CDOs and the Financial Crisis: Credit Ratings and Fair Premia*

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Abstract

We study risk and return characteristics of CDOs using the market standard models. We find that fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds. Our results imply that credit ratings are not sufficient for pricing, which is surprising given their central role in structured finance markets. This illustrates limitations of the rating methodologies that are solely based on real-world payoff prospects and do not incorporate risk premia. We also demonstrate that CDO tranches have large exposure to systematic risk and thus their ratings and prices are likely to decline substantially when credit conditions deteriorate.

Keywords: Collateralized debt obligations, Credit ratings, Fair premia, Structured finance, Rating agencies.

JEL Classification: C52, G01, G11.

I. Introduction

The spectacular growth of structured finance markets prior to the 2007-09 financial crisis was only possible because CDO tranches offered a seemingly attractive combination of risk and return. A critical role was played by the rating agencies that certified most CDO tranches with investment grade ratings assuring investors about their safety. Moreover, CDO tranches offered higher yields relative to similarly-rated corporate bonds, which was very appealing to investors who assumed that ratings represent a universal and robust indication of default risks. However, such a rating-based approach failed completely in 2008 when the CDO markets collapsed and even some of the triple-A rated tranches lost 90% of their value and were downgraded to junk.

Most commentators of the financial crisis argue that credit ratings were inflated due to mistakes made by the rating agencies and point out to incentive problems in the 'issuer pays' business model. Moreover, investment banks are accused of having marketed and sold CDO tranches to (unwitting) investors at yields that were too low to compensate for their true riskiness. This might have been possible because investors often lacked the sophistication needed to independently assess CDO tranches and therefore they had to rely on ratings for both risk management and pricing; for a discussion we refer to Brennan et al. (2009), Coval et al. (2009a), Crouhy et al. (2008), and Financial Crisis Report (2010).

There are several interesting questions to be addressed by the academic literature. Were credit ratings of CDO tranches indeed ex-ante incorrect? Were CDO tranches mispriced prior to the financial crisis? What is the meaning of credit ratings? Are ratings (reasonably) sufficient to capture risks of CDO tranches? What determines their risk-return properties? Is creditworthiness of a triple-A CDO tranche similar to that of a triple-A corporate bond? Can corporate bond yields be used as benchmarks for pricing similarly-rated CDO tranches?

One way to address these questions is to study historical data on CDO tranches. However, structured securities have only become popular in the late 1990s, which means the sample period is too short to assess probabilities of rare events such as defaults. We therefore take a different approach and we consider the market standard models for rating and for pricing CDO tranches and we use them to evaluate several stylized CDOs. The market standard rating model is in principle very similar to the market standard pricing model as they both rely on Gaussian copula to capture default dependence. The crucial difference is that the rating model calculates tranche losses implied by historical default probabilities of the collateral bonds, whereas the pricing model uses risk-neutral probabilities.

The main result of this paper is that fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds. In other words, CDO tranches can offer substantial *yield enhancement* relative to bonds. In our stylized example, a CDO tranche rated 'BBB-' has a fair spread almost three times higher compared to a 'BBB-' bond. Even more striking, an equally-rated CDO-squared tranche has a fair spread roughly seven times higher than the bond. This illustrates that credit ratings are by far insufficient for pricing. How can this be explained? Credit ratings reflect real-world (historical) default probabilities or expected losses. In contrast, fair spreads reflect not only such pure default risk, but also risk premia.

Our results demonstrate that the current rating system can be gamed if it is used for pricing purposes. The yield enhancement on tranches creates vast possibilities for rating arbitrage, which explains why the structured finance industry was so profitable. Excess spreads on tranches can be distributed between investors who are able to boost their returns on highly rated portfolios and issuers who are compensated for their efforts and risks associated with originating CDOs. A similar point is made by Brennan et al. (2009) who use an analytical approach based on the CAPM and the Merton model to analyze possible gains from rating arbitrage.

We then investigate risk properties of CDO tranches. We show that expected losses on CDO tranches are highly sensitive to changes in default probabilities of the underlying bonds. This feature turns out to be critical for understanding CDOs because it explains how tranches can combine having relatively low real-world expected losses (i.e. qualify for investment grade ratings) with much higher risk-neutral expected losses (i.e. offer attractive spreads). However, this feature also implies that ratings and prices of CDO tranches have low stability. In particular, CDO tranches are likely to perform poorly whenever the economy enters a recession. In such a case, market participants typically revise upwardly their estimates of collateral bond default probabilities and also demand higher risk premia. Due to the aforementioned sensitivity of tranches, this will have a much stronger effect on ratings and prices of CDO tranches than of corporate bonds. It also means that CDO tranches have large exposure to systematic risk as their defaults are concentrated in economic recessions.

Our results provide a new perspective in the discussion about the meaning of credit ratings and their performance during the crisis. Most studies argue that credit ratings were incorrect as a result of overly optimistic rating assumptions (e.g. too low correlations) and limitations of the rating models (such as failure to account for parameter uncertainty or reliance on the Gaussian copula), see, among others, Coval et al. (2009b), Fender et al. (2008), Griffin and Tang (2011), Hull and White (2010), and Mason and Rosner (2007). While these factors certainly contributed to the recent crisis, we show that even if credit ratings were fully correct, they would not be sufficient to ensure robust performance of CDO tranches. Specifically, even if credit ratings of CDO tranches represented accurate and unbiased estimates of real-world default probabilities or expected losses, then fair spreads on CDO tranches would still be much higher than fair spreads on similarly-rated corporate bonds. Moreover, highly rated CDO tranches would still be prone to large downgrades and losses in economic recessions. We can interpret our results in this way because they are obtained by evaluating stylized CDOs using the market-standard models. If we suppose that the models and assumptions are correct, then so are the obtained credit ratings of CDO tranches. In other words, the market-standard models already predict that CDO tranches have very different properties relative to corporate bonds and credit ratings are simply insufficient to capture risks and returns of CDO tranches. On the positive side, our results imply that inference based on the market standard models can describe risks and returns of CDO tranches to a much greater extent than credit ratings. The rating agencies' assessment of CDO tranches can therefore be improved using the existing methods.

This paper contributes to the growing literature analyzing pricing of CDO tranches. Coval et al. (2009a) find that market yields on CDX tranches before the crisis were too low to compensate for their high level of systematic risk because investors priced these securities based on credit ratings. In contrast, Collin-Dufresne et al. (2012) use the same data to show that CDX index tranches were actually priced correctly if the model incorporates more dynamic features. These studies price CDX tranches relative to the S&P options market, which provides insights into integration between these two correlation markets, but requires strong assumptions and accurate calibration. In practice, however, CDX tranches were traded between professional parties (such as banks) who relied on the simpler market standard models based on Gaussian copulas. Since we show that model-implied fair spreads on CDO tranches are much larger than compensation for pure default risk (i.e. credit ratings), our results are consistent with the findings of Collin-Dufresne et al. (2012). Mispricing was however likely in less transparent markets for non-index based CDO tranches sold directly to institutional investors (such as pension funds) who relied heavily on ratings.

Our paper is related to several strands of literature. In addition to the papers mentioned earlier, Longstaff and Rajan (2008), Bhansali et al. (2008), and Eckner (2007) analyze pricing of CDO index tranches, while Stanton and Wallace (2011) examine pricing of ABX tranches. Franke et al. (2012), Benmelech and Dlugosz (2009a), and Firla-Cuchra (2005) examine properties of CDO transactions and study their implications. The performance of CDO ratings during the crisis is studied by Benmelech and Dlugosz (2009b) and Cordell et al. (2011). The issue of liquidity in structured finance markets is examined by Dang et al. (2012) and Pagano and Volpin (2012), while financial regulation is discussed by Jarrow (2012) and Opp et al. (2012). A growing number of papers examine incentive problems within the rating industry, see Bolton et al. (2012), Bar-Isaac and Shapiro (2012), Griffin and Tang (2011), Griffin and Tang (2012), He et al. (2011) and Skreta and Veldkamp (2009). Our paper adds to the existing literate by presenting a comprehensive examination of the risks and returns of CDO tranches based on the market-standard models. An important finding is that risk-neutral expected losses on CDO tranches are much higher than real-world expected losses, thus demonstrating inherent limitations of credit ratings. Our analysis takes into account incentives and standard practices of different market participants, which is particularly important for understanding properties of CDO tranches and for policy-making implications.

The rest of the paper is organized as follows. Section II discusses the background of the structured finance markets. Section III explains the modeling approach, while Section IV discusses our assumptions and defines the stylized CDOs. In Section V we present our findings on the CDO yield enhancement and in Section VI we analyze the sensitivity of tranche payoffs. In Section VII we examine the stability of ratings and prices of CDO tranches and we discuss regulatory implications, while Section VIII concludes.

II. Background

Structured finance transforms corporate bonds and other assets into securitized tranches characterized by different risk-return properties. A CDO is created by pooling underlying securities into a well-diversified collateral portfolio and then allocating the cash flows from this portfolio between the CDO tranches in a prioritized manner. A given tranche incurs losses only after all subordinate tranches are wiped out. Most of the credit risk is thus concentrated in the first-loss equity tranche, which also provides the highest coupon. More senior tranches have lower default risks and accordingly offer lower coupons.

Structured finance has the ability to produce large volume of highly rated tranches. For example, tranches rated ‘AA’ and ‘AAA’ constituted about 60% of the volume of CDOs rated by Fitch (2007). Moreover, Benmelech and Dlugosz (2009a) document a large disparity between average credit ratings of the collateral assets and the tranches, which they describe as the ‘alchemy’ of CDO ratings. They find that nearly 70% of the volume of CDO tranches backed by loans (CLOs) was rated ‘AAA’ at issuance, while the average credit rating of the collateral was slightly above the ‘B’ grade.

