Term Structure of Volatilities and Estimation Method of the Term Structure of Interest Rates

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Abstract

In this paper, we proceed to estimate term structure of interest rate volatilities finding that these estimates depend significantly on the model used to estimate the term structure (Nelson and Siegel or Vasicek and Fong) and the heteroskedasticity structure of errors (OLS or GLS weighted by duration). We conclude in our empirical analysis that there are significant differences between these volatilities in the short (less than one year) and long term (more than ten years). Finally, we can detect that three principal components explain the 90 % of the changes in volatility term structure. These components are related with level, slope and curvature.

Keywords: Volatility Term Structure; GARCH; Principal Components

JEL Classification: E43; F31; G12; G13; G15

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1. Introduction

We define the term structure of volatilities as the relationship between the volatility of interest rates and their maturities. The importance of this concept has been growing during the last decades particularly as interest rate derivatives have developed and interest rate volatility has become the key factor for the valuation of assets such as caplets, caps, floors, swaptions, etc. Moreover, interest rate volatility is one of the inputs needed to implement some term structure models such as Black, Derman and Toi (1990) or Hull and White (1987) which are particularly popular among practitioners.

However, one of the main problems concerning the estimation of the volatility term structure arises from the fact that zero coupon rates are unobservable. So they must be previously estimated and this requires adopting a particular methodology. The problem of the term structure estimation is an old question widely analysed in the literature and several procedures have been suggested for the last thirty years.

Probably, among the most popular methods are those developed by Nelson and Siegel (1982) and Vasicek and Fong (1982).[1] In Spain, these methods have been applied in Núñez (1994) and Contreras et al. (1996) respectively.

A large body of literature focus on the bond valuation ability of these alternative models without analysing the impact of the term structure estimation method on other moments of the zero coupon rates[2]. Nevertheless, in this paper we focus on the second moment of interest rates derived from alternative term structure methods. So, the aim of this paper is to analyze if there are significant differences between the estimates of the volatility term structure (VTS) depending on the model used for estimating the term structure of interest rates (TSIR).

In this study we compare Nelson and Siegel (1987), NS^O, Vasicek and Fong (1982), VF^O, and both models using two alternative hypotheses about the error variance. First we assume homoscedasticity in the bond price errors and so do the term structure is estimated by OLS. Alternatively, a heteroskedastic error structure is employed estimating by GLS weighting pricing errors by the inverse of its duration, NS^G and VF^G.[3]

[1] There is another kind of researches use, for instance, swap rates (Cejas and Morini, 1999, and Abad and Novales, 2005a and 2005b) or Legendre polinomials (Morini, 2003) to estimate the term structure of interest rates.
[2] Some examples concerning the Spanish market are Morini (2003) and Benito (2004).
[3] See, for example, Díaz and Skinner (2001), Díaz et al. (2006) and Díaz and Navarro (1997), (2002a) and (2002b) for a more detailed explanation.

In the literature, to minimize errors in prices is usual in order to optimize any model for estimating the TSIR. Nevertheless, this procedure tends to misestimate short term interest rates. This is because an error in a short term bond prices induces an error in the estimation of short interest rates greater than the error in long term interest rates produced by the same error in long term bond prices. In order to solve this problem, it is usual to weight pricing errors by the reciprocal of bond Macaulay's duration.[4]

Once estimates of TSIR are obtained, we proceed to estimate interest rate volatilities using conditional volatility models (GARCH models).

In addition, we try to identify the three main components in the representation of the volatility term structure for each model. Some researchers have studied this subject, finding a small number of factors able to represent the behaviour of the term structure of interest rates (Núñez, 1994, Navarro and Nave, 1997, Domínguez and Novales, 2000, Benito, 2004, and Benito and Novales, 2007). Nevertheless, this analysis has not been applied, to a large extent, to the volatility term structure (except, e.g. Abad and Novales, 2005a).

Litterman and Scheinkman (1991), Matzner-Løber and Villa (2004) and Piazzesi (2005) assert that standard principal component analysis provides much of the intuition for the dynamics of financial time series. Empirical analysis generally determines that three principal components are needed to almost fully explain the dynamics of the term structure of interest rates, but there are not a lot of evidences in case of VTS. The interpretation of these principal components in terms of level, slope, and curvature describes how the yield curve shifts or changes shape in response to a shock on a principal component. These principal components are extremely useful in thinking about the driving forces of the yield curve and they have important macroeconomic and monetary policy underpinnings.

According to Matzner-Løber and Villa (2004), the latent factors implied by estimated affine term structure models behave like the first principal components (Duffee, 1996, and Dai and Singleton, 2000), and, in accordance with Diebold and Li (2003) and Matzner-Løber and Villa (2004), the parsimonious term structure model introduced by Nelson and Siegel (1987) can be reinterpreted as a modern three-factor model of level, slope and curvature (Bliss, 1997, and Diebold and Li, 2003). However, neither their approach nor their results are identical. Recently, Diebold, Ji and Li (2004)

[4] This correction is usual in official estimations of the central banks (BIS, 2005).

have shown that the sensitivities (or loadings) of the financial time series to the level, slope and curvature state variables (or factors) explain very well these series and they capture systematic risk.[5]

We apply our methodology to the Spanish volatility term structure (VTS) using the Spanish term structure of interest rates (TSIR). The data used in this empirical analysis are the Spanish Treasury bill and bond prices of actual transactions from January 1994 to December 2006.

We show statistically significant differences between estimates of the term structure of interest rate volatilities depending on the model used to estimate the term structure and the heteroskedasticity structure of errors (NS^O, NS^G, VF^O and VF^G), mainly in the short-term (less than one year) and in the large-term (more than ten years). This inspection could have significant consequences for a lot of issues related to risk management in fixed income markets. On the other hand, we find three principal components that can be interpreted as level, slope and curvature and they are not significantly different among our eight proposed models.

The rest of our paper is organized as follows: The next section describes the data used in this paper. The third section describes Nelson and Siegel (1987), NS, Vasicek and Fong (1982), VF, and both models weighted by duration, and conditional volatility models (GARCH models). The fourth section analyses the differences in the volatility term structure from our eight different models. Finally, the last two sections include a principal component analysis of volatility term structure and, finally, summary and concluding remarks.

2. Data

The database we use in this research consists of estimates that contains volume-weighted average of all the spot transaction prices and yields in each day corresponding to each Spanish Treasury bill and bond traded and registered in the dealer market or Bank of Spain's book entry system.[6] They are obtained from annual files available at

[5] Diebold and Li (2003) have pointed out that, despite the fact that their estimated factors are closed to the first three principal components, their approach improves standard principal component analysis.

[6] The secondary market for Spanish Treasury debt is known as "Mercado de Deuda Pública Anotada" or MDPA.

"Banco de España" website.[7] We focus on 27 different maturities between 1 day and 15 years. Our sample runs from January 1994 to December 2006.

First of all, in order to refine our data, we have eliminated from the sample those assets with a trading volume less than €3 million (500 million pesetas) in a single day and bonds with term to maturity less than 15 days or larger than 15 years. Besides, in order to obtain a good adjustment in the short end of the yield curve, we always include in the sample the one-week interest rate from the repo market.

From the price (which must coincide with the quotient between effective volume and nominal volume of the transaction) provided by market, we obtain the yield to maturity on the settlement day. Sometimes this yield diverges from the yield reported by the market.[8] Controlling for these conventions,[9] we recalculate the yield always using compound interest and the year basis ACT/ACT for both markets.

We estimate the zero coupon bond yield curve using two alternative methods. The first one we use fits the Nelson and Siegel's (1987) exponential model for the estimation of the yield curve.[10] The second methodology is developed in Contreras et al. (1996) where the Vasicek and Fong (1982) term structure estimation method (VF^O) is adapted to the Spanish Treasury market. VF^O uses a non-parametric methodology based on exponential splines to estimate the discount function. From the original method, Contreras et al. (1996) drop one of the parameters as the Spanish Treasury does not issue callable bonds and redefine another one according to the Spanish tax system. It takes the present value of the coupon cash flows discounted by the actual yield to maturity in the case of coupon bearing bonds and zero otherwise. A unique variable knot is used to adjust exponential splines, knot which is located to minimize the sum of squared residuals.

With respect to the estimation methodology we apply both OLS and GLS. In the second case we adjust the bond price errors by the inverse of the bond Macaulay duration in order to avoid to penalise more interest rate errors in the short end of the term structure.

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[7] http://www.bde.es/banota/series.htm. Information reported is only about traded issues. It contains the following daily information for each reference: number of transactions, settlement day, nominal and effective trading volumes, maximum, minimum and average prices and yields.

[8] These divergences are due to simple or compound interest and 360-day or 365-day year basis depending on the security term to maturity.

[9] http://www.bde.es/banota/actuesp.pdf

[10] See, for example, Díaz and Skinner (2001), Díaz et al. (2006) and Díaz and Navarro (1997), (2002a) and (2002b) for a more detailed explanation. Also, a number of authors have proposed extensions to NS model that enhance flexibility, for example, Svensson (1994) and Bliss (1997b).

