Optimal capital structure with time-to-build and the impact of financing constraints[1]

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Abstract

We develop a dynamic investment options framework that captures "time-to-build" and realistic features for multiple classes of debt, e.g. debt seniority and interacting debt issues with various maturities. The study investigates the effect of debt and equity constraints on firm value, dynamic leverage choice and the effect of "time-to-build" on firm value and leverage choice. It is shown that a firm is more likely to face financing constraints with short term debt. With "time-to-build" the firm increases leverage in order to reduce the impact of delayed cash flow receipts resulting from "time-to-build". The joint impact of "time-to-build" and financing constraints cause a significant decrease in firm values.

Key words: investment options, optimal capital structure; time-to-build; financing constraints; binomial lattice models; real options.

JEL Classification: G3;G32;G33; G1

1. Introduction

Recent theoretical developments in corporate finance building on Leland (1994) have provided a unified framework for the analysis of investment and financing decisions of the firm. An example is Mauer and Sarkar (2005), extending Leland (1994) to include a single investment option. Furthermore, Sundaresan and Wang (2007) develop a two-stage investment valuation with two sequential debt issues. They provide insights on the interaction between investment and financing decisions discussing debt overhang and leverage choices of firms in the presence of growth options. Hackbarth and Mauer (2010) use a similar dynamic model to analyze the joint choice of debt priority and capital structure with an expansion option. In this paper we develop a comprehensive model along the lines of this literature using a numerical binomial tree approach that extends Broadie and Kaya (2007), allowing for optimal capital structure, multiple debt issues with different priority rules and investment option stages. We use this model to analyze the effect of equity and debt financing constraints. Moreover, we also analyze how time-to-build affects leverage choices and how the joint presence of time-to-build and financing constraints would affect firm values and leverage choices over time. To our knowledge, this problem and this framework have not been tackled within the literature so far.

There is an extensive empirical literature documenting the existence of financing constraints (see for example, Rauh (2006) and Hubbard et al., 1995, and Whited and Wu 2006). Gomes et al. (2006) show that debt constraints represent a risk factor of firm returns. Hirth and Uhrig-Homburg (2010) and Koussis and Martzoukos (2010) analyze exogenous debt constraints impact on the timing of investment, showing a U-shape pattern of the investment threshold and leverage levels. Titman et al. (2004) investigate the impact of financing constraints on default spreads and Lensik and Sterken (2002) discuss conditions under which credit rationing by banks may apply in a real option model. Financing constraints may be the result of asymmetric information and moral hazard between debt and equity holders (see Jensen and Meckling, 1976 and Myers and Majluf, 1984). Financial institutions may often engage in credit rationing in response to adverse selection or moral hazard problems, i.e., set limits to available funding (see for example Fazzari et al., 1988 and Stiglitz and Weiss, 1981). Berger and Udell (2002) point out that banks use several methods (like financial statement analysis, credit scoring, asset-based lending and relationship banking) in an effort to alleviate these problems. And yet, empirical evidence shows that financial constraints are pervasive in particular for small firms. In this paper we introduce debt constraints by imposing ceilings on the level of installment based on the revenue levels at the time of the decision to lend.

Besides the existence of bank financing constraints firms may face equity financing constraints[2]. Uhrig-Homburg (2004) explores costly equity issue that can lead to a cash flow shortage restriction. Fazzari and Petersen (1993) explain that low net worth may induce or exasperate financing constraints. The argument is that low or negative values of net worth make it substantially more costly for the company to obtain financing from outside sources since the value of collateralizable assets is lower. In this paper we also investigate the impact of non-negative net worth constraints on equity, on the firm value and the levels of debt used by the firms. Our results confirm that non-negative net worth (equity) constraints affect the firm's choice of debt by reducing debt levels in order to mitigate the possibility of future negative net worth as revenue uncertainty unfolds.

A real options approach is adopted in this paper to examine a firm's decision to invest in a large capital project, which takes several years to build. For long construction period projects the firm has the option to halt construction, permanently or temporarily at various stages. The investment represents an upgrade to an existing plant and once the upgrade is installed, the firm has the option to suspend operation if the benefits do not cover the variable costs. Time-to-build reflecting the time it takes for the completion of a project characterizes many investment decisions and exists at different intensities, depending on the industry the firm operates. For example, a typical technology project may take 5 years or less for completion (see for example, Pennings and Lint, 1997, for a particular case of Philips in multimedia). The short horizon may be attributed to inherent small product cycles or increased competition. Land development projects may be delayed until a construction permit is issued, which takes time in particular in countries with less developed governmental procedures (Bar-Ilan and Strange, 1996). The development of land itself takes time to complete which varies with the complexity and

[2] Liquidity constraints may also affect the firm's investment decisions (see for example Boyle and Guthrie, 2003 and Cleary et al., 2007). We do not explore issues of liquidity in this paper.

size of the project. In some industries delays in the completion of a project and uncertainties are even more important. For example, new drug development may take more than 11 years for completion (Schwartz and Moon, 2000)[3]. Investment in power plants or aerospace projects may take about 6-10 years to complete (see Bar-Ilan and Strange, 1996 and references therein) and, typically, the installation of scrubbers to control pollution emissions and other abatement technologies require at least 3-4 years to complete (Insley, 2003). However, there are very few theoretical studies considering the impact of time-to-build on the valuation of an investment project. Theoretical work about real option valuation and time-to-build has focused on the case without optimal capital structure (see Majd and Pindyck, 1987, Bar-Ilan and Strange, 1996 and 1998). In Majd and Pindyck (1987) there is a maximum rate at which construction proceeds, so that it takes time before the project is completed and begins to generate revenue. Investment proceeds continuously until the project is completed, although construction can be stopped and later restarted without a cost. In contrast, in our paper investment decisions are made discretely, rather than continuously, the investment option comes to the end of its useful life, instead of being infinitely lived, and optimal capital structure and financial constraints are introduced. Koussis et al. (2007) analyze a similar case called "time-to-learn" where the firms learn new information about the project with a time lag. An interesting result which emerges from the above mentioned papers is that the usual relationship between volatility and the opportunity cost with the timing of investment may be reversed in the presence of time-to-build. It is shown that an increase in volatility or a decrease in the opportunity cost may accelerate investment in the presence of time-to-build. This result holds because an increase in time-to-build causes a reduction in the value (moneyness) of the option, so that an increase in volatility or a decrease in the opportunity cost may sufficiently increase the value of the project triggering earlier investment. We show that low volatility may cause a substantial decrease in value with net worth constraints and time-to-build since it reduces the possibility of the firm to use more debt to alleviate time-to-build constraints. Lower competitive erosion however acts favorably since the firm may borrow more heavily.

[3] Certain stages need to be completed which include preclinical testing of about 2 years, phase I clinical trials for 2 years, phase II clinical trials of 2 more years, phase III clinical trials of 3 years and FDA approval of about 2.5 years.

In the case of long term debt we find that the firm optimally sets low coupon levels which make the impact of debt financing constraints and the possibility for negative net worth less probable. In the case of short term debt the firm optimally sets coupon levels to be high often exceeding revenue levels at the time of debt issue. It is observed that with short term debt and in the absence of financing constraints firm values are higher than the long term cases because the firm accelerates the tax benefits of debt. In the presence of non-negative net worth (equity) constraints an increase in volatility hurts firm value by decreasing both the value of unlevered assets and the tax benefits of debt. This decrease in the tax benefits of debt is caused by the decrease in the use of debt in early stages, although subsequent leverage may increase. With debt financing constraints higher volatility increases firm value for out-of-the money options by increasing the option value component of firm value (since the firm cannot exploit high tax benefits of debt). For in-the-money options an increase in volatility has little impact on firm values.

With time-to-build debt levels increase to alleviate the impact of delayed cash flow receipt due to time-to-build. With low volatility the impact of time-to-build on firm value is reduced since the firm can borrow more heavily in order to alleviate the impact of time-to-build. A similar effect holds for low opportunity cost-competitive erosion. Equity financing constraints have an important effect on firm values in the presence of time-to-build reaching 31% for 5 year time-to-build horizons and 43% to 10 year time-to-build horizon). For low volatility and high opportunity cost the impact of equity financing constraints with time-to-build is more significant. Debt financing constraints impact on firm values is highly significant reaching 42% for plausible parameter values. Time-to-build is similar between high and low volatility (in contrast to equity constraints where the impact at low volatility is more significant). At low opportunity cost debt financing constraints significantly reduce bankruptcy risk (because of the high debt capacity in the unconstrained case) and the firm balances the leverage levels between the initial and the subsequent debt issue.

