Conditional Volatility Persistence

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Abstract

This study presents empirical findings on the determinants of daily volatility persistence. We show that the volatility persistence is strongly influenced by large (negative) returns. After controlling the impact of return, short-term volatility persistence is negatively related to volatility level. There are large variations in the conditional volatility persistence, especially when markets are under stress in 2008-09 and late 2011. We offer an economic explanation for volatility persistence based on information shocks and price discovery. Models with conditional volatility persistence significantly improve one-day volatility forecasts.

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I. Introduction

In financial markets, volatility is synonymous with risk. Asset pricing, portfolio selection, and risk management are centred on measuring and forecasting volatility. Volatility is known to be highly persistent: today's volatility is significantly correlated with volatility over 100 days ago. While a large number of models have been developed to capture the statistical characteristics of volatility dynamics, it remains true today that "a consensus economic model producing persistence in conditional variance does not exist," as stated by Diebold and Lopez (1995). This study presents new evidence on the empirical characteristics of volatility persistence and examines its economic origins. We show that (1) volatility persistence varies daily with market state variables, e.g. return and volatility level; (2) price discovery, the process of incorporating information into asset prices, plays a key role in determining future volatility persistence; (3) models incorporating these features significantly improve volatility forecasts.

The initial evidence linking stock return with volatility persistence can be seen from Figure 1. Let RV_t be the daily realized variance and $\Delta RV_{t+1} \equiv \frac{RV_{t+1}-RV_t}{RV_t}$. The absolute value of ΔRV_{t+1} is inversely related to RV persistence: high RV persistence implies a small change in RV therefore low $|\Delta RV_{t+1}|$, and vice versa for low RV persistence. Figure 1 plots $|\Delta RV_{t+1}|$ against daily return r_t for the S&P 500 index ETF (ticker SPY) and the S&P 600 small cap ETF (ticker IJR). The bulk of the data indicates that as r_t becomes larger, positive or negative, $|\Delta RV_{t+1}|$ becomes smaller, indicating more stable or persistent RV.

The relationships in Figure 1 suggest that volatility persistence as captured by the inverse of $|\Delta RV|$ varies with return size. In a GARCH(1,1) model with variance equation $\sigma_{t+1}^2 = \omega + \alpha\sigma_t^2 + \beta r_t^2$, volatility persistence is measured by α+β. If persistence parameters α and β are an increasing function of return size as suggested in Figure 1, the propagation from σ_t^2 to σ_{t+1}^2 depends not only on σ_t^2 and r_t^2 but also on the time-varying α and β. This represents a new channel through which volatility propagates over time. Metaphorically the flow of water from one tank depends not only on the water level ($\sigma_t^2,\ r_t^2$) but also on the time-varying size of the pipe (α+β).

The idea that volatility persistence is affected by returns is implicit in models that allow returns to have an asymmetric impact on volatility. In the GARCH model of Glosten, Jagannathan, and Rankle (GJR, 1993), the variance equation is $\sigma_{t+1}^2 = \omega + \alpha\sigma_t^2 + (\beta + \lambda I_{(r_t<0)})r_t^2$ and volatility persistence is α+β+λ/2. Thus $\lambda > 0$ implies that negative returns increase tomorrow's volatility, as well as the dependence of tomorrow's volatility on today's volatility. In general volatility persistence may be dependent on a set of market state variables M_t: $VP_t = f(M_t)$. We term this persistence measure the _*conditional volatility persistence*_ (CVP), akin to the conditional volatility in the GARCH-family models. This study identifies a set of market state variables and estimates daily CVP.

The success of GARCH models has motivated many studies to explore the economic mechanisms underlying volatility persistence. A partial list of potential explanations for volatility persistence include (1) persistence in exogenous information arrival, e.g. Laux and Ng (1993), Andersen and Bollerslev (1997), Fleming, Kirby, Ostdiek (2006, RFS); (2) endogenous trading-generated information arrival, e.g. Cao, Coval, and Hirshleifer (2002); (3) heterogeneous trading frequencies by different investors, e.g. Müller, et al. (1997), Xue and Gençay (2012, JBF); (4) volatility regime shifts, e.g. Lamoureux and Lastrapes (1990, JBES), Hamilton and Susmel (1994); (5) parameter uncertainty, e.g. Johnson (2000, MF), Timmermann (2001); (6) information cost as in de Fontnouvelle (2000); (7) learning about market state or trading strategies in agent-base models, e.g. Brock and LeBaron (1996), He, Li, and Wang (2015); (8) persistence of wealth distributions, e.g. Cabrales and Hoshi (1996); (9) time-varying risk aversion, e.g. McQueen and Vorkink (2004); and (10) investor attention (Andre and Hasler, 2015) and information percolation (Andre, 2013).

Our study makes two contributions to the literature on the economic origins of volatility persistence. First, we present evidence that market state variables, e.g. return and volatility, have significant impact on future volatility persistence. As a result, volatility persistence varies on daily basis, a significant departure from the existing literature. The variables affecting daily volatility persistence shed new light on the economic mechanism leading to volatility persistence. Not surprisingly, a calibration of volatility persistence from today to tomorrow significantly improves volatility forecasts.

Second, we offer a new explanation for volatility persistence based on information shocks and price discovery. Intraday returns can be decomposed into a random-walk component reflecting the changes in the efficient price, and a serially correlated component reflecting the price impact of liquidity and noise trading. The sum of the intraday random-walk components captures the net price impact of positive and negative information shocks over a trading day and can be viewed as a proxy for the direction and size of the aggregate information shock. The sum of the *squared* random-walk components is widely used in microstructure literature as a measure for information flow or price discovery, e.g. Hasbrouck (1991, 1993, and 1995). It is the dominant component of daily realized variance. Empirically we find that information shocks increase volatility persistence with negative shocks having greater impact than positive shocks. Large shocks are associated with greater uncertainty (Panel C of Figure 2) and take longer to be fully priced in. They tend to have a spill-over effect on tomorrow's volatility, increasing volatility persistence. On the other hand, we find that price discovery reduces volatility persistence. Greater price discovery means more information has been priced in by the end of a trading day, reducing information spill-over from today to tomorrow and the autocorrelation of daily volatility.

Price discovery is determined by information flow, information quality, and investor behaviour. Our price discovery-based explanation for volatility persistence is closely related to earlier information-based explanations. One prominent theory is the mixture of distribution hypothesis (MDH) developed by Clark (1973) and Tauchen and Pitts (1983) and extended by Andersen (1996). MDH is centred on a mixing variable $I_t > 0$ representing the (latent) number of information events. Let r_i be the return associated with the i^{th} information arrival. When r_i is iid $N(0,\sigma^2)$, daily return is $r_t = \sum_{i=1}^{I_t} r_i$ with variance $\sigma^2 I_t$. Therefore variation and persistence in I_t leads to variation and persistence in return variance. Laux and Ng (1993), Andersen and Bollerslev (1997), and He and Velu (2014) find empirical support for MDH while Lamoureux and Lastrapes (1994), Liesenfeld (1998), and Watanabe (2000) show that MDH fails to explain volatility persistence. Our explanation is consistent with MDH in that the exogenous information arrival I_t is a key determinant of price discovery. Persistence in I_t *across periods* leads to persistence in price discovery *across periods*, which in turn leads to persistence in volatility. Our explanation differs from MDH in that volatility persistence is not solely determined by exogenous information arrivals. Price discovery involves learning, information searching, and strategic trading by investors. A large information shock may take a few days to be priced in, leading to a spill-over effect *across periods* and volatility persistence even in the absence of new information.[1] While information arrivals increase uncertainty, price discovery is the process of absorbing information shocks and resolving uncertainty. Price discovery *within a period* reduces information spill-over to future periods therefore reduces volatility persistence.

Our price discovery-based explanation for volatility persistence is consistent with endogenous information arrivals, either through information costs as in de Fontnouvelle (2000), or through validation of private signals as in Cao, Coval, and Hirshleifer (2002), or though investor attention (Andre and Hasler, 2015) and information percolation (Andre, 2013). Barriers to information flow or participation by informed investors reduce price

[1] An example is the well-known post-earnings-announcement drift where price continues to drift in absence of new information.

discovery, induce delayed price reaction to correlated information, and increase volatility persistence. When investors have heterogeneous trading frequencies, e.g. Müller, et al. (1997) and Xue and Gençay (2012, JBF), infrequent traders learn from past prices when they are absent, similar to the validation of private signals in Cao, Coval, and Hirshleifer (2002). We differ from agent-based models, e.g. He, Li, and Wang (2015), which typically have agents switching between fundamental or trend-following traders. There are two equilibriums, each having a different volatility level and constant volatility persistence. As agents switch between fundamentalists and trend followers, market equilibrium changes and volatility persistence is disrupted.

Berger, Chaboud, and Hjalmarsson (2009) find that the sensitivity to information, as opposed to information flow itself, accounts for a large portion of volatility persistence in the foreign exchange markets. Conceptually one would expect the sensitivity to information to be closely related to price discovery in our study. Patton and Shappard (2015) show that "bad volatility", defined as the sum of squared negative returns, accounts for most of the volatility persistence. We show that negative returns are the dominant component of the conditional volatility persistence (CVP).

Our study is directly related to Ning, Xu, and Wirjanto (2015), who measure volatility persistence by the tail dependence of RV_t and RV_{t+1}, i.e. the probability of both RV_t and RV_{t+1} being in the left or right tails of the RV distributions. Using a set of copulas, they show that the right-tail dependence is much higher (75% for S&P 500) than the left-tail dependence (6%). They conclude that high volatility level is associated with high volatility persistence. We document a positive unconditional correlation between our CVP and RV. However, after controlling for the impact of daily returns, our estimated CVP is *inversely* related to RV_t. Two factors may have contributed to the different conclusions. First, we estimate CVP from a model for volatility dynamics that incorporates long memory as well as asymmetric return impact. A jump in volatility always reduces the estimated CVP. This is not the case when persistence is measured by tail dependence. For example, Table 1 shows that the mean and standard deviation of the daily realized variance (RV) of SPY are 1.13 and 2.43 (scaled by 10^4) respectively. On October 10 and 11, 2008, RV was 60.6 and 6.2 respectively. While these RV values are in the right tails of the RV distribution, a 10-time drop in RV should not be regarded as a case of volatility persistence. Second, we control for the impact of return on volatility persistence which is not examined in Ning, Xu, and Wirjanto (2015). Our estimated CVP is high during the financial crisis of 2008 because of the large negative returns, not the high volatility.

We estimate volatility persistence from the heterogeneous autoregressive (HAR) model which was proposed by Corsi (2009) and has been extensively used in many volatility studies. Our analyses are based SPY and IJR, as well as 87 of the S&P 100 index constituent stocks. Our empirical findings are summarized as following:

(1) Consistent with Figure 1, volatility persistence increases with the size of daily returns. Negative returns increase volatility persistence more than positive returns. The evidence indicates information shocks as a source for the propagation of volatility over time.

(2) After controlling the impact of return, we find that volatility persistence decreases with daily RV. The positive impact of return is generally larger than the negative impact of RV, resulting in a right-skewed distribution of the estimated CVP.

(3) After taking into account of the impact from return and RV, we fail to find consistent impact from other market state variables on volatility persistence, including volatility jumps, the number of trades, illiquidity, and the imbalance of buy and sell volumes.

(4) Using the Beveridge-Nelson decomposition, we estimate daily information shocks and price discovery. We find that price discovery accounts for over 90% of daily RV of market indices and over 80% of daily RV of individual stocks. The size of information

shocks increases volatility persistence, with greater impact from negative shocks. Most of the impact on volatility persistence comes from shocks above the median value. Non-information returns have no impact on volatility persistence. Non-information RV has little impact on future RV.

(5) Out-of-sample forecast comparisons show that models with conditional persistence significantly outperform models with constant persistence. The average reductions in loss function values are in the range of 30 to 50%. The CVP models outperform across all size categories for return and RV and in all forecasting sub-periods.

This paper has the following sections. Section II explains the sample and variable construction and presents the summary statistics of the key variables. Based on an intuitive measure for daily volatility persistence, Section III presents preliminary evidence, which guides the selection of conditioning variables. The empirical evidence on the conditional volatility persistence and robustness tests are presented in Section IV. Section V links price discovery to volatility persistence. Section VI compares volatility forecasts of models with conditional or constant volatility persistence. We conclude in Section VII.

II. Data Sample and Summary Statistics

Our analyses are based on two index ETFs and 87 of the S&P 100 constituent stocks. The first index ETF is the SPDR S&P 500 ETF (ticker SPY) representing a portfolio of large stocks. The second is the iShare S&P Small Cap 600 ETF (ticker IJR) representing a portfolio of small stocks. The S&P 100 constituent stocks are selected to avoid the issue of thin trading and large bid-ask bounce in small stocks.

Data Sample

SPY was incepted in 1993 and IJR was incepted in May 2000. Both are traded on NYSE. Our sample for SPY starts from 2 January 2000 and ends on 30 May 2014. To avoid the period of thin trading and high tracking errors immediately after the inception, the sample for IJR starts on 2 January 2002 and ends on 30 May 2014. From the S&P 100 constituent stocks, we remove seven stocks with less than five years of intraday data and six stocks with share prices dropping below $5 during the sample period. Upon inspecting the stock data, we find that in 2000 and 2001, several stock-months have less than 15 days of intraday data. Our sample of 87 stocks starts on or after 2 January 2002 and ends on 31 December 2014.

