Wavelet based factor analysis of implied volatilities

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This version 14 January 2013

Abstract

In this paper we explore the long memory properties of the implied volatility series of four European countries: France, Germany, UK, Switzerland and the US, by using wavelets analysis. The aim of this paper is twofold: first to investigate in the time-frequency domain the effect of common and idiosyncratic shocks on the dynamics of implied volatility indexes, second to assess if the relative importance of each shock is dependent on the degree of turbulence of the market. To this end we divide the sample period into two sub-periods (before and after the Lehman Brothers' collapse) which are characterized by a low and high degree of turbulence respectively.
The wavelet based maximum likelihood estimator shows that all volatilities are non-stationary long memory processes. A Full Information Maximum Likelihood analysis applied jointly to all five markets gives evidence of an increasing role played by the common shock (relative to idiosyncratic shocks) in shaping the variability of each market, especially after the Lehman Brothers' collapse and for the higher scales (e.g. those associated with lower frequency range). We interpret these findings in terms of an increasing role of systemic risk which characterize the period after the Lehman Brothers' collapse and the behavior of investors with long term horizons.

JEL: C32, C38, C58, G13

Keywords: Implied volatility, Long Memory, Wavelets, FIML**.**

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1. Introduction

Evidence of long memory in volatility measures is well documented. The studies of Baillie et al. (1996), Andersen and Bollerslev (1997), Comte and Renault (1998) give evidence of long-run dependencies, described by a fractionally integrated process, in GARCH, realized volatilies, and stochastic volatilities models, respectively. More recently, empirical studies show that the volatility implied from option prices exhibits properties well described by fractionally integrated process. In particular, the studies of Bandi and Perron (2006) and Christensen, and Nielsen (2006), concentrate on relationship between implied and realized volatilies through fractional cointegration (using monthly and weekly data, respectively). Moreover, Bollersev et al. (2011) employ a Fractional Integrated Vector Error Correction Model to study the relationship between risk (proxied by the implied volatility) and return using intra-daily data. All the aforementioned studies focus on the US stock market (hence on the VIX index of implied volatility). The only study analyzing spillovers effects across different implied volatility indices for the US and for Europe (with emphasis on the role played by news) is the one of Jiang, et al. (2012). However, the authors (op. cit.) focus is on the relationship between first differences of implied volatilities, and not on the levels.

The aim of this paper is twofold: first to investigate in the time-frequency domain the effect of common and idiosyncratic shocks on the dynamics of implied volatility indexes, second to assess if the relative importance of each shock is dependent on the degree of turbulence of the market. To this end we divide the sample period into two sub-periods (before and after the Lehman Brothers' collapse) which are characterized by a low and high degree of turbulence respectively.

In this study we use wavelet based estimators to re-examine the long memory properties in the implied volatility series (see Elder and Jin, 2007 and Gencay et al., 2010 on applications of wavelet based decomposition of realized volatility) of five countries: France, Germany, UK, US and Switzerland.

Wavelets analysis allow to study the time-series in the time-frequency domain. By providing a localized frequency decomposition, wavelets are more appropriate than Fourier analysis when the object under study is locally stationary and inhomogeneous (Percival and Walden (2000)). Wavelets are particularly useful when fractional order of integration lies in a non-stationary region ($1/2 < d < 1$), as reported for volatility series by Kellard et al. (2010) and Bandi and Perron (2006).

We first, use the Maximum Likelihood wavelet based estimator of the fractional integration parameter (see Jensen, 2000) and confirm the findings of Bandi and Perron (2006) regarding the existence of non-stationary long memory. In a second stage of the analysis, we use Full Information Maximum Likelihood to explore the contribution of common and idiosyncratic shocks to the variability of the level of each volatility index at different scales (each associated with a given frequency range). For this purpose, we exploit the decomposition, at different scales, of the covariance matrix of fractionally integrated time series developed by Witcher et al. (2000). The empirical evidence suggests that the common shock play a more important role than the idiosyncratic shock, especially for the higher scales (e.g. those corresponding to the lowest frequency range) and after the Lehman Brothers' collapse.

We contribute to the literature by investigating the different behavior of four European markets and the US market, where states with a different currency than the Euro (UK and Switzerlad) are expected to be less impacted by the common shocks than Germany and France. We use a factor model where we do not need to specify through an autoregressive model the dynamic of the factors, neither of the volatility. Differently from Jiang et al. 2012 we work directly on the volatility levels rather than on the first differences, which is more interesting from the financial and economic viewpoint, since the determinants of the volatility level are linked both to domestic factors such as the level of the economy and to international factors such as the international exchanges.

The structure of the paper is as follows. Section 2 describes the empirical methodology; section 3 focusses on the empirical evidence and section 4 concludes.

2. Wavelet based univariate and multivariate analysis of long memory series

2.1 Definition of long memory and traditional estimators

Let the implied volatility series, imp_t , be described by an *ARFIMA(p,d,q)* process:

$$\Phi(L)(1-L)^d imp_t = \Theta(L)\varepsilon_t \quad (1)$$

where ε_t is an *iid* Gaussian process with variance σ_ε^2. The AR component is given by a polynomial of degree p (with roots outside the unit circle):

$$\Phi(L) = 1 + \varphi_1 L + \varphi_2 L^2 + ... + \varphi_p L^p \quad (2)$$

and the MA component is described by a polynomial of degree q (with roots outside the unit circle):

$$\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + ... + \theta_q L^q \quad (3)$$

The fractional differencing operator $(1 – L)^d$ can be derived from a power series expansion as follows:

$$(1-L)^d = 1 + \sum_{z=0}^{\infty} \frac{\Gamma(z-d)}{\Gamma(-d)\Gamma(z+1)} L^z \quad (4)$$

It turns out that, for $–0.5 < d < 0.5$, the process imp_t is stationary and invertible. For such processes, the effect of a shock ε at time t on *imp* at time $t+h$ decays as h increases, but the rate of decay is much lower than for a process integrated of order zero, hence the autocorrelation function for a fractionally integrated process decays hyperbolically. If $0.5 < d < 1$, then the process is non-stationary long-memory and it is characterized by an infinite variance.