Figure 1.- TSIR estimated by NS^O and NS^G (01.07.1994)

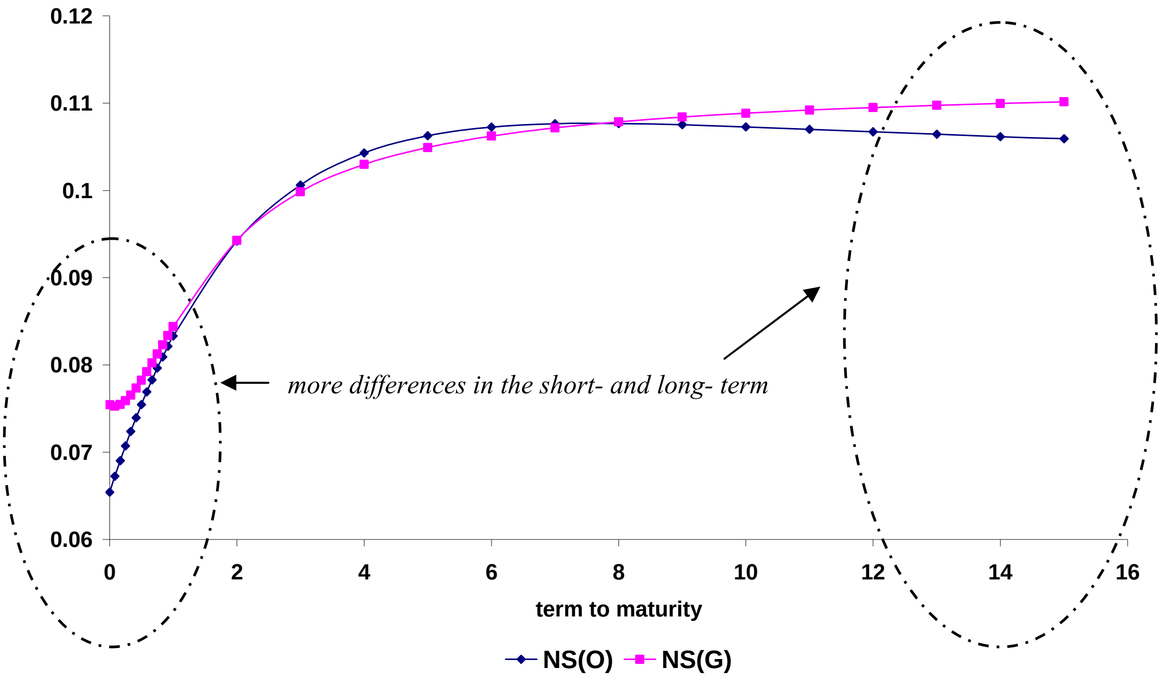
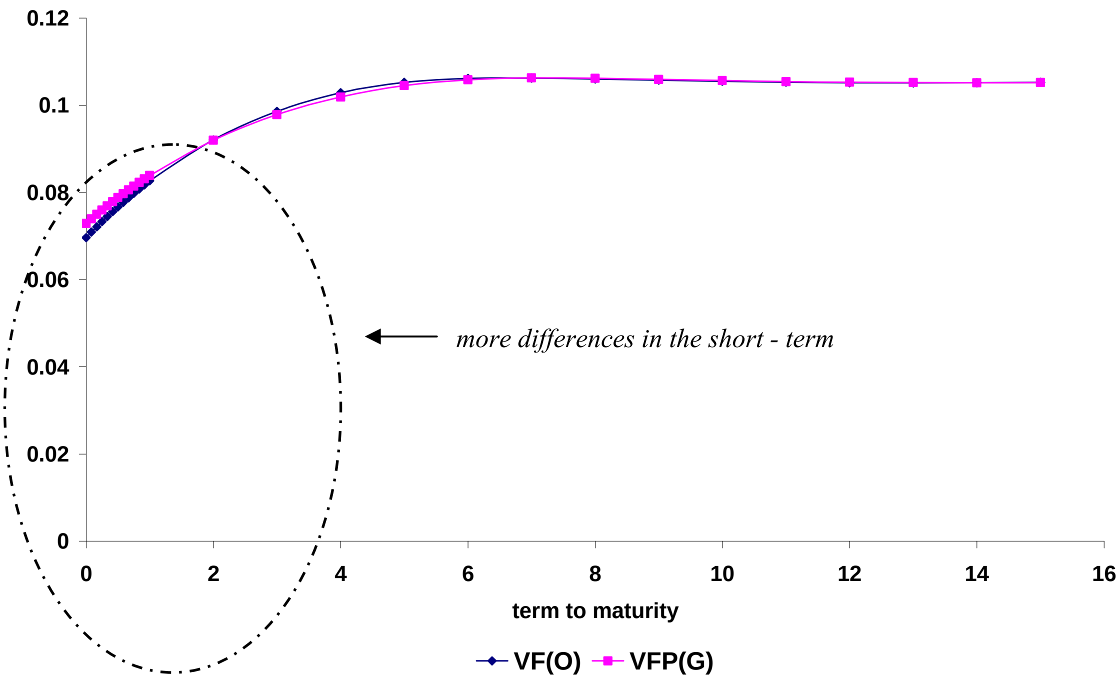


Figure 2.- TSIR estimated by VF^O and VF^G (01.07.1994)



In figure 1 we illustrate the resulting estimations of the term structure in a single day depending on the weighting scheme applied to the error terms. It can be seen how assuming OLS or GLS affects mainly to the estimates to the short and long ends of the term structure of interest rates even though in both cases we use Nelson and Siegel model. In case of using Vasicek and Fong model (figure 2), these differences are mainly shown in the short-term.

Finally, in figure 3 we show some of the differences that can be observed depending on the model employed (VF or NS) even when the same error weighting scheme is used.

In summary, we use four different estimation models: Nelson and Siegel (1987), NS^G, and Vasicek and Fong (1982), VF^G, which take into account residuals weighted by the reciprocal of maturity, and NS^O and VF^O, that is, with non weighted residuals. These alternative estimation procedures provide the input of the subsequent functional principal component analysis.

3. GARCH models

Volatility term structure is an essential issue in Finance, so it is important to have good volatility forecasts, which are based on the fact that volatility is time-varying in high-frequency data. In general, we can think that there are several reasons to model and forecast volatility. First of all, it is necessary to analyze the risk of holding an asset[11] and the value of an option which depends crucially of the volatility of the underlying asset. Finally, more efficient estimators can be obtained if heteroskedasticity in the errors is handled properly.

In order to achieve these forecasts, extensive previous literature has used autoregressive conditional heteroskedasticity (ARCH) models, as introduced by Engle (1982) and extended to generalized ARCH (GARCH) in Bollerslev (1986).[12] These models normally improve the volatility estimates, to a large extent, compared with a constant variance model and they provide good volatility forecasts, so they are widely used in various branches of econometrics, especially in financial time series analysis. In fact, it is usually assumed that interest rate volatility can be accurately described by GARCH models.

Autoregressive Conditional Heteroskedasticity (ARCH) models are specifically designed to model and forecast conditional variances. The variance of the dependent variable is modeled as a function of past values of the dependent variable and independent or exogenous variables.

[11] In fact, VaR estimates need as the main input the volatility of portfolio returns.
[12] See Bollerslev et al. (1992) for a review of the GARCH models family.

In developing a GARCH model, it is necessary to provide two distinct specifications: for the conditional mean and variance. The representation of the GARCH (p, q) specification is the following:

Conditional mean equation: $$y_t = \phi\, x_t^{'} + \varepsilon_t \qquad [1]$$

$x_t^{'}$ is a exogenous variable vector and ε_t is the error term.

Conditional Variance: $$\sigma_t^2 = \omega + \sum_{j=1}^{p} \beta_j\, \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i\, \varepsilon_{t-i}^2 \qquad [2]$$

where p is the order of the GARCH terms and q is the order of the ARCH term.

The conditional variance equation specified is a function of three terms: the mean (ω), news about volatility from the previous periods, measured as the lag of the squared residual from the mean equation (ε^2_{t-i}, the ARCH term), and last period's forecast variance (σ^2_{t-j}, the GARCH term).

ARCH-M models

If we introduce the conditional variance into the mean equation, we get the ARCH-in-Mean (ARCH-M) model (Engle et al., 1987):[13]

$$y_t = \phi\, x_t^{'} + \gamma\, \sigma_t^2 + \varepsilon_t \qquad [3]$$

The ARCH-M model is often used in financial applications where the expected return on an asset is related to the expected asset risk. The estimated coefficient on the expected risk is a measure of the risk-return tradeoff. A variant of the ARCH-M specification uses the conditional standard deviation in place of the conditional variance.

Asymmetric ARCH models

For equities, it is often observed that downward movements in the market are followed by higher volatilities than upward movements of the same magnitude. To account for this phenomenon, Engle and Ng (1993) describe a News Impact Curve with asymmetric response to good and bad news.

We can estimate two models that allow for asymmetric shocks to volatility: TGARCH and EGARCH.

[13] Note that [3] substitutes [1] in the definition of GARCH-M.

TGARCH or Threshold GARCH was introduced independently by Zakoïan (1994) and Glosten et al. (1993). The specification for the conditional variance is the following:[14]

$$\sigma_t^2 = \omega + \sum_{j=1}^{p} \beta_j \, \sigma_{t-j}^2 + \gamma \, \varepsilon_{t-1}^2 d_{t-1} + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 \qquad [4]$$

where $d_{t-1} = 1$ if $\varepsilon_{t-1} < 0$, and 0 otherwise.

