Econometric approach for Basel II Loss Given Default Estimation: from discount rate to final multivariate model

Abstract: LGD – defined as credit loss when extreme events occur influencing the obligor ability to repay debts - has a high relevance into credit and recovery process because of its direct impact on capital savings. Its relevance has been recently stressed also by the regulatory changes embedded in Technical Documents and Consultation Papers from Basel Committee and EBA[1]. While on PD models an extensive academic and practitioner's literature exists, LGD studies are in a less advance status because of the lack of available internal data on recoveries and differences in recovery process among commercial banks. This paper shows the results of a study developed on real banks data under Basel 2 framework starting from a workout approach with particular focus on discount rate and choice of multivariate model. Finally performance and backtesting tests have been computed in order to verify the predictive power and the accuracy of the model.

The model has been developed on 10 years of historical workout real data of Corporate and Retail loans from a panel of European Commercial Banks supervised by ECB. Our paper contributes to the strand of the literature studying the determinants of recovery rates on real portfolios from European Banks under the new Credit Risk Regulatory environment. In addition, this paper provides a benchmark analysis on different approaches for discounting cash flows and linear regression for multivariate estimation model starting from the main results of literature review. Finally, this paper is really important since very few analyses on recovery rates of bank loans focus on continental Europe, having found that the most part of research on recoveries are focused on Bond US Market.

Keywords: Loss Given Default, Rating Model, Basel2, Credit Risk Modeling, Quantitative Finance.

JEL Reference: C13, C18, C51, C52, C53, G21

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[1] Cfr. Consultative Document *"Reducing variation in credit risk-weighted assets – constraints on the use of internal model approaches"*(Basel Commettee, March 2016) and *"Guidelines on PD estimation, LGD estimation and the treatment of defaulted exposures"* (EBA, November 2016)

1. Introduction

In the last years the biggest European Banking Groups started to assess the possibility of adopting the Advanced Internal Rating Based Approach (AIRBA) under Basel2, in order to save capital thanks also to the possibility of a larger use of Credit Risk mitigators with respect to the Standardized Approach. The AIRBA framework requires banks to develop statistical models for estimating probability of default (PD), Loss Given Default (LGD) and Exposure at Default (EAD). The New Basel2 Accord, implemented throughout the banking world starting from 1 January 2007, made a significant difference to the use of modeling within financial organizations, by highlighting the relevant role of Loss Given Default (LGD) modeling[2]. While on PD models an extensive academic and practitioner's literature exists, LGD studies are in a less advance status because of the lack of data on recoveries and differences in recovery process among commercial banks. The existing literature on LGD is for the most part related to US market of Corporate Bonds given the public availability of data thus in most cases the existing papers try to test different statistical approaches applying them on external data (e.g. recoveries data from Rating Agencies). Given the relevance of LGD in capital requirements calculation, it is very important for banks, bankers, Regulators and academics to understand that depending on the methodology chosen, results can lead to different estimated values and different drivers combination. Our paper can contributes to the strand of the literature studying the determinants of recovery rates identifying the real recovery drivers on Corporate and Retail loans under the new Credit Risk Regulatory environment. For our study we have used a large panel dataset comprising of around 25,000 defaulted loans from 2005 to 2015 coming from different Italian banks under the European Central Bank Supervision and belonging to Corporate and Retail segments of client. We have first of all applied different approaches for discounting cash flows, confirming the goodness of Capital Asset Pricing Model for spread estimation. Finally, we have applied an ordinary least squares regression (OLS) for identifying the best combination of variables in predicting recovery rates.

This study also provides a comparative analysis among different ways of defining discount rate, finally chosing CAPM for discounting cash flows on the portfolio used for the application. This paper is very important since very few analyses on recovery rates of bank loans are focused on continental Europe Banking market, having found that the most part of research on recoveries are focused on Bond US Market.

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[2] Basel Committee on Banking Supervision (2004)

2. Literature Review

As previously mentioned, most empirical research of the last 10 years focuses on modeling and estimating the determinants of recoveries on US Market of Corporate Defaulted Bonds and only few studies on recoveries or lossess on loans from European countries are available.

I. Approaches for identifying modeling approach and recovery drivers

Table 1 – Overview of existing studies on recoveries for loans portfolios

Title	*Authors*	*Year of publication*	*Journal*	*Methodology chosen*
Bank-Loan Loss Given Default	Lea V. Carty, Daniel Gates, Greg M. Gupton,	2000	Moody's Investor Service, Global Credit Research	Empirical analysis
Default Recovery Rates in Credit Risk Modeling: A Review of the Literature and Empirical Evidence	Edward Altman, Andrea Resti, Andrea Sironi	2003	NYU Working Paper	Literature review
Incorporating Collateral Value Uncertainty in Loss Given Default Estimates and Loan-to-value Ratios	Esa Jokivuolle and Samu Peura	2003	European Financial Management	Literature review
Measuring LGD on Commercial Loans: An 18-Year Internal Study	Michel Araten, Michael Jacobs Jr., Peeyush Varshney	2004	The RMA Journal	Empirical analysis Linear regression Log Linear regression
What Do We Know About Loss Given Default?	Shuermann T.	2004	Wharton Financial Institutions Center WP N. 04-01	Look-up tables Linear regression Log Linear regression
LossCalc: Model for Predicting Loss Given Default (LGD)	Gupton. G.M., Stein R.M.	2005	Moody's KMV	Linear regression
Bank Loss Given Default: a Case Study	J. Derminea, C. Neto de Carvalho	2006	Journal of Banking & Finance	Linear regression Log Linear regression
Default Recovery Rates and LGD in Credit Risk Modeling and Practice: An Updated Review of the Literature and Empirical Evidence	Edward I. Altman	2008	in Altman "Advances in Credit Risk Modeling and Corporate Bankruptcy Prediction", Cambridge University Press	Literature review
Modeling Bank Loan LGD of Corporate and SME Segments: A Case Study	Chalupa R. et al.	2008	Czech Journal of Economics and Finance	Linear regression Log Linear