Another intriguing aspect of CDOs is that tranches offered higher coupons relative to similarly-rated bonds. For example, triple-A tranches issued in 2006 often had spreads above 50 bps in the case of CDO-squareds, while triple-A corporate bonds had yields of less than 10 bps.[1] Such yield

[1]The quoted spread of 50 bps for triple-A tranches is based on a sample of a dozen CDOs rated by S&P (Ratings Direct database).

enhancement on tranches was particularly appealing to certain institutional investors (such as pension or money-market funds) who were constrained to invest only in highly rated 'AA' or 'AAA' securities and therefore could not obtain such attractive coupons in other markets. Crouhy et al. (2008) point out that although "the yields on structured instruments exceeded those on equivalently rated corporations, the market knew they were not of the same credit and/or liquidity risk. But investors still misjudged the risk".

The rating agencies advertised credit ratings as "a uniform measure of credit quality globally and across all types of debt instruments" (S&P (2007)).[2] The same document further reads, "In other words, an 'AAA' rated corporate bond should exhibit the same degree of credit quality as an 'AAA' rated securitized debt issue". While the rating agencies also indicated that credit ratings are not sufficient for pricing, they did not explain fundamental differences in risks between similarly-rated bonds and securitized assets.

Credit ratings are an assessment of a security's credit quality. In case of corporate markets, bonds are categorized into a number of grades according to their relative payoff prospects. However, these rating grades are not meant to represent precise estimates of default probabilities. Actual default performance of bonds varies between years. For example, 'BBB' bonds rated by S&P have an average annual default rate of 0.26% with a standard deviation of 0.27% (based on the 1985-2009 period, see S&P (2010)). The rating agencies also publish longer term default probabilities of bonds, which are more stable than annual default rates. For example, triple-A bonds rated by S&P have a historical 10-year default probability of 0.36% (S&P (2005)). Similarly, triple-A bonds rated by Moody's have a historical 10-year expected loss of 0.0055% (Moody's (2007)). Such statistics provide investors with intuition behind the 'relative ranking of payoff prospects' represented by ratings.

The rating methodologies for structured securities are based on the principle that CDO tranche ratings should be comparable to bond ratings. The default likelihood of tranches is assessed by

[2]This quote and a broader discussion about the meaning of credit ratings is given by Ashcraft and Schuermann (2008).

quantitative models. S&P aims to ensure that CDO tranches have the same (real-world) default probabilities as equally-rated bonds, while Moody's aims to match (real-world) expected losses. This means that a tranche rated 'AAA' by S&P should have a 10-year default probability not exceeding 0.36%, while a tranche rated 'AAA' by Moody's should have a 10-year expected loss not exceeding 0.0055%. We refer to S&P (2002) and S&P (2005) for details on the S&P rating methodology and to Moody's (2005) and Moody's (2007) for details on the Moody's methodology.

III. Model

In this section, we provide an overview of the market standard methods for rating and pricing CDO tranches. A more detailed description is given by Mounfield's (2009), Benmelech and Dlugosz (2009a) and S&P (2005). Throughout the paper, we consider an unfunded CDO backed by a reference pool of CDS contracts, but for expositional convenience we refer to it as a collateral portfolio of (synthetic) bonds. This means that all securities under consideration pay only credit spreads, while there is no upfront exchange of notionals and the Libor rate is not paid. Nevertheless, our analysis is equally applicable to funded CDOs.

Let a CDO collateral pool consist of $i = 1, ..., n$ (synthetic) bonds with each bond i having a notional N_i. The CDO's maturity time is T. Default times of the obligors are denoted by $\tau_1, \tau_2, ..., \tau_n$ and the corresponding recovery rates are denoted by R_i. The cumulative losses on the collateral pool up to time t are given by:

$$L(t) = \sum_{i=1}^{n} N_i(1 - R_i)\mathbf{1}_{\tau_i<t}, \tag{1}$$

where $\mathbf{1}$ is the indicator function defined as usual.

Each tranche of a CDO is defined by its attachment point K_L (also known as the tranche subordination level) and its detachment point K_U. CDO tranche losses up to time t are calculated as:

$$L(K_L, K_U, t) = \min\left[\max\left(L(t) - K_L, 0\right), K_U - K_L\right]. \tag{2}$$

It follows from the above formula that modeling CDO tranches requires capturing both the univariate risk properties of the underlying bonds as well as the correlations between their defaults.

The univariate risk properties are summarized by the cumulative distribution functions of obligors' default times τ_i:

$$F_i(t) = Pr(\tau_i < t). \tag{3}$$

The specification of F_i depends on the purpose of modeling and it can reflect probabilities either under the physical measure or under the risk-neutral measure. For simplicity, we assume that F_i has a functional form given by $F_i(t) = 1 - e^{-t\lambda_i}$, where λ_i is the default intensity parameter.[3] The intensity parameter is calibrated based on the assumed default probability until maturity time T.

Default dependence is modeled using the one-factor Gaussian copula model. This model assumes that the scaled value of an obligor's assets, V_i, is normally distributed and driven by a combination of the common (systematic) factor, $Y \sim N(0,1)$, and the idiosyncratic factor, $X_i \sim N(0,1)$, such that:

$$V_i = \sqrt{\rho}Y + \sqrt{1-\rho}X_i, \tag{4}$$

where $\rho \in [0,1]$ is the pair-wise asset value correlation between any two obligors. It is further assumed that there is a one-to-one relation between the default time τ_i and the realization of V_i, which is given by:

$$\tau_i = F_i^{-1}(\Phi(V_i)), \tag{5}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function.

For modeling CDO-squareds, we assume a more complex correlation structure captured by the two-factor model. CDO-squared tranches are rated according to the same principles as CDOs. The market standard is to use the 'the bottom up' approach, which derives the cash-flows on the underlying tranches directly from the cash flows on their collateral bonds. A CDO-squared collateral pool is composed of $j = 1, ..., K$ underlying CDO tranches. In turn, each of these underlying tranches is backed by a portfolio of bonds indexed $i = 1, ..., n$. The scaled asset value of obligor i belonging to the reference portfolio of the (underlying) jth CDO is denoted as $V_{i,j}$ and can be

[3]This approximation is very accurate compared to exact default probabilities given by S&P (2005) for various time horizons.

expressed as:

$$V_{i,j} = \sqrt{\alpha\rho_i}Y + \sqrt{(1-\alpha)\rho_i}Z_j + \sqrt{1-\rho_i}X_{i,j}, \tag{6}$$

where Y and ρ_i are as defined below Eq. 4, $X_{i,j} \sim N(0,1)$ is the idiosyncratic (obligor-specific) component, $Z_j \sim N(0,1)$ is the factor specific to the reference portfolio of the jth CDO, and finally, parameter $\alpha \in [0,1]$ determines the relative exposure to the common factor Y and to the CDO-specific factor Z_j. The credit risk of the underlying tranches is thus partly driven by tranche-specific factors, which provides additional diversification.

A. Rating Measures

Standard and Poor's ratings are based on tranche default probabilities defined by:

$$PD_{tranche} = \mathbb{E}^P\left(\mathbf{1}_{L(T)>K_L}\right), \tag{7}$$

where P denotes the physical default probability measure. Moody's ratings are based on expected tranche losses given by:

$$EL_{tranche} = \frac{\mathbb{E}^P L(K_L, K_U, T)}{K_U - K_L}. \tag{8}$$

B. Fair Premia

To value a CDO tranche, we move to the risk-neutral measure Q and we consider its default leg and premium leg. The value of the default leg is given by:

$$V_{default} = \mathbb{E}^Q \int_0^T B(0,t)\, dL(K_L, K_U, t), \tag{9}$$

where $B(0,t)$ is the discount factor for the time interval $(0,t)$. The value of the premium leg paying an annual spread s on the outstanding notional can be calculated as:

$$V_{premium}(s) = \mathbb{E}^Q\left[\sum_{i=1}^{qT} B\left(0,\frac{i}{q}\right)\frac{s}{q}\left((K_U - K_L) - L\left(K_L, K_U, \frac{i}{q}\right)\right)\right], \tag{10}$$

where q is the frequency of coupon payments, e.g. $q = 4$ for quarterly payments. Finally, setting the two legs equal to each other and solving for s gives the fair tranche spread.

IV. Manufacturing CDO tranches

Manufacturing structured assets consists of two steps. The first step is to select the collateral portfolio. The second step is to choose the optimal capital structure (*tranching*) for a given collateral portfolio. For convenience, we start by discussing the second step and only afterwards discuss the first one.

The originator aims to capture the difference between coupons received on the collateral bonds and coupons paid out on the tranches. The originator has therefore incentives to produce tranches with as high ratings as possible because they can be sold to investors at lower yields. This means that manufacturing CDOs is largely influenced by the rating agencies, which effectively regulate the market. While the methodologies and assumptions of different rating agencies are not the same, their principles are very similar. For convenience, in this paper we mostly rely on the S&P methodology.

When asked by the originator to provide ratings for a CDO, S&P assesses the collateral pool to derive inputs for its rating model described in the previous section. Firstly, each collateral bond is assigned a default probability estimate based on its credit rating according to the S&P's studies of historical default rates. Secondly, the rating agencies classify collateral bonds among a dozen of industry sectors and geographic regions to derive correlation assumptions. All assumptions used by S&P are summarized in several tables disclosed on its website (S&P (2005)), which means that originators can (approximately) replicate the rating process themselves.[4]

For the S&P rating methodology, there is a simple way for the originator to optimally construct CDO tranches. Let us assume that the originator wants to issue five tranches rated 'AAA', 'AA', 'A', 'BBB' and 'BB'.[5] To maximize the volume of tranches with the highest ratings, the

[4]The S&P website even provides a software, which calculates tranche ratings for user-specified CDO structure (tranching scheme) and collateral portfolio characteristics.

