

EQUILIBRIUM IMPLICATIONS OF DELEGATED ASSET MANAGEMENT UNDER BENCHMARKING*

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Abstract

Despite the enormous growth of the asset management industry during the past decades, little is known so far about the asset pricing implications of investment intermediaries. Investment objectives of professional asset managers such as mutual funds differ from those of private households. However, standard models of investment theory do not address the distinction between direct investing and delegated investing. Our objective is to get a formal understanding of equilibrium implications of delegated asset management. In a model with endogenous delegation, we find that delegation under benchmarking leads to more informative prices, to a beta adjustment, and to significantly lower equity premia. Empirical results support the model's predictions.

Keywords: Portfolio Delegation, Benchmarking, General Equilibrium, Equity Risk Premia

JEL Classification: G11, G14, D83

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In this paper, we study equilibrium implications of benchmarking and delegated portfolio management. Delegation refers to the situation, in which the investor mandates a professional asset manager to make investment decisions on her behalf on a discretionary basis. Since the demand for risky assets of institutional investors differs from that of individuals, it raises the question of how asset prices are affected in equilibrium. In a model with two groups of investors, individual investors and professional asset managers, we provide a formal model of delegation under benchmarking and we illustrate how endogenous delegation optimally arises in general equilibrium. Furthermore, we establish an expected return-beta relationship, which allows us to study the cross-sectional implications of portfolio management delegation and benchmarking. Our analysis highlights that delegation and benchmarking are two related issues that need to be examined simultaneously.

Over the past decades, the asset management industry has faced rapid growth. Nowadays, a substantial part of wealth is invested through investment intermediaries such as banks, investment funds, pension funds, and other institutional investors. As an example, total assets for U.S. mutual funds grew by an annual rate of 15 % over the past twenty years and have reached a level of USD 7.88 trillion by the end of 2007, which corresponds to a fraction of 56% of the U.S. GDP. Also, despite the current financial crisis, the global hedge fund industry has still assets under management of an estimated USD 1.9 trillion in 2008.¹ One reason for these impressive numbers is a structural shift in the investment behavior of private households, as an increasing number of individual investors is delegating their investment decision to professional asset managers.

In Table 1, we report the ownership structure of U.S. equity over the past six decades. Back in 1952, individuals directly held over 90 % of corporate equities. By 2008, this proportion was down to less than 25 %. At the same time, the fraction of equities held by investment funds (including mutual funds, closed-end funds, and exchange traded funds) rose from 2.9 % to 28.5 %. The share of equities held by pension funds grew from 0.9 % to 23.2 %. This structural shift of investment discretion from private households to institutional investors may have implications on equilibrium asset prices and returns, since investment objectives of institutional investors and private investors differ in many respects. Along this line, Gompers and Metrick (2001) argue that the presence of institutional investors tends to increase the demand for large and liquid stocks, which could explain the disappearance of the small-stock premium. Further, many institutional asset managers are bound to performance objectives relative to a benchmark portfolio, which increases the demand for those stocks included in the benchmark.

Despite the growing importance of institutional investors, the resulting asset pricing

¹Data from the Bureau of Economic Analysis, The Federal Reserve Board, and McKinsey & Company.

U.S. Equity Ownership

Sector	1952	1970	1980	1990	2000	2008
Household sector	90.8	79.0	66.0	56.1	49.4	25.3
Banking sector	0.4	0.4	0.6	0.7	0.5	1.2
Insurance companies	3.0	3.2	5.6	4.7	5.8	8.2
Pension funds	0.9	8.5	19.5	24.2	17.9	23.2
Investment funds	2.9	5.7	3.3	6.9	18.6	28.5
Other	2.0	3.2	5.0	7.3	7.7	13.7
Total equities outstanding (in billions of USD)	170	815	1,070	3,613	20,229	19,361

Table 1: Holdings of corporate equities in the United States, as of the first quarter of the year indicated. All numbers (except total equities outstanding) are in percent. The banking sector consists of the subgroups commercial banking, savings institutions and brokers and dealers; insurance companies consist of property-casualty insurance and life insurance companies; pension funds consist of private pension funds and federal, state and local government retirement funds; investment funds consists of mutual funds, closed-end funds and exchange-traded funds. Source: The Federal Reserve Board *“Flow of Funds Accounts for the United States.”*

implications are poorly understood so far. Since traditional financial market theory is based on the representative investor paradigm, it has nothing to say about the impact of financial institutions on asset pricing. Or as Allen and Santomero (1998) state affirmatively:

“The fact that there is such extensive intermediation suggests that the approach of traditional asset pricing may miss important features of actual markets. [...] Given the importance of intermediaries’ trading in financial markets, asset pricing theories and intermediation theories need to be better integrated.”

Given the tremendous growth of the asset management industry, it is obvious that by neglecting the impact of portfolio delegation, we may miss some important aspects of asset pricing.

One question to be addressed is why investment decisions are delegated to professional asset managers in the first place. Apart from reasons related to market frictions and economies of scale, the main justification for the employment of asset managers is their investment skill and their superior capacity to observe and process information. We follow this route and introduce information asymmetry between managers and investors as a potential trigger for delegation. We endow the investor with a certain amount of

information capacity that allows her to observe a noisy but unbiased signal about the true risky asset payoff. In contrast, the manager faces costly information capacity and needs to decide how much additional capacity is optimal to acquire. This setup gives rise to a two-step investment process, where agents solve an information allocation problem in the first stage and a portfolio choice problem in the second stage.²

Another important issue to be addressed is the role of benchmarking. We model managers as relative return investors, who try to outperform a passive benchmark portfolio such as the S&P 500 index. The magnitude of the effects on asset prices due to benchmarking is particularly driven by the number of delegating investors in an economy. Contrariwise, the composition of the benchmark affects the portfolio holdings of managers and the utility from delegation, which in turn determines the number of delegating investors. Therefore, to consistently model the impact of delegation in an equilibrium model, we need to address delegation and benchmarking simultaneously. We do so by modelling the manager as a strategic agent who takes into account the actions of the investor. Hence, there is a two-way relation between delegation and benchmarking.

In partial equilibrium, we find that delegation is most valuable when the manager's risk aversion is equal to that of the investor. A high discrepancy between manager and investor risk aversion lowers the utility gain from delegation. Furthermore, we find that benchmarking reduces utility from delegation as soon as the manager exhibits lower risk aversion than the investor. This reduction in utility is increasing in the risk level of the benchmark portfolio. Our findings confirm the conclusions in Basak, Pavlova, and Shapiro (2007) on the suboptimality of benchmarking.

For the general equilibrium analysis, we use the concept of a noisy rational expectations equilibrium as proposed by Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982). We develop our model along the lines of the multiple risky asset framework of Admati (1985). We show that the expected return of an asset is a linear function of the asset's covariance with the market portfolio and its covariance with the benchmark portfolio. For assets included in the benchmark portfolio, only active management risk is priced in equilibrium. The risk premium turns out to be proportional to the covariance with a residual portfolio orthogonal to the benchmark. We further show that the presence of delegated agents leads to a more informative price system and to lower equity risk premia for those assets the manager acquires information about. All these equilibrium effects are amplified by the fraction of delegating agents in the economy. Since the seminal contribution of Mehra and Prescott (1985) about the equity risk

²To keep the analysis simple, we do not consider conflicts of interest between the investor and the manager. Rather, we assume that the manager's investment strategy is fully transparent for the investor. Contributions on optimal contracting are, e.g., Bhattacharya and Pfleiderer (1985), Stoughton (1993), and Ou-Yang (2003).

premium, many authors document that this premium actually is declining over time.³ To this end, our theory offers a theoretical explanation that is empirically supported by the fact that the volume of institutionally managed money sharply increased over the same period of time.

Our paper is related to different streams in the literature. Brennan (1993) studies a static mean-variance model with two types of investors, private investors holding the standard market portfolio and benchmark investors with performance objectives relative to a benchmark portfolio. He shows that equilibrium expected returns are characterized by a two-factor model, the two factors being the assets' covariance with market returns and the assets' covariance with the returns on the benchmark portfolio. A similar analysis is performed by Stutzer (2003) and Cornell and Roll (2005). Brennan (1995), Gomez and Zapatero (2003), and Brennan and Li (2008) find empirical support for those types of models.

In the same vein, Cuoco and Kaniel (2007) examine the equilibrium impact of symmetric and asymmetric relative performance evaluation contracts. They find that symmetric contracts tilt portfolio choice towards stocks that are part of the benchmark portfolio, while asymmetric contracts lead fund managers to choose portfolios that maximize tracking error. Probably the most rigorous equilibrium analysis in this field is Ross (2005). He shows that the noisy rational expectations equilibrium, in which investors have information of varying degrees of precision, is unstable to the formation of better informed asset management institutions. He further shows that markets will exhibit lower equity premia if dominated by institutional investors with higher precisions than individual investors.

However, none of the above contributions addresses the issue of endogenous delegation. Rather, they take the fraction of delegating investors and direct investors as exogenously given. While this is no restriction to find empirical support for the theory, these models are not able to explain why delegation occurs. An exception in this area is Kapur and Timmermann (2005). Similar to ours, they establish a decision rule for delegation based on expected utility to generate endogenous delegation. Agents are exogenously endowed with a fixed amount of signal precision and the manager is assumed to have a higher precision than the investor. However, their model has only one risky asset. In our model, we consider multiple risky assets. Therefore, we are able to address cross-sectional implications of delegation. In addition, we endow only individual investors with a fixed amount of information capacity. Managers face costly information and have to decide on their own how much information they should optimally acquire. Hence, not only delegation, but also the managers' signal precisions are endogenously

³See, e.g, Mehra and Prescott (2003) and DeSantis (2007). Jagannathan, McGrattan, and Scherbina (2000) attribute the decline in equity premia to institutional changes in the U.S. as a possible explanation.

specified in our model. Another paper related to ours is Petajisto (2007), who demonstrates that the existence of active managers has implications on the cross-sectional pricing of assets. However, he does not explicitly specify the benchmark portfolio nor the information allocation process.

Also related to our paper is the literature on relative performance objectives in asset management. Roll (1992) shows how relative performance objectives can be incorporated into the traditional mean-variance framework. He states that benchmarking is generally not mean-variance efficient, unless the benchmark corresponds to a portfolio on the efficient frontier. Other papers in this direction include Wagner (2001), Basak, Shapiro, and Tepla (2006), Basak, Pavlova, and Shapiro (2007), and Basak, Pavlova, and Shapiro (2008). Garcia and Strobl (2008) and Gomez, Pristley, and Zapatero (2008) derive equilibrium implications of relative wealth concerns with a single risky asset and multiple risky assets, respectively. They both find that relative performance objectives lower equilibrium risk premia.

The paper is organized as follows. In Section 1, we introduce the basic setup and discuss the sequence of events in the model. Section 2 presents partial equilibrium results on optimal delegation and information acquisition. We compute expected utility resulting from direct and delegated investment and establish a decision rule for delegation. In Section 3, we discuss equilibrium implications of our model. We also derive an expected return-beta relationship and analyze the cross-sectional implications of delegated portfolio management and benchmarking. Section 4 provides empirical evidence for the main prediction of the model. Section 5 concludes. All proofs are delegated to the appendix.

1 Basic Setup

We consider a simple economy with two types of agents, a private investor (the ‘investor’) and a professional portfolio manager (the ‘manager’). The private investor faces the decision problem of whether to delegate the management of her wealth to the manager or to invest directly. We motivate delegation in our model by assuming asymmetric information between the investor and the manager. Delegation results from the investor’s quest for better information and can be rational when the manager has superior private information. Once the investor has decided to delegate, the manager has the final decision about which assets to hold in the portfolio. In case of direct investment, the investor has the final decision about the portfolio composition.⁴

⁴Indexing is referred to as direct investing since no active portfolio decision on the part of the manager is involved. Indexing is not based on private information and absent any market frictions, the individual investor is able to replicate a passive investment strategy.

1.1 Preferences and Beliefs

The investment universe consists of N risky assets with price vector \mathbf{p} and one risk-free asset with constant rate of return r . Final payoffs of the risky assets are captured by the $N \times 1$ random vector \mathbf{f} . Agents have some initial information about the distribution of \mathbf{f} , which we assume to be the normal distribution. Both the investor and the manager start with the same set of prior beliefs

$$p(\mathbf{f}) \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \quad (1)$$

where $\boldsymbol{\mu}$ is the $N \times 1$ mean vector and $\boldsymbol{\Sigma}$ is the $N \times N$ covariance matrix. All agents observe a noisy but unbiased signal \mathbf{s}_k about the true realization of \mathbf{f} ,

$$\mathbf{s}_k = \mathbf{f} + \boldsymbol{\eta}_k, \quad (2)$$

where $\boldsymbol{\eta}_k \sim \mathcal{N}(0, \boldsymbol{\Omega}_k)$, and $k \in \{j, m\}$ stands for investor j and manager m , respectively. $\boldsymbol{\Omega}_k$ is a covariance matrix that determines the precision of the signal, which is defined as the inverse of the signal variance. $\boldsymbol{\Omega}_k$ is not restricted to be diagonal. Therefore, the signal about one asset might contain information about another asset.

With the signal observed, agents combine the prior belief and the signal using Bayes' law. The posterior belief of agent k about the realization of the asset payoffs is

$$p_k(\mathbf{f} | \boldsymbol{\mu}, \mathbf{s}_k) \sim \mathcal{N}(\hat{\boldsymbol{\mu}}_k, \hat{\boldsymbol{\Sigma}}_k), \quad (3)$$

with posterior mean

$$\hat{\boldsymbol{\mu}}_k := (\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Omega}_k^{-1})^{-1} (\boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} + \boldsymbol{\Omega}_k^{-1} \mathbf{s}_k) \quad (4)$$

and posterior variance

$$\hat{\boldsymbol{\Sigma}}_k := (\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Omega}_k^{-1})^{-1} \quad (5)$$

Given $\boldsymbol{\Omega}_k$ positive-semidefinite, the posterior variance is always lower (or equal) than the prior variance, reflecting the fact that the signal generates additional information.

We next define the utility functions for the investor and the manager. For investor j , we assume a standard mean-variance utility U_j with constant absolute risk aversion parameter ρ_j , i.e., for a given arbitrary set of beliefs $(\hat{\boldsymbol{\mu}}_j, \hat{\boldsymbol{\Sigma}}_j)$, we define

$$U_j(\hat{\boldsymbol{\mu}}_j, \hat{\boldsymbol{\Sigma}}_j) = \mathbf{q}_j' (\hat{\boldsymbol{\mu}}_j - \mathbf{p}r) - \frac{1}{2} \rho_j \mathbf{q}_j' \hat{\boldsymbol{\Sigma}}_j \mathbf{q}_j, \quad (6)$$

where \mathbf{q}_j is the vector of portfolio weights. For the manager, we impose a slightly different utility function, since most active portfolio managers are concerned with per-

formance objectives relative to a benchmark.⁵ The benchmarked manager derives utility from relative return between portfolio and benchmark minus the tracking error of the portfolio:⁶

$$U_{bm}(\phi, \hat{\mu}_m, \hat{\Sigma}_m) = \underbrace{(\mathbf{q}_{bm} - \phi)'(\hat{\mu}_m - \mathbf{p}^r)}_{\text{relative return}} - \frac{1}{2}\rho_m \underbrace{(\mathbf{q}_{bm} - \phi)'\hat{\Sigma}_m(\mathbf{q}_{bm} - \phi)}_{\text{tracking error}}. \quad (7)$$

ρ_m is the manager's aversion to the risk of deviating from the benchmark, \mathbf{q}_{bm} is the manager's portfolio given the performance objective relative to the benchmark portfolio ϕ . Note that usually we have $\rho_j \neq \rho_m$. While ρ_j measures the aversion to absolute risk, ρ_m measures the risk of deviating from the benchmark portfolio and is often referred to as regret aversion (see, e.g., Wagner (2001)). Empirically, the risk aversion parameter under benchmarking is usually higher than the risk aversion in an absolute return setting.

1.2 Information Acquisition

Agents not only have to solve for the optimal portfolio weights, but they also have to decide about which assets they want to learn about prior to observing the signal, i.e., how to acquire and allocate signal precision. Hence, agents face a two-level investment process. First, they decide what they want to learn about and how much they want to learn. This phase is referred to as information allocation. To observe a signal, agents need information capacity. For the investor, we assume that information capacity is fixed. For the manager, information capacity has to be acquired through costly effort. Therefore, on the one hand, agents need to decide on how much information capacity to acquire (manager). On the other hand, they decide on which assets to spend the available information capacity (manager and investor). Second, having observed the signal, agents form a posterior belief about future asset payoffs and select the optimal portfolio. This phase is referred to as portfolio choice.

⁵Active managers are primarily concerned about attracting new money. Recent empirical evidence suggests that money tends to flow into the fund, if it performs well relative to a benchmark. See, e.g., Gruber (1996), Chevalier and Ellison (1997), and Sirri and Tufano (1998). Given the manager is compensated on the basis of total assets under management (which is generally the case for mutual funds), the manager has an implicit incentive to focus on relative performance. By using the utility function in (7), we capture this type of behavior.

⁶Some authors define the tracking error as the difference between portfolio and benchmark return and refer to $(\mathbf{q}_{bm} - \phi)'\hat{\Sigma}_j(\mathbf{q}_{bm} - \phi)$ as the tracking error variance (see, e.g., Roll (1992)). The definition we use in this paper can be found in Meucci (2005).