				regression
Improvements in loss given default forecasts for bank loans	Marc Gürtler e Martin Hibbeln	2009	Journal of Banking &Finance	Linear regression
Recovery Rates of Commercial Lending: Empirical Evidence for German Companies	Jens Grunert , Martin Weber	2009	Journal of Banking &Finance	Linear regression
Modelling LGD for unsecured personal loans: Decision tree approach	Lyn C. Thomas, Christophe Mues, Anna Matuszyk	2010	Journal of the Operational Research Society	Decision trees
Forecasting bank loans Loss Given Default	Bastos, J.A.	2010	Journal of Banking and Finance	Decision trees Log Linear
Comparison of modeling methods for loss given default	Qi, M., Zhao, X	2011	Journal of Banking and Finance	Linear regression Log Linear Decision Tree Neural Network
The determinants of bank loan recovery rates	Hinh D. Khieu, Donald J. Mullineaux, Ha-Chin Yi	2012	Journal of Banking & Finance	Linear regression Log Linear regression
Predicting loss given default (LGD) for residential mortgage loans: A two-stage model and empirical evidence for UK bank data	Mindy Leow, Christophe Mues	2012	International Journal of Forecasting	Linear regression Log Linear regression
Loss given default models incorporating macroeconomic variables for credit cards	Tony Bellotti, Jonathan Crook	2012	International Journal of Forecasting	Linear regression Decision trees Log Linear
A realistic approach for estimating and modeling loss given default	Rakesh Malkani	2012	The Journal of Risk Model Validation	Literature review
Modeling exposure at default and loss given default: empirical approaches and technical implementation	Yang H.B et al.	2012	Journal of Credit Risk	Look-up tables Linear regression Log Linear regression
Loss given default modeling: a comparative analysis	Yashkir G. et al.	2012	The Journal of Risk Model Validation	Tobit model Linear regression Log Linear regression

Survival analysis approach in Basel2 Credit Risk Management: modelling Danger Rates in Loss Given Default parameter	Bonini S., Caivano G.	2013	Journal of Credit Risk	Survival Analysis
Estimating Loss Given Default (LGD) through advanced credibility theory	Bonini S., Caivano G.	2014	European Journal of Finance	Credibility Theory

One of the first studies aimed to identify a possible framework defining the best determinants of recoveries is the one of Shuermann 2004, in which the author tries to define a possible set of statistical techniques to be used for developing LGD and depending on the level of knowledge each bank has about this risk parameter. In particular, the author identify a look-up table as an easy way to compute LGD by producing single cells of values (a cell for example might be LGD for subordinated loans belonging to automotive industry during a recession). Another way to estimate LGDs could be the use of basic regressions (with a medium level of sophistication) in which all variables are treated as dummies of only one regression (for example dummies for Senior/Junior loans, collateral type, industry group, expansion / recession). A medium – high sophisticate approach is to use advanced regressions, in which each dummy variable can be treated as part of a specific regression. The author identify also some most complicated approaches as neural net, decision trees, machine learning. Starting from this work, other authors have tried to compare different approaches for developing LGD model.

Araten et al 2004 try to investigate the role of collaterals in explaining recovery rates analyzing 18 years of loan loss history at JPMC for 3,761 defaulted borrowers. Using a look-up table methodology and testing the following variables as main drivers: business unit, industry group, geographic region, cohort year, and collateral type. They find that economic LGD on unsecured loans is expected to be higher. They also try to test the link between LGDs and business cycle, observing that LGDs for unsecured loans exhibit relatively high correlation with the economic cycle.

Gupton 2005 tests LossCalc model (Moody’s KMV) to predict loss given default, applying multiple predictive factors at different information levels: collateral, instrument, firm, industry, country and the macroeconomy to predict LGD. Data used is represented by a global dataset of 3,026 recovery observations for loans, bonds and preferred stock from 1981 to 2004. He finds that recovery rates worldwide are predictable within a common statistical framework, which suggests that the estimation of economic firm value is a dominant step in LGD determination.

Sabato et al 2005 analyze the link between recoveries and the credit cycle for retail assets, to investigate the issue of estimating conservative LGD for retail exposures and to propose a solution for the common lack of retail recoveries data to cover a full credit cycle (including an economic downturn). They calculate LGD using ultimate recoveries observed one year after the default event and we analyze them at regulatory asset class level: the adverse dependencies between default rates and recovery rates are not present in all asset classes. They also propose two different techniques for secured and unsecured exposures. For the former, they stress the value of the collateral and the cure rate in order to find the expected increase of the LGD in a downturn period. For the latter instead, they use the existing correlation between default rates and recovery rates, if any, to quantify the amount of conservatism to be added to the LGD during a period of default rates significantly higher than the mean (i.e., downturn period).