Intraday 5-minute data are extracted from the Thomson Reuters Tick History (TRTH) database. Variables extracted include the first, the high, the low, and the last prices, as well as the volume and the number of trades for each 5-minute interval. Trading on NYSE ends at 1 pm on the day before July 4 and Christmas and the day after Thanksgiving. These days are excluded from the sample. We also remove days with less than 36 5-minute intervals (3 hours) possibly due to missing data or slow trading. There were 23 such days for SPY and 16 such days for IJR, all before 2005. The final samples have 3570 days for SPY and 3082 days for IJR. Data outside the NYSE trading hours are removed. To filter out data errors, we apply a filter similar to those of Barndorff-Nielsen, et al. (2009). For each 5-minute return, we calculate the standard deviation of the remaining returns on the same day. If a return is outside 6 standard deviations from zero, it is removed. The filter removes 246 intervals for SPY and 236 for IJR, representing 0.088% and 0.102% of the sample size respectively. The filter has no effect on 96.3% of the trading days. Of the remaining 3.7% trading days, 2.9% have unfiltered realized variances larger than the filtered ones by 50% or more. The filter removes very large price changes not present in the rest of the trading day.

Variable Construction

Our measure for daily volatility is the realized variance (RV). Let p_s be the log-price of an asset at time s which is assumed to follow a continuous stochastic process with a continuous component and a pure jump component. Let n be the number of intraday intervals on a trading day. Define $r_{i,t} = p_{i,t} - p_{i-1,t}$ as the return over interval i on day t. RV is defined as $RV_t = \sum_{i=1}^{n} r_{i,t}^2$ and is a consistent estimator of the true variation of $p_{i,t}$ over day t. We sample at 5-minute intervals therefore n = 78 for a trading day on NYSE. Measures of daily RV persistence are described in section III.

We aim to demonstrate that the time-varying RV persistence can be partially explained by a set of observed variables. The variables we consider include daily return, RV, volatility jump, number of trades, illiquidity, and the imbalance between buyer- and seller-initiated volumes. Volatility jumps have been shown to help forecast future volatility. The continuous component of RV_t is termed the bipower variation and is defined as $BV_t = \frac{\pi}{2}\sum_{i=2}^{n}|r_{i,t}||r_{i-1,t}|$. It converges to the integrated variance as $n \to \infty$. Following Huang and Tauchen (2005) and Patton and Sheppard (2015), BV_t is calculated using the skip-4 method to improve its statistical properties. The jump component of RV_t is $J_t = RV_t - BV_t$. Barndorff-Nielsen and Shephard (2006) suggest the following statistic for testing $J_t = 0$:

$$Z_t = \frac{n^{1/2}(BV_t/RV_t - 1)}{(\pi^2/4 + \pi - 5)^{1/2} \times \max\{1, QV_t^{1/2}/BV_t\}} \sim N(0,1)$$

where $QV_t \equiv \frac{\pi^2 n}{4}\sum_{i=4}^{n}|r_{i,t}||r_{i-1,t}||r_{i-2,t}||r_{i-3,t}|$ is known as quad-power variation. Let z_α be the left tail of the standard normal distribution with $P(Z<z_\alpha) = \alpha$. Volatility jump on day t is given by $J_t = I_{(Z_t<z_\alpha)}(RV_t - BV_t)$ where $I_{(*)}$ is an indicator function. We choose $\alpha = 1\%$ therefore $z_\alpha = -2.326$. SPY has jumps on 9% and IJR has jumps on 13% of trading days.

Daily illiquidity is measured by the Amihud (2000) measure. It is defined as $IL_t = |r_t|/Vol_t$ where Vol_t is trading volume in unit of million. We use the bulk volume classification of O'Hara, et al. (2012) to partition the 5-minute trades into buyer- and seller-initiated portions. The difference between the two portions is termed the trade imbalance ($TImb_t$). Recently Barclay? and O'Hara, et al. (2015) show that bulk volume classifications are better linked to proxies of information-based trading.

Summary Statistics

In this paper returns are calculated in percentage as $100\times(p_{i,t}-p_{i-1,t})$; therefore realized variance is inflated by 10^4. Panel A of Figure 2 presents the time series plots of RV for SPY and IJR. The surges in RV occurred during the height of the global financial crisis around October 2008. The distributions of daily RV have very long right tails, as seen in Panel B of Figure 2. Panel C of Figure 2 shows the contemporaneous relationships between daily RV and return, often referred as the news impact curve (Engle and Ng, 1993). The lower bound of RV rises as return increases in size. It rises faster for negative returns, a well-known feature of volatility in equity markets. Panel C shows that for a given return size, the realized variance has a wide range of values. Therefore the absolute return is not a measure of return uncertainty, even though it has been used in many studies as a proxy for return standard deviation. We argue that daily return is a proxy for the aggregate information shock. It may not have a monotonic relationship with the associated uncertainty.

Panel A of Table 1 reports the summary statistics of daily variables.[2] The average daily RV is 1.13 for SPY, 1.75 for IJR, and 2.50 for S&P 100 stocks. The medians of RV are much lower than means due to a small number of high RV days. The highest RV is 60.3 for SPY and 77.1 for IJR, both occurred on 10 October 2008. On days with volatility jumps, the average jump size is around 20% (=0.221/1.13) of the average RV for SPY, 24% for IJR, and 29% for stocks. SPY is actively traded and has low illiquidity, while IJR has less trades and higher illiquidity than stocks. Panel B of Table 1 reports the daily correlations across variables. Most correlations are consistent with those documented in the literature. RV is negatively correlated with contemporaneous return and positively correlated with trades, trade imbalance, and illiquidity. There is a significant positive correlation between return and trade imbalance.

[2] For stocks, the summary statistics are calculated for each stock and then are averaged across stocks.

III. Preliminary Evidence

This section proposes a proxy for volatility persistence and presents preliminary evidence on time-varying persistence. We show an asymmetric impact from positive and negative returns to volatility persistence. On most days with mild volatility, persistence decreases when volatility increases. The visual evidence in this section helps to select conditioning variables for estimating the conditional volatility persistence in section IV.

A Proxy for Daily Volatility Persistence

The first-order autocorrelation of RV can be estimated from $RV_{t+1} = \alpha + \rho \times RV_t + \varepsilon_{t+1}$. The coefficient ρ is defined as

$$\rho = \frac{E(RV_{t+1}-\mu)(RV_t-\mu)}{E(RV_t-\mu)^2}$$

where $\mu = E(RV_t)$. The unconditional ρ can be written as $\rho = E[\rho^e_{t,t+1}]$ where $\rho^e_{t,t+1} = E(\rho_{t,t+1}|I_{t-1})$ is the time-varying conditional expectation and I_{t-1} is the information set at t-1. The unconditional ρ is estimated as

$$\hat{\rho} = \frac{1}{T-1}\sum_{t=1}^{T-1}\frac{(RV_{t+1}-\overline{RV})(RV_t-\overline{RV})}{s^2}$$

from a sample of T days, with $\overline{RV} = \frac{1}{T}\sum_{t=1}^{T} RV_t$ and $s^2 = \frac{1}{T-1}\sum_{t=1}^{T}(RV_t - \overline{RV})^2$. We define daily volatility persistence as

$$\tilde{\rho}_{t,t+1} \equiv \frac{(RV_{t+1}-\overline{RV})(RV_t-\overline{RV})}{s^2},$$

therefore $\hat{\rho} = \frac{1}{T-1}\sum_{t=1}^{T-1}\tilde{\rho}_{t,t+1}$. Since $E(\hat{\rho}) = E[\frac{1}{T-1}\sum_{t=1}^{T-1}\tilde{\rho}_{t,t+1}] = \rho = E[\rho^e_{t,t+1}]$, $\tilde{\rho}_{t,t+1}$ can be viewed as a random draw from the underlying distribution for $\rho_{t,t+1}$ with $\rho^e_{t,t+1} = E(\rho_{t,t+1}|I_{t-1})$. We treat $\overline{RV}$ and s^2 as constants and use $\tilde{\rho}_{t,t+1}$ as a proxy for the time-varying daily RV persistence. While $E(\tilde{\rho}_{t,t+1}) = \frac{1}{T-1}\sum_{t=1}^{T-1}\tilde{\rho}_{t,t+1}$ is bounded between -1 and +1, $\tilde{\rho}_{t,t+1}$ as a noisy reflection of the expectation can be outside these bounds. For both SPY and IJR, there are close to 5% of daily $\tilde{\rho}$ greater than one. Some have extremely large values.

Return, RV, and the time-varying RV persistence

Our proxy for RV persistence allows us to explore how volatility persistence changes with market conditions without specifying a model for volatility dynamics. Since $\tilde{\rho}_{t,t+1}$ involves RV_t and RV_{t+1}, we examine the impact of the market condition on day t-1. Panel A of Figure 3 plots $\tilde{\rho}_{t,t+1}$ against r_{t-1} and RV_{t-1} for SPY and IJR. A striking feature is the asymmetric responses of $\tilde{\rho}$ to negative and positive returns: while $\tilde{\rho}$ increases with the size of return, its values are much higher after negative returns than positive returns. For SPY, the average $\tilde{\rho}$ following negative returns is 1.036 and the average $\tilde{\rho}$ following positive returns is 0.329. This feature is similar to the asymmetric impact of return on volatility level in Panel C of Figure 2. It suggests that large returns, especially large negative returns, are associated with greater future *volatility persistence*. Intuitively large returns are associated with major news arrivals which take longer for the market to analyse and price. Large negative news generates not only greater uncertainty but also longer persistence of uncertainty.

Panel B of Figure 3 depicts the relationship between $\tilde{\rho}_{t,t+1}$ and RV_{t-1} when RV_{t-1} is below 3. This is the normal range of daily RV, accounting for 93% of trading days for SPY and 88% of trading days for IJR. We see another striking feature that has not been documented in the volatility literature: higher RV_{t-1} is associated with lower $\tilde{\rho}_{t,t+1}$. This inverse relationship is more pronounced for IJR. Therefore on majority of trading days, higher volatility is associated with lower future volatility persistence! Only unusually high RVs are associated with high $\tilde{\rho}$. This is contrary to the common perception that high volatility leads to high persistence, as well as the findings of Ning, Xu, and Wirjanto (2015). However, the finding should not be surprising given the time-series plot and histogram of daily RV in Figure 2: low volatility is the norm on most days therefore is more persistent; high volatility is very rare and does not last very long.

IV. Long Memory, Asymmetric Volatility, and Conditional Persistence

In section III, the daily volatility persistence is measured from a simple dynamic model for RV: $RV_{t+1} = \alpha + \rho \times RV_t + \varepsilon_{t+1}$. It provides a measure of the time-varying daily volatility persistence using only realized variance. However it does not take into account some well-known features of volatility dynamics. Daily volatility has long memory, i.e. it is correlated with volatility in distant past. Taking into account of the long-run dependence may alter the short-run persistence measured by $\tilde{\rho}$. In addition, daily volatility is affected by lagged returns. Controlling such impact may also affect the short-term volatility dependence. In this section, we take a regression-based approach to measure daily volatility persistence. It allows us to incorporate the well-known features of volatility dynamics. We then estimate the time-varying volatility persistence conditional on the lagged return and RV. The characteristics of the time-varying volatility persistence are examined. The robustness of our results is tested with additional conditioning variables and sub-period analyses.

A Model of Volatility Dynamics

We use the heterogeneous autoregressive (HAR) model to represent RV dynamics. Proposed by Corsi (2009), it is a simple model to capture long memory, is parsimonious and easy to estimate, and has good out-of-sample forecasting performance. For our study, the model offers an easy way to isolate daily persistence from long-run persistence. We adopt the specification in Patton and Sheppard (2015) where the self-dependence of RV_{t+1} is captured by RV on day t (RV_t), the average RV from t-1 to t-4 ($RV_{t,W} \equiv \frac{1}{4}\sum_{i=1}^{4} RV_{t-i}$), and the average RV from t-5 to t-21 ($RV_{t,M} \equiv \frac{1}{17}\sum_{i=5}^{21} RV_{t-i}$). These are termed the lagged daily, weekly, and monthly RVs even though they are non-overlapping. The non-overlapping variables allow us to separate short-term daily volatility persistence from longer-term dependence. Similarly the lagged daily, weekly, and monthly returns are given by r_t,

$r_{t,W} \equiv \frac{1}{4}\sum_{i=1}^{4} r_{t-i}$ and $r_{t,M} \equiv \frac{1}{17}\sum_{i=5}^{21} r_{t-i}$ and they are non-overlapping. Corsi and Reno (2012) suggest that r_t, $r_{t,W}$, and $r_{t,M}$ have heterogeneous effects on future volatility. Our baseline model of RV dynamics is

(1) $$RV_{t+1} = \alpha + \beta_D RV_t + \beta_W RV_{t,W} + \beta_M RV_{t,M} + \theta_D r_t + \theta_W r_{t,W} + \theta_M r_{t,M} + \varepsilon_{t+1}$$

The daily volatility persistence is captured by β_D. Variations of the model in (1) have been extensively used in studies of volatility dynamics.[3] Recent studies show that the linear structure in (1) cannot be rejected (Lahaye and Shaw, 2014) and the deviations from linearity are very small (Fenger, Mammen, and Vogt, 2015). Previous studies of asymmetric volatility associated with lagged returns find that the asymmetry is generally larger for broad market indices than for individual stocks, e.g. Tauchen et al. (1996, JEtrics), and Andersen et al. (2001, JFE). We allow stock returns and market (S&P 500) returns to affect stock RV by estimating the model in (1) with separate and joint effects from stock and market returns. October 10, 2008, has extremely high RV, 60.3 for SPY and 77.1 for IJR, resulting in the extremely low estimates of daily persistence $\hat{\beta}$ compared to those of Andersen, et al. (2007) and Corsi and Reno (2012).[4] We treat this day as an outlier and remove it from the analyses.

