Interest Rate Risk, Term Spreads and the Mortgage Contract Term

Melanie Sturm and Bertram I. Steininger

RWTH Aachen University, School of Business and Economics, Templergraben 64, 52062 Aachen, Germany and ZEW Mannheim, L7, 1, 68161 Mannheim, Germany Tel.: +49 241 80 93653, Fax: +49 241 80 693653, bertram.steininger@rwth-aachen.de (corresponding author)

January 16, 2018

Abstract

A borrower of a mortgage can choose between fully bearing the interest rate chance risk and paying a term spread to be protected against fluctuating mortgage rates. By using a one-period model, we study the choice between a fully adjustable and a fully fixed-rate mortgage. Furthermore, we examine with a life cycle model whether a mortgage is best broken down into several short-to-medium-term fixed-rate mortgages – a common form in various mortgage markets but only rarely analyzed in research. We are among the first, who show that borrowers with high risk aversion, non-amortizing mortgages, a large mortgage, and a low probability of moving are better off with long-term contracts. Amortizing mortgages are best broken down into several contracts with the optimal contract term generally declining as the mortgage ages. Initial contracts may be shorter than following contracts, only if borrowers expect to benefit from decreasing interest rates. For non-amortizing mortgages, a fully fixed-rate mortgage is superior, unless interest rates are expected to decrease significantly.

Introduction

A mortgage is usually a long-term contract that requires the borrower to make interest and amortization payments for more than 20 or even 30 years. When the contract is concluded, the mortgage amortization schedule is generally fixed and future amortization payments are known. Optionally, borrowers may (partially) prepay the mortgage. However, they cannot be forced to make such unscheduled amortization payments. Depending on the strategy for managing the interest rate risk between the lender and the borrower, the mortgage rate and consequently future interest payments are known in advance or stochastic. A more detailed overview of strategies for managing the interest rate risk of a mortgage in various countries is provided in Scanlon and Whitehead [2004] and in a study conducted by the European Mortgage Federation [2006].

In general, mortgage rates can be classified into two main categories: the mortgage rate adjusts periodically to a reference rate of interest or is fixed and remains unchanged until maturity. The former shifts the interest rate risk onto the borrower, who therefore benefits from decreasing and suffers from increasing interest rates. For exapmle, this system is predominnat in Australia, Ireland, and Spain. Holders of fixed-rate mortgages are locked into the mortgage rate and the lender shoulders the interest rate risk, but may then benefit from favorable movements in interest rates. The USA is the most prominent example of a country in which the mortgage rate is usually fixed until the mortgage is paid off. Even if fully fixed-rate mortgages are unusual in other industrialized countries, they are also common in Canada, France and Denmark.

The borrower's choice between an adjustable-rate mortgage and a fixed-rate mortgage has been researched several times. Campell and Cocco [2003] found that a household with a large mortgage, precarious income, high risk aversion, a high cost of default, and a low probability of moving, are better off with a fixed-rate mortgage. Dhillon et al. [1987] confirm these findings by analyzing economic data on mortgage borrowing. They find that households with a more stable income and higher moving probability are more likely to choose adjustable-rate mortgages.

Besides these two extremes, further contracts are offered which split the risk of interest rate changes between the borrower and the lender.

On the one hand, the mortgage rate may vary to a limited extent in terms of the size of change. Adjustable-rate mortgages, for example, are often combined with caps and floors. Interest rate fluctuations are then borne partly by, or partly benefit both the lender and the borrower.[1] Iceland is unusual in capping the interest payments relative to borrower income by indexing the mortgage rate to inflation. Effectively, the borrower pays a determined real interest rate plus inflation.[2]

On the other hand, the mortgage rate may vary to a limited degree in terms of the frequency. In numerous markets, the mortgage rate is fixed for an initial period, which is shorter than the amortization period. Thereafter, the mortgage rate is renegotiated whenever the current contract expires and is renewed. As a result, the borrower takes out a series of contracts until the mortgage is amortized. Canada, the Netherlands, Germany and Switzerland, for example, are dominated by such short-to-medium-term fixed-rate mortgages which allow frequent rate adjustments. In these markets, the mortgage is commonly broken down into different contracts with a term of 5 to 10 years on average. [3]

Figure 1 provides an overview of the contract terms in various countries, including 1 to 5 years, 5 to 10 years and longer than 10 years, as well as fully fixed-rate mortgages.[4]