The most prevalent method for estimating the fractional differencing parameter is the method proposed by Geweke and Porter-Hudak (1983, hereafter GPH) which is based on the low frequency spectral behavior of the time series, exploiting the property that the spectral density of a long memory processes is infinite at frequency zero. In practice, the GPH estimator is simply the slope of the sample log periodogram:

$$\ln P(\lambda_s) = c - d \ln(4(\sin^2(\lambda_s / 2))) + \varepsilon(\lambda_s) \quad (5)$$

where $P(.)$ is the periodogram of the data computed at the harmonic frequencies $\lambda_s = \frac{2\pi s}{T}$, with

$s = 1,\ldots,\omega \prec T/2$, and T is the sample size. In particular, in line with the study of Bandi and Perron (2006), the maximum number of frequencies ω involved in log periodogram regression is set either to $T^{0.5}$, or to $T^{0.6}$, or to $T^{0.7}$. The asymptotic standard error of the parameter d, equal to $\sqrt{\left(\frac{\pi^2}{24}\right)\frac{1}{\sqrt{\omega}}}$, was obtained by Robinson (1995a) in the presence of stationary data and by Velasco (1999) in the presence of non-stationary data with $\frac{1}{2} \leq d \prec \frac{3}{4}$.

The local Whittle estimator developed by Kunsch (1987) and by Robinson (1995b) maximizes a frequency-domain Gaussian likelihood for frequencies in the neighborhood of zero, i.e.:

$$\log\left[\frac{1}{\omega}\sum_{s=1}^{\omega}\lambda_s^{2d}P(\lambda_s)\right]-\frac{2d}{\omega}\sum_{s=1}^{\omega}\lambda_s \quad (6)$$

The asymptotic standard error of the parameter d has been derived by Robinson (1995b) and it is equal to $\sqrt{\frac{1}{4}\frac{1}{\sqrt{\omega}}}$. (7)

2.2 Wavelet estimator of long memory parameter

Frequency domain approaches provide an insightful representation of econometric data by decomposing it into sinusoidal components at various frequencies, which have intensities that vary across the frequency spectrum. The shortcoming of Fourier analysis is related to the assumption of intensities constant through time. This feature makes Fourier methods ineffective in analysing signals containing local irregularities, such as spikes or discontinuities, which, we argue, are a feature of financial time series. Wavelets can be a particular useful tool when the signal is localized in time as well as frequency. Consequently, wavelet transforms can localize a process in time and scale, revealing long-run, or high-scale, features of the process in a more flexible manner than the Fourier analysis underlying the GPH estimator. Through wavelet analysis (see Appendix for more details) we provide a decomposition of a time series into time series components, each reproducing the evolution over time of the original series for a given scale λ_j (associated to a given frequency range).

The lower scales are associated to the highest frequency range and the highest scales (up to maximum level of decomposition J) correspond to the lowest frequency range. Two Discrete Wavelet Transform, DWT, based estimators of the fractional differencing parameter were introduced by Jensen (1999 and 2000), and earlier by Wornell and Oppenheim (1992) and McCoy and Walden (1996). The use of DWT implies that such a filter can be applied to time series with length equal to 2^J. Since we have a sample of 3264 daily observations for the implied volatility series, we can at most retrieve a decomposition up to level J equal to 11. Jensen (1999 and 2000) uses Monte-Carlo analysis to show that the wavelet estimators are superior to other long-memory estimators, including the more popular GPH estimator, on the basis of MSE. More specifically, Jensen (1999) shows that the wavelet coefficients (for a given scale j) associated with a fractionally integrated white noise process, with $|d| < 0.5$, are distributed approximately as $N(0,\ \sigma_\varepsilon^2 2^{-2d(J-j)})$. Consequently, the DWT based wavelet estimator is obtained by maximizing the log likelihood function:

$$\sum_{j=1}^{J} \sum_{t=t_1}^{t_2} L\left(w_{jt}, \sigma_\varepsilon^2 2^{-2d(J-j)}\right) \tag{8}$$

where $t_1 = 1$ and $t_2 = 2^{J-1}$ if the focus is on the first scale (e.g. for j = 1), implying that the first 1024 observations for the wavelet coefficients are employed. Moreover, $t_1 = 2^{J-1}+1$ and $t_2 = 2^{J-2}$ for j equal to 2, that is the second 526 observations for the wavelet coefficients are employed, till we reach higher level of decomposition for which the last wavelet coefficients observation is employed. The coefficients standard errors are then obtained by inverting the Hessian.

2.3 Wavelet based multivariate analysis

Once we have investigated the long memory properties of the implied volatility series, we turn our focus on multivariate analysis. In particular, we are interested in assessing the contribution of common vs. idiosyncratic shocks in shaping the variability of the different market implied volatilities for different time horizons. For this purpose, we apply a factor decomposition of the covariance matrix of the implied volatility series at different scales. The analysis is split into two stages.

In the first stage, we apply the Maximal Overlapping Discrete Wavelet Transform, MODWT (see Percival and Walden, 2000; Whitcher, 2000) to obtain a decomposition of each time series into different scales (each associated to a given frequency range) localized in time (see Appendix for more details). Unlike the DWT, the MODWT, by producing a decomposition of a given time series into components having the same size as the original time series, is capable to explore potential structural breaks. Beyond full sample analysis, in this paper, we are interested in exploring the contribution of common and idiosyncratic shocks before and after the Lehman Brothers' collapse occurred in September 15th, 2008.