1.2.1 Learning About Risk Factors

Borrowing from Van Nieuwerburgh and Veldkamp (2008a,b), we simplify the information allocation problem by letting agents learn about orthogonal risk factors or principal components. We decompose the covariance matrix Σ into a diagonal eigenvalue matrix Λ and an eigenvector matrix Γ ,

$$\Sigma = \Gamma \Lambda \Gamma'. \quad (8)$$

The matrix Γ captures the factor loadings and may include systematic risk factors as well as firm specific risks. By Γ_l we denote the l -th column of Γ that contains the loading of each asset on the l -th risk factor. The l -th row of Γ captures asset l 's factor loadings on each factor. The eigenvalue matrix $\Lambda = (\lambda_l)_{l=1,\dots,L}$ captures the variance of the l -th risk factor λ_l on its diagonal. Basically, we could reduce the dimensionality of the problem by only considering the L -th largest eigenvalues ($L < N$). However, without loss of generality, we can assume that the number of orthogonal risk factors equals the number of assets, i.e., $L = N$.

Instead of signal \mathbf{s} , we let agent k observe a signal about the payoff of the risk factor given by $\Gamma' \mathbf{s}$. The variance of the signal is $\Lambda_k = (\lambda_{k,l})_{l=1,\dots,L}$ and serves as choice variable in the agent's information acquisition problem. Since the factor loadings Γ remain constant and only eigenvalues change, the posterior variance of asset payoffs is

$$\widehat{\Sigma}_k = \Gamma \widehat{\Lambda}_k \Gamma'. \quad (9)$$

The eigenvector matrix Γ is known to all agents. Information allocation is referred to the choice of optimal signal precision Λ_k that helps to reduce uncertainty about the realization of final payoffs. With Λ_k being positive semidefinite, we implicitly assume that agents cannot forget what they already know about factors. The agents' information allocation problem consists of selecting the optimal posterior variance $\widehat{\Lambda}_k$ subject to different informational constraints to be discussed below.

1.2.2 Information Capacity and Information Cost

To operationalize the process in terms of how agents acquire information, we introduce two additional concepts, information capacity and the cost of information. First, to become informed, agents need information capacity, which we measure as the increase in total signal precision.

Definition 1.1. *Information capacity K_k for agent k is defined as the sum of the*

differences between the posterior and the prior precision of each risk factor,⁷

$$K_k := \sum_{l=1}^L \left(\hat{\lambda}_{k,l}^{-1} - \lambda_l^{-1} \right), \quad k \in \{j, m\}. \quad (10)$$

For investor j , we assume a fixed amount of information capacity K_j and argue that portfolio management is not the investor's profession nor is it her main activity. Hence, the private investor only has to decide how to allocate the available capacity optimally among the different assets and does not have to decide how much information capacity to acquire. In contrast, the manager can acquire further information by increasing her information capacity. However, information capacity comes at a cost. Each manager faces a tradeoff between the monetary cost of capacity and the benefits of more accurate information. We define the monetary cost of capacity as

$$C(K_m) = c \sum_{l=1}^L \left(\hat{\lambda}_{m,l}^{-1} - \lambda_l^{-1} \right)^\psi, \quad (11)$$

with constants $c > 0$ and $\psi \geq 1$. The constant c translates capacity expressed in terms of differences between posterior and prior precision into monetary costs and ψ specifies the curvature of the cost function. For $\psi = 1$, $C(K_m)$ is linear in capacity. For values $\psi > 1$, $C(K_m)$ is a convex function of information capacity and the marginal cost for information is increasing.

Two properties of equation (11) are worth mentioning. First, no costs occur if the agent does not receive any private information, i.e., information capacity is zero. From equation (5) and the specification of the correlation structure, we know that $\hat{\Lambda}_m = \Lambda_m$ if $(\lambda_{\eta,l}^m)^{-1} = 0, \forall l$. Second, if $\psi > 1$, the cost function assures that the signal can never reveal the true asset payoffs, because $\lim_{\hat{\lambda}_{m,l}^{-1} \rightarrow 0} c \left(\hat{\lambda}_{m,l}^{-1} - \lambda_l^{-1} \right)^\psi = \infty$. In addition, the convexity of the cost function makes it unattractive to acquire too much information about a single risk factor due to the decreasing returns of information acquisition implied by $\psi > 1$.

1.3 Sequence of Events

Having discussed the basic building blocks, we briefly outline the sequence of events. We may think of our framework as a static model with four time periods. In the first period,

⁷This definition of information capacity corresponds to the linear precision constraint in Van Nieuwerburgh and Veldkamp (2008a). They also discuss other types of capacity constraints such as an entropy based constraint. For our paper and to obtain analytical tractability, we restrict the analysis to the linear case. Also note that the prior precision λ_l is not agent specific.

the manager and the investor solve the information acquisition problem. The investor solves the information allocation problem to maximize expected utility of final wealth, while the manager solves the information allocation problem to maximize management fees. The manager does the information allocation with full anticipation of the investor’s information choice, whereas the investor chooses information independently. In the second period, after having observed the manager’s information choice and to make the delegation decision, the investor compares the magnitude of management fees and the ex-ante utility gain from delegation expressed in monetary units (labeled CED). In period 3, both agents observe a signal about the payoff of risk factors based on their information choice in period 1. Depending on the investor’s delegation decision, either the portfolio choice of the manager (‘delegated investment’) or the portfolio choice of the investor (‘direct investment’) takes place. Terminal wealth is realized in period 4.

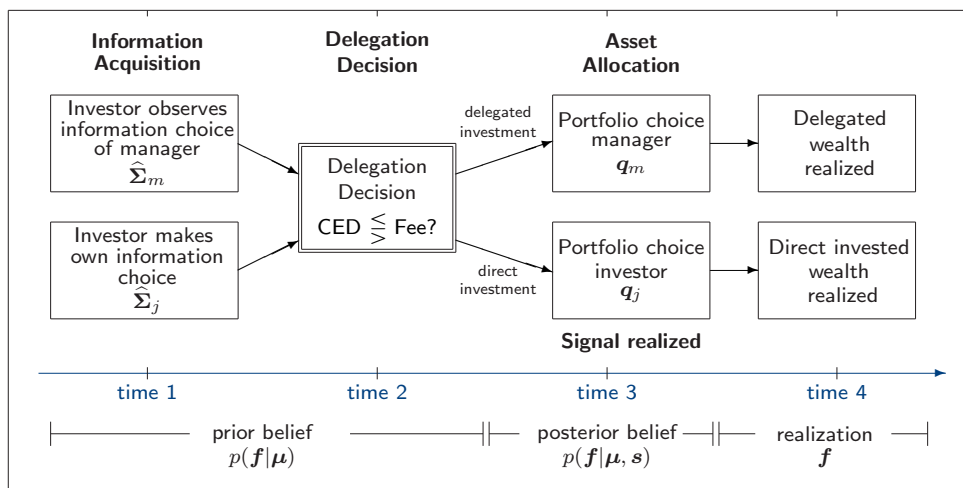


Figure 1: Sequence of events in the model with endogenous delegation.

Figure 1 summarizes the sequence of events in the model. The distinction between period 1 and period 2 is for illustrative purposes only and serves to point out the chronology of information allocation and the delegation decision. However, since the signal is received in period 3, there is no difference between period 1 and period 2 in terms of information sets.

The manager’s information choice $\hat{\Lambda}_m$ is a key variable in the model. It reveals managerial skill and determines the investor’s delegation decision. Comparing the manager’s posterior risk factor variances, the investor can assess where the manager has precise private information. A risk factor’s posterior variance relative to other risk factors reveals how much capacity the manager devotes to learning about this specific risk factor. If we define managerial skill as the manager’s ability to receive a precise signal, we may

say that a more precise private signal leads to higher (ex-ante) managerial skill. The quality of the manager's information is thus completely determined by $\hat{\Lambda}_m$: the smaller the posterior variance, the better the forecasting abilities and the higher manager skill.⁸ Given our setting with delegation, manager skill can also be interpreted as the total gain in precision relative to the investor's signal, i.e., as a direct measure of relative information capacity.

1.4 Solving the Model

We solve the model by backward induction. First, we solve the portfolio choice problem of the investor and the manager in period 3. Given an arbitrary posterior belief $(\hat{\mu}_j, \hat{\Sigma}_j)$, the investor maximizes the mean-variance utility in equation (6). Similarly, the manager maximizes the utility function including the benchmark portfolio in equation (7) given $(\hat{\mu}_m, \hat{\Sigma}_m)$.

In period 2, the investor has to make the delegation decision. An informed manager can add value for a less informed investor by selling her superior information. Since information is costly to produce but can be replicated almost without any cost, it is not attractive to sell information directly, e.g., via newsletters. A newsletter can be reproduced relatively easy and many investors can get access without paying for the service. An indirect way of selling information is to establish a mutual fund and manage wealth for others.⁹ Superior information is then needed to construct active portfolios. If the manager acts exactly in the interest of the investor, direct and indirect sale of information create the same utility for the investor.

To operationalize the delegation problem, we establish a decision rule for optimal delegation based on a certainty equivalent argument. In the context of portfolio management delegation, the certainty equivalent can be considered as the maximum fee an investor is willing to pay for the manager skills. We will refer to this measure as the certainty equivalent of delegation (CED) and define it as:

$$\text{CED} := \sup \left[\delta \left| U_{dir}(\hat{\mu}_j, \hat{\Sigma}_j) \leq U_{del}(\hat{\mu}_m - \delta, \hat{\Sigma}_m) \right. \right], \quad (12)$$

where $U_{dir}(\hat{\mu}_j, \hat{\Sigma}_j)$ is the investor's period 2 utility from direct investment and the period 2 utility from delegation given the belief of the manager is given by $U_{del}(\hat{\mu}_m, \hat{\Sigma}_m)$. The

⁸Results from Van Nieuwerburgh and Veldkamp (2008b) on optimal learning show that agents learn about one or a small set of risk factors, thus specializing in certain areas leading to concentrated and under-diversified portfolios. Their model prediction is consistent with recent empirical evidence. Kacperczyk, Sialm, and Zheng (2005) have shown that active managers do show stock selection abilities if they focus on relatively few industries. This finding stresses the importance of the manager's posterior covariance matrix in defining the area of specialization and managerial skill.

⁹Already Admati and Pfleiderer (1990) point out that delegated portfolio management is an indirect way of selling information.

definition in (12) allows for a direct comparison with the fee effectively charged for active management. A rational investor delegates investment decisions to the portfolio manager if and only if the fee for active management is smaller (or equal) than her CED.

In period 1, the posterior covariance matrices of investor j and the manager m are determined. As the investor chooses information independently of the manager's action, the manager acts strategically and optimizes the information allocation with full anticipation of the investor's information choice. Given the mean-variance utility in equation (6) and the optimal portfolio choice \mathbf{q}_j from period 3, the investor's period 1 problem can be written as

$$\max_{\hat{\Lambda}_j} E \left[\mathbf{q}_j' (\hat{\boldsymbol{\mu}}_j - \mathbf{p}r) - \frac{1}{2} \rho \mathbf{q}_j' \hat{\boldsymbol{\Sigma}}_j \mathbf{q}_j \mid \boldsymbol{\mu} \right], \quad (13)$$

subject to the capacity constraint

$$\sum_{l=1}^L (\hat{\lambda}_{j,l}^{-1} - \lambda_l^{-1}) \leq K_j \quad (14)$$

and the nonnegative-learning constraint

$$\hat{\lambda}_{j,l}^{-1} \geq \lambda_l^{-1} \quad \forall l. \quad (15)$$

The capacity constraint (14) limits the signal precision to a certain level of information capacity K_j and ensures that the investor cannot reduce posterior variance beyond that level. The nonnegative-learning constraint (15) prevents the investor from forgetting what she already knows at the expense of a more precise signal about another risk factor.

For the manager, information capacity is costly. However, the manager can acquire an arbitrary level of information capacity K_m . He faces a cost-benefit tradeoff between the cost of becoming informed and management fees earned from delegation. Higher manager skill enables the manager to charge a higher fee.

The maximum fee the manager can charge depends on the investor's willingness to delegate. When deciding how much information capacity to acquire and what assets to learn about, the manager acts strategically and takes the investor's response (in terms of CED) into account. Following Petajisto (2007), we assume that the manager observes the investor's posterior variance $\hat{\boldsymbol{\Sigma}}_j$ and maximizes period 1 utility with full anticipation of the investor's response in period 2. In this sense, the manager is a Stackelberg leader who chooses information such that management fees are maximized under costly information capacity. The period 1 utility of the manager U_m depends on

the investor's delegation decision,

$$U_m = \begin{cases} \alpha - C(K_m) & \text{if CED} \geq \eta \text{ (delegation),} \\ 0 & \text{if CED} < \eta \text{ (direct investment),} \end{cases} \quad (16)$$

where α denotes the management fee and η the manager's reservation utility. The manager establishes an investment fund, if the remuneration through management fees α outweighs the information costs $C(K_m)$. As long as the certainty equivalent of delegation is higher than the manager's reservation utility, both agents are better off when they engage in delegation. If the cost of information acquisition necessary for the manager skill required to induce the investor to delegate is higher than the fees from managing delegated wealth, the manager does not engage in wealth management and his utility is zero.

For simplicity, we assume that the manager observes the investor's certainty equivalent of delegation, i.e., the manager is aware of the investor's marginal willingness to pay for delegation. A rational manager will then charge the highest fee possible such that $\alpha = \text{CED}$. Hence, in period 1 the manager chooses information such that the investor's CED (and therefore his fee) is maximized:

$$\max_{\mathbf{\hat{\Lambda}}_m} (\text{CED} - C(K_m)), \quad (17)$$

subject to the nonnegative-learning constraint. Therefore, the manager fully discriminates management fees among investors.

2 Partial Equilibrium Results

We now discuss the partial equilibrium results of our model. We first explore the investor's optimal portfolio choice and information allocation in case of direct investment. Then, we consider the situation in which the individual investor has two investment opportunities, delegation and direct investment. Finally, we introduce a benchmarking portfolio for the asset manager.

2.1 Direct Investment

Given the investor's mean-variance utility function in (6), the solution to the period-3 maximization problem is simply given by

$$\mathbf{q}_j = \frac{1}{\rho_j} \widehat{\Sigma}_j^{-1} (\hat{\boldsymbol{\mu}}_j - \mathbf{p}r). \quad (18)$$

Since the signal realization is unknown in period 1, only expected portfolio holdings can be predicted:

$$E[\mathbf{q}_j|\boldsymbol{\mu}] = \frac{1}{\rho_j} \widehat{\boldsymbol{\Sigma}}_j^{-1}(\boldsymbol{\mu} - \mathbf{p}r). \quad (19)$$

The expected utility that results from the optimal investment strategy given in equation (18) is summarized in the following proposition. All proofs are delegated to the Appendix.

Proposition 1. *Consider an investor j with coefficient of absolute risk aversion ρ_j and posterior beliefs $\hat{\boldsymbol{\mu}}_j, \widehat{\boldsymbol{\Sigma}}_j$. If this investor chooses to invest directly, the period-2 expected utility implied by portfolio policy \mathbf{q}_j is*

$$U_{dir}^j = \frac{1}{2\rho_j} \left(\text{tr}(\widehat{\boldsymbol{\Sigma}}_j^{-1}\boldsymbol{\Sigma} - \mathbf{I}) + (\boldsymbol{\mu} - \mathbf{p}r)' \widehat{\boldsymbol{\Sigma}}_j^{-1}(\boldsymbol{\mu} - \mathbf{p}r) \right), \quad (20)$$

where \mathbf{I} is the identity matrix.

To solve the optimal information allocation problem, we substitute (18) in equation (13) and write the investor's period-1 maximization problem as

$$\max_{\hat{\boldsymbol{\Lambda}}_j} \frac{1}{2\rho_j} E \left[(\hat{\boldsymbol{\mu}}_j - \mathbf{p}r)' \widehat{\boldsymbol{\Sigma}}_j^{-1}(\hat{\boldsymbol{\mu}}_j - \mathbf{p}r) \middle| \boldsymbol{\mu} \right], \quad (21)$$

subject to the capacity constraint (14) and the nonnegative-learning constraint (15). Given the result from Proposition 1 and substituting for the eigen decomposition of the covariance matrix, we can alternatively write the maximization problem as¹⁰

$$\max_{\hat{\boldsymbol{\Lambda}}_j} \frac{1}{2\rho_j} \left(\text{tr}(\boldsymbol{\Gamma} \hat{\boldsymbol{\Lambda}}_j^{-1} \boldsymbol{\Gamma}' \boldsymbol{\Gamma} (\boldsymbol{\Lambda} - \hat{\boldsymbol{\Lambda}}) \boldsymbol{\Gamma}') + (\boldsymbol{\mu} - \mathbf{p}r)' \boldsymbol{\Gamma} \hat{\boldsymbol{\Lambda}}_j^{-1} \boldsymbol{\Gamma}' (\boldsymbol{\mu} - \mathbf{p}r) \right), \quad (22)$$

or, in summation notation,

$$\max_{\{\hat{\lambda}_{j,1}, \dots, \hat{\lambda}_{j,L}\}} \frac{1}{2\rho_j} \left(\sum_{l=1}^L \hat{\lambda}_{j,l}^{-1} \left(\lambda_l + [(\boldsymbol{\mu} - \mathbf{p}r)' \boldsymbol{\Gamma}_l]^2 \right) - L \right), \quad (23)$$

subject to (14) and (15). The optimal information allocation depends on the value of the constant term $\lambda_l + [(\boldsymbol{\mu} - \mathbf{p}r)' \boldsymbol{\Gamma}_l]^2$ in equation (23). We next define

$$X_l := \lambda_l + [(\boldsymbol{\mu} - \mathbf{p}r)' \boldsymbol{\Gamma}_l]^2, \quad (24)$$

which Van Nieuwerburgh and Veldkamp (2008a,b) call the learning index, because the value of X_l determines how much is learned about the risk factor. Each risk factor l has

¹⁰To derive this result, recall that if \mathbf{A} is a symmetric matrix that admits a diagonal factorization of the form $\mathbf{A} = \mathbf{V} \mathbf{C} \mathbf{V}'$ where \mathbf{V} is a matrix whose columns correspond to the eigenvectors of \mathbf{A} , and \mathbf{C} is a diagonal matrix whose entries are the eigenvalues corresponding to the columns of \mathbf{V} , the inverse of \mathbf{A} can be written as $\mathbf{A}^{-1} = \mathbf{V} \mathbf{C}^{-1} \mathbf{V}'$ (see, e.g., Gentle (2007)). Hence, $\widehat{\boldsymbol{\Sigma}}_j^{-1} = \boldsymbol{\Gamma} \hat{\boldsymbol{\Lambda}}_j^{-1} \boldsymbol{\Gamma}'$.

a unique X_l associated with it. The index X_l depends on prior information only and is independent of the signal realization and the investor's optimal information choice.

