Chartists and Fundamentalists in the U.S. Housing Market

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Abstract
In this paper we develop and estimate a heterogeneous agents model for the U.S. housing market. There are two groups of investors, fundamentalists and chartists. Fundamentalists expect the house price to revert to its fundamental value based on rents, while chartists simply extrapolate past price changes. Investors are allowed to switch between groups, depending on recent forecasting performance. The empirical results show significance presence of both fundamentalists and chartists in the market, usually with roughly equal proportions. From 1992 until 2005, however, the weight of chartists was substantially above the long-term average, while the house price level climbed far above it rent-based fundamental value. In an out-of-sample assessment the model outperforms competing timeseries models and predicts the decline of the housing market from 2006 onwards.

JEL – classifications: G12, G17

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1. Introduction

The busting of the housing bubble in the U.S. has often been mentioned as the factor triggering the financial crisis in 2007 and 2008, leading to the most severe recession in the developed world since the Great Depression. By lending to individuals with poor credit scores, the so called sub-prime market, financial institutions and investors in mortgage-backed securities were effectively betting on ever increasing house prices (Gorton, 2009). In retrospect, the U.S. housing market seems to have been driven by speculation, fueled by moral hazard induced lending, for a prolonged period of time. The housing market may be more vulnerable to inefficiencies and occasional crashes than other markets due to lack of effective short selling mechanisms that prevent bearish investors from participating (Hong and Stein, 2003).

Case and Shiller (1989, 1990) already provided evidence of the inefficiency of the market for single-family homes based on the existence of positive serial correlation in year-to-year changes in prices, and negative serial correlations at lags of two to four years. Englund and Ioannides (1997) provide similar evidence for housing prices in 15 OECD countries.[1] Case and Shiller (1990) also show that future house price changes can be predicted with rents and other lagged fundamental variables. This confirms to the general mean reversion pattern of asset returns found by Cutler, Poterba and Summers (1991) for stocks, bonds, exchange rates and precious metals.

What may explain this pattern of short-term return momentum and long-term mean reversion? Cutler, Poterba and Summers (1990) show that interactions between rational investors and noise traders following positive feedback strategies – buy when prices rise, sell when prices fall – can reproduce these stylized facts. De Long, Shleifer, Summers and Waldmann (1990) show that rational traders in such a model can actually destabilize the market by initially driving up prices beyond fundamentals and then later selling out at even higher prices to the feedback traders.[2] Frankel and Froot (1991) build a similar heterogeneous agents model for the foreign exchange market with trend chasers

[1] Levin and Wright (1997) show that past house price changes in the UK forecast future price changes. See also Cho (1996) for a survey on house price dynamics.
[2] A more recent paper by Abreu and Brunnermeier (2003) shows that bubbles created by noise traders can persist even though rational agents jointly have the ability to correct the mispricing, due to dispersion of opinion among the rational agents about the exact timing of the bubble. In this setting it can also be optimal for rational agents to jump the bandwagon and follow the strategy of the positive feedback traders.

and investors trading on mean reversion to fundamentals, and show that it can generate prolonged periods of overvaluation as observed in practice.

A crucial ingredient of the models of Cutler, Poterba and Summers (1990), Frankel and Froot (1991), Brock and Hommes (1997, 1998), and others, is the presence of a core of non-rational positive feedback traders – sometimes called chartists – that expect past price changes to continue in the future. Representativeness bias (Tversky and Kahneman, 1974) may explain why investors ignore probability rules and consider recent events to be representative of what to expect in the future (De Bondt, 1993). Bange (2000) shows that stock portfolio adjustments of individual investors reflect past market movements, consistent with positive feedback trading. Keim and Madhaven (1995) document momentum trading by institutional investors.

In the housing market Case and Shiller (1988) find that individuals base their expectations largely on past price movements, and not on fundamentals.[3] Other papers show evidence of trend chasing behavior in commercial banks' investments in real estate (Mei and Saunders, 1997) and among professional forecasters of the commercial real estate market (Ling, 2005). Given the widespread evidence of positive feedback trading among market participants, and the apparent failure of financial institutions, credit agencies, investors and regulators to foresee the disastrous housing bubble burst in the U.S., in this paper we try to improve forecasts for housing market prices by estimating a heterogeneous agents model with positive feedback traders.

The contribution of this paper is that we are the first to empirically estimate a heterogeneous agent model for the housing market. The model includes chartists who are positive feedback traders and fundamentalists who expect mean-reversion to a rent-based fundamental value. We use data on the repeat-sales house price index published by Freddie Mac until 2000, and the S&P/Case-Shiller U.S. National Home Price Index from 2000 onwards, together with a compatible index for rents developed by Davis, Lehnert and Martin (2008). In-sample estimation results indicate that expectations based on short-term momentum and mean reversion to fundamentals can both predict future changes the

[3] Hjalmarsson and Hjalmarsson (2009) show that buyers of apartment units in a cooperative housing association in Sweden do not properly discount future maintenance fees and capital costs. Clayton (1997) finds that prices in the apartment market in Canada move opposite to predictions based on rational expectations, probably due to the influence of noise traders and trend chasing.

in U.S. house price index well. The model coefficients for the two forecasting rules have signs as predicted by theory.

We find that allowing agents to switch between the two forecasting rules based on recent prediction performance, following Brock and Hommes (1997, 1998), is very beneficial for the fit of the model. In the latter part of the sample period, 1992-2005, the proportion of investors following the positive feedback trading rule is consistently above average, while prices move far above the rent-based fundamental value. From 2006 onwards, however, the mean reversion rule regains importance during the housing market downturn. Simulation results show that the estimated model produces regular boom-bust cycles. Out-of-sample forecasting results indicate that the model outperforms competing vector error correction and ARIMA timeseries models. The latter finding illustrates that the heterogeneous agents model may be not just of theoretical interest, but also a useful forecasting tool for housing market participants.

Our paper builds on the literature on heterogeneous agents models, following Cutler, Poterba and Summers (1990), Frankel and Froot (1991) and Brock and Hommes (1997, 1998), amongst others. Several studies have shown that these models can replicate many of the well-known stylized facts of financial market data. For example, Cutler, Poterba and Summers (1990) show that a model with positive feedback traders, fundamentalists and rational agents can generate price dynamics displaying short-momentum and long-term mean reversion. Lux (1998) demonstrates that a model with fundamentalists and positive feedback traders is capable of generating equity market returns with heavy tails, excess kurtosis and volatility clustering. De Grauwe and Grimaldi (2006) derive similar results for the foreign exchange market. See also Hommes (2006) for an overview.