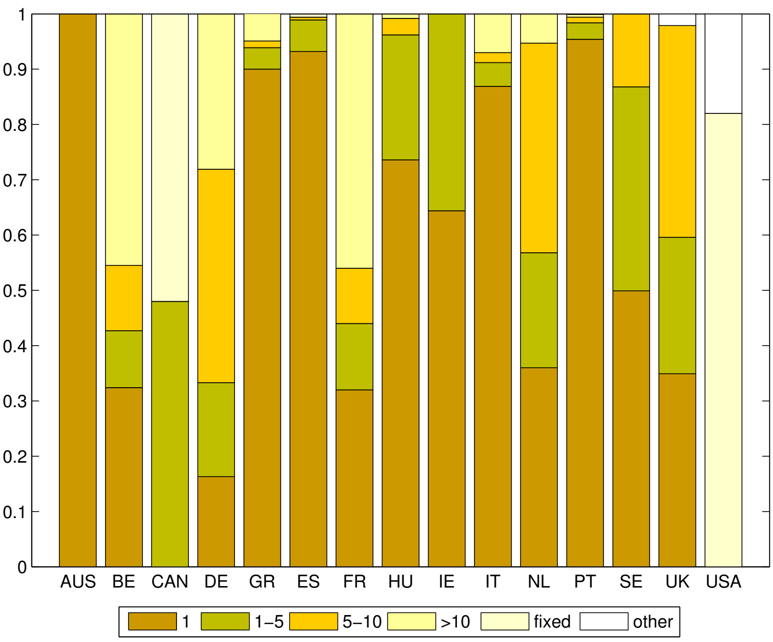

Figure 1: Contract term as a percentage of gross lending, Source: Scanlon and Whitehead [2004] and European Mortgage Federation [2006]

Two main factors motivate borrowers to break down the mortgage into several contracts rather than take out a fully adjustable-rate or fixed-rate mortgage. First, locking into the current rate causes the borrower to pay a term spread. For a steep yield curve, fixing the mortgage rate for the term to maturity of more than 20 or even 30 years can become very costly. Second, an adjustable-rate mortgage is usually associated with a lower mortgage rate, but exposes the borrower to substantial interest rate risk if, during the mortgage term of up to 30 years, interest rates vary substantially. Splitting the mortgage into several short-to-medium-term fixed-rate contracts, on the on hand, limits the costs of locking into the mortgage rate, because, in the presence of an upward sloping yield curve, a lower term spread is charged for shorter-term contracts. On the other hand, breaking down the mortgage into several contracts limits the interest rate risk, relative to an adjustable-rate mortgage, because the mortgage rate adjusts less frequently to the market rate of interest. However, the borrower is shielded from interest rate fluctuations only during the term of each single contract and therefore still faces interest rate risk. The mortgage contract is renewed each time a contract expires while the balance has not yet been paid off, so that the mortgage rate adjusts to the market rate of interest.

This article examines the optimal breakdown of a mortgage in a series of short-to-medium-term fixed-rate mortgages from the borrower perspective, with a focus on the trade-off between bearing the interest rate risk and paying a term spread in order to be protected against rate fluctuations. To the best of our knowledge, there is no previous research addressing this contract design, even though it is widespread in numerous markets.

Both the borrower and the lender may favor short or long-term contracts, depending on their preferences. Therefore, even though this article focuses on the borrower's perspective, not only the borrower's choice of contract term is considered, but also the lender's influence on the borrower's behavior. A one-period model is used to examine the trade-off between bearing interest rate risk or paying a term spread when interest rates follow a mean-reverting process. A basic model rules out follow-up financing and allows the borrower to either take out a fully fixed-rate mortgage or a fully adjustable one. The model is used to examine the impact of the model parameters, such as borrower risk aversion, mortgage balance and parameters determining the interest rate process on the borrower choice of the contract design. A longer period model examines whether, due to the long horizon of a mortgage, the borrower is better off breaking down the mortgage into several contracts, rather than taking out a fully fixed-rate or adjustable-rate mortgage. The mortgage agreement is concluded for an initial period. The borrower and lender renegotiate the mortgage conditions on the expiration date, and fix the term of the new contract. Attention is paid to how many contracts the borrower should take out, as well as to their optimal term. In other words, it is shown whether borrowers are better off sharing the interest rate risk with the lender through entering a series of contracts, rather than taking out a fully adjustable or fixed-rate mortgage.

The contract term from the borrower and lender perspective

Interest rate volatility and the spread between short-term and long-term rates are important determinants of borrower choice between adjustable and fixed-rate mortgages. Long-term rates mostly exceed short-term rates, meaning that yield curves are usually upward sloping. This is explained by investors demanding a risk premium or a liquidity premium, respectively, for locking into the current rate, because long-term investments are generally associated with greater risk. Downward sloping yield curves rarely occur, but may do so in economic depressions when there are high risks in the short run.

In the presence of an upward sloping yield curve, fixing the mortgage rate requires the borrower to pay a term spread, but ensures that future mortgage payments are known in advance. From the borrower's perspective, a wide and positive rate spread makes locking into a mortgage rate very costly. Smith [1987] and Templeton et al. [1996] showed that an adjustable-rate mortgage is then more favorable.[5] However, with an adjustable-rate mortgage, the rate is tied to the volatile market rate of interest and varies. Therefore, the borrower is exposed to interest rate risk. Over the course of American history, in the great depression of the early 1930s as well as during the recent subprime crisis, adjustable-rate mortgages led to substantial stress when borrowers could not afford the reset mortgage rate. Previous research by VanderHoff [1996] also revealed that borrowers who take out adjustable-rate mortgages and are exposed to interest rate risk, default more often than fixed-rate mortgage borrowers. By contrast, in several countries, the dominance of adjustable-rate mortgages is noted as a reason for lower default rates [see Lea, 2010]. Vandell [1978] predicted that, in the first four years following mortgage origination, borrowers with adjustable-rate mortgages default less often than those with fixed-rate mortgages. Overall, at least for moderately fluctuating interest rates, the default risk of adjustable-rate mortgages is found to be similar to that of a fixed-rate mortgage.