Overall, we demonstrate the flexibility of the lattice method to incorporate several features that are embedded in many investment decisions characterized by lengthy construction periods and financing constraints. Our paper is organized as follows. Section 2 describes the model, Section 3 presents the numerical results and Section 4 concludes.

2. The model

The model without financing constraints

We assume that price (or revenue) follows a geometric Brownian motion of the form:

$$\frac{dP}{P} = adt + \sigma dZ$$

where α , σ > 0 are constant parameters, r is the risk-free interest rate and *dZ* is the increment of a standard Wiener process. The firm pays an operational cost C so that total earnings before interest and taxes is *P-C*. The firm decides whether to exercise its investment option at time T_1 by paying an irreversible fixed cost I_1 and choosing a mix of debt $D_1(P)$ and equity ($I_1 - D_1(P)$) to finance the investment cost. At investment the equity holders receive a levered equity position denoted by $E_1(P)$. The firm has a useful life (firm maturity) T_F. After the first investment stage, subsequent investment stages may follow with maturities T_i, $i = 2,3,...S$ relative to the prior stage (so that accumulated time for the i option is $T_1 + T_2 + ...T_i$). At each investment stage the firm may decide to issue new debt and rebalance its capital structure[4]. Debt issue i demands a tax-deductible coupon payment R_i per period and a final principal debt (face value) F_i at maturity. Debt maturity for each debt issue is specified by T_{D_i} with $T_{D_1} \leq T_F$, $T_{D_2} \leq T_F - (T_1 + T_2)$, $T_{D_3} \leq T_F - (T_1 + T_2 + T_3)$ etc.

In order to accommodate the choice of different coupon levels at each investment stage we employ forward-backward algorithm. The algorithm proceeds by first creating the pre-investment stage tree with N_1 steps. At each price level at the end nodes of the first investment stage, several lattices are created that capture the next operational phase and default decisions conditional on the choice of the coupon level. Coupon levels

[4] We assume that existing debt cannot be repaid early at this stage so the firm has the option only to increase its debt usage.

depend on the level of revenues P at each state which is discretized through the choice of n_C points and the use of a maximum of $c_{\max}$ points. This implies a coupon grid of:

$$coupon = \{0, \frac{1}{n_c} \cdot P, \frac{2}{n_c} \cdot P, ... \frac{c_{\max}}{n_c} P\}$$

Figure 1 illustrates (using a two-stage example) how the lattice algorithm is applied for multiple investment stages and multiple debt issues. The operational phase is initiated at the time of the first investment maturity T_1 and is assumed to have duration of T_F periods. It may however be terminated if operational costs or coupon payments cause the firm to default (we explain endogenous default decisions in greater detail below). Operations may also be terminated at the subsequent investment stages if the firm decides not to proceed with new investment. At the end of the first investment horizon a first debt issue can be made. At this stage the first coupon selection process starts with new lattice trees created. Depending on the maturity of the first debt issue, the coupon payments may continue to exist after the second investment stage (or the third if one exists). At the time of the second investment stage, the firm may decide on a new debt issue. At this stage a new coupon search process will start *conditional* on the earlier coupon selection. Similarly, the debt maturity of the second option may or may not overlap with other stages and should mature before the end of the operational phase of the firm.

[Insert Figure 1 here]

Investment stages are approximated by lattices with sizes that are defined relative to the tree used for the pre-investment stage which is used as benchmark. Denoting the pre-investment stage with N_1 the size of the i subsequent interval ($i = 2,3,...$) will thus be $N_i = \left(\frac{T_i}{T_1}\right) \cdot N_1$. The last period (after T_S) is approximated by $N_F = \left(\frac{T_F - (T_1 + T_2 + ...T_{S-1})}{T_1}\right) \cdot N_1 = \left(\frac{T_S}{T_1}\right) N_1$.

If new investment leads to expansion of revenues then this is modeled using $e_1, e_2 ... e_S$ expansion factors multiplying the revenue variable. The revenue level at stage i equals $(e_1 + e_2 ... e_i)$ times the revenue of that stage. When investments do not expand cash flows the expansion factors will be set to zero with only $e_1 = 1$. In the event of bankruptcy proportional costs b to the value of the firm are supposed to be incurred. Priority rules for debt holders in case of default need also to be specified. Here both absolute priority (APR) and pari-passu (PPR) rules will be considered. Under APR of the early debt issues debt holders value i at time t, D_{it}, in case of default is specified by:

$$D_{1t} = \min\left[(1-b)V_t^u, R_1 \Delta t + \tilde{D}_{1t}\right]$$

$$D_{jt} = \min[(1-b)V_t^u - \sum_{i=1}^{j-1} D_{it}, R_j \Delta t + \tilde{D}_{jt}], \quad j = 2, \ldots S \tag{1a}$$

where $\tilde{D}_{it}$ denotes the expected continuation value for debt issue i in case the firm does not default at t calculated as $\tilde{D}_{it} = (p_u D_{t+dt,h} + (1-p_u) D_{t+dt,l}) e^{-rdt}$.[5] Notice that this rule is slightly different from the one specified in Sundaresan and Wang (2007), where debt holders' recovery value is based on the face value of debt and not on the continuation value of debt ($\tilde{D}_{it}$) at the default date. In the case of PPR debt value j in the event of bankruptcy will be determined by:

$$D_{jt} = \left(\frac{D_{jt}}{\sum_{i=1}^{S} D_{it}} \right) \cdot (1-b) V_t^u, \qquad j = 1, 2, \ldots S. \tag{1b}$$

[5] The continuation value can be thought of the value of an otherwise identical loan made to a firm with same characteristics in case it does not default at t. This takes into account that equity holders may choose to default at a subsequent stage.

A standard formulation of the lattice parameters for the up and down jumps and the up and down probabilities (see Cox, Ross and Rubinstein, 1979) requires that $u = e^{\sigma dt}$, $d = e^{-\sigma dt} = \frac{1}{u}$, $p_u = \frac{e^{(r-\delta)dt} - d}{u-d}$, $p_d = 1 - p_u$, where $dt = \frac{T_F}{N_F}$ and δ is an opportunity cost parameter. We keep track of the following information at each node of the binomial tree: unlevered assets (V^U), tax benefits of debt (*TB*), bankruptcy costs (BC), equity (*E*), debt issues ($D_1, D_2, ...$) and levered firm value (V^L).

Cash inflows (revenues) and outflows (costs and interest payments) as well as decisions occur every time step Δt. Δt can be controlled by a variable N_{dec} that specifies the number of decision points within each unit period[6]. Let the corporate tax rate be denoted by τ >0. At the end of the operational phase T_F equity and the other variables are calculated as follows:

$$E_{T_F} = \max\left[(P - C - \sum_{i=1}^{S} R_i I_i^{debt})(1-\tau)\Delta t - \sum_{i=1}^{S} F_i I_i^{debt},\, 0 \right] \quad (2a)$$

where I_i^{debt} is an indicator that takes the value of 1 if debt issue i has not expired and zero otherwise[7].

If $E_{T_F} > 0$, then

$$V_{T_F}^{u} = (P - C)(1-\tau)\Delta t$$

$$TB_{T_F} = \tau\left(\sum_{i=1}^{S} R_i I_i^{debt} \right)\Delta t$$

$$BC_{T_F} = 0 \quad (2b)$$

[6] Thus, Δt = 1/Ndec. Each Δt interval is approximated by a sub-tree $N_{\Delta t}$. To maintain accuracy discounting occurs for the interval dt = T_i/N_i. In principle, the decisions can be made as dense as possible approximating the continuous decision limit when N_{dec} tends to infinity.

