Acquisitive Investment, Costly Financing and Competition under Uncertainty

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Abstract

We analyse the effects of combining nonsynergistic operational activities when an acquirer is subject to a fear of preemption. Unlike other papers we show that a phase of a development of a target firm affects terms and timing of an acquisition. When a bidder is subject to the fear of preemption it is more likely to acquire the target firm in an early development stage. Financial constraints binding the target firm accelerate a timing of the acquisition by creating synergies related to an asymmetric financing effect. The fear of preemption speeds up the timing of a merger agreement, while the timing of a hostile takeover converges to a global optimizer merger threshold. Takeover premiums to the target firm are higher when the firm is acquired in the early development stage and when the bidder is subject to the fear of preemption.

Keywords: Friendly Mergers, Hostile Takeovers, Real Options, Asymmetric Financing, Fear of Preemption
1. Introduction

Alterations in the boundaries of the firm may create positive or negative synergies. Most papers in the financial literature focus on operational synergies that arise due to revenue increase or cost efficiency when firms combine their assets in place (e.g. Lambrecht (2004)). There are also a few studies that analyse financial synergies of merging assets in place of two firms together (e.g. Lewellen (1971), Leland (2007)). This paper studies financial synergies that arise due to asymmetric access to financing and attempts to complement the existing literature by analysing the effects of acquiring growth options.

The decision of the acquirer is a complex process where the bidder has to determine the optimal terms and timing of the acquisition based on operational or financial synergies.\(^1\) We consider another trade-off that the acquirer faces in a dynamic environment. In particular, whether to acquire the target firm early as a growth option (i.e., when it is in a form of a developed idea which is still not yet commercialized) or to wait and acquire the target firm with assets in place already installed and generating profit.

In this study we address a number of novel research questions. First, we study how the acquisition of firms that are in the early development stage affect merger synergies, successful consummation and takeover premium? How does the investment and financing strategy of the target firm affect the investment strategy of the acquirer? What sort of characteristics determine the choice of acquiring the target as a growth option or assets in place? Second, we explore whether synergies differ depending on the stage of development of the target firm.

\(^1\)For example, Lambrecht (2004), Morellec and Zhdanov (2005), and Gorbenko and Malenko (2013)
Third, we analyse whether the type of the offer, either friendly merger or hostile takeover, affects the equilibrium.

The core prediction of the model is that the effect of the fear of preemption depends on the negotiation process between the acquirer and the target firm. In the case of a merger agreement, where the bidder and the target decide on terms and timing simultaneously, competition erodes the option value and forces a merger’s timing to converge to the break-even threshold. However, in the case of a hostile takeover the timing of an acquisition converges to the first-best merger threshold (the global optimizer merger threshold). The intuition is that if the target firm acts as Stackelberg leader, and decides on the terms first, it preserves the option value and obtains an efficient ownership share. Then the acquirer decides on the timing of the acquisition which converges to the global optimizer first-best merger threshold for the infinite fear of preemption.

To address the research questions we study a decision to acquire a firm in a dynamic competitive environment. We develop a model where firms make investment and financing decisions. The model is framed within a real options literature. The target firm is an innovative firm, which is initially financed by equity and has an investment option. The profit of the target firm is subject to corporate tax, which gives an incentive to the firm to finance the cost of investment with debt due to tax shield benefits. However, debt issuance for the target firm is costly and it has to pay a proportional issuance cost. This in turn affects optimal capital structure and investment.

The acquirer has an option to buy the target firm. Both firms can create financial synergies when the acquirer facilitates access to financing for the target firm and changes its optimal
capital structure. The acquisitive investment decision is modelled as a compound option. The acquirer can exercise the option to buy the target firm as an investment opportunity or as assets in place. The acquirer in this model is a financially unconstrained firm and can raise financing costlessly as in McDonald and Siegel (1986) setting.

The decision of the acquirer is subject to the underlying diffusion process which is the cash flow risk. When issuing financing for the target firm is costly it may delay the investment exercise. The acquirer can take over the target firm and provide financing. The acquirer is subject to the risk of preemption, modelled as a jump process. The decision whether to acquire the target firm as a growth option or assets in place is an outcome of financial constraints binding the acquiree and competition among interested acquirers.

We solve the model using continuous time techniques. First, we solve the investment and financing problem of the target firm. Second, we solve the optimization problem of the acquirer. We find closed-form solutions for investment and acquisition thresholds. We determine when the bidder acquires growth opportunities and when physical assets. If the target firm is financially constrained, the decision to invest is delayed or never exercised, which strengthens the incentives of the acquirer to buy the target firm and enhances synergies by providing financing.

The remainder of this paper is organized as follows. Section 2 summarizes the related literature. Section 3 presents the outline and assumptions of the model. Section 4 presents closed-form valuation formulas and investment threshold when it is costly to issue financing. Section 5 discusses the financial synergies of acquisitive investment and the optimal timing of the global optimizer. Section 6 analyses the effect of the fear of preemption on acquisition
terms and timing. Section 7 discusses the decision to acquire growth option or assets in place and presents numerical examples. Section 8 discusses takeover premiums. Section 9 concludes.

2. Literature Review

The paper relates to the literature on financial synergies of mergers and acquisitions. Financial synergies arise when changes in the scope of the firm affect optimal capital structure. First arguments towards the existence of purely financial synergies were pointed out by Lewellen (1971). He suggested that when two firms, with imperfectly correlated cash flows merge, they decrease the risk of a combined firm through portfolio diversification. This is the so called coinsurance effect. In a world with a tradeoff between the tax shield and bankruptcy cost, a decrease in the level of risk reduces the expected default costs. Therefore, a firm can increase its optimal debt level with the potential tax benefits that accrue to shareholders. Stapleton (1982) proves a positive effect of a merger on debt capacity even for a merger between two firms with perfectly correlated cash flows.

The importance of financial synergies was recognized by Myers and Majluf (1984). They show that "slack-rich" bidders pair with "slack-poor" targets to create value. Given asymmetric information, they claim that a slack-poor firm might not undertake all investment opportunities. Therefore, a slack-rich firm can finance the projects of firms with high informational asymmetries.

Some papers discuss the issue of facilitating financing for a target firm during distress periods as a motive of a merger as in Fluck and Lynch (1999) and Almeida, Campello, and
Hackbarth (2011). Fluck and Lynch (1999) develop a theory of mergers and divestitures. They focus on conglomerate mergers that can diminish financing problems of distressed firms. In particular, they claim that distressed firms are unable to finance marginally profitable projects as stand-alone values due to agency problems between shareholders and claimholders. Conglomerate mergers help them to survive a distress period. Then, the assets are divested due to an increase in coordination cost. They present their model in the light of extensive empirical evidence which shows that many acquires divest assets subsequently the merger. Almeida, Campello, and Hackbarth (2011) present a model where financially distressed firms are acquired by relatively liquid firms within the industry. They claim that firms that are not able to raise capital, when liquidity shock hits, might be inefficiently terminated. Therefore, a solution to that problem is reallocation of assets from a firm that is illiquid to a firm that is liquid.

Our paper shares some features with Leland (2007) who derives a model where only financial synergies motivate the merger decision. He disentangles financial synergies into two components that affect the value of the merged entity: leverage effect (related to tax shield and default costs) and limited liability effect. He claims that the magnitude of this effect depends on the firm’s characteristics like default costs, firm size, taxes, and riskiness of cash flows.

In contrast to the previous literature, we focus on financial synergies that arise due to asymmetric financing. When markets are imperfect, the financing cost of investment for firms might be different. Heterogeneity between firms with respect to financing possibilities causes that production opportunities can be financed better by financially unconstrained firms.
There is some empirical evidence that firms forgo lucrative investment opportunities as a result of binding financial constraints as for example in Ivashina and Scharfstein (2010) and Campello, Graham, and Harvey (2010). The reasons why companies might have asymmetric access to financing are widely covered in the finance literature. For example, the relationship with the institution providing external financing is important for a firm and has a profound effect on corporate investment. The better the relationship the easier the access to financing (examples include Petersen and Rajan (1994), Berger and Udell (1995), Puri, Rocholl, and Steffen (2011)). Also firms may forgo profitable investment opportunities as a result of poor financial slack (Myers and Majluf (1984)) or excessive pre-existing debt (the debt overhang problem of Myers (1977)).

This paper is close to a recent study of Martos-Vila, Rhodes-Kropf, and Harford (2011), who analyse a pattern in the dominance of financial or strategic bidders during merger waves. Following the observation of Harford (2005) they point out that the misvaluation of debt and some specific characteristics of bidders determine their relative position. According to Harford (2005) the spread between the average rate charged for commercial and industrial loans and the fed funds rate (C&I spread) proxies for ease of financing. He provides evidence that this variable is statistically significant in explaining merger waves. Martos-Vila, Rhodes-Kropf, and Harford (2011) develop the analysis and show that when the C&I spread increases, the proportion of financial bidders in all mergers deals increases during merger waves.

This is consistent with the prediction of our model that mergers motivated by financial synergies arise during economic upturns only when the spread in the cost of issuing financing between the target and the acquirer is high enough. However, we provide different funda-
mentals in our model.

Our model also relates to the literature on mergers and acquisitions within real options frames. Lambrecht (2004), Lambrecht and Myers (2007), Bernile, Lyandres, and Zhdanov (2011) study terms and timing of mergers and acquisitions motivated by operational synergies and strategic reasons. Lambrecht (2004) provides a comprehensive theoretical framework of a pro-cyclical merger which is motivated by economies of scale. Lambrecht and Myers (2007) model the disinvestment decision in declining markets and claim that takeovers impose efficient closure. Bernile, Lyandres, and Zhdanov (2011) analyse strategic incentives in the case of horizontal mergers which explain takeover activity during economic booms and recessions.

Some papers analyse the mergers and acquisitions when the firm has debt in place. Morellec and Zhdanov (2008) highlight the strategic role of debt in the bidding process. They claim that the bidding contest is won by the firm with lower leverage and after a successful consummation an acquirer levers up. Hege and Hennessy (2010) present an analysis where the level of debt plays a strategic role in benefiting from larger merger share. However, there exists a trade-off between higher surplus and the resulting debt overhang which precludes efficient mergers.

Our model also relates to the literature on mergers and acquisitions in the presence of competition. Ruback (1983) is an early example of empirical studies that examine the presence of competition in the acquisitions market by looking at returns. Boone and Mulherin (2007b), Boone and Mulherin (2007a) and Boone and Mulherin (2009) are first studies that provide comprehensive evidence of competition in the takeovers market. Aktas, de Bodt, and Roll (2010) show that the probability of rival bidders appearing affects the negotiation
process and reduces bid premium. Dittmar, Li, and Nain (2012) show the impact of financial sponsor competition on corporate buyers.

Morellec and Zhdanov (2005) extend the behavioural analysis of Shleifer and Vishny (2003) by constructing a two-factor model based on stock market valuations of integrating firms. They show that the competition for the target firm speeds up the takeover process and erodes the ownership stake of bidding shareholders. Calcagno and Falconieri (2011) study the impact of competition on the outcomes of takeovers.

The theoretical framework that we develop in this paper contributes to the previous literature in several ways. First, we provide a novel motive for two firms to merge by creating financial synergies. Second, we disentangle the merger decision into two choices; an acquisition of production opportunities or an acquisition of physical assets. Third, we discuss the effect of competition on the terms and timing of the merger decision conditional on the type of offer, friendly merger or hostile takeover.

3. Model and Assumptions

This paper considers an economy where there are two types of infinitely-lived firms, which are continuously traded. Acquirers are defined as firms without any internal growth potential but with the ability to finance projects at a low cost. The acquirer’s growth strategy is to buy unexercised production opportunities or assets in place of other firms. The acquirer acts as in McDonald and Siegel (1986) setting and can raise funds costlessly. It is however subject to the fear of preemption. Competitors that have the same corporate strategy to acquire lucrative investment opportunities can arrive randomly with intensity $\delta$ (the fear
of preemption as in Hackbarth, Mathews, and Robinson (2012) and Morelec, Valta, and Zhdanov (2012)).

Targets are defined as innovators with production opportunities that are associated with an exercise sunk cost \( \kappa \). The target firm is subject to underinvestment due to high cost of issuing financing to cover the capital outlay. It can either issue debt or equity to finance its project and has to pay proportional issuance cost (\( \iota_d \) or \( \iota_e \)). However, a high issuance cost delays the investment exercise. Then, the target firm can be acquired by another company that facilitates the access to cheap financing. The acquirer can either propose a merger agreement or make a tender offer to the target shareholders. The acquirer can buy the firm as a growth option when the optimal acquisition threshold is lower than the investment trigger of the financially constrained target firm. Otherwise, the bidder can buy the target firm after the production opportunity is commercialized.

