\alpha, \beta, \tau > 0$

C. Statistical moments of *Weibull* distribution.

Moments	*Expression*
Mean	$\mu = \beta\Gamma(1+\alpha^{-1})$ where Γ corresponds to *Gamma function* $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$
Variance	$\sigma^2 = \beta^2 \left[\Gamma(1+2\alpha^{-1}) - \Gamma^2(1+\alpha^{-1})\right]$
Skewness	$\gamma_1 = \frac{\Gamma(1+3\alpha^{-1}) + 3\Gamma(1+2\alpha^{-1})\Gamma(1+\alpha^{-1}) + 2\Gamma^3(1+\alpha^{-1})}{\left[\Gamma(1+2\alpha^{-1}) - \Gamma^2(1+\alpha^{-1})\right]^{3/2}}$
Kurtosis	$\gamma_2 = \frac{-6\Gamma_1^4 + 12\Gamma_1^2\Gamma_2 - 3\Gamma_2^2 - 4\Gamma_1\Gamma_3 + \Gamma_4}{\left[\Gamma_2 - \Gamma_1^2\right]^2}$, where $\Gamma_i = \Gamma(1 + i/\alpha)$

D. CaR estimates in trial 2.

CaR (thousand Euro)	β=100					
	α					
λ	**0.3**	**0.5**	**0.6**	**0.75**	**0.9**	**1**
10	174,725	113,94	6,405	3,997	3,100	2,764
50	343,801	25,325	16,460	11,291	9,228	8,452
100	458,104	39,700	26,370	19,003	15,863	14,831
500	1,055,914	139,039	98,128	74,228	63,987	60,154
1000	1,683,747	252,293	181,877	139,254	121,028	114,104

E. CaR estimates in trial 3.

CaR (thousand Euro)	α=0.75					
	β					
λ	**10**	**50**	**100**	**500**	**1,000**	**2,000**
10	402	1,990	3,997	19,887	39,610	79,632
50	1,113	5,628	11,296	56,391	112,629	222,369
100	1,886	9,455	19,003	94,617	189,913	3,782,24
500	7,419	37,170	74,228	370,763	742,438	1,485,667
1000	13,936	69,682	139,254	695,165	1,391,160	2,785,010

F. Loss Data Collection Exercise.

In June 2002, the Risk Management Group (RMG) of the Committee conducted a compilation of data, on a global scale, of the operational losses incurred during the year 2001 in the banking sector. This process of compilation of operational losses, included in the *Quantitative Impact Study* (QIS), is known as the *Operational Risk Loss Data Collection Exercise* (LDCE). The study was conducted on a sample of 89 banks that operate internationally. The following matrix illustrates, in percentage terms, the losses classified by type of operational risk and business line. In absolute values, around 47,000 events of operational failure were recorded, and the total amount of losses incurred reached 7,800 million euros.

Matrix of distribution of losses by business line and event type.

Event Type (j) / Business Line (i)	Internal Fraud	External Fraud	Employment Practices and Workplace Safety	Clients, Products & Business Practices	Damage to Physical Assets	Business Disruption and System Failures	Execution, Delivery & Process Management
Corporate Finance	0.04 0.85	0.04 0.01	0.19 0.03	0.19 0.74	0.04 0.14	0.02 0.01	0.47 0.57
Trading & Sales	0.10 0.87	0.25 0.58	0.22 0.32	0.23 1.21	0.07 0.48	0.25 0.19	10.27 8.10
Retail Banking	2.87 4.03	38.86 10.82	5.01 3.61	4.47 3.16	0.56 1.14	0.35 0.19	11.55 5.61
Commercial Banking	0.19 0.34	3.68 4.20	0.17 0.33	0.60 2.09	0.11 18.19	0.09 0.20	1.96 9.23
Payment & Settlement	0.05 0.31	0.81 0.29	0.13 0.18	0.03 0.01	0.01 0.16	0.14 0.05	2.91 1.39
Agency Services	0.01 0.01	0.03 0.06	0.01 0.01	0.02 0.01	0.00 0.00	0.03 0.01	2.43 1.98
Asset Management	0.07 0.07	0.09 0.07	0.10 0.17	0.20 1.09	0.01 0.03	0.01 0.02	1.65 1.19
Retail Brokerage	0.15 0.98	0.04 0.02	2.14 0.86	1.35 2.56	0.02 8.79	0.11 0.02	4.06 1.45

*The top figure in each cell represents the frequency, and the bottom figure the severity, both units in percentage terms.
** the percentage of events that, due to lack of information, could not be classified by line of business or type of risk, do not appear included in the table.