In the second stage of the analysis, we employ a factor decomposition of the covariance matrix for the implied volatility series across different scales. As shown by Percival and Walden (2000) (see also Whitcher, 2000) the wavelet covariance between two fractionally integrated time series X and Y (with the orders of integration d_1 and d_2, respectively) for scale λ_j (which equal to 2^{j-1}) is defined as: $\gamma(\lambda_j)$ and it is given by:

$$\gamma(\lambda_j) = \frac{1}{N_j} \sum_{l=L_j-1}^{N-1} \bar{W}^{X}_{j,t} \bar{W}^{Y}_{j,t} \qquad (9)$$

where $\bar{W}_{j,t}$ are the wavelet coefficients of each series, and $N_j = N - L_j + 1$ and $L_j = (2^j-1)(L-1)+1$; L stands for the filter length. Given that the order of fractional integration is less than 1, we use, for a scale by scale decomposition, short filters such as Haar or a Daubechies filter of length L equal to two and four respectively . This choice is motivated by the requirement, when selecting the wavelet coefficients to be included in setting up the log-likelihood function, of avoiding trimming too many initial observations for the wavelet coefficients (especially those associated with higher scales) affected by the boundary. Trimming is the price to pay when using a relatively long filter which, on the other hand, guarantees to rely on the Central Limit Theorem, hence on standard asymptotics, when drawing inference.

The factor decomposition of each scale covariance matrix through maximization of the following Gaussian log-likelihood function:

$$\sum_{i=1}^{5} \sum_{j=j_1 j}^{j_2} \sum_{t=1}^{T} L(\bar{W}_{jt}^{i};\Omega_j^i + \Gamma\Gamma') \tag{10}$$

where *L(.)* is the Gaussian log-density at time *t* , for scale *j* and for country *i*. If the focus is on the short-run horizon, then we set j_1 to 1 and j_2 to 2 when we solve for the maximum likelihood (the coefficients standard errors are then obtained by inverting the Hessian). If the focus is on the medium-run horizon, then j_1 =3 and j_2 = 5. Finally, the focus on the long-run horizon, implies setting j_1 to 6 and j_2 to 7. The observables entering the log-density are given by $\bar{W}_{jt}$, the five dimensional vector of wavelets coefficients for the implied volatilities. The unknown coefficients enter Γ and Ω_j^i matrices. More specifically, for each scale *j*, the country specific unknown coefficients are in the diagonal covariance matrix of structural form shocks:

$$\Omega_j^i = \begin{bmatrix} \sigma_{FRA,j} & & & & \\ & \sigma_{GER,j} & & & \\ & & \sigma_{UK,j} & & \\ & & & \sigma_{US,j} & \\ & & & & \sigma_{SWI,j} \end{bmatrix} \tag{11}$$

This implies eight idiosyncratic shocks (and corresponding standard deviation) when the focus is on the short run horizon; twelve idiosyncratic shocks (and corresponding standard deviation) when the focus is on the medium run horizon; eight idiosyncratic shocks (and corresponding standard deviation) when the focus is on the long run horizon. The coefficients of the 5×1 factor loading matrix Γ measure the impact of the white noise common shock on each country implied volatility:

$$\Gamma = \begin{bmatrix} \gamma_{FRA} \\ \gamma_{GER} \\ \gamma_{UK} \\ \gamma_{US} \\ \gamma_{SWI} \end{bmatrix} \qquad (12)$$

We consider the first two scales as those able to capture the short run dynamics of the volatility series. More specifically, since at the j-th stage of the decomposition, one can extract cycles of period up to 2^{j+1}, and since we use daily data, the time series component at the first scale capture the dynamics of a time series over a time horizon between two and four days, whereas the time series component at the second scale capture the dynamics of a time series over a time horizon ranging between four and eight days. Overall, the focus on the first two scales corresponds to a horizon up to eight-days. Moreover, the medium term dynamics of the volatility series is captured by the time series components at scale three, four and five, in order to pick the time series evolution over a time horizon ranging between eight and sixty-four days. Finally, the time series components at scale six and seven, describing the time series evolution over a time horizon ranging between sixty-four days and two hundred and fifty six days, would be able to capture the long term dynamics of the volatility series.

3. Data and empirical evidence

The data-set is made of five implied volatility indexes/series for France (VCAC), Germany (VDAX), UK (VFTSE), US (VIX) and Switzerland (VSMI), observed at daily frequency from 4/1/2000 till 6/7/2012. Descriptive statistics are reported in Table 1. The implied volatility indexes/series represent a measure of market expectations of near-term volatility conveyed by the underlying stock index option prices. They are deemed by market participants to capture the so-called “market fear”: high index values are associated with high uncertainty in the underlying market, low index values with stable conditions.

We report the GPH and Local Whittle estimates of the long memory parameter using both the full sample of 3264 observations and the sample with only the last 2^{11} = 2048 observations. The choice of this sub-sample is motivated by comparison of the traditional estimators of d with the Discrete Wavelet based estimator developed by Jensen (1999; 2000). From Table 2, 3, and 4 there is evidence of long memory non stationarity in all five series. Moreover, the Maximum Likelihood wavelet based estimate of the fractional integration parameter *d* is lower than the one obtained from the GPH and local Whittle estimator. We are also aware of the possibility of structural breaks affecting the long memory parameter estimator and we leave this issue to be investigated with further research.

We now turn our focus on the multivariate analysis. From Table 5, 6, and 7 we can observe that all the coefficients are statistically significant, and, in particular, there is a pronounced increase in the common shock factor loading once we move from lower to higher scales. To ease the interpretation of the empirical results provided in Table 5, 6 and 7, we compute the ratio σ/γ (see Table 8, 9 and 10). The numerator and denominator of this ratio are (see Table 5, 6 and 7) the estimated factor loading of the idiosyncratic shock (e.g. one of the coefficient entering the main diagonal of the covariance matrix $a\Omega_i^j$ in eq. 11) and the factor loading of the common shock (e.g. one of the component of the column vector Γ in eq. 12), respectively. This ratio measures the contribution of the idiosyncratic shock relative to the common shock in explaining the dynamics of each implied volatility series. From the full sample (and, especially, from the post-break sub-sample) factor decomposition of the covariance matrix for the five markets we can observe that the common shock contributes the most to the variability of each implied volatility series across different scales. Moreover, the higher the scale (e.g. the lower the frequency range), the more important is the role played by the common shock. From Table 10, we can observe that, after the Lehman's collapse, at scale 7 (e.g. the highest frequency range considered), the common shock contributes to 80% of the variability of the German and Swiss implied volatility index, to 85% of the variability of the French and US implied volatility index, and to 95% of the UK implied volatility dynamics. These empirical findings can be interpreted in terms of the dominance of systemic risk over time horizons reflecting the behavior of short-term investors (whose time horizon is associated with lower scales, e.g. those related to higher frequency ranges) and, especially, the trading strategies of long term investors (whose time horizon is related to the higher scales, e.g. those associated with lower frequency ranges). Furthermore, the post-Lehman's collapse regime is the one mostly characterized by the dominance of systemic risk.