Dermine et al 2006, starting from their previous studies (2004), try to investigate the role of guarantees and collaterals in explaining the link between cumulative recovery rates and dynamic provisions. Using a log-log regression applied to the same sample used in their previous study, they find that bad and doubtful loans with no guarantee/collateral exhibit better recoveries than loans with personal guarantee (this could be due to the fact that the decision to lend without guarantee took into account the higher expected recovery rates). They also find that the past recovery history has a highly significant positive impact on future recovery and finally, a comparison with the Bank of Portugal mandatory provisioning rules indicates some regulatory conservatism in calling for 100% provision 31 months after the default date, when, in fact, significant amounts are still recovered after that date.

Altman et al 2008 presents a detailed review of approaches for credit risk models, developed during the last thirty years, treat the recovery rate and, more specifically, its relationship with the probability of default of an obligor. They also review the efforts by rating agencies to formally incorporate recovery ratings into their assessment of corporate loan and bond credit risk and the recent efforts by the Basel Committee on Banking Supervision to consider “downturn LGD” in their suggested requirements under Basel II. Their literature review shows that there are recent empirical evidence concerning these issues and the latest data on high-yield bond and leverage loan defaults are also presented and discussed.

Caselli et al 2008 verify the existence of a relation between loss given default rate (LGDR) and macroeconomic conditions on an Italian portfolio of bank loans by using both logistic and linear regression models. The authors pinpoint different macroeconomic explanatory variables for LGDR on loans to households and SMEs. For households, LGDR is more sensitive to the default-to-loan ratio, the unemployment rate, and household consumption. For SMEs, LGDR is influenced by the total number of employed people and the GDP growth rate.

Acharya et al 2008 show that creditors of defaulted firms recover significantly lower amounts in present-value terms when the industry of defaulted firms is in distress. The authors use data on defaulted firms in the United States over the period 1982–1999 and test it through a linear regression model using contract-specific characteristics (seniority and presence of collateral), industry of defaulting firm (utility sector or other sector), and macroeconomic condition (aggregate default intensity).

Couwenberg et al 2008 test the relationship between Dutch bankruptcy law system (that is a liquidation-based system) and firm recovery rates by using data from files of bankrupt companies in The Netherlands and collecting all information about process of resolving financial distress (time taken for asset sales to complete, type of buyer, managerial involvement, employees laid off, involvement of prior lenders, length of automatic stay, conflicting rights on assets, procedures started and resolution of the bankruptcy procedure) and on the pay-out on all the debts. Using a linear regression, the authors find that the firm recovery rate is higher when firms have more fixed assets, a higher quick ratio, are not liquidated and continue their operations in bankruptcy.

Davydenko et al 2008 study the effects of bankruptcy codes in France, Germany, and the United Kingdom on recovery rates of defaulted firms. The authors apply linear regressions to a unique data set of small to-medium size almost privately owned, defaulted on their bank debt. The data include detailed information on the terms of the loan contracts, the event of default and its resolution (either bankruptcy or workout), collateral values and the proceeds from asset sales, and banks’ recovery rates. The authors observe that collateral requirements at loan origination directly reflect the bank’s ability to realize assets upon default. Thus, because the proceeds from collateral sales are lower in France, at loan origination French banks demand higher levels of collateral per dollar of debt. Moreover, the composition of different types of collateral reflects their expected value in default.

Chalupa et al 2009 apply different statistical techniques (classic linear regression models, models with fractional responses and models with ordinal responses) to a set of firm loan micro-data of an anonymous Czech commercial bank in order to test empirically the determinants of LGDs. They find that LGD is driven primarily by the period of loan origination, relative value of collateral, loan size and length of business relationship. They also find that in more complex models, log-log models appear to perform better, implying an asymmetric response of the dependent variable.

Bastos 2010 try to test on a Portuguese sample of 347 loans two methodologies: linear regression and decision tree, with the following variables as drivers: loan amount, presence and amount of collateral, sector of activity, interest rate, rating, duration of loan. The results show a higher predictive power of decision tree.

Witzany et al 2010 propose an application of the survival time analysis methodology to estimations of the Loss Given Default (LGD) parameter. The authors identify a main advantage of the survival analysis approach compared to classical regression methods that is the possibility of exploiting partial recovery data. The empirical tests performed show that the Cox proportional model applied to LGD modeling performs better than the linear and logistic regressions.

Yashkir et al 2012 investigate several of the most popular loss given default (LGD) models (least-squares method, Tobit, three-tiered Tobit, beta regression, inflated beta regression, censored gamma regression) in order to compare their performance. They show that for a given input data set the quality of the model calibration depends mainly on the proper choice (and availability) of explanatory variables (model factors), but not on the fitting model. Model factors have been chosen based on the amplitude of their correlation with historical LGDs of the calibration data set. Numerical values of non-quantitative parameters (industry, ranking, type of collateral) are introduced as their LGD average. They show that different debt instruments depend on different sets of model factors (from three factors for revolving credit or for subordinated bonds to eight factors for senior secured bonds).