Recently, Malpezzi and Wachter (2005), Sommervoll et al. (2009) and Dieci and Westerhoff (2009), have developed specialized heterogeneous agents models for the housing market. However, these models have not been calibrated or estimated with housing market data. The empirical literature on heterogeneous agents models in general is relatively scarce. Boswijk, Hommes and Manzan (2007) rewrite a heterogeneous agents model as a smooth-transition auto regressive (STAR) model and estimate it for the U.S. stock market using S&P500 data. De Jong, Verschoor and Zwinkels (2009a) set up a heterogeneous agents model with multiple assets, and estimate it for two Asian stock market indices during the Asian crisis, while De Jong et al. (2009b) estimate a similar model for the EMS exchange rates. Westerhoff and Reitz (2003), Reitz and Westerhoff (2006), and Manzan and Westerhoff (2007) introduce time variation in the impact of either chartists or fundamentalists conditional on the distance of the market price to the fundamental value. In a recent contribution, Franke (2009) estimates a heterogeneous agents model using the simulated method of moments.

The remainder of the paper is organized as follows. Section 2 presents the heterogeneous agents model for the housing market. Section 3 describes the data and the methodology employed to estimate the heterogeneous agents model for the U.S. housing market. In addition, Section 3 introduces the fundamental house price, based on the present value of rents, employed by fundamentalists. Section 4 subsequently presents the estimation results and Section 5 concludes the paper.

2. A Heterogeneous Agents Model for the Housing Market

We develop a simple and stylized heterogeneous agents model for the housing market, following Cutler, Poterba and Summers (1990), Frankel and Froot (1991) and Brock and Hommes (1997, 1998) and Dieci and Westerhoff (2009). As in the model for the housing market of Dieci and Westerhoff (2009) the market is populated by three types of agents, namely consumers, constructors and investors. Consumers and investors are on the demand side of the market, while constructors are on the supply side. Consumers buy houses for the sole purpose of living. We assume that aggregate consumer demand for housing (D_t^C) depends on the value of the house price index at time t:

$$D_t^C = a + bP_t , \qquad (1)$$

where t is time measured in quarters, P_t is the logarithm of the real house price index at time t. We expect $b < 0$, as higher prices should reduce the demand for housing. Higher house prices also have large wealth effects for most consumers, as a house typically represents a large fraction of household net worth (Stein, 1995). Further, the majority of house sales are to repeat buyers (about 60%, see Stein 1995), for whom a substantial portion of the down payment on a new home typically comes from the proceeds of the sales of the old home. The model of Stein (1995) shows that self-reinforcing effects can run from prices to down payments back to the demand for housing. These effects may reduce the price elasticity of the demand for housing.

Following Brock and Hommes (1997, 1998), investors in our model are mean variance optimizers who invest their wealth in either the housing market or in the risk free asset with constant return *r*. Note that investors are only interested in short-term capital gains and do not rent out houses. Investor demand for houses D_t^I is then given by

$$D_t^I = \frac{E(R_{t+1}) - r}{\mu V(R_{t+1})}, \tag{2}$$

in which $E(R_{t+1})$ is the expected return of housing, $\mu > 0$ is the risk aversion parameter and $V(R_t) > 0$ is the risk associated with investing in the housing market. Return R_{t+1} is defined as the log-price change $P_{t+1} - P_t$.

Investors are boundedly rational in the way they form expectations. As in Frankel and Froot (1991), investors choose among two forecasting rules for determining the expected return $E(R_{t+1})$, called fundamentalist and chartist. The first rule, fundamentalist, is based on the expectation of mean reversion of the market price towards the long-term fundamental price

$$E_t^f(R_{t+1}) = \alpha(P_t - F_t), \tag{3}$$

in which F_t is the (log) fundamental price and $\alpha < 0$ the speed of mean reversion. The second rule, which we call chartist, takes advantage of the stickiness of house prices (positive autocorrelation), documented by Case and Shiller (1989):

$$E_t^c(R_{t+1}) = \beta\left(\sum_{l=1}^{L} R_{t-l+1}\right), \tag{4}$$

in which $\beta > 0$ is the extrapolation parameter and $L > 0$ is a positive integer indicating the number of lags. Chartists expect past price changes to continue in the future and are therefore positive feedback traders.

We assume that investors can switch between the two expectation formation rules based on historical forecasting performance, following Brock and Hommes (1997, 1998).[4] A strong motivation for switching among forecasting rules can be found Frankel and Froot (1991). Frankel and Froot (1991) find that professional market participants in the foreign exchange markets expect recent price changes to continue in the short term, while they expect mean reversion to fundamental value in the long term. Further, Frankel and Froot (1991) report survey evidence showing that professional forecasting services in the foreign exchange markets rely both on technical analysis (the chartist rule) and fundamental models, but with changing weights through time. The weights appear to depend strongly on recent forecasting performance.

To model the dependence of the weights on recent forecasting performance we use a logit switching rule, as introduced by Manski and McFadden (1982) and applied in Brock and Hommes (1997, 1998), such that the weight of fundamentalists W_t is given by

$$W_t = \left(1 + \exp\left[\gamma\left(\frac{\pi_t^f - \pi_t^c}{\pi_t^f + \pi_t^c}\right)\right]\right)^{-1}, \tag{7}$$

and the chartist weight is equal to $(1 - W_t)$, in which π_t^f and π_t^c are the historical performance measures of fundamentalist and chartist rules at time t, respectively. The parameter γ denotes the intensity of choice, or the sensitivity of investors to differences in forecast error between the two rules. A positive (negative) γ causes agents to move towards the better (worse) performing rule. With $\gamma = 0$, agents are completely insensitive to differences in performance and the market is split evenly between fundamentalists and chartists. In the other extreme, as $\gamma \rightarrow \infty$, investors are infinitely sensitive to $\pi_t^f - \pi_t^c$ such that the investors are perfectly adaptive and W will always be equal to zero or one.

[4] Since all investors compare the performance rules, all agents have the necessary knowledge and skill to use them. As such, we can assume without loss of generality that agents can switch between rules without any costs.

Alternatively, $1/\gamma$ can be interpreted as the status quo bias of investors; see Kahneman et al. (1982). In this behavioral setting, investors adhere to their strategy even though objective measures indicate they should switch.

Strategy performance, captured by π_t^f and π_t^c, is based on the absolute forecast errors in the previous K periods. That is,

$$\pi_t^f = \sum_{k=1}^{K} \left| E_{t-k}^f (R_{t-k+1}) - R_{t-k+1} \right|, \tag{5}$$

$$\pi_t^c = \sum_{k=1}^{K} \left| E_{t-k}^c (R_{t-k+1}) - R_{t-k+1} \right|, \tag{6}$$

in which $K > 0$ is an integer, and π_t^f and π_t^c denote the historical forecasting performance of the fundamentalists and chartists rules over the past K periods, respectively.

Total demand by investors is the weighted average of demand by fundamentalists and chartists, and can be written as follows:

$$D_t^I = \frac{W_t E_t^f (R_{t+1}) + (1 - W_t) E_t^c (R_{t+1}) - r}{\mu V(R_{t+1})}. \tag{8}$$

Apart from demand for housing by consumers and investors, constructors build new residential structures and sell them in the market. The supply by constructers (S_t) depends positively on the value of the house price index at time t:

$$S_t = c + dP_t, \tag{9}$$

in which $c > 0$ and $d > 0$.