As for the US implied volatility series, the contribution of the idiosyncratic shock is more important than the one related to the common shock over the first two scales. We can observe that the importance of the idiosyncratic shock to the VIX index dynamics decreases after the Lehman Brothers' collapse. In particular, from Table 8 we can observe that, as far as the first scale is concerned, the switch from the pre- to the post-Lehman's collapse implies a decrease of σ/γ, from 1.24 to 1.15 and, from 1.42 to 1.29, according to the Haar and LA4 filters, respectively. The decrease of σ/γ is more pronounced for the second scale (see Table 8), given a reduction from 1.31 to 1.01 and, from 1.57 to 1.14, according to the Haar and LA4 filters, respectively. As for the medium term horizon, that is, for scales 3, 4 and 5 (see Table 9), the full sample analysis shows approximately a contribution of the idiosyncratic shocks to the VIX index dynamics equal to the one associated to the common shock. More specifically, while the full sample estimate of σ/γ (using the Haar filter) is equal to 0.48, 0.45 and 0.50 for scale 3, 4 and 5, respectively, the LA4 filter produces an estimate of σ/γ equal to 0.57, 0.51 and 0.55 for scale 3, 4 and 5, respectively. The sub-sample estimates of σ/γ (using the Haar filter) show a decrease of this ratio from 0.59 to 0.42 over the third scale. This decrease is more pronounced for the fourth and fifth scales, since there is a switch from 0.57 to 0.36 and from 0.60 to 0.36, respectively. This finding is confirmed by the LA4 filter. In particular, while there is a shift in σ/γ for the fourth and fifth scales (from 0.65 to 0.42 and from 0.64 to 0.43, respectively), the ratio σ/γ is estimated to diminish from 0.69 to 0.52 over the third scale. A further decrease in the contribution of the idiosyncratic shocks relative to the common shock is recorded for a long term horizon, that is for scale 6 and 7 (see Table 10).

Switzerland and UK display a similar pattern. For both countries the contribution of the idyosincratic shock raises from the pre-break to the post-break sub-sample period (if the focus is on the first five scales, as we can observe from Table 8 and 9). However, the ratio σ/γ is well below unity for both sub-sample periods. In particular for UK, as for the first scale (see Table 8), according to the Haar filter, 0.56 and 0.74 are the values of this ratio for the pre- and post-break, respectively; 0.65 and 0.83 are the corresponding values if we use the LA4 filter. As for the second scale (see Table 8), according to the Haar filter, 0.49 and 0.68 are the values of this ratio for the pre- and post-break, respectively; 0.56 and 0.78 are the corresponding values if we use the LA4 filter. We find the same pattern for scales 3-4. Once we move to the scale 6 and 7 (see Table 10), we can observe that, after the Lehman's collapse, there is a decrease in the role played by the idiosyncratic shock in explaining implied volatility in UK and Switzerlad. More specifically, the common shock nearly contributes to 90 and 95%, of the variability of UK implied volatility series , at scale 6 and 7, respectively and to 80-85% of the variability of Swiss implied volatility.

France is the only European country where the idiosyncratic shock is more important than the common shock when the focus is on the first two scales and we consider the pre-Lehman's collapse sub-sample (see Table 8). However, once we observe a switch to the post-Lehman's collapse regime, we observe an increased contribution from the common shock across different scales (see Tables 9 and 10).

For France, from the pre-break to the post-Lehman's collapse, we find a decrease of the importance of the idiosyncratic shock for all the scales. The same happens for Germany for the first five scales (Tables 8 and 9). However, for scales 6 and 7 we observe for Germany an inversion of the tendency: a decrease of the importance of the common shock.

In the short run implied volatility indexes are driven by idiosyncratic components more than in the long run, when effects of market frictions disappear. In fact traders react to changes in implied volatility of other markets in the short run more than in the long run, thus causing artificial noise which vanishes in the long run. Overall, the empirical evidence suggest that systemic risk contributes the most in explaining the trading strategies of various class of investors, particularly those with long term view, especially after the Lehman Brothers' collapse.

4. Conclusions

In this paper we explore the long memory properties of five implied volatility indices. While previous fractional integration studies of implied volatility focus only on the US markets, we also consider the implied volatility indices of four European markets: France, Germany, UK and Switzerland. Our main contribution to previous studies of long memory properties of implied volatility relies on the use of univariate and multivariate wavelet based Full Information Maximum Likelihood analysis. The univariate analysis employs, beyond the GPH and the local Whittle estimator, the wavelet based maximum likelihood estimator (developed by Jensen, 2000) of the fractional integration parameter. This estimator produces estimates of the long memory parameter d lower than those of GPH and of the local Whittle estimator, but still lying in the non-stationary region (e.g. $0.5 < d < 1$). When we employ multivariate analysis, we concentrate on a factor decomposition of the covariance matrix of the implied volatility series at different scales (each associated to a given frequency range), to assess the role played by common shock vs. idiosyncratic shocks in explaining the variability of each volatility index at different scales. The empirical evidence points at an increasing role of the common shock (hence, of systemic risk) underlying the dynamics of the different series, especially, for the higher scales (associated with low frequency ranges) and after the collapse of Lehman Brothers.