Yang et al 2012 apply different statistical techniques (mainly the ones described by Schuermann) on a real portfolio of commercial loans by using the following information as drivers: borrower level utilization, facility level of collateralization, facility authorized amount, ratio of limit increase to undrawn, total assets value, industry code, facility level utilization, total facility collateral percentage value. The authors perform a comparison in accuracy power of the applied techniques, showing that Naïve Bayes methods have a lower R-square (26%) if compared to logit models (27%) or neural network (35%).

Thomas et al 2012 try to investigate what are the main drivers of LGDs under two different collection policies: In house and 3rd party. The authors apply different statistical tecniques on two different samples: the first one, related to in house collection procedures, is represented by 11,000 defaulted consumer loans defaulted over a two year period in the 1990s; the second one, concerning 3rd party collection policies, is given by 70,000 loans where the outstanding debts varied from £10 to £40,000. They try to apply different statistical tecniques on the two samples, by obtaining a higher predictive power of all models applied (linear regressions, Box Cox, Beta distribution, Log Normal transformation, WOE approach) on sample related to In house procedures.

Finally, *Bonini et al 2013* apply survival analysis techniques on a sample of Italian Retail Loans portfolio in estimating danger rate correction factor for computation of LGDs on doubtful loans. The authors take into account this statistical methodology above all because the censoring phenomenon ensures consistent forecasting independent from the size of the credit loan portfolio. The variables considered for their analysis are mainly related to the kind of employer, by detecting a R-square of more than 75%. In another work (2014) the authors apply the credibility theory in order to estimate LGD values embedding the quality level of credit risk mitigators.

The literature review on this topic has highlighted that also if there are statistical techniques ensuring a higher predictive power (decision trees in Bastos 2009), it is not possible to define which is the best methodology because the choice can be affected both by bank knowledge (Shuermann 2004) and data availability (Khieu et al 2012, Qi et al 2012).

The literature review has been also highlighted that the best determinants of recoveries on loans portfolios of Corporate and Retail clients are related to loan attributes as amount (Derminea et al 2006, Chalupa et al 2008, Grunert et al 2009 and Gürtler et al 2009), kind of credit risk mitigators covering the loan (Bastos 2009 and Araten et al 2004), collateral amount (Yang et al 2012, Grunert et al 2009 and Leow et al 2012) and duration of default (Araten et al 2004, Chalupa et al 2008 and Carty et al 2000), but also the attributes of client, such as the rating (Bastos 2009), age of corporate (Bastos 2009 and Chalupa et al 2008), duration of loan (Dermine et al 2006 and Bastos 2009) and industry sector (Araten et al 2004, Dermine et al 2006 and Bastos 2009).

Following these findings, we will try to identify the best combination of variables in predicting recovery rates through OLS regression.

II. Approaches for identifying discount rate

It can be remarked that for the estimation of LGD, Regulators requires that "*The measures of recovery rates used in estimating LGDs should reflect the cost of holding defaulted assets over the workout period, including an appropriate risk premium. (….) In establishing appropriate risk premiums for the estimation of LGDs consistent with economic downturn conditions, the institution should focus on the uncertainties in recovery cash flows associated with defaults that arise during an economic downturn…(...)*".[3] In addition, the recent evolutions on Credit Risk Modeling – as contained in EBA Consultative Paper[4] highlights how choosing the appriopriate discount with contractual rate or funding rate could not really aligned with Regulatory requirements, supporting the choice of a discount rate based on a risk-free rate and a spread.

On this specific topic some authors in the last years have tried to evaluate possible solutions on this topic.

Table 2 – Overview of existing studies on discount rate

Title	*Authors*	*Year of publication*	*Journal*	*Methodology chosen*
Measuring LGD on Commercial Loans: An 18-Year Internal Study	Michel Araten, Michael Jacobs Jr., Peeyush Varshney	2004	RMA Journal	Vulture rate
Choosing the Discount Factor for Estimating LGD	Ian Machlahan	2005	in Altman E., Resti A. and Sironi A. (ed), *Recovery risk. The next challenge in credit risk management*, Risk Books, London	CAPM with monofactorial risk premium
Bank Loss Given Default: A Case Study	J. Dermine, C. Neto de Carvalho	2005	Journal of Banking & Finance	Loan Interest rate
The link between default and recovery rates: implication for credit risk models and prociclicality	Edward I. Altman, Brooks Brady, Andrea Resti, Andrea Sironi	2005	The Journal of Business	Risk free rate
Discount Rate for Workout Recoveries: An Empirical Study	Brooks Brady, Peter Chang, Peter Miu, Bogie Ozdemir, David Schwartz	2006	FDIC Working paper	Returns on defaulted bonds
The selection of the discount rate in estimating the Loss Given Default	Lucia Gibilaro, Gianluca Mattarocci	2007	Global Journal of Business Research	CAPM with multifactorial risk premium

[3] Bank of Italy: *Nuove disposizioni di vigilanza prudenziale per le banche. Circolare n. 263, Titolo II, Capitolo 1*

[4] *"Guidelines on PD estimation, LGD estimation and the treatment of defaulted exposures"* (EBA, November 2016)