The overall change in the log real house price is linearly dependent on the excess demand plus a random noise term ε_t

$$P_{t+1} - P_t = f(D_t^c + D_t^I - S_t) + \varepsilon_t, \tag{10}$$

where $f > 0$ is a positive reaction parameter. Filling in the different elements from equations (1) to (9) into (10) yields the following relation

$$R_{t+1} = f\begin{pmatrix} (a-c) - \dfrac{r}{\mu V(R_{t+1})} + (b-d)P_t + \\ W_t \dfrac{\alpha}{\mu V(R_{t+1})}(P_t - F_t) + (1-W_t)\dfrac{\beta}{\mu V(R_{t+1})}\sum_{l=1}^{L} R_{t-l+1} \end{pmatrix} + \varepsilon_t. \tag{11}$$

The full model, finally, can be simplified without loss of generality to

$$\begin{cases} R_{t+1} = c' + d' P_t + W_t \alpha'(P_t - F_t) + (1-W_t)\beta' \sum_{l=1}^{L} R_{t-l+1} + \varepsilon_t \\ W_t = \left(1 + \exp\left[\gamma\left(\dfrac{\pi_t^f - \pi_t^c}{\pi_t^f + \pi_t^c}\right)\right]\right)^{-1} \\ \pi_t^f = \sum_{k=1}^{K} \left|E_{t-k}^f(R_{t-k+1}) - R_{t-k+1}\right| \\ \pi_t^c = \sum_{k=1}^{K} \left|E_{t-k}^c(R_{t-k+1}) - R_{t-k+1}\right| \end{cases} \tag{13}$$

in which the combined intercept is given by $c' = f\left(a - c - \dfrac{r}{\mu V(R_{t+1})}\right)$, the consumers versus constructors price elasticity is $d' = f(b-d)$, the fundamentalists' market impact is $\alpha' = \alpha \dfrac{f}{\mu V(R_{t+1})}$, and the chartist's market impact is $\beta' = \beta \dfrac{f}{\mu V(R_{t+1})}$.

We will later on estimate the heterogeneous agent model (13) empirically. In this model c' is a constant. The coefficient d' represents the sensitivity of the house price change to the current house price, driven by the real demand and supply by consumers and constructers. We expect this coefficient to be negative ($d' < 0$), assuming $b < 0$ and $d > 0$, but the magnitude may depend on the size of the wealth and liquidity effects of higher house prices on demand described by Stein (1995). We also estimate an alternative model with the coefficient d' restricted to zero (d' = 0). In this model we effectively assume that the demand by consumers and supply by constructors are always in balance, $D_t^C = S_t$, and that the marginal demand by investors drives the housing price. The empirical advantage of this restricted model is that it does not include the non-stationary variable P_t as an exploratory variable, which may otherwise lead to biased estimates and incorrect statistical inference.

The coefficient α' equals the speed of mean reversion parameter of the fundamentalists, scaled by a positive constant. We expect α' to be negative, otherwise the fundamentalists do not expect the price to revert to its fundamental value. If α' is between minus one and zero, then $-1/\alpha'$ (> 1) denotes the number of periods the price takes to revert to the fundamental value. The coefficient β' is the past return extrapolation parameter of the chartists, scaled by a positive constant. We expect β' to be positive for the chartists to be positive feedback traders exploiting the positive correlation in house price changes.

3. Data and Methodology

3.1. Data sources

We will estimate the model using quarterly time-series data on prices and rents for the aggregate stock of owner-occupied housing in the United States developed by Davis, Lehnert and Martin (2008) and made available by the Lincoln Institute of Land Policy.[5] The data covers the period 1960Q1 until 2009Q1, a total of 197 quarterly observations. The underlying source for the house price changes is the repeat-sales house price index published by Freddie Mac (CMHPI) until 2000, and the S&P/Case-Shiller U.S. National Home Price Index after 2000. The price data used to construct the house price and rent indices is published with a delay of two months in the relevant out-of-sample prediction period after 2000. Hence, the timing in the model does not coincide with calendar time,

[5] Data located at "Land and Property Values in the U.S.", Lincoln Institute of Land Policy, http://www.lincolninst.edu/resources/

but with the time of the release of the latest S&P/Case-Shiller U.S. National Home Price Index value.[6]

3.2. Fundamental value estimate

The expectation formation rule of the fundamentalists requires a fundamental value estimate F_t for the U.S. house price index. The real estate literature broadly poses two methods for calculating a fundamental real estate price. Both methods are based on the notion that the total return to housing, to speak in financial market terms, is the sum of the expected capital gain plus the dividend yield from owning a house. They differ, however, in how to calculate the dividend yield part. Himmelberg, Mayer and Sinai (2005) advocate the use of the so-called user cost of housing. This measure consists of a broad range of factors that affect the cost of living relevant to the owner, such as mortgage rates, taxes, and maintenance costs. Hott and Monnin (2008), on the other hand, theoretically show that there should be no arbitrage possible between renting and buying in equilibrium. As a result, the user cost of housing should be equal to the rental rate, such that the fundamental house price can be represented as the present value of all expected future rent payments.

Given that the fundamental price is a benchmark for investors in our model who do not intend to live in the house but keep it for the sole purpose of monetary profits, we proceed in constructing a fundamental price based on rents (instead of the user cost of housing). Hott and Monnin (2008) define the fundamental price as

$$F_t = E_t\left[\sum_{i=0}^{\infty} \frac{\delta^i H_{t+i}}{\prod_{j=0}^{i} DR_{t+j}}\right], \tag{14}$$

where H_t is the rent in period t, $(1 - \delta)$ is the rate of depreciation of the house, and DR_t the discount rate, consisting of the mortgage rate, maintenance costs and a risk premium.

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[6] The S&P/Case-Shiller U.S. National Home Price Index is published quarterly with a two-month lag. New levels are released at 9am Eastern Standard Time on the last Tuesday of the 2nd month after the end of the quarter. The underlying data for rents is based on 'the rent of primary residence' series, published monthly by the Bureau of Labor Statistics (BLS), which is published within three weeks after the end of the month.

Now suppose that rents increase by a fixed proportion g per period and that the mortgage rate is constant. The former assumption is motivated by the fact that rents are typically indexed, while the rate of inflation is targeted at a constant level in the long run by the Federal Reserve. The latter assumption follows from the observation that home buyers tend to use fixed-rate mortgages. As a result, Equation (14) reduces to

$$F_t = \frac{1+g'}{DR-g'} H_t \tag{15}$$

where $g' = g - \delta$.

Within the no-arbitrage framework of Hott and Monnin (2008), the discount rate of rents *DR* is equal to the unconditional expected return to housing. The expected return to housing consists of the expected return due to capital gains after depreciation, plus the expected rent yield ($E(H/P)$). Equilibrium implies that the long-run rate of capital gains after depreciation is equal to the adjusted growth rate of rents g'. This implies $DR = g' + E(H/P)$, and our final expression for the fundamental house price reduces to

$$F_t = \frac{1+g'}{E(H/P)} H_t \tag{16}$$

in which $E(H/P)$ is the unconditional (i.e. long-term) expected rent yield. See also Fama and French (2002) for a similar derivation of the fundamental price in an equity market setting.