Investors are assumed to be risk neutral and can borrow and lend at the risk-free rate of interest \( r \). Managers incentives are aligned with maximizing equityholders wealth. Irreversibility of investment implies that once exercised the decision cannot be costlessly reversed.

Assets in place and growth options are subject to the same source of uncertainty \( X_t \) that

\[2\] There are theories in the finance literature that explain why firms forgo positive NPV investment opportunities and underinvest. For example, financial constraints as in Fazzari, Hubbard, and Petersen (1988) and Boyle and Guthrie (2003) or debt overhang problem (which is a conflict between shareholders and debtholders as part of the project’s NPV financed with equity is captured by debtholders when debt is risky) as in Myers (1977).
follows a geometric Brownian motion:

\[ dX_t = \mu X_t dt + \sigma X_t dW_t \]  

(1)

where \( \mu < r \) is a deterministic drift, \( \sigma > 0 \) is volatility, and \( dW_t \) is the standard Brownian motion process.

Taxes affect the firm’s capital structure. The optimal coupon is chosen to balance the tax advantage of debt with expected bankruptcy costs. When the firm has debt in place the default threshold is chosen endogenously by equityholders.

4. Investment when Financing is Costly

To work out the optimal investment threshold of the innovating target firm we solve the game backwards. First, we solve for the post-investment values when it is costly to issue debt or equity. Second, we derive closed form solutions for the optimal investment threshold.

4.1. Value of the Firm after Investment

Before the firm exercises its production opportunity it is continuously traded and its shareholders receive capital gains over each time interval. Throughout the paper, we denote the equity value of the innovative firm before investment as \( V_{I,j} \) and the post-investment values of debt, equity and the firm value as \( B_{I,j}^+, E_{I,j}^+ \) and \( V_{I,j}^+ \), where \( j \) stands for the type of financing of the growth option, which can be equity (E) or debt (D).

\[ ^3 \text{We can assume that the firm issued equity to finance the know-how necessary to develop the production opportunity which does not yet generate any cash flows as the firm has to expand its operations.} \]
When the growth option is exercised by issuing equity at a cost \( \kappa_e \), the innovating firm starts to generate after tax cash flows of \((1 - \tau) \pi x_t\) at each instant of time. The unlevered value of the innovating firm after investment exercise is:

\[
V_{I,E}^+(x) = E_{I,E}^+(x) = \frac{(1 - \tau) \pi x}{r - \mu}
\]  

(2)

The value of the equity after the firm exercises the growth option at a cost \( \kappa \) is the discounted present value of cash flows \((1 - \tau) \pi x_t\) and \( \mu \) can be interpreted as the growth rate from Gordon growth model. The issuance cost of equity \( (\kappa_e) \) does not affect the post-investment equity value as it is sunk cost that is paid at the time of investment.

The target firm can also issue risky debt to cover the investment cost \( (\kappa) \) and pay the proportional issuance cost of debt \( (\kappa_d) \). After the investment option is exercised the cash flows and tax benefits accrue till default. Debt is risky, thus equityholders are left with nothing when the firm is liquidated. Bondholders are entitled to the scrap value of the firm’s assets that is left at default, which is \((1 - \varphi) \phi x_t I_D / (r - \mu)\) and we assume that \( \phi < \pi \), where \( \phi x_t \) is the first best firm value when the firm is liquidated and \( \varphi \) is a haircut on firm’s assets at bankruptcy and \( \varphi \in (0, 1) \). When \( \varphi = 0 \) debtholders can recover the assets of the firm at bankruptcy. When \( \varphi = 1 \) the investment cannot be reversed costlessly. \( x_{I,D} \) is the default threshold selected by equityholders. The coupon \( c_{I,D}(x, \kappa_d) \) maximizes the firm value net of the issuance cost of debt after investment exercise, which is given by \( V_{I,D}^+(x, \kappa_d) - \kappa_d B_{I,D}^+(x, \kappa_d) \).

Using standard techniques we calculate claims values and the optimal closure threshold. We present the results in the following Lemma.
Lemma 1  The value of the firm after investment $x \geq \pi_{I,D}$, net of issuance costs is:

$$V_{I,D}^+(x, \iota_d) = \frac{(1 - \tau)\pi x}{r - \mu} + \frac{\tau c_{I,D}}{r} \left[ 1 - \left( \frac{x}{\pi_{I,D}} \right)^\lambda \right] - \left[ \frac{(1 - \tau)\varphi \pi_{I,D}}{r - \mu} \right] \left( \frac{x}{\pi_{I,D}} \right)^\lambda - t_d B_{I,D}^+(x, \iota_d)$$

(3)

The optimal equityholders closure threshold is:

$$x_{I,D} = \frac{\lambda}{(\lambda - 1)} \left( \frac{r - \mu}{r} \right) c_{I,D}$$

(4)

where $\lambda$ is a negative root of the quadratic equation $1/2\sigma^2 z(z - 1) + \mu z - r = 0$. The optimal coupon maximizing the firm value is:

$$\frac{\partial[V_{I,D}^+(x, \iota_d) - t_d B^+(x, \iota_d)]}{\partial c} = 0 \Rightarrow c_{I,D}(x, \iota_d) = x\pi \Omega^{1/\lambda} \frac{r}{r - \mu} \frac{\lambda - 1}{\lambda}$$

(5)

where

$$\Omega = 1 - \lambda \frac{(1 - \iota_d)(\pi - (1 - \varphi)\phi)}{\pi(\tau - \iota_d)}$$

(6)

The firm value when issuing financing is costly ($\iota_d > 0$) is:

$$V_{I,D}^+(x, \iota_d) = x \Psi$$

(7)

where $\Psi = \left[ \pi \left( 1 - \tau + (\tau - \varphi)\Omega^{1/\lambda} \right) \right] / (r - \mu)$. The firm value when issuing financing is costless ($\iota_d = 0$) is:

$$V_{I,D}^+(x, 0) = x \Phi$$

(8)

where $\Phi = \Psi(\iota_d = 0)$

Proof. See Appendix A. □

The issuance cost of debt imposes financial frictions. When the firm uses debt to finance
the cost of investment, the proportional issuance cost ($\iota_d$) decreases the coupon that the firm
can issue, which is lower than the first-best coupon \( c_{I,D}(x, \tau_d) < c_{I,D}(x, 0) \). This in turn lowers the value of the firm after investment exercise and \( V_{I,D}(x, \tau_d) < V_{I,D}(x, 0) \). The higher the issuance cost, the more severe the underinvestment problem of the target firm might be. Presence of an external investor, who can provide access to cheap financing can facilitate this problem.

4.2. Optimal Investment Exercise and Financing Terms

Before investment, the management of the target firm decides on the timing and financing strategy. When the investment project is financed by issuing equity, the firm value is affected by the proportional sunk issuance cost at the time of investment. In the case of issuing debt the issuance cost affects the coupon and also the value of the firm after investment. Thus, the form of financing affects the investment surplus and the optimal exercise.

The shareholders of the innovative firm prior to investment exercise get capital gains only \( \mathbb{E}[dV_{I,i}(x)] \) over each time interval \( dt \) as the firm does not generate any cash flows. The standard ordinary differential equation (ODE) is solved subject to the boundary conditions. The solution to this problem provides the following results.

**Lemma 2** The investment threshold of the target firm when the investment is financed with equity is:

\[
\underline{\pi}_{I}(\tau_e > 0) = \frac{\beta}{\beta - 1} \frac{(r - \mu)\kappa}{(1 - \tau)(1 - \tau_{\pi})}
\]

The investment threshold of the target firm when the investment is financed with debt is:

\[
\underline{\pi}_{D}(\tau_d > 0) = \frac{\beta}{\beta - 1} \frac{(r - \mu)\kappa}{(1 - \tau + (\tau - \tau_{d})\Omega^{1/\lambda})\tau_{\pi}}
\]
where $\beta$ is a positive root of the quadratic equation $1/2\sigma^2 z(z - 1) + \mu z - r = 0$. The value of the innovative firm over the continuation interval $x < x_{I,j}$ is:

$$V_{I,E}(x, \tau_e) = x^\beta \frac{\kappa}{\beta - 1} \left[ \frac{\beta - 1}{\beta} \frac{(1 - \tau)(1 - \tau_e)\pi}{(r - \mu)\kappa} \right]^\beta$$

(11)

when the investment is financed with equity, and:

$$V_{I,D}(x, \tau_d) = x^\beta \frac{\kappa}{\beta - 1} \left[ \frac{\beta - 1}{\beta} \frac{\Psi\kappa}{\kappa} \right]^\beta$$

(12)

when the investment is financed with debt. The above can be written as the value of growth option and the future tax shield:

$$V_{I,D}(x, \tau_d) = GO(x, \tau_d) + \text{FTS}(x, \tau_d)$$

(13)

where the value of the growth option is:

$$GO(x, \tau_d) = \left[ \frac{(1 - \tau)\pi x}{r - \mu} - \kappa \right] \left( \frac{x}{x_{I,D}} \right)^\beta$$

(14)

and the value of the future tax shield is:

$$\text{FTS}(x, \tau_d) = \left[ \frac{\pi x(\tau - \tau_d)(\Omega)^{1/\lambda}}{r - \mu} \right] \left( \frac{x}{x_{I,D}} \right)^\beta$$

(15)

**Proof.** See Appendix B. ■

Lemma 2 provides a closed-form solution for the optimal investment exercise threshold when the cost of investment is financed with issuing equity or raising risky debt. Comparative statics show the standard predictions of the real options literature. In particular, the investment threshold is delayed when the cost of investment ($\kappa$) increases or when the investment payoff ($\pi$) decreases. Higher corporate taxes delay investment. Although there is
an increase in the tax shield when the \( (\tau) \) increases, however this effect is offset by decrease in the present value of after-tax cash flows. The uncertainty delays the investment exercise due to the value of waiting.

When the innovative firm chooses to finance the investment cost with equity then it does not benefit from the tax shield related to debt. We show that when the investment is financed with debt the value of the firm is the sum of the value of the growth option and the discounted value of the tax shield that starts accruing after the investment option is exercised. We demonstrate how much value is destroyed by sub-optimal financial structure in the following equation.

\[
\frac{V_{I,E}(x, \iota_e)}{V_{I,D}(x, \iota_d)} = \left( \frac{(1 - \iota_e)(1 - \tau)}{1 - \tau + (\tau - \iota_d)\Omega^{1/\lambda}} \right)^{\beta} < 1
\]

When the investment is financed with debt, the value of the firm is not only the growth option but also the net present value of future tax benefits. Higher future cash flows speed up the investment threshold, therefore: \( \pi_{I,D} < \pi_{I,E} \). This effect is consistent with the accelerated investment effect of Lyandres and Zhdanov (2010). They show that when shareholders do not have incentives to underinvest due to wealth transfers related to the presence of debt (the standard debtoverhang problem of Myers (1977)) they rather speed up investment.

5. Acquisitive Investment and Optimal Timing

In this section we present an alternative form of financing which is provided to the target firm by an external investor who can raise funds costlessly. We assume that the external
investor provides debt financing as the cost of issuing debt is lower than the cost of issuing equity.

We define the external investor as a firm whose growth strategy is to acquire production opportunities or assets of other firms. Most of acquisition deals are subsidiary mergers. The acquirer creates a shell subsidiary whose stock is then used to acquire the stock of the target company. In this setting the acquirer acts as a financial shell and can finance the target firm as a separate project.

The investment problem of the acquiring firm is a two-stage project. The bidder has an option to acquire the target firm, which is worth $OM(x)$. In the first stage the acquirer firm buys the target firm at an optimal merger threshold $x_M$ and pays the sunk cost of $K_M$ to cover restructuring costs. In addition, the target firm pays $K_T$, which is the sunk cost associated with the transaction processing. The second stage requires exercising the growth option at an optimal first-best investment threshold $x_I(t_d = 0)$ at a sunk cost $κ$, given the acquisition takes place before the investment exercise threshold of the financially constrained target firm $x_I(t_d > 0)$.