Problem (23) reduces to maximizing a weighted sum of posterior precision subject to a constraint on the sum of posterior precisions weighted by the index X_l . Analyzing the index X_l , we make two important observations concerning the optimal information allocation. First, it is optimal to learn about the risk factor(s) with a high prior variance λ_l . This means that the investor wants to learn about a risk factor with high initial uncertainty. Second, optimal learning comes from an anticipation of future portfolio positions. It is optimal to learn about the risk factor on which the expected portfolio strategy will heavily load. Therefore, the investor optimally uses all information capacity K_j to learn about the risk factor with the highest index X_l . In case the investor has more capacity available than is needed to fully reduce the uncertainty of the respective risk factor, the remaining capacity is used to reduce posterior variance of the risk factor with the second highest index X_l . However, with the correlation structure proposed, it is generally not possible to fully reveal the true payoff of an asset unless there is enough information capacity to reduce posterior variance of all risk factors to zero. The investor's linear capacity constraint always leads to a corner solution and information capacity is never allocated partially among different risk factors. We can now make the following claim.

Proposition 2. *With fixed capacity and under a linear capacity constraint, the investor maximizes period-1 utility, if she allocates all available information capacity to the risk factor with the highest index X_l .*

2.2 Delegated Investment

Absent any principal agent conflicts between the investor and the manager, we can derive the investor's utility from delegated investment as in Proposition 1 by substituting \mathbf{q}_m in the investor's utility function.

Proposition 3. *Consider an investor j who delegates the investment to a fund manager with risk aversion ρ_m and optimal investment strategy \mathbf{q}_m according to equation (18). Absent any market frictions, the period-2 utility from delegated investment is*

$$U_{del}^j = \left(1 - \frac{1}{2}\rho\right) \frac{1}{\rho_m} \left(\text{tr}(\widehat{\Sigma}_m^{-1}\Sigma - \mathbf{I}) + (\boldsymbol{\mu} - \mathbf{p}r)' \widehat{\Sigma}_m^{-1} (\boldsymbol{\mu} - \mathbf{p}r) \right), \quad (25)$$

where

$$\rho := \frac{\rho_j}{\rho_m}$$

is the ratio of the risk aversions of the investor and the manager.

In contrast to the case with direct investment, the posterior belief of the manager determines utility. Further, we observe that utility from delegation is decreasing in the risk aversion ratio ρ . The first order condition of equation (25) implies that utility from delegation is maximized if $\rho_m = \rho_j$. Hence, given the investor's risk aversion, her utility is maximal if the manager has the same level of risk aversion.

2.2.1 Certainty Equivalent of Delegation

In period 2, the investor observes the manager's information choice $\hat{\Sigma}_m$ and decides whether to delegate or not by comparing the CED with the management fee.

Proposition 4. *A rational investor delegates her investment decision to a portfolio manager if and only if*

$$\text{CED} \geq \alpha,$$

where α denotes the management fee and

$$\text{CED} := \sup \left[\delta \mid \delta \leq \sum_{l=1}^L \left(a \hat{\lambda}_{m,l}^{-1} - b \hat{\lambda}_{j,l}^{-1} \right) X_l - L(a - b) \right], \quad (26)$$

with constants a , b , and X_l defined as

$$a := \left(1 - \frac{\rho}{2} \right) \frac{1}{\rho_m}, \quad b := \frac{1}{2\rho_j}, \quad X_l := \lambda_l + [(\boldsymbol{\mu} - \boldsymbol{pr})' \boldsymbol{\Gamma}_l]^2.$$

X_l is a constant and thus does not depend on any private information. The only two parameters driven by private signals are the posterior variances of the manager and the individual investor, $\hat{\Lambda}_{m,l}$ and $\hat{\Lambda}_{j,l}$. From Proposition 4, two dimensions of optimal delegation can be observed. First, delegation is driven by ex-ante manager skill, i.e., the manager's signal quality relative to that of the investor. The bigger the difference between the signal precision of the manager and the signal precision of the investor, the higher the CED and the more likely the investor decides to delegate for a given level of management fees. In other words, the higher the difference in information between manager and investor, the higher the maximum fee the investor is willing to pay for delegation. Second, the certainty equivalent of delegation is influenced by the ratio $\rho = \rho_j/\rho_m$. Because $\lambda_{m,l}^{-1}$ is itself a function of ρ , this effect cannot be directly observed from equation (39). However, we know from Proposition 3 that the utility from delegation is at a maximum when $\rho = 1$.

In Figure 2, we plot the CED for different levels of the ratio $\rho = \rho_j/\rho_m$ using the numerical example with three assets reported in Table 2. Any deviation of the ratio of

Summary Statistics of Three-Asset Example

<i>Panel A: Assets</i>					
	Mean return (% p.a.)	Volatility (% p.a.)	Correlations		
Southwestern Energy (SWN)	23.39	37.96	1.00		
Merck & Co. (MRK)	16.93	25.09	-0.03	1.00	
Pfizer (PFE)	15.76	24.84	0.06	0.57	1.00

<i>Panel B: Factors</i>					
	$X_l \times 100$	Volatility (% p.a.)	Factor loadings $\mathbf{\Gamma}'$		
			SWN	MRK	PFE
Factor 1	0.22	16.34	-0.05	-0.70	0.71
Factor 2	0.83	31.22	-0.05	0.72	0.70
Factor 3	1.23	38.02	-1.00	0.00	-0.07

Table 2: Summary Statistics for the three-asset example. The sample period is January 1982 to June 2008. Calculations are based on monthly returns from Datastream. The annual risk-free rate is fixed at 2.40%.

risk aversion from one leads to a loss in terms of certainty equivalent wealth. Panel a) plots the CED for three different levels of investor information capacity.¹¹ We observe that the CED decreases with the investor's information capacity. A well informed investor (high information capacity) has a lower incentive to delegate. Therefore, the CED is smaller. Panel b) plots the CED for different values of investor risk aversion. The CED is decreasing in ρ_j . The more risk averse the investor, the lower her willingness to delegate for a given level of manager skill. Hence, an investor with a low risk aversion will be inclined to pay higher management fees.

2.2.2 The Manager's Information Acquisition Problem

So far, we have taken $\hat{\Sigma}_m$ as given. We now solve the manager's information choice problem. Substituting for CED and $C(K_m)$ in equation (17), the manager's period-1 problem can be written as

$$\max_{\{\hat{\lambda}_{m,1}, \dots, \hat{\lambda}_{m,L}\}} \sum_{l=1}^L \left(a \hat{\lambda}_{m,l}^{-1} - b \hat{\lambda}_{j,l}^{-1} \right) X_l - L(a - b) - c \left(\hat{\lambda}_{m,l}^{-1} - \lambda_l^{-1} \right)^\psi, \quad (27)$$

subject to the nonnegative-learning constraint, $\lambda_l \geq \hat{\lambda}_l, \forall l$. With a convex cost function, problem (27) is well defined and there exist extremum points. Since risk factors are uncorrelated, the information choice is independent among different factors. The

¹¹To get an idea about the magnitude of information capacity, we note that $K_j = 10$ is equal to a reduction in posterior standard deviation for one risk factor from 20% to 16.9%.

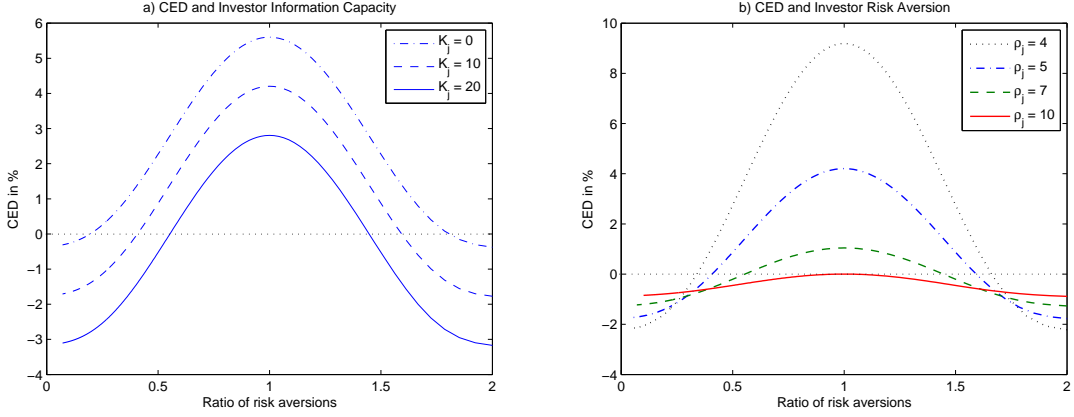


Figure 2: The certainty equivalent of delegation as a function of the ratio of risk aversions ρ_j/ρ_m for three different levels of investor information capacity K_j . Calculations are based on the numbers of the three-asset example to be presented in Table 2. In Panel a) and b), we set $\rho_j = 5$ and $K_j = 10$, respectively.

proposition below determines the optimal solution. The proof follows directly from the first-order condition for equation (27).

Proposition 5. *With uncorrelated risk factors, the information choice for risk factor l is independent of other risk factors. For convex cost functions ($\psi > 1$), an optimal solution to the manager's information acquisition problem is*

$$\hat{\lambda}_{m,l}^* = \left(\lambda_l^{-1} + \left(\frac{a}{c\psi} X_l \right)^{\frac{1}{\psi-1}} \right)^{-1} \quad \forall l. \quad (28)$$

In contrast to the situation of a fixed level of information capacity (investor's information allocation problem) in which information choice is a strategic substitute, information acquisition with costly information capacity is independent from other assets. Optimal information allocation is a decreasing function of risk aversion and information cost, and increasing in the index X_l . For the manager, it is optimal to learn about risk factor(s) for which she expects to have a large exposure and where she has a large initial uncertainty about. However, the restriction of costly information capacity weakens this effect. As the coefficient a decreases, i.e., the discrepancy between ρ_m and ρ_j grows, the manager becomes more cautious about investing in information and thus decreases the level of information capacity acquired. When the discrepancy between ρ_m and ρ_j is high, the CED tends to be low. A low CED implies that the maximum fee will be low as well and the manager will invest less in information. Furthermore, we note that the optimal posterior precision is a concave function of $\frac{a}{c\psi} X_l$. Therefore, it is never optimal to learn too much about the same risk factor.

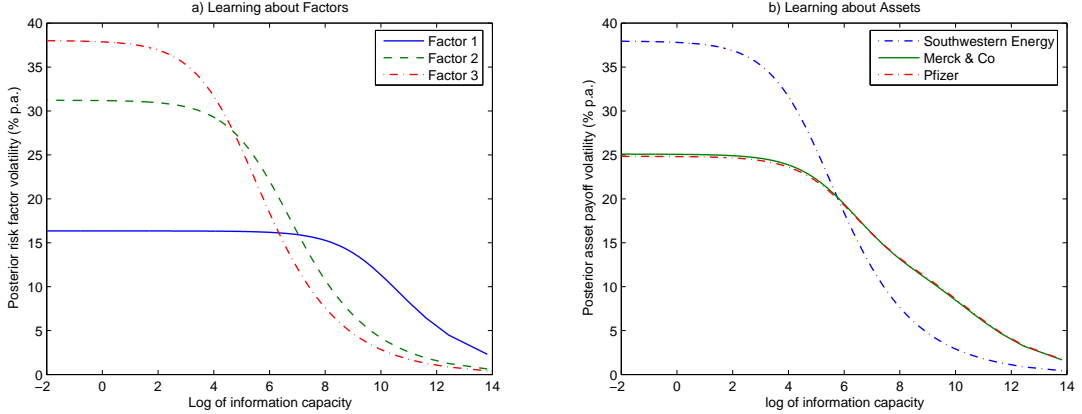


Figure 3: Optimal information allocation with three assets. We plot posterior volatilities of risk factors and assets as a function of information capacity $K \in [0, +\infty]$. Each level of information capacity K is associated with a cost $C(K)$. The ratio of risk aversion is chosen to be $\rho_j/\rho_m = 1$. Calculations are based on the numbers reported in Table 2.

Summarizing the information allocation for the investor and the manager, we conclude that learning with costly information capacity leads to more diversified learning than with free but constrained capacity. The capacity constraint leads to a corner solution, making it optimal to use all information capacity on the risk factor with the highest index X_l . In contrast, costly information leads to more balanced learning for the manager.

2.2.3 Numerical Illustration of Optimal Information Allocation

A numerical example with three assets illustrates the effect of learning on posterior variance of the risky asset payoffs with costly learning according to Proposition 5. The summary statistics of the three assets and the results of the eigen decomposition are given in Table 2. We have chosen *Southwestern Energy* (SWN), *Merck & Co.* (MRK), and *Pfizer* (PFE). Table 2, Panel b) reports the eigen decomposition into three orthogonal factors. Factor 3 exhibits the highest learning index. Since SWN has a low correlation with the two other assets, it loads relatively strong on one specific risk factor (Factor 3), while MRK and PFE load mainly on the two other factors.

Figure 3 exhibits the effect of optimal learning on posterior risk factor and asset volatilities. The manager starts learning about the risk factor with the highest index X_l (factor 3). Since SWN loads heavily on factor 3, the impact of learning is most pronounced for this asset. Because returns to learning are decreasing, there is a point when it becomes reasonable to reduce learning about factor 3 to start learning instead about factor 2 and eventually about factor 1.

Optimal Learning with Costly Information

<i>Panel A: Standard deviations of risk factors (in % p.a.)</i>							
	K_m	Factor 1		Factor 2		Factor 3	
		$\rho = 1$	$\rho = 1.5$	$\rho = 1$	$\rho = 1.5$	$\rho = 1$	$\rho = 1.5$
Prior	-	16.34	16.34	31.22	31.22	38.02	38.02
c high	22	16.30	16.31	30.22	30.46	35.48	36.07
c moderate	114	16.15	16.19	26.99	27.88	28.81	30.48
c low	228	15.95	16.05	24.11	25.43	24.13	26.16

<i>Panel B: Standard deviations of assets (in % p.a.)</i>							
	K_m	SW Energy		Merck & Co.		Pfizer	
		$\rho = 1$	$\rho = 1.5$	$\rho = 1$	$\rho = 1.5$	$\rho = 1$	$\rho = 1.5$
Prior	-	37.96	37.96	25.09	25.09	24.84	24.84
c high	22	35.43	36.01	24.44	24.60	24.19	24.35
c moderate	114	28.78	30.44	22.37	22.94	22.15	22.71
c low	228	24.11	26.13	20.54	21.37	20.37	21.18

Table 3: Impact of learning on posterior variance with costly information. Numerical example with three assets with different levels of information capacity costs c . K_m denotes the manager's capacity implied by the choice of a particular c . The other parameters are $\psi = 2$ and $\rho_j = \rho_m = 5$.

Table 3 shows optimal learning for the same assets by means of three different levels of information costs. We report the results on learning about risk factors in Panel a) and on learning about assets in Panel b). Note that since the manager's learning problem is a strategic decision based on the investors certainty equivalent of delegation, ρ (or a , equivalently) matters for optimal learning. Everything else constant, we have seen that the CED takes its highest value when $\rho = 1$. A high CED increases the investor's willingness to pay a high management fee. Hence the manager spends a high effort to acquire information, because this effort is rewarded by a high management fee. If there is a mismatch between risk aversions ($\rho \neq 1$), the investor's willingness to pay high fees diminishes independently of manager skill. In this case, a rational manager spends less effort on information acquisition. Table 3 reports the standard deviation of risk factors and of the asset payoffs under optimal learning when $\rho = 1$ and in the case when there is a mismatch in risk aversion. When $\rho \neq 1$, the manager always acquires less information as is the case for $\rho = 1$. We also note that the effect of the risk aversion ratio is symmetric around $\rho = 1$.

2.2.4 Information Acquisition and Optimal Portfolios

How does learning affect portfolio choice? To answer this question, we compare the learning portfolios of the investor and the manager with the prior portfolio. Since the

prior portfolio is based on the prior belief, we can interpret it as a special case with $K = 0$. With zero information capacity, there would be no signal and $\hat{\Lambda} = \Lambda$. It is indeed exactly the difference in these matrices that drives a wedge between the learning portfolio and the prior portfolio. Taking a closer look at investor j , we first define the expected period-1 portfolio difference as

$$\Delta(\mathbf{q}_j) := E[\mathbf{q}_j - \mathbf{q}_{j,prior} | \boldsymbol{\mu}] = E[\mathbf{q}_j | \boldsymbol{\mu}] - \mathbf{q}_{j,prior}, \quad (29)$$

which we can write in terms of risk factor variance as

$$\Delta(\mathbf{q}_j) = \frac{1}{\rho_j} \mathbf{\Gamma}(\hat{\Lambda}_j^{-1} - \Lambda^{-1}) \mathbf{\Gamma}'(\boldsymbol{\mu} - \mathbf{pr}).$$

We know from Proposition 2 that $\Delta(\mathbf{q}_j)$ is solely driven by the variance of the factor λ_l the investor decides to learn about. For the other factors the investor does not acquire information. Assuming that the investor learns only about risk factor l , we can write

$$\Delta(\mathbf{q}_j) = \frac{1}{\rho_j} (\hat{\lambda}_{j,l}^{-1} - \lambda_l^{-1}) \mathbf{\Gamma}_l \mathbf{\Gamma}_l'(\boldsymbol{\mu} - \mathbf{pr}) = \frac{K_j}{\rho_j} \mathbf{\Gamma}_l \mathbf{\Gamma}_l'(\boldsymbol{\mu} - \mathbf{pr}), \quad (30)$$

where the second equation arises due to the fact that $\hat{\lambda}_{j,l} - \lambda_l$ is the additional capacity K_j available to the investor to reduce posterior variance of risk factor l . However, we cannot make any general claim about the direction of the portfolio changes, since the loadings in the column vector $\mathbf{\Gamma}_l$ may have positive or negative signs.