In this model, good news, $\varepsilon_t > 0$, and bad news, $\varepsilon_t < 0$, have differential effects on the conditional variance. In particular, good news has an impact of α_i, while bad news has an impact of $(\alpha_i + \gamma)$. If $\gamma > 0$, we say that a "leverage effect" exists in that bad news increases volatility. If $\gamma \neq 0$, the news impact is asymmetric.

The EGARCH or Exponential GARCH model was proposed by Nelson (1991). The specification for the conditional variance is:

$$\log \sigma_t^2 = \omega + \beta \log \sigma_{t-1}^2 + \alpha \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right| + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \qquad [5]$$

Note that the left-hand side is the *log* of the conditional variance. This implies that the leverage effect is exponential, rather than quadratic, and that forecasts of the conditional variance are guaranteed to be nonnegative. The presence of leverage effects can be tested by the hypothesis that $\gamma > 0$. The impact is asymmetric if $\gamma \neq 0$.

Taking into account this variety of models, we identify the best one for each estimate of the term structure of interest rates: Nelson and Siegel (NS^O), Vasicek and Fong (VF^O) and both models weighted by duration (NS^G and VF^G), using Schwarz and Akaike Information Criterion (SIC and AIC respectively).[15] Table 1 collects the selected model for each maturity and estimation model of the term structure of interest rates (TSIR).

In particular, GARCH models fit very well when we use NS^O and VF^G. Nevertheless, T-GARCH and E-GARCH seem to be the best models for VF^O and NS^G estimations, respectively.[16]

[14] Equation [4] supplements the equation for the conditional mean.
[15] We use the minimum average error criterion.
[16] Although we could simplify our analysis using a GARCH (1, 1) model in all cases (because, in general, this is the best model), we finally apply the best particular model for each estimate.

Table 1.- The best fitted GARCH model for each maturity

Maturity (years)	NS^O	NS^G	VF^O	VF^G
0.0001	GARCH (2,1)	GARCH (2,4)-M	GARCH (2,2)	GARCH (1,1)
0.08333333	T-GARCH (1,1)	GARCH (1,1)	E-GARCH (1,1)-M	GARCH (1,1)
0.16666667	E-GARCH (3,2)	E-GARCH (2,1)	E-GARCH (1,1)-M	GARCH (1,1)
0.25	T-GARCH (1,1)	E-GARCH (2,1)	T-GARCH (2,1)-M	GARCH (1,1)
0.33333333	T-GARCH (1,1)	E-GARCH (2,1)	T-GARCH (2,1)	GARCH (2,1)
0.41666667	T-GARCH (1,1)	GARCH (2,3)	T-GARCH (2,1)-M	T-GARCH (2,1)
0.5	GARCH (1,1)	E-GARCH (2,1)	T-GARCH (2,1)	T-GARCH (2,2)
0.58333333	GARCH (2,1)	E-GARCH (2,1)	T-GARCH (2,1)	T-GARCH (2,2)
0.66666667	GARCH (2,1)	E-GARCH (2,1)	T-GARCH (2,1)	T-GARCH (2,2)-M
0.75	GARCH (2,2)	E-GARCH (2,1)	T-GARCH (2,2)	T-GARCH (2,2)
0.83333333	GARCH (2,2)	E-GARCH (2,1)	T-GARCH (2,2)	T-GARCH (2,2)
0.91666667	T-GARCH (2,2)-M	E-GARCH (2,1)	T-GARCH (2,2)	GARCH (2,2)
1	GARCH (2,2)-M	E-GARCH (2,1)	T-GARCH (2,2)	T-GARCH (2,2)
2	GARCH (2,2)-M	E-GARCH (2,2)	GARCH (2,2)	GARCH (2,2)-M
3	GARCH (1,1)-M	GARCH (2,2)	GARCH (2,1)	GARCH (2,2)
4	GARCH (1,1)	GARCH (2,1)	GARCH (2,2)	GARCH (2,2)
5	E-GARCH (2,2)-M	GARCH (2,2)-M	GARCH (2,2)	GARCH (2,2)-M
6	E-GARCH (2,2)-M	GARCH (2,2)-M	GARCH (2,2)	E-GARCH (2,2)
7	GARCH (1,1)	GARCH (2,2)-M	GARCH (2,2)	E-GARCH (2,2)
8	GARCH (1,1)	GARCH (4,3)	GARCH (2,2)-M	GARCH (2,2)
9	GARCH (2,2)	GARCH (2,2)	GARCH (1,1)	E-GARCH (2,1)
10	GARCH (2,2)-M	GARCH (2,3)	GARCH (2,2)	E-GARCH (2,2)
11	E-GARCH (2,2)-M	E-GARCH (2,3)	GARCH (2,1)	GARCH (2,2)
12	E-GARCH (2,2)-M	E-GARCH (1,1)	GARCH (2,1)-M	GARCH (2,2)-M
13	GARCH (2,2)	E-GARCH (1,1)	E-GARCH (2,1)-M	GARCH (2,2)-M
14	GARCH (2,1)	E-GARCH (1,1)	GARCH (1,2)-M	GARCH (1,1)
15	GARCH (2,1)	E-GARCH (1,1)	E-GARCH (2,1)	GARCH (2,2)

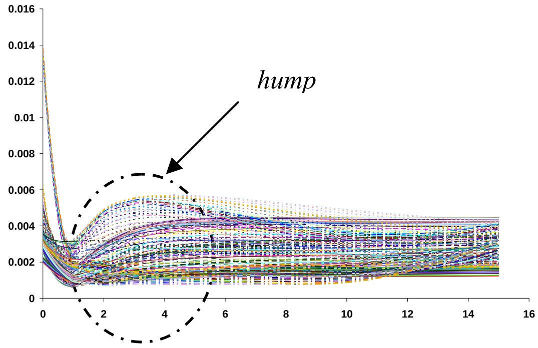
4. Differences in the volatility from different models

In this section we study the differences between the volatility term structure from different estimation models of the TSIR (NS^O, VF^O, NS^G and VF^G) and conditional volatility models (GARCH models in each previous case). In the first type of models, we obtain the historical volatility using 30-, 60-, and 90- day moving windows and the standard deviation measure. We show the results with a 30-days moving window.

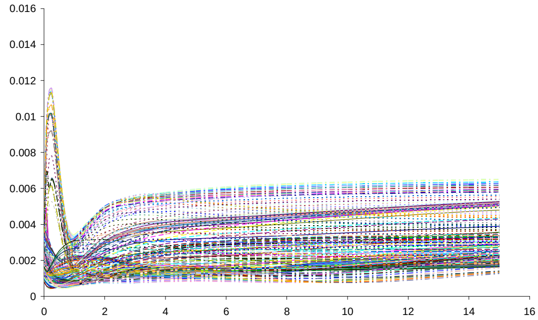
As a whole we can see a repeating pattern in the shape of the VTS: initially decreasing, then it increases until one to two years term and finally we can observe a constant or slightly decreasing interest rate volatility as we approach the long term of the curve. This is consistent with Campbell et al. (1997), who argue that the hump of the VTS in the middle-run can be explained by reduced forecast ability of interest rate movements at horizons around one year. They argue that there is some short-run forecastability arising from Federal Reserve operating procedures, and also some long-run forecastability from business-cycle effects on interest rates.

Figure 3.- Volatility Term Structure (VTS) among different models (1994)