There are three regions where the acquirer can buy the target firm, which are specified as follows. First, when the merger threshold is lower than the optimal first-best investment threshold, $x_M < x_I$, the acquirer buys the target firm as a growth option and waits to exercise at this production opportunity at the optimal first-best trigger $x_I$.

Second, when the merger threshold is higher than the first-best investment trigger but lower than the the optimal threshold of the constrained firm $x_I < x_M < x_I$ the acquirer buys

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4Gaughan (2011)
the target firm as the growth option and immediately exercises investment.

Third, when the merger threshold is higher than the investment threshold of the constrained firm $x_I < x_M$ the acquirer buys the target firm as assets in place.

In each region the synergies created depend on the phase of development of the target firm. We decompose the synergies created in the subsequent subsection. Then, we determine the first-best merger threshold where we capture the dynamic aspects of the acquirer’s decision.

5.1. Decomposition of Synergies

An acquisition can change the value of the target firm by relaxing financial constraints. We identify the sources of synergies that depend on whether the acquirer buys the target firm as a growth option or assets in place. We abstract in this paper from synergies due to revenue increase or cost efficiencies. If they exist they will be supplementary to the synergies that we analyse. We therefore focus on synergies that can be created by facilitating the access to financing for the target firm.

We define the option to invest $OI(x, 0)$ as a production opportunity where the financing is raised costlessly as in McDonald and Siegel (1986) setting and $OI(x, \iota_d)$ when financing is costly. Total synergies that accrue to the acquirer are the difference between the value of the option to invest when financing is costless $OI(x, 0)$ and the value of the option to invest when financing is costly $OI(x, \iota_d)$. The acquirer compares the value of the target under these two extremes. The characteristics of synergies depend on whether the acquirer buys the target firm as a growth option or as assets in place.

To disentangle the effects of the alteration in the company ownership we separate total
synergies into three categories: the limited liability effect, the leverage effect and the asymmetric financing effect. For the first two effects we follow the notation of Leland (2007) to facilitate a direct comparison with his findings. Therefore, we identify the synergies that arise due to the limited liability effect (LL) and the leverage effect (LE). The limited liability effect is associated with the changes in the value of the option to abandon the firm if the cash flows fall below the critical level. The leverage effect is associated with the changes in the value of the tax shield (TS) and default costs (DC). The third effect we identify in this paper is related the asymmetric financing effect (AF). This effect arises when the target firm is in an early development stage and subject to financial constraints. The acquirer by providing access to cheaper financing ensures that the option to invest is exercised effectively. Total synergies can be decomposed as follows:

$$\Delta(x, \tau_d) = LL(x, \tau_d) + TS(x, \tau_d) + DC(x, \tau_d) + AF(x, \tau_d)$$

We show in this paper only incremental effects of synergies on the acquirer’s value. We analyse total synergies over three regions when it is optimal to acquire the target firm depending upon its development stage.

When $$t_M < t_I$$, the target firm is in a form of a growth option. The acquirer compares the value of the target firm under costless financing $$V_{I,D}(x, 0)$$ and costly financing $$V_{I,D}(x, \tau_d)$$. Gains over this region are associated with the undervalued growth option and the future tax shield. Synergies depend on the asymmetric financing effect (AF) and the leverage effect (LE). There is no contribution of the limited liability effect (LL). Synergies can be
decomposed into:

$$\Delta(\bar{x}_M < \bar{x}_I) = \left[ (1 - \tau) \pi x \frac{x}{r - \mu} - \kappa \right] \left( \frac{x}{\bar{x}_{I,D}(x, 0)} \right)^\beta - \left[ (1 - \tau) \pi x \frac{x}{r - \mu} - \kappa \right] \left( \frac{x}{\bar{x}_{I,D}(x, \iota_d)} \right)^\beta (AF)$$

$$+ \left[ \frac{\pi x (\tau)(\Omega(0))^{1/\lambda}}{r - \mu} \right] \left( \frac{x}{\bar{x}_{I,D}(x, 0)} \right)^\beta - \left[ \frac{\pi x (\tau - \iota_d)(\Omega(\iota_d))^{1/\lambda}}{r - \mu} \right] \left( \frac{x}{\bar{x}_{I,D}(x, \iota_d)} \right)^\beta (TS) \quad (18)$$

The first term is related to the asymmetric financing effect (AF). It shows the change in the value of the growth option due to facilitated access to financing provided by the acquirer. The asymmetric financing effect is positive as long as the discrepancy in the access to financing prevails. The second term (TS) shows the change in the future tax benefits if the firm issues debt to finance the growth option. These synergies are always positive if there is a wedge between the cost of financing for the target and the bidder.

When $\bar{x}_I < \bar{x}_M < \bar{x}_I$, the acquirer buys the target firm as a growth option and immediately exercises the investment. However, this occurs later than the first-best investment trigger. The acquirer compares the value of the target firm when it would have been optimal to exercise investment $V_{I,D}^+(x, 0)$ and the value of the target firm that still continues as a growth option due to capital market frictions $V_{I,D}(x, \iota_d)$. The synergies can be decomposed into:

$$\Delta(\bar{x}_I < \bar{x}_M < \bar{x}_I) = \frac{(1 - \tau) \pi x}{r - \mu} - \kappa - \left[ \frac{(1 - \tau) \pi x}{r - \mu} - \kappa \right] \left( \frac{x}{\bar{x}_{I,D}(x, \iota_d)} \right)^\beta (AF)$$

$$+ \frac{\tau c_{I,D}(0)}{r} - \left[ \frac{\pi x (\tau - \iota_d)(\Omega(\iota_d))^{1/\lambda}}{r - \mu} \right] \left( \frac{x}{\bar{x}_{I,D}(\iota_d)} \right)^\beta (TS)$$

$$+ \left[ \frac{(1 - \varphi) \phi x_{I,D}(0)}{r - \mu} - c_{I,D}(0) \right] \left( \frac{x}{x_{I,D}(0)} \right)^\lambda - 0(DC)$$

$$+ (1 - \tau) \left[ \frac{c_{I,D}}{r} - \frac{\pi x_{I,D}(0)}{r - \mu} \right] \left( \frac{x}{x_{I,D}(0)} \right)^\lambda - 0(LL) \quad (19)$$
The first term presents the synergies related to asymmetric financing effect (AF). The acquirer compares the value of the optimally exercised option with the value of the option when managed by the financially constrained firm. The value is positive as the probability of an option exercise by the financially constrained firm is lower than one over this region, 
\[(x/\bar{x}_{I,D}(x, t_d))^\beta < 1.\] Next we show the leverage effect in this region, the positive effect of the tax shield (TS) and the negative effect of default costs (DC). The last effect is the positive contribution of the value of limited liability (LL).

When \(\bar{x}_I < \bar{x}_M\), the acquirer buys the target firm as assets in place. The acquirer compares the value of the target firm after the investment exercise \(V_{I,D}^+(x, 0)\) and the value of the financially constrained target firm \(V_{I,D}^+(x, t_d)\). Ex-post the investment option exercise synergies are related to two sources: the leverage effect (tax shield and default costs) and the limited liability effect.

\[
\Delta(\bar{x}_I < \bar{x}_M) = \frac{(1 - \tau)p x}{r - \mu} - \frac{(1 - \tau)p x}{r - \mu} (AF)
\]
\[
+ \frac{\tau c_{I,D}(0)}{r} - \frac{\tau c_{I,D}(t_d)}{r} (TS)
\]
\[
+ \left[\left(1 - \phi\right)\frac{\pi_{I,D}(0)}{r - \mu} - c_{I,D}(0)\right] \left(\frac{x}{\bar{x}_{I,D}(0)}\right)^\lambda - \left[\left(1 - \phi\right)\frac{\pi_{I,D}(t_d)}{r - \mu} - c_{I,D}(t_d)\right] \left(\frac{x}{\bar{x}_{I,D}(t_d)}\right)^\lambda (DC)
\]
\[
+ (1 - \tau) \left[\frac{c_{I,D}(0)}{r} - \frac{\pi_{I,D}(x, 0)}{r - \mu}\right] \left(\frac{x}{\bar{x}_{I,D}(0)}\right)^\lambda - (1 - \tau) \left[\frac{c_{I,D}(t_d)}{r} - \frac{\pi_{I,D}(t_d)}{r - \mu}\right] \left(\frac{x}{\bar{x}_{I,D}(t_d)}\right)^\lambda (LL)
\]

The first term shows the change due to the asymmetric financing effect. When the assets in place are installed the effect of the efficient investment exercise disappears. Positive synergies, when the target firm is under the management of the acquirer, might only be created when assets generate synergistic cash flows. The second term shows a change in synergies due to
the tax shield effect (TS). We predict this effect to be positive as \( c_{I,D}(0) > c_{I,D}(t_d) \). The third term is associated with default costs (DC). The incremental effect on synergies is negative, which is consistent with Leland (2007). A higher coupon in the absence of operational synergies increases the probability of default and decreases the bankruptcy value. We do not make any assumption on the correlation between the assets of the acquirer and the target. However, when assets of two firms are imperfectly correlated then the portfolio combining these activities can actually reduce the risk of the merged firm. This argument was made by Lewellen (1971). The last term is the incremental effect on the combined value of the firm due to limited liability (LL) on the synergies.

5.2. Merger Surplus and Globally Optimal Timing

The acquirer facilitates the access to financing for the target firm. Therefore, the value of the option to invest for the acquirer is \( OI(x, 0) \), when financing is raised costlessly. The acquirer has to pay the price which is the stand-alone value of the target firm \( OI(x, t_d) \) when it has to raise costly financing. The benefit of combining these two firms together is the difference between the value of the target firm with different cost of issuing the financing. When the cost of financing for the target firm is higher than for the acquirer then the benefit of merging these firms together is positive. Both firms have to cover the costs of merging which are \( K_M \) and \( K_T \). The combined surplus from merging defined from a global optimizer perspective when two firms are pooled together is:

\[
S(x, t_d) = OI(x, 0) - OI(x, t_d) - K_M - K_T
\]  (21)

23
This payoff resembles a call option characteristics. The firm can exercise this option to merge at an optimal threshold \( x_M \) and receive the payoff \( S(x, \tau_d) \) or leave the option unexercised. We denote the value of this option as \( OM \). The merger decision from a global optimizer point of view is taken in isolation. However, the timing and terms of the acquirer’s decision depend on the investment and financing policy of the target firm. The goal is to facilitate the flow of financing and reduce the underinvestment problem.

Over the continuation region the value of the option to merge (\( OM \)) satisfies a second-order differential equation. We solve this equation subject to boundary conditions that are discussed in Appendix. We define the first-best merger threshold in Proposition 1.

**Proposition 1** The first-best merger threshold of a global optimizer is defined as follows. If \( \frac{K_M + K_T}{\kappa} < \frac{\Phi^\beta - \Psi^\beta}{\beta \Psi^\beta} \), the target firm is acquired as a growth option (i.e. \( x_M < x_I < x_I^c \)) and the merger threshold is:

\[
\bar{x}_M = H_\beta \kappa \left[ \frac{\kappa}{\beta} \frac{\beta (K_M + K_T)}{(\Phi^\beta - \Psi^\beta)} \right]^{\frac{1}{\beta}} \tag{22}
\]

If \( \frac{\Phi^\beta - \Psi^\beta}{\beta \Psi^\beta} \leq \frac{K_M + K_T}{\kappa} < \frac{\Phi - \Psi}{\Psi} \), the target firm is acquired as a growth option, which is immediately exercised (i.e. \( x_I^o < x_M < x_I^c \)). The merger threshold is a solution to the following equation:

\[
\bar{x}_M \Psi - \bar{x}_M^\beta \frac{\kappa}{\beta - 1} \left( \frac{\Psi}{\kappa H_\beta} \right)^\beta - H_\beta (K_M + K_T) = 0 \tag{23}
\]

If \( \frac{\Phi - \Psi}{\Psi} \leq \frac{K_M + K_T}{\kappa} \), the target firm is acquired as assets in place (i.e. \( x_I^o < x_I^c < x_M \)) and the merger threshold is:

\[
\bar{x}_M = H_\beta \frac{K_M + K_T}{\Phi - \Psi} \tag{24}
\]
where \( H_{\beta} = \beta/(\beta - 1) \) and \( \beta > 1 \) is the positive root of the quadratic equation \( 1/2\sigma^2 z(z - 1) + \mu z - r = 0 \).