[5]While it might seem optimal to construct as many tranches as there are rating categories, in practice most CDOs had no more than 6 tranches, see Firla-Cuchra and Jenkinson (2006) and

originator must structure these tranches to have default probabilities exactly equal to the historical default probabilities of similarly-rated bonds. This ensures there is no excess subordination and all tranches just meet the criteria for their respective ratings. The attachment point of a tranche targeting a default probability of $P_{tranche}$ is simply given as a $(1 - P_{tranche})$ quantile of the portfolio loss distribution. Note that according to the S&P methodology, detachment points of all tranches are implied by attachment points of the more senior tranches.

Let us consider the choice of the collateral portfolio. In practice, collateral portfolios were characterized by substantial heterogeneity because CDOs were created for different reasons and from various types of collateral. Sometimes originating banks were securitizing assets from their balance sheets, while in other cases CDOs were created for specific investors who had a say in selecting the collateral. Nevertheless, there are several broad principles that allow for large rating arbitrage gains. The collateral portfolio should not include too many highly rated bonds because tranching such bonds gives little improvement of rating quality. According to Griffin and Tang (2012) collateral portfolios of CDOs had an average rating around the 'BB+' grade.[6] In terms of diversification, the originator should aim to have a well diversified portfolio because it allows for a larger portion of highly rated tranches. Finally, the originator has incentives to choose bonds with relatively high yields for their credit ratings because this does not influence tranche ratings while it generates larger cash inflows to the CDO.

A. CDO collateral portfolio

To create the stylized CDO, we assume a hypothetical collateral portfolio of one hundred bonds with a 'BBB-' rating by S&P and 'Ba1' by Moody's. According to the S&P assumptions, 'BBB-' bonds have a default probability of roughly 10% over a 10-year horizon. According to Moody's,

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Benmelech and Dlugosz (2009a).

[6]Although there were also high-grade CDOs (typically ABS CDOs) backed by collateral assets rated at or above the 'A' grade. These CDOs typically contained only 'AA' and 'AAA' rated tranches with very low subordination levels.

bonds rated 'Ba1' have a 10-year expected loss of roughly 5%.[7] We also assume that the collateral bonds have a recovery rate of 50%, which makes the assumed default probability consistent with the expected loss.[8]

In making the correlation assumptions, we follow the guidelines of the rating agencies. According to the S&P, the asset value correlation between U.S. corporate obligors belonging to different industry sectors is 5%, while the asset value correlation within an industry sector is 15% (S&P (2005)). We set the asset value correlation to 12.5%, which is a cautious average correlation if the bonds belong to several industries. In practice, the average correlation is typically lower than 12.5% because collateral portfolios include not only U.S. bonds, but also other assets. For asset value correlation lower than 12.5%, fair spreads on CDO tranches are even higher as shown in the sensitivity analysis in Section V.

We also assume that each collateral bond has a market-implied default probability of 20% until maturity of 10 years, which is double the physical default probability. This is equivalent to a market spread of 111.95 bps on the collateral bonds.[9] Moreover, this assumption is cautious since it corresponds to the lower bound of estimates given in the literature suggesting that risk-neutral default probabilities of 'BBB-' rated bonds are 2 to 5 times higher than their physical default probabilities, see Berndt et al. (2005), Delianedis and Geske (2003), Driessen (2005) or Hull et al. (2005). For higher risk-neutral default probabilities, the yield enhancement on tranches is even stronger as shown in Section V.

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[7]We use a slight approximation since a 'BBB-' bond rated by S&P has a 10-year historical default probability of 10.64% (S&P (2005)). Similarly, a 'Ba1' bond rated by Moody's has a 10-year expected loss of 5.64% (Moody's (2007)).

[8]The assumption of 50% recovery rate is supported by Elkamhi et al. (2012) who analyze historical data on corporate bonds and find that the average recovery rate is 54%. This is larger than the standard assumption of 40% recovery often used for pricing credit derivatives.

[9]Given the assumed recovery rate of 50%.

B. CDO structuring

As for structuring of the CDO, we first describe the junior mezzanine tranche which is tailored to have the same (real-world) default probability and (real-world) expected loss as the collateral bonds, respectively, 10% and 5% over a 10-year horizon. From the perspective of the rating agencies, this tranche and the collateral bonds have therefore identical credit quality and of course the same credit ratings (i.e. 'BBB-' by S&P and 'Ba1' by Moody's). This is very convenient for our further analysis of the rating-premia relationship although it departs slightly from the typical structuring process. To construct the junior mezzanine tranche, we first choose its attachment point such that its 10-year default probability meets the assumed target of 10%. Next, we fix the detachment point such that the 10-year expected tranche loss is 5%. For our portfolio, this implies that the attachment and detachment points of this tranche are 9.90% and 14.75%, respectively.

We then construct the senior and super-senior tranches aiming at 'AA' and 'AAA' ratings by S&P, which means their attachment points are chosen to meet the benchmark default probabilities of 0.87% and 0.36%, respectively, required by S&P for these rating grades. For our portfolio, these two tranches have attachment points of 17.08% and 19.45%. While the CDO structuring is based on the S&P methodology, we also report ratings calculated according to the Moody's methodology. Based on the expected loss criteria, Moody's would assign an 'A2' rating to the senior tranche and an 'Aa1' rating to the super-senior tranche.

Finally, we also obtain two tranches, which have both attachment and detachment points implied by the tranches defined so far. The first one is the unrated equity tranche at the bottom of the capital structure. Its attachment is 0% and its detachment is given by the attachment point of the junior mezzanine tranche, i.e. 9.90%. Another tranche is in between the junior mezzanine tranche and the senior tranche, so it goes from 14.75% to 17.08% of the CDO notional. It would be rated 'A-' by S&P and 'Baa1' by Moody's.

C. Creating a CDO-squared

We also analyze a stylized CDO-squared backed by a collateral pool constructed as follows. We first create thirty CDOs that are all identical to the stylized CDO described in the previous subsection; however, we assume that these CDOs reference portfolios of different bonds implying no overlap among their collateral portfolios. Consequently, each of these CDOs contains a junior mezzanine tranche with an attachment of 9.90% and detachment of 14.75%, which would be rated 'BBB-' by S&P and 'Ba1' by Moody's. The CDO-squared collateral portfolio is then obtained by pooling these thirty mezzanine CDO tranches.

We further assume that the pair-wise asset value correlation between obligors within the same CDO collateral pool is 12.5%, while the asset value correlation between obligors belonging to collaterals of different underlying CDOs is 3.5%. This is equivalent to assuming that $\rho = 12.5\%$ and $\alpha\rho = 3.5\%$ in Eq. 6 of Section III. We thus obtain additional diversification at the level of the underlying tranches, which is critical for producing large volume of highly rated CDO-squared tranches. A similar approach is used by Hull and White (2010) in the analysis of ABS CDOs. Such diversification can be achieved by selecting tranches backed by collateral bonds belonging to different industry sectors and geographic regions. In addition, collateral pools of CDO-squareds in most cases include tranches of asset backed securities (RMBS or ABS).

To construct CDO-squared tranches, we apply the same structuring process as in the CDO case. Therefore, the corresponding CDO and CDO-squared tranches have very similar (real-world) default probabilities and (real-world) expected losses as well as the same credit ratings. In particular, the junior mezzanine tranche of the CDO-squared is tailored to have a 10-year default probability of 10% and an expected loss of 5%, which means this tranche has identical rating quality as the collateral bonds.

V. Credit ratings and fair premia

In this part, we investigate the relation between credit ratings and fair premia. The results of structuring and rating of the CDO tranches as described in the previous section are reported in columns

(1)-(2) of Table 1. In addition, columns (3)-(5) report tranche default probabilities, expected losses and spreads calculated under the physical measure corresponding to collateral default probability of 10% over a 10-year horizon. We can see how CDO prioritization of cash flows determines the risks of tranches. The default probabilities and expected losses of the tranches are decreasing with tranche seniority. Most of the credit risk is borne by the equity tranche, which absorbs losses up to the limit of 9.90% of the portfolio. The lowest default risks are associated with the super-senior 'AAA' tranche, which has almost 80% of the CDO notional.

[TABLE 1 ABOUT HERE]

To price CDO tranches, we move to the risk-neutral measure corresponding to the market-implied default probability of the collateral bonds of 20% over a 10-year horizon. This is equivalent to a bond market spread of 111.95 bps. Columns (6) and (7) report risk-neutral default probabilities and expected losses of the tranches, while column (8) gives the fair market spread. It is clear that the transition from the physical to the risk-neutral measure corresponds to a huge increase in the default probabilities, expected losses and spreads for all tranches.

From a pricing perspective, the most important observations follow from comparing the junior mezzanine tranche with the similarly-rated collateral bonds. In column (8) we see that while the underlying portfolio of 'BBB-' bonds has a fair spread of 111.95 bps, the 'BBB-' CDO tranche has a fair spread of 320.69 bps. Hence the fair spread is almost three times as high. This result clearly demonstrates that fair spreads on CDO tranches are much higher than fair spreads on corporate bonds even when there are absolutely no differences in real-world default probabilities and real-world expected losses.

Since the collateral bonds and the junior-mezzanine tranche have the same credit ratings, they must also have very similar compensation for pure default risk as reported in column (5). This means that the yield enhancement on CDO tranches can be entirely attributed to higher risk premia. The risk premia are given as the difference between the fair (market) spread reported in column (8) and the pure default risk compensation in column (5). In our case, the 'BBB-' tranche has a risk premium of 272.44 bps, which is almost six times higher than the risk premium of 58.89

bps on the ‘BBB-’ bonds.