As pointed out by Patton and Sheppard (2015), because the dependent variable is a volatility measure, OLS estimates tend to overweigh periods with high volatility and under-weigh periods with low volatility. As a result, the OLS residuals have heteroskedasticity related to the level of RV. Patton and Sheppard (2015) use the weighted least squares (WLS) to overcome this problem. We carry out the WLS estimation by using the inverse of the squared OLS residuals as the diagonal terms of the weight matrix. For index ETFs, the standard errors are estimated using the Newey-West robust covariance with automatic lag

[3] A partial list of volatility studies using the HAR model includes ABDL (2003), ABD (2007), Busch, et al (2011), Bauer and Vorkink (2011), Park (2011), and Maheu and McCurdy (2011), ABH (2011), McAleer and Medeiros (2011), Patton and Sheppard (2015).

[4] Without removing 10 October 2008, the estimated daily persistence $\hat{\beta}$ is not significantly different from zero at 10% significance for SPY.

selection using Bartlett kernel. For individual stocks, the reported coefficients are the cross-sectional averages. Following Hameed, Kang, and Viswanathan (2010), the standard error of the average coefficient $\overline{\hat{\beta}}_k$ is given by

$$\text{StDev}\left(\overline{\hat{\beta}}_k\right) = \text{StDev}\left(\frac{1}{N}\sum_{i=1}^{N}\hat{\beta}_{i,k}\right) = \frac{1}{N}\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}\widehat{\omega}_{i,j}\sqrt{Var(\hat{\beta}_{i,k})Var(\hat{\beta}_{j,k})}}$$

where $Var(\hat{\beta}_{i,k})$ is based on the Newey-West standard error of the regression of stock i and $\widehat{\omega}_{i,j}$ is the correlation between the regression residuals for stocks i and j. We term the t statistic from the above equation the HKV t-statistic.

Table 2 reports the estimation results for the model in (1). For SPY and IJR, all lagged RVs and all lagged returns are significant at 5%, consistent with the presence of long memory and heterogeneous effects of returns. For individual stocks, we find that stock returns become insignificant when market returns are included. The HKV t-statistics indicate that the asymmetry in stock volatility is largely driven by market returns. Only 16% of the stocks have negative and significant coefficients for daily returns and 10% for weekly returns. On the other hand, 91-94% of stocks have negative and significant coefficients for daily and weekly market returns and 60% for monthly return. In the subsequent analyses, we include only the S&P 500 return when estimating volatility dynamics of individual stocks.[5]

While the coefficients in Table 2 are broadly similar to those of Andersen, et al. (2007) and Corsi and Reno (2012), we note that daily RV persistence captured by $\hat{\beta}$ is much smaller than the sample first-order autocorrelation of RV $\hat{\rho}$ from section III. The ratio $\hat{\beta}/\hat{\rho}$ is 0.48 for SPY, 0.58 for IJR, and 0.42 for individual stocks: after controlling the long-run dependence and the impact of lagged returns, the daily dependence of RV is much lower than the first-order autocorrelation of daily RV.

[5] This finding contributes to the discussion on the economic mechanisms underlying the asymmetry in stock volatility. By showing that the asymmetry is unrelated to stock returns, our finding supports asymmetry being mostly driven by the volatility feedback effect, not the firm-level financial leverage.

Conditional Volatility Persistence

Evidence from Section III suggests that the expected daily persistence is time-varying and depends on market conditions. We modify the model in (1) to allow the daily persistence coefficient β_D to be conditional on market state variables, i.e. $\beta_t = \rho^e_{t,t+1} = E(\rho_{t,t+1}|I_t)$. This is termed the conditional volatility persistence (CVP_t), with conditioning variables motivated by the findings in section III:

(2) $$CVP_t = \beta_0 + \beta^- r_t^- + \beta^+ r_t^+ + \beta^{RV} RV_t$$
$$CVP_t = \beta_0 + \beta^- r_t^- + \beta^+ r_t^+ + \beta^L RV_t^L + \beta^H RV_t^H$$

To further assess the differential impacts from high and low RVs, daily RV is classified as RV_t^L or RV_t^H, with $RV_t^L = RV_t$ if $RV_t < \delta$, 0 otherwise; $RV_t^H = RV_t$ if $RV_t \geq \delta$, 0 otherwise. In the first specification in (1), δ is set to 0. In the second specification, δ is chosen by a grid-search procedure that minimizes the regression sum of squared residuals (SSR). The CVP parameters in (1) are estimated from the modified HAR model:

(3) $$RV_{t+1} = \alpha + CVP_t RV_t + \beta_W RV_{t,W} + \beta_M RV_{t,M} + \theta r_t + \theta_W r_{t,W} + \theta_M r_{t,M} + \varepsilon_{t+1}$$

Table 3 presents the estimated CVP parameters in (2) from the modified HAR model in (3).[6] Under the first specification in (2), the CVP coefficients ($\hat{\beta}_0$, $\hat{\beta}^-$, $\hat{\beta}^+$, and $\hat{\beta}^{RV}$) are all statistically significant at 1% for SPY, IJR, and individual stocks. Large returns increase RV persistence ($\hat{\beta}^- < 0$ and $\hat{\beta}^+ > 0$). Negative returns have about twice the impact of positive returns ($|\hat{\beta}^-|/\hat{\beta}^+ \approx 2$). High RV is associated with low future persistence ($\hat{\beta}^{RV} < 0$). Individual stock regressions provide clear support to the sign and the significance of the conditional persistence coefficients: $\hat{\beta}^-$ is negative and significant for 75% of the stocks, $\hat{\beta}^+$ is positive and significant for 66% of the stocks, and $\hat{\beta}^{RV}$ is negative and significant for 90% of the stocks. While not reported here, the F statistics resoundingly reject the null hypothesis

[6] To conserve space, we do not report the estimated coefficients of the control variables $Z_t = (RV_{t,W},\ RV_{t,M},\ r_t,\ r_{t,W},\ r_{t,M})'$. They are qualitatively the same as in Table 2, but numerically much smaller.

of β^{-}=β^{+}=β^{RV}=0; both the Aikaike and the Bayesian information criteria are heavily in favour of the conditional persistence model in (3) over the constant persistence model in (1).

When daily RV is classified as RV_t^L or RV_t^H, the threshold parameter δ is determined by grid-search. We set the search range from 10 to 90 percentiles of daily RV with step size = 0.01. The optimal value of δ is determined by the lowest SSR for (3). Table 3 shows that the optimal δ is 1.03 for SPY and 1.90 for IJR. For SPY, there are 72% of daily RV below and 28% above the threshold. The split is 76% and 24% for IJR. Both low and high RVs have negative coefficients and are associated with lower future RV persistence. The slope is much steeper for low RV than it is for high RV. For SPY, $\hat{\beta}^L$ = -0.175 and the median RV 0.55 < 1.03 (Table 1). On a typical day, RV's impact on future persistence is -0.175*0.55 = -0.0963. Assuming return is close to zero on a typical day, this represents a 33% reduction relative to $\hat{\beta}_0$ = 0.291. Similarly for IJR, the impact from a median RV (0.99) is a 16% reduction in persistence relative to $\hat{\beta}_0$ = 0.393. For high RV days, i.e. $RV_t > \delta$, the coefficients are numerically smaller but statistically highly significant. High RV reflects high uncertainty, which takes longer to resolve. Higher RV is still associated with lower persistence, but at slower rate of reduction in persistence. We note that $\hat{\beta}^H$ is very similar in value to $\hat{\beta}^{RV}$. This is not surprising since the ranges for RV_t^H are almost the same as the ranges for RV_t. From Table 1, $RV_t^H \in$ [1.03, 60.3] and $RV_t \in$ [0.033, 60.3] for SPY, $RV_t^H \in$ [1.9, 77.1] and $RV_t \in$ [0.095, 77.1] for IJR. The very high overlapping in value between RV_t^H and RV_t results in very similar estimated coefficients.

For individual stocks, the results in Table 3 are based on the S&P 500 index returns and are qualitatively similar to those of SPY and IJR. Large returns are associated with high CVP, while high RV is associated with low CVP. Although the average $\hat{\beta}^L$ is not significant, we note that 31% of stocks have negative and significant $\hat{\beta}^L$ and only 8% of stocks have positive and significant $\hat{\beta}^L$. The average $\hat{\beta}^H$ is highly significant and reduces CVP. The thresholds for individual stocks range from 0.43 to 10.13 with an average of 3.24. Again the values for RV_t^H and RV_t are highly overlapping, resulting in similar estimated coefficients. While not reported here, the F test strongly rejects the CVP coefficients β^-=β^+=β^L=β^H=0 for SPY, IJR, and individual stocks; the Aikaike and the Bayesian information criteria are very similar for the two models of RV persistence in (2). The adjusted R^2s in Table e are almost identical between the two CVP models. We therefore take the more parsimonious specification $CVP_t = \beta_0 + \beta^- r_t^- + \beta^+ r_t^+ + \beta^{RV} RV_t$ as the baseline model going forward.

For robustness check, we divide the sample into two-year sub-periods. The results are not reported to conserve space. Negative returns are significant in almost all sub-periods. The coefficients of RV is negative significant in most sup-periods. The model in (3) works particularly well during the crisis period of 2008-09, with all conditioning variables highly significant and $\bar{R}^2$ = 0.726 for SPY, 0.741 for IJR, 0.645 for stocks.

Characteristics of Conditional Volatility Persistence

Figure 3 depicts the estimated $\widehat{CVP}_t = \hat{\beta}_0 + \hat{\beta}^- r_t^- + \hat{\beta}^+ r_t^+ + \hat{\beta}^{RV} RV_t$ for SPY and IJR with the estimated coefficients given in Table 3. $\widehat{CVP}$ is very high during the financial crisis in the second half of 2008. There are a few $\widehat{CVP}$ > 1, 3 in 3547 days (0.08%) for SPY and 5 in 3059 days (0.16%) for IJR. The summary statistics for $\widehat{CVP}$ is reported in Panel A of Table 1. The mean of $\widehat{CVP}$ is 0.354 for SPY, 0.456 for IJR, and 0.434 for stocks. $\widehat{CVP}$ is highly persistent, but is less persistent than RV as indicated by the Ljung-Box statistic, consistent with the large impact from returns.

Panel B of Table 1 reports the correlations of $\widehat{CVP}$ with other variables. $\widehat{CVP}$ has a significant positive correlation with return. The effect of r_t on $\widehat{CVP}_t$ can be written as $\frac{\hat{\beta}^+ - \hat{\beta}^-}{2}|r_t| + \frac{\hat{\beta}^+ + \hat{\beta}^-}{2} r_t$. Because $\hat{\beta}^- < 0$ and $\hat{\beta}^+ > 0$, $\frac{\hat{\beta}^+ - \hat{\beta}^-}{2} > 0$ and is much larger than $\frac{\hat{\beta}^+ + \hat{\beta}^-}{2}$; therefore the return size effect is larger than the return direction effect. The unconditional correlation between $\widehat{CVP}$ and RV is positive, which is consistent with Ning, Xu, and Wirjanto (2015). With the exception of illiquidity for IJR, $\widehat{CVP}$ is positively correlated with trade and liquidity variables.