Given these diverging experiences, both lenders and mortgage regulators need to understand how borrowers self-select the contract term in order to correctly evaluate and price the risk associated with a mortgage.[6] Basically, when choosing the contract term, the borrower decides on the term spread to be paid and the interest rate risk to be borne, which equates to balancing between paying costly protection against fluctuating rates or facing volatile mortgage rates. This article addresses several issues influencing borrower behavior, including not only the yield curve and the interest rate process, but also borrower risk aversion and the initial mortgage balance.

In principle, borrowers tend to choose the contract design which leads to the lowest costs, meaning that low term spreads lead borrowers to favor fixed-rate or long-term mortgages, and less volatile interest rates lead borrowers to bear the interest rate risk of adjustable-rate or short-term mortgages. For example, in an extreme case, the market rate of interest may be constant, meaning that there is no need to fix the rate at high cost. By contrast, interest rates may follow a random walk and be unpredictable given historic rates, so that adjustable-rate mortgages become very risky, or very costly. Borrowers then choose to protect themselves against interest rate shocks by locking into the rate. Furthermore, in the presence of a downward sloping yield curve, protection against interest rate risk is free, making borrowers potentially better off with a fixed-rate mortgage.

Not only the yield curve and interest rate volatility, but also expectations about future rates impact on the borrower's behavior. If borrowers believe interest rates are mean-reverting and declines in interest rates are expected to be followed by increases, then it may be rational to lock into a rate that is currently low, relative to the recent past, even though the yield curve is steep. The argument is that the borrower is more willing to pay for a spread which ensures not only known interest payments, but also shields the borrower from increasing rates in future periods. Similarly, in a high interest-rate environment, adjustable-rate mortgages ensure that the borrower benefits from decreasing market rates, thus becoming more beneficial; although locking into the rate would be reasonable owing to a flat yield curve.

It should be also kept in mind that the interest payment depends on both the charged mortgage rate and the outstanding balance. For a high balance, borrowers are more vulnerable to interest rate shocks, leading risk-averse borrowers to demand fixed-rate mortgages, especially in the case of missed amortizing payments. However, in the subprime crisis, many borrowers were attracted by mortgages associated with variable-rate loans, but requiring low or no amortizing payments. Many defaults were caused because borrowers had been exposed to substantial interest rate risk through non-amortizing or negative-amortizing adjustable-rate mortgages [see Foote et al., 2008]. Borrowers who choose such mortgage designs are risk-loving and less willing or less able to pay for protection against fluctuating rates. For example, a wide term spread constrains some borrowers from qualifying for a fixed-rate mortgage. In order to attain homeownership, these borrowers are more likely to accept the interest-rate risk of a short-term or adjustable-rate mortgage. Mori et al. [2009], for example, argued that borrowers focus heavily on pricing factors and ignore the risk factors associated with an adjustable-rate mortgage. They demonstrated that adjustable-rate mortgages are on average significantly larger than fixed rate mortgages, meaning that borrowers attempt to qualify for a mortgage with a high principal by taking out short-term mortgages. Borrowers may also decide on the mortgage rate with an imprudently short-sighted attitude. On the one hand, impatient borrowers accept the interest-rate risk of an adjustable-rate mortgage in order to gain a more favorable time path of payments [see Brueckner, 1993]. On the other hand, borrowers who are likely to move and shortly resell the house and prepay the mortgage take out an adjustable rate in order to limit short-term costs [see Dhillon et al., 1987, Brueckner, 1992, Campell and Cocco, 2003]. In principle, when choosing the contract term, the borrower considers a term to maturity which is shorter than the amortization period of a mortgage.

As mentioned, rather than taking out a fully adjustable-rate or fully fixed-rate mortgage, borrowers often break the mortgage down into several short-to-medium-term fixed-rate contracts, meaning that they balance between bearing interest rate risk and paying for protection against fluctuating rates. As long as the rate is fixed, the borrower is shielded from interest rate movements. A long-term contract provides protection against fluctuating rates for longer than a short-term one, but mostly requires the borrower to pay a greater spread. In return, a series of long-term contracts is associated with less frequent rate adjustments than one of short-term contracts. However, interest rate volatility increases over time. Therefore, the interest rate adjustment is more likely to be larger when a long-term contract is renewed, rather than a short-term one. Thus, with the balance being constant for non-amortizing mortgages, the borrower should shield from interest rate fluctuations by fixing the mortgage rate. The borrower may be better off breaking down the mortgage into two contracts only if rates are expected to decrease. A contract which is to be extended should be, however, shorter termed than the contract expiring at the term to maturity, in order to minimize the risk that interest rates move against the borrower and the mortgage rate adjusts to an increased market rate. In other words, the optimal term of a contract in which a non-amortizing mortgage is broken down tends to lengthen as the mortgage ages. For an amortizing mortgage, mostly the converse applies; the optimal term of the contracts in which a mortgage is broken down shortens as the mortgage ages. This is explained by the borrower becoming less vulnerable to interest rate shocks, due to a decreasing balance, and therefore being able to increasingly bear interest rate fluctuations as the mortgage ages. Thus, amortization payments serve as a substitute for locking into the rate.