We also analyze CDO-squared securities, which incurred particularly large losses during the financial crisis. The results for the CDO-squared are reported in Table 2. In columns (3)-(5), we see that the real-world properties of the CDO-squared tranches are very similar to those of the corresponding CDO tranches. In contrast, in columns (6)-(8) we observe that the risk-neutral default probabilities, expected losses and fair spreads for the CDO-squared are much higher compared to the CDO case. For example, the junior-mezzanine CDO-squared tranche rated ‘BBB-’ has a fair spread of 749.52 bps, which is almost seven times higher than the fair spread on the ‘BBB-’ collateral bonds.

[TABLE 2 ABOUT HERE]

The results for the junior-mezzanine tranches are striking. They demonstrate that model-implied fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds, which means that credit ratings are by far insufficient for pricing. Such yield enhancement is possible because the rating methodologies capture solely pure default risk. In other words, CDO tranches are tailored to have very low real-world default probabilities and expected losses and thus qualify for good credit ratings, while they have much higher risk-neutral expected losses corresponding to very high fair spreads.

The foregoing analysis can be generalized to the case of the more senior tranches; however, it requires consideration of similarly-rated corporate bonds as benchmarks. For this purpose, we create corporate bonds that have identical (real-world) default probabilities and (real-world) expected losses as the corresponding tranches. We call these bonds *risk-equivalent* to the respective tranches. To determine fair spreads on these risk-equivalent bonds, we assume that their risk-neutral default probabilities are double the historical probabilities (robustness to this assumption is explained further down). The obtained results are summarized in Table 3. In column (6) we report fair spreads on the risk-equivalent bonds, while columns (1)-(5) summarize tranche ratings and fair premia previously shown in Tables 1 and 2.

[TABLE 3 ABOUT HERE]

The main message from Table 3 is that fair spreads on the tranches are much higher than fair spreads on their risk-equivalent bonds. The magnitude of the yield enhancement critically depends on tranche seniority and on whether a particular tranche is a CDO or CDO-squared. For example, the spread on the super-senior CDO tranche is 2.52 bps, while the spread on the corresponding risk-equivalent 'AAA' bond is 0.22 bps. For the CDO-squared, the spread on the super-senior tranche is as high as 71.83 bps, while the corresponding risk-equivalent 'AAA' bond yields 0.83 bps.

We check robustness of the above results with respect to the ratio of risk-neutral to real-world default probabilities of the risk-equivalent bonds. The market evidence suggests that for highly rated bonds this ratio can be much higher than the assumed value of 2. For example, Hull et al. (2005) report a ratio of 16.8 for 'AAA' bonds. Under such assumption, the 'AAA' bonds that are risk-equivalent to the super-senior CDO and CDO-squared tranches would have fair spreads of 1.88 bps and 7.01 bps, respectively. Since these spreads are lower than fair spreads of 2.52 bps and 71.83 bps on the corresponding super-senior tranches, we still obtain yield enhancement on these tranches.[10]

The large differences in fair spreads between similarly-rated tranches and bonds create opportunities for rating arbitrage, which means that excess spreads on tranches can be distributed between tranche investors and CDO issuers. For this reason, CDO tranches can be very appealing to both investors and issuers. Since investors are risk averse and CDO tranches are tailored to their risk appetites, the total risk compensation paid on the tranches of a CDO can be lower than the total spread received on the collateral portfolio. The remaining share of the yield is then allocated to CDO issuers compensating them for the risks associated with their part of structured finance activities. These risks arise because the originators are often unable to sell the total notional of

[10]We conduct similar robustness checks for the other tranches using the ratio of risk-neutral to real-world default probabilities reported by Hull et al. (2005) for their corresponding rating grades. We find that CDO tranches offer yield enhancement relative to their risk-equivalent bonds in all cases.

all CDO tranches and have to retain and hedge remaining risks. There are also reputational risks, which can lead financial institutions to even bail-out their CDOs as in the case of Bear Stearns.

Let us further discuss the meaning, implications and robustness of our results. We have shown that credit ratings are not sufficient for pricing CDO tranches. To a certain extent the same is true in the corporate bond markets, but what is really surprising in the case of CDOs is the very large magnitude of yield enhancement on tranches. Our results are obtained by evaluating stylized CDO tranches using the market standard rating and pricing methods, which rely on the Gaussian copula to capture default dependence. These methods were and still are used universally not only by the rating agencies, but also by traders for pricing purposes. Such approach therefore allows us to look at CDOs from the perspective of market participants.

In the aftermath of the recent crisis, a number of studies point out that credit ratings were based on overly optimistic assumptions and that rating models did not accurately capture the true default behavior of collateral portfolios, which likely led to underestimation of default risks and biased ratings (references are given in Section I). However, we show that even if the rating agencies could derive accurate and unbiased estimates of real-world default probabilities and expected losses, then credit ratings would still be insufficient for pricing. To understand this point, note that our results are obtained without using market data, so possible inaccuracies in how the models capture the actual default behavior of assets are not an issue. We simply investigate the relationship between credit ratings and fair spreads as implied by the models - i.e. the relationship that would hold in a stylized world where the true default behavior of assets perfectly 'agrees' with the model and assumptions of the rating agencies. The reason why even accurate and unbiased credit ratings are not informative for pricing CDO tranches is that they solely reflect real-world default probabilities or real-world expected losses, which are much lower than risk-neutral expected losses.

The results presented in this paper depend on a number of assumptions about the stylized CDO tranches. This does not mean that our approach is unrealistic because the rating agencies have to make similar assumptions when rating CDOs and we closely follow their guidelines. Our analysis can also be generalized to a broader universe of CDOs backed not only by corporate bonds, but also by mortgages, loans, and other debt securities. While our stylized CDO does not correspond to any specific CDO issued in the market, we select collateral portfolio and tranching scheme in line with the market practice as discussed in Section IV. Moreover, we made very cautious assumptions and therefore our results likely underestimate the possible magnitude of yield enhancement. Given a collateral portfolio of 'BBB-' bonds, the yield enhancement on tranches is mostly dependent on the size of the risk premia on the collateral bonds and also correlations between their defaults. Let us therefore consider alternative assumptions.[11]

In Table A.I in the appendix we replicate the results of Table 3 assuming a market spread of 180.72 bps on the collateral 'BBB-' bonds, i.e. higher bond risk premia as suggested by Hull et al. (2005).[12] It is seen, for example, that fair spreads on the junior mezzanine CDO and CDO-squared tranches rated 'BBB-' are as high as 784.92 bps and 1717.63 bps, respectively, compared to 320.69 bps and 749.52 bps in the baseline case. This shows how the yield enhancement on tranches increases with higher risk premia on the collateral bonds. A CDO issuer aiming to maximize tranche spreads should therefore select collateral bonds with relatively high CDS spreads for their credit ratings. While higher CDS spreads indicate larger risks, they have no impact on tranche ratings that are derived solely based on real-world default probabilities. There is anecdotal evidence that such adverse selection was one of the reasons for the poor performance of CDOs during the 2007-09 crisis (Fitch (2008)).

[11] In unreported results, we also check robustness with respect to the assumed rating quality and heterogeneity of the collateral pool, recovery rate, and maturity time. The results are qualitatively similar to the baseline case and we obtain substantial yield enhancement on tranches under every reasonable specification.

[12] This is equivalent to the risk-neutral default probability of the collateral 'BBB-' bonds of 30%, which is triple the real-world default probability. According to Hull et al. (2005) the ratio of risk-neutral to physical default probabilities is equal to 5.1 for 'BBB' bonds and 2.1 for 'BB' bonds.

To illustrate the impact of the correlation assumptions, in Table A.II in the appendix we report tranche spreads calculated using an asset value correlation of 5%.[13] In this analysis, we recalculate tranche subordination levels according to this lower correlation parameter, which results in lower tranche subordinations. In such a case, the junior-mezzanine CDO tranche rated 'BBB-' has a fair spread of 522.85 bps and the corresponding CDO-squared tranche has a fair spread of 1271.46 bps compared to, respectively, 320.69 bps and 749.52 bps under the baseline assumption of 12.5% correlation. This shows that tranches backed by more diversified collateral portfolios have higher fair spreads than tranches backed by less diversified portfolios, ceteris paribus.

The foregoing sensitivity analysis demonstrates not only the robustness of our results, but also shows that even similarly-rated and seemingly identical CDO tranches can have very different fair spreads depending on collateral portfolio characteristics. It implies that CDO originators can game the rating agencies by constructing tranches that have very high fair spreads relative to their credit ratings, but also larger risks as discussed in the following sections. When investors rely on credit ratings for pricing, originators have clear incentives to game the ratings because they can retain excess spreads.

VI. Sensitivity analysis

In this section we examine the sensitivity of tranche payoffs to default probabilities of the underlying bonds with the aim of providing a clear-cut explanation of the yield enhancement on tranches and illustrating their associated risk properties. We depict sensitivity of default probabilities and expected losses of CDO tranches because they determine credit ratings and, moreover, expected

[13]We choose a correlation of 5% because the baseline assumption of 12.5% is quite large compared to the S&P benchmark correlations. S&P uses the following assumptions about pair-wise correlations between corporate bonds: 0% for different geographic regions and different industries, 5% for the same region and different industries, 15% for the same region and the same industry.

losses under the risk-neutral measure are closely related to fair spreads.[14]

Figure 1 presents the sensitivity results for the CDO tranches. To benchmark tranche sensitivities, we plot curves corresponding to the underlying portfolio of bonds. To facilitate inference, we add vertical lines representing the real-world measure, i.e. at the collateral default probability of 10%, and the risk-neutral measure, i.e. at the collateral default probability of 20%. Note that these lines cross the CDO tranche curves at the values corresponding to the results of Table 1.

