Dynamic Wrong-Way Risk in CVA Pricing

Yeying Gu∗


Abstract

Wrong-way risk is a fundamental component of derivative valuation that was largely neglected prior to the 2008 financial crisis because it was considered immaterial. One important lesson learned from the crisis is that correlation between counterparty default risk and risk factors need not be static, but can rise sharply during periods of stress. In this paper, we propose a new method for capturing wrong-way risk based on a relatively recent innovation - the dynamic factor copula. Our method extends the traditional copula-based approach for modelling wrong-way risk by incorporating state-dependent correlation dynamics. In addition, our approach is flexible enough to accommodate high dimensions and computationally efficient (we bypass matrix decompositions). These attractive features enable the modelling of a bank’s entire derivative portfolio credit value adjustment (CVA) at once. Through numerical examples, this study points to the importance of a better approach for modelling wrong-way risk as both CVA and CVA delta are sensitive to its estimates and it has implications on derivative pricing and regulatory capital requirement.

1 Introduction

The risk of default by a derivative trading counterparty has always been known to market participants. However, it was not until the 2008 financial crisis (GFC) when counterparty credit risk, in particular, credit valuation adjustment (CVA) volatility risk, ascended to the centre of attention for both the regulators and market participants.

During the GFC, the Basel Committee on Banking Supervision (BCBS) observed that approximately two-thirds of the trading losses incurred by financial institutions came not from actual defaults but from writing down the fair value on their derivative positions as counterparties became less likely than expected to meet their obligations[1]. Recognising the significance of CVA variability risk, the Basel III accord introduces a new CVA risk framework stipulating mandatory capital charges for CVA variability risk[1].

∗PhD Student, Discipline of Finance, Codrington Building (H69), The University of Sydney, NSW 2006, Australia (E-mail: yeying.gu@gmail.com)

[1]The current Basel III requires capital charges for CVA variability arising from credit spread volatility. At the time of writing this paper, the BCBS is in the process of introducing a new CVA framework which also incorporates CVA variability contributed by volatility of market risk factors.
The definition of CVA implies that the variability risk is driven mainly by three factors -
(i) variability in the credit risk of the counterparty, (ii) variability in the market risk factors
underlying the transaction, and (iii) the correlation between the (i) and (ii). Any correlation
which gives rise to increased CVA is referred to as wrong-way risk while any favourable
 correlation generates right-way risk.\footnote{Since the expected loss due to a counterparty default is also a function of how much can be recovered,
wrong-way risk may also be present if a counterparty’s recovery rate falls as its default probability jumps.
We eliminate the unnecessary degree of freedom in the model by assuming a constant recovery rate.}

Prior to the crisis, wrong-way risk was largely neglected. A costly lesson learnt from the
global financial meltdown is that correlations between risk factors are not static - they can
rise sharply during periods of crisis. In fact, there is empirical evidence supporting the
presence of time-varying correlation. For instance,\footnote{3} finds that corporate defaults tend
to cluster during periods of falling interest rates, most likely caused by a recession leading
to central bank intervention and high default rates. Sovereign credit crisis is usually ac-
companied by a strong weakening in the domestic currency. These observations call for the
modelling of the dynamic and state-dependent nature of the correlation matrix.

Existent literature modeling the wrong-way risk can be classified into two major branches,
namely the copula approach and the parametric approach. The copula-based approach (also
known as exposure sampling approach) proposed by\[2\] maps pre-computed exposure and
counterparty default time onto chosen distributions (for example, Gaussian) and correlate
the two using a Gaussian (or other) copula structure. The focus is on computing the ex-
pected exposure conditional on default time. In contrast, the parametric approach by\[5\]
tackles the problem by letting the hazard rate depend on the evolution of exposures. Both
approaches are parsimonious and easy to use stress-testing purposes. The correlation co-
efficients, however, are very difficult to interpret and estimate, as exposures are portfolio
specific.