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Table 1: Descriptive Statistics

Country	Mean	Std Dev	Minimum	Maximum
FRA	24.69	9.60	9.24	78.05
GER	23.79	9.28	10.98	74.00
UK	21.79	9.15	9.09	75.54
US	22.16	9.28	9.89	80.86
SWI	20.44	8.95	9.23	84.89

Table 2: GPH estimates of parameter *d* (Full sample)

FRA	GER	UK	US	SWI
$m = T^{0.5}$				
0,689 (0,084)	0,784 (0,084)	0,602 (0,084)	0,735 (0,084)	0,661 (0,084)
$m = T^{0.6}$				
0,900 (0,056)	0,949 (0,056)	0,848 (0,056)	0,960 (0,056)	0,893 (0,056)
$m = T^{0.7}$				
0,841 (0,002)	0,891 (0,002)	0,856 (0,002)	0,868 (0,002)	0,926 (0,002)

Table 2´: GPH estimates of parameter *d* (sub-sample $T_1 = 2^{11}$; last 2048 observations)

FRA	GER	UK	US	SWI
		$m = T_1^{0.5}$		
0,889 (0,095)	0,905 (0,095)	0,892 (0,095)	0,993 (0,095)	0,892 (0,095)
		$m = T_1^{0.6}$		
0,826 (0,065)	0,876 (0,065)	0,828 (0,065)	1,015 (0,065)	0,893 (0,065)
		$m = T_1^{0.7}$		
0,833 (0,044)	0,788 (0,044)	0,890 (0,044)	0,905 (0,044)	0,926 (0,044)

Table 3: Whittle estimates of parameter *d* (Full sample)

FRA	GER	UK	US	SWI
		$m = T^{0.5}$		
0,764 (0.066)	0,804 (0.066)	0,708 (0.066)	0,775 (0.066)	0,755 (0.066)
		$m = T^{0.6}$		
0,889 (0.044)	0,925 (0.044)	0,884 (0.044)	0,972 (0.044)	0,950 (0.044)
		$m = T^{0.7}$		
0,830 (0.029)	0,879 (0.029)	0,843 (0.029)	0,863 (0.029)	0,935 (0.029)

Table 3´: Whittle estimates of parameter *d* (sub-sample $T_1 = 2^{11}$; last2048 observations)

FRA	GER	UK	US	SWI
		$m = T_1^{0.5}$		
0,854 (0.074)	0,928 (0.074)	0,855 (0.074)	0,951 (0.074)	0,881 (0.074)
		$m = T_1^{0.6}$		
0,819 (0.050)	0,928 (0.050)	0,829 (0.050)	0,992 (0.050)	0,894 (0.050)
		$m = T_1^{0.7}$		
0,815 (0.034)	0,840 (0.034)	0,833 (0.034)	0,875 (0.034)	0,897 (0.034)

Table 4: ML wavelet based estimates of parameter *d* (sample $T_1 = 2^{\wedge 11}$; last 2048 observations)

FRA	GER	UK	US	SWI
HAAR filter				
0.741 (0.015)	0.795 (0.014)	0.782 (0.014)	0.765 (0.014)	0.843 (0.014)
LA4 filter				
0.758 (0.014)	0.824 (0.014)	0.774 (0.014)	0.754 (0.014)	0.852 (0.014)

Table 5: Factor decomposition of scale 1 and 2 covariance matrices for the implied volatilities time series

	HAAR filter			**LA4 filter**		
Parameter	**Full sample**	**Pre-break**	**Post-break**	**Full sample**	**Pre-break**	**Post-break**
$\sigma_{FRA,1}$	0.620 (0.009)	0.645 (0.016)	0.855 (0.023)	0.585 (0.009)	0.589 (0.014)	0.822 (0.023)
$\sigma_{GER,1}$	0.380 (0.007)	0.401 (0.013)	0.526 (0.017)	0.368 (0.007)	0.370 (0.012)	0.522 (0.017)
$\sigma_{UK,1}$	0.513 (0.008)	0.454 (0.014)	0.751 (0.021)	0.491 (0.008)	0.440 (0.014)	0.720 (0.020)
$\sigma_{US,1}$	0.738 (0.010)	0.600 (0.014)	1.054 (0.025)	0.709 (0.009)	0.552 (0.013)	1.020 (0.024)
$\sigma_{SWI,1}$	0.379 (0.006)	0.331 (0.011)	0.532 (0.016)	0.347 (0.006)	0.315 (0.010)	0.481 (0.015)
$\sigma_{FRA,2}$	0.608 (0.009)	0.697 (0.017)	0.810 (0.022)	0.587 (0.008)	0.648 (0.016)	0.810 (0.022)
$\sigma_{GER,2}$	0.326 (0.007)	0.411 (0.013)	0.394 (0.017)	0.306 (0.007)	0.389 (0.012)	0.376 (0.017)
$\sigma_{UK,2}$	0.478 (0.008)	0.397 (0.014)	0.690 (0.019)	0.463 (0.007)	0.383 (0.014)	0.673 (0.018)
$\sigma_{US,2}$	0.676 (0.009)	0.635 (0.015)	0.927 (0.022)	0.654 (0.009)	0.612 (0.015)	0.900 (0.022)
$\sigma_{SWI,2}$	0.397 (0.006)	0.337 (0.011)	0.561 (0.016)	0.375 (0.006)	0.306 (0.010)	0.536 (0.016)
γ_{FRA}	0.801 (0.011)	0.583 (0.018)	1.174 (0.027)	0.700 (0.010)	0.477 (0.017)	1.040 (0.026)
γ_{GER}	0.718 (0.008)	0.679 (0.015)	0.979 (0.019)	0.622 (0.007)	0.568 (0.013)	0.858 (0.017)
γ_{UK}	0.776 (0.009)	0.814 (0.016)	1.012 (0.023)	0.662 (0.009)	0.682 (0.015)	0.865 (0.022)
γ_{US}	0.610 (0.011)	0.485 (0.017)	0.920 (0.027)	0.512 (0.010)	0.390 (0.015)	0.789 (0.025)
γ_{SWI}	0.647 (0.008)	0.610 (0.013)	0.908 (0.020)	0.539 (0.007)	0.486 (0.011)	0.770 (0.018)

Note: Standard error in parenthesis. The pre-break sample period runs from 4/1/2000 until 12/9/2012. The post-break sample period runs from 15/9/2012 until 6/7/2012