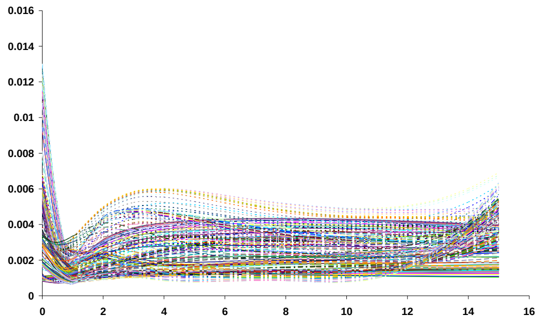
NS^O + Historical Volatility



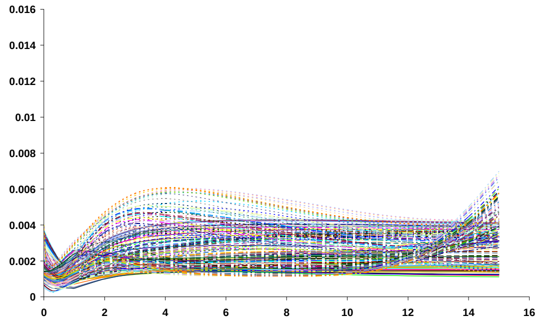
NS^G + Historical Volatility



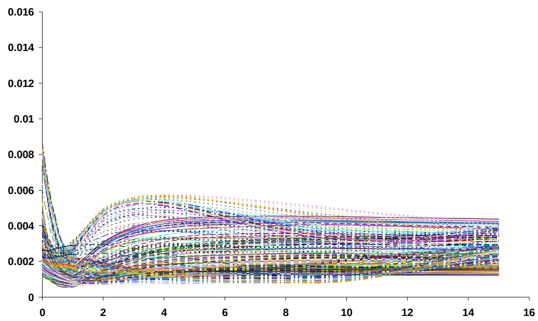
VF^O + Historical Volatility



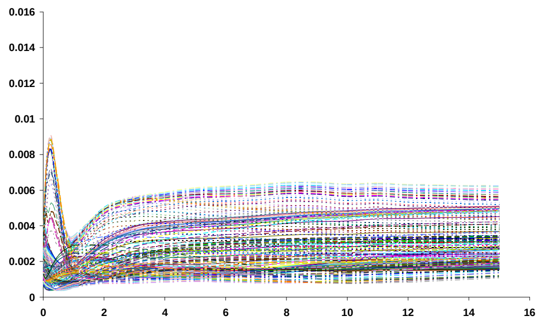
VF^G + Historical Volatility



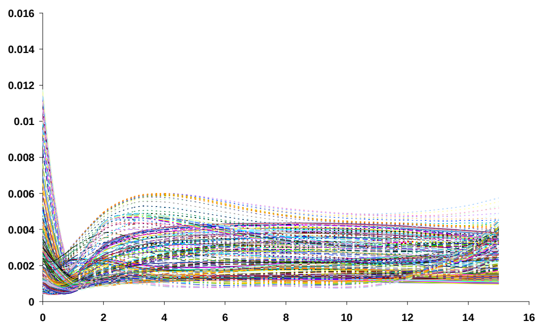
NS^O + Conditional Volatility



NS^G + Conditional Volatility



VF^O + Conditional Volatility



VF^G + Conditional Volatility

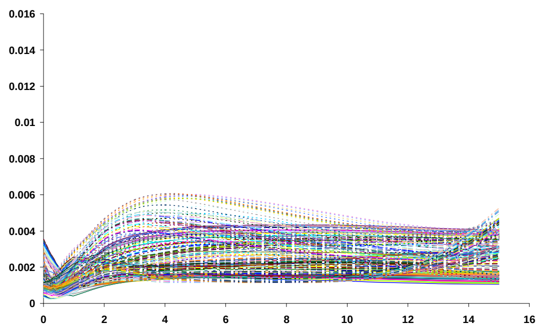
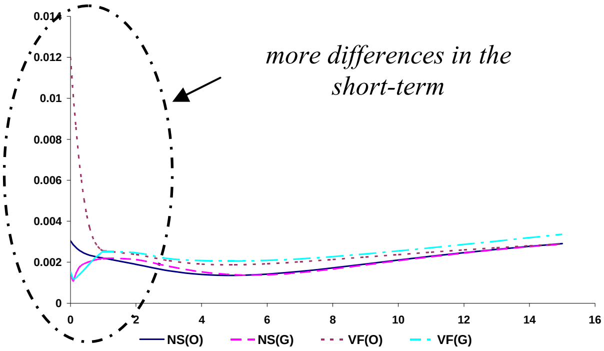


Figure 3 describes volatility estimates for 1994 for the different models used to estimate the interest rate term structure. It shows, at first glance, how the methodology employed to estimate zero coupon bonds may have an important impact, both in level and shape, on the subsequent estimate of the VTS.

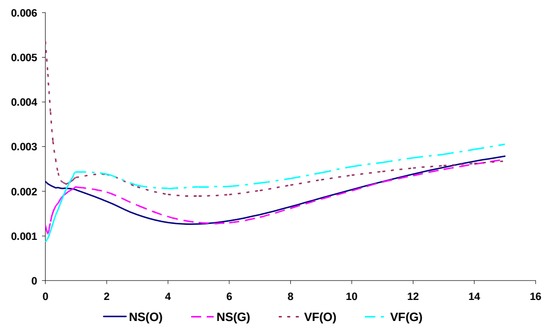
This can be more clearly seen in figure 4, where we show the VTS for our 8 cases in some particular days:

Figure 4.- Volatility Term Structure (VTS) among different models

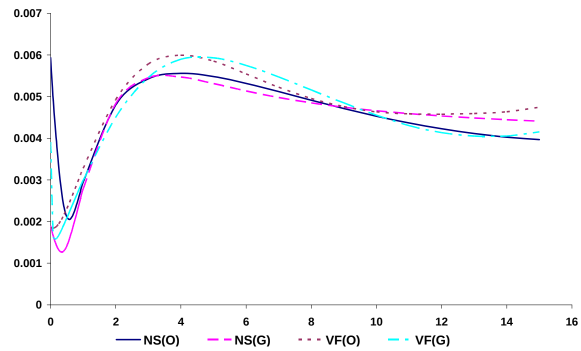
Historical Volatility (03.01.94)



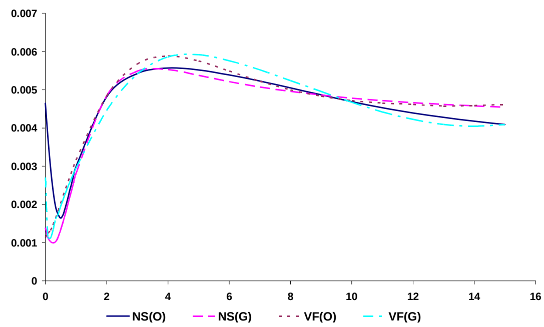
Conditional Volatility (03.01.94)



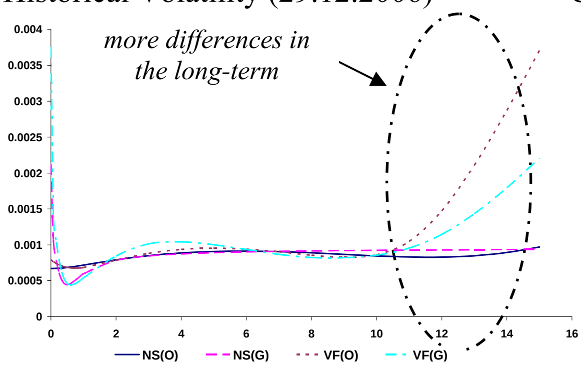
Historical Volatility (01.07.94)



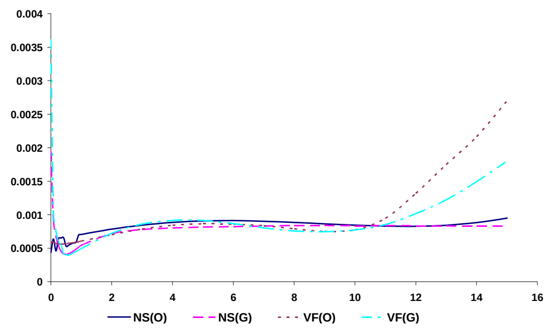
Conditional Volatility (01.07.94)



Historical Volatility (29.12.2006)



Conditional Volatility (29.12.2006)

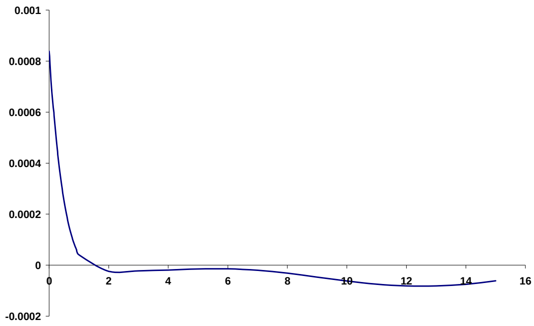


In order to improve our analysis, we proceed to measure the average differences between volatility estimates using two alternative and different methods. We can see that these differences seem to be higher in the short-term (less than one year) and in the long-term (more than ten years). We can corroborate these results in figure 5.

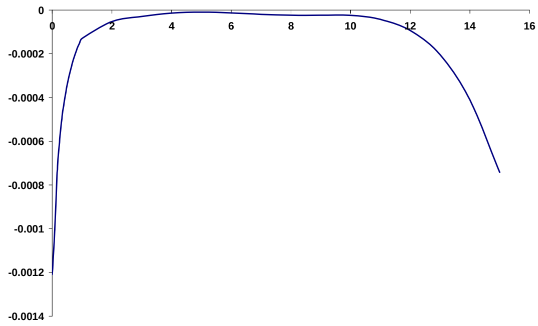
Figure 5.- Average differences in VTS between each model