In this paper, we propose a new method for capturing wrong-way risk. Our formulation is a
natural extention of the copula-based approach by incorporating a dynamic correlation with
state-dependence. Inspired by the authors of\[6\] who introduced a new class of copula-based
dynamic models for high dimension conditional distributions, our model is equally flexi-
ble enough to accommodate high dimensions and possesses the attractive feature of being
computationally efficient. This allows systematic modelling of the bank’s entire derivative
portfolio at once.

The remainder of the paper is set out as follows. Section 2 briefly reviews the definition
and pricing formula for CVA, and Section 3 provide details on model formulation. A few
numerical examples are given in Sections 4. Finally, Section 5 concludes the paper.

2 Credit Valuation Adjustment

It is assumed throughout the paper that all processes are well defined on a filtered probabil-
ity space \((\Omega, F, (F_t)_{0 \leq t \leq T}, P)\) satisfying the usual conditions where\footnote{2} denotes the risk-neutral
probability measure.

Credit valuation adjustment (CVA) is the expected loss resulting from the potential future default of the counterparty. Let $T$ be the expiry of the derivative contract and $\tau$ the stopping time corresponding to the random counterparty default time. The CVA at time 0 is given by

$$CVA = \mathbb{E}_\mathcal{F}[B_\tau^{-1} \gamma_\tau V_\tau \mathbf{1}_{\tau \leq T} | \mathcal{F}_0]$$

where $B_\tau$ is the bank account numeraire, $\gamma_\tau$ is the counterparty’s loss given default, and $V_\tau$ is the market value of the derivative at time of counterparty default. The indicator function $\mathbf{1}_{\tau \leq T}$ takes the value unity on the set $\{\tau \leq T\}$ and zero otherwise.

3 Model Formulation

3.1 Dynamic Factor Copulas

Consider an observable random vector $Y_t = (Y_{1,t}, Y_{2,t}, \ldots, Y_{n,t})^\top$ satisfying the n-dimensional data generating process given by

$$Y_t = \mu_t + \Sigma_t \eta_t$$

where $\mu_t = (\mu_{1,t}, \mu_{2,t}, \ldots, \mu_{n,t})^\top$ \hspace{1cm} (2)

$$\Sigma_t = \text{diag}(\sigma_{1,t}, \sigma_{2,t}, \ldots, \sigma_{n,t})$$ \hspace{1cm} (3)

$$\eta_t = (\eta_{1,t}, \eta_{2,t}, \ldots, \eta_{n,t})^\top$$ \hspace{1cm} (4)

and $\eta_{i,t} | \mathcal{F}_{t-1} \sim F_{i,t}$ for $i = 1, 2, \ldots, n$ \hspace{1cm} (5)

where $\mu_t$ and $\Sigma_t$ are conditional means and conditional standard deviations, respectively. Conditional on $\mathcal{F}_{t-1}$, each $\eta_{i,t}$ follows its own conditional distribution $F_{i,t}$ with zero mean and unit variance. Applying the probability integral transform, we have

$$u_{i,t} = F_{i,t}\left(\frac{Y_{i,t} - \mu_{i,t}}{\sigma_{i,t}}\right)$$ \hspace{1cm} (6)

By Sklar’s Theorem, the conditional copula of $Y_t$ is therefore equal to the conditional joint distribution $U_t$, that is,

$$U_t \sim C_t(u_{1,t}, u_{2,t}, \ldots, u_{n,t})$$ \hspace{1cm} (7)

Consider also a vector of (latent) random variables $X_t = (X_{1,t}, X_{2,t}, \ldots, X_{n,t})^\top$ whose dependence structure is governed by a set of common factors $Z_t = (Z_{1,t}, Z_{2,t}, \ldots, Z_{k,t})^\top$ and a set of idiosyncratic random noises $\xi_t = (\xi_{1,t}, \xi_{2,t}, \ldots, \xi_{n,t})^\top$. A Gaussian one-factor version
is given by the following structure,

\[ X_t = \Lambda_t Z_t + C_t \Xi_t \]  

(8)

where \( \Lambda_t = (\lambda_{1,t}, \lambda_{2,t}, \ldots, \lambda_{n,t})^T \)  

(9)

\[ Z_t = Z_t, \quad Z_t \sim F_{Z,t} = \mathcal{N}(0, 1) \]  