Table 6: Factor decomposition of scale 3. 4 and 5 covariance matrices for the implied volatilities time series

parameter	HAAR filter			LA4 filter		
$\sigma_{FRA,3}$	0.606 (0.009)	0.792 (0.019)	0.686 (0.020)	0.606 (0.009)	0.780 (0.019)	0 .683 (0.021)
$\sigma_{GER,3}$	0.348 (0.007)	0.430 (0.013)	0.428 (0.017)	0.323 (0.007)	0.406 (0.012)	0.426 (0.018)
$\sigma_{UK,3}$	0.465 (0.008)	0.381 (0.014)	0.638 (0.020)	0.466 (0.008)	0.354 (0.014)	0.642 (0.020)
$\sigma_{US,3}$	0.650 (0.009)	0.668 (0.016)	0.852 (0.022)	0.653 (0.009)	0.673 (0.016)	0.857 (0.023)
$\sigma_{SWI,3}$	0.436 (0.007)	0.374 (0.012)	0.636 (0.019)	0.406 (0.007)	0.316 (0.011)	0.620 (0.020)
$\sigma_{FRA,4}$	0.599 (0.009)	0.855 (0.021)	0.598 (0.018)	0.603 (0.009)	0.875 (0.021)	0.577 (0.017)
$\sigma_{GER,4}$	0.394 (0.007)	0.494 (0.015)	0.415 (0.014)	0.369 (0.007)	0.515 (0.015)	0.352 (0.013)
$\sigma_{UK,4}$	0.430 (0.008)	0.397 (0.017)	0.523 (0.017)	0.432 (0.008)	0.362 (0.018)	0.548 (0.017)
$\sigma_{US,4}$	0.611 (0.008)	0.645 (0.016)	0.726 (0.019)	0.577 (0.008)	0.634 (0.016)	0.692 (0.018)
$\sigma_{SWI,4}$	0.482 (0.007)	0.497 (0.014)	0.654 (0.017)	0.470 (0.007)	0.455 (0.013)	0.642 (0.017)
$\sigma_{FRA,5}$	0.574 (0.009)	0.807 (0.021)	0.592 (0.017)	0.592 (0.009)	0.870 (0.023)	0.606 (0.017)
$\sigma_{GER,5}$	0.470 (0.008)	0.490 (0.017)	0.575 (0.016)	0.444 (0.007)	0.527 (0.018)	0.534 (0.015)
$\sigma_{UK,5}$	0.427 (0.008)	0.451 (0.019)	0.427 (0.016)	0.410 (0.008)	0.367 (0.023)	0.438 (0.016)
$\sigma_{US,5}$	0.670 (0.009)	0.677 (0.017)	0.727 (0.020)	0.624 (0.009)	0.629 (0.016)	0.710 (0.019)
$\sigma_{SWI,5}$	0.545 (0.008)	0.633 (0.017)	0.710 (0.019)	0.509 (0.008)	0.633 (0.017)	0.626 (0.017)
γ_{FRA}	1.536 (0.013)	1.470 (0.025)	2.078 (0.029)	1.358 (0.012)	1.252 (0.024)	1.829 (0.026)
γ_{GER}	1.379 (0.011)	1.489 (0.021)	1.773 (0.025)	1.208 (0.010)	1.309 (0.020)	1.499 (0.021)
γ_{UK}	1.574 (0.012)	1.729 (0.024)	2.034 (0.028)	1.396 (0.011)	1.561 (0.022)	1.765 (0.025)
γ_{US}	1.350 (0.012)	1.135 (0.019)	2.024 (0.030)	1.141 (0.010)	0.982 (0.018)	1.647 (0.026)
γ_{SWI}	1.428 (0.011)	1.485 (0.022)	1.885 (0.028)	1.224 (0.010)	1.256 (0.019)	1.587 (0.024)

Note: Standard error in parenthesis. The pre-break sample period runs from 4/1/2000 until 12/9/2012. The post-break sample period runs from 15/9/2012 until 6/7/2012

Table 7: Factor decomposition of scale 6 and 7 covariance matrices for the implied volatilities time series

Parameter	HAAR filter			LA4 filter		
	Full sample	**Pre-break**	**Post-break**	**Full sample**	**Pre-break**	**Post-break**
$\sigma_{FRA,6}$	0.535 (0.009)	0.708 (0.020)	0.528 (0.015)	0.530 (0.009)	0.766 (0.023)	0.499 (0.015)
$\sigma_{GER,6}$	0.563 (0.009)	0.496 (0.017)	0.689 (0.018)	0.515 (0.008)	0.419 (0.017)	0.648 (0.017)
$\sigma_{UK,6}$	0.449 (0.009)	0.578 (0.019)	0.338 (0.016)	0.454 (0.009)	0.548 (0.021)	0.376 (0.016)
$\sigma_{US,6}$	0.793 (0.011)	0.842 (0.021)	0.744 (0.020)	0.698 (0.010)	0.681 (0.019)	0.739 (0.020)
$\sigma_{SWI,6}$	0.611 (0.009)	0.636 (0.019)	0.840 (0.021)	0.600 (0.009)	0.743 (0.023)	0.826 (0.021)
$\sigma_{FRA,7}$	0.522 (0.010)	0.696 (0.022)	0.578 (0.017)	0.460 (0.009)	0.668 (0.026)	0.476 (0.013)
$\sigma_{GER,7}$	0.705 (0.011)	0.647 (0.020)	0.831 (0.022)	0.634 (0.010)	0.574 (0.022)	0.700 (0.017)
$\sigma_{UK,7}$	0.413 (0.010)	0.569 (0.022)	0.142 (0.033)	0.420 (0.009)	0.547 (0.025)	0.203 (0.015)
$\sigma_{US,7}$	0.953 (0.013)	0.897 (0.024)	0.634 (0.021)	0.870 (0.013)	0.724 (0.024)	0.623 (0.016)
$\sigma_{SWI,7}$	0.644 (0.010)	0.646 (0.021)	0.739 (0.020)	0.572 (0.009)	0.735 (0.027)	0.627 (0.015)
γ_{FRA}	2.989 (0.027)	3.333 (0.058)	3.982 (0.064)	2.887 (0.027)	3.401 (0.067)	3.781 (0.061)
γ_{GER}	2.765 (0.026)	3.160 (0.055)	3.654 (0.061)	2.628 (0.025)	3.085 (0.060)	3.444 (0.057)
γ_{UK}	2.955 (0.027)	3.345 (0.057)	3.927 (0.063)	2.881 (0.027)	3.465 (0.067)	3.753 (0.060)
γ_{US}	2.855 (0.028)	2.388 (0.045)	4.520 (0.073)	2.781 (0.028)	2.563 (0.052)	4.238 (0.069)
γ_{SWI}	2.971 (0.028)	3.416 (0.059)	3.956 (0.065)	2.935 (0.028)	3.567 (0.070)	3.866 (0.063)