Panel A: Historical Volatility

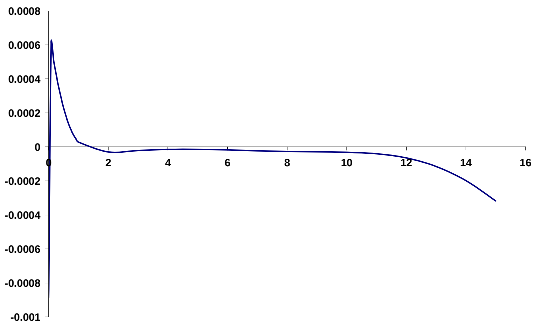
NS^{O}-NS^{G}



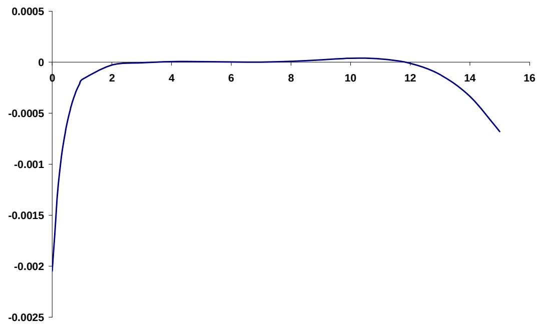
NS^{O}-VF^{O}



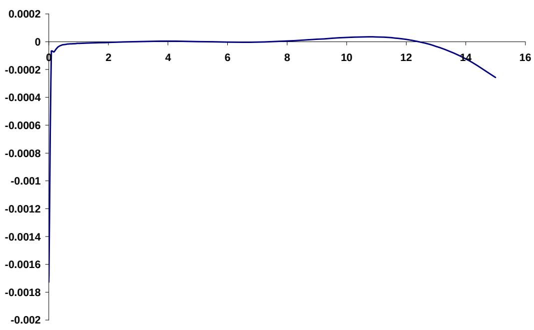
NS^{O}-VF^{G}



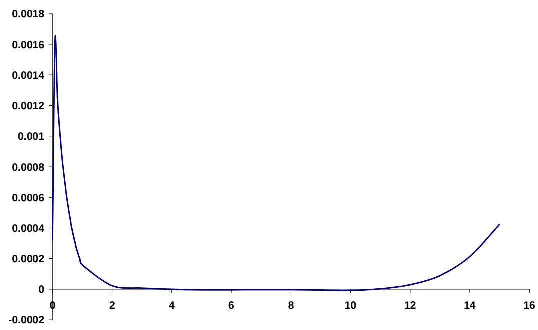
NS^{G}-VF^{O}



NS^{G}-VF^{G}

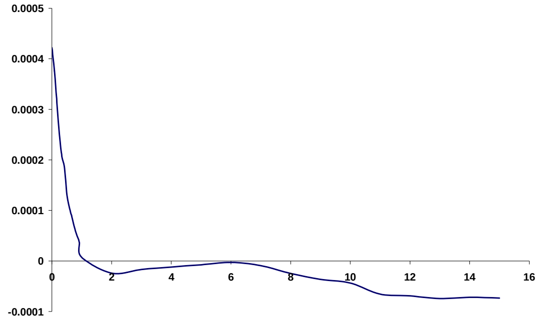


VF^{O}-VF^{G}

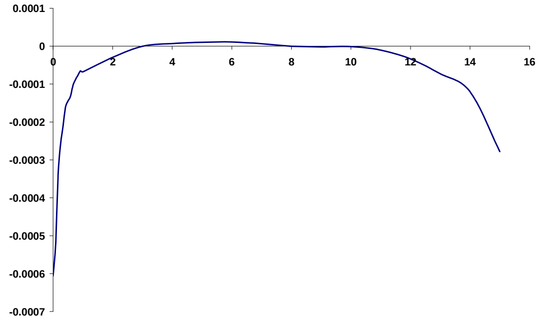


Panel B: Conditional Volatility

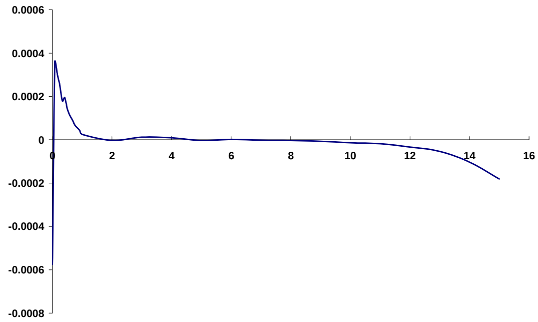
GNS^O-GNS^G



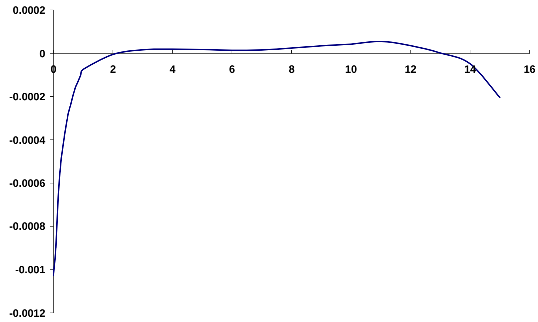
GNS^O-GVF^O



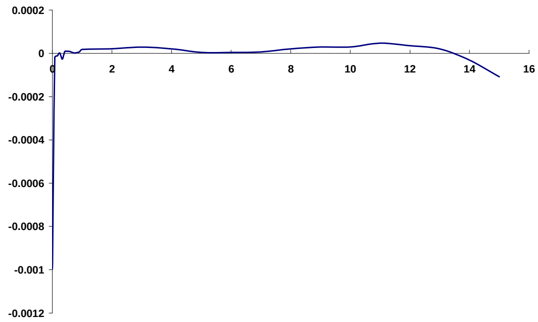
GNS^O-GVF^G



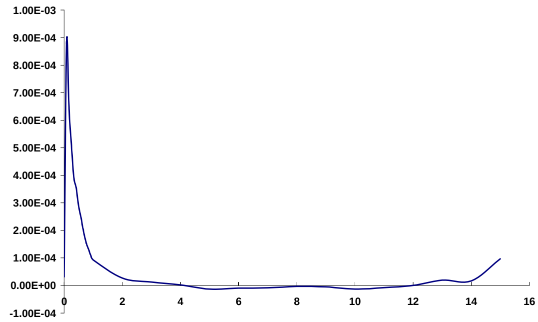
GNS^G-GVF^O



GNS^G-GVF^G



GVF^O-GVF^G



G-before the name of the model indicates that we have used a GARCH model

Finally, we use some statistics to test whether volatility series have the same mean, median and variance (table 2). In order to do this analysis, we obtain Anova-F Test for the mean analysis, Kruskal-Wallis and van der Waerden Test for the median analysis and, finally, Levene and Brown-Forsythe Test for analysing the significance of the VTS variance.

Table 2.- Tests of equality of means, medians and variances among different models for each maturity

	Maturity (years)								
TEST	**0.0001**	**0.0833333**	**0.1666667**	**0.25**	**0.3333333**	**0.4166667**	**0.5**	**0.5833333**	**0.6666667**
F	469.2349^{c}	650.5977^{c}	591.6347^{c}	477.8088^{c}	383.9666^{c}	323.2406^{c}	254.7131^{c}	196.0654^{c}	141.1546^{c}
K-W	3710.024^{c}	4902.543^{c}	4724.034^{c}	4349.893^{c}	3754.263^{c}	3285.614^{c}	2636.512^{c}	2083.659^{c}	1565.177^{c}
vW	4031.904^{c}	5261.365^{c}	4937.412^{c}	4454.727^{c}	3818.899^{c}	3296.418^{c}	2607.419^{c}	2038.510^{c}	1521.237^{c}
L	650.0991^{c}	428.2810^{c}	296.2978^{c}	194.7067^{c}	140.8215^{c}	108.8034^{c}	80.67102^{c}	56.18532^{c}	35.35199^{c}
B-F	345.5845^{c}	306.6132^{c}	221.2285^{c}	145.3684^{c}	104.0462^{c}	80.13080^{c}	58.94565^{c}	40.60811^{c}	25.05816^{c}

	Maturity (years)								
TEST	**0.75**	**0.8333333**	**0.9166667**	**1**	**2**	**3**	**4**	**5**	**6**
F	97.67177^{c}	65.88030^{c}	43.22459^{c}	27.64625^{c}	1.618184	0.653847	0.393028	0.305357	0.211659
K-W	1151.682^{c}	842.5733^{c}	608.9702^{c}	433.3387^{c}	19.05171^{c}	7.505526	4.937434	3.589879	2.603751
vW	1100.201^{c}	787.4118^{c}	554.6464^{c}	379.1995^{c}	20.32857^{c}	8.914100	6.539913	4.463184	2.947742
L	20.38274^{c}	11.18151^{c}	6.302970^{c}	4.522192^{c}	0.204407	0.106095	0.072056	0.259973	0.554394
B-F	14.20114^{c}	7.421875^{c}	3.705566^{c}	2.158965^{b}	0.043454	0.092483	0.067191	0.217367	0.379985

	Maturity (years)								
TEST	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
F	0.346454	0.779177	1.420462	2.175614^{b}	3.657369^{c}	5.938809^{c}	19.66521^{c}	67.75681^{c}	175.7461^{c}
K-W	2.921245	3.336836	4.567510	6.862232	15.29165^{b}	44.18141^{c}	167.8568^{c}	481.3393^{c}	1141.098^{c}
vW	3.426414	5.068104	8.060296	11.93060	25.36176^{c}	55.15105^{c}	183.2679^{c}	508.3304^{c}	1170.865^{c}
L	1.217599	2.339045^{b}	3.528918^{c}	4.682544^{c}	6.565824^{c}	6.543890^{c}	15.40305^{c}	61.29457^{c}	165.7889^{c}
B-F	0.688643	1.243352	1.874787^{a}	2.481528^{b}	3.008177^{c}	3.134396^{c}	7.535485^{c}	32.52243^{c}	91.89411^{c}

^{a} $p < 0.10$, ^{b} $p < 0.05$, ^{c} $p < 0.01$

F: Anova-F Test, K-W: Kruskal-Wallis Test, vW: van der Waerden Test, L: Levene Test, B-F: Brown-Forsythe Test

On the one hand, statistics offer evidence against the null hypothesis of homogeneity for the shorter maturities (below to 1 year) and also for the longer maturities (more than 10 years), in mean and median.