(10)

\[ C_t = \text{diag}(\sqrt{1 - \lambda_{1,t}^2}, \sqrt{1 - \lambda_{2,t}^2}, \ldots, \sqrt{1 - \lambda_{n,t}^2}) \]  

(11)

\[ \Xi_t = (\xi_{1,t}, \xi_{2,t}, \ldots, \xi_{n,t})^T, \quad \xi_{i,t} \sim \text{iid } F_{\xi,t} = \text{iid } \mathcal{N}(0, 1), \text{ for } i = 1, 2, \ldots, n \]  

(12)

Consequently, the vector \( X_t \) follows a multivariate Gaussian distribution, and we denote the conditional distribution function of \( X_t \) as \( G_t \). Since the conditional distribution of \( X_t \) is also the conditional copula function \( C_t \) of marginal uniforms at time \( t \), we have

\[ X_t \sim G_t = C_t (u_{1,t}, u_{2,t}, \ldots, u_{n,t}) \]  

(14)

It can be shown that the Pearson’s correlation between \( X_{i,t} \) and \( X_{j,t} \) is given by

\[ \rho_{ij,t} = \lambda_{i,t} \lambda_{j,t} \]  

(15)

### 3.2 Generalised Autoregressive Score (GAS) Dynamics

Generalised autoregressive score (GAS) dynamics is an observation-driven model where time variation of the parameters is introduced by letting parameters be functions of lagged dependent variables as well as contemporaneous and lagged exogenous variables. Although the parameters are stochastic, they are perfectly predictable given the past information. The GAS model updates the parameters over time based on the score function of the predictive model density at time \( t \).

We assume that a vector of latent factors \( \Gamma_t = (\gamma_{1,t}, \gamma_{2,t}, \ldots, \gamma_{n,t})^T \) evolve over time according to a GAS(1,1) model given by

\[ \Gamma_{t+1} = \Omega + A \nabla_t + B \Gamma_t \]  

(16)

where \( \Omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \)  

(17)

\[ A = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_n), \quad B = \text{diag}(\beta_1, \beta_2, \ldots, \beta_n) \]  

(18)

\[ \nabla_t = \left( \frac{\partial \log c_t(u; \Lambda_t(\Gamma_t))}{\partial \Gamma_t} \right)^T \]  

(19)

where \( c_t(u; \Lambda_t(\Gamma_t)) \) is the copula density.  

(20)

The link between the factor loadings \( \Lambda_t \) and latent factors \( \Gamma_t \) is established using the \( \tanh(x) \) function such that \( \Lambda_t \) is bounded between \((-1, 1)\),

\[ \Lambda_t = \tanh(\frac{1}{2} \Gamma_t) \]  

(21)
4 Numerical Examples - Interest Rate Swaps

4.1 CVA for Interest Rate Swaps

In this section, we provide a simulation study to illustrate the impact of dynamic correlation on CVA. In particular, we compute the CVA for a ten-year fixed-for-floating interest rate swap.

We assume the short rate and counterparty default intensity follow the SDEs given by

\[
\begin{align*}
    dr(t) &= (\theta(t) - r(t)) dt + \sigma_r(t) dW_r(t) \\
    d\lambda(t) &= \alpha (\beta - \lambda(t)) dt + \sigma_\lambda \sqrt{\lambda(t)} dW_\lambda(t), \quad 2\alpha\beta > \sigma_\lambda^2
\end{align*}
\]

where the correlation is given by \(d\langle W_r, W_\lambda \rangle_t = \rho dt\).

The uniform marginals are estimated via GJR-GARCH model and the GAS parameters are estimated using maximum likelihood. Hull-White and CIR parameters are calibrated to relevant market data.

4.1.1 CVA with Constant Correlation Coefficient

Figures 1 and 2 plot the dollar amount CVA as a function of constant correlation for a unit notional ten-year interest rate swap. The sensitivity of CVA to correlation (i.e., the impact of wrong-way risk) is more significant for in-the-money swaps. This is expected since more paths enter the CVA calculation for an in-the-money swap compared to an otherwise identical at-the-money or out-of-the-money swap.
Figure 1: Unilateral CVA for a ten-year payer interest rate swap as a function of constant correlation.