Modelling Bank Loan LGD of Corporate and SME Segments: A Case Study	Radovan Chalupka, Juraj Kopecsni	2008	Czech Journal of Economics & Finance	CAPM with monofactorial risk premium
Loss given default of high loan-to-value residential mortgages	Min Qi, Xiaolong Yang	2008	Journal of Banking & Finance	Cost of funding
Forecasting bank loans loss-given-default	Joao A. Bastos	2009	Journal of Banking & Finance	Loan Interest rate
Unexpected Recovery Risk and LGD Discount Rate Determination	Witzany, J.	2009	European Financial and Accounting Journal	CAPM with monofactorial risk premium
An Empirical Study of the Returns on Defaulted Debt and the Discount Rate for Loss-Given-Default	Michael Jacobs, Jr.	2010	Journal of Advanced Studies in Finance	Returns on defaulted bonds
The Impact of discounting Rate Choice in Estimating the Workout LGD	Gibilaro, L. - Mattarocci, G.	2011	The Journal of Applied Business Research	CAPM with multifactorial risk premium
Improvements in loss given default forecasts for bank loans	Marc Gürtler, Martin Hibbeln	2013	Journal of Banking & Finance	Loan Interest rate
Discounting Long and Uncertain Workout Recoveries for Estimating Loss Given Default	Subarna, R.	2013	Journal of Risk Management in Financial Istitutions	CAPM with monofactorial risk premium

Cost of funding is studied in Yang et al 2008, using 1 year Libor (plus a spred of 3%) for discounting cash flows coming from mortgages with high Loan to Value, considering this rate much more useful for reflecting the recoveries uncertainty and the presence of an undiversifiable risk component. For what concernes the return of defaulted bonds (the market price of bonds after issuer default), Jacobs et al 2010 calculate discount rate as the annual return of defaulted bonds between the opening and closing of default event through the application of a maximum-likehood function starting from the pricing normalized error. Other authors, as Brady et al 2006 adopt a regressive approach for modeling discount rate on defaulted bonds based on their market price and sector dummies. The use of loan interest rate can be found in Bastos 2009 applied to a Portuguese bank portfolio. The author highlights that there is no substantial difference in discounted recoveries considering a fixed discount rate not changing over time. Also Dermine et al 2005 adopt a loan interest rate, but they discover that the outstanding capital could be different from the loan amount at the moment of default because of the absence of an adequate risk premium. For Gürtler et al 2013 the loan interest rate is an approach approved by Regulators. Other authors, as Araten et al 2004 adopt a “vulture” rate opposite to the loan interest rate or cost of funding, because it is considered much more reflecting the riskness of defaulted cash flows.

Altman et al 2005 study the adoption of risk-free rate analyzing historical time series on American defaulted Bonds, considering this rate able to reflect to volatility of defaulted Bonds implied returns.

Finally, Machlahan 2005 assumes that cash flows observed after the default can be a proxy of financial assets. Based on this assumption, he says that the best discount rate must take into account also a systematic risk component acting on assets after default and he applies the framework of Capital Asset Pricing Model (Sharpe 1994). Also Gibilaro et al 2007 evaluate the estimation of a risk premium component (in addition to the risk – free rate) through the estimation of a monofactorial rate based on market index or macroeconomic variables. In 2011 they try to evolve their model estimating a multifactorial rate obtaining LGDs with a low volatility. Also Witzany et al 2009 and Subarna 2013 apply CAPM for defining discount rate and they introduce the correlation factor deriving from the Basel 2 risk-weights functions for calculating β. Finally, Chalupa et al 2008 apply CAPM framework defining different risk premium according to the type of collateral covering the portfolio.

Here we show our considerations on each approach analyzed aimed to justify and reinforce the choice of CAPM model:

- Contractual loan doesn't make possible a diversification of returns pre and post default (it is not so common a frequent negotiation of contractual conditions during the whole life of a loan, maybe except for mortgages);
- The adoption of the only risk-free rate is subject to the identification of the reference market;
- The adoption of funding cost is based on the assumption that systemic risk of defaulted asset replaces the risk of bank;
- Ex-post Defaulted Bonds returns are influenced by the volatility embedded in the chosen index.

In addition to these considerations, in order to choose the best discount rate to be applied to our data, we have compared the results deriving from the adoption of a CAPM spread with the outcomes deriving from the use of a discount factor based only on risk-free rate or using internal spreads / funding rates. We have found that recovery rates based on discounted cash flows adopting a risk –free rate added to a spread derived from CAPM are less volatiles (given an average value not so different with what can be obtained adopting the other proposed approaches).

In this paper we have applied and compared three different approaches for discounting cash flows for Economic Loss Given Default calculation: contractual loan rate, risk-free rate and risk-free rate plus a spread derived from CAPM. We have found that the adoption of a risk-free rate plus a spread ensure a lower volatility of LGDs and in the same time a higher conservatism of estimates.