[7] This requires that we keep track of elapsed time for each debt issue.

$$D_{iT_F} = R_i \Delta t + F_i$$

$$V^L_{T_F} = E_{T_F} + \sum_{i=1}^{S} D_i I_i^{debt} ,$$

otherwise if $E_{T_F} = 0$[8] (i.e., bankruptcy occurs):

$$V^u_{T_F} = (P - C)(1 - \tau)\Delta t$$

$$TB_{T_F} = 0 \tag{2c}$$

$$BC_{T_F} = bV^u_{T_F}$$

$$V^L_{T_F} = E_{T_F} + D_{T_F} .$$

Debt values at maturity in the event of default depend on the priority rule. If APR is defined then:

$$D_{1T_F} = \min\left[(1-b)V^U_{T_F}, R_1 \Delta t + F_1\right] \tag{2d}$$

$$D_{jT_F} = \min[(1-b)V^u - \sum_{i=1}^{j-1} D_{iT_F}, R_j \Delta t + F_j], \ j = 2,\ldots .S$$

In the case of PPR:

$$D_{i,T_F} = \left(\frac{D_{i,T_F}}{\sum_{i=1}^{S} D_{iT_F}} \right) \cdot (1-b)V^U_{T_F} \tag{2e}$$

Prior to the maturity of the operational phase (and after all investments have taken place) the values of each of these variables are calculated as follows:

$$E_t = \max\left[(P - C - \sum_{i=1}^{S} R_i I_i^{debt})(1-\tau)\Delta t + \tilde{E}_t , 0 \right] \tag{3a}$$

8 If the value of unleveled assets turns negative then the value of all variables are set to zero.

If $E_t > 0$, then

$$\begin{aligned} V_t^u &= (P - C)(1-\tau)\Delta t + \tilde{V}_t^u \\ BC_t &= 0 + \tilde{BC}_t \\ TB_t &= \tau\left(\sum_{i=1}^{S} R_i I_i^{debt}\right)\Delta t + \tilde{TB}_t \\ D_{it} &= R_i \Delta t + \tilde{D}_{i,t} \\ V_t^L &= E_t + \sum_{i=1}^{S} D_{it} I_i^{debt}, \end{aligned} \tag{3b}$$

whereas, if $E_t = 0$ then

$$\begin{aligned} V_t^u &= (P - C)(1-\tau)\Delta t + \tilde{V}_t^u \\ BC_t &= bV_t^u \\ TB_t &= 0 \\ V_t^L &= E_t + \sum_{i=1}^{S} D_{it} I_i^{debt}, \end{aligned} \tag{3b}$$

where $\tilde{x}_t$ denotes the expected discounted value of variable x and equals $\tilde{x}_t = (p_u x_{t+dt,h} + (1-p_u)x_{t+dt,l})e^{-rdt}$. Debt values are determined similarly depending on the priority structure. Under APR expression (1a) can be applied while for the case of PPR one would use equation (1b) for each debt value.

For points within the lattice not involving a decision to default or not, which are used for increased accuracy, the values of each variable are the discounted expected values of the variables of the following period.

At the maturity of each investment option stage i occurring at time t, where t takes values according to the specified investment maturities, the levered firm value includes the equity value plus the amount of debt received at i plus the expected values of debt raised in the future minus the total cost which includes the investment paid at i stage and the expected cost to be paid in the future:

$$V_t^L = \max[E_t + D_{it} + \sum_{k=i+1}^{S} \tilde{D}_{kt} - (I_i + \sum_{k=i+1}^{S} \tilde{I}_k),0] = \max[V_t^u + TB_t - BC_t - (I_i + \sum_{k=i+1}^{S} \tilde{I}_k),0]$$

(4)

For example, in the first investment stage and assuming two stages only, the condition at the maturity of the investment stage would be:

$$V_{T_1}^L = \max[E_{T_1} + D_{T_1} + \tilde{D}_{T_2} - I_1 - \tilde{I}_2, 0] = \max[V_{T_1}^u + TB_{T_1} - BC_{T_1} - I_1 - \tilde{I}_2, 0]$$

while at the second stage the condition at investment becomes:

$$V_{T_2}^L = \max[E_{T_2} + D_{T_2} - I_2, 0] = \max[V_{T_2}^u + TB_{T_2} - BC_{T_2} - I_2, 0]$$

Bankruptcy in periods between investment stages (and prior to the final investment) is triggered when the earnings net of cost and coupon payments plus the expected levered firm value (which includes expected equity value, expected cash received by debt issues and expected costs to be paid) are negative. Thus, the bankruptcy condition for any time *t* prior to the last investment stage is:

$$V_t^L = \max\left[(P - C - \sum_{i=1}^{I} R_i I_i^{debt})(1-\tau)\Delta t + \tilde{V}_t^L,\ 0 \right] \tag{5}$$

If $V_t^L > 0$ then the values at that stage are calculated as in equations (3a) and (3b) while in case of bankruptcy using equations (3c) and (3d) (or 1b in the case of PPR).

The values obtained at the first investment stage are discounted at $t = 0$. The value of the firm at time zero involves the sum of the present value of equity, all expected debt issues minus the expected present value of the investment costs. This is equivalent to the expected present value of the unlevered assets plus the expected present value of the tax benefits minus the expected present value of bankruptcy and the investment costs.

Finally, in order to evaluate the amount of leverage used one can calculate the proportion of each debt value over the total value of equity plus debt. In the case of two stages considered in our numerical results we calculate three measures. All measures use the time 0 values of equity and debt (despite the fact that some debt may be issued at a future date). The first measure (Lev_1) includes only the first debt issue over the total value of equity plus all debt. The second measure (Lev_2) includes the proportion of the second debt issue over the value of equity plus all debt issues. The third measure is total leverage over the total value of equity plus debt (Lev_T).

Incorporating financing constraints

A firm may face equity or debt financing constraints or both. We first note that the bankruptcy trigger decision between investment stages described in (5) allows that equity value between stages becomes negative implying an equity infusion of cash (or negative net worth) if the continuation value is sufficiently large to justify it. In order to incorporate equity financing constraints in our numerical simulations we investigate the case where equity holders face constraints where net worth remains positive at all times, i.e., $E_t \geq 0$ [9]. We note that this condition does not imply that there is no equity financing since this condition still allows that equity holders finance part or all of the investment cost at the time investments take place. However, if at any point in time after investment equity value drops to zero default is triggered without considering additional infusion of cash. This type of constraints may reflect difficulties in raising new external finance in the case where the firm is not performing well or the inability of current equity holders to infuse new cash because of personal financing constraints.

Debt constraints are also easily incorporated in this framework by limiting the possible coupon level at the borrowing date. A reasonable way to formulate this is to allow coupon rates to be only a fraction of the revenue level at which the leverage decision is

[9] Alternative constraints where partial infusion of cash prior to investment equity can be easily analyzed. We analyze the most serious form of these constraints.

made. This is a particularly reasonable assumption especially for bank loans since banks normally take into consideration that the coupon (installment) is only a fraction of the current revenue levels of the firm before setting up a new loan in order to limit the risk of default.

3. Numerical results and discussion

Numerical accuracy of the algorithm

Table 1 describes the numerical accuracy of the model. In general our investigation reveals that firm values exhibit small oscillations between small and larger number of lattice steps. This is particularly important since the differences between 12 and 24 steps do not exceed 3.6% and are not affected by the volatility used.

[Insert table 1 here]

Similar results apply for the value of unlevered assets, tax benefits and bankruptcy costs which exhibit some oscillations which are in general small. Equity and debt values exhibit the largest oscillations. This is expected based as pointed out by the analysis of Broadie and Kaya (2007) and Agliardi and Koussis (2010) working with lattice models in a similar context. As pointed out in these papers debt values are oscillatory because the approximation of the default boundary is particularly important. In general, equity values are not very oscillatory when investment options are not involved but in our case approximation of the investment trigger exasperates equity oscillations as well. Despite the oscillatory nature of the solution with respect to these values important conclusions can be reached about several issues as discussed below.

Long term debt without constraints

In this section we investigate the case of long term debt where the early debt issue horizon interacts with the subsequent debt issue. In all our numerical simulations in this and the following sections we adopt APR of the first debt issue. Results are not materially altered with PPR, with only slight increases in the second debt issue value[10].

Table 2 presents the sensitivity results for the long term debt case with respect to volatility. We first observe that with long term debt firms optimally select the coupon levels to be much lower than the current revenue levels. For this reason the debt financing constraints (of the form analyzed in this paper as a percentage of current revenues) are not binding. Similarly, because of the low coupon levels negative net worth and equity infusion is also not important. This can be contrasted with the short term debt case that will be investigated in the next subsection, where firms would optimally choose high coupon levels in the unconstrained case which would make debt constraints binding and also increase the likelihood of negative net worth as revenue uncertainty unfolds.

We observe an interesting U-shape pattern of firm values as a function of volatility for relatively out-of-the-money to at-the-money options[11]. This result is due to the trade-offs involved: a higher volatility increases the option value on unlevered assets (net of investment costs) but reduces the net benefits of debt (tax benefits minus bankruptcy costs). This result is similar to the result one obtains for one-stage investment problem with one debt issue (e.g., using Mauer and Sarkar, 2005). If the firm's revenue levels are high (in-the-money case) then an increase in volatility not only hurts the net benefits of debt but also reduces the option value on unlevered assets thus resulting in a monotonic decrease in firm value.

[Insert table 2 here]

[10] The differences in firm value between the different priority rules are negligible and hard to distinguish using the numerical approach employed in this paper (since small oscillations exist in firm values not enabling one to make a clear statement). Hackbath et al. (2007), however, show that firm values are slightly higher under equal priority.