Note: Standard error in parenthesis. The pre-break sample period runs from 4/1/2000 until 12/9/2012. The post-break sample period runs from 15/9/2012 until 6/7/2012

Table 8: Factor decomposition of scale 1 and 2 covariance matrices for the implied volatilities time series: σ/γ

HAAR filter			LA4 filter			parameter
Full sample	Pre-break	Post-break	Full sample	Pre-break	Post-break	
0.77	1.11	0.73	0.84	1.23	0.79	$\sigma_{FRA,1}/\gamma_{FRA,1}$
0.53	0.59	0.54	0.59	0.65	0.61	$\sigma_{GER,1}/\gamma_{GER,1}$
0.66	0.56	0.74	0.74	0.65	0.83	$\sigma_{UK,1}/\gamma_{UK,1}$
1.21	1.24	1.15	1.38	1.42	1.29	$\sigma_{US,1}/\gamma_{US,1}$
0.59	0.54	0.59	0.64	0.65	0.62	$\sigma_{SWI,1}/\gamma_{SWI,1}$
0.76	1.20	0.69	0.84	1.36	0.78	$\sigma_{FRA,2}/\gamma_{FRA,2}$
0.45	0.61	0.40	0.49	0.68	0.44	$\sigma_{GER,2}/\gamma_{GER,2}$
0.62	0.49	0.68	0.70	0.56	0.78	$\sigma_{UK,2}/\gamma_{UK,2}$
1.11	1.31	1.01	1.28	1.57	1.14	$\sigma_{US,2}/\gamma_{US,2}$
0.61	0.55	0.62	0.70	0.63	0.70	$\sigma_{SWI,2}/\gamma_{SWI,2}$

Note: The pre-break sample period runs from 4/1/2000 until 12/9/2012.
The post-break sample period runs from 15/9/2012 until 6/7/2012

Table 9: Factor decomposition of scale 3, 4 and 5 covariance matrices for the implied volatilities time series: σ/γ

HAAR filter			LA4 filter			Parameter
Full sample	Pre-break	Post-break	Full sample	Pre-break	Post-break	
0.39	0.54	0.33	0.45	0.62	0.37	$\sigma_{FRA,3}/\gamma_{FRA,3}$
0.25	0.29	0.24	0.27	0.31	0.28	$\sigma_{GER,3}/\gamma_{GER,3}$
0.30	0.22	0.31	0.33	0.23	0.36	$\sigma_{UK,3}/\gamma_{UK,3}$
0.48	0.59	0.42	0.57	0.69	0.52	$\sigma_{US,3}/\gamma_{US,3}$
0.31	0.25	0.34	0.33	0.25	0.39	$\sigma_{SWI,3}/\gamma_{SWI,3}$
0.39	0.58	0.29	0.44	0.70	0.32	$\sigma_{FRA,4}/\gamma_{FRA,4}$
0.29	0.33	0.23	0.31	0.39	0.23	$\sigma_{GER,4}/\gamma_{GER,4}$
0.27	0.23	0.26	0.31	0.23	0.31	$\sigma_{UK,4}/\gamma_{UK,4}$
0.45	0.57	0.36	0.51	0.65	0.42	$\sigma_{US,4}/\gamma_{US,4}$
0.34	0.33	0.35	0.38	0.36	0.40	$\sigma_{SWI,4}/\gamma_{SWI,4}$
0.37	0.55	0.28	0.44	0.69	0.33	$\sigma_{FRA,5}/\gamma_{FRA,5}$
0.34	0.33	0.32	0.37	0.40	0.36	$\sigma_{GER,5}/\gamma_{GER,5}$
0.27	0.26	0.21	0.29	0.24	0.25	$\sigma_{UK,5}/\gamma_{UK,5}$
0.5	0.60	0.36	0.55	0.64	0.43	$\sigma_{US,5}/\gamma_{US,5}$
0.38	0.43	0.38	0.42	0.50	0.39	$\sigma_{SWI,5}/\gamma_{SWI,5}$

Note: The pre-break sample period runs from 4/1/2000 until 12/9/2012.
The post-break sample period runs from 15/9/2012 until 6/7/2012

Table 10: Factor decomposition of scale 6 and 7 covariance matrices for the implied volatilities time series: σ/γ

HAAR filter			LA4 filter			parameter
Full sample	Pre-break	Post-break	Full sample	Pre-break	Post-break	
0.18	0.21	0.13	0.18	0.23	0.13	$\sigma_{FRA,6}/\gamma_{FRA,6}$
0.2	0.16	0.19	0.20	0.14	0.19	$\sigma_{GER,6}/\gamma_{GER,6}$
0.15	0.17	0.09	0.16	0.16	0.10	$\sigma_{UK,6}/\gamma_{UK,61}$
0.28	0.35	0.16	0.27	0.22	0.21	$\sigma_{US,6}/\gamma_{US,6}$
0.21	0.19	0.21	0.21	0.21	0.22	$\sigma_{SWI,6}/\gamma_{SWI,6}$
0.17	0.21	0.15	0.16	0.20	0.13	$\sigma_{FRA,7}/\gamma_{FRA,7}$
0.25	0.20	0.23	0.24	0.19	0.20	$\sigma_{GER,7}/\gamma_{GER,7}$
0.14	0.17	0.04	0.15	0.16	0.05	$\sigma_{UK,7}/\gamma_{UK,7}$
0.33	0.38	0.14	0.33	0.24	0.18	$\sigma_{US,7}/\gamma_{US,7}$
0.22	0.19	0.19	0.20	0.21	0.17	$\sigma_{SWI,7}/\gamma_{SWI,7}$