3. Methodological Framework

3.1 Workout LGD calculation

The New Basel2 Accord, implemented by the European Banking System starting from 1 January 2007, has highlighted the relevant role of Loss Given Default (LGD) modeling for its impact in facility ratings, approval levels and setting of loss reserves, as well as developing credit capital (as in Machlahan 2004). The concept of LGD in Basel 2 Accord is quite close to the one used by researchers and practitioners: it can be defined as the share of a defaulted exposure that will never be recovered by the lender. The efficiency levels (in terms of costs and time) of bank's workout department may affect LGD quite significantly, and must be reflected in the estimates used to assess recovery risk on future defaulters. Thus an improvement in recovery procedure can lead to a reduction in empirical LGDs and subsequently may reduce capital requirements for the following years.

The best practice on European Banks, in particular on Retail Portfolios, is to use a workout approach. Infact the same CEBS Guidelines[5] specifies that: *"LGD estimates based on an institution's own loss and recovery experience should in principal be superior to other types of estimates, all other things being equal, as they are likely to be most representative of future outcomes". (....) "The market and implied market LGD techniques can currently be used only in limited circumstances. They may be suitable where capital markets are deep and liquid. It is unlikely that any use of market LGD can be made for the bulk of the loan portfolio". (...) "The implied historical LGD technique is allowed only for the Retail exposure class (...). The estimation of implied historical LGD is accepted in case where institutions can estimate the expected loss for every facility rating grade or pool of exposures, but only if all the minimum requirements for estimation of PD are met .*

The workout LGD estimation is based on economic notion of loss including all the relevant costs tied to the collection process, but also the effect deriving from the discount of cash flows, as required by CEBS Guidelines: *"The measures of recovery rates used in estimating LGDs should reflect the cost of holding defaulted assets over the workout period, including an appropriate risk*

[5] CEBS - GL10 Guidelines on the implementation, validation and assessment of Advanced Measurement (AMA) and Internal Ratings Based (IRB) Approaches (4 April 2006).

premium. When recovery streams are uncertain and involve risk that cannot be diversified away, net present value calculations should reflect the time value of money and an appropriate risk premium for the undiversifiable risk. (…) *In establishing appropriate risk premiums for the estimation of LGDs consistent with economic downturn conditions, the institution should focus on the uncertainties in recovery cash flows associated with defaults that arise during an economic downturn. When there is no uncertainty in recovery streams (e.g., recoveries are derived from cash collateral), net present value calculations need only reflect the time value of money, and a risk free discount rate is appropriate."*

We have chosen to adopt a workout approach, based on economic notion of loss including all the relevant costs tied to the collection process, but also the effect deriving from the discount of cash flows. The workout LGD calculation consists in the calculation of empirical loss rates through the observation of each charge-off at the end of recovery process, according to the following formula:

$$LGD_C = 1 - RR = 1 - \frac{\sum Rec_i \delta_i^T - \sum A_i \delta_i^T - \sum Cost_i \delta_i^T}{EAD} \quad (1)$$

All the parameters used in the previous formulas and their meaning are shown in the Table below:

Table 3 - List of factors for Workout LGD calculation

Parameter	***Description***
LGD_C	LGD estimated on charge-offs positions
RR	Recovery rate on charge-offs
REC_i	Recovery flow at date i
A_i	Increase flow at date i
COST	Costs of litigation, collection procedures (e.g. legal expenses) at date i
EAD	Exposure at default at the charge-off opening date
i	Date in which each cash flow has been registered
T	Time before the charge-off opening date
δ_i^T	Discount rate of each flow at date i and opened before T (to be applied on a doubtful position, next moved into charge-off)

3.2 Discount factor

As previously mentioned, in order to find the best discount factor to be applied to workout cash flows we have compared three different approaches:

a. Discounting at the risk-free rate: we have performed a linear interpolation of time curves of the following interest: Eonia, EUR001w, EUR001M, EUR003M, EUR006M, EUR012M, I05302Y, I05303Y, I05304Y, I05305Y, I05310Y, I05315Y, I05320Y, I05330Y in order to apply the correct discount rate on each loan, depending on the cash flow date and the start date of workout process;

b. Adoption of a discount rate based on average funding cost of each bank of the panel on the market.

c. Adoption of a discount rate that considers risk-free rates plus spreads estimated through the Capital Asset Pricing Model (CAPM framework). In this case the discount factor is given by:

$$r = r_f + \beta \cdot (r_m - r_f) = r_f + \beta \cdot MRP \quad (2)$$

In particular, β is computed as:

$$\beta = \rho_{i,m} \frac{\sigma_i}{\sigma_m} \quad (3)$$

The Table below shows the meaning of each parameter and how we have calculated them:

Table 4 – CAPM parameters description

Parameter	*Description*
Market volatility (σ_M)	Standard deviation of logaritmic returns of market index MIB30 and FTSEMIB
Asset volatility (σ_i)	Standard deviation of logaritmic returns of cumulative annual recoveries
Asset and market correlation (R_i)	Assumption of Basel2 asset correlation for capital requirements calculation (k)
Market Risk Premium (MRP)	MRP has been set to 5.6%, as in Fernandez et al. 2012

3.3 Modeling approach

In order to identify the best determinants of recoveries, we have examinate a long list of factors classifying them into the following groups: borrower characteristics, loan characteristics, recovery process determinants, and external/macroeconomic factors. We have tested different factors depending on the portfolio segment (Retail or Corporate).