Of course, lender behavior is also influenced by the yield curve and the interest rate risk which, in return, influences borrower behavior. On the one hand, the lender benefits from an upward sloping yield curve by granting long-term contracts which are funded with short-term capital. In Germany, for example, the business model of many banks relies on this term transformation, encouraging lenders to promote long-term mortgages in the presence of an upward sloping yield curve.[7] However, the mismatch between long-term lending and short-term funding, which is not hedged through financial contracts, causes severe losses when the costs of the short-term funds increase and the profits on long-term lending remain unchanged.[8] Nevertheless, this article focuses on the borrower's choice. The argument is that lenders are more likely to have greater expertise in and more tools for hedging interest rate risk than the borrower. The lenders are able to decrease their interest rate risk exposure by the use of covered bonds and loan sales in the secondary market, both of which reduce the maturity mismatch between lending and funding. On the other hand, lenders have incentives to promote short-term contracts when term spreads are wide. As mentioned, a steep yield curve reduces housing affordability [see Scanlon et al., 2008]. By offering mortgages with non-standard features, such as variable interest rates or interest-only payments, lenders provide borrowers with access to a mortgage, thereby increasing lending activity in order to gain short-term profits [see Pavlov and Wachter, 2006, Linneman and Wachter, 1989, Barakova et al., 2003]. In the long run, however, mortgages granted through unduly relaxed lending standards are associated with a greater default risk [see Demyanyk and van Hemert, 2011, Maddaloni and Peydr, 2011, Scanlon et al., 2008].

This also suggests that lenders are responsible for charging reasonable and sufficient risk premiums. The mortgage conditions offered by the lender may influence the borrower's behavior, either encouraging them to take out fixed-rate mortgages if borrowers facing volatile mortgage rates are more likely to default, or preventing borrowers with an insufficient credit rating from acquiring a mortgage at all. For example, the impact of mortgage pricing could equate to a parallel shift of the yield curve for more risky borrowers, who should have less access to a mortgage. Furthermore, lender behavior effectively flattens the yield curve if higher premiums are charged for adjustable-rate mortgages. As a result, adjustable-rate mortgages become less attractive and fixed-rate mortgages become more attractive.

The Model

As mentioned, this article focuses on the borrower's choice between bearing the interest rate risk and paying a spread for locking into the current rate. The optimal choice is examined for a utility-maximizing borrower, when interest rates follow a mean-reverting process. First, the interest rate process is shown. Then, a basic model of the optimal choice between a fully adjustable and a fully fixed-rate mortgage is presented, which enables an analysis of the impact of the interest rate process, borrower risk aversion and the mortgage balance on the optimal contract design. This is especially important, as later on the interest rate process is calibrated to match the basic features of the US market, so as to relate the subsequent results to other countries. For example, market rates of interest can be more or less volatile or revert more slowly or more quickly to the long-term mean in other countries.

Interest Rates and Yield Curve

The short-term mortgage rate at time t is assumed to be the sum of the market rate and a positive lending premium, and to follow a mean-reverting process[9] according to Vasicek [1977][10]

$$dr_t = \alpha[\nu - r_t]dt + \theta dW, \quad t \in R_0^+, \quad \alpha > 0 \tag{1}$$

with ν denoting the long-term mean to which the mortgage rate reverts and α being the mean reversion parameter, expressing the speed with which the mortgage rate reverts to its mean.[11] θ denotes the standard deviation and W is a standard Brownian Motion. At time 0, the short-term mortgage rate at time t is therefore normally distributed with a mean of

$$\mu_t = e^{-\alpha t} r_0 + \nu \left(1 - e^{-\alpha t}\right) \tag{2}$$

and a variance of

$$\sigma_t^2 = \frac{\theta^2}{2\alpha}\left(1 - e^{-2\alpha t}\right) \tag{3}$$

Modeling a Fully Adjustable and Fully Fixed-Rate Mortgage

At $t = 0$, the borrower chooses between a fully adjustable and a fully fixed-rate mortgage amounting to Q_1, leading to mortgage payments at future points in time $t = 1, ..T$. The loan agreement is concluded for the term to maturity T, meaning that borrowers cannot revise their initial decision for an adjustable-rate or a fixed-rate mortgage by switching to the other option. The borrowers maximize their CARA utility gained until the mortgage matures at T, by consuming at each time $t \in [1, T]$ the residual income after having paid the mortgage rate r_t and the term spread Δ on the outstanding balance Q_t. This equates to minimizing the utility which is lost due to paying interest on the outstanding balance. Rather than interest payments, amortization payments A_t result in the borrower gaining (housing) consumption utility through building up housing wealth and thus do not reduce borrower utility.