To derive the expected portfolio changes $\Delta(\mathbf{q}_m)$ for the manager when there is no benchmarking, we can substitute optimal posterior precision $\hat{\lambda}_{m,l}^*$ given in (5) into the optimal portfolio demand. Since, in contrast to investor j , the manager learns not only about one single asset, the portfolio differences will be influenced by all variances of the different risk factors. Again, due to positive and negative signs in the eigenvectors, we cannot make any general claim about the sign of the elements in $\Delta(\mathbf{q}_m)$.

Figure 4 exhibits the predicted portfolio weights in the three-assets example reported in Table 2. Since we want to focus on the impact of learning, we assume that there is no benchmarking. The manager and the investor mostly learn about the risk factor with the highest variance. Therefore, we observe the most significant shift in portfolio holdings for SWN, since this asset loads the most on factor 3. While the information capacity of the investor is exogenous, the manager's information acquisition (and thus expected portfolio holdings) are endogenously determined and depend on the investor's information choice. The manager's expected demand for risky assets turns out to be higher than the one of the investor. Obviously, this simple exercise already shows the importance of including the potential impact of asset delegation and learning when we extend our analysis to a general equilibrium framework.

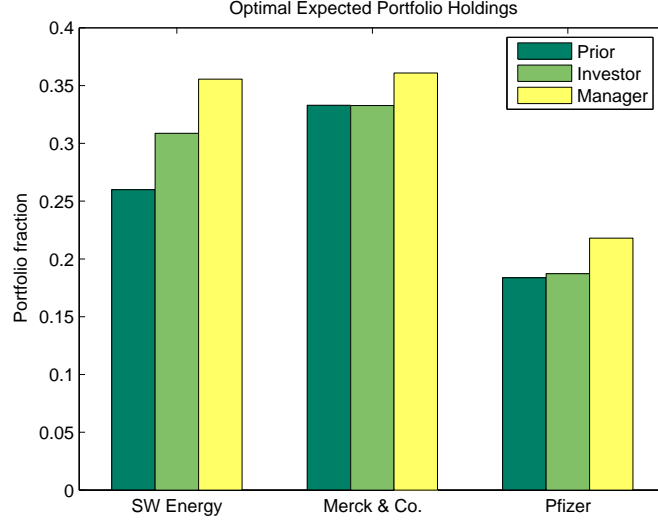


Figure 4: Optimal expected portfolio holdings under different posterior beliefs (prior Σ , manager $\hat{\Sigma}_m$ and investor $\hat{\Sigma}_j$) for the three-assets example in Table 2 and no benchmarking.

2.3 Delegated Investment with Benchmarking

We now turn to the situation, in which the manager has a performance objective relative to a prespecified benchmark portfolio. How does the presence of a benchmark influence the investor’s delegation decision? And how is the manager’s information acquisition affected? First, we note that the optimal portfolio with benchmarking is

$$\mathbf{q}_{bm} = \boldsymbol{\phi} + \frac{1}{\rho_m} \hat{\Sigma}_m^{-1} (\hat{\boldsymbol{\mu}}_m - \mathbf{p}r). \quad (31)$$

The optimal portfolio of risky assets in (31) has two components, a passive component that consists of the benchmark portfolio $\boldsymbol{\phi}$ and an active component $\frac{1}{\rho_m} \hat{\Sigma}_m^{-1} (\hat{\boldsymbol{\mu}}_m - \mathbf{p}r)$. The deviation from the benchmark portfolio is controlled by the risk aversion parameter ρ_m . The lower the manager’s risk aversion, the stronger the deviation from the benchmark. A manager with an infinitely high coefficient of risk is referred to as a passive manager, since such an investor has no active exposure to risky assets and completely matches the benchmark portfolio. The active part of the portfolio is independent of the benchmark. Absent any principal agent conflicts between the investor and the manager, the investor’s utility from delegated investment with benchmarking is summarized below. The result follows directly from substituting for \mathbf{q}_{bm} in the investor’s utility function.

Proposition 6. *Consider an investor j who delegates the investment to a fund manager with a relative performance objective against an exogenously benchmark portfolio $\boldsymbol{\phi}$, risk aversion ρ_m , and optimal investment strategy \mathbf{q}_{bm} according to equation (31). The period*

2 utility from delegated investment is

$$\begin{aligned}
U_{del}^j = & \underbrace{\left(1 - \frac{1}{2}\rho\right) \frac{1}{\rho_m} \left(\text{tr}(\widehat{\Sigma}_m^{-1}\Sigma - \mathbf{I}) + (\boldsymbol{\mu} - \mathbf{p}r)' \widehat{\Sigma}_m^{-1} (\boldsymbol{\mu} - \mathbf{p}r)\right)}_I \\
& + \underbrace{(1 - \rho) \phi'(\boldsymbol{\mu} - \mathbf{p}r)}_{II} - \underbrace{\frac{1}{2}\rho_j \phi' \widehat{\Sigma}_m \phi}_{III}, \tag{32}
\end{aligned}$$

where $\rho := \frac{\rho_j}{\rho_m}$.

From Proposition 6, we observe that the expected utility from delegated investment has three different components. The first term is the utility that arises from delegated investment in the absence of a benchmark (see, Proposition 3). The second and third terms are induced by the presence of a benchmark. The second term accounts for the expected return of the benchmark portfolio $\phi'(\boldsymbol{\mu} - \mathbf{p}r)$. If $\rho > 1$ ($\rho < 1$), a positive expected excess return of the benchmark portfolio decreases (increases) utility. This is to say that when the investor is more (less) risk averse than the manager, the presence of a benchmark decreases (increases) utility. This effect cancels out if $\rho = 1$. The third term is a penalty for the additional variance of the benchmark portfolio, which is increasing with the risk aversion of the investor. Therefore, it is not clear whether benchmarking increases or decreases expected utility. However, the impact of benchmarking is negative as long as the investor is more risk averse than the manager, i.e., when we have $\rho < 1$.

Proposition 7. *When the manager has a performance objective relative to a benchmark portfolio ϕ , the certainty equivalent of delegation is defined as*

$$\text{CED}_{bm}^j := \sup \left[\delta \mid \delta \leq \sum_{l=1}^L \left(a \hat{\lambda}_{m,l}^{-1} - b \hat{\lambda}_{j,l}^{-1} \right) X_l + L(b - a) + (1 - \rho) \mu_\phi - \frac{1}{2} \rho_j \widehat{\Sigma}_\phi \right], \tag{33}$$

where $\mu_\phi := \phi'(\boldsymbol{\mu} - \mathbf{p}r)$ is the expected return and $\widehat{\Sigma}_\phi := \phi' \widehat{\Sigma}_m \phi = \sum_{l=1}^L \hat{\lambda}_{m,l} (\phi' \boldsymbol{\Gamma}_l)^2$ is the variance of the benchmark portfolio.

As before, the investor delegates investment when the certainty equivalence is higher than the cost for delegation and invests directly otherwise. In contrast to the case without benchmarking, the expected return and the variance of the benchmark portfolio matter for investment delegation. In particular, CED_{bm} is decreasing in the variance of the benchmark portfolio. A riskier benchmark decreases the likelihood of delegation.

Figure 5 plots the certainty equivalent as a function of the risk aversion ratio ρ . The size of the impact of benchmarking depends on the riskiness of the benchmark. We assume three cases. In the first case, we do not include benchmarking, i.e., we set $\phi = 0$. In the second case, we consider a ‘normal’ benchmark ϕ_2 , which equals

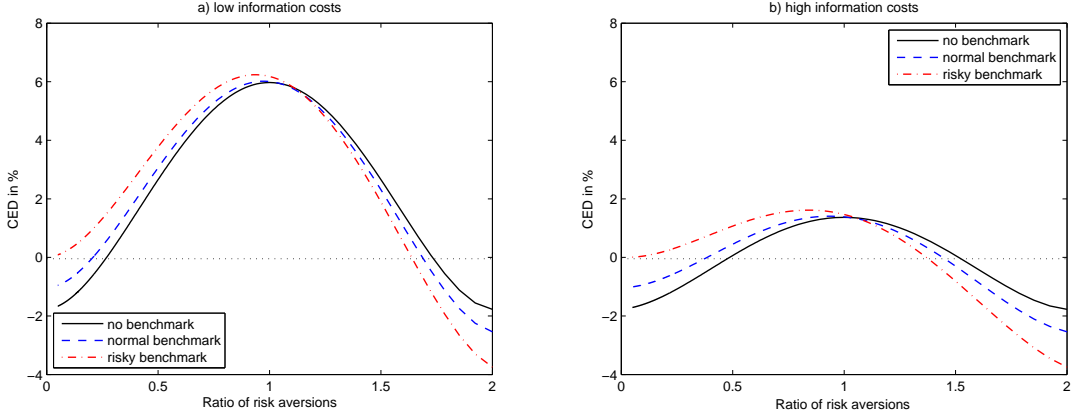


Figure 5: The certainty equivalent of delegation as a function of ρ for three different types of benchmarks. The investor’s risk aversion is 5 and information capacity $K_j = 10$. The solid line denotes the case without benchmarking; the normal benchmark corresponds to the prior mean-variance efficient portfolio where the risk aversion equals the investor’s risk aversion. The risky benchmark is a prior mean-variance efficient portfolio with risk aversion coefficient equal to two. Panel a) plots the CED for low information costs and panel b) plots the CED for high information costs. Calculations are based on the three-assets example in Table 2.

the prior mean-variance efficient portfolio when using ρ_j as risk aversion coefficient. In our numerical example, this choice implies an annualized volatility of 13.45% for the benchmark portfolio. In the third case, we consider the prior mean-variance efficient portfolio as ‘risky’ benchmark by setting the risk aversion coefficient equal to two, which implies an annualized benchmark volatility of 33.54%. In Figure 5, we observe that adding a benchmark has an asymmetric impact on the certainty equivalent wealth. The presence of a benchmark tends to increase the certainty equivalent if the risk aversion ratio is smaller than one and tends to lower the CED for a risk aversion ratio above one.

2.3.1 Information acquisition in presence of a benchmark

As in the case without benchmarking, the manager’s information choice problem amounts to maximizing the certainty equivalent of delegation minus the cost of information capacity,

$$\max_{\{\hat{\lambda}_{m,1}, \dots, \hat{\lambda}_{m,L}\}} \sum_{l=1}^L \left(a \hat{\lambda}_{m,l}^{-1} - b \hat{\lambda}_{j,l}^{-1} \right) X_l - L(a-b) + (1-\rho)\mu_\phi - \frac{1}{2}\rho_j \hat{\Sigma}_\phi - c \left(\hat{\lambda}_{m,l}^{-1} - \lambda_l^{-1} \right)^\psi, \quad (34)$$

subject to the nonnegative-learning constraint. Unfortunately, the extra term of the variance of the benchmark portfolio leads to additional complexity and we cannot solve the maximization problem (34) in closed-form.

In Table 4, we report some numerical results for the manager’s information ac-

Optimal Learning with Benchmarking

<i>Panel A: Standard deviations of risk factors (in % p.a.)</i>									
	Factor 1			Factor 2			Factor 3		
	ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3
Prior	16.34	16.34	16.34	31.22	31.22	31.22	38.02	38.02	38.02
c high	16.30	16.30	16.30	30.22	30.20	30.09	35.48	35.44	35.25
c moderate	16.15	16.15	16.14	26.99	26.94	26.71	28.81	28.77	28.55
c low	15.95	15.95	15.95	24.11	24.07	23.86	24.13	24.10	23.98

<i>Panel B: Standard deviations of asset payoffs (in % p.a.)</i>									
	SW Energy			Merck & Co.			Pfizer		
	ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3
Prior	37.96	37.96	37.96	25.09	25.09	25.09	24.84	24.84	24.84
c high	35.43	35.39	35.20	24.44	24.43	24.36	24.19	24.18	24.11
c moderate	28.78	28.73	28.52	22.37	22.34	22.19	22.15	22.12	21.99
c low	24.11	24.08	23.96	20.54	20.52	20.39	20.37	20.35	20.22

Table 4: Optimal learning with benchmarking. All numbers are based on the three-assets example in Table 2. The columns denoted with ϕ exhibit different benchmark regimes: ϕ_1 is the case without benchmarking; ϕ_2 is the case with normal benchmarking, where the benchmark portfolio is the prior mean-variance efficient portfolio with risk aversion equal to 5, implying an annualized benchmark volatility of 15.22%; ϕ_3 is the case with a risky benchmark, where the risk aversion coefficient is 2, implying an annualized benchmark volatility of 38.07%. The other parameters are: $\rho_j = 5$, $\rho_m = 5$, and $\psi = 2$.

quisition problem with benchmarking. We find that the impact of the benchmark on information acquisition is marginal. A possible explanation lies in the independence of the active component of the portfolio from the benchmark (see equation (31)). Furthermore, the active component of the benchmarking portfolio \mathbf{q}_{bm} equals the optimal portfolio without benchmarking. Hence, it is intuitively reasonable that benchmarking has no dominant effect on the information acquisition process. The small differences between the three benchmarking regimes given in Table 4 might be explained by the fact that the optimal information acquisition strategy reacts to the investor’s CED, which is slightly different across the three benchmarking regimes.

3 Equilibrium Impact of Delegation under Benchmarking

Equilibrium models with asymmetric information have a long tradition in finance. Grossman and Stiglitz (1980) establish the concept of a *noisy rational expectations equilibrium*. A noisy rational expectations equilibrium has two important characteristics. First, the price system makes publicly available the information obtained by informed individuals to uninformed or less-informed individuals. Thus, all individuals

do recognize any information comprised in asset prices when specifying their asset demand. Second, the price has a noisy and unlearnable component such that information is not fully revealed by prices. This property is a necessary condition for an equilibrium to exist. A price system that perfectly aggregates information is not robust. The more informative prices become, the lower are the incentives to acquire private information. However, the price can not be informative if nobody collects private information. This paradox on the impossibility of informationally efficient markets was first pointed out by Grossman (1976).

Many subsequent papers in the literature on asymmetric information have built on the framework of Grossman and Stiglitz (1980). Hellwig (1980) and Verrecchia (1982) extend the base model to allow agents to choose the precision of their private signals. Admati (1985) extends the noisy rational expectations equilibrium model to the case with multiple risky asset. Instead of choosing their signal precision, agents are endowed with heterogenous information. In this section, we base our analysis on the equilibrium analysis of Admati (1985).

3.1 The Economy

We assume a continuum of $j \in [0, 1]$ investors. We denote by n the fraction of delegating investors. If $j \in [0, n]$, then investor j delegates the portfolio allocation. If $j \in (n, 1]$, the investor invests directly in the market. The fraction of delegating investors is determined by the certainty equivalent rule, i.e., $\text{CED}_j > \alpha_j \Leftrightarrow \forall j \in [0, n]$, and $\text{CED}_j < \alpha_j \Leftrightarrow \forall j \in (n, 1]$, where α_j is the management fee faced by the j th investor. For simplicity, we assume that each investor has a specific manager associated with her. Each manager charges an individual fee α_j .

Asset prices \mathbf{p} are determined by market clearing. The per capita risky asset supply is $\bar{\mathbf{x}} + \mathbf{x}$, with $\mathbf{x} \sim \mathcal{N}(0, \sigma_x^2 \mathbf{I})$. The noise in the asset supply assures that the price never fully reveals all private information. This extra source of randomness can be, e.g., due to liquidity traders. The market clears when the demand of all agents, direct investors and delegated managers, equals supply:

$$\int_0^n \mathbf{q}_{del}^j dj + \int_n^1 \mathbf{q}_{dir}^j dj = \bar{\mathbf{x}} + \mathbf{x}, \quad (35)$$

where $\mathbf{q}_{del}^j = \phi + \frac{1}{\rho_m} \widehat{\Sigma}_m^j (\hat{\boldsymbol{\mu}}_m^j - \mathbf{p}r)$ denotes the demand for risky assets of the delegating investor's manager and $\mathbf{q}_{dir}^j = \frac{1}{\rho_j} \widehat{\Sigma}_j (\hat{\boldsymbol{\mu}}_j - \mathbf{p}r)$ is the demand of the direct investor.¹² Following Admati (1985), we prove in Corollary 5.1 in the appendix that the price is a

¹²Since we assume that each investor j is associated with one specific manager, we index the manager's posterior with the index j .

linear function of the asset payoff and the unexpected component of the asset supply,

$$\mathbf{p} = \frac{1}{r} (\mathbf{A} + \mathbf{B}\mathbf{f} + \mathbf{C}\mathbf{x}). \quad (36)$$

In contrast to the partial equilibrium model, where the investor has two pieces of information (prior and signal) to form the posterior belief, there is an additional source of information in general equilibrium, the information inferred from equilibrium prices. Therefore, by observing equilibrium prices, agents can infer part of the other agents' private information. The noise due to liquidity traders, $\bar{\mathbf{x}} + \mathbf{x}$, prevents the price system from fully revealing all private information.