To further assess the impact of return and RV on $\widehat{CVP}$, we decompose the variance of $\widehat{CVP}$ into components associated with the variances of the orthogonalized r_t^-, r_t^+, and RV_t.[7] The contributions of r_t^-, r_t^+, and RV_t to the variance of $\widehat{CVP}$ are denoted as w(r_t^-), w(r_t^+), and w(RV_t) respectively. The decomposition outcome depends on the order of the variables in the orthogonalization process. Table 4 reports the summary of w(r_t^-), w(r_t^+), and w(RV_t) across 3! = 6 permutations for SPY and IJR. For individual stocks, we first calculate the average w(r_t^-), w(r_t^+), and w(RV_t) of each stock, then present the summary statistics across all stocks. The most striking feature of Table 4 is the dominant impact of negative returns on the variations of volatility persistence: negative returns on average account for 72~76% of $\widehat{CVP}$ variance; positive returns account for 16~23%; RV accounts for only 5~7%. The median w(r_t^-), w(r_t^+), and w(RV_t) across stocks are even more skewed toward negative returns, with the median w(r_t^-) = 85%. For SPY and IJR, the pecking order w(r_t^-) > w(r_t^+) > w(RV_t) holds in 5 out of the 6 permutations. In both exceptions, w(r_t^-) > 82% but w(r_t^+) < w(RV_t) ≈ 10%. Based on the average w(r_t^-), w(r_t^+), and w(RV_t), 57 out of 87 stocks (66%) have the same packing order, and 77 out of 87 stocks (89%) have w(r_t^-) being the highest of the three components. Overall we see that the lagged returns, negative and positive, account for almost 95% of the variation in volatility persistence, while the lagged volatility explains the

[7] Let $y = ax_1 + bx_2 + cx_3$. The variance of y is decomposed into components attributed to x_1, x_2, and x_3 based on the following orthogonalization process:

1) Take residuals from regressions $x_2 = \alpha_0 + \alpha_1 x_1 + u_{21}$ and $x_3 = \beta_0 + \beta_1 x_1 + u_{31}$;
2) Run $\hat{u}_{31} = \lambda\hat{u}_{21} + u_{32}$ to get $\hat{u}_{32}$
3) $y = ax_1 + b(\hat{\alpha}_0 + \hat{\alpha}_1 x_1 + \hat{u}_{21}) + c(\hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{u}_{31})$
   $= (a+b\hat{\alpha}_1+c\hat{\beta}_1)x_1 + (b+c\hat{\lambda})\,\hat{u}_{21} + c\hat{u}_{32} + \text{constant}$
   $= Ax_1 + B\hat{u}_{21} + c\hat{u}_{32} + \text{constant}$
4) $var(y) = A^2 var(x_1) + B^2 var(u_{21}) + c^2 var(u_{32})$;
5) $w(x_1) \equiv \frac{A^2 var(x_1)}{var(y)}$, $w(x_2) \equiv \frac{B^2 var(x_2)}{var(y)}$, and $w(x_3) \equiv \frac{c^2 var(x_3)}{var(y)}$.

remaining 5%. The evidence in Tables 3 and 4 suggests that the propagation of volatility over time is not due to high volatility itself, but rather the shocks to the broad market embedded in the current market return.

Additional Conditioning Variables

In addition to daily return and RV, we examine whether daily volatility persistence is affected by volatility jumps (J), number of trades (NT), illiquidity (IL), and the imbalance of buyer- and seller-initiated volumes (VI). The construction and statistical descriptions of these variables are given in section II. Let Y_t be one of these variables on day t. To assess the impact of these variables on volatility persistence, we extend the model in (3) to include Y_t and its interaction with RV_t:

(4) $$RV_{t+1} = \alpha + (\beta + \beta^- r_t^- + \beta^+ r_t^+ + \beta^{RV} RV_t + \beta^Y Y_t) RV_t + \gamma^Y Y_t + \varphi Z_t + \varepsilon_{t+1}$$

Table 5 reports the impact of the additional conditioning variables. For individual stocks, none of the additional conditioning variables has a significant impact on volatility level (γ^Y) or volatility persistence (β^Y). For index ETFs, none of them is significant for both ETFs. Volatility jumps reduces volatility persistence for SPY but increases persistence for IJR. Illiquidity and volume imbalance have no effect on the level and the persistence of IJR volatility. The impact of return and RV on volatility persistence remains intact in Table 3. Our findings are consistent with those of Gillemot, Farmer, and Lillo (2006), who conclude that "the long-memory of volatility is dominated by factors other than transaction frequency or total trading volume."

Long-run Volatility Persistence

As a final robustness check, we examine whether including long-run conditional persistence alters the results for daily conditional persistence. Persistence at weekly and monthly lags is estimated using the regression below:

$$RV_t = \alpha + \sum_{f=D}^{M} \left[\left(\beta_f + \beta_f^- r_{t-1,f}^- + \beta_f^+ r_{t-1,f}^+ + \beta_f^{RV} RV_{t-1,f} \right) RV_{t-1,f} + \theta_f r_{t-1,f} \right] + \varepsilon_t$$

where *f* = day (D), week (W), and month (M). Results presented in Table 6 show that the daily conditional persistence in Table 3 remain largely intact. There is strong evidence of weekly and monthly conditional persistence for individual stocks, but only weak evidence for SPY and IJR. However, unlike positive daily returns which increase daily persistence, positive weekly and monthly returns reduce the corresponding volatility persistence.

V. Information Flow and Volatility Dynamics

In this section we show that the findings in section IV can be explained by the daily price discovery and net information flow. Intraday returns are decomposed into a random-walk component and a serially-correlated component. The random-walk component is traditionally attributed to the arrival of new information on asset value and is termed the information component. However it also captures the price effects from shocks to liquidity demand and supply. The serially-correlated component is termed the liquidity component and captures all other price impact from trading, e.g. feedback trading, inventory management, etc. Price discovery or gross information flow is measured by the variance of the information component. The sum of the information component over a trading day captures the net effect of positive and negative information flows. We show that (1) net information flow increases but price discovery decreases volatility persistence; (2) large negative net information flow has a dominant effect on volatility persistence; (3) the source of volatility persistence is primarily the gross information flow and the covariance between information and non-information returns; (4) the source of the so-called "leverage effect" is primarily the net information flow. These findings shed new light on the mechanisms for volatility persistence.

Measuring Price Discovery and Information Flows

Let there be S sub-periods on a trading day t, and p_s be the logarithmic price observed at the end of sub-period s.[8] It can be viewed as having two components: $p_s = m_s + n_s$, where m_s is the efficient price and n_s is the noise term. The return is $r_s \equiv p_s - p_{s-1} = \Delta m_s + \Delta n_s$. The changes in the efficient price Δm_s are independent over time and represent a permanent shift in the asset value. The term Δn_s is serially correlated over time and captures a wide range of transitory factors, e.g. bid-ask bounce, inventory management, feedback trading, etc. In this study, Δm_s and Δn_s are termed the information and the liquidity returns respectively. Starting from Hasbrouck (1991ab, 1993, 1995), price discovery is measured by the variance of Δm_s in microstructure studies. We follow that tradition and use $\sum_{s=1}^{S} \Delta m_s^2$ as a measure of price discovery or gross information flow, and $\sum_{s=1}^{S} \Delta m_s$ as a measure of net information flow, i.e. the outcome of price discovery over a trading day.

To decompose r_s into Δm_s and Δn_s, r_s is fitted in an AR(K) model: $r_s = A(L)r_s + u_s$ where $A(L) = a_1L + \ldots + a_KL^K$ and L is the lag operator.[9] Since $A(L)r_s$ captures the autocorrelations in r_s, u_s captures the innovations in r_s and is serially independent. A simple return decomposition is to set $\Delta m_s = u_s$ and $\Delta n_s = A(L)r_s$ where Δm_s and Δn_s are orthogonal. We use the Beveridge-Nelson (1980) decomposition which allows Δm_s and Δn_s to be correlated. Let $B(L) = (1-A(L))^{-1}$ thus $r_s = B(L)u_s$. B(L) can be decomposed as $B(L) = B(1) + C(L)(1-L)$ where $C(L) = \sum_{j=0}^{\infty} c_j L^j$ has exponentially decaying coefficient as j increases. The Beveridge-Nelson return decomposition is given by $r_s = B(1)u_s + C(L)\Delta u_s$. Price at time s is $p_s = p_{s-1} + r_s = p_{s-1} + B(1)u_s + \sum_{j=0}^{\infty} c_j \Delta u_{t-j}$. Let I_s be the information set at time s which includes u_s: $\lim_{q\to\infty} E(p_{s+q}|I_s) = p_{s-1} + B(1)u_s$. Therefore the random-walk component $\Delta m_s \equiv B(1)u_s$ captures the permanent price shocks from new information. The serially-correlated

[8] The t subscript is suppressed when it does not cause confusion.
[9] As in Hasbrouck (1991ab and 1993), the AR(K) model is estimated without a constant so that the sum of the estimated residuals, i.e. the aggregate shocks over a trading day, is non-zero.

component $\Delta n_s \equiv C(L)\Delta u_s$ captures the transitory impact of non-information trading. The covariance between Δm_s and Δn_s is $B(1)[1-B(1)]Var(u_s) = -A(1)Var(\Delta m_s)$.

For each trading day t, we calculate 5-minute returns $r_{s,t}$, s = 1,…,78. The maximum number of lags for the AR model is 6, i.e. we allow return autocorrelation up to 30 minutes. The optimal lag length K_t is determined by the average of the Akaike and Bayesian information criteria. The AR coefficients are estimated for each trading day t via OLS. Let A_t be the sum of the estimated AR coefficients on day t: $A_t \equiv \sum_{k=1}^{K_t} \hat{a}_{k,t}$ therefore $B_t(1) = \frac{1}{1-A_t}$. The information return is $\Delta m_{s,t} = \frac{\hat{u}_{s,t}}{(1-A_t)}$ and the liquidity return is $\Delta n_{s,t} = r_{s,t} - \frac{\hat{u}_{s,t}}{(1-A_t)}$. The net information flow for day t is defined as $r_{inf,t} \equiv \sum_{s=1}^{78} \frac{\hat{u}_{s,t}}{(1-A_t)}$. The net liquidity return is $r_{liq,t} \equiv \sum_{s=1}^{78} (r_{s,t} - \frac{\hat{u}_{s,t}}{(1-A_t)})$. The daily $RV_t = \sum_{s=1}^{78} r_{s,t}^2$ is decomposed into three components. The information RV is defined as $RV_{inf,t} \equiv \sum_{s=1}^{78} \Delta m_{s,t}^2$. It measures price discovery or gross information flow for day t. The liquidity RV is defined as $RV_{liq,t} \equiv \sum_{s=1}^{78} \Delta n_{s,t}^2$. The covariance between $r_{inf,t}$ and $r_{liq,t}$ is given by $Cov_t \equiv -A_t RV_{inf,t}$. One can easily verify $RV_{inf,t} + RV_{liq,t} + 2Cov_t = RV_t$.[10]

Table 7 reports the summary statistics of the return and RV decompositions. It shows that following features:

(1) The information and liquidity returns, r_{inf} and r_{liq}, have very different characteristics. For example, r_{inf} has higher standard deviation than r_{liq}, consistent with information trading having greater price impact than liquidity trading. While r_{inf} is negatively skewed and has low kurtosis, r_{liq} is positively skewed and has much higher kurtosis. Even though r_{inf} is the sum of the intraday random-walk returns, it has higher Ljung-Box statistic, thus higher autocorrelations across trading days, than r_{liq}. Since the

[10] Since the AR model is estimated without the first K observations, one has to add the first K returns in $r_{s,t}$ to the vector of $\hat{u}_t$ to make the return and the RV decompositions precise.

AR(K) model does not have a constant, the daily constant is partially embedded in the estimated residual $\hat{u}_{s,t}$, creating autocorrelation in $r_{inf,t} \equiv \sum_{s=1}^{78} \frac{\hat{u}_{s,t}}{(1-A_t)}$. If r_{inf} is viewed as the net outcome of price discovery, i.e. the net information flow, perhaps news arrival is not random: it may persist in one direction for several days.

(2) Daily RV is dominated by its information component. On average RV_{inf} accounts for 93% of daily RV for SPY and IJR, and 83% for large stocks. Therefore daily RV is mostly driven by price discovery or gross information flow. RV_{inf} is relatively more stable than RV_{liq} with lower coefficients of variation. It has much higher Ljung-Box statistic, thus higher persistence. This suggests that price discovery is highly persistent. While liquidity trading may account for a significant portion of daily trading, its gross and net price impacts, RV_{liq} and r_{liq}, appear to be relatively small and less persistent.

(3) The mean and median of daily covariance between r_{inf} and r_{liq} are around zero. Stocks have much higher covariance variations than SPY and IJR. The median size of covariance varies from 5.2% to 8.8% of the daily RV. Covariance is highly negatively skewed but only mildly persistent. It does not appear that information and liquidity trading consistently leads to significant co-movements in returns.

Information Flows and Volatility Persistence

To examine the differential impacts of the information and liquidity components on volatility persistence, we re-estimate the HAR-CVP model in (3) with CVP_t specified as

$$CVP_t = \beta_0 + \beta_{inf}^{-} r_{inf,t}^{-} + \beta_{inf}^{+} r_{inf,t}^{+} + \beta_{liq}^{-} r_{liq,t}^{-} + \beta_{liq}^{+} r_{liq,t}^{+} + \beta_{inf}^{RV} RV_{inf,t}$$

The negative and positive return components are defined as in eq (2). We do not include $RV_{liq,t}$ because of the high correlations between $RV_{inf,t}RV_t$ and $RV_{liq,t}RV_t$ in regression (2), e.g. 0.886 for SPY. Panel A of Table 8 reports the estimated CVP coefficients. We see three features in the table:

(1) Large information returns r_{inf} increase RV persistence, with negative information returns r_{inf}^{-} having greater impact than positive information returns r_{inf}^{+}. Therefore the strong positive impact of return size on volatility persistence, as reported in Tables 3 and 4, is largely driven by r_{inf}, the outcome of price discovery. Intuitively, large returns are associated with high volatility via the news impact curve of Engle and Ng (1993). Large net information flow r_{inf} leads to greater price discovery (RV_{inf}) effort thus high volatility on the next day.

(2) Liquidity returns r_{liq}, positive or negative, do not affect RV persistence. Note that the size of r_{liq} is significant relative to r_{inf}. Using standard deviation (Table 8) as a proxy for return size, the size of r_{liq} is around 28% (stocks) to 46% (SPY) of the size of r_{inf}. The strong impact from returns on CVP (Tables 3 & 4) and the lack of impact from r_{liq} on CVP further supports net information flow as an important determinant of volatility persistence.