With a fixed-rate mortgage, the borrower pays a constant rate. The interest payments include the initial short term rate r_0, and a spread $\Delta(T)$, depending on the term to maturity T. Therefore, future mortgage payments are independent of the market rate volatility and future market rates. Assume the borrower's utility at times $t = 1, .., T$ sum to the overall utility. Under constant absolute risk aversion, the utility for a fixed-rate mortgage with maturity T at time $t = 0$ is given by[12]

$$U_{F,0} = \sum_{t=1}^{T} -e^{-\gamma[-Q_t(r_0+\Delta(T))]} \tag{4}$$

whereby $\gamma > 0$ denotes the risk aversion coefficient.

In an adjustable-rate mortgage, the borrower pays the current market rate on the outstanding balance at each time t. No term premium is charged. Future mortgage rates r_t therefore depend on the volatility of the market rate and are normally distributed with mean μ_t and variance σ_t^2, as given by equations 2 and 3. Under constant absolute risk aversion, the borrower's utility with an adjustable-rate mortgage with maturity T at time $t = 0$ is given by

$$U_{A,0} = \sum_{t=1}^{T} \int_{-\infty}^{\infty} -e^{-\gamma[-Q_t r_t]} \frac{1}{\sigma_t \sqrt{2\pi}} e^{-\frac{(r_t-\mu_t)^2}{2\sigma_t^2}} dx \tag{5}$$

By simplifying the integral, equation 5 can be rewritten as[13]

$$U_{A,0} = \sum_{t=1}^{T} -e^{-\gamma\left[-Q_t\left(\mu_t+0.5\gamma\sigma_t^2 Q_t\right)\right]} \tag{6}$$

Model Parameters and Borrower Choice of the Mortgage Design

If $U_{F,0} > U_{A,0}$, a utility maximizing borrower chooses a fixed-rate mortgage rather than an adjustable-rate. For $T = 1$ and a non-amortizing mortgage, meaning that $Q_t = Q$, this is equal to[14]

$$(\nu - r_0)\left(1 - e^{-\alpha}\right) + 0.5Q\gamma\frac{\theta^2}{2\alpha}\left(1 - e^{-2\alpha}\right) - \Delta(1) \geq 0 \tag{7}$$

Yield Curve (Δ) The borrower is more likely to lock into the rate if term spreads are low. A fixed-rate mortgage is then more reasonable relative to the adjustable-rate mortgage, as the yield curve determines the costs of fixing the mortgage rate.

Interest Rate Level ($\nu - r_0$) The first term of equation 7 determines the impact of the interest rate level on the borrower's choice between an adjustable and a fixed-rate mortgage. For a low interest-rate environment, when the short-term rate r_0 is below its long-term mean ν, interest rates are expected to increase. To be shielded from interest rate increases, the borrower is better off with a fixed-rate mortgage. Conversely, in a high interest-rate environment, when the short-term rate is above its long-term mean and expected to decrease, adjustable-rate mortgages become more favorable in order for the borrower to benefit from decreasing rates. If the current rate equals its long-term mean, the interest rate is expected to remain unchanged and the first term of equation 7 becomes zero, meaning that expectations about future interest rates have no impact on the borrower's choice.

Interest Rate Volatility (θ) For low θ, interest rates are less volatile, decreasing the risk associated with an adjustable-rate mortgage. The adjustable-rate mortgage becomes less costly and more beneficial relative to the fixed-rate mortgage. By contrast, in a more volatile context, the borrower tends to lock into the current market rate.

Mean Reversion Parameter (α) Basically, the mean reversion parameter determines the speed with which the interest rate reverts to its mean. When the borrower expects to benefit from decreasing rates, strong mean reversion favors adjustable-rate mortgages. Strong mean reversion also prevents mortgage rates from rising indefinitely in the long run and limits the interest rate risk, therefore further encouraging the borrower to take out an adjustable-rate mortgage if interest rates are expected to decrease. By contrast, weak mean reversion is associated with more volatile interest rates. If furthermore, interest rates are expected to increase, the borrower is better off locking into the rate.

There is an ambiguous relationship between the mean reversion parameter and the borrower's choice between an adjustable and a fixed-rate mortgage in two cases: Firstly, if strong mean reversion limits interest rate risk but rates are expected to increase, and secondly, if weak mean reversion results in a more volatile environment but interest rates are expected to decrease. The optimal contract design depends on whether α has a stronger impact in determining expectations about future rates or in limiting interest rate risk. The former is especially significant for historically low or high interest rates, meaning that the distance from the current interest rate to its mean is wide, because, for high α, interest rates revert back to the mean more rapidly. By contrast, the impact of the mean reversion parameter on the interest rate volatility has to be considered jointly with the standard deviation of the interest rate process, the mortgage balance and borrower risk aversion. A low α increases interest rate volatility, which is more significant for a high balance, high standard deviation and a more risk-averse borrower.