**Proof.** See Appendix C. ■

The above shows that the form of assets of the target firm is important for the acquisition strategy of the bidder. Depending whether the acquirer buys the target firm as a growth option or as assets in place the optimal first-best merger is exercised at different triggers.

The merger threshold, conditional on acquiring the target firm as assets in place, has a standard form, well-known in the real options literature. It depends on the sunk transaction cost and synergies that can be realized upon exercising. The merger threshold is delayed if uncertainty increases due to hysteresis factor \( H_{\beta} \) that reflects the value of an option to wait.

Acquiring the target firm as a growth option, before the optimal investment exercise, is associated with high uncertainty over future synergies, which is reflected by additional factor \( \beta \). The higher the uncertainty the lower the synergies \( (\Phi_\beta - \Psi_\beta) \) are. The economic intuition behind these results is that synergies from acquiring the target firm as the growth option can be only materialized after investment exercise. Therefore, even after the merger exercise there is still uncertainty over the synergies. Their value is lower as the realization of synergies is postponed in time. Therefore, the acquirer discounts for higher risk associated with acquiring growth options.

It is therefore optimal to acquire firms in an early development stage for a lower level of volatility. An increase in volatility limits the region over which the acquirer decides to buy the target firm as a growth option, as \( (\Phi_\beta - \Psi_\beta)/\beta \Phi_\beta \) decreases if \( \sigma \) increases. All else equal,
acquiring growth options becomes unlikely.

6. Fear of Preemption and Acquisition Terms and Timing

In this section we analyse the terms and timing of friendly mergers and hostile takeovers in a competitive environment. The recent empirical literature suggests that there is strong competition among bidders. Boone and Mulherin (2007a) and Boone and Mulherin (2007b) claim that half of takeover deals in their sample are subject to competition from other public or private bidders. Moreover, friendly mergers in a form of negotiation are not free from competition. Aktas, de Bodt, and Roll (2010) provide evidence of latent competition (the likelihood that rival bidders could appear), which increases the bid premium. The fear of preemption in the case of a merger agreement can be explained by a fiduciary out clause. It gives the target the right to terminate the merger agreement if a better deal arises before the target board gives the full approval.

Therefore, based on this compelling evidence we introduce competition in our model in a form of the fear of preemption. We assume that the bidder that enters negotiation process or makes a tender offer is subject to preemption risk \( \delta \) due to an arrival of a competing bidder, where \( \delta > 0 \) is the hazard rate related to the Poisson process. The probability of preemption per unit of time is \( \delta dt \). This is reflected in an ordinary differential equation by an additional term which changes the value of the firm.

The form of an offer has a fundamentally different effect on the timing and how the synergies are shared between firms. Lambrecht (2004) shows that the negotiation game affects the terms and timing of the acquisition when mergers are motivated by the economies
of scale. We follow his setting and analyse friendly mergers and hostile takeovers.

Each firm during the merger negotiation process has an option on the fraction of merger synergies. The acquirer obtains a share \( s_B \) of the new entity and the target firm obtains \( s_T \) of the new entity. We assume that at the time of the acquisition the whole merger surplus is shared between two firms, thus \( s_B + s_T = 1 \). We define the payoff accruing to the acquirer and the target as follows. The bidder’s surplus is equal to a share \( s_B \) in a new entity, which can now raise the funds costlessly, less the fixed acquisition cost \( K_M \):

\[
S_B(x, \tau_d, s_B) = s_B \text{OI}(x, 0) - K_M
\]

(25)

The target exchanges its firm value when the financing is costly into a share in the new entity when the financing is costless, less the fixed acquisition cost \( K_T \) and its surplus is:

\[
S_T(x, \tau_d, s_T) = s_T \text{OI}(x, 0) - \text{OI}(x, \tau_d) - K_T
\]

(26)

These payoffs have a call option characteristics and ensure the existence of an optimal exercise threshold for each firm, the bidder and the target, where synergies are maximized.

In the next subsections we analyse how the terms and timing of the merger depend on the form of the acquisition. The merger agreement refers to a negotiated deal while the hostile takeover in a form of a tender offer usually means that one firm is making an offer directly to shareholders to sell their shares at specified prices.\(^5\)

6.1. Friendly Merger

In this subsection we analyse a friendly merger in a form of an agreement where firms negotiate over the division of the surplus. The payoff that both firms obtain is uncertain and contingent on the future realization of merger synergies. Firms have an interest to maximize merger synergies and to exercise their options at an optimal threshold. The bidder and the target firm negotiate over the terms and timing of the merger simultaneously.

At the optimal merger threshold each firm compares the value of the merger option, $OM_B(x)$ for the bidder and $OM_T(x)$ for the target, with the payoff realized at the optimal exercise threshold. One can show that the value of the merger option for the bidder is:

$$OM_B(x, \pi_M(s_B), t_d) = (s_B OI(\pi_M(s_B), 0) - K_M) \left( \frac{x}{\pi_M(s_B)} \right)^\xi$$  \(27\)

where $\pi_M(s_B)$ is the reaction function of the bidder dependent on the share $s_B$. For the target the value of the option is:

$$OM_T(x, \pi_M(s_T), t_d) = (s_T OI(\pi_M(s_T), 0) - OI(\pi_M(s_T), t_d) - K_T) \left( \frac{x}{\pi_M(s_T)} \right)^\beta$$  \(28\)

where $\pi_M(s_T)$ is the reaction function of the target dependent on the share $s_T$. Above equations show the discounted value of synergies accruing to each company. In the presence of competition the discount factor of the bidder $(x/\pi_M)^\xi$ is higher than the discount factor of the target $(x/\pi_M)^\beta$. The bidder having preemption fears becomes impatient and discounts its payoff at a higher hurdle rate.

Subsequently, from Eq. \(27\) and \(28\) we calculate the reaction function of the bidder $\pi_M(s_B)$ and the target $\pi_M(s_T)$. A friendly merger is executed when $\pi_M(s_B) = \pi_M(s_T)$.
and the ownership is divided according to the unique sharing rule \((s_B, s_T)\). The optimal merger threshold is derived for the regions that were discussed in Section 5. The results are summarized in the following proposition.

**Proposition 2** The exercise trigger of a merger agreement modelled as Cournot negotiation game, where the target and the acquirer firm decide on the terms and timing simultaneously, is defined as follows. If \(\frac{(H_\xi/H_{\beta})K_M + K_T}{\kappa} < \frac{\Phi - \Psi}{\Phi - \Psi_{\kappa}}\), the target firm is acquired as a growth option (i.e. \(\overline{x}_M < \overline{x}_T < \overline{x}_I\)) and the merger threshold is:

\[
\overline{x}_M = H_{\beta \kappa} \left[ \beta \left( \frac{H_\xi K_M + K_T}{\kappa (\Phi - \Psi_{\beta})} \right) \right]^{\frac{1}{\beta}}
\]  

(29)

If \(\frac{\Phi - \Psi}{\Phi - \Psi_{\kappa}} \leq \frac{(H_\xi/H_{\beta})K_M + K_T}{\kappa} < \frac{\Phi - \Psi}{\Phi - \Psi}\), the target firm is acquired as a growth option, which is immediately exercised (i.e. \(\overline{x}_T < \overline{x}_M < \overline{x}_I\)). The merger threshold is a solution to the following equation:

\[
\overline{x}_M \Phi - \overline{x}_M^\beta \frac{\kappa}{\beta - 1} \left( \frac{\Psi}{\kappa H_{\beta}} \right)^\beta - H_{\beta} \left( \frac{H_\xi}{H_{\beta}} K_M + K_T \right) = 0
\]

(30)

If \(\frac{\Phi - \Psi}{\Phi - \Psi_{\kappa}} \leq \frac{(H_\xi/H_{\beta})K_M + K_T}{\kappa}\), the target firm is acquired as assets in place (i.e. \(\overline{x}_I < \overline{x}_M < \overline{x}_I\)) and the merger threshold is:

\[
\overline{x}_M = H_{\beta} \left( \frac{H_\xi}{H_{\beta}} K_M + K_T \right)
\]

(31)

where \(H_\xi = \xi/(\xi - 1)\) and \(\xi > 1\) is the positive root of the quadratic equation \(1/2\sigma^2 z(z - 1) + \mu z - r - \delta = 0\).

**Proof.** See Appendix D. ■

The fear of preemption speeds up the exercise trigger of the acquirer. This is reflected by the additional factor \(H_\xi/H_{\beta}\), that decreases the cost of the acquirer \(K_M\). It suggests that
the acquirer accepts a lower or even negative NPV projects to preempt competition.

If there is no fear of preemption and \( \delta = 0 \), expressions from Proposition 2 coincide with the first-best merger threshold defined in Proposition 1. Then, it is optimal to merge at the first-best threshold \( \pi_M(\delta = 0) \) and the target firm obtains a share in the new entity equal to \( s_T(\delta = 0) \). When the fear of preemption increases, the bidder is willing to offer a higher share in the new entity to the target firm: \( s_T(\delta > 0) > s_T(\delta = 0) \). Firms merge at a different threshold \( \pi_M(\delta > 0) \), which is lower than the first-best merger threshold. The fear of preemption changes the terms and timing of the acquisition. These are summarized in the following proposition.

**Proposition 3** *When the bidder is subject to the fear of preemption, an acquisition that is an outcome of a merger agreement occurs earlier than the first-best merger threshold and the target firm obtains a higher ownership share.*

### 6.2. Hostile Takeover

In contrast to friendly mergers where both firms decide on the terms and timing simultaneously, during the hostile takeover process the target dictates the terms and the acquirer subsequently decides on the timing. The hostile takeover can be modelled as Stackelberg game, where the target firm determines the ownership share \( (s_T) \) and the acquirer decides subsequently on the optimal merger exercise \( (\pi_M) \).

Both firms have a takeover option, where by \( OT_B(x) \) and \( OT_T(x) \) we denote the option
of the bidder and the target. One can show that the value of the option for the bidder is:

\[ OT_B(x, \pi_M(s_B)) = (s_B OI(\pi_M(s_B), 0) - K_M) \left( \frac{x}{\pi_M(s_B)} \right)^\xi \] (32)

and for the target:

\[ OT_T(x, s_T(\pi_M), s_T)) = (s_T(\pi_M) OI(\pi_M, 0) - OI(\pi_M, \tau_d) - K_T) \left( \frac{x}{\pi_M} \right)^\beta \] (33)

The fear of preemption, the bidder is subject to, is reflected in the discounting factor \((x/\pi_M(s_B))^\xi\). We solve the game by backward induction. First, the acquirer chooses the timing of the acquisition, \(\pi_M\), given the ownership share \(s_B\), by maximizing its option:

\[ \max_{\pi} OT_B(x, \pi_M, s_B) \] (34)

Second, the target firm maximizes its option:

\[ \max_{s_T} OT_T(x, \pi_M(s_T), s_T) \] (35)

The optimal takeover threshold is derived for the regions discussed in Section 5. The results are summarized in the following proposition.

**Proposition 4** The exercise trigger of a hostile takeover modelled as Stackelberg game, where the target firm decides on the terms and the acquirer then decides on the timing, is defined as follows. If \(\frac{H \epsilon K_M + K_T}{\kappa} < \frac{\Phi - \Psi}{\beta \Phi}\), the target firm is acquired as a growth option (i.e. \(\pi_M < \pi_T^g < \pi_T\)) and the merger threshold is:

\[ \pi_M = H \beta \kappa \left[ \frac{\beta (H \epsilon K_M + K_T)}{\kappa (\Phi - \Psi)} \right]^{\frac{1}{\beta}} \] (36)
If $\frac{\Phi - \Psi}{\Phi - \Psi} \leq \frac{\xi K_M + K_T}{\kappa} < \frac{\Phi - \Psi}{\Phi - \Psi}$, the target firm is acquired as a growth option, which is immediately exercised (i.e. $x_I^0 < x_M < x_I^1$). The merger threshold is a solution to the following equation:

$$x_M^\Phi - x_M^\beta \frac{\kappa}{\beta - 1} \left[ \frac{\Psi}{H_\beta \kappa} \right]^\beta - H_\beta (\xi K_M + K_T) = 0$$  \hspace{1cm} (37)

If $\frac{\Phi - \Psi}{\Phi - \Psi} \leq \frac{\xi K_M + K_T}{\kappa}$, the target firm is acquired as assets in place (i.e. $x_I^0 < x_I^1 < x_M$) and the merger threshold is:

$$x_M = H_\beta \frac{\xi K_M + K_T}{\Phi - \Psi}$$  \hspace{1cm} (38)

In contrast to friendly mergers, hostile takeovers are associated with a delay due to increased costs. This is reflected by a premium, the acquirer has to pay, in a form of the additional hysteresis factor ($H_\xi$) that increases the acquisition cost of the acquirer ($K_M$).