[11] We loosely define moneyness by observing the small firm values in the case $P = 10$ and the large firm values for $P = 30$. One can calculate the implied value of unlevered assets (without default) in each case which gives further indication of this insight. When $P = 10$ then $V = 10/0.06\ (1-0.35) = 108.33$ while in the in-the-money case $V = 30/0.06\ (1-0.35) = 325$. The investment costs add up to 100.

Equity values appear to be decreasing in volatility for out-of-the-money options but may be increasing for in-the-money options (unless a high volatility exists). The behavior of equity value as a function of volatility can be better understood if one looks at its components. Equity value is the sum of unlevered value (before netting the costs) and the net benefits of debt minus the amount of first and second debt value due. We note that the value of unlevered assets and the net benefits of debt are both decreasing in volatility for the out-of-the-money case which is the main driving force for the inverse relationship of equity value with volatility. Furthermore, this inverse effect is exasperated in the out-of-the-money case by the increase of the first debt value with higher volatility which increases the amount that equity holders need to pay.

For the out-of-the-money case the present value of the first debt issue is increasing in volatility while the present value of the subsequent debt issue is decreasing in volatility. When the revenue levels are high (in-the-money case) higher volatility reduces the first debt issue (except for high volatility levels) while a decreasing relationship with volatility is maintained for the second debt issue.

Besides these directional effects of volatility on debt values, it is also interesting to note the relative levels of debt used under low or high volatility levels between the first and the second debt issue. As can be seen, when the option is relatively out-of-the-money and the volatility is low the firm borrows conservatively at first while the present value of the second debt issue is much higher. The reverse is true for high volatility levels where the firm would optimally borrow heavily in the first debt issue and less (in present value terms) in the second debt issue.

It is a well-known result that a low volatility enhances debt values when the firm is at the investment trigger and no future investment stages follow (see for example, Leland, 1994). This is because of lower bankruptcy risk which enhances the net benefits of debt. The relatively surprising result observed in table 2 for the out-of-the-money case is that the first debt issue increases with volatility. This result exists because another investment stage follows and initial debt matures after the maturity of the second investment stage.

This creates an option effect: an increase in volatility gives the incentive for the firm to borrow more heavily initially since the option value of repaying the remaining debt (and obtaining remaining tax benefits net of bankruptcy costs) at the time of second stage investment is now higher. This option effect obviously exists for both the out-of-the-money case and the in-the-money case but it is more important in the former case since a higher volatility makes the potential of the option ending up in-the-money more likely. On the other hand, when the option is in-the-money a low volatility implies lower bankruptcy risk while also keeps a larger probability that the option stays in-the-money. This allows the firm to borrow more heavily. An increase in volatility in the in-the-money case not only increases bankruptcy risk but also increases the probability that the option becomes out-of-the-money; these effects result in the firm borrowing more conservatively on the first debt issue. With very high levels of volatility the high upside option potential may dominate even for the in-the-money case and the firm chooses to borrow heavily on the first debt issue.

We observe that the second debt issue is decreasing in volatility for both the out-of-the-money and the in-the-money case. With no more stages remaining creating a similar option effect as the one existing for the first debt issue, an increase in volatility causes a decrease in second debt values for a given revenue level reached at the second stage investment. However, it is important to notice that there are still some other effects that could affect this result. First, a higher volatility may result in a potentially higher revenue levels reached at the second stage and thus higher potential debt value for the second issue. However, the decision on how much debt to obtain on the second debt issue is also affected by the initial leverage level decision. Second, an increase in volatility affects the probability of exercising the second stage investment and thus the expected present value of the second debt issue. Similarly, however, this probability is also endogenously affected by the firm’s choice of the first stage debt level. The observed results show that the optimal firm’s choice of initial debt with higher volatility results in a decrease in the probability of exercising the second stage investment. This is indicated by the lower expected present value of the second stage costs which decrease for both the out-of-the-money and the in-the-money case.

Overall, with respect to leverage ratios we observe that the proportion of the first debt in firm value as described by lev_1 is increasing in volatility for out-of-the-money options and may have a U-shape for in-the-money options. Lev_2 - capturing the proportion of the second debt issue on firm value - shows to be decreasing in volatility for out-of-the-money options while it may be decreasing at least for medium levels of volatility for in-the-money options. Total leverage as captured by Lev_T exhibits a U-shape for both out-of-the-money and in-the-money options in relation to volatility.

Short term debt with and without debt constraints

In this subsection we focus on the short term debt case and analyze the impact of financing constraints. Table 3 presents sensitivity results with respect to volatility for the unconstrained case with debt maturity for both debt issues set short term ($TD_1 = TD_2 = 5$). For the short-term debt case we observe that the firm optimally selects high coupon levels which exceed current revenue levels. As we shall subsequently see, it implies that debt financing constraints become binding. We also observe that equity values (net worth) is negative in this case. This is because of the high coupon and debt levels used. Firm values are higher than the long term debt case for all levels of volatility and for different levels of moneyness. This is confirmed with more detailed tests of debt maturity choice we have performed which shows that short term debt is optimal for both debt issues. This result confirms an earlier result of Agliardi and Koussis (2010) for a single debt issue, showing that for bank loans (no principal payment involved) short term maturity is preferred. This is also consistent with empirical evidence by Altman, Gande and Saunders (2006) and Rauh and Sufi (2008), who find that loans typically are of shorter maturity than bonds and MacKay (2003) who found that firms relying more on shorter term loans have higher leverage (although he relates this to agency reasons).

[Insert table 3 here]

In contrast to the long term case, we observe that the U-shape in firm value for the low initial revenue case (P =10) does not appear. This result holds because with short term debt firm values are at highest levels for the same initial level of revenue used (a more in-the-money project). Thus the U-shape would appear for lower values of the current revenue level. For short-term debt equity values are increasing in volatility for both low and high revenue level (in contrast to the long term debt case). Debt values are strictly decreasing and thus total leverage is reduced.

In the long-term case the second debt issue appeared more important for out-of-the-money options with low volatility. In contrast, with short term debt, the initial and subsequent debt values are at their highest levels at low volatility and are decreasing in volatility (with initial debt having a more important contribution in value). The results thus indicate that with short term debt equity holders want to accelerate the receipt of tax benefits of debt. The fact that both debt issues are now decreasing in volatility reinforces the interpretation provided earlier regarding the option effect of repaying initial debt which exists in a multi-stage setting. Since in the case of short-term debt the initial debt expires before the exercise of the second stage investment this option effect is no longer present.

Table 4 shows sensitivity results with respect to volatility when the firm faces equity financing constraints. We observe that for both the out-of-the-money and the in-the-money case an increase in volatility reduces firm values by reducing the value of unlevered assets and the tax benefits of debt. In order to avoid subsequent negative net worth that would result in higher default risk, initial debt levels remain rather flat. Firm values decrease as a function of volatility because the debt levels are more significantly decreased than the equity increase. In contrast to the unconstrained case where the firm borrows heavily initially (lev_1 high), with equity constraints the firm avoids early default by borrowing less heavily initially (lev_1 reduced) and increases leverage at the second stage investment (lev_2 increase).

[Insert table 4 here]

In Table 5 we observe the results in the presence of debt financing constraints. In the out-of-the-money case an increase in volatility causes an increase in firm value which is caused by an increase in the option value of unlevered assets (value of unlevered assets net of expected costs). In this case a higher volatility helps alleviate the impact of financing constraints which reduce the tax benefits of debt. We observe that both the initial and subsequent debt are decreased but lev_1 and lev_2 remain rather flat with lev1 being higher than lev_2 (like in the unconstrained case).

[Insert table 5 here]

In the in-the-money case an increase in volatility has little impact on firm value when the firm faces debt financing constraints. This occurs because both the value of unlevered assets and the net benefits of debt remain flat. An increased volatility keeps equity and debt values practically unchanged. Lev1 is higher than lev2 as in the case of out-of-the-money option.

Compared to the unconstrained case the percentage drop in value is more substantial for the out-of-the-money case. For in-the-money the absolute value drop is significant while the percentage drop is less important than out-of-the-money case. In the unconstrained case we observe that lev_1 is much higher than lev_2. While this remains in the constrained debt case this gap is now reduced.

The effect of time-to-build

In this subsection we investigate the impact of time-to-build on firm values in the presence and in the absence of financing constraints. First, without financing constraints the financing choices of the firm facing time-to-build restrictions are examined, in order to understand whether the firm chooses to raise more debt in the presence of time-to-build and whether the firm raises more debt at the beginning or at subsequent debt issues.

Our second goal is to investigate the importance of financing constraints when the firm faces time-to-build restrictions and analyze the adjustments in the financing policy.