Table 5 – Long List description

Information group	*Factors*	*Factors descriprion*	*Segment of application*	*% missing Retail*	*% missing Corporate*
Borrower characteristics	Geographical area	North – West, North – East, Center, South, Islands	Corporate, Retail	0%	0%

	Industry		Corporate, Retail	45%	0%
	Type of counterparty	(e.g. Public Admin, Private sector, Individuals, etc.)	Corporate, Retail	0%	0%
	Sector of business		Corporate, Retail	12%	0%
	Corporate size	Annual turnover	Corporate	N.a.	56%
	Legal form	(Individuals, LtD etc.)	Corporate, Retail	0%	0%
Loan characteristics	Loan size at the moment of default		Corporate, Retail	0%	0%
	Type of product	Current accounts, Term loans, Commercial loans, Financial loans	Corporate, Retail	0%	0%
	Type of credit risk mitigators (CRM)	(Mortgage, guarantee, pledge, not covered)	Corporate, Retail	0%	0%
	Credit Risk Mitigation amount	Mortgage amount	Corporate, Retail	15%	11%
	Credit Risk Mitigation amount	Guarantee amount	Corporate, Retail	5%	8%
	Credit Risk Mitigation amount	Pledge amount	Corporate, Retail	7%	6%
	Value to loan (VTL)	Value to loan (VTL) mortgage	Corporate, Retail	15%	11%
	Value to loan (VTL)	Value to loan (VTL) guarantees	Corporate, Retail	5%	8%
	Value to loan (VTL)	Value to loan (VTL) pledge	Corporate, Retail	7%	6%
	Dummy variable on presence / absence for each type of CRM	Presence / Absence Mortgage	Corporate, Retail	0%	0%
	Dummy variable on presence / absence for each type of CRM	Presence / Absence Guarantees	Corporate, Retail	0%	0%
	Dummy variable on presence / absence for each type of CRM	Presence / absence Pledge	Corporate, Retail	0%	0%
Recovery process determinant	Type of recovery procedure	In court, out of court, no procedure	Corporate, Retail	0%	0%
	Vintage of recovery process		Corporate, Retail	0%	0%
Macroeconomic factors	Italian GDP		Corporate, Retail	0%	0%

We have performed an univariate analysis in order to select the drivers with a higher predictive power in explaining recovery rates to be used as the base for finding the best multivariate combination. We have thus removed variables with a percentage of missing values higher then 15% and checked the correlation coefficient between the pairs of remaining variables (in order to avoid the presence of correlation greater than |0.5|. Missing values lower than 15% have been replaced with average value of the variable (for continuous variables) and with a specific cluster for categorical variable). In addition, outliers have been censored at 2^ and 98^ percentile of distribution.

3.3.1 Multivariate econometric model specification

Starting from results of other research studies, we have applied on our data an Ordinary Least Square Linear regression in order to identify the best combination of variables in predicting recovery rates. The linear regression model is specified as:

$$y_i = \beta_0 + \beta_{x_i}^T + \varepsilon_i \quad (4)$$

$$\varepsilon_i = N(0, \sigma^2) \quad (5)$$

Here the list of final variables selected for the multivariate analysis:

Table 6 – Variables selected for multivariate analysis

Information group	*Factors*	*Factors descriprion*	*Segment of application*	*% missing Retail*	*% missing Corporate*
Borrower characteristics	Geographical area	North – West, North – East, Center, South, Islands	Corporate, Retail	YES	YES
	Industry		Corporate, Retail	NO	YES
	Type of counterparty	(e.g. Public Admin, Private sector, Individuals, etc.)	Corporate, Retail	YES	YES
	Sector of business		Corporate, Retail	YES	YES
	Corporate size	Annual turnover	Corporate	N.a.	NO
	Legal form	(Individuals, LtD etc.)	Corporate, Retail	YES	YES
Loan characteristics	Loan size at the moment of default		Corporate, Retail	YES	YES
	Type of product	Current accounts, Term loans, Commercial loans, Financial loans	Corporate, Retail	YES	YES
	Type of credit risk mitigators (CRM)	(Mortgage, guarantee, pledge, not covered)	Corporate, Retail	YES	YES
	Credit Risk Mitigation amount	Mortgage amount	Corporate, Retail	YES	YES
	Credit Risk Mitigation amount	Guarantee amount	Corporate, Retail	YES	YES
	Credit Risk Mitigation amount	Pledge amount	Corporate, Retail	YES	YES
	Value to loan (VTL)	Value to loan (VTL) mortgage	Corporate, Retail	YES	YES
	Value to loan (VTL)	Value to loan (VTL) guarantees	Corporate, Retail	YES	YES
	Value to loan (VTL)	Value to loan	Corporate,	YES	YES

		(VTL) pledge	Retail		
	Dummy variable on presence / absence for each type of CRM	Presence / Absence Mortgage	Corporate, Retail	YES	YES
	Dummy variable on presence / absence for each type of CRM	Presence / Absence Guarantees	Corporate, Retail	YES	YES
	Dummy variable on presence / absence for each type of CRM	Presence / absence Pledge	Corporate, Retail	YES	YES
Recovery process determinant	Type of recovery procedure	In court, out of court, no procedure	Corporate, Retail	YES	YES
	Vintage of recovery process		Corporate, Retail	YES	YES
Macroeconomic factors	Italian GDP		Corporate, Retail	YES	YES