Thus, in general equilibrium, the posterior belief about the asset payoff \mathbf{f} , is conditional on prior information, $\boldsymbol{\mu} \sim \mathcal{N}(\mathbf{f}, \boldsymbol{\Sigma})$, information observed from the signal $\mathbf{s}_j \sim \mathcal{N}(\mathbf{f}, \boldsymbol{\Omega}_j)$, and information inferred from equilibrium prices, $\mathbf{f}|\mathbf{p} \sim \mathcal{N}(\mathbf{B}^{-1}(\mathbf{p}r - \mathbf{A}), \boldsymbol{\Sigma}_p)$ with

$$\boldsymbol{\Sigma}_p := \text{Var}[\mathbf{f}|\mathbf{p}] = \sigma_x^2 \mathbf{B}^{-1} \mathbf{C} \mathbf{C}' (\mathbf{B}^{-1})'.$$

Standard Bayesian updating implies that the posterior mean about \mathbf{f} is

$$\hat{\boldsymbol{\mu}}_j := \left(\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Omega}_j^{-1} + \boldsymbol{\Sigma}_p^{-1} \right)^{-1} \left(\boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} + \boldsymbol{\Omega}_j^{-1} \mathbf{s}_j + \boldsymbol{\Sigma}_p^{-1} \mathbf{B}^{-1} (\mathbf{p}r - \mathbf{A}) \right), \quad (37)$$

with variance

$$\hat{\boldsymbol{\Sigma}}_j := \left(\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Omega}_j^{-1} + \boldsymbol{\Sigma}_p^{-1} \right)^{-1}. \quad (38)$$

Note that all investors (and managers) $j \in [0, 1]$ start with identical priors. Information inferred from observed prices is equal for all investors as well. Only the signal \mathbf{s}_j and the signal precision matrix $\boldsymbol{\Omega}_j^{-1}$ are investor specific. Therefore, heterogeneity of investors in the model enters only through the signal and signal precision.

3.2 Certainty Equivalent of Delegation

Optimal delegation in general equilibrium follows the same reasoning as in partial equilibrium. The only difference is the fact that prices are not constant and contain information about the private information of other market participants. The existence of so called noise traders or liquidity traders prevents the equilibrium price from being fully revealing. This property of the general equilibrium will eventually influence the optimal information choice of the investor and of the manager, which in turn influences the certainty equivalent of delegation.

Proposition 8. *In general equilibrium, the certainty equivalent of delegation for in-*

vestor j with risk aversion ρ_j is given as

$$\begin{aligned} \text{CED}_E^j := \sup \left[\delta \mid \delta \leq \sum_{l=1}^L \left(a_j (\hat{\lambda}_{m,l}^j)^{-1} - b_j \hat{\lambda}_{j,l}^{-1} \right) X_l^E \right. \\ \left. + L(b_j - a_j) + \left(1 - \frac{\rho_j}{\rho_m} \right) \mu_\phi - \frac{1}{2} \rho_j \hat{\Sigma}_\phi^j \right], \end{aligned} \quad (39)$$

where

$$X_l^E := \lambda_l (1 - \lambda_{B,l})^2 + \lambda_{p,l} + [((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{A})' \boldsymbol{\Gamma}_l]^2 \quad (40)$$

with $a_j := \left(1 - \frac{1}{2} \frac{\rho_j}{\rho_m} \right) \frac{1}{\rho_m}$, $b_j := \frac{1}{2\rho_j}$, $\mu_\phi := \boldsymbol{\phi}'(\boldsymbol{\mu} - \mathbf{p}r)$, $\hat{\Sigma}_\phi^j := \boldsymbol{\phi}' \hat{\Sigma}_m^j \boldsymbol{\phi}$, and $\lambda_{B,l}$ and $\lambda_{p,l}$ denote the eigenvalues of \mathbf{B} and $\boldsymbol{\Sigma}_p$, respectively.

Investor j delegates if and only if the certainty equivalent of delegation is higher than the management fee, $\text{CED}_E^j > \alpha_j$. As in partial equilibrium, X_l^E is independent from any private information, i.e., every investor faces the same learning index. Furthermore, the CED in general equilibrium is also increasing in manager skill and the index X_l^E . However, in contrast to the partial equilibrium result, the learning index X_l^E and hence the CED is additionally driven by the information efficiency of prices.

First, note that X_l^E depends on $\lambda_{B,l}$. The eigenvalue $\lambda_{B,l}$ captures the dependence of the l th risk factor's price with its true payoff. A high value indicates a strong dependence between the price and the future payoff. Equation (40) predicts a negative relation between $\lambda_{B,l}$ and X_l^E . In other words, when the price reflects future asset payoffs accurately, delegation is less attractive. The utility from delegation is thus higher for a financial market in which prices are imprecise predictors of future asset payoffs. When $\lambda_{B,l}$ is small, the first term of the index X_l^E is high, which makes delegation more valuable.

Second, X_l^E also depends on the eigenvalue $\lambda_{p,l}$ of $\text{Var}[\mathbf{f}|\mathbf{p}]$, the conditional variance of risk factor l . This eigenvalue depends not only on \mathbf{B} , but also on the variance of the supply shock σ_x^2 and on \mathbf{C} , the correlation between the price and the supply shocks \mathbf{x} . The higher $\lambda_{p,l}$, the noisier the l th risk factor's price. Therefore, the price contains less information about future payoffs. In such an environment, it is more difficult to predict future payoffs and delegation becomes more valuable. When $\lambda_{p,l}$ is high, the second term of the index X_l^E is also high, which makes delegation more valuable.

3.3 The Equilibrium Price in Presence of Delegation

How does delegation under benchmarking impact risky asset prices in equilibrium? Inspired by the model of Admati (1985), we derive the equilibrium price function in presence of delegated portfolio management with benchmarking.

Proposition 9. *In presence of delegation, the equilibrium price is a function of the ‘average’ investor’s posterior mean and the covariance with the residual portfolio, where the residual portfolio is the difference between the market portfolio and the benchmark portfolio,*

$$\mathbf{p} = \frac{1}{r} \left(\hat{\boldsymbol{\mu}}_a - \rho_a \hat{\boldsymbol{\Sigma}}_a (\mathbf{x}_{mkt} - n\boldsymbol{\phi}) \right). \quad (41)$$

We present the expressions for $\hat{\boldsymbol{\mu}}_a$, $\hat{\boldsymbol{\Sigma}}_a$ and ρ_a in the appendix and the market portfolio is defined as $\mathbf{x}_{mkt} := \bar{\mathbf{x}} - \mathbf{x}$.

It turns out that the belief of the ‘average’ investor is driven by both groups of agents, the individual investors and the managers. The higher the number of delegating investors, the more the ‘average’ investor reflects manager beliefs, and vice versa. As long as there is no benchmark ($\phi_i = 0 \forall i$), prices are determined by their expected payoff minus their covariances with the market portfolio, which corresponds to the classic result of the CAPM. Proposition 9 shows that in presence of a benchmark, asset prices are determined by their covariance with the residual portfolio, i.e. the difference between the market portfolio and the weighted benchmark portfolio. Therefore, benchmarking leads to a positive shift in asset demand for those assets included in the benchmark portfolio. A high weight of an asset in the benchmark creates a steady demand for this asset independent of any beliefs. A high and positive weight in the benchmark portfolio decreases the residual portfolio and leads to a higher price. This effect increases in the fraction of delegating agents n .

The ‘average’ investor in our model reflects the average of the actions taken by all agents in the economy. Furthermore, $\hat{\boldsymbol{\mu}}_a$ and $\hat{\boldsymbol{\Sigma}}_a$ are not observable. The agents act based upon their individual signals and the information revealed by equilibrium prices but do not observe the true market portfolio, due to the additional noise in total supply, \mathbf{x} . Therefore, in our framework, the market portfolio is not observable ex ante. As pointed out by Biais, Bossaerts, and Spatt (2004), such a setup is in line with the critique of Roll (1977), who emphasized that the market portfolio is not observable in general.

3.4 Expected Returns

Rearranging equation (41), we can write the expected dollar excess return from the viewpoint of the ‘average’ investor as

$$E_a[\mathbf{f} - \mathbf{p}r | \boldsymbol{\mu}] = \rho_a \left(\hat{\boldsymbol{\Sigma}}_a \bar{\mathbf{x}} - n \hat{\boldsymbol{\Sigma}}_a \boldsymbol{\phi} \right), \quad (42)$$

where we use E_a to indicate the expectation operator conditional on the information set of the ‘average’ investor. Equation (42) expresses the expected excess return on an asset

as a linear function of the asset's covariance with the portfolio $\bar{\mathbf{x}}$ and its covariance with the benchmark portfolio ϕ . Note that the expected return is increasing in the covariance with the market portfolio and decreasing with the benchmark portfolio. The higher the asset's correlation with the benchmark, the stronger this effect. The negative impact due to benchmarking is driven by, first, the asset's weight in the benchmark portfolio and, second, by the fraction of delegating investors n . When $n = 0$, equation (42) collapses to the standard CAPM. Furthermore, the equilibrium expected return is increasing with the risk aversion ρ_a of the 'average' investor.

From the viewpoint of the manager, the benchmark portfolio is the risk-free asset. Holding a portfolio of assets that perfectly matches the composition of the benchmark is thus a risk free portfolio for the manager. In this case, the manager does not demand a premium for systematic risk. However, holding some assets in other proportions than provided in the benchmark, the manager would require a risk premium for the fraction that deviates from the benchmark weight. The higher the fraction of benchmarked managers in an economy, the stronger this effect will be. Therefore, benchmarking directly reduces the risk that must be borne and reflected in security prices and, as a consequence, lowers the expected returns of assets included in the benchmark portfolio.¹³ We summarize this important finding in the following proposition.

Proposition 10. *In an economy with delegation under benchmarking, only the risk arising from active portfolio management is priced in equilibrium and it is priced proportional to the covariance with the residual portfolio $\hat{\Sigma}_a(\bar{\mathbf{x}} - n\phi)$.*

So far, we have not specified the per capita market capitalization of the benchmark portfolio ϕ . The capitalization of the benchmark can take any value between zero and total market capitalization \mathbf{x}_{mkt} . If the benchmark capitalization were equal to that of the market portfolio, a passive manager would hold exactly the market portfolio. However, we can also think of a benchmark with constituents that have a lower market capitalization than their counterparts in the market portfolio. In this case $\mathbf{x}'_{mkt}\mathbf{1} > \phi'\mathbf{1}$ and a passive manager (with risk aversion $\rho_m = \infty$) would only invest a fraction of his wealth into the benchmark portfolio, since $\phi'\mathbf{1} < 1$. In this case, the remainder is invested into the risk free asset. Denote the fully invested benchmark portfolio as ϕ_{full} . We can then define the market capitalization of the actual benchmark portfolio as:

$$\phi = h \phi_{full}, \tag{43}$$

where h is a scaling factor $0 \leq h \leq 1$ that determines the total market capitalization of the benchmark portfolio. h is referred to as the benchmark scaling factor.

¹³Brennan (1993) and Cornell and Roll (2005) find similar theoretical results. Brennan (1995), Gomez and Zapatero (2003) and Brennan and Li (2008) provide empirical evidence for their model.

3.4.1 A Two-Factor CAPM

Based on the equilibrium expected return given in equation (42), we derive now an expected return-beta relationship as in the standard CAPM. For a single asset i the expected excess return R_i can be written as

$$E_a[f_i - p_i r] = \rho_a \sum_{k=1}^N Cov_a(f_i, f_k) \bar{x}^k - \rho_a n \sum_{k=1}^N Cov_a(f_i, f_k) \phi^k := E_a[R_i], \quad (44)$$

where \bar{x}^k and ϕ^k is the weight of asset k , respectively, in the market portfolio and the benchmark portfolio. Define the payoff of the market portfolio $f_{mkt} := \sum_{i=1}^N f_i \bar{x}^i$, and the payoff of the benchmark portfolio $f_\phi := \sum_{i=1}^N f_i \phi^i$. Then, we can write the expected return as

$$E_a[R_i] = \rho_a Cov_a(f_i, f_{mkt}) - \rho_a n Cov_a(f_i, f_\phi). \quad (45)$$

Recall the definition of the market beta $\beta_{mkt}^i = Cov(f_i, f_{mkt})/Var_a(f_{mkt})$ and the benchmark beta $\beta_\phi^i = Cov(f_i, f_\phi)/Var_a(f_\phi)$, where $Var_a(f_{mkt})$ and $Var_a(f_\phi)$ denotes the volatility of the market portfolio and the benchmark portfolio, respectively. With these definitions, we can write:

$$E_a[R_i] = \rho_a Var_a(f_{mkt}) \beta_{mkt}^i - \rho_a n Var_a(f_\phi) \beta_\phi^i. \quad (46)$$

Following Brennan (1993), we can define the return of the benchmark portfolio R_ϕ as a linear combination of the return on the market portfolio and a residual return component orthogonal to the market return, i.e., $R_\phi = \alpha_\phi + \beta_{mkt}^\phi R_{mkt} + \varepsilon$. Using this result, the expected return may be expressed in terms of the market beta and the beta of the residual portfolio. This gives rise to a two-factor CAPM:

$$E_a[R_i] = \underbrace{\rho_a Var_a(f_{mkt}) (1 - n \beta_{mkt}^\phi)}_{\lambda_{mkt}} \beta_{mkt}^i - \underbrace{\rho_a n Var_a(f_\varepsilon)}_{\lambda_\varepsilon} \beta_\varepsilon^i, \quad (47)$$

where $Var_a(f_\varepsilon)$ is the variance of the residual portfolio and $\beta_\varepsilon^i = Cov_a(f_i, f_\varepsilon)/Var_a(f_\varepsilon)$ is the beta of the residual portfolio for asset i . We can interpret λ_ε as the price of active management risk and λ_{mkt} as the market price of risk. For $\beta_{mkt}^\phi > 0$ and $\beta_{mkt}^i > 0$, the market price of risk is decreasing in the number of delegating investors and in the beta between the market and the benchmark portfolio. The higher the (positive) correlation between the market portfolio and the benchmark, the stronger this effect. Furthermore, we notice that λ_{mkt} increases with a higher average risk aversion ρ_a . If the fraction of delegation is equal to one and the benchmark perfectly matches the market portfolio, the market price of risk would degenerate to zero.

3.4.2 The Adjusted Beta

Another way to see the impact of portfolio management delegation is to express expected returns as a function of the excess return of the market portfolio. This gives rise to an *adjusted beta*. Recall that the expected excess return of asset i can be written as

$$E_a[R_i] = \rho_a (Cov_a(f_i, f_{mkt}) - nCov_a(f_i, f_\phi)). \quad (48)$$

If this relationship holds true for asset i , it also holds true for the market portfolio. Thus,

$$E_a[R_{mkt}] = \rho_a (Var_a(f_{mkt}) - nCov_a(f_{mkt}, f_\phi)), \quad (49)$$

where $E_a[R_{mkt}]$ is the expected excess return on the market portfolio. Solving for ρ_a in equation (49) and substituting into equation (48), we find in a setting with delegation under benchmarking a linear expected return-beta relationship. The resulting beta is an adjusted version of the standard market beta:

$$E_a[R_i] = \beta_{adj}^i E_a[R_{mkt}] \quad (50)$$

where

$$\beta_{adj}^i = \frac{Cov_a(f_i, f_{mkt}) - nCov_a(f_i, f_\phi)}{Var_a(f_{mkt}) - nCov_a(f_{mkt}, f_\phi)}. \quad (51)$$

The adjusted beta, β_{adj}^i , differs from the standard CAPM beta β_{CAPM}^i in two respects. First, β_{adj}^i accounts for the number of delegated agents n and the covariance of the benchmark portfolio with the market and asset i . A high positive covariance between the market and the benchmark leads to a high beta for asset i . Also, a strong positive covariance between the asset and the benchmark tends to lower the adjusted beta.

Second, we notice that the beta without delegation under benchmarking but with information acquisition, say β_a^i , is not equal to the standard CAPM beta β_{CAPM}^i . Although the correlation structure of the assets is not changed through learning, the standard deviation of the assets is what agents in our model can reduce through learning. Since $\beta_a^i = Std_a(f_i)Corr(f_i, f_{mkt})/Std_a(f_{mkt})$, we observe that β_a^i tends to be lower than the standard CAPM beta for those assets for which agents acquire additional information. Since learning decreases the uncertainty about the future asset payoff, assets held by agents with high information capacity tend to exhibit a lower expected return than assets held by less sophisticated investors. Therefore, our model predicts a lower risk premium for those assets held by the asset managers.

Table 5 summarizes the different cases for the beta adjustment under the assumption $Var_a(f_{mkt}) > Cov_a(f_{mkt}, f_\phi) > 0$, which covers most of the practical cases. In some situations it is not possible to make a general statement about the direction of

Beta Adjustments

β_a^i	Benchmark-effect	Impact on β_{adj}^i
$\beta_a^i > 1$	$Cov_a(f_i, f_\phi) > Cov_a(f_{mkt}, f_\phi)$	ambiguous
	$Cov_a(f_i, f_\phi) = Cov_a(f_{mkt}, f_\phi)$	positive
	$Cov_a(f_i, f_\phi) < Cov_a(f_{mkt}, f_\phi)$	positive
$\beta_a^i = 1$	$Cov_a(f_i, f_\phi) > Cov_a(f_{mkt}, f_\phi)$	negative
	$Cov_a(f_i, f_\phi) = Cov_a(f_{mkt}, f_\phi)$	no impact
	$Cov_a(f_i, f_\phi) < Cov_a(f_{mkt}, f_\phi)$	positive
$\beta_a^i < 1$	$Cov_a(f_i, f_\phi) > Cov_a(f_{mkt}, f_\phi)$	negative
	$Cov_a(f_i, f_\phi) = Cov_a(f_{mkt}, f_\phi)$	negative
	$Cov_a(f_i, f_\phi) < Cov_a(f_{mkt}, f_\phi)$	ambiguous

Table 5: Impact of beta adjustment for different cases of benchmark correlation of asset i and the market portfolio. The coefficient β_a^i is defined as $\beta_a^i = Cov_a(f_i, f_{mkt})/Var_a(f_{mkt})$, i.e., the beta without delegation, but with information acquisition. ‘Ambiguous’ means that the effect of the adjustment can either be positive, negative or neutral. Further, we assume that $Var_a(f_{mkt}) > Cov_a(f_{mkt}, f_\phi) > 0$.

the adjustment. However, we notice that the correlation between the market and the benchmark usually takes values close to one for most widely used benchmarks such as, e.g., the S&P 500 Index for the U.S. stock market. Such a practice implies that for assets with moderate variance, we generally have $Cov_a(f_i, f_\phi) < Cov_a(f_{mkt}, f_\phi)$. But for assets with a high variance, it is possible to have $Cov_a(f_i, f_\phi) > Cov_a(f_{mkt}, f_\phi)$. Furthermore, assets with a big weight in the benchmark tend to have a higher covariance with the benchmark than other assets. When β_a^i takes value greater than one, the adjustment due to the presence of delegated agents and benchmarking is positive as long as the covariance of asset i with the benchmark is lower or equal to the covariance between the market and the benchmark, i.e., as long as $Cov_a(f_i, f_\phi) \leq Cov_a(f_{mkt}, f_\phi)$. Contrary, when the standard beta β_a^i takes values smaller than one, the adjustment is negative as long as $Cov_a(f_i, f_\phi) \geq Cov_a(f_{mkt}, f_\phi)$.