[FIGURE 1 ABOUT HERE]

Panel A of Figure 1 explores the sensitivity of tranche default probabilities to changes in default probabilities of the collateral bonds. It is seen that the sensitivity of tranche default probabilities is generally higher than the corresponding sensitivity of the collateral bonds. In Panel B we present the sensitivity of expected tranche losses and we observe qualitatively similar results.

Figure 1 is the key to understanding the mechanics of yield enhancement on tranches. To illustrate the argument, let us consider the junior mezzanine tranche and the collateral bond. Clearly, these two securities have equal default probabilities and expected losses at the real-world level of the collateral default probability ('0.1' line). Consequently, they have identical credit ratings. In contrast, the expected tranche loss at the risk-neutral level of the collateral default probability ('0.2' line) is considerably higher than the expected bond loss. This is only because the curve of the expected tranche loss is steeper than the curve of the expected bond loss in the 10-20% interval of the collateral default probability. Consequently, the fair spread on the junior-mezzanine tranche is much higher than the fair spread on the corporate bonds.

Similar reasoning applies to the case of the more senior tranches. On the one hand, these tranches must be tailored to have sufficiently low default probabilities and expected losses under the physical measure ('0.1' line) to qualify for their credit ratings. On the other hand, yield enhancement on the tranches is achieved when their risk-neutral expected losses ('0.2' line) are higher than risk-neutral expected losses of similarly-rated corporate bonds. These two proper-

[14]It follows from Eq. 8 and Eq. 9 that the value of the tranche default leg is equal to the expected tranche loss multiplied by the tranche notional and adjusted for discounting.

ties can only be combined due to the high sensitivity of expected tranche payoffs to collateral default probabilities illustrated in Figure 1.[15] For CDO-squared tranches, the yield enhancement mechanics is analogous to that for CDO tranches. In Figure 2 we can see that the sensitivity of the CDO-squared tranches is much higher compared to the CDO case, which explains the large magnitude of yield enhancement on the CDO-squared tranches.

[FIGURE 2 ABOUT HERE]

The high sensitivity of CDO tranches seen in Figures 1 and 2 is a distinguishing feature of CDOs, which also determines their risk properties. It implies that even modest changes in collateral default probabilities can lead to very large changes in expected payoffs on CDO tranches. It means that CDO tranches and in particular CDO-squared tranches are highly leveraged securities. In Figure 2 we can see that CDO-squareds are structured at the ‘critical’ points such that expected tranche losses are still low, but they rise dramatically if the collateral default probability increases beyond the real-world level. This feature implies that CDO-squared tranches have little upside potential relative to their real-world expected payoffs, while adverse market changes are likely to result in huge losses. Similar asymmetry of payoffs is observed for CDO tranches although it is of lesser magnitude. In contrast, collateral bonds are characterized by symmetry in payoff prospects.

The results of this section imply that credit ratings convey very limited information about properties of CDO tranches. Credit ratings reflect default probabilities or expected losses under the real-world measure. However, Figures 1 and 2 demonstrate that risks and returns of CDO tranches are also critically dependent on the shapes of the sensitivity curves, which determine not only the level of fair spreads on tranches, but also potential losses when credit conditions deteriorate. Overall, our results suggest that extending the rating methodologies to incorporate more information

[15]Only the expected loss of the super-senior tranche appears to be fairly insensitive to a modest increase in the collateral default probability. However, the relative (percentage) increase in the expected loss of the super-senior tranche is very large in the 10-20% interval of the collateral default probability. Table 1 shows that the expected tranche loss increases 27 times (from 0.01% to 0.27%) when the collateral default probability doubles (from 10% to 20%).

provided by Figures 1 and 2 is necessary to more fully capture risk and return properties of CDO tranches.

VII. Rating and price stability of CDO tranches

Figures 1 and 2 of the previous section show that even a highly rated tranche, which is structured to have a minute expected loss under the physical measure, can incur heavy losses if the realized default rate of the collateral pool exceeds the assumed threshold. In a dynamic setting, tranche prices are likely to become depressed even prior to the realization of actual losses. When credit conditions deteriorate, then CDS spreads widen as a consequence of a rise in actual default probabilities as well as in corresponding risk premia. In such a case, investors should reprice CDO tranches using the revised market-implied default probabilities of the collateral bonds. The changes in prices of CDO tranches are typically much higher than the changes in prices of corporate bonds due to high sensitivity of tranche payoffs to collateral default probabilities. This might go a long way in explaining the dramatic decline in prices of structured securities during the 2007-09 crisis.

In case of unfavorable market conditions, tranche ratings can be expected to come under severe stress as well. Firstly, credit ratings are highly sensitive to credit enhancement levels, which are reduced once defaults hit the underlying portfolios. Secondly, downgrades within collateral portfolios correspond to an increase of the rating agencies' estimates of real-world default probabilities of the collateral bonds. This can have a disproportionately larger impact on tranche ratings again due to higher sensitivity of tranche payoffs. Thirdly, a large decline in prices of CDO tranches might create pressure on the rating agencies to further downgrade CDO tranches due to reputational concerns.

To illustrate the divergence between stability of CDO tranches and corporate bonds, we consider the following scenario corresponding to a fairly severe deterioration in credit conditions. The CDO and CDO-squared are structured and rated under the baseline assumptions given in Section IV. We further assume that soon after the issuance, the collateral bonds are downgraded by one notch from the 'BBB-' to the 'BB+' grade, which is equivalent to an increase in their 10-year default probabilities from 10% to 13%. Finally, we suppose that the rating agencies promptly re-rate the tranches using the revised default probabilities of the collateral bonds. The obtained results presented in Table 4 demonstrate a large deterioration in the credit quality of the tranches, particularly for the senior and super-senior tranches.[16] (For ease of exposition, in this section we only consider S&P ratings.) The super-senior CDO tranche is downgraded from the initial rating of 'AAA' to 'AA-' and the corresponding CDO-squared tranche is downgraded as far as to the 'BBB+' grade. In other words, a one notch downgrade of the collateral pool triggers downgrades of the super-senior tranches by as many as 3 and 7 notches.[17]

[TABLE 4 ABOUT HERE]

An argument can be made that the scenario analyzed in Table 4 is not very realistic as the rating agencies would have to downgrade the entire collateral pool and subsequently re-run their rating models. However, a similar deterioration in credit quality of the tranches can occur if a large portion of the collateral bonds is downgraded by more than one notch. Moreover, we show that a large increase in default probabilities of the collateral bonds can be explained by default contagion, i.e. collateral bond defaults revealing information about the realization of the common economic factor Y in Eq. 4 or Eq. 6.[18]

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[16]The values reported in Table 4 correspond to those at which tranche curves in Figures 1 and 2 would cross vertical lines at the collateral default probability of 10% and 13%.

[17]During the crisis, as predicted by the models, the severity of downgrades of CDO tranches was much larger compared to corporate bonds. Benmelech and Dlugosz (2009b) find that the average downgrades of CDO tranches in 2007 and 2008 were 4.7 and 5.6 notches, respectively, while for corporate bonds they were 1.4 and 2.2 notches. Most severe downgrades by 8 notches or more were concentrated among tranches rated originally 'AAA' (so called 'fallen angels').

[18]We consider information-driven default contagion, which arises because obligors' defaults reveal information about the common factor Y driving the riskiness of all obligors. This definition is different from a more standard one according to which default contagion is associated with "a

To study the impact of default contagion, we consider a scenario when a single default within the collateral portfolio occurs soon after CDO origination. The creditworthiness of the tranches is then reduced not only due to a partial loss of credit enhancements, but also due to the increased likelihood of a market-wide deterioration in credit conditions. For illustration, we calculate conditional default probabilities of the surviving obligors given that the first default in the collateral portfolio occurs at time t within one year after CDO issuance. In this analysis, we use the same model and assumptions about the collateral portfolio as in the previous sections.

The results plotted in Figure 3 show that a sudden and early default event can lead to a large increase in default probabilities of the surviving names. If the first default occurs after 1, 3 or 6 months, then the conditional default probabilities of the surviving bonds jump to, respectively, 16.19%, 13.31%, 11.14%.[19] This means that a deterioration of credit conditions considered in Table 4 corresponds to a single default within the collateral pool after roughly 3 months, which is not an unlikely scenario under stressful market conditions.

[FIGURE 3 ABOUT HERE]

Our analysis of default contagion shows that abrupt and severe downgrading of CDO tranches can be explained according to the market standard models. Let us generalize this finding to the actual market setting. The key question is under what circumstances the rating agencies will revise default probabilities of the collateral bonds and re-rate CDO tranches. We argue that the critical role is played by the business cycle. On the one hand, credit ratings of both corporate bonds and CDO tranches are supposed to be through-the-cycle and therefore should not fluctuate with transitory changes in default risks. On the other hand, the realized performance of companies with low investment grade ratings (such as 'BBB') is well-established to be cyclical with much higher de-

direct causal relationship between two obligor's defaults" (Schonbucher (2004)).

[19]In this analysis, it is critical that the first default occurs relatively early such that it signals a low realization of the common factor Y. The first default occurring after roughly 1 year is neutral information about the realization of Y.

fault rates in economic recessions (S&P (2012)).[20] For example, our stylized CDO tranches would be rated by S&P under the assumption that collateral bonds have a 10-year default probability of 10%, which corresponds to the long-term historical default frequency of similarly-rated 'BBB-' bonds. However, a conditional (point-in-time) default probability of 'BBB-' bonds is (much) higher than the assumed 10% in economic recessions and (much) lower than 10% in economically good times. This means that when the economy enters a recession, the increase in collateral default probabilities analyzed in Table 4 (i.e. from 10% to 13%) is realistic and it could potentially lead to severe (mark-to-market) losses and downgrades of CDO tranches.