Note: The pre-break sample period runs from 4/1/2000 until 12/9/2012.
The post-break sample period runs from 15/9/2012 until 6/7/2012

Appendix

Wavelets can be a particular useful tool when the signal is localized in time as well as frequency. Discontinuities in signals can be described in terms of very short (compressed) basis functions with a high-frequency content, whereas a fine analysis at low frequencies can be achieved using highly dilated (stretched) basis functions. In other words, the wavelet is contracted or dilated to change the scale at which one looks at a signal. The wavelet is then shifted or translated in time to correspond to different part of the signal. The procedure is called multiresolution analysis. In particular, in case of a dyadic multiresolution analysis, the dilated and translated family of wavelets functions can be defined as[1]:

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t - k);\; j,k \in I \tag{A1}$$

Where *j* and *k* are the integer parameters governing the scale resolution (i.e. 2^{-j}) and translation in time, respectively.
All the wavelet basis functions, $\psi_{j,k}$, are self-similar, namely, they differ only by translation and change of scale from one another. These functions result from a *mother* wavelet, $\psi(t)$, which is any oscillating function with unit energy, i.e.:

$$\int_{-\infty}^{+\infty} \psi(t)dt = 0 \qquad \int_{-\infty}^{+\infty} |\psi(t)|^2 dt = 1 \tag{A2}$$

The object of a wavelet analysis is to associate an amplitude coefficient to each of the wavelet. The task is accomplished by the Discrete Wavelet Transform which is implemented via the pyramid algorithm of Mallat (1987). If certain conditions are satisfied, these coefficients completely characterize the signal which is resolved in terms of a coarse approximation and the sum of fine details:

$$x(t) = \sum_k v_{J,k}\phi_{J,k}(t) + \sum_j \sum_k w_{j,k}\psi_{j,k} \tag{A3}$$

Here J is the highest possible level of decomposition; $\phi_{J,k}$ is the set of translated orthogonal scaling functions spanning the lower frequency range [0, $\pi/2^{(J)}$). Therefore, the first term $\sum_k v_{J,k}\phi_{J,k}(t)$ in (A3) is the coarse approximation of the signal, and the second term $\sum_j \sum_k w_{j,k}\psi_{j,k}$ in A(3) is the sum of fine details.

[1] Given a time series with T observations, conventional dyadic multiresolution analysis applies to a succession of frequency intervals in the form of ($\pi/2^{(j)}$, $\pi/2^{(j-1)}$), with the decomposition level j running from 1 to J. The bandwidths are halved (downsampled by 2) repeatedly descending from high to low frequencies. By the j^{th} round, there will be j wavelet bands and one accompanying scaling function band . At the decomposition level j, one obtains a set of $T/2^j$ mutually orthogonal wavelets functions given by equation (A.1), separated from each other by 2^j points.

The scaling and wavelet coefficients $v_{j,k}$ and $w_{j,k}$ are the following projections of *x(t)* on the bases $\phi_{j,k}$ and $\psi_{j,k}$ respectively:

$$v_{j,k} = \int x(t)\phi_{j,k}(t)dt \qquad \text{(A4)}$$

$$w_{j,k} = \int x(t)\psi_{j,k}(t)dt \qquad \text{(A5)}$$

The signal can then be written as a set of orthogonal components at resolutions 1 to J:

$$x(t) = S_J + D_J + D_{J-1} + ... + D_1 \qquad \text{(A6)}$$

An important feature of a wavelet analysis consists in the fact that it is an energy-preserving transform; as a consequence, the variance of the signal is perfectly captured by the variance of the wavelet coefficients, *w*. In other words, the overall variance of the data can be expressed as a sum of the variances within the frequency bands, which may be indexed by j:

$$\sigma^2 = \sum_{j=1}^{\infty} \sigma_j^2 \qquad \text{(A7)}$$

where σ_j^2 is the contribution of the variability at scale 2^{-j} to the overall variability of the process:

$$\sigma_j^2 = \frac{1}{2^j} Var(w_{j,t}) \qquad \text{(A8)}$$

Similarly, as shown by Whitcher (1998) and by Whitcher et al. (2000), the wavelet covariance decomposes the covariance between two stochastic processes on a scale-by-scale basis. For a bivariate stochastic process $X_t = (x_{1,t}, x_{2,t})$, there will be:

$$\sum_{j=1}^{\infty} Cov_x(j) = Cov(x_{1,t}, x_{2,t}) \qquad \text{(A9)}$$

where

$$Cov_x(j) = \frac{1}{2^j} Cov(w_{1,j,t}, w_{2,j,t}) \qquad \text{(A10)}$$

A disadvantage of the conventional dyadic wavelet analysis is the restriction on the sample size T which has to be a power of 2. A further problem lies in the fact that the DWT depends upon a non-symmetric filter that is liable to induce a phase lag in the processed data. These difficulties can be circumvented by the Maximum Overlapping Discrete Wavelet Transform (MODWT), which represents an attempt to generate a transform that is not sensitive to the choice of the starting point for the data series. In order to avoid such sensitivity, the filtered output at each stage of the pyramid algorithm is not subjected to downsampling. As a consequence, the number of coefficients generated at the j-th stage of the decomposition are in number equal to the sample size, T, instead that equal to $T/2^j$. An important feature of the MODWT is that, besides handling any sample size, the detail and smooth coefficients of the multiresolution analysis are associated with linear phase filters. The consequence is that it is possible to align the features of the original time series with those of the multiresolution analysis.

The DWT, as well as its variants, the Partial DWT and the MODWT, makes use of circular filtering. The series under investigation is treated as if it is a portion of a periodic sequence with period N. In other words, the transform considers x_{N-1}, $x_{N-2\ldots}$ as useful surrogates for the unobserved x_{-1} , x_{-2} …. . This can be a questionable assumption for some time series. The effects of this assumption, and solutions to the problems created, are fully explored in Percival and Walden (2000). A problem with the periodic extension can occur when there is a large discontinuity between the end of one replication of the sample and the beginning of the next. In such cases the coefficients produced by the transform result remarkably high and the reconstructed details are affected. To reduce this problem the data should be suitably de-trended. The aforementioned criticism related to filter circularity would not apply to financial time series.