Davis et al. (2008) construct quarterly rent data for owner occupied housing in the United States, which we will use for the calculation of our fundamental price. Prices and rents are deflated using seasonally adjusted CPI data from the IMF International Financial Statistics database. Both the growth rate g' and the unconditional expected rent yield are estimated every quarter t as rolling averages of the available historical observations on the growth of rents (H_t / H_{t-1}) and the rental yield (H_t / P_t).[7] We choose this methodology such that the fundamental price does not incorporate any future information. Figure 1 presents the resulting log-real fundamental price, and the actual log-real price for comparison.

Insert Figure 1 Here

Figure 1 shows that the actual house price generally oscillates around the fundamental price, which supports our method for deriving the fundamental value. Clearly recognizable is the recent run-up and crash in U.S. house prices, which according to our definition of fundamental value looks like a housing price bubble. The log-real house price reached a maximum of 10.7 in the first quarter of 2006, an overvaluation of 48% compared to the rent-based fundamental value. This was an unprecedented situation, as can be seen in the right panel, since the misalignment had never exceeded the 10% mark before. In the first quarter of 2009 the prolonged period of overvaluation ends, as the price eventually drops below the fundamental value. Striking also is the observation that the decrease in house prices during the beginning of the 1990's was not enough to offset the overvaluation created during the second half of the 1980's.

3.3. Descriptive Statistics

Table 1 presents the descriptive statistics of the data used for the estimation of the model. The descriptive statistics confirm the image arising from Figure 1. The U.S. national house price index is on average above its fundamental value (the difference is statistically significant, with *t*-statistic 8.06), which is mainly due to the latter part of the sample

[7] We set the expected rate of house depreciation δ equal to zero, as we lack historical data on depreciation rates. The impact on the fundamental value estimate is small, as changes in F_t in (16) are mainly driven by changes in rents (H_t) and the long-term expected rental yield $E(H/P)$. Using a different value for the depreciation rate δ (for example, 0.5% or 1% per quarter) shifts all fundamental values downwards by the same small fraction and would not materially affect the empirical results in the paper.

(1985-2009). Quarterly changes in the house price index display high positive autocorrelation at lags of 1 to 4 quarters and significantly negative autocorrelation at lags of 3 to 5 years, confirming the mean reversion pattern found by Case and Shiller (1990) and Cutler, Poterba and Summers (1991).

Price changes are twice as volatile as fundamental value changes, confirming the excess volatility puzzle in the housing market noted by Shiller (1981). The correlation between actual price changes and fundamental value changes is only 0.1758 (t = 2.48). However, the Johansen cointegration test indicates that house prices and fundamental values are cointegrated.[8] Hence, the data indicates that there is a long-term equilibrium relation between house prices and fundamental values based on rents. The existence of this equilibrium relation is not driven by the bursting of the housing bubble in the last few years of the sample: we also find a significant cointegration relation if we repeat the Johansen test in the period 1960-2000.

Insert Table 1 Here

The model for the quarterly house price change given by Equation (13) can be estimated directly using quasi-maximum likelihood estimation, as it is a non-linear polynomial of R_t, with the fundamental price F_t as an exogenous variable. We first estimate the model with constant weights, i.e. with $\gamma = 0$ and $W_t = W = 1/2$ (50% chartists and 50% fundamentalists) to study the validity of the functional forms for the different groups of market participants.[9] Subsequently, we estimate the unrestricted model such that the added value of switching based on historical forecasting performance

[8] Results are not shown in the table. The p-value for the null hypothesis of no cointegrating relation is 0.000, while the p-value for the null hypothesis of at most one cointegrating relation is 0.732, based on a VAR model with 3 lags estimated in the period 1961Q2 until 2009Q1.
[9] Estimating the unconditional weight as a free parameter in the constant weight case is not possible as it would only serve as a scaling parameter. As such, the weight parameter would not be identified.

can be determined. The optimal number of lags investors use in their switching decision, *K*, as well as the optimal number of lagged returns used by chartists, *L*, is calibrated using the Box-Jenkins methodology. We check the robustness of the results by also estimating the model without the recent bubble period, using only data from 1961 until 1994. The next section presents the estimation results of the heterogeneous agents model for the U.S. housing market.

4. Results

4.1. In-sample estimation results

Table 2 presents the in-sample estimation results.

Insert Table 2 Here

The first column in Table 2 presents the estimation results over the full sample period for the case without switching between chartists and fundamentalists (the weights are 50%). The coefficient for the current house price on the change in price, *d'*, is positive and significant. This may suggests that the price elasticity of supply is relatively low, while wealth and liquidity effects push up the demand for houses by existing home owners when prices rise, as described in Stein (1995). The investors' coefficients α' and β' are highly significant, with the expected sign. The estimated (scaled) mean reversion parameter α' is negative, indicating that fundamentalists expect the house price to return to the fundamental value. The estimated (scaled) past return extrapolation parameter β' is positive, confirming that chartists are positive feedback traders who extrapolate last quarter's price change. The optimal number of lags for the chartists is $L = 4$.

The second column of Table 2 shows the results for the model that allows switching among the chartist and fundamentalist forecasting rules. The positive sign and the significance of the intensity of choice parameter γ (p-value < 0.01) implies that investors switch to the better performing forecast rule, based on past performance. The optimal number of lags for measuring past performance is K=2. That is, if fundamentalists (chartists) have a more accurate price forecast in period t and t-1, more investors will follow that expectation formation rule in period $t+1$. The added value of switching is further illustrated by the significant increase of the log-likelihood value. The other estimates in the second column are similar to those in the first column, except that the fundamentalists' speed of mean reversion is larger.

The third column shows estimation results for a model with switching, but with the coefficient *d'* for the lagged house price restricted to zero (*d'* = 0). In this model we effectively assume that the demand by consumers and supply by constructors are always in balance, $D_t^C = S_t$, and that the marginal demand by investors drives housing prices. The advantage of this restricted model is that it does not include the non-stationary variable P_t as an exploratory variable, which otherwise may lead to spurious results.[10] The results show that the coefficient estimates for the chartists and fundamentalist rule are not much affected by the inclusion or exclusion of P_t. The switching parameter is somewhat higher, while model fit deteriorates only slightly.

The last three columns of the table show estimation results for the pre-1995 period. Excluding the recent period 1995-2008 does not affect the estimates much, except for the coefficient *d'*. The coefficient *d'* is no longer significant at the 5% level, while the model with switching and *d'* restricted to zero (last column) has the best fit based on AIC. These results suggest that the significant positive value of *d'* in the full sample period may be an artifact of the bubble episode after 1995. We further observe that the scaled extrapolation coefficient of the chartists is lower in the pre-1995 period, in comparison to the full sample period 1961-2009. This fits the image that in the most recent period (1996-2009) the U.S. housing market was driven more strongly than usual by speculators chasing positive price momentum.