When the economy enters a recession, the rating agencies have to balance between rating stability and responding to new information about deteriorating creditworthiness of obligors. The rating agencies aim to provide through-the-cycle ratings and therefore might be hesitant to downgrade tranches that no longer qualify for 'AA' or 'AAA' ratings, but still have rather limited default risks (as the senior or super-senior tranches in Table 4). However, there is no doubt that investors will reprice CDO tranches as soon as CDS spreads of their collateral bonds widen. This repricing will depend on the magnitude of a deterioration in credit conditions and also the sensitivity of tranches. If tranche prices fall substantially, then the rating agencies will feel the pressure to downgrade CDO tranches as otherwise the disparity between high credit ratings and low market value will at some point undermine the reliability of the rating agencies. Market prices of CDO tranches can influence the rating agencies because their downgrading decisions are fairly arbitrary. While the assessment of CDOs is based on quantitative models, the rating agencies can make special adjustments and take into account overall credit conditions including the level of credit spreads (Fitch (2008)).

The aforementioned interplay between prices and ratings is consistent with market develop-

[20]An S&P definition of the 'BBB' rating states: "An obligor rated 'BBB' has adequate capacity to meet its financial commitments. However, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity of the obligor to meet its financial commitments" (S&P (2009)).

ments during the 2007-09 financial crisis. The massive wave of downgrading of 'AAA' tranches backed by subprime debt started early in 2008 when the ABX.HE.AAA index[21] (vintage 07-1 and 07-2) traded below 70% of par for a few months already. Subsequently, the value of the aforementioned ABX.HE.AAA indices reached a low of less than 30% of par in mid-2009 and almost 85% of RMBS tranches rated originally 'AAA' were downgraded by Moody's (Financial Crisis Report (2010)). However, despite these large mark-to-market losses and massive downgrades, only around 10% of RMBS tranches rated originally 'AAA' eventually suffered principal losses (Financial Crisis Report (2011), p. 229). Stanton and Wallace (2011) argue that the prices of ABX tranches plummeted below levels that could be explained by any reasonable expectations about future defaults suggesting that their prices were influenced by other factors (e.g. capital constraints).

Our analysis demonstrates that CDO tranches have large exposure to systematic risk. It is clear that non-equity tranches incur losses only when collateral default rates are very high, which in case of large and well-diversified collateral portfolios is almost surely associated with severe economic recessions. Consequently, even if the unconditional real-world default probabilities of a CDO tranche and a similarly-rated corporate bond are the same, the losses on CDO tranches are more heavily concentrated in adverse economic states (see Coval et al. (2009a) for a study of systematic risk of CDO tranches and its pricing relative to the S&P options). The exposure to systematic risk is also the reason why CDO markets are prone to experiencing very strong boom and bust cycles. During favorable economic conditions, prices and ratings of investment grade CDO tranches tend to perform robustly as they have little exposure to idiosyncratic risks associated with defaults of individual obligors. Therefore, prolonged periods of good economic conditions are likely to result in excessive optimism of market participants (including the rating agencies) about the riskiness of

[21]The ABX.HE.AAA index is a CDS contract on a pool of 20 representative subprime mortgage backed tranches rated 'AAA' (see Stanton and Wallace (2011) for details). There are analogous indices for other rating categories.

CDOs.[22] Conversely, during periods of economic recessions, CDO tranches can be expected to significantly underperform relative to similarly-rated collateral assets. A rise in default probabilities of collateral bonds leads to increased uncertainty about eventual payoffs of CDO tranches, which can quickly result in large mark-to-market losses and downgrades.[23] The results of Table 4 show that even the highest-rated 'AAA' tranches can have their creditworthiness severely undermined as they also 'inherit' cyclicality of performance from their lower rated collateral. In contrast, corporate bonds rated 'AAA' tend to perform robustly throughout the business cycle. Overall, our results show a clear trade-off between risks and returns of CDO tranches. While CDO tranches can offer attractive yields, they have inherently large leverage and exposure to systematic risk, so their ratings and prices become unstable when credit conditions deteriorate.

The results presented in this paper have several regulatory implications. In the period of historically low corporate yields preceding the financial crisis, some institutional investors such as pension funds tried to boost their portfolio returns by investing in highly rated CDO tranches. Due to rating-based restrictions, they could not obtain similar returns by investing in lower rated corporate bonds. However, the regulatory goals are likely better addressed if such investors hold well-diversified portfolios of lower rated bonds with much less systematic risk compared to highly rated CDO tranches. A possible solution is to relax restrictions based on credit ratings and to regulate risk taking differently as suggested by the Dodd-Frank Act. Another approach would be to ensure that CDO tranches are structured to perform robustly throughout the business cycle. This

[22]Bolton et al. (2012) and Bar-Isaac and Shapiro (2012) propose theoretical models of the rating industry according to which credit ratings tend to be more inflated during economic booms when fees earned from rating CDOs are high and when more investors trust the ratings. He et al. (2011) find empirical confirmation that ratings of mortgage-backed securities were more favorable during the recent boom of 2004-2006 compared to the earlier periods.

[23]Large mark-to-market losses on CDOs caused distress of many financial institutions, most notably AIG (see Financial Crisis Report (2011), p. 243).

could be achieved by choosing appropriately higher tranche subordination levels such that CDO tranches would not incur large downgrades and mark-to-market losses even during economic recessions (i.e. stress testing). However, such approach would significantly reduce both the volume of highly rated tranches and the level of fair spreads on tranches.

Structured finance markets can only function properly when investors have a reliable method for pricing CDO tranches. Our results clearly demonstrate that credit ratings are not sufficient for pricing. However, given the complexity of securitized transactions and often even the lack of available information about collateral characteristics, many investors did not have capabilities to independently price these securities and therefore they had to rely on ratings.[24] Moreover, when the systematic risk associated with CDO tranches realizes, creditworthiness of CDO tranches becomes uncertain and these securities are no longer information insensitive. If information about collateral composition and performance can be obtained and processed only by sophisticated market participants, then adverse selection arises and liquidity is likely to dry up.[25] A possible remedy is to introduce greater transparency and disclosure of information in structured finance markets. It is also worth considering whether the rating agencies should provide more information about pricing of CDO tranches and more detailed analysis of their risk profiles, which could benefit unsophisticated investors. The rating agencies disclose their risk assessment of CDO tranches in 'Presale' and 'New Issue' reports, which briefly discuss the assigned ratings and summarize information

[24]Coval et al. (2009a) argue that investors did not appreciate the systematic nature of risks inherent in CDO tranches and therefore creating CDOs was "the optimal mechanism for exploiting investors who rely on ratings for pricing".

[25]Dang et al. (2012) derive a theoretical model of debt, liquidity and information in which the lack of incentives for private information production facilitates trading, but an aggregate shock can trigger information production and result in adverse selection. Pagano and Volpin (2012) show that more information disclosure benefits liquidity in the secondary market, but reduces liquidity in the primary market.

about structure and collateral portfolio of a newly-issued CDO. Such analysis could be extended to discuss, among other issues, the pricing of CDO tranches, the sensitivity analysis, and the projected performance of tranches under different economic scenarios.

The distress of many banks with large exposure to CDOs during the recent crisis highlights the need for more robust regulation of bank risk taking and capital requirements. In fact, more than 50% of losses associated with MBSs and CDOs (i.e. $250 billion) "were borne by investment banks, commercial banks, and thrifts" (Financial Crisis Report (2011), p. 228). A possible explanation as to why banks invested in CDOs is related to the findings of Iannotta and Pennacchi (2012) who show that when risk weights under the Basel Accords and FDIC insurance premia are based on credit ratings (i.e. real-world expected losses), banks can boost their market value by investing in corporate bonds that have large systematic risk and thus offer relatively high coupons for their credit ratings.[26] Our results show that for CDOs this moral hazard is substantially greater because CDO tranches have much higher systematic risk and fair spreads compared to similarly-rated bonds. It suggests that more effective regulation of bank risk taking in the context of CDOs could be achieved by linking capital ratios and bank insurance premia to risk-neutral expected losses as also proposed by Iannotta and Pennacchi (2012) for corporate bonds.

VIII. Conclusions

We show that fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds implying that credit ratings are not sufficient for pricing. Furthermore, fair spreads on similarly-rated CDO tranches can be very different depending on collateral portfolio characteristics, which means that the criteria of the rating agencies can be gamed. The recipe for maximizing yield enhancement on tranches is to select collateral portfolios that are highly diversified and consist of bonds with relatively high CDS spreads for their credit ratings. Our results illustrate

[26] Erel et al. (2011) examine other possible explanations as to why banks invested in highly-rated CDO tranches. Opp et al. (2012) show that ratings-based regulation provides incentives for the rating agencies to inflate ratings and allow for regulatory arbitrage benefiting regulated investors.

limitations of credit ratings that are based on real-world payoff prospects as opposed to risk-neutral payoff prospects corresponding to prices. On the one hand, CDO tranches are tailored to have sufficiently low real-world default probabilities and expected losses to meet the criteria for the highest credit ratings. On the other hand, CDO tranches are tailored to have sufficiently large risk-neutral expected losses to provide higher coupons relative to similarly-rated corporate bonds. These two features can be combined only because expected tranche payoffs are highly sensitive to changes in default probabilities of the collateral bonds. However, the downside of such risk profile is that CDO tranches are inherently prone to incurring large losses and massive downgrades when credit conditions deteriorate (i.e. systematic risk). In the periods of stress, uncertainty about tranche payoffs increases and investors with low tolerance for risk as well as investors with limited financial sophistication might turn away from CDOs. Therefore, assessing risks and returns of CDO tranches requires looking not only at the likelihood of eventual principal losses, but also the extent of downgrades and mark-to-market losses during economic recessions.