(3) RV_{inf} reduces CVP for SPY, IJR, and stocks: more price discovery increases the information content of price and reduces the spillover of uncertainty over time. Overall the evidence indicates that gross and net information flows have significant impact on volatility persistence, while liquidity trading and its price impact do not affect volatility persistence.

Table 4 shows that negative returns account for up to 87% of the variation of CVP. Panel A of Table 8 shows that negative information flow r_{inf}^{-} has a dominant impact on CVP. These findings motivate us to examine the size effect of r_{inf}^{-} using three dummy variables:

D_t^{-} = 1 if $r_{inf,t} < 0$, 0 otherwise;

$D_{v,t}^{-}$ = 1 if $|r_{inf,t}^{-}| > \text{Median}(|r_{inf,t}| \mid r_{inf,t} < 0)$, 0 otherwise;

$D_{\%,t}^{-}$ = 1 if $\frac{|r_{inf,t}^{-}|}{|r_{inf,t}+r_{oth,t}|} > \text{Median}\left(\frac{|r_{inf,t}^{-}|}{|r_{inf,t}+r_{oth,t}|}\right)$, 0 otherwise.

The size dummies are based on the absolute return value ($D^-_{v,t}$) or the relative size to total return ($D^-_{\%,t}$). The conditional volatility persistence is specified as

$$CVP_t = \beta_0 + (\beta_0^- + \beta_v^- D^-_{v,t} + \beta^-_{\%} D^-_{\%,t} + \beta^-_{v\%} D^-_{v,t} D^-_{\%,t}) D^-_t$$

We estimate the impact of r^-_{inf} when it is small (β_0^-), when it is high in value (β_v^-), when it is high as relative size ($\beta^-_{\%}$), and when it is high in both value and relative size ($\beta^-_{v\%}$). Panel B of Table 8 reveals some interesting features:

(1) β_v^- is positive and highly significant: the positive impact of $|r^-_{inf}|$ on CVP is from large $|r^-_{inf}|$ above the median value.

(2) $\beta^-_{\%}$ and $\beta^-_{v\%}$ are not significant for IJR and stocks. The same is true for SPY if its top 2% daily RV were winsorized. The relative size dummy and the interaction $D^-_{v,t} D^-_{\%,t}$ have no effect on most days.

(3) β_0^- is not significant for IJR and stocks, nor is it for SPY if its top 1% daily RV were winsorized. Therefore when r^-_{inf} is below its median value, it has no effect on volatility persistence. Only large information shocks increase the spillover of volatility from today to tomorrow.

Information Flow and Volatility Level

The distinct characteristics of the information and liquidity components, r_{inf} versus r_{liq} and RV_{inf} versus RV_{liq}, motivate us to explore their impact on future RV level as well as their own dynamics. We estimate a variation of the standard HAR model:

$$\begin{aligned} Y_{t+1} &= \alpha + \beta_{inf} RV_{inf,t} + \beta_{liq} RV_{liq,t} + \beta_{cov} Cov_t + \beta_W RV_{t,W} + \beta_M RV_{t,M} \\ &+ \theta_{inf} r_{inf,t} + \theta_{liq} r_{liq,t} + \theta_W r_{t,W} + \theta_M r_{t,M} + \varepsilon_{t+1} \end{aligned}$$

where Y = RV, RV_{inf}, or RV_{liq}. The model separately estimates the impact on future volatility from the three components of the lagged daily RV. If price discovery or the gross information flow is highly persistent, β_{inf} is expected to be positive and significant at least for Y = RV and RV_{inf}. RV_{liq} is small and far less persistent relative to RV and RV_{inf}, therefore may not have a significant impact on these variables. It is still highly persistent; therefore β_{liq} is expected to be significant for Y = RV_{liq}. While the covariance between r_{inf} and r_{liq} is mildly persistent, positive co-movements between information and liquidity trading tends to make price discovery more difficult, resulting in higher uncertainty and high volatility tomorrow. We expect β_{cov} to be positive. The separation of the lagged daily return to r_{inf} and r_{liq} is aimed at testing whether they both lead to the leverage effect or asymmetric volatility.

Table 9 reports the estimated coefficients of the information and liquidity variables. It shows several new features in volatility dynamics:

(1) β_{inf} is positive and significant for Y = RV, RV_{inf}, and RV_{liq}: RV_{inf} is an important determinant of tomorrow's volatility and its components. Note that a positive and significant β_{inf} indicates that RV_{inf} increases tomorrow's volatility level, even though it simultaneously reduces RV persistence as shown in Panel A of Table 9. While CVP is reduced by RV_{inf}, it remains positive, leading to a positive impact from RV_{inf} to future volatility level. This is consistent with Table 8, i.e. price discovery measured by RV_{inf} is the dominant component of daily RV and it is highly persistent. Taken together with Table 9, we conclude that volatility dynamics is dominated by the gross and net information flows.

(2) RV_{liq} has limited impact on future volatility: it is marginally significant for SPY and stocks and Y = RV. Although the contemporaneous correlations between RV_{inf} and RV_{liq} are quite high, 0.681 for SPY and 0.627 for IJR, RV_{liq} has no impact on future RV_{inf}. After controlling for the effects of other variables, RV_{liq} shows no daily persistence for SPY and IJR, contrary to the LB5 in Table 8. While it is persistence for stocks, the impact from RV_{inf} and Cov appears to be equal or larger.

(3) The daily covariance between r_{inf} and r_{liq} has a strong positive impact on future volatility, especially for IJR and stocks. There is a strong negative correlation between

Cov and RV_{inf}, -0.465 for SPY and -5.81 for IJR. Therefore price discovery (RV_{inf}) is low when information and liquidity trading are in the same direction (Cov > 0). Low price discovery today leads to higher volatility tomorrow.

(4) The net information flow r_{inf} is the main driver of asymmetric volatility or the leverage effect in volatility dynamics. However r_{liq} also contributes to the leverage effect in IJR.

VI. Conditional Persistence and Volatility Forecast

This section provides further evidence on the importance of the conditional volatility persistence. Building on the above analyses, we compare the pseudo-out-of-sample volatility forecasts based on the standard HAR model (HAR) against those based on the conditional HAR model (CHAR). In volatility forecast, model parameters are estimated using an expanding or rolling window. Therefore even in models with constant volatility persistence, persistence is re-estimated every day. CHAR explicitly allows persistence to be dependent on return and volatility. If the true persistence indeed varies with return and volatility as demonstrated in the preceding analyses, the CHAR model should lead to superior out-of-sample forecasts.

Evaluation of volatility forecast performance is based on two loss functions: the negative quasi-likelihood function QLIKE$(RV_t, \widehat{RV}_t) = \frac{RV_t}{\widehat{RV}_t} - \ln\left(\frac{RV_t}{\widehat{RV}_t}\right) - 1$ and the logarithmic mean-squared errors LMSE$(RV_t, \widehat{RV}_t) = (\ln(RV_t) - \ln(\widehat{RV}_t))^2$ where $\widehat{RV}_t$ is the forecasted value of RV_t. Patton (2011) shows that QLIKE is robust to the noise in the empirical volatility measures. Patton and Sheppard (2009) show that QLIKE has the best size-adjusted power among robust loss functions. The usual mean-squared error (MSE) is often affected by a few extreme observations. We use the logarithmic MSE to mitigate this problem. Forecasts are based on 6-year rolling windows, starting in 2006 for SPY and in 2008 for IJR and stocks.

Forecast performance is evaluated by the Diebold-Mariano (1995, DM) test. Taking the HAR model as the benchmark, a negative DM statistic indicates a reduction in loss value by CHAR relative to HAR. While HAR is nested in CHAR, Giacomini and White (2006) show that the DM test remains asymptotically valid when the estimation period is finite.

Panel A of Table 10 provides a summary of QLIKE and LMSE values. For both loss functions and across SPY, IJR, and stocks, CHAR has lower mean and median loss values than HAR. The reductions in loss value of CHAR are substantial: e.g. for SPY, CHAR reduces the average QLIKE by 51% and the average LMSE by 45%. Across SPY, IJR, and stocks and for both loss functions, the reduction is 44% for the average loss values and 39% for the median loss values. The DM tests show that the reductions are statistically significant at 10% for stocks with QLIKE, and are significant at 1% for all other cases. CHAR has lower average QLIKE for 85% of the stocks and lower average LMSE for 97% of the stocks. It has lower median loss values than HAR for all stock (100%). We note that the differences between the average and the median QLIKE are quite large, indicating the presence of a few very large values. The problem is not as extreme for LMSE but is still prominent.

Panel B of Table 10 compares forecast performance of HAR and CHAR under different market conditions. Trading days are divided into the high and low RV days based on the median daily RV. The high and low RV days are further divided into thirds: days in the bottom third have large negative returns ($r \ll 0$), days in the top third have large positive returns ($r \gg 0$), and days in the middle third have small returns ($r \approx 0$). For each of the six categories we calculate $(L_{CHAR}-L_{HAR})/L_{HAR}$ where L_{CHAR} and L_{HAR} are the median loss function values of HAR and CHAR. Panel B shows that $L_{CHAR} < L_{HAR}$ for all three asset types and two loss functions. CHAR performs better on low RV days and positive return days ($r \gg 0$). Although negative returns ($r \ll 0$) increase persistence more than positive returns, they are associated with high RVs which reduce volatility persistence. On days with small returns ($r \approx 0$), RV is less persistent therefore more difficult to forecast.

Panel C of Table 10 reports the median loss function values in different sub-periods. CHAR has lower median loss values in all sub-periods for both QLIKE and LMSE. Even during the global financial crisis, CHAR is able to reduce the forecast losses of HAR by 16% to 32%. It is interesting to note that CHAR performs much better than HAR after 2010, even though the in-sample fit of CHAR is not particularly strong in these periods (Table 5).

VII. Conclusion

Contrary to the current literature which views volatility persistence as constant or slow moving, this study shows that the persistence of daily RV has large variations associated with daily return and RV. The propagation of volatility over time not only depends on the level of volatility but also on the time-varying persistence. The finding of return and volatility level as systematic factors affecting volatility persistence should help guide theoretical research on the economic mechanisms for volatility persistence.

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Table 1: Data Summary

Return is the percentage change in log daily closing prices. RV is daily realized variance based on 5-minute returns in percentage. CVP is the conditional volatility persistence described in section IV. Jump is volatility jump. Its statistics are based on days with significant positive jumps. Trades are the number of transactions. TImb is the difference between buyer- and seller-initiated trades. Illiq is the Amihud illiquidity measure. LB5 is the Ljung-Box statistic for 5 lags. The asterisks ***, **, * indicate significance at 1%, 5%, and 10% respectively.

Panel A: Summary statistics

	Mean	Median	St Dev	Skew	Kurt	LB5
SPY						
Return	0.008	0.065	1.32	0.029	12.3	38***
RV	1.13	0.551	2.43	11.4	210	6714***
Jump	0.221	0.101	0.468	8.41	97.4	-
Trades	23.2	13.0	26.3	1.91	8.36	14596***
TImb	-0.091	-0.002	15.4	0.771	18.2	29***
Illiq	0.226	0.059	0.487	4.25	25.7	5462***
CVP	0.354	0.330	0.076	3.01	19.5	709***
IJR						
Return	0.034	0.100	1.52	-0.306	7.30	20***
RV	1.75	0.990	3.05	9.79	167	5853***
Jump	0.420	0.243	1.26	13.2	189	-
Trades	4.58	3.58	4.73	1.49	6.32	10462***
TImb	-0.011	-0.002	0.411	0.451	14.7	6.1
Illiq	2.08	0.828	3.90	4.92	40.1	2517***
CVP	0.456	0.431	0.082	2.86	17.2	203***
Stocks						
Return	0.018	0.037	2.31	-5.37	247	20***
RV	2.50	1.21	5.60	9.64	172	6065***
Jump	0.719	0.354	1.67	7.29	88.2	-
Trades	11.8	9.94	7.69	2.34	17.7	9090***
TImb	0.015	0.020	1.37	0.00	18.2	26***
Illiq	1.30	0.772	1.68	8.57	289	870***
CVP	0.434	0.412	0.071	2.71	25.4	356***

Panel B: Correlations

	Return	RV	CVP	Trades	TImb
SPY					
RV	-0.088***				
CVP	0.064***	0.495***			
Trades	-0.069***	0.458***	0.292***		
TImb	0.387***	0.05***	0.046***	0.013	
Illiq	-0.041**	0.089***	0.042**	-0.319***	0.005
IJR					
RV	-0.039**				
CVP	0.075***	0.419***			
Trades	-0.049***	0.472***	0.278***		
TImb	0.285***	0.036**	0.048***	0.001	
Illiq	-0.03*	0.027	-0.02	-0.316***	0.003
Stocks					
RV	-0.058***				
CVP	0.033*	0.281***			
Trades	-0.048***	0.432***	0.193***		
TImb	0.514***	0.011	0.043**	-0.02	
Illiq	-0.127***	0.146***	0.037**	0.016	0.009

Table 2: Heterogeneous Autoregressive Models

This table reports the coefficients of the following regression:

$$RV_{t+1} = \alpha + \beta RV_t + \beta_W RV_{t,W} + \beta_M RV_{t,M} + \theta r_t + \theta_W r_{t,W} + \theta_M r_{t,M} + \varphi r_t^{SP} + \varphi_W r_{t,W}^{SP} + \varphi_M r_{t,M}^{SP} + \varepsilon_{t+1}$$

where RV_t and r_t are the realized variance and return of SPY, IJR, and individual stocks. The returns of the S&P 500 index r_t^{SP}, $r_{t,W}^{SP}$, and $r_{t,M}^{SP}$ are included only for individual stocks. The t-statistics are based on the Newey–West robust covariance with automatic lag selection using Bartlett kernel. The asterisks ***, **, * indicate significance at 1%, 5%, and 10% respectively.