4.2. Investor weights

Figure 2 displays several characteristics of the weight W, the percentage of investors following the fundamentalist forecasting rule. In Panel (A) we show a time series plot of

[10] As P and F are cointegrated, the term ($P – F$) is stationary and does not cause similar problems. Further, if we add a coefficient b_F for F in the cointegration relation in Equation (13), i.e. ($P – b_F F$), the coefficient estimate is not significantly different from 1 in all model specifications in Table 2.

the weights, and the distance between the actual price and the fundamental price ($P - F$). During the first part of the sample the weight oscillates around the 50% mark, which implies that investors are equally divided between the fundamentalist and chartist groups. The fluctuations around the mean of 50% are driven by the relative performance of the two forecasting rules, which varies from quarter to quarter. The structural break with this regular pattern in the second part of the sample is very striking: in the period 1993-2007 chartists dominate continuously, with a weight of roughly 65 to 70%, eventually accompanied by a house price level that is far above the fundamental value. Soon after the difference between price and fundamental value reaches its peak in 2006, the estimated proportion of fundamentalists shoots back up to almost 70%, while the price level reverts back to its fundamental value.[11] In the last quarter we finally observe a decline in *W*, because the fundamentalist rule loses its forecast accuracy when the market price drops below its fundamental value in the first quarter of 2009.

Insert Figure 2 Here

Panel (B) of Figure 2 presents a scatter plot of the relative performance of the fundamentalist forecast rule, $(\pi^f - \pi^c)/(\pi^f + \pi^c)$, versus the fraction of fundamentalist investors, *W*. Due to the positive estimated value of γ this line slopes downwards, such that a more accurate fundamentalist forecast results in a higher weight *W*. Furthermore, we observe a slight S-shape, induced by the logit function in Equation (7).

Panel (C), finally, shows the histogram and descriptive statistics of *W*. On average, the majority of investors uses the chartist forecasting rule (1-46% = 54%). The

[11] Note that *W* first drops in 2007 to 30% before climbing to its top of 66% in 2008. This initial decline is caused by the extreme overvaluation. The large overvaluation implies that fundamentalists also expect a large drop in price, due to Equation 3. If this does not materialize, or at least not in the order of magnitude that fundamentalists expect, chartists temporarily gain momentum because they start to extrapolate the negative trend. When price comes closer to its fundamental, fundamentalists' expectation does materialize and they start dominating the market.

spread between the minimum and maximum, though, indicates that the market is never fully dominated by either group of investors. The autocorrelation of the series *W*, 0.81, indicates that the weight is fairly stable; agents do not quickly change their strategy.

4.3. Model simulation

To learn more about the behavior of agents in our model, we simulate house prices by generating a sequence of price changes from the estimated heterogeneous agents model with switching. The log-real fundamental price is set equal to 10 and kept constant. Figure 3 shows the limiting behavior of the log price *P* and the fundamentalist weight *W* for 200 periods of the simulation process.[12]

Interestingly, irrespective of the starting values, the model does not converge to a stable point, as is usually the case in economic models, but to a stable limit cycle. The interaction of fundamentalists and chartists causes the market not to have an equilibrium point. Prices regularly oscillate between just below the fundamental value of 9.993, and the empirical upper limit of 10.154; because these are log-prices, this constitutes a non-negligible range of over 16%. Fundamentalists bring the price back to the fundamental value, after which the price is pushed upwards again by the real side of the market (coefficient *d'*) and extrapolated by chartists. As such, the fraction of fundamentalists in the market ranges from 0.296 to 0.732. A full cycle takes 42 periods, which corresponds to 10.5 years. In other words, also in the absence of external shocks the calibrated heterogeneous agents model generates regular boom and bust price cycles.

Insert Figure 3 Here

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[12] Using $P = F = 10$ as starting values, the model directly sets off in the limit cycle.

4.4. Forecasting

As a final test of the validity of our heterogeneous agents model for the housing market, we study its forecasting power. We contrast the forecasting accuracy of the heterogenous agents model (HAM) with two alternative models: a vector error correction model (VECM) and an ARIMA timeseries model. All models are initially estimated over the in-sample period 1962Q1–2000Q4, and evaluated in the out-of-sample period 2001Q1–2009Q1. The VECM has one lag, indicated by the AIC criterion, and exploits the difference between the actual house price index and its fundamental value based on rents to make forecasts, as well as lagged price changes and lagged fundamental value changes. The ARIMA model does not use fundamental values and purely exploits the (partial) autocorrelation pattern of the historical house price returns. The best fitting ARIMA model in the in-sample period is an ARIMA(4,0,0) model, which is subsequently used to generate out-of-sample forecasts.

Forecasts are created using an expanding window. That is, each model is first estimated on the sample 1962Q1–2000Q4.[13] Subsequently, prices are forecasted up to one year ahead depending on the forecast horizon, which we vary from 1 to 4 quarters.[14] The models are then re-estimated on the expanded sample 1962Q1–2001Q1, and a new set of forecasts is generated. This process is repeated and eventually results in 30 out-of-sample forecasts. Table 3 shows the out-of-sample forecasts made by the models for a horizon of one quarter, and compares them to the actual change in the log real house price and the fundamental value.

Insert Table 3 Here

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[13] Results are qualitatively insensitive to this choice of sample period.

[14] When forecasting more than one period ahead the fundamental value is held constant (equal to the last in-sample observation), such that there is no informational advantage.

Table 3 shows that the HAM and the simple ARIMA model correctly predict the decline in real national U.S. house prices from the second quarter of 2006 onwards, while the VECM model predicts the decline one quarter too early. The HAM and the VECM model also correctly predict the decrease in *nominal* U.S. house prices from the third quarter of 2006 onwards (quarterly inflation rates in 2006Q2 and 2006Q3 were 0.83% and 0.89%, respectively). We do not want to celebrate the success of these forecasts after the fact. Still, it is very interesting to see that relatively simple econometric models, even a plain timeseries model using only the last four lagged returns, could have predicted the big turnaround in the U.S. housing market in the beginning of 2006 and the large nominal price declines that followed.

The difference in forecasting accuracy between the models is assessed using the ratio of the average forecasting accuracy of the HAM over the average forecasting accuracy of the alternative models. A ratio less than one implies better performance for the HAM. Forecast performance is measured using the mean error, mean absolute error, and mean squared error. Table 4 presents these forecast performance ratios, and corresponding *t*-statistics (see Diebold and Mariano, 1995) using a rectangular lag window with *k-1* sample auto-covariances for the *k*-step ahead forecast error.

Insert Table 4 Here

The results in Table 4 show that the HAM forecasts are most accurate: all ratios are below one, apart from the VECM at a horizon of one quarter using MAE. The advantage of the HAM versus the benchmark models generally increases as the forecast horizon increases. The difference is typically significant compared to the VECM (apart from the 1-quarter horizon). Compared to the ARMA model, the difference is of similar magnitude, but mostly insignificant.

In Table 5 and 6 we provide additional evidence of the forecasting power of the HAM. Table 5 presents results of a biasedness and efficiency test of the forecasts. That is, we estimate the equation $\Delta_{t-k}P = \alpha + \beta E_{t-k}(\Delta_{t-k}P_t) + \varepsilon_t$ for each model. Theoretically, unbiasedness of forecasts is represented by $\alpha = 0$, while efficiency is given by $\beta = 1$.