Table 6 analyzes the case without financing constraints with sensitivity with respect to volatility and the opportunity cost-competitive erosion. With time-to-build we assume that the firm receives no cash flows until the project is completed, i.e., cash flows initiate after the second investment is implemented. It is also assumed that the useful life of the project remains the same after completion of the project at 20 years so as to isolate the effect of time-to-build. In all our numerical simulations in this section we use short maturity for both debt 1 and debt 2. In the base case, we observe that firm value, the value of unlevered assets and net benefits of debt are reduced with time-to-build. Interestingly, debt 1 and debt 2 increase with time-to-build and are only decreased in very long time-to-build horizons. Lev_1, Lev_2 and Lev_T reflect this pattern of debt values since they are increasing as time-to-build increases and then remain flat for long time-to-build horizons. Equity values are decreasing with time-to-build reflecting the increase in the debt due (except for very long time-to-build horizons where debt due is decreased).

[Insert table 6 here]

Investigating the case of low volatility reveals some interesting insights. Since we are using short-term debt an increase in volatility (as was shown in previous section) reduces firm value. Based on standard real options model (e.g., Majd and Pindyck, 1987 and Bar-Ilan and Strange, 1996 and 1998) with time-to-build one could expect that higher volatility may have been beneficial in order to increase the option value of completing the project successfully. The results however show that with low volatility the firm borrows more heavily to alleviate the impact of time-to-build. This is reflected in the large differentials between the low and high volatility case with respect to the tax benefits of debt. The ability of the firm to raise more debt under low volatility reduces the percentage decrease in firm values at higher time-to-build horizons (compared to the high volatility case). A similar effect exists for the case of low δ. A low δ reduces the impact of time-to-build on firm values since the debt capacity is higher and the increase in the tax benefits is more significant.

Table 7 investigates the impact of equity financing constraints in the presence of time-to-build. The results show that equity financing constraints can cause severe reductions in firm values in the presence of time-to-build. For the base case the decrease in firm value reaches 31% for 5 year time-to-build horizon and 43% for 10 years of time-to-build.

[Insert table 7 here]

For lower volatility the decrease in values is more severe. The reason is that at low σ the unconstrained firm would optimally choose to borrow more at $t = 0$ but that would require additional equity financing (since for some states the revenues would not suffice to cover coupon payments). For this reason equity financing constraints indirectly impose constraints on the optimal amount of leverage causing these severe reductions in firm values. Interestingly, the firm in this case will optimally shift emphasis on obtaining higher leverage in subsequent debt issues.

For lower opportunity cost δ the percentage reduction in firm values due to equity constraints are less important (but still significant). The reason for this smaller impact is that with lower δ the firm can more easily retain positive equity values at the unconstrained levels (despite borrowing more heavily). With lower δ and for short time-to-build the firm can retain positive equity values even using high initial debt levels. However, as the time-to-build becomes longer the subsequent leverage level becomes more important.

Table 8 analyzes the case with debt financing constraints and no equity constraints. Debt financing constraints cause a decrease in value of about 15-16% even for the case without time-to-build. Time-to-build makes the impact of debt financing constraints more significant reaching firm reductions of 33% for time-to-build of 5 years for the base case, 42% reduction for the low volatility case and 36% for the low opportunity cost case. The results also show that the initial leverage appears more significant than the subsequent leverage choice.

[Insert table 8 here]

With debt constraints we observe that the firm value differences between low and high volatility are not significant. Low coupon levels due to the constraints reduce the risk of bankruptcy (as indicated by the low bankruptcy costs) even for the high volatility (base case) thus values of unlevered assets, and tax benefits do not change with different volatility levels. Leverage choices remain very similar to the higher volatility case with lev_1 being more important than lev_2.

When δ is small bankruptcy is practically non-existent because of the constrained low levels of coupons used and the high debt capacity if the firm was not constrained. Debt appears to be equally balanced between the first and the second debt issue.

4. Conclusions

Using a dynamic investment options model with optimal capital structure we show that firms borrow more heavily when debt maturity is short and are thus more likely to face financing constraints compared with long term debt. With time-to-build the firm increases leverage in order to reduce the impact of delayed cash flow receipts due to time-to-build. Time-to-build causes more significant drop in value when non-negative net worth constraints exist in the case of low volatility and high opportunity cost. With debt constraints we observe that the firm value differences between low and high volatility are not significant. The joint impact of time-to-build and financing constraints causes significant decreases in firm values.

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Figure 1: A graphical illustration of the forward-backward algorithm for multi-stage investment issues with multiple debt issues

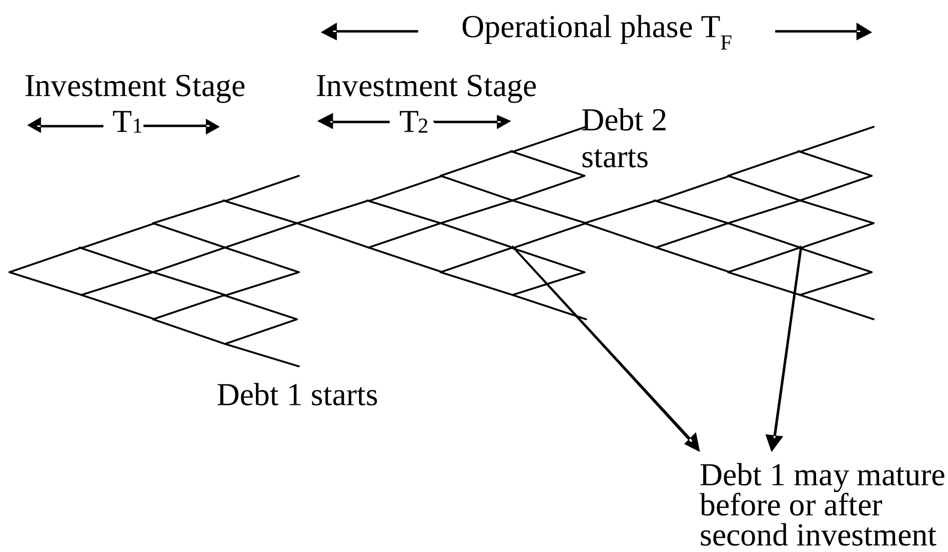

Table 1. Accuracy of the numerical algorithm

N = 6

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	Coupon 1	Lev1	Lev2	Lev_T
σ = 0.10	11.676	75.777	19.824	1.381	36.198	22.602	35.419	50	32.544	2	0.24	0.38	0.62
σ = 0.20	8.404	68.726	14.561	1.692	38.300	18.379	24.915	50	23.189	2	0.23	0.31	0.53
σ = 0.30	10.697	64.573	14.658	2.312	32.726	25.630	18.563	50	16.222	3.5	0.33	0.24	0.57
σ = 0.40	13.856	66.514	15.279	3.416	31.306	26.405	20.666	50	14.521	4	0.34	0.26	0.60

N = 12

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	Coupon 1	Lev1	Lev2	Lev_T
σ = 0.10	11.271	74.790	19.356	1.270	36.302	22.213	34.361	50	31.604	2	0.24	0.37	0.61
σ = 0.20	8.202	66.153	14.706	1.748	35.347	21.613	22.152	50	20.909	2.5	0.27	0.28	0.55
σ = 0.30	10.229	63.590	14.279	2.171	32.731	25.257	17.710	50	15.469	3.5	0.33	0.23	0.57
σ = 0.40	13.560	60.847	16.662	3.863	22.177	37.816	13.652	50	10.085	7	0.51	0.19	0.70

N = 24

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	Coupon 1	Lev1	Lev2	Lev_T
σ = 0.10	11.431	77.510	19.580	1.385	38.375	17.557	39.772	50	34.273	1.5	0.18	0.42	0.60
σ = 0.20	8.122	64.171	14.948	1.743	32.926	24.712	19.738	50	19.254	3	0.32	0.26	0.57
σ = 0.30	9.852	62.883	13.999	2.094	32.698	24.921	17.169	50	14.937	3.5	0.33	0.23	0.56
σ = 0.40	13.131	61.555	15.728	3.466	25.414	33.889	14.514	50	10.686	6	0.46	0.20	0.66

Notes: The model with no equity or debt constraints is used with parameters are: P =10, C = 0, risk-free rate r = 0.06, competitive erosion δ = 0.06, investment cost I_1 = 50, I_2 = 50, b = 0.5, tax rate τ = 0.35 and T_1 = 0, T_2 = 5 (time of second option relative to the first), T_F = 20 and debt maturity T_{D1} = 20 and T_{D2} = 15 assuming zero principal. An optimal coupon is chosen based on n_c =20 discretization points for each price level with maximum coupon level points being equal to the price levels (c_{max} = 40). The table results investigate the accuracy of the model at different volatility levels using assume N_{dec} = 1 (1 yearly decisions) and using $N_{\Delta t}$ = 6, 12 and 24 steps per year.