In the absence of the fear of preemption, when $\delta = 0$ implying that $H_\xi = H_\beta$, hostile takeovers are exercised later than the first-best threshold and friendly mergers. This result is consistent with Lambrecht (2004) where precommitment delays the timing of the takeover.

We contribute by showing that when the bidder is subject to the fear of preemption, if $\delta > 0$ implying that $H_\xi < H_\beta$, the hostile takeover threshold is exercised earlier. If $\delta \to \infty$ implying that $H_\xi \to 1$ the takeover is exercised at the optimal global optimizer merger threshold ($\delta = 0$) defined in Proposition 1. A subsequent proposition follows.

**Proposition 5** When the bidder is subject to the fear of preemption mergers are exercised too early, while takeovers converge to the global optimizer first-best merger threshold when the fear of preemption increases.

**Proof.** See Appendix E. □
It is very intuitive to predict that the competition erodes the option value and speeds up the merger threshold. For example, Morellec and Zhdanov (2005) in their model for takeovers show that the competition speeds up the timing and increases the ownership share of the target firm. Our analysis suggests that it is important to distinguish how the competition affects a particular type of an acquisition offer. We show that when there is no competition a friendly merger offer is an optimal choice that gives the first-best outcome. However, in the presence the infinite fear of preemption hostile takeover threshold goes to the first-best merger threshold.

We show a novel result that the effect of competition depends on the negotiation process between the acquirer and the target firm. In the case of a merger agreement competition erodes the option value and forces a merger’s timing to converge the break-even threshold. However, in the case of a hostile takeover the timing of an acquisition converges to the first-best merger threshold (the global optimizer merger threshold). The intuition is that if the target firm acts as Stackelberg leader it preserves the option value and obtains an efficient ownership share.

7. Model Predictions

To illustrate our results and understand better the dynamics of the model we present a number of numerical examples. Parameters’ values for each figure are based on the estimates from the corporate finance literature. Volatility is assumed to be at the level of \( \sigma = 20\% \), which is roughly consistent with the estimates for volatility of Schaefer and Strebulaev (2008). We set the growth rate of cash flow to \( \mu = 5\% \), the risk-free rate to \( r = 6\% \), and the corporate
tax rate is $\tau = 25\%$. The default cost is based on the recent estimates of Glover (2012) and $\varphi = 45\%$. It implies that around 65\% of the revenue stream is recovered in the case of the alternative use of firm’s assets. The size of the growth option is $\pi = 10$ and the costs of investment and acquisition exercise are as follows: $\kappa = 140$, $K_M = 10$, and $K_T = 5$. The arrival rate of competitors is set to be $\delta = 1$.

7.1. Risk Analysis

In this subsection we present the implications of the parameters that define risk in our model, uncertainty over future cash flows ($\sigma$) and preemption risk ($\delta$). In Figure 1 we depict when it is optimal to acquire the target firm as a growth option or assets in place.

We present the solution in a form of a region plot, where $\iota_d$ is a critical level of switching between regimes. We define three regions: (i) where it is optimal for the acquirer to buy the target firm as assets in place (white region), (ii) as a growth option and immediately exercise (light shaded region) or (iii) as a growth option and wait to exercise at the optimal investment threshold (dark shaded region).

Panels A and D define the conditions for the global optimizer merger threshold defined in Proposition 1. Panels B and E define the conditions for the friendly merger defined in Proposition 2. and Panels C and F define the conditions for the hostile takeover defined in Proposition 3.

In Panel A, we present the global optimizer solution. The first-best solution of the global optimizer is not affected by the preemption fear. The critical level of $\iota_d$ is monotonically increasing in $\sigma$. This means that the acquirer is more likely to buy the target firm as
Figure 1: Region Plot. In the dark shaded area the bidder acquires the target firm as the growth option and waits to exercise the investment option at the optimal investment threshold. In the light shaded area the bidder acquires the target firm as the growth option and immediately exercises the investment option. In the white area the bidder acquires the target firm as assets in place. We vary cash flow risk ($\sigma$), preemption risk ($\delta$) and the issuance cost of financing ($\iota_d$). Panels A and D define the conditions for the global optimizer merger threshold defined in Proposition 1. Panels B and E define the conditions for the friendly merger defined in Proposition 2. Panels C and F define the conditions for the hostile takeover defined in Proposition 4.

A growth option for lower levels of volatility. When the acquirer buys the target firm as a growth option the decision is not only affected by uncertainty surrounding the acquisition but also by subsequent uncertainty associated with the investment option exercise and synergies that will be materialized at that time. Financial synergies in the case of buying the target firm as a growth option are associated with greater uncertainty. This is reflected by $\beta$ in the expression defining synergies from acquisition $\Phi^\beta - \Psi^\beta$. The latter decreases as $\sigma$ increases.
Therefore, for higher levels of uncertainty the acquirer is more likely to buy assets in place already generating profit.

In Panel B, in the case of a solution for the friendly merger offer subject to the fear of preemption the effect of $\sigma$ is non-monotonic. To understand this, first we consider the optimal policy of the acquirer and the target. When uncertainty increases it delays merger and investment exercise as the hysteresis factor $H_\beta$ increases. However, in the case of a friendly merger threshold, defined in Proposition 2, the cost of the acquisition is associated with an additional factor which is a multiple of $K_M$. It depends on hysteresis factors $H_\xi / H_\beta$ and decreases in $\sigma$. In the presence of significant competition, the fear of preemption can erode the value of waiting. The acquirer is then forced to buy the target firm at the break-even threshold.

In Panel C, in the case of a solution for the hostile takeover offer the effect of $\sigma$ on the critical level of $\iota_d$ is monotonically increasing. It is delayed in comparison to the friendly merger offer. This delay depends on additional hysteresis factor $H_\xi$ imposed on the merger cost $K_M$, that increases in $\sigma$. This premium might be associated with a higher entry cost for hostile bidders that have to face more uncertainty than in the case of a negotiated bid.

The first-best solution of the global optimizer is not affected by preemption and in Panel D the relationship is constant. We however show it for comparison with the friendly merger offer and the hostile takeover offer.

In Panel E, we show that the acquirer is more likely to buy the target firm as a growth option when the fear of preemption increases. This effect is associated with the erosion of the option value due to competition.
In Panel F, we illustrate the effect of the fear of preemption on the hostile takeover offer. The competition accelerates the optimal exercise. The takeover threshold converges to the first-best threshold of the global optimizer.

7.2. Capital Structure

Next, we analyse the parameters that are of the main importance for the capital structure choice of the target firm in our model, the tax rate ($\tau$) and the bankruptcy cost ($\varphi$). In Figure 2 we depict region plots of the decision to acquire the target firm as a growth option or assets in place.

In Panels A to C the critical level $\iota_d$ decreases in $\tau$. This means that the acquirer is more likely to buy the target firm as a growth option when corporate taxes are higher. In the model taxes affect financial synergies. The corporate tax increases the importance of the future tax shield in the case of acquiring growth options. Therefore, higher taxes increase financial synergies and speed up the exercise decision of the acquirer. When taxes are high the reduction in after-tax profit delays the decision to invest, which makes the underinvestment problem of the constrained target firm even more severe. In countries with high tax rates acquiring firms as growth options might be the solution to relaxing capital constraints these firms are facing.

In Panels D to F the critical level $\iota_d$ increases in the bankruptcy cost $\varphi$. Acquiring the target firm as a growth option is more likely when the target firm has tangible assets, that in bankruptcy can be sold or converted into an alternative use. The high bankruptcy cost delays the investment threshold of the constrained target firm. The bankruptcy cost also has
Figure 2: Region Plot. In the dark shaded area the bidder acquires the target firm as the growth option and waits to exercise the investment option at the optimal investment threshold. In the light shaded area the bidder acquires the target firm as the growth option and immediately exercises the investment option. In the white area the bidder acquires the target firm as assets in place. We vary the tax rate ($\tau$), the bankruptcy cost ($\phi$) and the issuance cost of financing ($\iota_d$). Panels A and D define the conditions for the global optimizer merger threshold defined in Proposition 1. Panels B and E define the conditions for the friendly merger defined in Proposition 2. Panels C and F define the conditions for the hostile takeover defined in Proposition 4. The parameters are set as in the previous figure.

an effect on the acquisition threshold. It increases the probability of default of the target firm which decreases financial synergies and delays the exercise trigger of the acquirer.

7.3. Profitability of Target Firm

In this subsection we show how the parameters related to the profitability of the target firm affect the choice between acquiring the target firm as a growth option or assets in place. In
Figure 3: We depict the effect of the growth rate of cash flows ($\mu$) and the size of the growth option ($\pi$) for the decision to acquire the target firm.

**Figure 3:** Region Plot. In the dark shaded area the bidder acquires the target firm as the growth option and waits to exercise the investment option at the optimal investment threshold. In the light shaded area the bidder acquires the target firm as the growth option and immediately exercises the investment option. In the white area the bidder acquires the target firm as assets in place. We vary the growth rate of cash flows ($\mu$), the size of the growth option ($\pi$), and the issuance cost of financing ($\iota_d$).

Panels A and D define the conditions for the global optimizer merger threshold defined in Proposition 1. Panels B and E define the conditions for the friendly merger defined in Proposition 2. Panels C and F define the conditions for the hostile takeover defined in Proposition 4. The parameters are set as in the previous figure.

In Panels A to C the critical level $\iota_d$ decreases in the growth rate of cash flows ($\mu$). The higher $\mu$ increases the value of future cash flows. It accelerates the investment decision of the constrained firm. However, for the acquiring firm $\mu$ increases the financial synergies of acquiring a growth option and erodes the effect of uncertainty surrounding the option exercise.
In $\Phi^\beta - \Psi^\beta$ ($\beta$ decreases as $\mu$ increases increasing financial synergies).

In the case of the first-best and hostile takeover solution (Panels A and C) the increase in financial synergies speeds up the acquisition more when the acquirer buys growth options and waits to exercise the investment. These two effects are of the same magnitude when the acquirer buys a growth option with an immediate exercise trigger.

In the case of the friendly merger $\mu$ has an additional effect on the cost $K_M$. When $\mu$ increases it affects the factor $H_\xi / H_\beta$ and it erodes the effect of uncertainty associated with the acquisition. This make acquiring growth options more likely.

In Panels D to F the critical level $\iota_d$ increases in the size of the growth option ($\pi$). The size of the growth option speeds up the investment exercise given the sunk cost $\kappa$ is fixed. The underinvestment problem of the target firm is less severe. This in turn lowers financial synergies of acquiring growth options. It suggests that smaller firms are more likely to be acquired as growth options and larger firms are more likely to be acquired as assets in place.

8. **Premiums in Friendly Mergers and Hostile Takeovers**

In the existing literature there is no agreement on how the takeover premiums are affected by the competition and the type of the offer. For example, Boone and Mulherin (2007b) suggest that wealth effects for target shareholders do not differ depending on the form of an acquisition. Mandatory disclosure rules that increase expected competition among bidders possibly raise offer premiums. Eckbo (2008) shows that there are 'no conclusions as to whether offer premiums are higher, the same, or lower in tender offers than in merger bids'. We therefore present some insights of the effect of the fear of preemption and the type of the
offer on the target’s premiums from acquisition.

Many empirical studies approximate takeover premium by using a measure based on cumulative takeover return. We define the cumulative return as a change in the stand alone value due to the merger option. Therefore, we express the takeover premium as a cumulative return, that results from a merger option, as a fraction of the stand alone value:

\[ TP = \frac{OM(x, \pi_M, t_d)}{OI(x, t_d)} \] (39)

The stand alone value of the target firm \( OI(x, t_d) \) depends on the phase of development the target firm is in. When it is acquired as a growth option then \( OI(x, t_d) = V_{I,D}(x, t_d) \) and when as assets in place then \( OI(x, t_d) = V_{I,D}^+(x, t_d) \).

In Figure 4 we present the sensitivity of an acquisition premium to volatility \( \sigma \). We compare the premium to the target firm from the merger agreement and hostile takeover when the acquirer makes an offer in isolation \( (\delta = 0) \) or when it is subject to the fear of preemption \( (\delta > 0) \).