Insert Table 5 Here

Insert Table 6 Here

The results in Table 5 show that the forecasts of the HAM are unbiased and efficient for all forecast horizons. The same can be said for both the VECM and the ARMA model. However, the adjusted R^2 of the efficiency equation is notably higher for the HAM than the benchmark models. Table 6, finally, presents results for the encompassing test, showing the estimation results for the following test equation: $\Delta_{t-k}P_t = \alpha + \beta_1 E_{t-k}^{HAM}(\Delta_{t-k}P_t) + \beta_2 E_{t-k}^{ARMA}(\Delta_{t-k}P_t) + \beta_3 E_{t-k}^{VECM}(\Delta_{t-k}P_t) + \varepsilon_t$. The model with the most informative forecasts will have significant β's. The results in Table 6 are again in favor of the HAM for all four forecast horizons. The forecast of the HAM is the only one that yields a significant β. Moreover, the adjusted R^2's in Table 6 are not significantly higher compared those for the HAM in Table 5. Therefore, the forecasts of the VECM and ARMA models do not seem to contain any information not incorporated in the HAM forecasts.

5. Conclusions

The unprecedented rise and decline in the U.S. housing market in the last decade is broadly viewed as the trigger for the global credit crisis. In addition, an increasing amount of evidence is building that market participants do not always act rationally in the traditional definition. In this paper we develop and estimate a parsimonious model for the U.S. housing market with boundedly rational participants. In our model the market is driven by consumers, constructors and speculative investors. Investors in the housing market use two simple rules of thumb for forming expectations about future house prices: fundamentalist and chartist. The fundamentalist rule predicts that the house price will return to its fundamental value based on rents, while the chartist rule simply extrapolates recent house price changes.

To estimate the model, we first derive a fundamental value estimate for the aggregate U.S. housing market, represented by the Case-Shiller index, using data on rents. We show that the U.S. house price index has a long-term cointegration relation with the rent-based fundamental value. We then estimate the heterogeneous agents model and find that both the chartist rule and the fundamentalist rule explain actual house price changes well. The estimated model indicates that investors switch between these two rules, conditional on past forecasting performance. The results suggest that during the recent period 1992-2005 the housing market was dominated by chartists chasing short-term price momentum, while housing prices rose far above the fundamental value based on rents. Eventually, though, the price level did revert back to fundamental value in the period 2006-2009.

Interestingly, the estimated model can produce boom and busts cycles endogenously, induced by the behaviour of the investors. Although the model is extremely simple and stylized in nature, it is able to forecast the decline of the national U.S. house price index from 2006 onwards. In addition, the heterogeneous agents model outperforms several well-known benchmark models in an assessment of competing out-of-sample forecasts. Heterogeneous agent models may therefore not just be of theoretical interest, but also a useful forecasting tool for housing market participants and regulators.

At the most basic level, the model for the housing market put forward in this paper can be interpreted as follows: the expected change in house prices is driven by two main components, positive autocorrelation in price changes and reversion of the price index to its long-term fundamental value based on rents. The relative importance of these two expected return components varies over time, depending on the recent performance of the two forecasting rules. Empirically this model with dynamic weights fits the data well, providing more accurate out-of-sample forecasts than competing VECM and ARMA models. In addition, our paper can provide an economic interpretation for this empirical model in a heterogenous agents framework with positive feedback traders and traders that expect the price to mean revert to its fundamental value.

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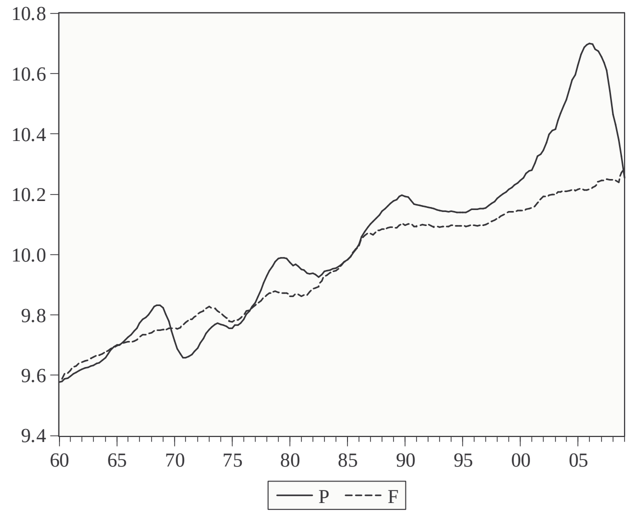
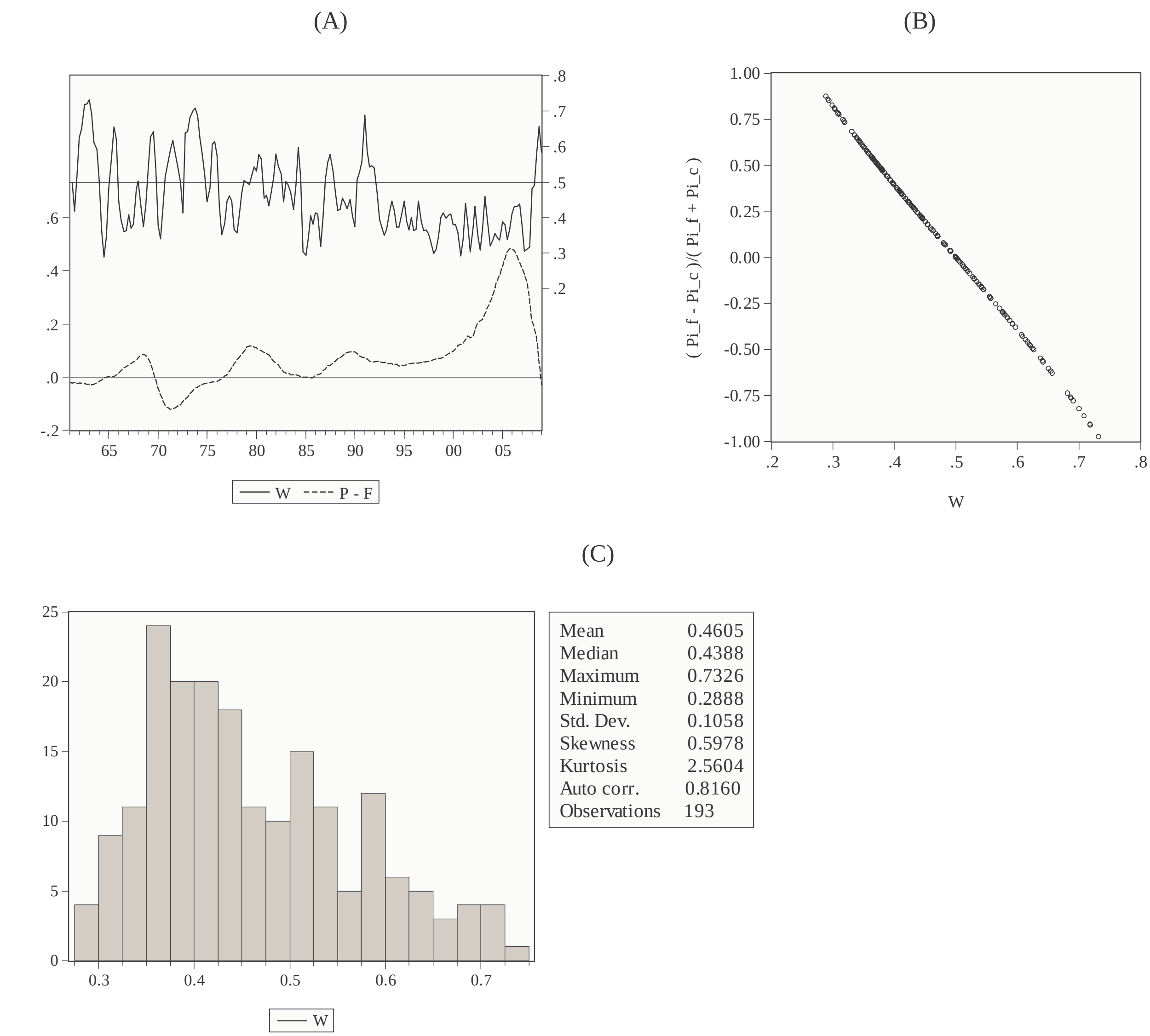
Figure 1: U.S. House Price Index and Fundamental Value Estimate