	β	β_W	β_M	θ	θ_W	θ_M	φ	φ_W	φ_M	α	$\bar{R}^2$
SPY	0.313***	0.394***	0.163**	-0.305***	-0.434***	-0.498***				0.154***	0.631
t-stat	*4.50*	*3.39*	*2.46*	*-3.70*	*-3.90*	*-2.92*				*3.53*	
IJR	0.390***	0.292***	0.193***	-0.306***	-0.439***	-0.459**				0.256***	0.673
t-stat	*9.58*	*4.31*	*3.10*	*-4.44*	*-3.67*	*-2.58*				*4.37*	
Stocks:											
Average	0.328***	0.387***	0.192***	-0.332***	-0.477***	-0.236				0.259***	0.571
Min	0.121	0.044	0.041	-0.799	-2.782	-1.005				0.073	0.155
Max	0.545	0.735	0.547	-0.071	-0.023	2.295				1.516	0.762
HKV t-stat	*5.90*	*5.63*	*3.99*	*-3.18*	*-3.16*	*-1.09*				*3.85*	
%(t≤-1.96)	*0%*	*0%*	*0%*	*76%*	*56%*	*14%*				*0%*	
%(t≥1.96)	*97%*	*95%*	*82%*	*0%*	*0%*	*0%*				*82%*	
Average	0.278***	0.371***	0.211***	-0.084	-0.180	-0.038	-0.569***	-0.927***	-0.957***	0.414***	0.591
Min	0.079	0.048	0.060	-0.677	-2.001	-0.928	-8.552	-3.667	-8.380	0.130	0.169
Max	0.530	0.721	0.506	2.338	0.464	4.165	-0.034	-0.292	-0.228	2.658	0.769
HKV t-stat	*4.76*	*5.66*	*4.62*	*-0.93*	*-1.36*	*-0.17*	*-3.64*	*-4.17*	*-2.95*	*5.28*	
%(t≤-1.96)	*0%*	*0%*	*0%*	*16%*	*10%*	*5%*	*91%*	*94%*	*60%*	*0%*	
%(t≥1.96)	*87%*	*95%*	*93%*	*0%*	*1%*	*0%*	*0%*	*0%*	*0%*	*100%*	

Table 3: Conditional Volatility Persistence

This table reports the daily persistence coefficients of the following models:

$$RV_{t+1} = \alpha + (\beta_0 + \beta^- r_t^- + \beta^+ r_t^+ + \beta^{RV} RV_t) RV_t + \varphi Z_t + \varepsilon_{t+1}$$
$$RV_{t+1} = \alpha + (\beta_0 + \beta^- r_t^- + \beta^+ r_t^+ + \beta^L RV_t^L + \beta^H RV_t^H) RV_t + \varphi Z_t + \varepsilon_{t+1}$$

RV_t^L = RV_t if $RV_t < \delta$; 0 otherwise; RV_t^H = RV_t if $RV_t \geq \delta$; 0 otherwise. The threshold parameter δ is determined by a grid search that minimizes the sum of squared residuals. $Z_t = (RV_{t,W},\ RV_{t,M},\ r_t,\ r_{t,W},\ r_{t,M})'$. For individual stocks, the returns are those of the S&P 500 index and the. The asterisks ***, **, * indicate significance at 1%, 5%, and 10% respectively.

	β_0	β^-	β^+	β^{RV}	β^L	β^H	δ	$\bar{R}^2$
SPY	0.291***	-0.096***	0.0646***	-0.0068***				0.681
t-stat	*3.58*	*-3.36*	*3.49*	*-5.73*				
	0.286***	-0.091***	0.0653***		-0.173***	-0.0069***	1.03	0.681
	3.40	*-3.13*	*3.63*		*-3.39*	*-5.60*		
IJR	0.392***	-0.10***	0.0462***	-0.0084***				0.717
t-stat	*5.56*	*-6.19*	*4.00*	*-6.22*				
	0.393***	-0.098***	0.047***		-0.065***	-0.0085***	1.90	0.718
	5.60	*-6.06*	*4.13*		*-2.67*	*-6.28*		
Stocks								
Average	0.390***	-0.090***	0.041***	-0.0060***				0.638
Min	0.097	-0.173	-0.031	-0.019				0.206
Max	1.034	-0.017	0.109	-0.001				0.825
HKV t-stat	*6.42*	*-4.26*	*3.35*	*-6.26*				
#(t≤-1.96)	*0%*	*75%*	*0%*	*90%*				
#(t≥1.96)	*91%*	*0%*	*66%*	*0%*				
Average	0.392***	-0.090***	0.041***		-0.001	-0.0061***	3.24	0.638
Min	0.115	-0.174	-0.031		-0.677	-0.020	0.430	0.205
Max	1.035	-0.016	0.109		1.147	-0.001	10.13	0.825
HKV t-stat	*6.47*	*-4.30*	*3.34*		*-0.03*	*-6.21*		
#(t≤-1.96)	*0%*	*72%*	*0%*		*31%*	*91%*		
#(t≥1.96)	*91%*	*0%*	*63%*		*8%*	*0%*		

Table 4: CVP Variance Decomposition

This table decomposes the variance of $\widehat{CVP}_t = \hat{\beta}_0 + \hat{\beta}^- r_t^- + \hat{\beta}^+ r_t^+ + \hat{\beta}^{RV} RV_t$ into components associated with the variances of the orthogonalized r_t^-, r_t^+, and RV_t. The estimated coefficients $\hat{\beta}_0$, $\hat{\beta}^-$, $\hat{\beta}^+$, and $\hat{\beta}^{RV}$ are from Table 3. The weights w(r_t^-), w(r_t^+), and w(RV_t) are the percentages of the variance of $\widehat{CVP}_t$ associated with the variances of the orthogonalized r_t^-, r_t^+, and RV_t respectively. For SPY and IJR, the summary statistics are across six permutations in the orthogonalization process. For stocks, the summary statistics are across all stocks based on the average w(r_t^-), w(r_t^+), and w(RV_t) of each stock.

	w(r_t^-)	w(r_t^+)	w(RV_t)
SPY			
Average	71.8%	22.7%	5.5%
Median	72.9%	22.4%	4.7%
Min	53%	6.9%	0.2%
Max	86.2%	36.9%	10.1%
IJR			
Average	86.7%	8.8%	4.5%
Median	87.0%	7.1%	5.0%
Min	75.8%	0.0%	0.6%
Max	95.0%	19.2%	5.9%
Stocks			
Average	76.5%	16.3%	7.3%
Median	84.6%	9.5%	4.4%
Min	2.6%	2.4%	1.5%
Max	94.4%	94.4%	49.9%

Table 5: Additional Conditioning Variables

This table reports the estimated coefficients of the following regression:

$$RV_{t+1} = \alpha + (\beta + \beta^{-} r_t^{-} + \beta^{+} r_t^{+} + \beta^{RV} RV_t + \beta^{Y} Y_t) RV_t + \gamma^{Y} Y_t + \varphi Z_t + \varepsilon_{t+1}$$

where $r_t^{-} = r_t D_t$ with $D_t = 1$ if $r_t < 0$ and 0 otherwise; $r_t^{+} = r_t(1 - D_t)$. Y_t is one of the following variables: volatility jump (J), the number of trades in thousand (NT), illiquidity (IL), and volume imbalance (VI). $Z_t = (RV_{t,W},\ RV_{t,M},\ r_t,\ r_{t,W},\ r_{t,M})$'. The asterisks ***, **, * indicate significance at 1%, 5%, and 10% respectively.

	β	β^{-}	β^{+}	β^{RV}	β^{Y}	γ^{Y}	$\bar{R}^2$
SPY:							
J	0.322***	-0.141***	0.070***	-0.011***	-0.070**	0.222	0.718
t-stat	*3.97*	*-2.99*	*4.11*	*-5.63*	*-2.02*	*0.65*	
NT	0.273***	-0.129***	0.06***	-0.014***	0.351	-0.574	0.713
	3.13	*-3.08*	*3.46*	*-3.85*	*1.24*	*-0.82*	
IL	0.361***	-0.136***	0.068***	-0.011***	-0.088*	0.381***	0.716
	4.60	*-3.15*	*4.02*	*-5.59*	*-1.79*	*2.78*	
VI	0.322***	-0.118***	0.092***	-0.009***	-0.003*	0.006**	0.720
	4.36	*-2.86*	*4.54*	*-6.52*	*-1.76*	*2.28*	
IJR:							
J	0.367***	-0.16***	0.073***	-0.013***	0.026***	-0.791**	0.713
t-stat	*4.09*	*-3.50*	*3.78*	*-5.56*	*2.86*	*-2.05*	
NT	0.278***	-0.154***	0.062**	-0.007***	-2.58	38.07***	0.712
	3.32	*-3.01*	*2.55*	*-8.32*	*-0.39*	*2.71*	
IL	0.312***	-0.151***	0.059***	-0.007***	-0.005	0.014	0.711
	2.62	*-3.25*	*3.26*	*-7.18*	*-1.21*	*1.15*	
VI	0.324***	-0.145***	0.067***	-0.009***	-0.112	-0.094	0.712
	3.09	*-3.18*	*3.20*	*-8.42*	*-1.50*	*-0.47*	
Stocks							
J	0.407***	-0.100***	0.040***	-0.006***	-0.014	0.011	0.651
HKV t-stat	*6.68*	*-4.69*	*3.27*	*-6.13*	*-0.77*	*0.04*	
NT	0.351***	-0.097***	0.041***	-0.006***	0.000	0.000	0.651
	4.86	*-4.89*	*3.44*	*-5.16*	*0.08*	*0.01*	
IL	0.470***	-0.100***	0.039***	-0.006***	-0.012	-0.220	0.654
	8.56	*-7.00*	*4.57*	*-3.98*	*-0.96*	*-1.03*	
VI	0.390***	-0.095***	0.042***	-0.006***	-0.012	-0.131	0.648
	7.49	*-5.52*	*3.35*	*-6.41*	*-0.21*	*-0.71*	

Table 6: Long-Run Conditional Persistence

This table reports the coefficients of the following regression:

$$RV_{t+1} = \alpha + \sum_{f=D}^{M}\left[\left(\beta_f + \beta_f^- r_{t,f}^- + \beta_f^+ r_{t,f}^+ + \beta_f^{RV} RV_{t,f}\right)RV_{t,f} + \theta_f r_{t,f}\right] + \varepsilon_{t+1}$$

where *f* = day (D), week (W), and month (M). The asterisks ***, **, * indicate significance at 1%, 5%, and 10% respectively.

	β_D	β_D^-	β_D^+	β_D^{RV}	β_W	β_W^-	β_W^+	β_W^{RV}	β_M	β_M^-	β_M^+	β_M^{RV}	$\bar{R}^2$
SPY	0.310***	-0.126***	0.061***	-0.01***	0.191***	-0.053	-0.041	0	0.349***	-0.156*	-0.16	-0.019***	0.721
	4.74	*-3.35*	*3.20*	*-4.63*	*2.27*	*-1.45*	*-0.83*	*0.07*	*4.59*	*-1.77*	*-1.59*	*-3.52*	
IJR	0.271**	-0.149***	0.058***	-0.008***	0.167*	-0.076*	-0.044	-0.001	0.361***	-0.088	-0.028	-0.017***	0.722
	2.32	*-3.41*	*3.11*	*-11.2*	*1.65*	*-1.75*	*-1.00*	*-0.28*	*4.36*	*-1.26*	*-0.31*	*-4.16*	
Stocks	0.344***	-0.094***	0.034**	-0.006***	0.388***	-0.071**	-0.077***	-0.005**	0.279***	-0.141***	-0.180**	-0.010***	0.659
HKV t-stat	*5.45*	*-4.92*	*2.43*	*-5.68*	*5.07*	*-2.36*	*-2.79*	*-2.46*	*4.19*	*-2.74*	*-2.31*	*-3.71*	
%(t≤-1.96)	*0%*	*83%*	*0%*	*86%*	*0%*	*31%*	*45%*	*44%*	*0%*	*57%*	*46%*	*69%*	
%(t≥1.96)	*89%*	*0%*	*41%*	*0%*	*94%*	*1%*	*2%*	*1%*	*84%*	*1%*	*0%*	*0%*	

Table 7: Information Components of Daily Return and Variance

This table reports the decomposition of daily return and variance into information and liquidity components. r_{inf} and RV_{inf} are the information components of daily return and RV. r_{liq} and RV_{liq} are the liquidity components. Cov is the covariance between r_{inf} and r_{liq}. CV is the coefficient of variation. LB5 is the Ljung-Box statistic for 5 lags. The asterisks ***, **, * indicate significance at 1%, 5%, and 10% respectively.