3.4.3 Numerical Illustration of Expected Returns

We illustrate the model’s implications using a numerical example for SWN (see Table 2). Figure 6 shows how the presence of better informed delegated agents affects equilibrium returns. To clearly isolate the effect of delegation, we assume that the individual investor does not observe any informative signal \mathbf{s}_j . Therefore, the investor can only use information observed from the equilibrium price to build up her posterior belief. For both panels, the annualized true excess return $(f_i - p_i r)/p_i$ equals 15.83% and the annualized prior expected excess return $(\mu_i - p_i r)/p_i$ equals 17.72% for all agents as long as $n = 0$, i.e., in the case without delegation. Note that f_i and μ_i are fixed parameters

in the model. However, because expected returns depend on realized prices p_i , not only posterior expected returns but also the true and the prior return vary with the other parameters such as the fraction of delegation n , the benchmark scaling factor h , and manager skill.

Panel a) in Figure 6 shows expected excess returns as a function of n for three different levels of manager skill. To isolate the effect of manager skill from the effect of benchmarking, we set $h = 0$, i.e., there is no benchmarking. The plot confirms that expected returns are decreasing in manager skill. Since delegation takes place only when the manager's posterior signal precision is higher than that of the investor, the presence of delegated agents lowers $\widehat{\Sigma}_a$ and leads to a more informative price system. Since, in addition the fraction of delegating investors increases with manager skill, the decrease of the expected return can be substantial. For instance, assuming high manager skills, the expected return for SWN drops from an expected excess return with 100% uninformed investors of 17.72% down to 7.01%, if 80% of the investors decide to delegate. Note that when there are only uninformed investors, the expected return implied by the model equals the prior expected return, because no agent acquires information and therefore no additional information is discovered through prices.

We now turn to the impact of benchmarking on expected returns. In Panel b) of Figure 6, we plot expected excess returns of SWN for different levels of the benchmark scaling factor h . Clearly, when the fraction of delegation n approaches 1, the price will rise such that expected excess returns fall to zero, because an active manager would not require any risk premium for systematic risk as long as ϕ equals \bar{x} . When $n < 1$, there will always be an agent demanding a premium for holding systematic market risk. Therefore, the expected excess return will be strictly positive. However, the expected excess return will be smaller the larger the market capitalization of the benchmark portfolio. Therefore, taking Panel a) and b) together, we conclude that delegation under benchmarking can lead to a large and economically significant decrease in expected excess returns.

4 Empirical Implications of the Model

The goal of this section is to find empirical support for some of the predictions of our theory. One of the main results from the equilibrium analysis is that the presence of institutional investors lowers a stock's expected return. The effect is more pronounced when institutional ownership is high. The theory further states that in presence of benchmarking, a two-factor CAPM holds. Besides market risk, the second risk factor is active management risk. The model predicts that the price of this new risk factor is

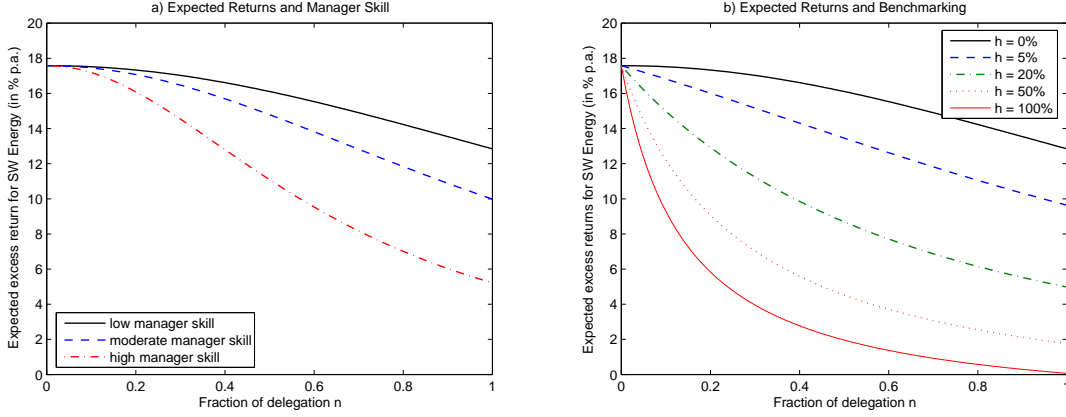


Figure 6: Impact of manager skill and benchmarking on equilibrium expected excess returns in a model with delegated agents. Panel a) exhibits expected excess returns as a function of n for three different levels of manager skill. Panel b) exhibits expected excess returns as a function of n for different levels of benchmark capitalizations h . The prior beliefs are based on historical data for Southwestern Energy, given in Table 2. Prices are normalized to one such that the historical return equals the expected payoff μ . The true payoff f_i is chosen to be 0.192, μ_i is 0.212, and the risk-free rate is 2.40% p.a.

negative, indicating that the expected return is lower for stocks with a high exposure to active management risk.

4.1 Hypothesis

Based on our theoretical results, we establish two hypothesis. Hypothesis 1 claims that institutional ownership (IO) has power to explain the cross-section of returns.

Hypothesis 1. *The fraction of institutional ownership (IO) impacts the cross-section of returns. Stocks with high institutional ownership exhibit lower returns than stocks with low institutional ownership.*

Hypothesis 1 follows from the observation of our model that delegation only takes place when the manager is skilled, i.e. when he possesses more precise information than the individual investor. The manager will primarily invest in assets where he has an informational advantage. This leads to a higher demand for those stocks institutional managers learn about and invest in, which in turn lowers expected returns.

Hypothesis 2 is motivated by the two-factor CAPM established in the previous section.

Hypothesis 2. *The presence of benchmarking gives rise to an additional risk factor that accounts for active management risk. The price of active management risk is negative*

and increases with the overall level of delegation in the economy. A positive loading on active management risk is associated with lower expected returns.

Gomez and Zapatero (2003) and Brennan and Li (2008) both provide empirical support for Hypothesis 2. Performing Fama and MacBeth (1973) type regressions, they find strong evidence that in the recent period the (negative) price of active management risk is statistically significant, i.e. that stocks whose returns covary stronger with the residual component of the S&P 500 index return exhibit significantly lower returns. The objective of this section is to demonstrate that institutional ownership matters for explaining cross-sectional differences in returns. We therefore focus on Hypothesis 1 and try to find empirical evidence for the relationship between a stock's return and its fraction of institutional ownership.

In an insightful paper, Nagel (2005) demonstrates that IO helps explaining stock return anomalies. More specifically, he finds that when controlling for size, the under-performance of stocks with high market-to-book ratios, analyst forecast dispersion, turnover, or volatility is most pronounced among stocks with low institutional ownership. More recently, Phalippou (2008) documents that the value premium exists only for stocks with low institutional ownership.

4.2 Data and Methodology

For our empirical analysis, we use a comprehensive sample of U.S. firms covering the time period from March 1980 to December 2007. The data come from three primary sources. Data on institutional ownership is from CDA/Spectrum 13F Institutional Holdings (s34) database, data on analyst coverage from I/B/E/S International Corp. and stock market data from the Center for Research in Security Prices (CRSP).

In the U.S. large investment managers such as banks, insurance companies, mutual funds, pension funds, and other investment advisors with assets under discretionary management of at least USD 100 million are required to report their equity holdings to the SEC by filing the 13F form¹⁴. All stock positions greater than 10,000 shares or USD 200,000 must be disclosed on a quarterly basis, which leads to a comprehensive data sample on U.S. institutional ownership. For every U.S. corporation, this data set provides us with the total number of shares held by institutions, which facilitates to calculate the exact fraction of institutional ownership (IO). The data is collected by CDA/Spectrum and made accessible to us by Thomson Reuters. Institutional holdings are available beginning in March 1980. Gompers and Metrick (2001) give a more detailed

¹⁴See United States Securities and Exchange Commission, Form 13F for more details, <http://www.sec.gov/about/forms/form13f.pdf>.

description of the CDA/Spectrum database¹⁵. In what follows, we refer to institutional investors as investment intermediaries or managers that are obligated to report their equity holdings to the SEC. Using this definition of institutional investors, it is important to notice that small investment intermediaries are classified as private investors, as long as they do not file the 13F reports.

To provide additional information, we also collect data on analyst coverage. Following Hong, Him, and Stein (2000), coverage in any given quarter is set equal to the number of I/B/E/S analysts who provide fiscal year 1 earnings estimates for that quarter. Data on stock returns, prices and shares outstanding is from the CRSP monthly stock file that includes NYSE, AMEX, and NASDAQ stocks. To calculate quarterly cumulative returns, we require a minimum track record of three month starting in the first month of the quarter for a stock to be included in the sample. Further, we require that prices and total shares outstanding as of the end of the quarter are available. The CRSP file with quarterly data on returns, prices, and total shares outstanding defines the universe of stocks included in the empirical analysis. Stocks that are in the CDA/Spectrum or I/B/E/S database but cannot be matched¹⁶ to a stock in the CRSP file are excluded. Stocks that are on the CRSP file but without any data on institutional holdings or analyst coverage are assumed to have zero IO and analyst coverage, respectively.

To arrive at our final sample, we perform different screens. First, we exclude all non-equity issues such as certificates, ADRs, SBIs, and Units. Second, we exclude companies incorporated outside the U.S., primes and scores, closed-end funds, and REITs. In particular, we exclude all stocks that do not have a CRSP share type code of 10 or 11. Analyzing institutional ownership with CDA/Spectrum holdings data, it is important to exclude foreign stocks listed in the U.S. from the analysis. For foreign stocks, we expect that U.S. institutional investors only account for a small fraction of IO. Hence, institutional holdings reported to the SEC is far below the true number of institutionally held stocks, leading to a significant underestimation of IO for foreign stocks listed in the U.S. Third, we exclude small stocks having market capitalization below USD 10 million and stocks with a share price smaller than five USD¹⁷. Applying these filters¹⁸,

¹⁵Note that holdings of mutual funds are reported on fund family level rather than for every fund separately. CDA/Spectrum also provides holdings data on a single fund level. Since the focus of our analysis is on the impact of aggregate institutional ownership (where mutual funds are just one subgroup), a breakdown of mutual fund families into single funds is not of interest for us.

¹⁶Stocks from CDA/Spectrum and I/B/E/S are assigned to the sample of CRSP stocks based on the cusip identification number.

¹⁷We repeat the analysis for a cutoff level of three USD and also for the case where no stocks are excluded due to a price level criterium. The main results are not affected. However, results for the first size, IO, and RO decile are much more stable when excluding stocks with a price below five dollars.

¹⁸After screening for late filers and stale data, for some stocks the number of institutionally held shares still exceeds total shares outstanding in certain quarters, which is obviously a problem in data quality. Officials from Thomson Reuters told us that this problem is ascribed to wrong filings by institutions and

we arrive at a final sample of 13,675 companies and 247,574 quarterly observations for the time period between March 1980 and December 2007. This is an average number of 2,210 observations per quarter.

Table 6 provides a year by year overview of the final sample. The second column gives the total number of observations per year. We notice that data availability for the first three years is very limited. Columns 3 and 4 exhibit mean and median quarterly returns. The median is robust to outliers and gives a more accurate picture of returns in a given quarter. Columns 5 and 6 exhibit mean and median numbers of institutional ownership (IO). IO is calculated by summing the stock holdings of all institutional investors and dividing it by the total amount of shares outstanding for each stock in each quarter. IO for stock i is defined as

$$IO_i := \frac{\sum_{j=1}^N SHARES_{i,j}}{SHROUT_i}, \quad (52)$$

where IO_i is the fraction of institutional ownership of stock i , $SHARES_{i,j}$ is the number of shares of stock i held by institution j , $SHROUT_i$ is the total number of shares outstanding for stock i and N is the total number of institutions holding stock i . The numbers on IO confirm the trend observed in Table 1 in the introduction. During the sample period, mean (median) IO increased from 17.3% (7.5%) to 50.4% (55.0%). Ovtcharova (2003) report similar results. Notice that Gompers and Metrick (2001) analyze only stocks that are included in the CDA/Spectrum database. Since we join CDA/Spectrum data with the entire universe of U.S. stocks in the CRSP monthly stock file and set IO equal to zero for those stocks not matched in the CDA/Spectrum database, our numbers on IO are significantly lower than those reported in Gompers and Metrick (2001). Other authors that have analyzed institutional ownership do not report IO over time.

Regarding our methodology to calculate IO, two issues should be noticed that have an impact on the results. First, since small institutions and foreign investment intermediaries are not required to file 13F reports and are considered as individual investors, reported numbers on IO underestimate the numbers disclosed by the Fed, as given in Table 1. Second, some of the growth in IO is due to small institutions that crossed the cutoff level for institutional investors only because an increase in market value pushed their assets under management above the threshold of USD 100 million. Hence, we assume that a small portion of growth in IO is not due to a ‘real’ change in institutional holdings, since the nominal threshold of USD 100 million did not change with the growth in market value of total equity. However, as pointed out by Gompers and

cannot be corrected by the database vendor. Therefore, we delete all observations where the number of institutionally held shares exceeds total shares outstanding. This is the case for about 4,145 observations in our sample.

Metrick (2001), this bias is very small.

Columns 7 and 8 exhibit mean and median analyst coverage. We observe a moderate increase in both columns. In 2007, the median group of firms has 4 analysts providing fiscal year 1 earnings forecasts. Since our sample is relatively large compared to other studies and includes many small capitalization stocks that are not followed by analysts, our numbers on analyst coverage are small. The average and median number of institutions holding a share of equity significantly increased over the past 28 years, as can be seen from columns 9 and 10 in Table 6. The big gap between mean and median figures indicates that there exists a small group of stocks with a high number of institutional investors whereas the main part of our sample stocks exhibit significantly lower numbers of institutional investors. Mean and median size are depicted in columns 11 and 12. Mean size dramatically increased over the sample period. Again, the big gap between mean and median figures indicates an increasing concentration in market capitalization among a small group of firms.

As we will see later on, IO strongly depends on size. Since our theory does not make any predictions about the influence of size, we will test whether IO can explain the cross-section of stock returns when holding size fixed. To control for size, we follow Hong, Him, and Stein (2000) and Nagel (2005) and sort stocks based on *residual institutional ownership (RO)*. RO is obtained as the residual of a cross-sectional regression of IO on size. Since IO is bounded by 0 and 1 we perform a logit transformation as follows:

$$\text{logit}(\text{IO}) = \log\left(\frac{\text{IO}}{1 - \text{IO}}\right). \quad (53)$$

As in Nagel (2005), values of IO below 0.0001 and above 0.9999 are replaced with 0.0001 and 0.9999, respectively. To get residual institutional ownership (RO), we estimate the following cross-sectional regression for each quarter t from March 1980 to December 2007,

$$\text{logit}(\text{IO}_{i,t}) = \delta_0 + \delta_1 \log(\text{SIZE}_{i,t}) + \delta_2 \log(\text{SIZE}_{i,t})^2 + \epsilon_{i,t} \quad (54)$$

where the residual $\epsilon_{i,t}$ is residual ownership for stock i in quarter t , $\text{RO}_{i,t}$.