Next, the volatility of both the short rate and intensity is increased by 50%, and the results are plotted in Figures 3 and 4. With increased volatility, CVA is substantially more sensitive to changes in correlation.
Figure 2: Unilateral CVA for a ten-year receiver interest rate swap as a function of constant correlation.
Figure 3: Unilateral CVA for a ten-year payer interest rate swap as a function of constant correlation. Both the short rate and intensity volatility structure is increased by 50%. The sensitivity of CVA to change in correlation is substantially larger for all three swaps.
Figure 4: Unilateral CVA for a ten-year receiver interest rate swap as a function of constant correlation. Both the short rate and intensity volatility structure is increased by 50\%. The sensitivity of CVA to change in correlation is substantially larger for all three swaps.
4.1.2 CVA with GAS Correlations

Figure 5 compares dynamic correlation CVA to constant correlation CVA for a ten-year payer swap. The solid curve is the same curve as the at-the-money swap from Figures 1. The dashed line is the resultant CVA quantities with GAS correlations for a payer swap. The x-axis value corresponding to the cross point can be viewed as the equivalent constant correlation. It is worth stressing that the equivalent constant correlation is not known a priori. Figure 6 shows the same for a ten-year receiver swap.

Figure 5: Constant correlation vs GAS correlation for a ten-year payer swap.
The equivalent constant correlations for both graphs appear to be fairly close to 0.2. This leads naturally to the question of whether the cross point represents the historical correlation. Table 4.1.2 presents the historical correlations using various estimation windows from 30 days up to 3 years. It can be seen that the historical correlation ranges from as 0.164 to 0.281. Moreover, there is no consensus on which historical correlation best represents the forward-looking correlation.

<table>
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<th>Window (days)</th>
<th>Correlation</th>
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<tr>
<td>30</td>
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<tr>
<td>60</td>
<td>0.2248</td>
</tr>
<tr>
<td>90</td>
<td>0.2807</td>
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<tr>
<td>180</td>
<td>0.2582</td>
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<td>360</td>
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<td>720</td>
<td>0.1640</td>
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<tr>
<td>1080</td>
<td>0.2077</td>
</tr>
</tbody>
</table>

Table 1: Historical correlation.
A closer look at the graphs shows that the cross points do not correspond to the same correlation value. The zoomed-in graphs are presented in Figures 7 and 8. The equivalent constant correlation for a payer swap is close to 0.24, while the equivalent constant correlation for a receiver swap is approximately 0.18. This is not a coincidence. In fact, this is a consequence of the state-dependent nature of the correlation dynamics in the GAS model, together with the non-linearity in the CVA pricing equation.

Figure 7: Constant correlation vs GAS correlation for a ten-year payer swap (zoomed-in).
The CVA variability capital charge is heavily based on CVA Greeks. To this end, we compute the CVA delta for both dynamic and constant correlation. Sensitivities of CVA delta to constant and dynamic correlations are depicted in Figures 9 and 10. The solid lines represent at-the-money, in-the-money, and out-of-the-money ten-year swaps, and the dashed line represents the CVA delta using dynamic correlation for the at-the-money swap.
Figure 9: Constant correlation vs GAS correlation for a ten-year payer swap (CVA delta).
Figure 10: Constant correlation vs GAS correlation for a ten-year receiver swap (CVA delta).
5 Conclusions

In this paper, we propose an improved framework for capturing time-varying wrong-way risk based on a relatively recent innovation - the dynamic factor copula. The approach has several ideal features. First, it generates dynamic and state-dependent correlations. Second, the model can be extended to high dimensions, allowing for systematic and efficient modelling of the entire derivative portfolio. Using numerical examples of CVA and CVA delta, our results demonstrate that both CVA and CVA delta are sensitive to changes in correlation between the short rate and default intensity dynamics. Further, the proposed Basel CVA risk framework relies heavily on CVA sensitivities and requires modelling of wrong-way risk. Subject to regulatory authority’s approval, our proposed framework has potential implications on bank’s regulatory capital requirement.

References