Our findings show limitations of credit ratings understood as a universal measure of credit quality. Even if credit ratings represent accurate and unbiased estimates of real-world default risks, they are by far insufficient for assessing risks and returns of CDO tranches. Despite being poor benchmarks for pricing CDOs, credit ratings were widely used for this purpose by investors with limited financial sophistication. The performance of highly rated CDO tranches can be improved by introducing more stringent rating criteria aimed at reducing their vulnerability during economic recessions. Structured finance markets could also benefit from greater transparency. While the issuance of CDOs virtually halted during the financial crisis and the current issuance rate is only a fraction of its pre-2007 levels, the market is showing some signs of recovery. The future of structured finance depends on whether the challenges in assessing risks and returns of CDO tranches are properly addressed. We demonstrate that analysis based on the market standard models can provide crucial information about risks and returns of CDO tranches, which is not conveyed by credit ratings implying that the current rating methodologies could be extended.

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Tables

Table 1: CDO tranche risk statistics, credit ratings and spreads.

This table reports the results of structuring, rating and pricing of CDO tranches. The first two columns summarize the capital structure of the CDO. The remaining columns report (10-year horizon) tranche default probabilities and expected losses as well as annualized spreads. These results are obtained separately under the physical and under the risk-neutral measure. The physical measure corresponds to the assumption of 10% default probability of the underlying bonds (over a 10-year horizon), whereas the risk-neutral measure corresponds to default probability of 20%. The results obtained under the physical measure are related to the rating process, so in columns (3) and (4) we also report credit ratings by S&P and Moody's. Column (5) reports the spreads compensating for pure default risk, while column (8) gives the fair (market) spreads. The last row of the table shows the statistics for the underlying corporate bonds.

		Physical measure (PD=10%)			Risk-neutral measure (PD=20%)		
Tranche	Tranche subordination	Default probability & S&P rating	Expected loss & Moody's rating	Spread (bps)	Default probability	Expected loss	Fair spread (bps)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
tranche 1 equity	0.00%	98.33% 'NR'	47.50% 'NR'	636.54	99.90%	78.53%	1475.40
tranche 2 junior mezz.	9.90%	10.00% 'BBB-'	5.00% 'Ba1'	48.25	44.64%	30.24%	320.69
tranche 3 senior mezz.	14.75%	1.97% 'A-'	1.35% 'Baa1'	12.76	18.43%	14.55%	143.83
tranche 4 senior	17.08%	0.87% 'AA'	0.58% 'A2'	5.43	11.09%	8.46%	81.81
tranche 5 super-senior	19.45%	0.36% 'AAA'	0.01% 'Aa1'	0.10	6.21%	0.27%	2.52
collateral bond	n.a	10.00% 'BBB-'	5.00% 'Ba1'	53.06	20.00%	10.00%	111.95

Table 2: CDO-squared tranche risk statistics, credit ratings and spreads.

This table reports the results of structuring, rating and pricing of CDO-squared tranches. The first two columns summarize the capital structure of the CDO-squared. The remaining columns report (10-year horizon) tranche default probabilities and expected losses as well as annualized spreads. These results are obtained separately under the physical and under the risk-neutral measure. The physical measure corresponds to the assumption of 10% default probability of the underlying bonds (over a 10-year horizon), whereas the risk-neutral measure corresponds to default probability of 20%. The results obtained under the physical measure are related to the rating process, so in columns (3) and (4) we also report credit ratings by S&P and Moody's. Column (5) reports the spreads compensating for pure default risk, while column (8) gives the fair (market) spreads. The last row of the table shows the statistics for the underlying corporate bonds.

		Physical measure (PD=10%)			Risk-neutral measure (PD=20%)		
Tranche	Tranche subordination	Default probability & S&P rating	Expected loss & Moody's rating	Spread (bps)	Default probability	Expected loss	Fair spread (bps)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
tranche 1 equity	0.00%	77.88% 'CCC-'	32.46% 'Caa2'	338.84	99.62%	91.87%	1498.01
tranche 2 junior mezz.	13.27%	10.00% 'BBB-'	5.00% 'Ba1'	46.89	80.60%	68.16%	795.71
tranche 3 senior mezz.	24.92%	2.07% 'A-'	1.38% 'Baa1'	12.86	55.67%	49.27%	520.66
tranche 4 senior	31.25%	0.87% 'AA'	0.58% 'A2'	5.36	43.09%	37.50%	379.86
tranche 5 super-senior	37.50%	0.36% 'AAA'	0.04% 'Aa1'	0.40	32.21%	7.61%	71.83
collateral bond	n.a	10.00% 'BBB-'	5.00% 'Ba1'	53.06	20.00%	10.00%	111.95

Table 3: Comparison of fair spreads on tranches and risk-equivalent bonds.

In columns (1)-(5) we summarize tranche ratings and fair spreads reported previously in Tables 1 and 2. Note that the corresponding tranches and their risk-equivalent bonds have the same credit ratings by S&P and Moody's. In column (6) we report fair spreads on the *risk-equivalent* corporate bonds. The *risk-equivalent* bonds are defined as having the same (real-world) default probabilities and (real-world) expected losses as the respective tranches. For calculating fair spreads on these bonds, we assume that their risk-neutral default probabilities are double the physical probabilities. Since the expected losses on the super-senior CDO and CDO-squared tranches are quite different (see Table 1 and Table 2), the risk-equivalent bonds for these tranches are constructed separately and they have fair spreads of 0.22 bps for the CDO and 0.83 bps for the CDO-squared.

Tranche	S&P rating	Moody's rating	Fair spread (bps)		
			CDO	CDO-squared	Corporate bond
(1)	(2)	(3)	(4)	(5)	(6)
tranche 2 junior mezz.	'BBB-'	'Ba1'	320.69	795.71	111.95
tranche 3 senior mezz.	'A-'	'Baa1'	143.83	520.66	28.13
tranche 4 senior	'AA'	'A2'	81.81	379.86	11.72
tranche 5 super-senior	'AAA'	'Aa1'	2.52	71.83	0.22 / 0.83

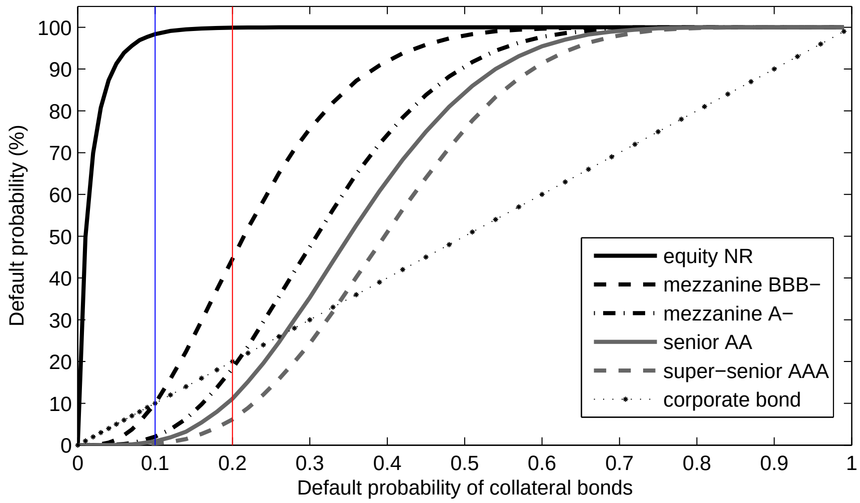
Table 4: Impact of a deterioration in collateral credit quality on tranche ratings.

This table analyzes the impact of a deterioration in credit conditions corresponding to an increase in 10-year default probabilities of the collateral bonds from 10% to 13%. This is equivalent to a one-notch downgrade from BBB- to BB+. The first three columns summarize the capital structure of the CDO and CDO-squared. Columns (4), (5) and (6) report default probabilities and S&P ratings under the standard market conditions (i.e. collateral default probability of 10%), while columns (7), (8) and (9) report the same statistics recalculated under the deteriorated market conditions (i.e. collateral default probability of 13%).

Tranche	Tranche subordination		Default probability under standard conditions (PD=10%)			Default probability under deteriorated conditions (PD=13%)		
	CDO	CDO-sq.	Bond	CDO	CDO-squared	Bond	CDO	CDO-squared
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
tranche 1 equity	0%	0%		98.33% 'NR'	77.88% 'CCC-'		99.34% 'NR'	92.23% 'NR'
tranche 2 junior mezz.	9.90%	13.27%	10.00% 'BBB-'	10.00% 'BBB-'	10.00% 'BBB-'	13.00% 'BB+' 1 notch	19.14% 'BB-' 3 notches	29.83% 'B+' 4 notches
tranche 3 senior mezz.	14.75%	24.92%		1.97% 'A-'	2.07% 'A-'		4.96% 'BBB' 2 notches	10.15% 'BBB-' 3 notches
tranche 4 senior	17.08%	31.25%		0.87% 'AA'	0.87% 'AA'		2.40% 'A-' 4 notches	5.46% 'BBB' 6 notches
tranche 5 super-senior	19.45%	37.50%		0.36% 'AAA'	0.36% 'AAA'		1.06% 'AA-' 3 notches	2.87% 'BBB+' 7 notches

Figures

Panel A: Sensitivity of CDO tranche default probabilities.



Panel B: Sensitivity of CDO expected tranche losses.