On the other hand, statistics to test for whether the volatility produced by the eight models has the same variance show the same results than mean and median analysis, that is, we find evidence against the null hypothesis for the shorter and longer maturities.

To summarize, this analysis shows that volatility estimates using different models and techniques display statistically significant differences, mainly in the shorter and longer maturities, as it would be expected.

5. A principal component analysis of volatility term structure (VTS)

In this section, we try to reduce the dimensionality of the vector of 27 time series of historical/conditional volatilities,[17] working out their principal components, because this analysis is often used to identify the key uncorrelated sources of information.

[17] Note that we analyze volatility changes (see, for example, Benito and Novales, 2007).

This technique decomposes the sample covariance matrix or the correlation matrix computed for the series in the group (see tables 3 and A1). The row labeled "eigenvalue" in table 3 reports the eigenvalues of the sample second moment matrix in descending order from left to right. We also show the variance proportion explained by each principal component. This value is simply the ratio of each eigenvalue to the sum of all eigenvalues. Finally, we collect the cumulative sum of the variance proportion from left to right, that is, the variance proportion explained by principal components up to that order.

Also, table A1 displays the eigenvectors corresponding to each eigenvalue. The first principal component is computed as a linear combination of the series in the group with weights given by the first eigenvector. The second principal component is the linear combination with weights given by the second eigenvector and so on.

Table 3.- Main results of the principal component analysis

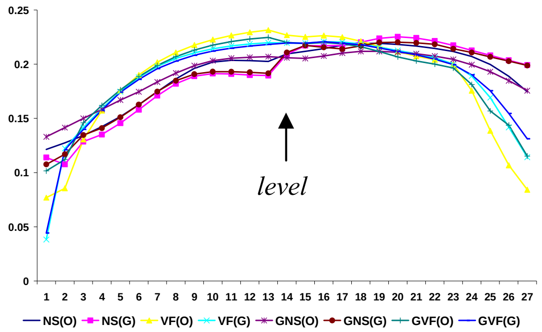
	NS^O	**NS^G**	**VF^O**	**VF^G**	**GNS^O**	**GNS^G**	**GVF^O**	**GVF^G**
	Historical Volatility	Historical Volatility	Historical Volatility	Historical Volatility	Conditional Volatility	Conditional Volatility	Conditional Volatility	Conditional Volatility
				FIRST PRINCIPAL COMPONENT				
Eigenvalue	14.47963	14.42727	12.50790	14.95705	15.11595	14.52248	13.30651	15.23433
Var. prop.	0.536283	0.534343	0.463255	0.553965	0.559850	0.537870	0.492834	0.564234
Cum. prop.	0.536283	0.534343	0.463255	0.553965	0.559850	0.537870	0.492834	**0.564234**
				SECOND PRINCIPAL COMPONENT				
Eigenvalue	8.191949	7.305261	7.484512	6.623200	7.767501	7.520611	7.930251	6.769219
Var. prop.	0.303406	0.270565	0.277204	0.245304	0.287685	0.278541	0.293713	0.250712
Cum. prop.	0.839688	0.804909	0.740460	0.799268	**0.847535**	0.816411	0.786547	0.814946
				THIRD PRINCIPAL COMPONENT				
Eigenvalue	2.440719	2.549777	2.997161	2.321565	2.149763	2.366136	2.400861	2.120942
Var. prop.	0.090397	0.094436	0.111006	0.085984	0.079621	0.087635	0.088921	0.078553
Cum. prop.	**0.930085**	0.899345	0.851466	0.885252	0.927156	0.904045	0.875467	0.893499
				FOURTH PRINCIPAL COMPONENT				
Eigenvalue	1.216678	1.318653	2.161253	1.388741	1.234067	1.270866	1.866241	1.237788
Var. prop.	0.045062	0.048839	0.080046	0.051435	0.045706	0.047069	0.069120	0.045844
Cum. prop.	**0.975147**	0.948184	0.931512	0.936687	0.972862	0.951114	0.944588	0.939343
				FIFTH PRINCIPAL COMPONENT				
Eigenvalue	0.500027	0.711464	0.755749	0.812430	0.473576	0.690566	0.677145	0.788932
Var. prop.	0.018520	0.026351	0.027991	0.030090	0.017540	0.025577	0.025079	0.029220
Cum. prop.	**0.993667**	0.974534	0.959503	0.966777	0.990402	0.976691	0.969667	0.968563

G-before the name of the model indicates that we have used a GARCH model

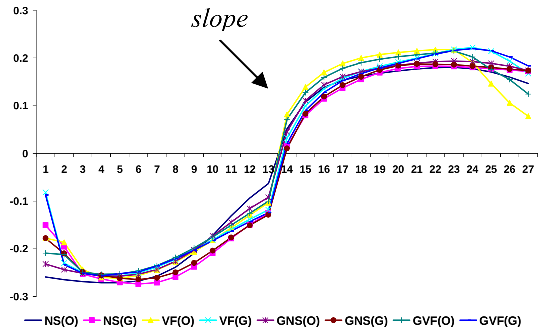
We can emphasize the best values about the percentage of cumulative explained variance for each principal component: 56 % in case of GVF^G (first principal component), 84 % in case of GNS^O (second principal component) and 93 % (third principal component), 97 % (fourth principal component) and 99 % (fifth principal component) in case of NS^O. Thus, the first five factors capture, at least, 97 % of the variation in the volatility time series.

Figure 6.- Representation of the main first five principal components (PC)

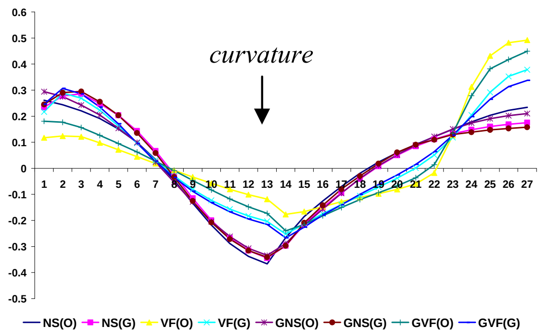
PC1



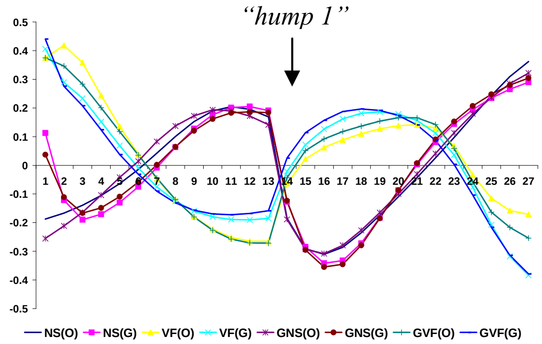
PC2



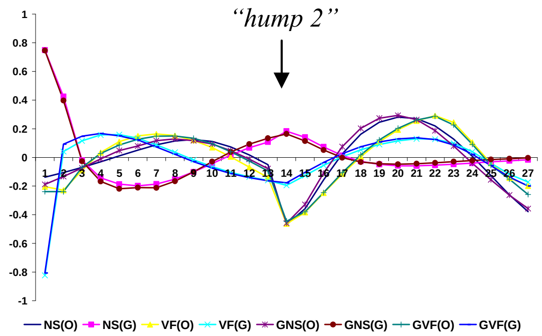
PC3



PC4



PC5



The eigenvectors in each case are shown in appendix, in Table A1. On the other hand, figure 6 plots the evolution of these five factors.

In this section, we can assert that the first three principal components (PC) are quite similar among different models. Particularly, the first PC keeps quasi constant over the whole volatility term structure (VTS) and the eight models. So we can interpret it as the general level of the volatility (level or trend). With respect to the second PC, it presents coefficients of opposite sign in the short-term and coefficients of the same sign in the large-term, so this component can be interpreted as the difference between the levels of volatility between the two ends of volatility the term structure (slope or tilt). Finally, the third PC shows changing signs of the coefficients, so this PC could be interpreted as changes in the curvature of the volatility term structure (curvature).

With regard to the fourth and fifth PC, they present some differences among each model; nevertheless, these PCs can be related with higher or lower hump of the volatility term structure (VTS).

The three first factor loadings are similar to those obtained by Litterman and Scheinkman (1991) and Piazzesi (2005), Matzner-Løber and Villa (2004), and Cornillon et al. (2008), who estimated loadings via a standard principal component analysis and variations about this analysis. Nevertheless, in this paper we focus on the second moment of interest rates. An important insight is that the three factors may be interpreted in terms of level, slope and curvature.

In order to finish this analysis, we want to test whether the first three principal components, which clearly reflect level, slope and curvature of the VTS, and the last two PCs are different among our eight models (historical and conditional volatilities).

Table 4.- Tests of equality of means, medians and variances among different models

TEST	PC1	PC2	PC3	PC4	PC5
F	0.012749	0.056012	0.020015	0.179951	0.024021
K-W	1.016249	2.452214	3.810190	11.82140	55.13159 ^{c}
vW	0.518985	0.795634	2.032438	8.070648	45.21040 ^{c}
L	4.033919 ^{c}	23.92485 ^{c}	16.57419 ^{c}	66.74642 ^{c}	67.33491 ^{c}
B-F	4.064720 ^{c}	23.87826 ^{c}	16.51119 ^{c}	65.80991 ^{c}	67.06584 ^{c}

^{a} $p < 0.10$, ^{b} $p < 0.05$, ^{c} $p < 0.01$

F: Anova-F Test, K-W: Kruskal-Wallis Test, vW: van der Waerden Test, L: Levene Test, B-F: Brown-Forsythe Test

Considering the results from table 4, we can assert that statistics related with differences in mean evidence homogeneity in mean for our eight models as we can not reject the null hypothesis. In case of differences in median, we find evidence against the null hypothesis of equal medians for the fifth PC. Nevertheless, the rest of PCs offer evidence in favour the null hypothesis.