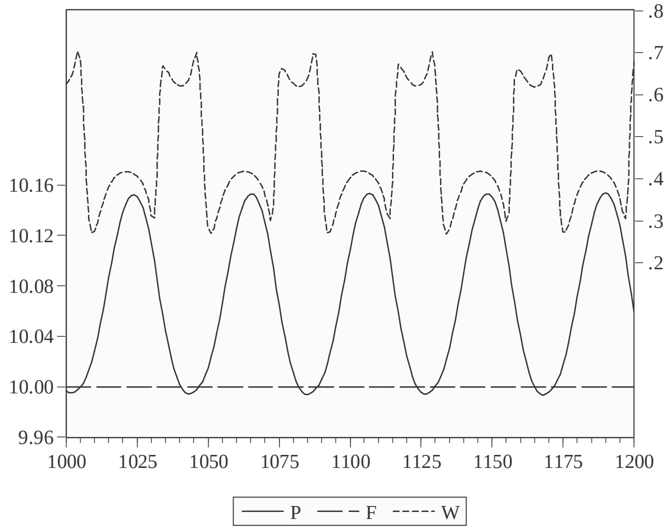
Notes: Figure 1 displays the log-real U.S. house price index *P* and the log-real fundamental value estimate *F* based on rents.

Figure 2: Fraction of Investors using the Fundamentalist Forecasting Rule



Notes: Figure 2 displays the evolution and characteristics of the weight W_t, the fraction of investors using the fundamentalist forecasting rule (assuming mean reversion to fundamental value F). The chartist weight is equal to $(1–W_t)$. Panel (A) presents the time-series of weight W_t and the price misalignment $P_t – F_t$. Panel (B) shows a scatter plot of the weights W_t versus the relative forecasting errors of fundamentalist rule, $(\pi_t^f - \pi_t^c)/(\pi_t^f + \pi_t^c)$. Panel (C) presents the histogram and descriptive statistics of the fundamentalist weight W_t.

Figure 3: Simulated House Price Index Values



Notes: Figure 3 displays the simulated behavior of the log real house price index *P* and the weight of investors using the fundamentalist forecasting rule (*W*), using the estimated model parameters (full sample period). The fundamental value *F* is fixed at the value 10.

Table 1: Descriptive Statistics

	P	*ΔP*	*F*	*ΔF*	*P - F*
Mean	10.0267	0.0035	9.9585	0.0036	0.0705
Median	9.9877	0.0061	9.9708	0.0032	0.0515
Maximum	10.7002	0.0373	10.2841	0.0309	0.4827
Minimum	9.5781	-0.0792	9.5882	-0.0108	-0.1211
Std. Dev.	0.2908	0.0161	0.1941	0.0057	0.1225
Skewness	0.4660	-2.1329	-0.1328	0.7911	1.6815
Kurtosis	2.4981	10.8716	1.6493	5.9840	5.9745
Auto-corr. Q(-1)	0.991	0.820	0.983	0.394	0.989
Auto-corr. Q(-4)	0.944	0.561	0.935	0.156	0.896
Auto-corr. Q(-8)	0.851	0.057	0.872	-0.011	0.682
Auto-corr. Q(-12)	0.747	-0.191	0.810	-0.110	0.460
Auto-corr. Q(-16)	0.650	-0.203	0.750	-0.029	0.295
Auto-corr. Q(-20)	0.569	-0.135	0.691	0.077	0.188
Observations	197	196	196	195	196

Notes: Table 1 presents descriptive statistics of the U.S. log real house price index *P*, the change in price *ΔP*, the log real fundamental value *F* based on rents, the change in fundamental value *ΔF*, and the deviation between the log-real price level and the fundamental value (*P* – *F*). Rows denoted ‘Auto-corr. Q(-*k*)’ display the autocorrelation of the series at quarterly lag *k*.

Table 2: Estimation Results

	1961 - 2009			1961 - 1994		
	Static	Switching	Switching	Static	Switching	Switching
c’	-0.1094**	-0.0874**	0.0011	-0.0831*	-0.0462	0.0009
	(0.0532)	(0.0415)	(0.0008)	(0.0505)	(0.0436)	(0.0006)
d’	0.0112**	0.0089**	-	0.0086*	0.0048	-
	(0.0054)	(0.0042)		(0.0052)	(0.0044)	
α’	-0.1080***	-0.1453***	-0.1080***	-0.1463***	-0.1317***	-0.1076***
	(0.0241)	(0.0189)	(0.0129)	(0.0250)	(0.0271)	(0.0210)
β’	0.5340***	0.4957***	0.4520***	0.4275***	0.3993***	0.3992***
	(0.0162)	(0.0220)	(0.0176)	(0.0245)	(0.0322)	(0.0324)
γ	-	1.0327***	1.4316***	-	1.1362***	1.2449***
		(0.2279)	(0.2066)		(0.3125)	(0.3176)
LL	649.46	659.89	657.28	495.50	504.69	503.88
2ΔLL	-	20.86***	5.21**	-	18.38***	1.62
AIC	-6.713	-6.811	-6.795	-7.267	-7.388	-7.391
Obs	192	192	192	135	135	135

Notes: Table 2 presents in-sample estimation results of the heterogeneous agents model, specified by Equation (13). Standard errors are reported in parentheses below the estimates; LL is the log likelihood of the model and AIC denotes the Akaike information criterion. For the full model with switching, 2ΔLL denotes the difference in log likelihood compared to the static model without switching ($\gamma = 0$). For the switching model with *d’*=0, 2ΔLL denotes the difference in log likelihood compared to the full model with switching. *, **, *** denotes significance at the 10%, 5%, and 1% level.