In contrast to previous studies, that predict a positive effect of volatility on the cumulative return (i.e., the premium for the target firm decreases with uncertainty \( \sigma \)) when the merger is exercised in isolation (e.g. Lambrecht (2004)) or in the presence of competition (e.g. Hackbarth and Miao (2012)), the uncertainty in this model decreases cumulative return. In previous studies the synergies of combining two firms together were studied in a setting where they were independent of uncertainty. Then the main driver of positive effect of volatility on the cumulative return was the hysteresis factor. In this paper we show that if synergies

\(^6\)Eckbo (2008)
depend on uncertainty, the increase in volatility can lower the cumulative return. This effect is mainly driven by the fact that synergies depend on capital structure. Optimal leverage decreases with uncertainty and therefore the synergies related to financial structure of the firm decrease.
The level of premium for the target firm depends on its development phase. The premium for the target is higher when it is acquired as a growth option (dotted line). The acquirer only decides to buy the target firm as the growth option when the level of uncertainty is relatively low.

We show the importance of the dynamics embedded into the acquisition decision of the acquirer. The premium to the target firm is higher in case of the merger agreement offer if the target firm is acquired earlier and in a form of a growth option. If the target firm is acquired as assets in place then the premium form the takeover offer is higher. The fear of preemption increases premiums of merger agreement. In the case of the takeover offer the premium increases due to the fact that now the bidder due to preemption fears acquirers target firm as a growth option.

9. Conclusion

This paper presents a dynamic theoretical model of mergers actuated by financial synergies. The target firm with an investment option is subject to the underinvestment problem, due to a constrained access to financing. Hence, financial frictions delay the option exercise. The acquirer can buy the target firm and provide an access to cheap financing.

We show that synergies created by providing cheap financing to the target firm can be decomposed into: the limited liability effect, the leverage effect and the asymmetric financing effect. The magnitude and importance of these synergies depend on whether the target firm is acquired as a growth option or as physical assets. When the target firm is acquired in an early development stage, synergies that the acquirer can realize, are related to asymmetric
financing effect and the leverage effect. However, once the target firm has commercialized its growth options and has assets in place generating profits, synergies are related to the leverage effect and the limited liability effect.

The optimal choice of the acquirer depends on the magnitude of financial synergies, the fear of preemption and the type of the merger offer. The phase of the development of the target firm affects the optimal merger threshold, ownership division and a takeover premium. The acquirer buys the target firm as a growth option for low levels of volatility.

When the bidder faces latent competition, in a form of a fear of preemption, it is more likely to acquire the target firm in an early development stage when the growth option still remains unexercised. Furthermore, our framework predicts that the fear of preemption speeds up the timing of the merger agreement, while the timing of the hostile takeover converges to the global optimizer merger threshold. Takeover premiums to the target firm are higher when the firm is acquired in the early development stage and when the bidder is subject to the fear of preemption.

The main empirical predictions of the model are as follows. First, our theory implies that if there are multiple financial bidders interested in a particular target firm the more likely the target will be acquired as a growth option (or production opportunity). Therefore, firms that have more R&D in their balance sheets are acquired earlier than firms that have physical assets. Second, the higher the issuance cost of financing for the target firm, the more likely mergers motivated by financial synergies are. We predict that there should be more merger activity of financial bidders during economic booms when there is a high spread in the cost of issuing financing between the acquirer and the target. Furthermore, we show that,
depending on the type of the offer, the timing of the acquisition is different. When the bidder
does not have any preemption fears, friendly mergers are exercised at the first-best threshold.
Our analysis reveals additional insights on the role competition plays in the merger process.
When the bidder has preemption fears then friendly mergers are exercised too early. When
the fear of preemption is high hostile takeovers converge to the first-best merger threshold.
Appendix A. Proof of Lemma 1 (Value of Target Firm)

To work out the optimal investment threshold of the innovating firm we have to solve the problem backwards. First, we solve for the post-investment values when the innovating target firm can rely on capital markets financing. Second, we derive the ex ante-investment values and then closed form solutions for the optimal investment threshold.

When the innovative firm financed the cost of investment with risky debt, its assets in place generate the after tax cash-flow \((1 - \tau)\pi\) less the fixed coupon \(c_{I,D}\) paid to bondholders.

Assuming that \(r\) is a risk free rate and agents are risk-neutral, the firm’s equity \(E_{I,D}^+\) and debt \(B_{I,D}^+\) must satisfy:

\[
\begin{align*}
    rE_{I,D}^+ &= (1 - \tau)\pi - c_{I,D} + \frac{d}{d\Delta} \mathbb{E}[E_{I,D,t+\Delta}^+]|_{\Delta=0} \\
    rB_{I,D}^+ &= c_{I,D} + \frac{d}{d\Delta} \mathbb{E}[B_{I,D,t+\Delta}^+]|_{\Delta=0}
\end{align*}
\]

Assuming \(E_{I,D}^+\) and \(B_{I,D}^+\) are twice-continuously differentiable functions of the state variable \(x_t\), then by applying Ito’s lemma we obtain:

\[
\begin{align*}
    rE_{I,D}^+(x) &= (1 - \tau)\pi - c_{I,D} + \frac{\partial E_{I,D}^+(x)}{\partial x} x\mu + \frac{\partial^2 E_{I,D}^+(x)}{\partial^2 x} x^2 \sigma^2 \\
    rB_{I,D}^+(x) &= c_{I,D} + \frac{\partial B_{I,D}^+(x)}{\partial x} x\mu + \frac{\partial^2 B_{I,D}^+(x)}{\partial^2 x} x^2 \sigma^2
\end{align*}
\]

The ordinary differential equations have solutions as follows:

\[
E_{I,D}^+(x) = (1 - \tau)\pi - c_{I,D} + A_1 x^3 + A_2 x^\lambda
\]
\[ B_{I,D}^+(x) = c_{I,D} + A_3 x^\beta + A_4 x^\lambda \]  

(A.6.)

where \( \beta > 1 \) and \( \lambda < 0 \) are the roots of the equation: \( 1/2 \sigma^2 z(z - 1) + \mu z - r = 0 \).

The constants \( A_1 \) and \( A_2 \) are determined by the value matching conditions for equityholders. First condition stipulates that at the default threshold \( \xi_{I,D} \) the equityholders are left with nothing and their claims are equal to: \( E_{I,D}^+(\xi_{I,D}) = 0 \). Second, a no-bubble condition states that when the state variable goes to infinity the equityholders claims approach the unlimited liability value: \( \lim_{x_t \to \infty} E_{I,D}^+(x_t) = (1 - \tau)\pi/(r - \mu) - c_{I,D}/r \).

The constants \( A_3 \) and \( A_4 \) are determined by the value matching conditions for bondholders. First, at the investment threshold \( \xi_{I,D} \) the bondholders are left with the liquidation value, \( B_{I,D}^+(\xi_{I,D}) = (1 - \varphi)\phi \xi_{I,D}/(r - \mu) \). Second, when the state variable goes to infinity the bondholders claims approach the unlimited liability value: \( \lim_{x_t \to \infty} B_{I,D}^+(x_t) = c_{I,D}/r \).

These necessary conditions yield the solutions for equity and debt when \( x \geq \xi_{I,D} \):

\[
E_{I,D}^+(x, \iota_d) = (1 - \tau) \left[ \frac{\pi x}{r - \mu} - \frac{c_{I,D}}{r} \right] - \left[ \frac{(1 - \tau)\pi \xi_{I,D}}{r - \mu} - \frac{(1 - \tau)c_{I,D}}{r} \right] \left( \frac{x}{\xi_{I,D}} \right)^\lambda \quad \text{(A.7.)}
\]

\[
B_{I,D}^+(x, \iota_d) = \frac{c_{I,D}}{r} - \left( \frac{c_{I,D}}{r} - \frac{(1 - \varphi)\phi \xi_{I,D}}{r - \mu} \right) \left( \frac{x}{\xi_{I,D}} \right)^\lambda \quad \text{(A.8.)}
\]

The firm value is the sum of equity and debt given in the equation \( 3 \) in Lemma \( 1 \).

Equityholders choose the default threshold that maximizes their claims value:

\[
\frac{\partial E_{I,D}^+(x)}{\partial x} \bigg|_{x = \xi_{I,D}} = 0 \quad \text{(A.9.)}
\]

\(^7\)Equityholders can cover operating loses by injecting more capital.
which gives the solution for the closure threshold in Lemma 1. Next, we substitute for $x_{I,D}$ in $V_{I,D}^+(x)$. We determine the closed-form solution for the optimal coupon $c_{I,D}(x)$ by maximizing the firm value (the root of the first order condition):

$$\frac{\partial[V_{I,D}^+(x,0) - \iota_d B_{I,D}^+(x,0)]}{\partial c_{I,D}(x)} = 0 \quad (A.10.)$$

Inserting values for $c_{I,D}$ and $x_{I,D}$ the post-investment equity and debt are:

$$E_{I,D}^+(x, \iota_d) = \pi x (1 - \tau) \frac{(1 - \lambda + \lambda \Omega^{-1/\lambda} - \Omega^{-1}) \Omega^{1/\lambda}}{\lambda (r - \mu)} \quad (A.11.)$$

$$B_{I,D}^+(x, \iota_d) = \frac{x \Omega^{1/\lambda} (\pi (1 - \iota_d)(1 - \lambda) + (1 - (1 - \iota_d)\lambda - \tau)\phi(-1 + \varphi))}{(r - \mu)(\tau - \iota_d)} \quad (A.12.)$$

where $1/\lambda [1 - \lambda + \lambda \Omega^{-1/\lambda} - \Omega^{-1}] > 0$ is always satisfied. The post-investment firm value net of issuance cost is:

$$V_{I,D}^+(x, \iota_d) = V_{I,D}^+(x,0) - \iota_d B_{I,D}^+(x, \iota_d) = x \Psi \quad (A.13.)$$

where $\Psi = \left[\pi x (1 - \tau + (\tau - \iota_d)(\Omega)^{1/\lambda})\right] / (r - \mu)$. Lemma 1 follows.
Appendix B. Proof of Lemma 2 (Investment Threshold)

In the continuation region, before the investment exercise, the innovative firm is a growth option and does not generate any cash flows. The equityholders only obtain capital gains. The firm value has to satisfy the following Bellman equation:

\[ rV_{I,j} = \frac{d}{d\Delta} E[V_{I,j,t+\Delta}] \bigg|_{\Delta=0} \text{ for } j = E, D. \] (B.1.)

Using Ito’s lemma one can show that the firm value before investment satisfies:

\[ rV_{I,j}(x_t) = \frac{\partial V_{I,D}(x)}{\partial x} x \mu + \frac{\partial^2 V_{I,D}(x)}{\partial x^2} x^2 \sigma^2 \] (B.2.)

The ordinary differential equation has the solution as follows:

\[ V_{I,j}(x_t) = A_5 x^\beta + A_6 x^\lambda \] (B.3.)

where \( \beta > 1 \) and \( \lambda < 0 \) are the roots of the equation: \( 1/2\sigma^2 z(z - 1) + \mu z - r = 0 \). The constants are derived as the solutions to no-bubble condition, limiting \( A_6 = 0 \), and the value matching condition that at the time of investment the value of equity has to be equal the payoff from investment. When the capital outlay is financed with equity the following expression has to be satisfied:

\[ V_{I,E}(x_{I,E}) = V_{I,E}^+(x_{I,E}) - \kappa \] (B.4.)

and when the investment cost is financed by raising debt:

\[ V_{I,D}(x_{I,D}) = V_{I,D}^+(x_{I,D}, \tau d) - \tau d B_{I,D}^+(x_{I,D}, \tau d) - \kappa = \Psi x_{I,D} - \kappa \] (B.5.)
Then, the smooth-pasting conditions to ensure that the investment occurs along the optimal path for investment financed with equity and debt respectively are:

\[ \frac{\partial V_{l,E}(x)}{\partial x} \bigg|_{x = \tau_{l,E}} = \frac{\partial (1 - \tau_e) V_{l,E}^+(x)}{\partial x} \bigg|_{x = \tau_{l,E}} \]

(B.6.)

\[ \frac{\partial V_{l,D}(x)}{\partial x} \bigg|_{x = \tau_{l,D}} = \frac{\partial (1 - \tau_d) V_{l,E}^+(x, \tau_d)}{\partial x} \bigg|_{x = \tau_{l,D}} = \Psi \]

(B.7.)