Mortgage Balance (Q) The mortgage balance influences borrower vulnerability to interest rate shocks. Borrowers are better off with a fixed rate for a large mortgage, because the potential effect of interest rate fluctuations is high. In other words, borrowers are shielded from the interest rate risk of a mortgages, not only by locking into the rate, but also by choosing low balances and amortizing. This means that borrowers with non-amortizing mortgages can only rely on a fixed-rate mortgage, as a means of protecting themselves from rising interest payments.

Borrower Risk Aversion (γ) Borrowers with greater risk aversion are more willing to pay for protection against interest rate fluctuations. They tend to choose fixed-rate mortgages in order to smooth their consumption path.

To summarize, the borrower demands an adjustable-rate mortgage when current interest rates are historically high and expected to decrease, mean reversion is strong, interest rate volatility is low and the yield curve is steep. Furthermore, borrowers with low risk aversion and a small mortgage tend to bear the interest rate risk of an adjustable-rate mortgage. In contrast, a fixed-rate mortgage is more beneficial if the yield curve is flat, interest rates are historically low, therefore being expected to increase, and more volatile. Borrowers with a high risk aversion and a large mortgage are also better off locking into the rate.

However, borrowers who believe that interest rates are mean-reverting may take out an adjustable-rate mortgage, even though interest rates are expected to increase slightly, when strong mean reversion limits interest-rate volatility, while locking into the rate is costly due to a wide term spread. They may also fix the mortgage rate when interest rates are historically high, but weak mean reversion results in more volatile and only slowly decreasing interest rates, from which the borrower expects to benefit.

Breaking Down a Mortgage in Single Contracts

For a long term to maturity, a fully fixed-rate mortgage can become very costly in the presence of an upward sloping yield curve. A fully adjustable-rate mortgage, by contrast, can be very risky if interest rates vary substantially. Therefore, the borrower may address the interest rate risk by breaking the mortgage down into a series of short-to-medium-term fixed-rate contracts with varying terms, lengthening or shortening them as the mortgage matures. These short-to-medium-term fixed-rate contracts require the borrower to pay lower term spreads than a fully fixed-rate mortgage. Further, the series of contracts is associated with lower interest rate risk than a fully adjustable-rate mortgage. The interest rate risk is actually shared by the lender and borrower. The lender shoulders the interest rate risk until a contract expires and the mortgage is renewed, meaning that the mortgage rate is renegotiated and adjusts to the market rate. Because of this adjustment, the borrower is also exposed to interest rate fluctuation. Each time a contract expires, the market rate of interest may have risen and moved against the borrower.

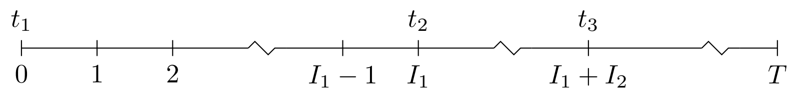

Figure 2: Series of mortgage contracts with the time t below and the time t_n at which a contract is entered above the timeline

Assume that N denotes the number of contracts taken out by the borrower and $I_n \in N^+$ is the length of the n-th contract, $n = 1, ..., N$, whereby the terms of the single contracts sum to the term to maturity $\sum_{n=0}^{N} I_n = T$ (equation 15). In this setting, a borrower renegotiates the mortgage conditions at times $t_n = \sum_{i=0}^{n-1} I_i$, n=1,...,N, (equation 14) meaning that at t_n, the borrower enters the n-th contract, with the first payment being due at $t_n + 1$. Figure illustrates the breakdown of a mortgage into a series of contracts.

Assume further that the market rate of interest is normally distributed with mean and variance according to equations 2 and 3 (equation 13). The borrower is able to reduce the exposure to market rate fluctuations by fixing the mortgage rate for terms $I_n > 1$. This requires the borrower to pay, at any time $t \in [t_n + 1, t_n + I_n]$, the current short-term mortgage rate r_{t_n} and a spread $\Delta(I_n)$ on the outstanding balance Q_t (equations 11 and 12). The yield spread depends on the term I_n for which the mortgage rate is fixed, with $\frac{\delta \Delta}{\delta I_n} > 0$ in the presence of an upward sloping yield curve. For simplicity, the yield curve is assumed to shift parallel and the spread $\Delta(I_n)$ is therefore independent of the time t_n at which the mortgage is renewed. The mortgage term I_n is required to be an element of the natural numbers (equation 16). This makes sense, as the mortgage rate used to be fixed for years, or at least months rather than weeks, or any other unnatural number. Furthermore, it simplifies the model and makes it computationally tractable.