Sample Statistics

Year	N	Qtr Return (%)		IO (%)		Analysts		Institutions		ln(Size)	
		Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median
1980	185	10.00	8.16	17.32	7.48	2.39	0.00	33.49	5.00	20.18	19.09
1981	228	7.54	7.27	15.74	8.02	2.05	0.00	28.96	5.00	20.11	18.68
1982	514	22.39	18.89	12.46	5.61	1.40	0.00	14.87	4.00	19.37	17.96
1983	3'258	9.15	7.15	12.10	5.89	0.57	0.00	11.31	4.00	18.91	17.99
1984	3'103	0.94	1.29	13.17	8.27	0.90	0.00	12.64	5.00	18.79	17.94
1985	4'133	11.16	10.16	15.63	10.93	1.25	0.00	15.11	7.00	18.89	18.03
1986	5'072	5.42	4.32	16.85	11.64	1.30	0.00	16.74	8.00	19.13	18.14
1987	5'046	2.66	1.32	17.82	12.22	1.47	0.00	19.48	9.00	19.24	18.18
1988	3'693	8.91	6.04	20.66	16.26	1.72	0.00	23.56	14.00	19.24	18.27
1989	3'370	7.22	6.28	21.30	17.63	1.99	0.00	25.41	16.00	19.42	18.31
1990	2'758	-0.81	-1.54	25.49	22.96	2.23	0.00	31.61	19.00	19.66	18.44
1991	2'640	15.89	12.63	24.70	20.79	2.16	0.00	30.64	19.00	19.67	18.55
1992	3'070	9.60	8.11	24.93	21.61	2.15	0.00	31.10	18.00	19.74	18.61
1993	3'835	7.61	5.56	24.70	20.42	2.67	0.00	34.73	18.00	20.21	18.80
1994	10'949	1.21	0.20	31.72	30.18	3.62	1.00	63.17	28.00	21.16	19.47
1995	11'907	8.86	7.47	32.82	30.96	3.62	1.00	65.80	29.00	21.27	19.50
1996	12'601	6.42	4.90	31.97	29.88	3.48	1.00	62.61	28.00	21.43	19.63
1997	15'413	7.45	6.96	34.02	32.91	3.56	2.00	64.56	30.00	21.53	19.65
1998	19'574	2.47	0.86	32.74	29.62	3.39	1.00	60.79	25.00	21.52	19.37
1999	18'376	7.47	1.35	33.44	30.51	3.89	2.00	69.79	31.00	21.78	19.44
2000	17'447	4.31	2.48	34.58	31.94	3.91	2.00	81.32	36.00	21.97	19.66
2001	14'722	7.72	6.13	40.26	40.72	4.34	2.00	95.31	53.00	21.96	19.75
2002	13'487	0.46	1.44	44.03	46.51	4.58	2.00	105.22	62.00	21.88	19.79
2003	13'573	12.46	9.95	45.82	48.62	4.86	2.00	113.81	71.00	21.88	19.83
2004	14'771	5.94	4.56	48.42	52.44	5.05	3.00	117.85	74.00	21.98	19.99
2005	14'551	2.81	1.77	50.00	54.69	5.53	3.00	121.25	74.00	22.07	20.12
2006	14'970	5.00	3.82	50.72	55.92	5.68	4.00	123.92	75.00	22.13	20.18
2007	14'328	0.30	-0.06	50.35	55.00	5.85	4.00	127.78	70.00	22.26	20.22

Table 6: Summary statistics for the entire sample, reported on an annual basis. Data on institutional ownership is from the CDA/Spectrum 13F database, data on stock returns are from the Center for Research in Security Prices (CRSP) monthly stock file that includes NYSE, AMEX, and NASDAQ stocks and data on analyst coverage are from I/B/E/S. We exclude all stocks of foreign companies listed in the U.S. as well as stocks with market capitalization below USD 10 millions and share price smaller than USD 5.00 from the analysis. Further, only stocks with available data on all variables for a given quarter are used. Stocks that are on CRSP but without any data on institutional holdings or analyst coverage are assumed to have zero IO and analyst coverage, respectively. All numbers are time series means and medians of quarterly cross-sectional data. N is the total number of observations for a given year, where a single stock can have a maximum of four observations per year. *Returns* are quarterly holding period returns reported in percent. *IO* denotes institutional ownership, calculated as the sum of stock holdings of all institutional investors divided by the total amount of shares outstanding for the respective stock. *Analysts* is the number of analysts providing fiscal year 1 earnings forecasts in a given quarter. *Institutions* is the number of institutional investors reporting stock holdings to the SEC. $\ln(\text{Size})$ is the natural logarithm of market capitalizations in USD.

4.3 Institutional Ownership and the Cross-Section of Returns

Table 7 reports descriptive statistics for equal-weighted quintile portfolios sorted on size and institutional ownership. The numbers are time series averages based on quarterly data for each quintile. Panel A sorts all stocks in the sample on size. The average number of observations is about 2,210 per quarter. Total average returns vary between 4.92% and 6.00%. The sort on size is not able to explain average returns in the cross-section. There is no regularity among different quintiles. IO clearly increases with size measured as the logarithm of total market capitalization. IO in the lowest size quintile is 16.04% and increases steadily to 51.08% for the highest size quintile. There is a clear relationship between IO and size. The same holds true for analyst coverage. The number of analysts following a stock increases with size. The average number of institutional investors per stock also increases with size from approximately 9 institutions per stock in the lowest quintile to 230 institutions in the highest quintile.

Panel A gives strong evidence that IO is related to size. Given the fact that institutional investors have a preference for large, liquid stocks, this result is not surprising. Therefore, to control for size, we sort stocks based on residual ownership (RO). Panel B of Table 7 exhibits the results for quintile portfolios sorted on RO. The results are striking. Returns seem unrelated to RO for the first two quintiles, but then monotonically decrease for quintiles 3, 4, and 5 from 5.97% per quarter down to only 4.81%. The highest quintile portfolio under-performs the first two quintile portfolios by more than one percent per quarter, about four percent per year. IO now ranges from 2.65% to 67.58%. Size is relatively constant among all RO-quintiles. Hence sorting on RO seems to be effective in creating variation in IO and holding size fixed. The number of analysts does not show substantial variation among RO-quintiles, indicating that analyst coverage is more related to size than to institutional ownership. The same holds true for the number of institutional investors holding shares of equity of a firm. The highest RO-quintile has on average 98 institutional investors, while the highest size-quintile has 230 institutional investors on average.

Do high RO-quintile portfolios consistently deliver lower than average returns? In other words, is the return differential between the highest RO-quintile portfolio and the rest of the sample statistically significant? To answer this question, we test the null hypothesis that the return differential between the sample mean return and the return of the highest RO-quintile portfolio is not significantly different from zero against the alternative hypothesis that the return differential is different from zero. At the end of each quarter t , we form five quintile portfolios sorted on RO. The return of this portfolios is the average return of all stocks included in the portfolio in the respective quarter. It is important to notice that we do not build portfolios based on past RO and calculate returns for the forthcoming period. End of quarter institutional holdings

Descriptive Statistics of Sorted Portfolios

Measure	Low	2	3	4	High
<i>Panel A: Quintile portfolios sorted on size</i>					
RET (%)	4.92	5.52	5.64	6.00	5.44
RETX (%)	4.60	5.27	5.39	5.70	5.04
DIV (%)	0.32	0.25	0.26	0.30	0.40
IO (%)	16.04	30.34	39.98	45.20	51.08
Analysts	0.55	1.77	3.11	4.91	9.26
Institutions	9.05	26.69	49.29	81.18	230.06
ln(SIZE)	17.68	18.79	19.69	20.68	23.16
<i>N</i>	687	688	688	688	687
<i>Panel B: Quintile portfolios sorted on residual ownership (RO)</i>					
RET (%)	5.87	5.97	5.64	5.23	4.81
RETX (%)	5.52	5.58	5.32	4.96	4.61
DIV (%)	0.35	0.38	0.32	0.27	0.21
IO (%)	2.65	21.37	38.72	52.32	67.58
Analysts	1.35	3.44	4.66	5.10	5.04
Institutions	12.40	73.31	102.96	109.40	98.09
ln(SIZE)	21.47	21.60	21.81	21.82	21.60
<i>N</i>	687	688	688	688	687

Table 7: Quarterly Returns to quintile portfolios sorted on size and residual ownership (RO). All numbers are time series averages based on quarterly data. At the end of each quarter t from March 1980 to December 2007, stocks are ranked by Size and RO as of quarter t . RET is the average holding period return, RETX is the average return ex dividends and DIV is the average contribution of dividends to total return. Institutional ownership (IO) is defined as the sum of the holdings of all reporting institutions divided by the total shares outstanding for the firm. Analyst coverage is the number of analysts providing fiscal year 1 earnings estimates in any given quarter. Institutions is the average number of institutional investors per stock. Size is the market capitalization at the end of the quarter, reported in millions of USD; N is the average number of quarterly observations for each decile. Only U.S. stocks were included that are traded on the NYSE, AMEX, and NASDAQ. Stock data is from CRSP, data on institutional ownership is from the CDA/Spectrum database and data on analyst coverage is from I/B/E/S. Data is filtered according to the screening procedure documented in section 4.2.

reflect which stocks institutions have bought in the previous quarter. It makes sense to compare quarter t holdings with quarter t returns rather than with quarter $t+1$ returns. We do not intend to test an investable investment strategy but rather want to capture the impact of institutional holdings on returns¹⁹. Based on the quarterly returns of quintile portfolios, we calculate the sample mean return and the return differentials for each quintile. Again, we additionally calculate average IO and size for the respective quintile portfolios.

¹⁹We have redone the analysis taking $t-1$ and $t-2$ RO to sort stocks and could not find any evidence of cross-sectional variation in returns for RO-sorted portfolios.

Quintile Portfolios Sorted on Residual Ownership					
	Low	2	3	4	High
<i>Panel A: Entire Sample March 1980 - December 2007</i>					
RET (%)	6.38	6.72	6.42	6.20	5.22
Δ RET (%)	0.20	0.53	0.23	0.01	-0.96
(t-statistic)	(0.83)	(2.00)**	(1.16)	(0.04)	(-3.46)***
IO (%)	1.62	15.27	31.04	43.43	56.32
ln(SIZE)	21.05	21.17	21.39	21.36	21.15
N	442	442	442	442	442
<i>Panel B: March 1980 - December 1993</i>					
RET (%)	7.28	8.00	7.56	7.40	5.79
Δ RET (%)	0.08	0.80	0.35	0.19	-1.42
(t-statistic)	(0.22)	(1.66)*	(0.94)	(0.42)	(-2.97)***
IO (%)	0.01	5.83	19.41	29.92	39.70
ln(SIZE)	19.61	19.41	19.77	19.61	19.44
N	146	146	146	146	146
<i>Panel C: March 1994 - December 2007</i>					
RET (%)	5.49	5.44	5.28	5.00	4.66
Δ RET (%)	0.31	0.26	0.11	-0.17	-0.51
(t-statistic)	(1.00)	(1.19)	(0.82)	(-0.83)	(-1.83)*
IO (%)	3.23	24.71	42.67	56.94	72.94
ln(SIZE)	21.62	21.77	21.97	21.96	21.75
N	738	738	738	738	738

Table 8: Average returns of RO-sorted quintile portfolios for different time periods. At the end of each quarter t stocks are ranked by RO as of quarter t and the average returns of each quintile is calculated for quarter t . The returns reported in the table are time-series averages of quarterly quintile-portfolio returns. Δ RET (%) denotes the return differential between the mean return of the quintile and the sample mean return. ln(Size) is the natural logarithm of the average market capitalization and N denotes the average number of observations. *, **, and *** denotes significance at the 10%, 5% and 1% level.

The results are shown in Table 8. Panel A exhibits average returns for the five quintile portfolios sorted on RO for the entire sample period from 1980 to 2007. Along the average returns also the return differentials between the quintile portfolios and the sample mean are reported with associated t-statistics in parentheses. We find that the differential is significantly negative for the 5th quintile. Again, size is relatively constant among deciles and indicates that our sorting technique effectively controls for size. Panel B and C report results for the two subperiods 1980-1993 and 1994-2007. We observe that the negative return differential persists for the two subperiods. In Panel B the mean return for the 5th quintile under-performs the sample mean by 1.42 percent per quarter. In Panel C the under-performance is 0.51 percent for the highest RO-quintile. All negative return differentials for the 5th quintile are statistically significant. Overall, we observe that the pattern we found in Table 7 seems to be robust to different periods, and that the highest RO-quintile significantly under-performs the sample mean in all

periods.

One concern regarding robustness of our results is that the low returns of high-RO stocks could be due to a lower compensation for market risk in this group of stocks. If for some reasons stocks in the 5th RO-quintile load less on market risk, this could be an explanation for the lower returns. To check on this issue, we calculate the historical beta for each stock over the entire sample period and divide the sample into three subgroups of beta before sorting on RO. The results not reported here indicate that the under-performance of the 5th quintile portfolio persists, i.e. we cannot find any different return pattern due to variation in betas.

Summing up, the above analysis demonstrates that there might be a relationship between institutional ownership and stock returns. Since IO is strongly related to size, it is important to control for the size effect. We notice that residual institutional ownership is a powerful tool on this. Our extensive sample on institutional holdings data provides some evidence in favor of Hypothesis 1. Holding size fixed, we observe that average returns of quintile portfolios monotonically decrease with increasing IO and that stocks with high IO exhibit significantly lower than average returns.

5 Conclusion

We analyze equilibrium implications of delegated portfolio management under benchmarking. A novel part of our paper is the simultaneous modeling of delegation and benchmarking. Delegation arises endogenously in equilibrium as a result of a given benchmarking policy. The main result from our partial equilibrium analysis is that the investor's utility from delegation is always maximized when the manager exhibits the same attitude towards risk than the investor. Whenever the manager is more risk tolerant than the delegating investor, benchmarking results in a utility loss compared to the situation without benchmarking .

In general equilibrium, we find that the impact of delegation is twofold. First, the presence of benchmarking gives rise to active management risk that lowers expected returns. For those assets included in the benchmark portfolio, the manager does not require a premium for systematic market risk. Only active management risk is priced, implying a lower market price of risk and lower expected excess returns. Second, delegated agents acquire more information than individual investors to induce investors to delegate and to justify the management fees. This motivation for information acquisition leads to a more informative price system and to lower expected returns, at least for those assets in which delegated managers invest. In a brief empirical analysis, we find support for the model's predictions that institutional ownership lowers returns.

On a final note, we provide theoretical and empirical arguments that the existence of investment intermediaries matters for asset pricing and significantly affects the cross-section of equilibrium returns. Finally, our model predictions might find empirical support given the decrease in returns for stocks with high institutional ownership over the past two decades, a period during which the volume of institutionally managed money has sharply increased. However, we leave a more rigorous empirical analysis for future research.

Appendix

Proof of Proposition 1

Substituting for (18) in equation (13), the investor's expected utility reads

$$U_j = \frac{1}{2\rho_j} E \left[(\hat{\boldsymbol{\mu}}_j - \mathbf{pr})' \widehat{\boldsymbol{\Sigma}}_j^{-1} (\hat{\boldsymbol{\mu}}_j - \mathbf{pr}) \mid \boldsymbol{\mu} \right] \quad (55)$$

The value of $\hat{\boldsymbol{\mu}}_j$ depends on the signal realization and therefore is not known in period 2. The term $(\hat{\boldsymbol{\mu}}_j - \mathbf{pr})' \widehat{\boldsymbol{\Sigma}}_j^{-1} (\hat{\boldsymbol{\mu}}_j - \mathbf{pr})$ follows a non-central χ_N^2 -distribution with N degrees of freedom. Taking the expected value of a χ_N^2 -distributed random variable, the investor's expected utility can be rewritten as

$$U_j = \frac{1}{2\rho_j} \left(\text{tr} \left(\widehat{\boldsymbol{\Sigma}}_j^{-1} \text{Var}[\hat{\boldsymbol{\mu}}_j \mid \boldsymbol{\mu}] \right) + E[\hat{\boldsymbol{\mu}}_j - \mathbf{pr} \mid \boldsymbol{\mu}]' \widehat{\boldsymbol{\Sigma}}_j^{-1} E[\hat{\boldsymbol{\mu}}_j - \mathbf{pr} \mid \boldsymbol{\mu}] \right), \quad (56)$$

where $E[\hat{\boldsymbol{\mu}}_j - \mathbf{pr} \mid \boldsymbol{\mu}] = \boldsymbol{\mu} - \mathbf{pr}$ and $\text{Var}[\hat{\boldsymbol{\mu}}_j \mid \boldsymbol{\mu}] = \boldsymbol{\Sigma} - \widehat{\boldsymbol{\Sigma}}_j$. Therefore,

$$U_j = \frac{1}{2\rho_j} \left(\text{tr} \left(\widehat{\boldsymbol{\Sigma}}_j^{-1} (\boldsymbol{\Sigma} - \widehat{\boldsymbol{\Sigma}}_j) \right) + (\boldsymbol{\mu} - \mathbf{pr})' \widehat{\boldsymbol{\Sigma}}_j^{-1} (\boldsymbol{\mu} - \mathbf{pr}) \right), \quad (57)$$

which is equivalent to the expression given in Proposition 1. \square

Proof of Proposition 4

$U_{dir}^j(\boldsymbol{\mu}, \widehat{\boldsymbol{\Sigma}}_j^j) \leq U_{del}^j(\boldsymbol{\mu} - \delta, \widehat{\boldsymbol{\Sigma}}_m^j)$ in equation (12) can be written as

$$E \left[(\hat{\boldsymbol{\mu}}_j - \mathbf{pr})' \mathbf{q}_j - \frac{1}{2} \rho_j \mathbf{q}_j' \widehat{\boldsymbol{\Sigma}}_j \mathbf{q}_j \mid \boldsymbol{\mu} \right] \leq E \left[(\hat{\boldsymbol{\mu}}_m - \mathbf{pr})' \mathbf{q}_m - \delta - \frac{1}{2} \rho_j \mathbf{q}_m' \widehat{\boldsymbol{\Sigma}}_m \mathbf{q}_m \mid \boldsymbol{\mu} \right]$$

$$\delta \leq \left(1 - \frac{1}{2} \rho \right) \frac{1}{\rho_m} E \left[(\hat{\boldsymbol{\mu}}_m - \mathbf{pr})' \widehat{\boldsymbol{\Sigma}}_m (\hat{\boldsymbol{\mu}}_m - \mathbf{pr}) \mid \boldsymbol{\mu} \right] - \frac{1}{\rho_j} E \left[(\hat{\boldsymbol{\mu}}_j - \mathbf{pr})' \widehat{\boldsymbol{\Sigma}}_j (\hat{\boldsymbol{\mu}}_j - \mathbf{pr}) \mid \boldsymbol{\mu} \right]$$

For notational simplicity, we drop superscript j . Because we are in partial equilibrium, it is not necessary to distinguish between different investors. Taking the expected value of a noncentral χ_N^2 -distributed random variable, we get

$$\begin{aligned} \delta \leq & \left(1 - \frac{1}{2} \rho \right) \frac{1}{2\rho_m} \left(\text{tr}(\widehat{\boldsymbol{\Sigma}}_m^{-1} \boldsymbol{\Sigma} - \mathbf{I}) + (\boldsymbol{\mu} - \mathbf{pr})' \widehat{\boldsymbol{\Sigma}}_m^{-1} (\boldsymbol{\mu} - \mathbf{pr}) \right) \\ & - \frac{1}{2\rho_j} \left(\text{tr}(\widehat{\boldsymbol{\Sigma}}_j^{-1} \boldsymbol{\Sigma} - \mathbf{I}) - (\boldsymbol{\mu} - \mathbf{pr})' \widehat{\boldsymbol{\Sigma}}_j^{-1} (\boldsymbol{\mu} - \mathbf{pr}) \right). \end{aligned}$$