Table 3: U.S. House Prices and Out-of-Sample Forecasts, 2001Q1-2009Q1

	Actual prices and fundamental values				Out-of-sample forecasts		
	P_t	F_t	% dif	$P_t - P_{t-1}$	HAM	VECM	ARMA
2001Q1	10.28	10.16	12.47%	0.31%	0.88%	0.52%	0.65%
2002Q1	10.34	10.19	15.23%	1.19%	1.20%	0.30%	1.30%
2003Q1	10.42	10.20	21.81%	0.45%	1.68%	0.81%	1.61%
2004Q1	10.51	10.21	30.23%	1.91%	1.75%	1.54%	3.12%
2005Q1	10.63	10.22	41.17%	3.15%	1.47%	0.73%	2.08%
2005Q2	10.66	10.22	44.54%	3.73%	1.78%	1.81%	2.62%
2005Q3	10.69	10.21	47.13%	2.10%	2.16%	2.23%	2.61%
2005Q4	10.70	10.21	48.27%	1.01%	1.48%	0.79%	2.08%
2006Q1	10.70	10.22	48.23%	0.43%	0.65%	-0.27%	1.03%
2006Q2	10.70	10.22	47.66%	-0.16%	-0.27%	-0.87%	-0.32%
2006Q3	10.68	10.23	45.35%	-1.85%	-1.21%	-1.37%	-0.58%
2006Q4	10.67	10.24	43.14%	-0.63%	-2.13%	-2.83%	-1.92%
2007Q1	10.65	10.25	40.86%	-1.89%	-2.12%	-1.80%	-0.40%
2007Q2	10.64	10.25	38.83%	-1.96%	-2.35%	-2.70%	-2.44%
2007Q3	10.61	10.25	36.08%	-2.37%	-2.78%	-2.66%	-0.94%
2007Q4	10.54	10.25	29.46%	-6.88%	-2.74%	-2.97%	-2.44%
2008Q1	10.46	10.25	21.62%	-7.92%	-4.12%	-6.75%	-6.26%
2008Q2	10.43	10.25	18.50%	-3.31%	-4.71%	-7.46%	-6.47%
2008Q3	10.38	10.24	13.95%	-5.07%	-4.50%	-3.20%	-3.72%
2008Q4	10.33	10.27	5.52%	-5.34%	-4.26%	-4.59%	-4.97%
2009Q1	10.25	10.28	-2.92%	-7.15%	-4.11%	-5.04%	-2.24%

Notes: Table 3 shows the real log house price index P_t, the real log fundamental value F_t based on rents, the deviation between price and fundamental value ($P_t - F_t$), the actual change in the log real house price index ($P_t - P_{t-1}$), versus the one-quarter-ahead out-of-sample forecast of the change in the log house price index based on three models: the heterogeneous agents model (HAM), the vector error correction model (VECM) and the ARIMA model (ARMA).

Table 4: Comparison of Out-of-Sample Forecast Errors

Horizon	ME		MAE		MSE	
k	VECM	ARMA	VECM	ARMA	VECM	ARMA
1	0.188	-0.473	1.016	0.952	0.862	0.953
	(0.132)	(-0.452)	(0.132)	(-0.452)	(-0.568)	(-0.265)
2	0.298***	-0.585	0.817***	0.890	0.680***	0.727*
	(-2.801)	(-1.597)	(-2.801)	(-1.597)	(-2.760)	(-1.757)
3	0.333***	-0.479	0.770***	0.818	0.623***	0.630
	(-4.490)	(-1.575)	(-4.490)	(-1.575)	(-2.717)	(-1.427)
4	0.337***	-0.306	0.754***	0.833	0.610**	0.5845
	(-6.810)	(-1.017)	(-6.810)	(-1.017)	(-2.372)	(-1.181)

Notes: Table 4 shows ratio of the forecast error of the HAM over the forecast error of the competing VECM and ARMA models; a number < 1 therefore represents better performance by the HAM. 'ME' is mean error; 'MAE' mean absolute error, and 'MSE' mean squared error. Diebold-Mariano *t*-statistics are reported in parentheses; *, **, *** denotes significance at the 10%, 5%, and 1% level, respectively.

Table 5: Forecast Bias and Efficiency

	HAM			VECM			ARMA		
k	α	β	R^2	α	β	R^2	α	β	R^2
1	0.0000	1.078***	0.700	0.0023	0.887***	0.663	-0.0005	0.960***	0.683
	(0.0028)	(0.139)		(0.0031)	(0.128)		(0.0032)	(0.123)	
2	0.0011	1.0616***	0.732	0.0055	0.878***	0.622	-0.0021	0.955***	0.630
	(0.0060)	(0.1408)		(0.0077)	(0.129)		(0.0084)	(0.145)	
3	0.0022	1.0842***	0.771	0.0083	0.923***	0.638	-0.0052	0.965***	0.633
	(0.0095)	(0.1507)		(0.0130)	(0.148)		(0.0144)	(0.161)	
4	0.0031	1.1307***	0.793	0.0105	0.991***	0.650	-0.0121	1.022***	0.636
	(0.0130)	(0.1514)		(0.0189)	(0.161)		(0.0209)	(0.170)	

Notes: Table 5 presents the results for the efficiency tests of the forecasts of the three competing models. The estimated equation is: $\Delta_{t-k}P = \alpha + \beta E^{i}_{t-k}(\Delta_{t-k}P_t) + \varepsilon_t$, with $k \in \{1,2,3,4\}$ the forecast horizon and $i \in \{HAM, VECM, ARMA\}$ the forecasting model. Due to overlapping data, Newey-West standard errors are used (shown in parentheses); *, **, *** denotes significance at the 10%, 5%, and 1% level, respectively.

Table 6: Forecast Encompassing Test

Horizon k	α	HAM β_1	VECM β_2	ARMA β_3	R^2
1	-0.0018	0.859***	0.761	-0.513	0.695
	(0.0041)	(0.305)	(0.693)	(0.682)	
2	0.0006	1.591***	-0.186	-0.320	0.726
	(0.0082)	(0.385)	(0.554)	(0.466)	
3	0.0011	1.633***	-0.137	-0.404	0.774
	(0.0155)	(0.363)	(0.770)	(0.669)	
4	-0.0077	1.662***	0.324	-0.844	0.797
	(0.0281)	(0.336)	(0.837)	(0.852)	

Notes: Table 6 shows the results of the encompassing tests for the forecasts of the three competing models: the heterogeneous agents model (HAM), the vector error correction model (VECM) and the ARIMA model (ARMA). The following equation is estimated: $\Delta_{t-k}P_t = \alpha + \beta_1 E_{t-k}^{HAM}(\Delta_{t-k}P_t) + \beta_2 E_{t-k}^{ARMA}(\Delta_{t-k}P_t) + \beta_3 E_{t-k}^{VECM}(\Delta_{t-k}P_t) + \varepsilon_t$, with $k \in \{1,2,3,4\}$ the forecast horizon. Due to overlapping data, Newey-West standard errors are reported (in parentheses); $^{*}, ^{**}, ^{***}$ denotes significance at the 10%, 5%, and 1% level.