The mortgage is gradually amortized over a period $M \geq T$. Note that $M = \infty$ yields a non-amortizing mortgage. For an amortizing mortgage, the mortgage payment P_t consists decreasingly of interest payment Z_t and increasingly of amortization payment, as the mortgage ages (equations 9 and 10). Then the borrower's maximization problem at time $t = 0$ is given by

$$\max_{I_1,\ldots,I_n} \sum_{n=1}^{N} \sum_{t=t_n+1}^{t_n+I_n} -e^{-\gamma[-Q_t(r_{t_n}+\Delta(I_n))]} \tag{8}$$

subject to

$$Q_t = Q_{t-1} - (P_{t-1} - Z_{t-1}) \tag{9}$$

$$P_t = \frac{[1+\kappa_{t_n}]^{M-t+1}[\kappa_{t_n}]\,Q_t}{[1+\kappa_{t_n}]^{M-t+1} - 1} \tag{10}$$

$$Z_t = \kappa_{t_n} Q_t \tag{11}$$

$$\kappa_{t_n} = r_{t_n} + \varDelta(I_n) \tag{12}$$

$$r_{t_{n+1}} \sim \mathrm{N}(\mu_{I_n}, \sigma^2_{I_n}) \tag{13}$$

$$t_n = \sum_{i=0}^{n-1} I_i, \quad t_1 = 0 \tag{14}$$

$$\sum_{n=1}^{N} I_n = T \tag{15}$$

$$I_n \in \mathbb{N}^+ \tag{16}$$

The optimal term of each contract depends not only on the current short-term rate and the yield spread, but also on expectations of future interest rates. Each time the mortgage conditions are renegotiated, the volatile market rate determines the rate of the new contract. Therefore, the optimal series of mortgage contracts has to be determined backwards, starting at T when the mortgage matures and no decision is made to extend the mortgage. Given that $V_t(.)$ denotes the borrower's value function at t, the optimization problem faced at any time t_n is defined using the recursive Bellman equation and is given by

$$V_{t_n}(Q_{t_n}, r_{t_n}) = \max_{I_n} \sum_{t=t_n+1}^{t_n+I_n} -e^{\gamma Q_t(r_{t_n}+\Delta(I_n))} + E\left[V_{t_{n+1}}(Q_{t_{n+1}}, r_{t_{n+1}})\right] \tag{17}$$

Parametrization

The Vasicek model is fitted to historical data of 1 year mortgage rates. The yield curve is determined by additionally considering longer term rates. We use weekly data from the FRED (Federal Reserve Economic Data) database of the Federal Reserve Bank of St. Louis, starting in September 1991 and ending in December 2010, including mortgage rates for terms of 1, 15 and 30 years. 5 year mortgage rates are available from January 2005 onwards. Table 1 shows descriptive statistics for the data series.

	$1y$	$5y$	$15y$	$30y$	$5y-1y$	$15y-1y$	$30y-1y$
mean	0.052	0.053	0.064	0.068	0.004	0.012	0.0162
median	0.054	0.056	0.065	0.069	0.005	0.012	0.0160
$99th$ percentile	0.073	0.064	0.088	0.092	0.011	0.027	0.032
$1th$ percentile	0.033	0.034	0.038	0.044	-0.002	-0.002	-0.001
std.dev.	0.009	0.008	0.012	0.012	0.003	0.007	0.008
skewness	-0.07	-0.72	-0.12	-0.07	-0.18	0.20	0.15
kurtosis	2.57	2.35	2.31	2.29	2.14	2.37	2.24
Jarque-Bera	8.53	32.74	22.55	22.03	11.55	23.72	28.01
prob.	0.017	0.000	0.000	0.000	0.009	0.000	0.000
observations	1012	315	1012	1012	315	1012	1012

Table 1: Descriptive statistics of mortgage rates and spreads based on weekly data from the FRED (Federal Reserve Economic Data) database of the Federal Reserve Bank of St. Louis. The data series for 1, 15 and 30-year mortgage rates start in September 1991 and end in December 2010. 5-year mortgage rates are available from January 2005 onwards

The Vasicek model parameters are estimated by maximum likelihood. The model parameters are as follows

$$\alpha = 0.201$$

$$\nu = 0.055$$

$$\theta = 0.010$$

The yield curve is based on historical spreads between 1 and 5-year, 1 and 15-year, as well as 1 and 30-year mortgage rates. The spread of terms for which no historical time series are available are calculated by cubic spline interpolation. A comparatively steep yield curve as of mid 1994 is chosen, where the 30-year mortgage rate exceeds the 1-year mortgage rate of more than 3 percentage points. This article therefore focuses on an upward sloping yield curve, with long-term rates exceeding the short-term rates. This is the most common form of yield curve and therefore the most relevant.

In the base case, the initial mortgage balance amounts to $Q_0 = 200000$, the term to maturity is $T = 30$ and the short-term rate equals its long-term mean, $r_0 = \nu = 0.055$. In the following analysis, one of these parameters is varied while keeping the others constant, in order to examine the sensitivity of borrower choice to these parameters. Both an amortizing mortgage with $M = T$ and a non-amortizing mortgage with $M = \infty$[15] are considered. The risk-aversion parameter equals $\gamma = 0.003$.