Define the constants

$$a := \left(1 - \frac{1}{2} \rho \right) \frac{1}{\rho_m}, \quad b := \frac{1}{2\rho_j}.$$

Substituting for the eigen decomposition yields:

$$\begin{aligned} \delta \leq & a \left(\text{tr}(\mathbf{\Gamma} \hat{\mathbf{\Lambda}}_m^{-1} \mathbf{\Gamma}' \mathbf{\Gamma} \mathbf{\Lambda} \mathbf{\Gamma}' - \mathbf{I}) + (\boldsymbol{\mu} - \mathbf{r}\mathbf{p})' \mathbf{\Gamma} \hat{\mathbf{\Lambda}}_m^{-1} \mathbf{\Gamma}' (\boldsymbol{\mu} - \mathbf{p}\mathbf{r}) \right) \\ & - b \left(\text{tr}(\mathbf{\Gamma} \hat{\mathbf{\Lambda}}_j^{-1} \mathbf{\Gamma}' \mathbf{\Gamma} \mathbf{\Lambda} \mathbf{\Gamma}' - \mathbf{I}) - (\boldsymbol{\mu} - \mathbf{p}\mathbf{r})' \mathbf{\Gamma} \hat{\mathbf{\Lambda}}_j^{-1} \mathbf{\Gamma}' (\boldsymbol{\mu} - \mathbf{p}\mathbf{r}) \right). \end{aligned}$$

Using the fact that the trace of a matrix is the sum of its diagonal values, the above expression reads in summation notation

$$\delta \leq a \frac{1}{\rho_m} \sum_{l=1}^L \hat{\lambda}_{m,l}^{-1} (\lambda_l + [(\boldsymbol{\mu} - \mathbf{p}\mathbf{r})' \mathbf{\Gamma}_l]^2) - b \sum_{l=1}^L \hat{\lambda}_{j,l}^{-1} (\lambda_l + [(\boldsymbol{\mu} - \mathbf{p}\mathbf{r})' \mathbf{\Gamma}_l]^2) + N(b - a).$$

Collecting terms yields the expression in Proposition 4. \square

Proof of Proposition 7

The derivation of this result is similar to the proof given in Proposition 4. Substituting for U_{del}^j and U_{dir}^j in equation (12) and solving for δ yields the expression stated in Proposition 7. \square

Proof of Proposition 8

Recall that the CED for investor j is defined as

$$\text{CED}^j := \sup \left[\delta \left| U_{dir}^j(\hat{\boldsymbol{\mu}}_j, \hat{\boldsymbol{\Sigma}}_j) \leq U_{del}^j(\hat{\boldsymbol{\mu}}_m^j - \delta, \hat{\boldsymbol{\Sigma}}_m^j) \right. \right].$$

We first derive $U_{dir}^j(\hat{\boldsymbol{\mu}}_j, \hat{\boldsymbol{\Sigma}}_j)$. The period 2 utility from direct investment, given optimal portfolio choice $\mathbf{q}_{dir}^j = \frac{1}{\rho_j} \hat{\boldsymbol{\Sigma}}_j (\hat{\boldsymbol{\mu}}_j - \mathbf{p}\mathbf{r})$ and optimal information choice $\hat{\boldsymbol{\Sigma}}_j$, is

$$U_{dir} = \frac{1}{2\rho_j} E \left[(\hat{\boldsymbol{\mu}}_j - \mathbf{p}\mathbf{r})' \hat{\boldsymbol{\Sigma}}_j^{-1} (\hat{\boldsymbol{\mu}}_j - \mathbf{p}\mathbf{r}) \mid \boldsymbol{\mu} \right]. \quad (58)$$

In period 2, $(\hat{\boldsymbol{\mu}}_j - \mathbf{p}\mathbf{r})$ is normal with mean

$$E[(\hat{\boldsymbol{\mu}}_j - \mathbf{p}\mathbf{r}) \mid \boldsymbol{\mu}] = (\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{A} \quad (59)$$

and variance

$$\text{Var}[(\hat{\boldsymbol{\mu}}_j - \mathbf{p}\mathbf{r}) \mid \boldsymbol{\mu}] = \boldsymbol{\Sigma} - \hat{\boldsymbol{\Sigma}}_j + \mathbf{B}\boldsymbol{\Sigma}\mathbf{B}' - 2\boldsymbol{\Sigma}\mathbf{B} + \boldsymbol{\Sigma}_p, \quad (60)$$

which follows from the equilibrium price function (36). Hence, we write the period-2 utility according to equation (56) as

$$U_{dir}^j = \frac{1}{2\rho_j} \left(Tr \left(\widehat{\Sigma}_j^{-1} (\Sigma - \widehat{\Sigma}_j + \mathbf{B}\Sigma\mathbf{B}' - 2\Sigma\mathbf{B} + \Sigma_p) \right) + ((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{A})' \widehat{\Sigma}_j^{-1} ((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{A}) \right). \quad (61)$$

Substituting for the correlation structure of the risky assets $\widehat{\Sigma} = \Gamma\widehat{\Lambda}\Gamma$ yields

$$U_{dir}^j = \frac{1}{2\rho_j} \left(Tr \left(\Gamma\widehat{\Lambda}_j^{-1}\Gamma' (\Gamma\widehat{\Lambda}\Gamma' - \Gamma\widehat{\Lambda}_j\Gamma' + \mathbf{B}\Sigma\mathbf{B}' - 2\Sigma\mathbf{B} + \Sigma_p) \right) + ((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{A})' \Gamma\widehat{\Lambda}_j^{-1}\Gamma' ((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{A}) \right). \quad (62)$$

Since the trace of a matrix is the sum of its diagonal elements, equation (62) can be written in summation notation as follows:

$$U_{dir}^j = \frac{1}{2\rho_j} \left(\sum_{l=1}^L \widehat{\lambda}_{j,l}^{-1} \underbrace{\left(\lambda_l(1 - \lambda_{B,l})^2 + \lambda_{p,l} + [((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{A})' \Gamma_l]^2 \right)}_{X_l^E} - L \right), \quad (63)$$

where Λ_B and Λ_p are the eigenvalues of \mathbf{B} and Σ_p . Define

$$X_l^E := \lambda_l(1 - \lambda_{B,l})^2 + \lambda_{p,l} + [((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{A})' \Gamma_l]^2.$$

Similarly, by substituting for \mathbf{q}_m^j , we write $U_{del}^j(\hat{\boldsymbol{\mu}}_m^j - \delta, \widehat{\Sigma}_m^j)$ as

$$U_{del}^j = \left(1 - \frac{1}{2} \frac{\rho_j}{\rho_m^j} \right) \frac{1}{\rho_m^j} E \left[(\hat{\boldsymbol{\mu}}_m^j - \mathbf{p}r)' (\widehat{\Sigma}_m^j)^{-1} (\hat{\boldsymbol{\mu}}_m^j - \mathbf{p}r) | \boldsymbol{\mu} \right] + \left(1 - \frac{\rho_j}{\rho_m} \right) \mu_\phi - \frac{1}{2} \rho_j \widehat{\Sigma}_\phi - \delta, \quad (64)$$

where ρ_m^j is the risk aversion of the manager of investor j . The above equation is equivalent to

$$U_{del}^j = \left(1 - \frac{1}{2} \frac{\rho_j}{\rho_m^j} \right) \frac{1}{\rho_m^j} \left(\sum_{l=1}^L (\widehat{\lambda}_{m,l}^j)^{-1} X_l^E - L \right) + \left(1 - \frac{\rho_j}{\rho_m} \right) \mu_\phi - \frac{1}{2} \rho_j \widehat{\Sigma}_\phi - \delta. \quad (65)$$

Combining equations (63) and (65) and using the definition of the certainty equivalent of delegation yields the expression in Proposition 8. \square

Proof of Proposition 9

The market clearing condition implies that the total demand for risky assets of delegating investors, \mathbf{q}_{del}^j , and direct investing investors, \mathbf{q}_{dir}^j , equals total supply of risky

assets, $\bar{\mathbf{x}} + \mathbf{x}$,

$$\int_0^n \mathbf{q}_{del}^j dj + \int_n^1 \mathbf{q}_{dir}^j dj = \bar{\mathbf{x}} + \mathbf{x}, \quad (66)$$

where $n \in [0, 1]$. Therefore,

$$\int_0^n \left(\frac{1}{\rho_m^j} (\hat{\Sigma}_m^j)^{-1} (\hat{\boldsymbol{\mu}}_m^j - \mathbf{p}r) + \phi \right) dj + \int_n^1 \frac{1}{\rho_j} \hat{\Sigma}_j^{-1} (\hat{\boldsymbol{\mu}}_j - \mathbf{p}r) dj = \bar{\mathbf{x}} + \mathbf{x}, \quad (67)$$

where $(\hat{\boldsymbol{\mu}}_m^j, \hat{\Sigma}_m^j)$ denotes the set of posterior beliefs of the manager of the j -th investor and $(\hat{\boldsymbol{\mu}}_j, \hat{\Sigma}_j)$ the set of posterior beliefs of the j -th investor. Substituting (37) and (38) into the market clearing condition, we get

$$\begin{aligned} & \int_0^n \left(\frac{1}{\rho_m^j} (\hat{\Sigma}_m^j)^{-1} \left(\hat{\Sigma}_m^j (\Sigma^{-1} \boldsymbol{\mu} + (\Omega_m^j)^{-1} \mathbf{s}_m^j + \Sigma_p^{-1} \mathbf{B}^{-1} (\mathbf{p}r - \mathbf{A})) - r\mathbf{p} \right) + \phi \right) dj \\ & + \int_n^1 \frac{1}{\rho_j} \hat{\Sigma}_j^{-1} \left(\hat{\Sigma}_j (\Sigma^{-1} \boldsymbol{\mu} + \Omega_j^{-1} \mathbf{s}_j + \Sigma_p^{-1} \mathbf{B}^{-1} (\mathbf{p}r - \mathbf{A})) - r\mathbf{p} \right) dj = \bar{\mathbf{x}} + \mathbf{x}. \end{aligned} \quad (68)$$

Define the following two average quantities. The first is the average signal precision matrix,

$$\Psi := \int_0^n (\Omega_m^j)^{-1} dj + \int_n^1 \Omega_j^{-1} dj. \quad (69)$$

The second average quantity is the average risk tolerance of direct and delegating investors,

$$\rho_a^{-1} := \int_0^n \frac{1}{\rho_m^j} dj + \int_n^1 \frac{1}{\rho_j} dj. \quad (70)$$

With these two definitions, the market clearing condition becomes

$$\frac{1}{\rho_a} \Sigma^{-1} \boldsymbol{\mu} + \frac{1}{\rho_a} \Psi \mathbf{f} + \frac{1}{\rho_a} \Sigma_p^{-1} \mathbf{B}^{-1} (\mathbf{p}r - \mathbf{A}) - \frac{1}{\rho_a} \Sigma^{-1} \mathbf{p}r - \frac{1}{\rho_a} \Psi \mathbf{p}r - \frac{1}{\rho_a} \Sigma_p^{-1} \mathbf{p}r + n\phi = \bar{\mathbf{x}} + \mathbf{x}. \quad (71)$$

To derive the above result, we used the fact that signals \mathbf{s} are distributed *iid* around the true payoffs \mathbf{f} . Solving for $\mathbf{p}r$ yields

$$\mathbf{p}r = \rho_a (\Sigma^{-1} + \Psi - \Sigma_p^{-1} (\mathbf{B}^{-1} - \mathbf{I}))^{-1} \underbrace{\left(\frac{1}{\rho_a} \Sigma^{-1} \boldsymbol{\mu} - \frac{1}{\rho_a} \Sigma_p^{-1} \mathbf{B}^{-1} \mathbf{A} - \bar{\mathbf{x}} + n\phi - \frac{1}{\rho_a} \Psi \mathbf{f} + \mathbf{x} \right)}_{\text{constant}} \quad (72)$$

The above expression is clearly linear in \mathbf{f} and \mathbf{x} and confirms (36). Next step is to solve for the constants in (36). From (36) and (72), we know that \mathbf{B} takes the following form,

$$\mathbf{B} = (\Sigma^{-1} + \Psi - \Sigma_p^{-1} (\mathbf{B}^{-1} - \mathbf{I}))^{-1} \Psi. \quad (73)$$

Solving for \mathbf{B} yields

$$\mathbf{B} = (\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Psi} + \boldsymbol{\Sigma}_p^{-1})^{-1} (\boldsymbol{\Psi} + \boldsymbol{\Sigma}_p^{-1}). \quad (74)$$

In the same manner, one can solve for the other constants. \mathbf{C} can be expressed as:

$$\mathbf{C} = -\rho_a (\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Psi} - \boldsymbol{\Sigma}_p^{-1}(\mathbf{B}^{-1} - \mathbf{I}))^{-1}. \quad (75)$$

Solving for \mathbf{C} yields

$$\mathbf{C} = -\rho_a (\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Psi} + \boldsymbol{\Sigma}_p^{-1})^{-1} (\boldsymbol{\Sigma}_p^{-1} \boldsymbol{\Psi}^{-1} + \mathbf{I}). \quad (76)$$

Last step is to solve for the constant \mathbf{A} .

$$\mathbf{A} = \rho_a (\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Psi} - \boldsymbol{\Sigma}_p^{-1}(\mathbf{B}^{-1} - \mathbf{I}))^{-1} \left(\frac{1}{\rho_a} \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} - \frac{1}{\rho_a} \boldsymbol{\Sigma}_p^{-1} \mathbf{B}^{-1} \mathbf{A} - \bar{\mathbf{x}} + n\boldsymbol{\phi} \right), \quad (77)$$

$$\mathbf{A} = \rho_a (\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Psi} + \boldsymbol{\Sigma}_p^{-1})^{-1} \left(\frac{1}{\rho_a} \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} - \bar{\mathbf{x}} + n\boldsymbol{\phi} \right) \quad (78)$$

Note that $\boldsymbol{\Sigma}_p := \sigma_x^2 \mathbf{B}^{-1} \mathbf{C} \mathbf{C}' (\mathbf{B}^{-1})'$. Therefore, we can make the following claim.

Corollary 5.1. *Asset prices are linear functions of the asset payoffs and the unexpected component of asset supply. With identical priors, the price function is*

$$\mathbf{p} = \frac{1}{r} (\mathbf{A} + \mathbf{B}\mathbf{f} + \mathbf{C}\mathbf{x}), \quad (79)$$

with constants

$$\mathbf{A} = \rho_a (\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Psi} + \boldsymbol{\Sigma}_p^{-1})^{-1} \left(\frac{1}{\rho_a} \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} - \bar{\mathbf{x}} + n\boldsymbol{\phi} \right) \quad (80)$$

$$\mathbf{B} = (\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Psi} + \boldsymbol{\Sigma}_p^{-1})^{-1} (\boldsymbol{\Psi} + \boldsymbol{\Sigma}_p^{-1}) \quad (81)$$

$$\mathbf{C} = -\rho_a (\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Psi} + \boldsymbol{\Sigma}_p^{-1})^{-1} (\boldsymbol{\Sigma}_p^{-1} \boldsymbol{\Psi}^{-1} + \mathbf{I}) \quad (82)$$

where $\boldsymbol{\Psi} := \int_0^n \boldsymbol{\Omega}_j - 1 dj + \int_n^1 (\boldsymbol{\Omega}_{dir}^j)^{-1} dj$ and $\rho_a^{-1} := \int_0^n \frac{1}{\rho_m^j} dj + \int_n^1 \frac{1}{\rho_j} dj$.

The first term on the right-hand side in equations (80), (81) and (82), is the posterior covariance matrix of the ‘average’ investor $\widehat{\boldsymbol{\Sigma}}_a$,

$$\widehat{\boldsymbol{\Sigma}}_a := (\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Psi} + \boldsymbol{\Sigma}_p^{-1})^{-1}. \quad (83)$$

Like the individual investor, the ‘average’ investor has a posterior covariance matrix that is based on the prior and signal variance and the variance observed from the price level. Using the expression for the ‘average’ posterior variance, the price function can

be written as

$$\mathbf{pr} = \widehat{\Sigma}_a \left(\Sigma^{-1} \boldsymbol{\mu} - \rho_a \bar{\mathbf{x}} + \rho_a n \phi + (\Psi + \Sigma_p^{-1}) \mathbf{f} - \rho_a (\Sigma_p^{-1} \Psi^{-1} + \mathbf{I}) \mathbf{x} \right). \quad (84)$$

Rearranging terms yields

$$\mathbf{pr} = \widehat{\Sigma}_a \left(\Sigma^{-1} \boldsymbol{\mu} + \Psi \mathbf{f} + \Sigma_p^{-1} (\mathbf{f} - \rho_a \Psi^{-1} \mathbf{x}) - \rho_a \bar{\mathbf{x}} - \rho_a \mathbf{x} + \rho_a n \phi \right). \quad (85)$$

Note that $(\mathbf{f} - \frac{1}{\rho} \Psi^{-1} \mathbf{x}) = \mathbf{B}^{-1}(\mathbf{pr} - \mathbf{A})$ is the signal that investors observe from prices. The first three terms in (85) are equal to the posterior mean of the ‘average’ investor, as given by the Bayesian updating formulas. Defining

$$\hat{\boldsymbol{\mu}}_a := \widehat{\Sigma}_a \left(\Sigma^{-1} \boldsymbol{\mu} + \Psi \mathbf{f} + \Sigma_p^{-1} (\mathbf{f} - \rho_a \Psi^{-1} \mathbf{x}) \right) \quad (86)$$

the price function can be rewritten as given in Proposition 9. \square

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