3.4 Sample description

The framework proposed has been applied on a sample of 26,000 charge-offs with a closed recovery process between 30/09/2002 and 31/12/2012 so composed:

Table 7 – Sample description for customer segment (% obs and Average LGD)

Customer segment	*# OBS*	*% obs*	*Average LGD*
Individuals (Retail)	15,000	57,80%	44,00%
Small size Corporate (Retail)	3,500	13,50%	47,00%
Medium – Large size Corporate	7,500	28,70%	50,00%

Table 8 – Sample description for sector of activity

Sector of activity	*# obs.*	*% obs.*
Industry	18.341	70,54%
Commerce	3.000	11,54%
Building & Construction	889	3,42%
Services	2.500	9,62%
Transportation	383	1,47%
Agricolture	887	3,41%

Table 9 – Sample description for product type

Product type	*# obs.*	*% obs.*
Check accounts	16.453	63,28%

Term loans	7.138	27,45%
Advance invoices	1.611	6,20%
Other loans	677	2,60%
Credit commitments	121	0,47%

3.5 Empirical results

The spread estimated adopting CAPM framework assumes values in a range [0,8% - 1,67%] as shown in the table below:

Table 10 – Final spread estimated with CAPM framework

Segment	σ_i	$\rho_{i,m}$	σ_m	*MRP*	*beta*	*SPREAD*
Corporate	0,1747	0,0827	0,2425	0,056	0,2072	1,160%
Large Corporate	0,1747	0,1431	0,2425	0,056	0,2725	1,526%
Other Retail	0,2233	0,0750	0,2425	0,056	0,2522	1,412%
Retail Mortgages	0,1866	0,1500	0,2425	0,056	0,2979	1,668%
Retail Rotative	0,1772	0,0400	0,2425	0,056	0,1461	0,818%
Other	0,1772	0,0400			0,1461	0,818%

Table 11 – Final discount rate (comparison among different approaches)

Discount rate calculation	*Avg.rate*	*Std.Dev.Rate*	*Avg. LGD*	*Std.Dev.LGD*
Only *Risk-free*	2,347%	1,80%	48,74%	105,26%
Risk free + Spread (CAPM)	3,562%	1,82%	49,83%	103,92%
Cost of funding	2,975%	2,33%	49,81%	106,80%

The adopted approach ensures conservatism to LGD estimates and decrease the overall volatility of recovery distribution, mainly described in the next Table:

Table 12 – LGD Distribution

Metrics	*Economic LGD*
Mean	49,83%
# of Missing values	0
# obs.	26.000
Min	-487,28%
p1	0,4580%
p5	2,78%
p10	5,73%
p25	20,44%
p50	40,81%
p75	90,26%
p90	100,00%
p95	100,23%
p99	106,49%
Max	234,44%

Before estimating the model the distribution has been floored at 0%.

We have finally identified thatthe best determinants of recoveries are related to the segment of portfolio, geographical area, exposure at default, type of product,different types of credit risk mitigators and type of recovery procedure, has shown below:

Table 13 – Final model description

Variables	*Grouping*	*Coefficient*	*p-value*	*Variable weight*
	Intercept	0,1001	<,0001	
Macro-geographical area	**Center**	0,2145	<,0001	13,87%
	North East	0,1113		
	Sud & Island	0,0788		
	North West	0		
Exposure at Default	**EAD**	0,1567	<,0001	10,13%
Portfoglio segmentation	**Medium – Large Corporate**	0,5944	0,0033	38,4%
	Small Business (Retail)	0,377	0,0022	
	Individuals (Retail)	0	<,0001	
Type of product	**Mortgages**	0,1876	<,0001	12,13%
	Other products	0		
Presence of personal guarantess	**Absence**	0,1134	<,0001	7,33%
	Presence	0		
Presence of mortgages	**Absence**	0,1609	<,0001	10,40%
	Presence	0		
Type of recovery process	**Out of court**	0,1189	<,0001	7,69%
	In court	0,0533		
	No information	0		

The backtesting performed on the development sample has shown an Accuracy Ratio of final model of 57% and an AUROC (Area Under the ROC Curve) of 75%. The model has an Adjusted R-square of 31%.

4. Conclusions

This paper has presented a case study of LGD in which, according to the requirements of Basel2, the model has been developed on 10 years of historical real data of Corporate and Retail portfolio of a panel of commercial banks under ECB supervision. Giving a particular stress on the economic component of the model, the presented model highlights the determinant role of mitigators as recovery drivers, but also the geographical localization of loans, the type of product and the exposure at default. Our paper contributes to the strand of the literature studying the determinants of recovery rates on real portfolio of Corporate and Retail loans under the new Credit Risk Regulatory environment. The paper also provide a comparative analysis among different ways of defining discount rate and estimating recoveries, chosing CAPM for discounting cash flows and a linear regression approach for forecasting losses. Finally, this paper is really important since the existence of very few analyses on recovery rates of bank loans focused on continental Europe, having found that the most part of research on recoveries are focused on Bond US Market. A further development of this research could be the comparison of different approaches for multivariate model definition, starting from the main findings from literature review.

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