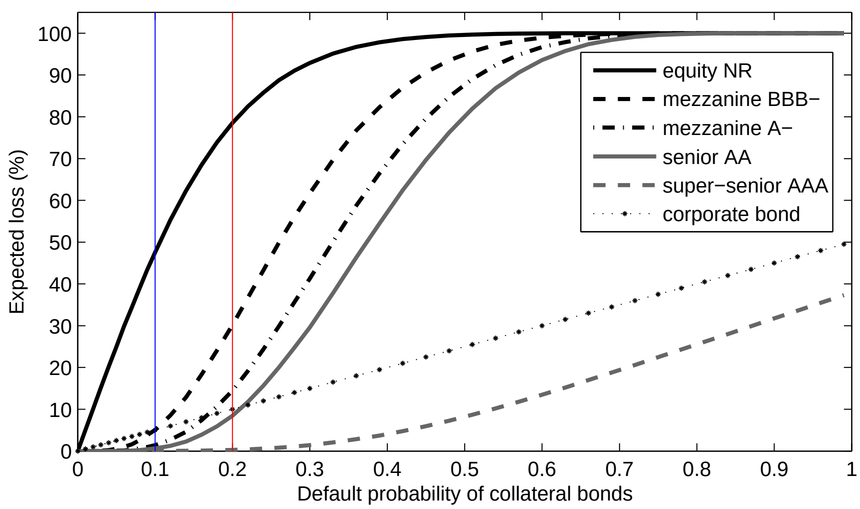
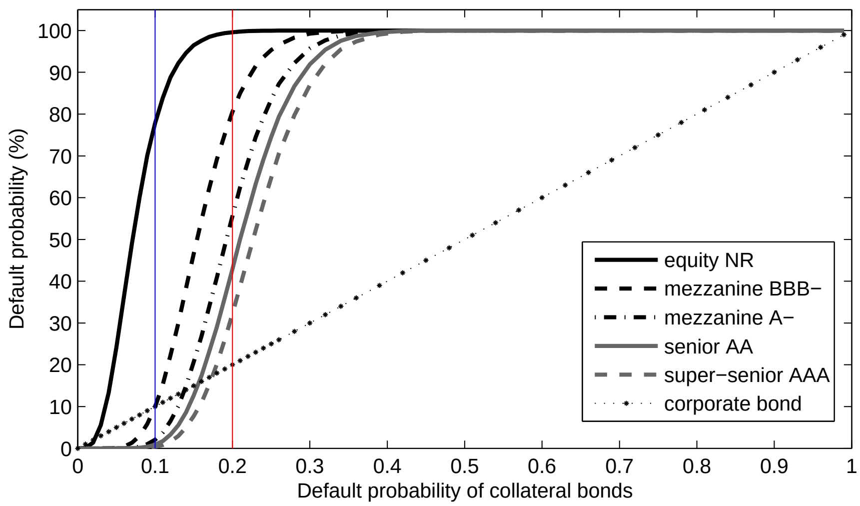


Figure 1: This figure illustrates sensitivity of payoff prospects of CDO tranches to default probabilities of the collateral bonds. Panel A presents sensitivity of tranche default probabilities, whereas Panel B presents sensitivity of expected tranche losses. The vertical lines at value ‘0.1’ correspond to the real-world level of collateral default probabilities, whereas the ‘0.2’ lines correspond to the risk-neutral level. For ease of comparison, we add curves representing the collateral ‘BBB-’ bonds.

Panel A: Sensitivity of CDO-squared tranche default probabilities.



Panel B: Sensitivity of CDO-squared expected tranche losses.

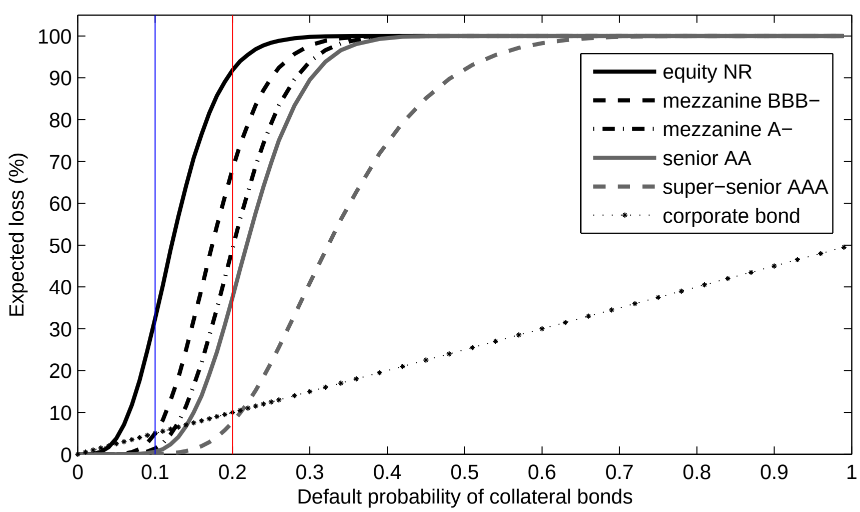


Figure 2: This figure illustrates sensitivity of payoff prospects of CDO-squared tranches to default probabilities of the collateral bonds. Panel A presents sensitivity of tranche default probabilities, whereas Panel B presents sensitivity of expected tranche losses. The vertical lines at value '0.1' correspond to the real-world level of collateral default probabilities, whereas the '0.2' lines correspond to the risk-neutral level. For ease of comparison, we add curves representing the 'BBB-' collateral bonds.

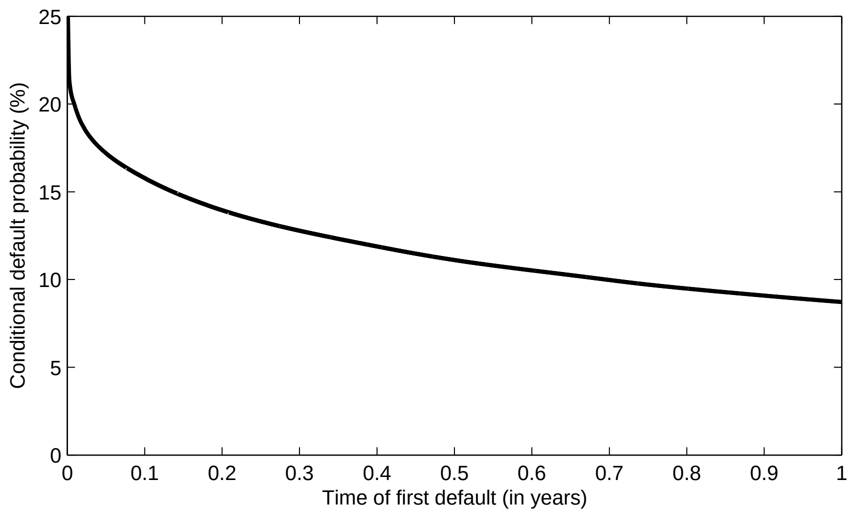


Figure 3: This figure plots the default probability of the surviving bonds conditional on the first default in the collateral pool occurring at time *t*.

Appendix

Table A.I: Comparison of fair spreads on tranches and risk-equivalent bonds under the assumption of 30% risk-neutral default probability of the underlying bonds.

In columns (1)-(3) we summarize tranche subordination levels and credit ratings. The structuring and rating is done under the baseline assumptions. For calculating fair spreads we assume that risk-neutral default probabilities of bonds are equal to 30%, which is triple the physical default probabilities as motivated by Hull et al. (2005). In columns (4) and (5) we report fair tranche spreads, while column (6) reports fair spreads on the corresponding *risk-equivalent* bonds. The *risk-equivalent* bonds are defined as having the same (real-world) default probabilities and (real-world) expected losses as the respective tranches. Since the expected losses on the super-senior CDO and CDO-squared tranches are different, the risk-equivalent bonds for these tranches are constructed separately and they have fair spreads of 0.33 bps for the CDO and 1.21 bps for the CDO-squared.

Tranche	Tranche subordination CDO/CDO-squared	S&P / Moody's ratings	Fair spread (bps)		
			CDO	CDO-squared	Corporate bond
(1)	(2)	(3)	(4)	(5)	(6)
tranche 2 junior mezz.	9.90% / 13.27%	'BBB-'/'Ba1'	784.92	1717.63	180.72
tranche 3 senior mezz.	14.75% / 24.92%	'A-'/'Baa1'	453.97	1407.68	42.62
tranche 4 senior	17.08% / 31.25%	'AA'/'A2'	306.75	1228.89	17.59
tranche 5 super-senior	19.45% / 37.50%	'AAA'/'AA1'	13.46	428.03	0.33 / 1.21

Table A.II: Comparison of fair spreads on tranches and risk-equivalent bonds under the assumption of 5% asset value correlation.

In columns (1)-(3) we summarize tranche subordination levels and credit ratings. The structuring and rating is done under the assumption that asset value correlation is 5% instead of 12.5% used in the baseline case. In columns (4) and (5) we report fair tranche spreads, while column (6) reports fair spreads on the corresponding *risk-equivalent* bonds. The *risk-equivalent* bonds are defined as having the same (real-world) default probabilities and (real-world) expected losses as the corresponding tranches. For calculating fair spreads we assume that risk-neutral default probabilities of bonds are double the physical probabilities. Since the expected losses on the super-senior CDO and CDO-squared tranches are different, the risk-equivalent bonds for these tranches are constructed separately and they have fair spreads of 0.12 bps for the CDO and 0.62 bps for the CDO-squared.

Tranche	Tranche subordination CDO/CDO-squared	S&P / Moody's ratings	Fair spread (bps)		
			CDO	CDO-squared	Corporate bond
(1)	(2)	(3)	(4)	(5)	(6)
tranche 2 junior mezz.	8.45% / 12.52%	'BBB-'/'Ba1'	522.85	1271.46	111.95
tranche 3 senior mezz.	11.40% / 22.07%	'A-'/'Baa1'	282.38	1057.71	27.75
tranche 4 senior	12.64% / 27.35%	'AA'/'A2'	186.13	906.41	11.28
tranche 5 super-senior	13.96% / 33.41%	'AAA'/'Aa1'	4.15	237.00	0.12 / 0.62