Table 2. Long-term debt with debt interactions: Sensitivity to volatility

Out-of-the-money case (P = 10)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
σ = 0.10	11.431	77.510	19.580	1.385	38.375	17.557	39.772	50	34.273	1.5	0.18	0.42	0.60
σ = 0.20	8.122	64.171	14.948	1.743	32.926	24.712	19.738	50	19.254	3	0.32	0.26	0.57
σ = 0.30	9.852	62.883	13.999	2.094	32.698	24.921	17.169	50	14.937	3.5	0.33	0.23	0.56
σ = 0.40	13.131	61.555	15.728	3.466	25.414	33.889	14.514	50	10.686	6	0.46	0.20	0.66

In-the-money case (P=30)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
σ = 0.10	219.464	238.614	73.664	6.318	89.174	140.302	76.483	50	36.495	12	0.46	0.25	0.71
σ = 0.20	199.632	234.084	57.494	8.306	110.699	97.100	75.473	50	33.640	9	0.34	0.27	0.61
σ = 0.30	191.659	230.086	51.183	9.896	115.241	86.003	70.129	50	29.715	9	0.32	0.26	0.58
σ = 0.40	188.034	218.655	53.130	12.985	94.016	108.592	56.192	50	20.766	15	0.42	0.22	0.64

Notes: The model with no equity or debt constraints is used. Base case parameters are: P =10 (out-of-money) or P = 30 (in-the-money), C = 0, risk-free rate r = 0.06, volatility σ = 0.2, competitive erosion δ = 0.06, investment cost I_1 = 50, I_2 = 50, b = 0.5, tax rate τ = 0.35 and T_1 = 0, T_2 = 5 (time of second option relative to the first), T_F = 20 and debt maturity T_{D1} = 20 and T_{D2} = 15 assuming zero principal. An optimal coupon is chosen based on n_c =20 discretization points for each price level with maximum coupon level points being equal to the price levels (c_{max} = 40). In all tables N_{dec} = 1 (yearly decisions) with $N_{\Delta t}$ = 24 steps per year.

Table 3. Short term debt with no constraints: Sensitivity to volatility

Out-of-the-money case (P = 10)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
σ = 0.10	42.776	79.275	48.971	2.497	-16.666	82.289	60.126	50	32.973	16.5	0.65	0.48	1.13
σ = 0.20	34.600	74.358	39.691	5.624	-10.602	71.392	47.635	50	23.826	16	0.66	0.44	1.10
σ = 0.30	31.324	71.545	34.110	7.928	-7.660	69.885	35.501	50	16.402	18	0.72	0.36	1.08
σ = 0.40	31.170	70.621	31.914	7.883	-4.414	66.008	33.057	50	13.481	18	0.70	0.35	1.05

In-the-money case (P=30)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
σ = 0.10	327.563	239.867	174.825	0.119	-85.046	311.372	188.246	50	37.009	60	0.75	0.45	1.21
σ = 0.20	305.948	239.331	158.112	8.114	-70.534	290.180	169.683	50	33.382	58.5	0.75	0.44	1.18
σ = 0.30	281.229	235.359	137.561	15.097	-50.305	270.946	137.182	50	26.594	60	0.76	0.38	1.14
σ = 0.40	261.456	232.054	120.325	18.968	-29.342	239.926	122.827	50	21.954	57	0.72	0.37	1.09

Notes: The model with no equity or debt constraints is used. Base case parameters are: P =10 (out-of-money) or P = 30 (in-the-money), C = 0, risk-free rate r = 0.06, volatility σ = 0.2, competitive erosion δ = 0.06, investment cost $I_1 = 50$, $I_2 = 50$, b = 0.5, tax rate τ = 0.35 and $T_1 = 0$, $T_2 = 5$ (time of second option relative to the first), $T_F = 20$ and debt maturity $T_{D1} = 5$ and $T_{D2} = 5$ assuming zero principal. An optimal coupon is chosen based on n_c =20 discretization points for each price level with maximum coupon level points being equal to the price levels ($c_{max} = 40$). In all tables $N_{dec} = 1$ (yearly decisions) with $N_{\Delta t} = 24$ steps per year.

Table 4. Short term debt with equity financing constraints: Sensitivity to volatility

Out-of-the-money case (*P* = 10)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	Coupon_1	Lev_1	Lev_2	Lev_T
σ = 0.10	31.037	79.813	37.319	1.766	6.973	41.641	66.751	50	34.328	8	0.36	0.58	0.94
σ = 0.20	26.521	75.918	31.386	4.739	8.151	43.397	51.017	50	26.044	9	0.42	0.50	0.92
σ = 0.30	25.687	73.651	28.337	3.745	13.534	41.104	43.604	50	22.556	9	0.42	0.44	0.86
σ = 0.40	24.783	70.621	25.231	5.394	12.975	41.002	36.481	50	15.675	10	0.45	0.40	0.86

In-the-money case (*P*=30)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	Coupon_1	Lev_1	Lev_2	Lev_T
σ = 0.10	262.944	239.867	111.901	3.165	25.721	117.089	205.792	50	35.659	22.5	0.34	0.59	0.93
σ = 0.20	246.437	239.845	97.930	7.525	42.926	108.555	178.769	50	33.813	21	0.33	0.54	0.87
σ = 0.30	238.636	238.742	90.632	10.115	50.195	119.431	149.634	50	30.624	24	0.37	0.47	0.84
σ = 0.40	227.516	236.392	81.142	12.759	60.181	107.942	136.652	50	27.259	22.5	0.35	0.45	0.80

Notes: The model with equity constraints (debt unconstrained) is used. Base case parameters are: P =10 (out-of-money) or P = 30 (in-the-money), C = 0, risk-free rate r = 0.06, volatility σ = 0.2, competitive erosion δ = 0.06, investment cost I_1 = 50, I_2 = 50, b = 0.5, tax rate τ = 0.35 and T_1 = 0, T_2 = 5 (time of second option relative to the first), T_F = 20 and debt maturity T_{D1} = 5 and T_{D2} = 5 assuming zero principal. An optimal coupon is chosen based on n_c =20 discretization points for each price level with maximum coupon level points being equal to the price levels (c_{max} = 40). In all tables N_{dec} = 1 (yearly decisions) with $N_{\Delta t}$ = 24 steps per year.

Table 5. Short term debt with debt financing constraints: Sensitivity to volatility

Out-of-the-money case (P = 10)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	Coupon_1	Lev_1	Lev_2	Lev_T
σ = 0.10	16.445	78.310	23.245	0.055	35.030	38.588	27.882	50	35.055	7.5	0.38	0.27	0.65
σ = 0.20	16.565	72.407	21.097	0.540	32.149	36.517	24.299	50	26.400	7.5	0.39	0.26	0.65
σ = 0.30	18.571	71.545	20.081	1.009	32.234	34.908	23.476	50	22.046	7.5	0.39	0.26	0.64
σ = 0.40	20.681	70.621	19.077	1.247	32.700	33.175	22.577	50	17.770	7.5	0.38	0.26	0.63

In-the-money case (P=30)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	Coupon_1	Lev_1	Lev_2	Lev_T
σ = 0.10	223.994	239.867	71.169	0.000	107.696	116.807	86.532	50	37.041	22.5	0.38	0.28	0.65
σ = 0.20	223.946	239.779	71.110	0.024	107.669	116.736	86.461	50	36.919	22.5	0.38	0.28	0.65
σ = 0.30	222.648	238.024	69.682	0.628	107.359	114.875	84.844	50	34.430	22.5	0.37	0.28	0.65
σ = 0.40	220.579	236.392	66.866	2.025	108.161	111.162	81.910	50	30.654	22.5	0.37	0.27	0.64

Notes: The model with debt constraints (without equity constraints) is used. Base case parameters are: P =10 (out-of-money) or P = 30 (in-the-money) , C = 0, risk-free rate r = 0.06, volatility σ = 0.2, competitive erosion δ = 0.06, investment cost I_1 = 50, I_2 = 50, b = 0.5, tax rate τ = 0.35 and T_1 = 0, T_2 = 5 (time of second option relative to the first), T_F = 20 and debt maturity T_{D1} = 5 and T_{D2} = 5 assuming zero principal. An optimal coupon is chosen based on n_c =20 discretization points for each price level with maximum coupon level points being equal to the price levels (c_{max} = 15) implying that coupons cannot exceed 75% of revenue (P) level at the time of the financing decision. In all tables N_{dec} = 1 (yearly decisions) with $N_{\Delta t}$ = 24 steps per year.