	r_{inf}	r_{liq}	RV_{inf}	RV_{inf}/RV	RV_{liq}	RV_{liq}/RV	Cov	2Cov/RV
SPY								
Mean	0.015	-0.021	1.06	0.936	0.09	0.081	-0.01	-0.017
Median	0	-0.001	0.477	0.917	0.023	0.044	0.009	0.052
StDev	0.977	0.452	2.68	0.462	0.298	0.183	0.607	0.59
Skew	-0.23	1.54	16.3	7.96	11.7	23.7	-16.5	-14.6
Kurt	12.8	54.6	448	168	187	846	446	405
CV	65.1	-21.5	2.5	0.5	3.3	2.3	-60.7	-34.7
LB5	39.0***	8.96	4240***	16.9***	741.2***	6.58	24.6***	12.9**
IJR								
Mean	-0.006	-0.015	1.68	0.934	0.196	0.111	-0.06	-0.045
Median	0	-0.003	0.838	0.882	0.066	0.07	0.031	0.088
StDev	1.28	0.389	3.54	0.504	0.76	0.154	1.20	0.586
Skew	-0.116	0.607	9.53	2.58	23.6	5.76	-20.2	-4.42
Kurt	11.9	60.8	133	16.8	750	64.2	574	35.5
CV	-213.3	-25.9	2.1	0.5	3.9	1.4	-20.0	-13.0
LB5	32.6***	19.8***	3521***	39.9***	83.5***	10.7*	20.2***	11.6**
Stocks								
Mean	-0.002	0.003	2.37	0.834	0.876	0.271	-0.33	-0.105
Median	0.065	0	0.874	0.745	0.221	0.189	0.038	0.081
StDev	1.01	0.284	20.0	2.47	16.3	2.25	16.4	4.60
Skew	-0.117	1.45	17.2	12.2	22.2	16.3	-24.8	-17.9
Kurt	13.2	29.5	558	474	862	694	1059	707
CV	-505	94.7	8.4	3.0	18.6	8.3	-49.7	-43.8
LB5	25.0***	20.8***	2685***	18.7***	799***	89.7***	48.3***	7.9

Table 8: Information Flow and Volatility Persistence

The impact of price discovery variables is estimated via the following regression:

$$RV_{t+1} = \alpha + CVP_t RV_t + \beta_W RV_{t,W} + \beta_M RV_{t,M} + \theta r_t + \theta_W r_{t,W} + \theta_M r_{t,M} + \varepsilon_{t+1}$$

The daily conditional volatility persistence is specified below. The coefficients in the CVP specification are estimated via the RV regression. The asterisks ***, **, * indicate significance at 1%, 5%, and 10% respectively.

Panel A: $CVP_t = \beta_0 + \beta_{inf}^{-} r_{inf,t}^{-} + \beta_{inf}^{+} r_{inf,t}^{+} + \beta_{liq}^{-} r_{liq,t}^{-} + \beta_{liq}^{+} r_{liq,t}^{+} + \beta_{inf}^{RV} RV_{inf,t}$

	β_0	β_{inf}^{-}	β_{inf}^{+}	β_{liq}^{-}	β_{liq}^{+}	β_{inf}^{RV}	$\bar{R}^2$
SPY							
Coeff	0.292***	-0.103***	0.037**	0.016	-0.016	-0.005*	0.664
t stat	*4.59*	*-4.85*	*2.21*	*0.26*	*-1.16*	*-1.67*	
IJR							
Coeff	0.381***	-0.062***	0.007	-0.045	0.034	-0.003**	0.692
t stat	*4.20*	*-3.10*	*0.518*	*-0.744*	*0.826*	*-2.47*	
Stocks							
Ave Coeff	0.267***	-0.151***	0.043***	0.071	-0.021	-0.003**	0.649
HKV t stat	*4.13*	*-5.79*	*2.83*	*1.01*	*-0.93*	*-2.22*	
t <= -1.96	*0%*	*94%*	*0%*	*1%*	*22%*	*44%*	
t >= 1.96	*68%*	*0%*	*56%*	*7%*	*5%*	*5%*	

Panel B: $CVP_t = \beta_0 + (\beta_0^{-} + \beta_v^{-} D_{v,t}^{-} + \beta_{\%}^{-} D_{\%,t}^{-} + \beta_{v\%}^{-} D_{v,t}^{-} D_{\%,t}^{-}) D_t^{-}$

where $D_t^{-} = 1$ if $r_{inf,t} < 0$, 0 otherwise; $D_{v,t}^{-} = 1$ if $|r_{inf,t}^{-}| >$ Median$(|r_{inf,t}| \mid r_{inf,t} < 0)$; $D_{\%,t}^{-} =$ 1 if $\frac{|r_{inf,t}^{-}|}{|r_{inf,t} + r_{oth,t}|} >$ Median$\left(\frac{|r_{inf,t}^{-}|}{|r_{inf,t} + r_{oth,t}|}\right)$, 0 otherwise.

	β_0	β_0^{-}	β_v^{-}	$\beta_{\%}^{-}$	$\beta_{v\%}^{-}$	$\bar{R}^2$
SPY						
Coeff	0.269***	-0.192**	0.438***			0.650
t stat	*3.55*	*-2.17*	*5.06*			
Coeff	0.268***	-0.191**	0.562***	0.147*	-0.465***	0.659
t stat	*3.34*	*-2.21*	*5.14*	*1.75*	*-2.95*	
IJR						
Coeff	0.328***	0.061	0.17**			0.683
t stat	*8.51*	*0.934*	*2.04*			
Coeff	0.333***	0.103	0.327**	-0.111	-0.219	0.695
t stat	*9.58*	*1.22*	*2.33*	*-1.32*	*-1.44*	
Stocks						
Ave Coeff	0.226***	-0.013	0.224***			0.596
HKV t stat	*3.61*	*-0.239*	*3.32*			
t <= -1.96	*0%*	*7%*	*0%*			
t >= 1.96	*67%*	*2%*	*69%*			
Ave Coeff	0.219***	-0.026	0.176***	0.041	0.077	0.602
HKV t stat	*3.44*	*-0.496*	*2.56*	*0.65*	*0.608*	
t <= -1.96	*0%*	*9%*	*0%*	*1%*	*3%*	
t >= 1.96	*66%*	*3%*	*41%*	*2%*	*9%*	

Table 9: Information Flow and Volatility Level

$$Y_{t+1} = \alpha + \beta_{inf}RV_{inf,t} + \beta_{liq}RV_{liq,t} + \beta_{cov}Cov_t + \beta_W RV_{t,W} + \beta_M RV_{t,M}$$
$$+\theta_{inf} r_{inf,t} + \theta_{liq} r_{liq,t} + \theta_W r_{t,W} + \theta_M r_{t,M} + \varepsilon_{t+1}$$

Y = RV, RV_{inf}, or RV_{liq}. The asterisks ***, **, * indicate significance at 1%, 5%, and 10% respectively.

	β_{inf}	β_{liq}	β_{cov}	θ_{inf}	θ_{liq}	$\bar{R}^2$
SPY						
RV	0.384***	0.255**	0.36	-0.388***	-0.239*	0.635
t stat	*6.20*	*1.96*	*1.54*	*-3.65*	*-1.67*	
RV_{inf}	0.28***	0.093	0.372***	-0.362***	-0.079	0.621
t stat	*6.29*	*0.822*	*2.74*	*-3.98*	*-0.657*	
RV_{liq}	0.052*	-0.006	0.011	-0.046***	-0.001	0.140
t stat	*1.85*	*-0.107*	*0.227*	*-2.70*	*-0.042*	
IJR						
RV	0.406***	0.151	0.492***	-0.323***	-0.402**	0.667
t stat	*9.44*	*0.757*	*4.49*	*-4.14*	*-1.99*	
RV_{inf}	0.461***	-0.239	0.364**	-0.367***	-0.703***	0.659
t stat	*7.82*	*-0.825*	*2.23*	*-3.86*	*-3.42*	
RV_{liq}	0.04**	-0.031	0.033	-0.021	-0.159	0.118
t stat	*1.99*	*-0.483*	*0.858*	*-1.31*	*-0.946*	
Stocks						
RV	0.335***	0.186*	0.469***	-0.743***	-0.372	0.592
HKV t stat	*5.83*	*1.65*	*3.78*	*-3.46*	*-1.37*	
t <= -1.96	*0%*	*5%*	*0%*	*90%*	*13%*	
t >= 1.96	*91%*	*36%*	*64%*	*0%*	*1%*	
RV_{inf}	0.309***	0.091	0.398***	-0.741***	-0.022	0.434
HKV t stat	*6.00*	*0.823*	*3.34*	*-3.77*	*-0.080*	
t <= -1.96	*0%*	*7%*	*0%*	*76%*	*1%*	
t >= 1.96	*87%*	*6%*	*49%*	*0%*	*0%*	
RV_{liq}	0.088***	0.091***	0.129***	-0.106***	0.035	0.233
HKV t stat	*6.77*	*3.45*	*4.58*	*-3.20*	*0.344*	
t <= -1.96	*0%*	*3%*	*1%*	*20%*	*3%*	
t >= 1.96	*54%*	*23%*	*38%*	*0%*	*1%*	

IJR RV_{inf} is winsorized to 3RV. β_{liq} becomes insignificant for RV_{inf}. β_{inf} and β_{cov} become smaller. Winsorizing SPY RV_{inf} does not make a qualitative difference.

Table 10: Volatility Forecast Comparison

Panel A: The average, median, and standard deviation of the loss functions QLIKE and LMSE for the standard HAR and the condition HAR (CHAR) models.

	QLIKE		LMSE	
	HAR	CHAR	HAR	CHAR
SPY				
Average	1.489	0.729	1.157	0.631
Median	0.170	0.098	0.363	0.201
St Dev	9.94	6.37	2.15	1.53
%Days(CHAR<HAR)	67%		67%	
DM statistic	-2.86		-8.56	
IJR				
Average	0.876	0.31	0.771	0.406
Median	0.112	0.079	0.232	0.160
St Dev	5.88	2.54	1.67	0.93
%Days(CHAR<HAR)	64%		63%	
DM statistic	-3.36		-7.31	
Stocks				
Average	1.467	1.040	0.927	0.667
Median	1.153	0.512	0.861	0.569
St Dev	18.9	15.5	1.98	1.48
%Stk(CHAR<HAR|mean)	85%		97%	
%Stk(CHAR<HAR|median)	100%		100%	
Average DM	-1.79		-5.19	
Median DM	-1.94		-5.59	
%Stk(DM<-1.96)	47%		90%	
%Stk(DM>1.96)	1%		1%	

Panel B: Loss function comparison: $(L_{CHAR}/L_{HAR})-1$ where L_{CHAR} and L_{HAR} are the median loss function values of the conditional and the standard HAR models respectively. High and low RVs are based on its median value. Returns are divided into lowest ($r<<0$), middle ($r\approx0$), and highest ($r>>0$) thirds.

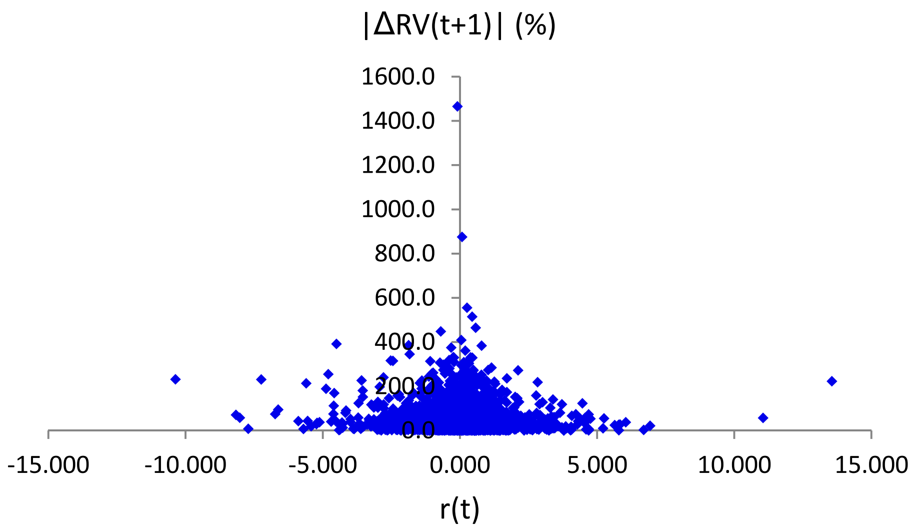
	QLIKE			LMSE		
	r<<0	r≈0	r>>0	r<<0	r≈0	r>>0
SPY						
Low RV	-37%	-33%	-79%	-42%	-37%	-77%
High RV	-31%	-39%	-48%	-32%	-35%	-44%
IJR						
Low RV	-43%	-28%	-74%	-44%	-29%	-63%
High RV	-23%	-0.7%	-50%	-22%	-4.9%	-45%
Stocks						
Low RV	-34%	-21%	-49%	-39%	-20%	-44%
High RV	-16%	-16%	-28%	-18%	-18%	-28%