Lemma 2 follows.
Appendix C. Proof of Proposition 1 (Globally Efficient Merger)

Over the continuation interval investors having an option to merge obtain capital gains and the following condition has to be satisfied:

\[ rOM = \frac{d}{d\Delta} \mathbb{E}[OMt + \Delta] \bigg|_{\Delta=0} \]  

(A.1.)

Assuming \( OM \) is a twice-continuously differentiable function of the state variable \( x_t \), then by applying Ito’s lemma we obtain a second-order ODE equation:

\[ rOM(x) = \mu X \frac{\partial OM(x)}{\partial X} + \frac{1}{2} \sigma^2 X^2 \frac{\partial^2 OM(x)}{\partial^2 X} \]  

(A.2.)

The ordinary differential equation has a solution as follows:

\[ OM(x) = A_7 x^\beta + A_8 x^\lambda \]  

(A.3.)

This equation can be solved subject to boundary conditions. The value matching condition stipulates that at the optimal merger threshold the value of the option equals to the merger payoff: \( OM(\overline{x}_M, t_d) = S(\overline{x}_M, t_d) \). A no-bubble condition implies that: \( \lim_{x_t \to 0} = 0 \). The resulting value of the merger option conditional on the merger trigger satisfies the following equation:

\[ OM(x, \overline{x}_M, t_d) = (OI(\overline{x}_M, 0) - OI(\overline{x}_M, t_d) - K_M - K_T) \left( \frac{x}{\overline{x}_M} \right)^\beta \]  

(A.4.)

The smooth-pasting condition ensures that the merger occurs along optimal path, and \( \partial OM(\overline{x}_M, t_d)/\partial \overline{x}_M = 0 \). The optimal threshold \( \overline{x}_M \) satisfies the following condition:

\[ -\beta (OI(\overline{x}_M, 0) - OI(\overline{x}_M, t_d) - K_M - K_T) + \overline{x}_M \frac{\partial (OI(\overline{x}_M, 0) - OI(\overline{x}_M, t_d))}{\partial \overline{x}_M} = 0 \]  

(A.5.)

We solve the above equation for the following cases when:
1) \( \bar{x}_M < \bar{x}_I^j < \bar{x}_I^c \)

When the acquirer buys the target firm as a growth option and waits to exercise the investment at the optimal trigger, the merger threshold is the solution to the following F.O.C.:

\[
\frac{\beta^\kappa}{\beta - 1} \left[ \frac{1}{\mathcal{H}_{\beta K}} \right]^\beta (\Phi^\beta - \Psi^\beta) - \mathcal{H}_\beta K_M - \mathcal{H}_\beta K_T = 0 \tag{C.6.}
\]

where \( \Phi = \Psi(\iota_d = 0) \). The optimal threshold when the acquirer buys the target firm as a growth option is:

\[
\bar{x}_M = \mathcal{H}_{\beta K} \left[ \frac{\beta(K_M + K_T)}{\kappa(\Phi^\beta - \Psi^\beta)} \right]^{\frac{1}{\beta}} \tag{C.7.}
\]

And is satisfied only when:

\[
\bar{x}_M < \bar{x}_I^j \iff \frac{K_M + K_T}{\kappa} < \frac{\Phi^\beta - \Psi^\beta}{\beta \Phi^\beta} \tag{C.8.}
\]

2) \( \bar{x}_I^j \leq \bar{x}_M < \bar{x}_I^c \)

When the acquirer buys the target firm as a growth option and exercises the investment option immediately, the merger threshold is the solution to the following F.O.C.:

\[
\bar{x}_M \Phi - \kappa - \bar{x}_M^\beta \frac{\kappa}{\beta - 1} \left[ \frac{\Psi}{\mathcal{H}_{\beta K}} \right]^{\beta} - \mathcal{H}_\beta K_M - \mathcal{H}_\beta K_T = 0 \tag{C.9.}
\]

The above equation requires a numerical solution for \( \bar{x}_M \). And is satisfied only when:

\[
\bar{x}_I^j \leq \bar{x}_M < \bar{x}_I^j \iff \frac{\Phi^\beta - \Psi^\beta}{\beta \Phi^\beta} \leq \frac{K_M + K_T}{\kappa} < \frac{\Phi - \Psi}{\Psi} \tag{C.10.}
\]

3) \( \bar{x}_I^j < \bar{x}_I^j \leq \bar{x}_M \)

When the acquirer buys the target firm as assets in place, the merger threshold is the solution to the following F.O.C.:

\[
\bar{x}_M(\Phi - \Psi) - \mathcal{H}_\beta K_M - \mathcal{H}_\beta K_T = 0 \tag{C.11.}
\]
The optimal threshold when the acquirer buys the target firm as assets in place is:

\[ \bar{x}_M = \frac{H_\beta K_M + H_\beta K_T}{\Phi - \Psi} \]  
(C.12.)

And is satisfied only when:

\[ \bar{x}_I \leq \bar{x}_M \iff \frac{\Phi - \Psi}{\Psi} \leq \frac{K_M + K_T}{\kappa} \]  
(C.13.)

To prove that \( \bar{x}_M \) is defined over all possible regions and no discontinuities are present one can check that: when \( \bar{x}_I = \bar{x}_M \) then \( \frac{K_M + K_T}{\kappa} = \frac{\Phi - \Psi}{\beta \phi} \), and when \( \bar{x}_I = \bar{x}_M \) then \( \frac{\Phi - \Psi}{\Psi} = \frac{K_M + K_T}{\kappa} \).

These lead to Proposition 11.
Appendix D. Proof of Proposition 2 and 3 (Friendly Merger)

The probability of one jump in the Poisson process is:

\[ Pr(C = 1) = \delta t e^{-\delta t} \]  

(D.1.)

Therefore, the expected time before competitor’s arrival is:

\[ E[T_C] = \frac{1}{\delta} \]  

(D.2.)

The option to merge with the target can be only exercised before the competitor arrives at time \( T_C \). Therefore, if \( t < T_C \) bidder can still exercise the option to merge. Otherwise, if \( t > T_C \) the competing bidder takes over the target firm. Bidder maximizes the value of the option to merge, at time \( T_M \), which is now:

\[ OM_B(x) = \max_{T_M} E[I_{T_M < T_C} e^{-rT_M}(s_B OI(\pi_M, 0) - K_M)] \]  

(D.3.)

\( I_a \) is the indicator function of an event \( a \). \( I_a = 1 \) if \( t < T_C \) and \( I_a = 0 \) otherwise. The Bellman equation over the continuation region is:

\[ r OM_B(x) dt = E[\Delta OM_B(x)] \]  

(D.4.)

Using Ito’s lemma one can show that the value of the option to acquire before exercise and project expiry has to satisfy (where the jump is of a fixed size 1):

\[ r OM_B(x) = \mu X \frac{\partial OM_B(x)}{\partial X} + \frac{1}{2} \sigma^2 X^2 \frac{\partial^2 OM_B(x)}{\partial^2 X} + \delta (OM^J_B(x) - OM_B(x)) \]  

(D.5.)

As \( OM^J_B(x) = 0 \) equation can be written as:

\[ r OM_B(x) = \mu X \frac{\partial OM_B(x)}{\partial X} + \frac{1}{2} \sigma^2 X^2 \frac{\partial^2 OM_B(x)}{\partial^2 X} - \delta OM_B(x) \]  

(D.6.)
The left hand side of the equation represents the required rate of return of investing in the option to acquire. The right hand side shows the capital gains that investors get per unit of time and the last term reflects on the effect of preemption. The ordinary differential equation has the solution as follows:

$$OM_B(x_t) = A_9x_t^\xi + A_{10}x_t^\nu$$  \hspace{1cm} (D.7.)

where $\xi > 1$ and $\nu < 0$ are the solutions of the equation: \(1/2\sigma^2z(z-1) + \mu z - r - \delta = 0\).

This equation can be solved subject to boundary conditions. The value matching condition stipulates that at the optimal merger threshold the value of the option equals to the merger payoff,

$$OM_B(\bar{x}_M) = s_BOI(\bar{x}_M, 0) - K_M$$  \hspace{1cm} (D.8.)

and no-bubble condition to eliminate constant $A_{10}$:

$$\lim_{x \to 0} OM_B(x) = 0$$  \hspace{1cm} (D.9.)

The value of the merger option conditional on the merger trigger satisfies the following equation for the bidder:

$$OM_B(x, \bar{x}_M(s_B), t_d) = (s_BOI(\bar{x}_M(s_B), 0) - K_M) \left( \frac{x}{\bar{x}_M(s_B)} \right)^\xi$$  \hspace{1cm} (D.10.)

where $\bar{x}_M(s_B)$ is the reaction function of the bidder dependent on the share $s_B$. Following similar arguments, where now $\delta = 0$, the merger option for the target firm satisfies the following ODE:

$$rOM_T(x) = \mu X \frac{\partial OM_T(x)}{\partial X} + \frac{1}{2} \sigma^2 X^2 \frac{\partial^2 OM_T(x)}{\partial^2 X}$$  \hspace{1cm} (D.11.)
The left hand side of the equation represents the required rate of return of investing in the option to acquire. The right hand side shows the capital gains that investors get per unit of time. The option value for the target firm is:

$$OM_T(x, \bar{\pi}_M(s_T), \tau_d) = (s_T OI(\bar{\pi}_M(s_T), 0) - OI(\bar{\pi}_M(s_T), \tau_d) - K_T) \left( \frac{x}{\bar{\pi}_M(s_T)} \right)^\beta$$

(D.12.)

where $\bar{\pi}_M(s_T)$ is the reaction function of the bidder dependent on the share $s_T$. To determine the optimal merger threshold we calculate reaction functions for the bidder and the target dependent on the share in the new entity. The F.O.C. for the bidder is as follows:

$$-\xi(s_B OI(\bar{\pi}_M(s_B), 0) - K_M) + \bar{\pi}_M(s_B) \frac{\partial (s_B OI(\bar{\pi}_M(s_B), 0)}{\partial \bar{\pi}_M(s_B)} = 0$$

(D.13.)

and for the target:

$$-\beta (s_T OI(\bar{\pi}_M(s_T), 0) - OI(\bar{\pi}_M(s_T), \tau_d) - K_T) + \bar{\pi}_M(s_T) \frac{\partial (s_T OI(\bar{\pi}_M(s_T), 0) - OI(\bar{\pi}_M(s_T), \tau_d))}{\partial \bar{\pi}_M(s_T)} = 0$$

(D.14.)

The optimal threshold $\bar{\pi}_M$ is the solution to the following condition: $\bar{\pi}_M(s_B) = \bar{\pi}_M(s_T)$. We solve the above optimization problem for the following cases discussed in Section 5.

1) $\bar{\pi}_M < \bar{\pi}_T^I < \bar{\pi}_T^J$

When the bidder buys the target firm as a growth option and waits to exercise the investment at the optimal trigger, the merger threshold is the solution to the following F.O.C.s, for the bidder:

$$\bar{\pi}_M^{\beta} \frac{\kappa}{\beta - 1} \left[ \frac{1}{\mathcal{H}_\beta K_M} \right]^\beta s_B \Phi^\beta - \mathcal{H}_\xi K_M = 0$$

(D.15.)

and for the target:

$$\bar{\pi}_M^{\beta} \frac{\kappa}{\beta - 1} \left[ \frac{1}{\mathcal{H}_\beta K_M} \right]^\beta (s_T \Phi^\beta - \Psi^\beta) - \mathcal{H}_\beta K_T = 0$$

(D.16.)
The reaction function of the bidder is:

$$\varpi_M(s_B) = \frac{H_\beta \kappa}{\Phi} \left[ \frac{(\beta - 1)H_\xi K_M}{\kappa s_B} \right]^{\frac{1}{\beta}}$$  \hspace{1cm} (D.17.)

and the reaction function of the target:

$$\varpi_M(s_T) = \frac{H_\beta \kappa}{\Phi} \left[ \frac{(\beta - 1)H_\beta K_T}{\kappa (s_T - (\Psi/\Phi)^\beta)} \right]^{\frac{1}{\beta}}$$  \hspace{1cm} (D.18.)

The optimal ownership is calculated as the solution to: \(\varpi_M(s_B) = \varpi_M(s_T)\) and \(s_T = 1 - s_B\).

$$s_B = \frac{H_\xi K_M (\Phi^\beta - \Psi^\beta)}{\Phi^\beta (H_\xi K_M + H_\beta K_T)}$$  \hspace{1cm} (D.19.)