Table 6. Time-to-build without financing constraints: Sensitivity to model parameters

Panel A: Base case

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
T_2=0	72.624	79.956	33.902	1.234	14.527	49.665	48.432	20	20.000	9.75	0.44	0.43	0.87
T_2=1	68.256	73.282	36.361	2.632	0.491	57.813	48.708	20	18.756	11.5	0.54	0.46	1.00
T_2=2	66.139	65.781	39.224	2.129	-11.322	69.032	45.166	20	16.737	14	0.67	0.44	1.11
T_2=3	63.855	60.223	40.815	1.798	-19.171	69.411	49.001	20	15.385	14	0.70	0.49	1.19
T_2=4	61.404	55.652	41.946	1.781	-25.811	67.493	54.136	20	14.413	13.5	0.70	0.56	1.27
T_2=5	59.215	50.846	43.985	3.122	-37.085	73.999	54.794	20	12.494	15.5	0.81	0.60	1.40
T_2=6	51.063	46.199	39.547	2.917	-33.080	66.920	48.990	20	11.766	14	0.81	0.59	1.40
T_2=7	43.361	41.622	35.513	2.706	-29.743	59.753	44.418	20	11.067	12.5	0.80	0.60	1.40
T_2=8	34.935	37.323	30.421	2.413	-24.000	50.301	39.030	20	10.397	10.5	0.77	0.60	1.37
T_2=9	28.838	33.188	27.453	2.176	-22.149	45.403	35.211	20	9.627	9.5	0.78	0.60	1.38
T_2=10	21.921	29.367	23.467	1.911	-18.037	38.298	30.661	20	9.001	8	0.75	0.60	1.35

Panel B: Lower volatility (σ = 0.1)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
T_2=0	79.406	79.956	40.118	0.668	4.115	57.980	57.312	20	20.000	11.25	0.49	0.48	0.97
T_2=1	77.070	73.440	43.070	0.611	-7.770	79.899	43.770	20	18.830	15.5	0.69	0.38	1.07
T_2=2	75.262	67.154	46.325	0.602	-20.083	84.985	47.976	20	17.616	16.5	0.75	0.43	1.18
T_2=3	72.927	61.477	48.520	0.876	-30.383	89.384	50.119	20	16.194	17.5	0.82	0.46	1.28
T_2=4	70.811	56.136	50.614	0.580	-39.022	87.111	58.080	20	15.358	17	0.82	0.55	1.37
T_2=5	68.645	51.027	52.587	0.504	-47.642	87.066	63.686	20	14.464	17	0.84	0.62	1.46
T_2=6	61.236	46.211	49.528	0.878	-47.526	79.386	63.002	20	13.625	15.5	0.84	0.66	1.50
T_2=7	53.048	41.677	44.997	0.796	-43.482	71.698	57.660	20	12.829	14	0.83	0.67	1.51
T_2=8	44.689	37.406	40.077	0.712	-38.448	64.018	51.200	20	12.082	12.5	0.83	0.67	1.50
T_2=9	35.833	33.384	34.460	0.633	-31.879	53.793	45.296	20	11.378	10.5	0.80	0.67	1.47
T_2=10	28.235	29.596	29.913	0.558	-27.072	46.118	39.905	20	10.715	9	0.78	0.68	1.46

Panel C: Lower opportunity cost (δ = 0.02)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
T_2=0	120.073	112.578	50.614	3.120	12.340	75.426	72.306	20	20.000	15	0.47	0.45	0.92
T_2=1	118.984	105.944	54.906	3.075	-2.173	93.384	66.564	20	18.791	18.5	0.59	0.42	1.01
T_2=2	118.172	98.573	59.218	2.453	-16.310	100.722	70.926	20	17.165	20	0.65	0.46	1.10
T_2=3	118.326	93.142	63.495	1.856	-28.489	100.222	83.049	20	16.455	19.5	0.65	0.54	1.18
T_2=4	116.906	87.102	66.529	1.291	-39.035	103.007	88.368	20	15.434	20	0.68	0.58	1.26
T_2=5	113.603	81.313	67.802	0.939	-46.483	103.140	91.518	20	14.573	20	0.70	0.62	1.31
T_2=6	106.761	75.458	66.179	1.429	-50.303	101.990	88.521	20	13.447	20	0.73	0.63	1.36
T_2=7	98.139	69.691	62.422	1.314	-48.864	99.401	80.262	20	12.661	19.5	0.76	0.61	1.37
T_2=8	89.146	64.039	59.925	3.666	-54.582	98.335	76.544	20	11.151	20	0.82	0.64	1.45
T_2=9	78.908	58.491	54.242	3.328	-48.901	90.935	67.369	20	10.496	18.5	0.83	0.62	1.45
T_2=10	68.655	53.058	48.475	2.999	-42.966	81.130	60.370	20	9.879	16.5	0.82	0.61	1.44

Notes: The model with no equity or debt constraints is used. Base case parameters are: P =10 , C = 0, risk-free rate r = 0.06, volatility σ = 0.2, competitive erosion δ = 0.06, investment cost I_1 = 20, I_2 = 20, b = 0.5, tax rate τ = 0.35 and T_1 = 0,. T_2 ranges from 0 (no-time-to-build) to time-to-build of 10 years. When time-to-build exists the firm foregoes all cash flows until full completion of the project. Useful life after the investment completion is constant at T_F = 20. Debt maturity T_{D1} = 5 and T_{D2} = 5 assuming zero principal. An optimal coupon is chosen based on n_c =20 discretization points for each price level with maximum coupon level points being equal to the price levels (c_{max} = 40). In all tables N_{dec} = 1 (yearly decisions) with $N_{\Delta t}$ = 12 steps per year.

Table 7. Time-to-build with equity financing constraints: Sensitivity to model parameters

Panel A: Base case

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
T_2=0	72.624	79.956	33.902	1.234	14.527	49.665	48.432	20	20.000	9.75	0.44	0.43	0.87
T_2=1	68.256	73.282	36.361	2.632	0.491	57.813	48.708	20	18.756	11.5	0.54	0.46	1.00
T_2=2	62.727	67.189	34.316	1.512	0.434	43.526	56.034	20	17.266	8.5	0.44	0.56	1.00
T_2=3	55.376	61.549	31.359	1.276	0.758	28.433	62.441	20	16.256	5.5	0.31	0.68	0.99
T_2=4	47.418	56.130	28.085	1.618	0.736	18.254	63.607	20	15.179	3.5	0.22	0.77	0.99
T_2=5	40.682	51.015	25.232	1.274	1.608	13.147	60.217	20	14.290	2.5	0.18	0.80	0.98
T_2=6	34.428	46.199	22.831	1.149	1.499	13.100	53.281	20	13.453	2.5	0.19	0.78	0.98
T_2=7	28.116	41.622	20.259	1.063	1.872	10.496	48.450	20	12.701	2	0.17	0.80	0.97
T_2=8	22.641	37.323	18.157	1.007	1.588	10.478	42.408	20	11.832	2	0.19	0.78	0.97
T_2=9	17.416	33.188	16.126	0.656	1.928	7.806	38.923	20	11.241	1.5	0.16	0.80	0.96
T_2=10	12.490	29.367	14.145	0.794	1.508	7.868	33.342	20	10.227	1.5	0.18	0.78	0.96

Panel B: Lower volatility (σ = 0.1)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
T_2=0	79.406	79.956	40.118	0.668	4.115	57.980	57.312	20	20.000	11.25	0.49	0.48	0.97
T_2=1	72.593	73.456	38.586	0.612	0.572	25.957	84.900	20	18.835	5	0.23	0.76	0.99
T_2=2	64.549	67.334	35.513	0.559	0.263	12.979	89.047	20	17.738	2.5	0.13	0.87	1.00
T_2=3	56.690	61.569	32.348	0.522	0.451	7.787	85.157	20	16.705	1.5	0.08	0.91	1.00
T_2=4	49.101	56.140	29.159	0.466	1.055	5.191	78.587	20	15.733	1	0.06	0.93	0.99
T_2=5	42.417	51.027	26.736	0.584	0.206	5.246	71.727	20	14.762	1	0.07	0.93	1.00
T_2=6	35.909	46.211	24.116	0.464	0.496	5.191	64.175	20	13.953	1	0.07	0.92	0.99
T_2=7	29.435	41.677	21.320	0.421	1.240	2.596	58.740	20	13.141	0.5	0.04	0.94	0.98
T_2=8	23.688	37.406	19.033	0.375	1.308	2.596	52.160	20	12.376	0.5	0.05	0.93	0.98
T_2=9	18.340	33.384	16.943	0.332	1.254	2.596	46.145	20	11.655	0.5	0.05	0.92	0.97
T_2=10	13.364	29.596	15.035	0.291	1.091	2.596	40.653	20	10.976	0.5	0.06	0.92	0.98