Panel C: Median daily loss function values in sub-periods

	QLIKE			LMSE		
	HAR	CHAR	Difference	HAR	CHAR	Difference
SPY						
2006-07	0.101	0.078	-23%	0.220	0.164	-26%
2008-09	0.098	0.069	-29%	0.196	0.134	-32%
2010-11	0.289	0.125	-57%	0.648	0.262	-60%
2012-14	0.303	0.117	-62%	0.738	0.245	-67%
IJR						
2008-09	0.062	0.047	-24%	0.126	0.102	-19%
2010-11	0.124	0.081	-35%	0.267	0.164	-39%
2012-14	0.169	0.104	-38%	0.378	0.231	-39%
Stocks						
2008-09	0.087	0.073	-16%	0.178	0.150	-16%
2010-11	0.168	0.114	-32%	0.359	0.238	-34%
2012-14	0.156	0.110	-29%	0.335	0.235	-30%

Figure 1: Return and RV Persistence

Panel A: S&P 500 ETF (ticker SPY)



Panel B: S&P 600 ETF (ticker IJR)

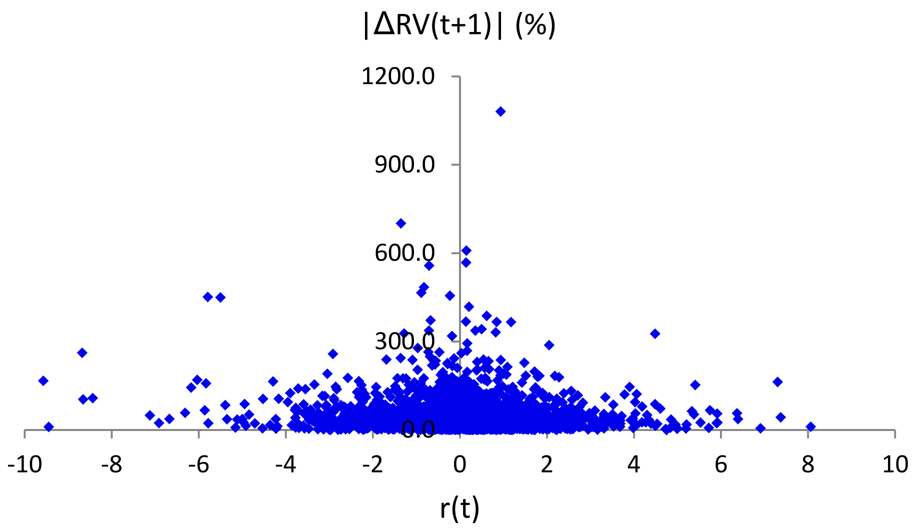
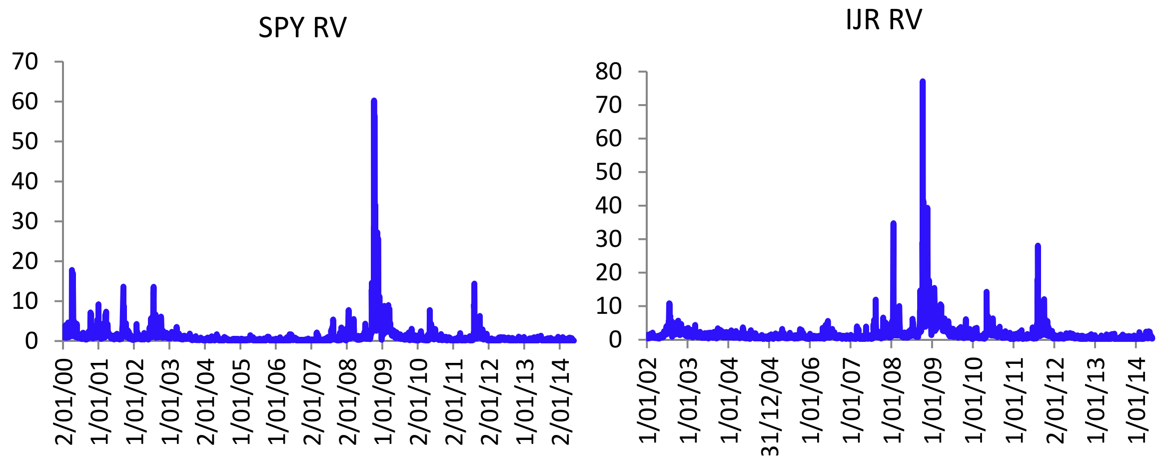
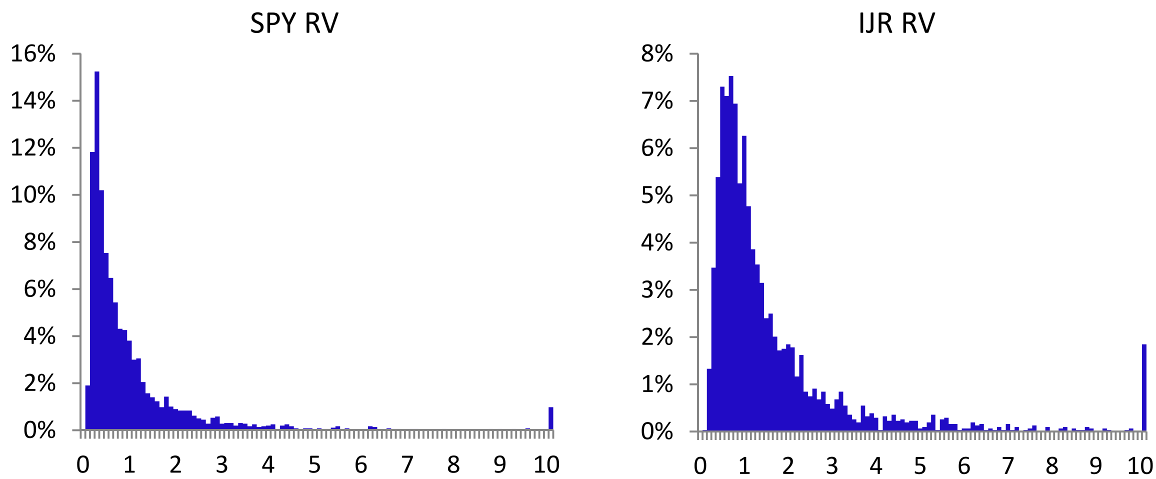


Figure 2: Daily Realized Variance

Panel A: Time series of RV



Panel b: Histogram of RV



Panel C: Daily return and RV

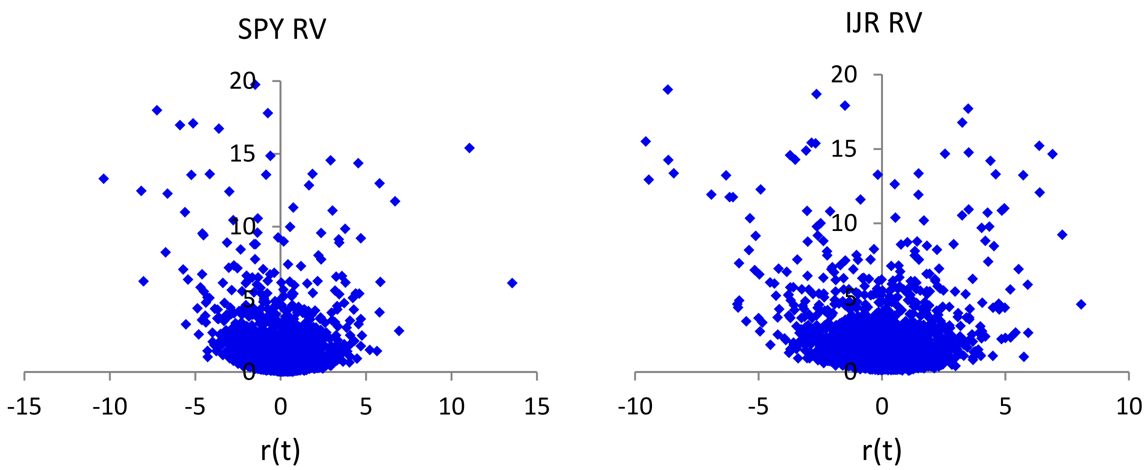
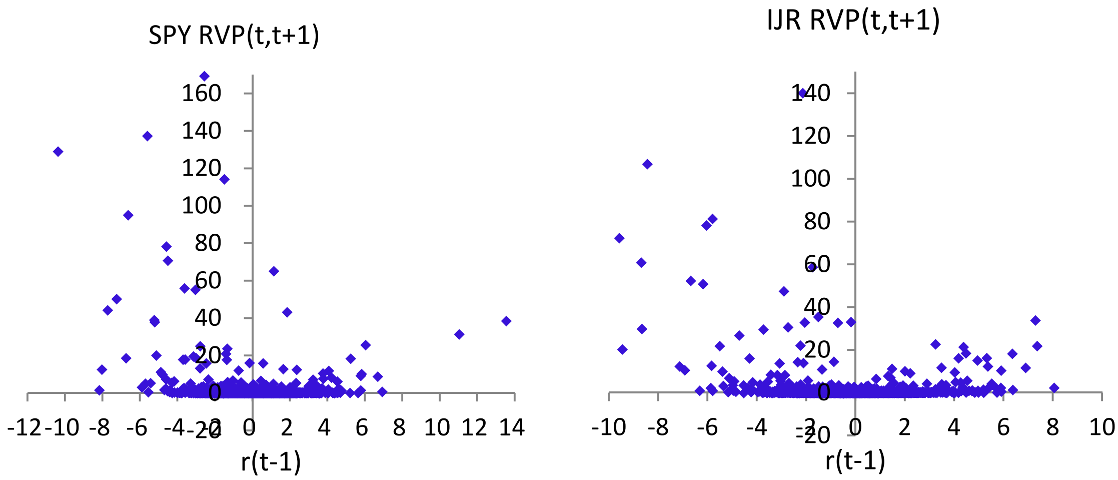


Figure 3: Return, RV, and the Proxy for RV Persistence

Panel A: Return and the proxy for RV persistence (RVP)



Panel B: RV and the proxy for RV persistence (RVP) when RV ≤ 3

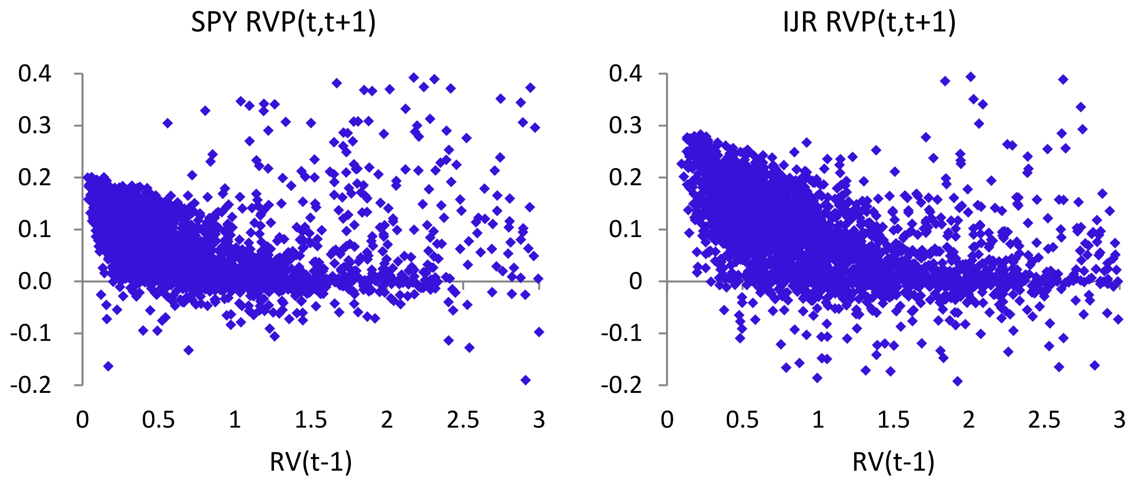
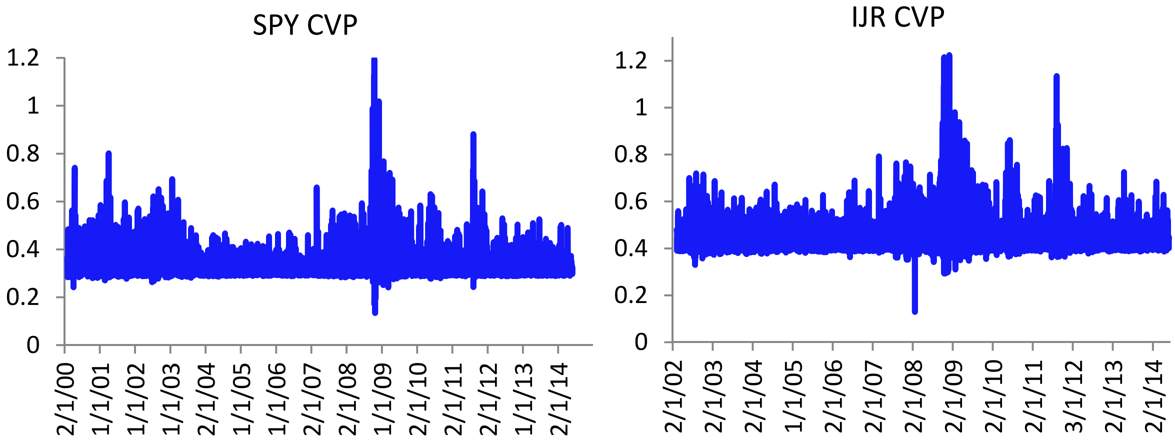


Figure 4: Daily Conditional Volatility Persistence

Panel A: Time series of CVP



Panel B: Histogram of CVP.