On the other hand, statistics to test for whether the PC variance produced by our eight models is the same or not also appears in table 4. For all the PCs, these statistics offer strong evidence against the null hypothesis.

Summarizing, in this section we have concluded that the first three principal components can be related with level, slope and curvature of the volatility term structure (VTS) and, besides, these PCs are not significantly different in mean and median among our eight models. Nevertheless, PC4 and PC5 are significantly different between our models.

6. Summary and conclusions

This paper is aimed to provide new insights into the behaviour of the volatility term structure (VTS) of interest rates by using historical volatility estimates from four different models of the term structure of interest rate (TSIR) and applying alternative conditional volatility specifications (using GARCH models) from 1994 to 2006. We have used the mentioned models, and we have worked out the volatility time series using 30-, 60-, 90-day moving windows in order to construct the volatility term structure (VTS).

First of all, the results of our analysis show that there are statistically significant differences between estimates of the term structure of interest rate volatilities depending on the model used to estimate the term structure and the heteroskedasticity structure of errors (NS^O, NS^G, VF^O and VF^G), mainly in the short-term (less than one year) and in the large-term (more than ten years), but these differences do not depend on procedures to estimate the volatility term structure.

Secondly, the previous evidence suggests that the dynamics of term structures of volatilities can be well described by relatively few common components. The possible interpretation of these principal components in terms of level, slope, and curvature can describe how the VTS shifts or changes shape in response to a shock on a principal component.

We find that the first three principal components (PC) are quite similar among different models and they can be identified as trend, tilt and curvature. Regarding fourth and fifth PC, they can be related with higher or lower hump of the volatility term structure. Also, the first three PCs are not significantly different in mean and median among our eight models. Nevertheless, PC4 and PC5 are significantly different between our models.

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Appendix

Table A1.- Representation of the principal components (PC): factor weights

Panel A: Historical Volatility

	NS^{O}	NS^{O}	NS^{O}	NS^{O}	NS^{O}	NS^{G}	NS^{G}	NS^{G}	NS^{G}	NS^{G}
	PC1	**PC2**	**PC3**	**PC4**	**PC5**	**PC1**	**PC2**	**PC3**	**PC4**	**PC5**
1	0.121397	-0.259249	0.261221	-0.187814	-0.138194	0.113942	-0.150523	0.234249	0.112743	0.748817
2	0.127484	-0.264761	0.244047	-0.166698	-0.105761	0.107654	-0.195499	0.277798	-0.121744	0.425546
3	0.134342	-0.268752	0.221091	-0.139226	-0.06958	0.128617	-0.253108	0.285545	-0.189594	-0.020031
4	0.14235	-0.271143	0.191057	-0.10512	-0.030515	0.135174	-0.263191	0.249016	-0.170776	-0.142228
5	0.151754	-0.271231	0.151607	-0.063686	0.010448	0.145574	-0.2706	0.203691	-0.13	-0.186363
6	0.162626	-0.267524	0.099883	-0.014271	0.051094	0.158059	-0.273952	0.14348	-0.074455	-0.19805
7	0.174608	-0.257603	0.033866	0.041884	0.08769	0.171016	-0.271009	0.067186	-0.007826	-0.185778
8	0.1865	-0.238721	-0.045664	0.100143	0.114505	0.18204	-0.259145	-0.021882	0.063126	-0.151983
9	0.196263	-0.209476	-0.133124	0.152586	0.124226	0.188972	-0.237608	-0.114734	0.12798	-0.101339
10	0.202024	-0.171744	-0.217996	0.189674	0.111063	0.191415	-0.209018	-0.199647	0.176215	-0.042221
11	0.203716	-0.131086	-0.288516	0.204216	0.073944	0.191012	-0.178223	-0.267598	0.201797	0.016358
12	0.203258	-0.093839	-0.337898	0.195536	0.017164	0.18997	-0.149475	-0.315279	0.205022	0.067578
13	0.202512	-0.063243	-0.366708	0.169053	-0.051113	0.18959	-0.124765	-0.34446	0.191075	0.107537
14	0.209538	0.051382	-0.265868	-0.184753	-0.468382	0.209218	0.014711	-0.29688	-0.12729	0.183392
15	0.211982	0.107458	-0.18496	-0.291188	-0.358059	0.217149	0.079669	-0.215525	-0.284608	0.140762
16	0.214489	0.13848	-0.125447	-0.310665	-0.156896	0.217154	0.114496	-0.150177	-0.341714	0.07367
17	0.217481	0.153438	-0.069554	-0.285884	0.025451	0.217344	0.137134	-0.093528	-0.332023	0.013242
18	0.219179	0.161991	-0.01922	-0.235883	0.161091	0.220351	0.155325	-0.040737	-0.272246	-0.027212
19	0.219275	0.168071	0.023808	-0.174282	0.245437	0.223836	0.169293	0.008056	-0.183311	-0.048827
20	0.218288	0.172948	0.06009	-0.109023	0.281353	0.225306	0.178064	0.05028	-0.086814	-0.057984
21	0.216693	0.176908	0.091702	-0.042633	0.271988	0.224296	0.182274	0.084808	0.003305	-0.05895
22	0.21461	0.179656	0.12124	0.025814	0.219512	0.221393	0.183263	0.111971	0.080669	-0.054619
23	0.211719	0.180288	0.150256	0.097474	0.128181	0.217364	0.182283	0.132789	0.143969	-0.047406
24	0.207176	0.177554	0.178378	0.171299	0.007766	0.212829	0.180209	0.1485	0.194505	-0.039082
25	0.199827	0.170696	0.203544	0.243769	-0.127305	0.208191	0.17759	0.160269	0.234374	-0.030805
26	0.188869	0.159963	0.222817	0.309274	-0.260831	0.203672	0.17475	0.169057	0.265636	-0.023234
27	0.174687	0.146502	0.234292	0.362238	-0.378401	0.199377	0.17186	0.175594	0.29005	-0.016643

	VF^{O}	VF^{O}	VF^{O}	VF^{O}	VF^{O}	VF^{G}	VF^{G}	VF^{G}	VF^{G}	VF^{G}
	PC1	**PC2**	**PC3**	**PC4**	**PC5**	**PC1**	**PC2**	**PC3**	**PC4**	**PC5**
1	0.076862	-0.177532	0.117497	0.374599	-0.204143	0.038251	-0.081649	0.216733	0.406232	-0.823221
2	0.085535	-0.186532	0.123966	0.418413	-0.228741	0.122404	-0.230248	0.288881	0.289442	0.041293
3	0.130671	-0.243434	0.121608	0.358295	-0.090576	0.141644	-0.249789	0.269506	0.233799	0.115692
4	0.157309	-0.259957	0.09794	0.244077	0.033962	0.159101	-0.256702	0.223374	0.152348	0.157072
5	0.175653	-0.261789	0.071493	0.135912	0.110097	0.17479	-0.256923	0.16297	0.068733	0.159254
6	0.190172	-0.256015	0.044827	0.038637	0.150682	0.1879	-0.25054	0.096515	-0.009205	0.1332
7	0.201836	-0.244209	0.01828	-0.047433	0.163435	0.197998	-0.238361	0.030899	-0.074983	0.08877
8	0.210893	-0.227272	-0.007827	-0.120803	0.152187	0.205364	-0.221778	-0.029062	-0.12507	0.035991
9	0.2177	-0.206247	-0.03316	-0.179737	0.119632	0.210612	-0.202188	-0.08063	-0.159253	-0.016904
10	0.222779	-0.18222	-0.057445	-0.223333	0.068805	0.214316	-0.180994	-0.122914	-0.179547	-0.064625
11	0.226621	-0.156294	-0.08022	-0.251472	0.004179	0.216907	-0.159417	-0.156555	-0.188949	-0.10457
12	0.229508	-0.129575	-0.10072	-0.26485	-0.067699	0.218705	-0.13832	-0.182913	-0.190191	-0.136104
13	0.231568	-0.103121	-0.118238	-0.265345	-0.139638	0.219941	-0.118238	-0.203349	-0.185361	-0.159987
14	0.226889	0.0817	-0.176722	-0.063281	-0.463132	0.21963	0.032169	-0.252261	-0.021641	-0.194524
15	0.225285	0.138928	-0.165744	0.022565	-0.383735	0.219405	0.099493	-0.214108	0.072085	-0.126334
16	0.226433	0.169705	-0.146884	0.062347	-0.248556	0.220774	0.135399	-0.174371	0.126337	-0.054982
17	0.225001	0.188583	-0.128132	0.088624	-0.114954	0.220526	0.157267	-0.138932	0.161967	0.005677
18	0.221253	0.200106	-0.111826	0.109826	0.005567	0.218648	0.171787	-0.105313	0.181576	0.053883
19	0.216394	0.20708	-0.097461	0.127279	0.109489	0.215806	0.18241	-0.072063	0.186732	0.09038
20	0.211643	0.211652	-0.080596	0.138557	0.192828	0.212728	0.191325	-0.037555	0.178782	0.116325
21	0.207496	0.214836	-0.05978	0.141689	0.255587	0.209661	0.199957	0.001449	0.156203	0.130149
22	0.20419	0.217557	-0.017421	0.129831	0.291293	0.205986	0.209188	0.05096	0.112386	0.126942
23	0.200108	0.217602	0.12654	0.067375	0.24487	0.200173	0.217965	0.118618	0.036338	0.098652
24	0.175586	0.190631	0.312657	-0.033949	0.105121	0.189154	0.221884	0.20488	-0.07672	0.039701
25	0.138686	0.146155	0.431757	-0.115057	-0.041807	0.169184	0.214129	0.291991	-0.207781	-0.040542
26	0.106733	0.106027	0.482087	-0.157968	-0.143171	0.14176	0.193356	0.352725	-0.31766	-0.117252
27	0.084227	0.077779	0.492251	-0.171303	-0.198566	0.114293	0.167248	0.37848	-0.383985	-0.171863