After substituting we obtain the merger threshold when the bidder is subject to competition as:

$$\varpi_M = H_\beta \kappa \left[ \frac{\beta (K_M + K_T)}{\kappa (\Phi^\beta - \Psi^\beta)} \right]^{\frac{1}{\beta}}$$  \hspace{1cm} (D.20.)

And is satisfied only when:

$$\varpi_M < \varpi_f^i \iff \frac{H_\xi K_M + K_T}{\kappa} < \frac{\Phi^\beta - \Psi^\beta}{\beta \Phi^\beta}$$  \hspace{1cm} (D.21.)

$$2) \varpi_f^i < \varpi_M < \varpi_f^f$$

When the acquirer buys the target firm as a growth option and exercises the investment option immediately, the merger threshold is the solution to the following F.O.C.:

$$s_B \varpi_M \Phi - H_\xi K_M = 0$$  \hspace{1cm} (D.22.)

The reaction function when the bidder buys the target firm as a growth option and immediately exercises is:

$$\varpi_M(s_B) = \frac{H_\xi K_M}{s_B \Phi}$$  \hspace{1cm} (D.23.)
The F.O.C. for the target firm is:

\[ s_T \bar{x}_M \Phi - \bar{x}_M^\beta \frac{\kappa}{\beta - 1} \left[ \frac{\Psi}{\mathcal{H}_\beta \kappa} \right]^\beta - \mathcal{H}_\beta K_T = 0 \]  \hspace{1cm} (D.24.)

The reaction function for the target \( \bar{x}_M(s_T) \) requires a numerical solution. Then the ownership share can be derived as the solution to: \( \bar{x}_M(s_B) = \bar{x}_M(s_T) \). And is satisfied only when:

\[ \bar{x}_I^0 \leq \bar{x}_M < \bar{x}_I^j \iff \frac{\Phi^\beta - \Psi^\beta}{\beta \Phi^\beta} \leq \frac{\mathcal{H}_\xi K_M + K_T}{\kappa} < \frac{\Phi - \Psi}{\Psi} \]  \hspace{1cm} (D.25.)

3) \( \bar{x}_I^j < \bar{x}_I^j < \bar{x}_M \)

When the acquirer buys the target firm as assets in place, the merger threshold is the solution to the following F.O.C.:

\[ s_B \bar{x}_A \Phi - \mathcal{H}_\xi K_M = 0 \]  \hspace{1cm} (D.26.)

The F.O.C. for the target firm is:

\[ s_B \bar{x}_M \Phi - \bar{x}_M \Psi - \mathcal{H}_\beta K_T = 0 \]  \hspace{1cm} (D.27.)

The reaction function when the bidder buys assets in place is:

\[ \bar{x}_M(s_B) = \mathcal{H}_\xi \frac{K_M}{s_B \Phi} \]  \hspace{1cm} (D.28.)

and for the target:

\[ \bar{x}_M(s_T) = \mathcal{H}_\beta \frac{K_T}{s_T \Phi - \Psi} \]  \hspace{1cm} (D.29.)

The share is calculated as the solution of \( \bar{x}_M(s_T) = \bar{x}_M(s_B) \):

\[ s_B = \frac{\mathcal{H}_\xi K_M (\Phi - \Psi)}{(\mathcal{H}_\xi K_M + \mathcal{H}_\beta K_T) \Phi} \]  \hspace{1cm} (D.30.)
The after substituting we obtain the merger threshold when the bidder is subject to competition:

$$\bar{x}_M(\delta > 0) = \frac{\mathcal{H}_\xi K_M + \mathcal{H}_\beta K_T}{\Phi - \Psi} < \bar{x}_M(\delta = 0)$$ \hspace{1cm} (D.31.)

And is satisfied only when:

$$\bar{x}_I < \bar{x}_M(\delta > 0) \iff \frac{\Phi - \Psi}{\Psi} \leq \frac{\mathcal{H}_\xi K_M + K_T}{\kappa}$$ \hspace{1cm} (D.32.)

These lead to Proposition 2 and 3.
Appendix E. Proof of Proposition 4 (Hostile Takeover)

Following similar arguments as in previous appendix for Proposition 2, we can show that a takeover option, denoted as $OT(x)$, satisfies the following equation, which is for the bidder:

$$OT_B(x, \bar{\tau}_M(s_B)) = (s_BOI(\bar{\tau}_M(s_B), 0) - K_M) \left( \frac{x}{\bar{\tau}_M(s_B)} \right)^\xi$$ \hspace{1cm} (E.1.)

and for the target:

$$OT_T(x, s_T(\bar{\tau}_M), s_T) = (s_T(\bar{\tau}_M)OI(\bar{\tau}_M, 0) - OI(\bar{\tau}_M, \tau_d) - K_T) \left( \frac{x}{\bar{\tau}_M} \right)^\beta$$ \hspace{1cm} (E.2.)

In contrast to merger agreement, the takeover is solved as a Stackelberg game, where the target decides on the ownership share ($s_T$) and then the bidder decides on the timing of the takeover conditional on the share that the target firm obtains. The smooth-pasting condition ensures that the takeover occurs along optimal path, and $\partial OT_B(\bar{\tau}_M)/\partial \bar{\tau}_M = 0$. The bidder’s takeover reaction function $\bar{\tau}_M(s_B)$ satisfies the following condition:

$$- \xi (s_BOI(\bar{\tau}_M, 0) - K_M) + \bar{\tau}_M \frac{\partial s_BOI(\bar{\tau}_M, 0)}{\partial \bar{\tau}_M} = 0$$ \hspace{1cm} (E.3.)

The target firm then decides on the share. Its optimization problem can be formulated as $\max_{s_T} OT_T(x, s_T(\bar{\tau}_M), s_T)$ or when we substitute for $s_T(\bar{\tau}_M)$ the value derived from Eq. E.3, it can be written as $\max_{\bar{\tau}_M} OT_T(x, \bar{\tau}_M, s_T(\bar{\tau}_M))$. Solving the latter optimization problem the takeover threshold is the solution to the following equation:

$$- \beta (s_TOI(\bar{\tau}_M, 0) - OI(\bar{\tau}_M, \tau_d) - K_T) + \bar{\tau}_M \frac{\partial (s_TOI(\bar{\tau}_M, 0) - OI(\bar{\tau}_M, \tau_d))}{\partial \bar{\tau}_M} = 0$$ \hspace{1cm} (E.4.)

We solve the above optimization problem for the following cases:
1) $\bar{x}_M < \bar{x}_I^j < \bar{x}_c^j$

When the acquirer buys the target firm as a growth option and waits to exercise the investment at the optimal trigger, the takeover threshold is the solution to the following F.O.C.:

$$s_B \bar{x}_M^\beta \frac{\kappa}{\beta - 1} \left[ \frac{1}{\phi_{\beta \kappa}} \right]^\beta \Phi^\beta - \phi_{\xi \phi} K_M = 0$$  \hspace{1cm} (E.5.)

The reaction function when the bidder buys the target firm as a growth option is:

$$\bar{x}_M(s_B) = \frac{\phi_{\xi \phi} K_M}{\phi} \left[ \frac{(\beta - 1)\phi_{\xi \phi} K_M}{\kappa s_B} \right]^\frac{1}{\beta}$$  \hspace{1cm} (E.6.)

or rewriting:

$$s_B(\bar{x}_M) = \frac{\phi_{\xi \phi} K_M}{\phi_{\beta \kappa}} \left[ \phi_{\beta \kappa} \right]^\beta - \phi_{\beta \kappa}$$  \hspace{1cm} (E.7.)

Now substituting for $s_B$ into the F.O.C. for the target firm:

$$\bar{x}_M^\beta \frac{\kappa}{\beta - 1} \left[ \frac{1}{\phi_{\beta \kappa}} \right]^\beta (\Phi^\beta - \Psi^\beta) - \phi_{\beta \phi} \phi_{\xi \phi} K_M - \phi_{\beta \phi} K_T = 0$$  \hspace{1cm} (E.8.)

The optimal takeover threshold when the acquirer buys the target firms as a growth option is:

$$\bar{x}_M = \phi_{\beta \kappa} \left[ \frac{(\beta - 1)(\phi_{\beta \phi} \phi_{\xi \phi} K_M + \phi_{\beta \phi} K_T)}{\kappa(\Phi^\beta - \Psi^\beta)} \right]^\frac{1}{\beta}$$  \hspace{1cm} (E.9.)

The optimal share $s_B$ is:

$$s_B = \frac{\phi_{\xi \phi} K_M (\Phi^\beta - \Psi^\beta)}{\phi^\beta (\phi_{\beta \phi} \phi_{\xi \phi} K_M + \phi_{\beta \phi} K_T)}$$  \hspace{1cm} (E.10.)

And the above is satisfied only when:

$$\bar{x}_M < \bar{x}_I^j \iff \frac{\phi_{\xi \phi} K_M + K_T}{\kappa} < \frac{\phi^\beta - \Psi^\beta}{\beta \phi^\beta}$$  \hspace{1cm} (E.11.)

2) $\bar{x}_I^j < \bar{x}_M < \bar{x}_c^j$

When the acquirer buys the target firm as a growth option and exercises the investment
option immediately, the takeover threshold is the solution to the following F.O.C.:

\[ \pi_M s_B \Phi - \mathcal{H}_\xi K_M = 0 \quad (E.12.) \]

The reaction function when the bidder buys the target firm as a growth option followed by immediate exercise is:

\[ \pi_M(s_B) = \frac{\mathcal{H}_\xi K_M}{s_B \Phi} \quad (E.13.) \]

or rewriting:

\[ s_B(\pi_M) = \frac{\mathcal{H}_\xi K_M}{\Phi \pi_M} \quad (E.14.) \]

Now substituting for \( s_B \) into the F.O.C. for the target firm:

\[ \pi_M \Phi - \pi_M^\beta \frac{\kappa}{\beta - 1} \left[ \frac{\Psi}{\mathcal{H}_\beta \kappa} \right]^\beta - \mathcal{H}_\beta \mathcal{H}_\xi K_M - \mathcal{H}_\beta K_T = 0 \quad (E.15.) \]

The above equation requires a numerical solution for the optimal merger threshold \( \pi_M \). And is satisfied only when:

\[ \pi_l^0 \leq \pi_M < \pi_l^1 \iff \frac{\Phi^\beta - \Psi^\beta}{\beta \Phi^\beta} \leq \frac{\mathcal{H}_\xi K_M + K_T}{\kappa} < \frac{\Phi - \Psi}{\Psi} \quad (E.16.) \]

3) \( \pi_l^1 < \pi_M \)

When the acquirer buys the target firm as assets in place, the takeover threshold is the solution to the following F.O.C.:

\[ - \xi(s_B \pi_M \Phi - K_M) + s_B \pi_M \Phi = 0 \quad (E.17.) \]

The reaction function when bidder buys assets in place is:

\[ \pi_M(s_B) = \frac{\mathcal{H}_\xi K_M}{s_B \Phi} \quad (E.18.) \]

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or rewriting:

\[ s_B(\bar{x}_M) = \frac{\mathcal{H}_\xi K_M}{\Phi \bar{x}_M} \]  \hspace{1cm} (E.19.)

Substituting into F.O.C. for the target firm gives:

\[ \bar{x}_M(\Phi - \Psi) - \mathcal{H}_\beta \mathcal{H}_\xi K_M - \mathcal{H}_\beta K_T = 0 \]  \hspace{1cm} (E.20.)

The optimal takeover threshold is:

\[ \bar{x}_M = \frac{\mathcal{H}_\xi \mathcal{H}_\beta K_M + \mathcal{H}_\beta K_T}{\Phi - \Psi} \]  \hspace{1cm} (E.21.)

The ownership is shared according to the following rule \( s_T = 1 - s_B \), where:

\[ s_B = \frac{\mathcal{H}_\xi K_M (\Phi - \Psi)}{\Phi \mathcal{H}_\beta(\mathcal{H}_\beta \mathcal{H}_\xi K_M + \mathcal{H}_\beta K_T)} \]  \hspace{1cm} (E.22.)

Above is satisfied only when:

\[ \bar{x}_f < \bar{x}_M \iff \frac{\Phi - \Psi}{\Psi} \leq \frac{\mathcal{H}_\xi K_M + K_T}{K} \]  \hspace{1cm} (E.23.)

These lead to Proposition 4.
References


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