Panel C: Lower opportunity cost (δ = 0.02)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
T_2=0	120.073	112.578	50.614	3.120	12.340	75.426	72.306	20	20.000	15	0.47	0.45	0.92
T_2=1	118.132	106.069	53.831	2.936	0.226	78.982	77.756	20	18.832	15.5	0.50	0.50	1.00
T_2=2	110.592	99.704	50.270	1.911	2.522	54.229	91.312	20	17.471	10.5	0.37	0.62	0.98
T_2=3	102.546	93.460	46.774	1.313	3.967	41.419	93.533	20	16.374	8	0.30	0.67	0.97
T_2=4	94.176	87.339	43.251	0.998	5.020	31.161	93.411	20	15.416	6	0.24	0.72	0.96
T_2=5	86.334	81.340	40.642	1.147	3.568	26.018	91.248	20	14.501	5	0.22	0.76	0.97
T_2=6	78.257	75.458	37.508	0.890	4.020	18.222	89.835	20	13.819	3.5	0.16	0.80	0.96
T_2=7	70.461	69.691	34.597	0.815	3.809	18.209	81.456	20	13.013	3.5	0.18	0.79	0.96
T_2=8	60.845	64.039	31.113	2.199	1.859	10.589	80.505	20	12.108	2	0.11	0.87	0.98
T_2=9	53.254	58.491	28.073	1.729	2.897	10.411	71.527	20	11.581	2	0.12	0.84	0.97
T_2=10	46.145	53.058	25.533	1.548	2.544	10.405	64.094	20	10.898	2	0.14	0.83	0.97

Notes: The model with equity constraints and no debt constraints is used. Base case parameters are: P =10 , C = 0, risk-free rate r = 0.06, volatility σ = 0.2, competitive erosion δ = 0.06, investment cost I_1 = 20, I_2 = 20, b = 0.5, tax rate τ = 0.35 and T_1 = 0,. T_2 ranges from 0 (no-time-to-build) to time-to-build of 10 years. When time-to-build exists the firm foregoes all cash flows until full completion of the project. Useful life after the investment completion is constant at T_F = 20. Debt maturity T_{D1} = 5 and T_{D2} = 5 assuming zero principal. An optimal coupon is chosen based on n_c =20 discretization points for each price level with maximum coupon level points being equal to the price levels (c_{max} = 40). In all tables N_{dec} = 1 (yearly decisions) with $N_{\Delta t}$ = 12 steps per year.

Table 8. Time-to-build with debt financing constraints: Sensitivity to model parameters

Panel A: Base case

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
T_2=0	66.997	79.956	27.140	0.098	29.357	38.869	38.771	20	20.000	7.5	0.36	0.36	0.73
T_2=1	60.943	73.456	26.387	0.065	24.322	38.895	36.561	20	18.835	7.5	0.39	0.37	0.76
T_2=2	55.246	67.302	25.677	0.017	19.583	38.896	34.483	20	17.717	7.5	0.42	0.37	0.79
T_2=3	49.812	61.500	24.963	0.012	15.116	38.864	32.470	20	16.639	7.5	0.45	0.38	0.83
T_2=4	44.654	56.057	24.238	0.065	10.913	38.805	30.511	20	15.575	7.5	0.48	0.38	0.86
T_2=5	39.809	50.948	23.535	0.148	6.945	38.757	28.633	20	14.526	7.5	0.52	0.39	0.91
T_2=6	35.137	46.144	22.788	0.278	3.266	38.590	26.798	20	13.517	7.5	0.56	0.39	0.95
T_2=7	30.505	41.566	21.837	0.579	-0.148	38.168	24.803	20	12.318	7.5	0.61	0.39	1.00
T_2=8	26.180	37.250	20.955	0.808	-3.283	37.730	22.951	20	11.218	7.5	0.66	0.40	1.06
T_2=9	21.076	33.049	19.182	1.617	-5.808	36.158	20.264	20	9.538	7.5	0.71	0.40	1.11
T_2=10	16.693	29.221	17.767	1.568	-6.911	33.501	18.830	20	8.727	7	0.74	0.41	1.15

Panel B: Lower volatility (σ = 0.1)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
T_2=0	67.210	79.956	27.255	0.000	29.339	38.936	38.936	20	20.000	7.5	0.36	0.36	0.73
T_2=1	61.082	73.456	26.461	0.000	24.313	38.936	36.668	20	18.835	7.5	0.39	0.37	0.76
T_2=2	55.310	67.334	25.714	0.000	19.580	38.936	34.533	20	17.738	7.5	0.42	0.37	0.79
T_2=3	49.874	61.569	25.010	0.000	15.122	38.936	32.522	20	16.705	7.5	0.45	0.38	0.83
T_2=4	44.754	56.140	24.347	0.000	10.924	38.936	30.628	20	15.733	7.5	0.48	0.38	0.86
T_2=5	39.933	51.027	23.723	0.000	6.970	38.936	28.844	20	14.816	7.5	0.52	0.39	0.91
T_2=6	35.393	46.211	23.135	0.000	3.247	38.935	27.164	20	13.953	7.5	0.56	0.39	0.95
T_2=7	31.117	41.677	22.581	0.000	-0.260	38.935	25.582	20	13.141	7.5	0.61	0.40	1.00
T_2=8	27.083	37.406	22.052	0.006	-3.561	38.928	24.085	20	12.370	7.5	0.65	0.41	1.06
T_2=9	23.224	33.383	21.497	0.053	-6.646	38.856	22.617	20	11.603	7.5	0.71	0.41	1.12
T_2=10	18.429	29.595	19.821	0.272	-7.760	35.929	20.975	20	10.715	7	0.73	0.43	1.16

Panel C: Lower opportunity cost (δ = 0.02)

	Firm	Unlevered	TB	BC	Equity	Debt 1	Debt 2	Inv1	Inv2	$Coupon_1$	Lev_1	Lev_2	Lev_T
T_2=0	99.832	112.578	27.254	0.000	61.962	38.935	38.935	20	20.000	7.5	0.28	0.28	0.56
T_2=1	94.227	106.078	26.985	0.000	55.963	38.935	38.164	20	18.835	7.5	0.29	0.29	0.58
T_2=2	88.689	99.707	26.720	0.000	50.083	38.935	37.409	20	17.738	7.5	0.31	0.30	0.60
T_2=3	83.216	93.460	26.460	0.000	44.319	38.934	36.667	20	16.704	7.5	0.32	0.31	0.63
T_2=4	77.813	87.337	26.205	0.001	38.669	38.933	35.940	20	15.729	7.5	0.34	0.32	0.66
T_2=5	72.480	81.337	25.956	0.002	33.130	38.933	35.228	20	14.811	7.5	0.36	0.33	0.69
T_2=6	67.215	75.456	25.707	0.007	27.700	38.931	34.525	20	13.940	7.5	0.38	0.34	0.73
T_2=7	62.024	69.688	25.467	0.007	22.378	38.930	33.840	20	13.124	7.5	0.41	0.36	0.76
T_2=8	56.905	64.022	25.230	0.006	17.155	38.929	33.162	20	12.341	7.5	0.44	0.37	0.81
T_2=9	51.851	58.477	24.988	0.014	12.042	38.918	32.491	20	11.600	7.5	0.47	0.39	0.86
T_2=10	46.876	53.015	24.750	0.018	7.014	38.913	31.820	20	10.871	7.5	0.50	0.41	0.91

Notes: The model with debt constraints (without equity constraints) is used. Base case parameters are: P =10 , C = 0, risk-free rate r = 0.06, volatility σ = 0.2, competitive erosion δ = 0.06, investment cost I_1 = 20, I_2 = 20, b = 0.5, tax rate τ = 0.35 and T_1 = 0,. T_2 ranges from 0 (no-time-to-build) to time-to-build of 10 years. When time-to-build exists the firm foregoes all cash flows until full completion of the project Useful life after the investment completion is constant at T_F = 20. Debt maturity T_{D1} = 5 and T_{D2} = 5 assuming zero principal. An optimal coupon is chosen based on n_c =20 discretization points for each price level with maximum coupon level points being equal to 75% of the price levels (c_{max} = 15) . In all tables N_{dec} = 1 (yearly decisions) with $N_{\Delta t}$ = 12 steps per year.