Panel B: Conditional Volatility

	GNS^{O} PC1	GNS^{O} PC2	GNS^{O} PC3	GNS^{O} PC4	GNS^{O} PC5	GNS^{G} PC1	GNS^{G} PC2	GNS^{G} PC3	GNS^{G} PC4	GNS^{G} PC5
1	0.133095	-0.231878	0.294572	-0.255244	-0.18813	0.107534	-0.177863	0.245492	0.03719	0.74622
2	0.141549	-0.243706	0.275368	-0.211485	-0.133614	0.116937	-0.209815	0.290919	-0.110932	0.397662
3	0.150027	-0.251323	0.243096	-0.164082	-0.073496	0.134689	-0.248504	0.294975	-0.166686	-0.026263
4	0.158364	-0.256879	0.205054	-0.104133	-0.010013	0.141089	-0.255525	0.255592	-0.148616	-0.167542
5	0.166975	-0.257253	0.153892	-0.041977	0.047895	0.151122	-0.261673	0.20376	-0.109323	-0.218733
6	0.17463	-0.25363	0.099534	0.01431	0.079359	0.162752	-0.264339	0.13547	-0.058507	-0.210391
7	0.183669	-0.24352	0.024016	0.083275	0.116732	0.174767	-0.261003	0.058584	0.001428	-0.211714
8	0.191784	-0.226234	-0.052882	0.137171	0.129434	0.184777	-0.249605	-0.033051	0.064435	-0.165529
9	0.198723	-0.202704	-0.128489	0.172435	0.117696	0.190912	-0.229677	-0.125135	0.120706	-0.101016
10	0.203252	-0.173113	-0.204432	0.193611	0.093336	0.193153	-0.203855	-0.207331	0.161693	-0.028981
11	0.205647	-0.144536	-0.261568	0.189775	0.047522	0.193119	-0.176393	-0.271556	0.183186	0.038201
12	0.206457	-0.115741	-0.307124	0.172328	-0.012008	0.192589	-0.150836	-0.315736	0.186365	0.092953
13	0.207053	-0.092125	-0.33418	0.141437	-0.078106	0.191652	-0.128569	-0.341219	0.183924	0.134229
14	0.206073	0.045451	-0.288687	-0.188633	-0.459106	0.210923	0.010573	-0.296954	-0.123588	0.164243
15	0.205358	0.110646	-0.21798	-0.292905	-0.328072	0.2174	0.083055	-0.208965	-0.295069	0.114224
16	0.207606	0.143926	-0.156935	-0.308258	-0.10939	0.215569	0.119036	-0.14189	-0.354966	0.05152
17	0.210335	0.161034	-0.095679	-0.279159	0.075436	0.214018	0.143037	-0.079571	-0.345324	-0.001795
18	0.211853	0.171408	-0.038611	-0.226859	0.202649	0.216918	0.160555	-0.029458	-0.279005	-0.031746
19	0.211922	0.178827	0.010845	-0.164907	0.273692	0.220048	0.174787	0.017958	-0.184801	-0.044345
20	0.211035	0.184673	0.052552	-0.099244	0.293368	0.220326	0.18415	0.061315	-0.086444	-0.047197
21	0.209625	0.189241	0.088892	-0.03054	0.263143	0.219748	0.187455	0.09094	0.007478	-0.043418
22	0.207565	0.192409	0.121922	0.041142	0.187213	0.218371	0.186869	0.109787	0.090043	-0.037217
23	0.204425	0.193585	0.150915	0.112992	0.079765	0.214199	0.186097	0.129574	0.153123	-0.028872
24	0.199499	0.192473	0.174269	0.17982	-0.040405	0.210911	0.182948	0.138974	0.20695	-0.021828
25	0.193051	0.188733	0.190811	0.237111	-0.155585	0.206815	0.180022	0.147692	0.247767	-0.014737
26	0.184799	0.18257	0.202417	0.284937	-0.262786	0.202819	0.176661	0.153113	0.279993	-0.008801
27	0.175618	0.173619	0.210408	0.32207	-0.357152	0.198849	0.173671	0.157843	0.304631	-0.003384

	GVF^{O} PC1	GVF^{O} PC2	GVF^{O} PC3	GVF^{O} PC4	GVF^{O} PC5	GVF^{G} PC1	GVF^{G} PC2	GVF^{G} PC3	GVF^{G} PC4	GVF^{G} PC5
1	0.10152	-0.209115	0.180743	0.375075	-0.238076	0.044211	-0.087372	0.246447	0.440701	-0.806486
2	0.112132	-0.212229	0.176866	0.346632	-0.23868	0.121155	-0.233448	0.306776	0.275852	0.092715
3	0.145956	-0.249138	0.156755	0.28335	-0.068751	0.139944	-0.251216	0.285775	0.207839	0.148226
4	0.162087	-0.253531	0.126507	0.201446	0.027599	0.157751	-0.25667	0.235409	0.123908	0.166591
5	0.176341	-0.252508	0.095606	0.11779	0.087363	0.174164	-0.251984	0.170945	0.037549	0.150586
6	0.188499	-0.24639	0.062558	0.035723	0.127335	0.186097	-0.248303	0.096107	-0.03171	0.12161
7	0.19881	-0.235101	0.027486	-0.044673	0.148414	0.195994	-0.23582	0.027228	-0.090061	0.073364
8	0.207036	-0.218444	-0.008493	-0.119451	0.149183	0.202966	-0.220809	-0.035023	-0.130967	0.022347
9	0.213181	-0.198301	-0.045918	-0.180447	0.133358	0.208282	-0.202773	-0.088122	-0.156749	-0.028828
10	0.217706	-0.175248	-0.082806	-0.226791	0.096328	0.212124	-0.182839	-0.131836	-0.169874	-0.074801
11	0.220989	-0.150434	-0.117584	-0.257144	0.040137	0.214843	-0.162813	-0.166882	-0.172514	-0.112268
12	0.223328	-0.125458	-0.147975	-0.270905	-0.02485	0.216811	-0.14314	-0.194315	-0.16827	-0.141253
13	0.224842	-0.100092	-0.172974	-0.271428	-0.094238	0.218243	-0.124162	-0.215491	-0.158591	-0.162942
14	0.220158	0.071582	-0.239823	-0.042902	-0.445221	0.219589	0.021089	-0.266722	0.025184	-0.177543
15	0.219012	0.127511	-0.215262	0.051141	-0.379538	0.219463	0.089954	-0.225547	0.114237	-0.104384
16	0.220089	0.15901	-0.181736	0.091882	-0.245264	0.220729	0.12788	-0.178802	0.15709	-0.034501
17	0.218874	0.178304	-0.150338	0.117961	-0.107	0.219416	0.152674	-0.139367	0.187605	0.026756
18	0.216148	0.189958	-0.120142	0.136523	0.017863	0.217874	0.167358	-0.099685	0.196648	0.074283
19	0.211683	0.197425	-0.093632	0.154138	0.122805	0.214959	0.179088	-0.062305	0.191654	0.107899
20	0.206821	0.202665	-0.067021	0.166427	0.203746	0.211785	0.188958	-0.024978	0.17354	0.129593
21	0.203007	0.20655	-0.036337	0.165758	0.261962	0.2086	0.19834	0.015369	0.140742	0.137297
22	0.200085	0.210614	0.013699	0.142213	0.287014	0.204726	0.208034	0.063873	0.087146	0.125384
23	0.196525	0.21451	0.135455	0.059105	0.227086	0.19962	0.215813	0.127096	0.003157	0.086896
24	0.181324	0.202363	0.278651	-0.058542	0.09162	0.190196	0.219787	0.198801	-0.104372	0.021774
25	0.156899	0.175793	0.381855	-0.163658	-0.054509	0.175568	0.21529	0.265068	-0.216411	-0.056749
26	0.143804	0.15499	0.417864	-0.216846	-0.151305	0.154526	0.202391	0.313637	-0.314076	-0.136343
27	0.115287	0.124477	0.449314	-0.253707	-0.25874	0.131227	0.183318	0.337612	-0.379402	-0.199237

G-before the name of the model indicates that we have used a GARCH model