As mentioned, the optimization problem is solved numerically, starting at T and using backward induction. The state space for the endogenous state variables r_{t_n} and Q_t is calculated using 101 grid points, which are equally distributed on $[0, 0.1]$ or $[1, 200000]$, respectively. For values of r_{t_n} and Q_t within the grid, cubic spline interpolation is performed. The integral of the expectation in equation 17 is computed using Gaussian quadrature.

Numerical Results

The optimal strategy for managing the interest rate risk of a mortgage is exhibited in figures showing a mortgage as a stacked bar. The number of a bar's elements equals the number of contracts into which the borrower breaks the mortgage down. The height of an element indicates the term I_n of a contract. Because of equation 15, the bar height equals the term to maturity T. Consequently, a fully adjustable-rate mortgage is represented by a bar with T elements of height $I = 1$. For a fully fixed-rate mortgage, the bar consists of only one element of height $I = T$.

Given the interest rate process, the yield curve and borrower risk aversion, the borrower chooses the contract term at time t, depending on the mortgage balance Q_t, the term to maturity T and the short-term rate r_t which determines not only the interest rate level, but, jointly with the interest rate process, also expectations of future interest rates.

An important finding is that for non-amortizing mortgages, fully fixed-rate mortgages are mostly superior, while amortizing mortgages are usually best broken down into several short-to-medium-term fixed-rate contracts. The argument is that interest rates become less predictable in the long run. Given a constant balance due to missed amortization payments, the borrower would be exposed to significant interest rate risk in a fully adjustable-rate mortgage and therefore is better off locking into the rate. By contrast, amortizing the mortgage decreases borrower vulnerability to interest rate shocks, because the decreasing balance protects the borrower from interest rate fluctuations or, in other words, interest rate shocks are less relevant for a lower outstanding mortgage balance.

Figure 3 depicts the crucial role of amortization payments in choosing the contract term by showing the optimal breakdown of a mortgage into single contracts for differing terms to maturity.

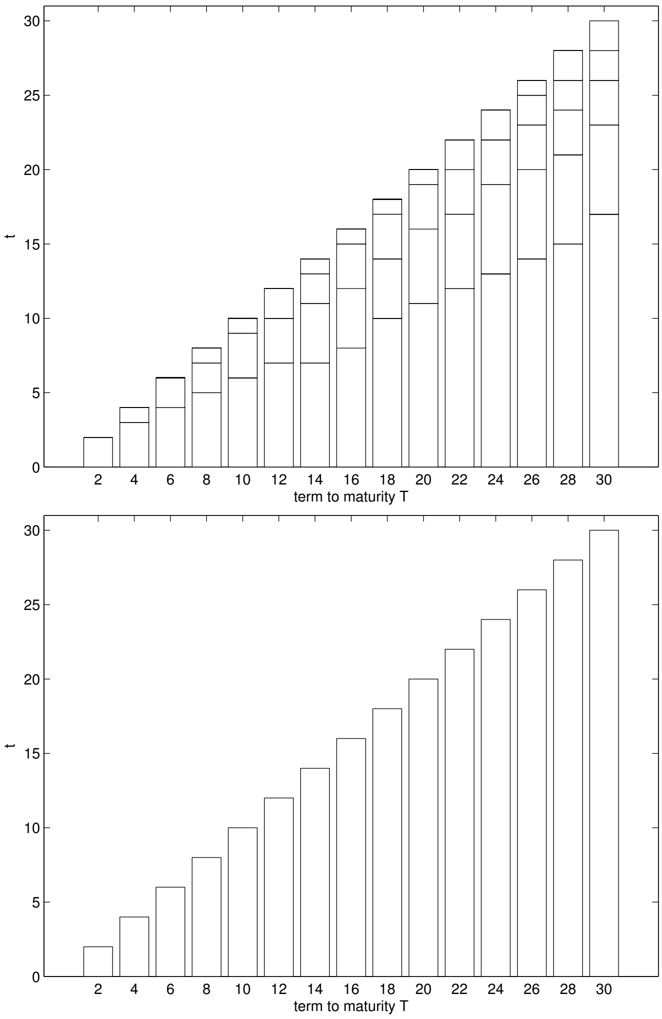

Figure 3: Optimal breakdown of a mortgage into single contracts for different terms to maturity T. Other parameters are calibrated as follows: $Q_0 = 200000$, $\gamma = 0.003$, $r_0 = \nu = 0.055$. The upper graph shows the results for an amortizing mortgage, the lower graph for a non-amortizing mortgage

As mentioned, amortizing mortgages are best split into several contracts. However, an increasing term to maturity means that the mortgage is amortized more slowly. For a longer term to maturity, the optimal term of single contracts therefore lengthens, especially for the initial contract, in order to fix the rate until the mortgage is amortized significantly. By contrast, the rate should be fixed for non-amortizing mortgages. Note that locking into the rate causes costs which make it irrational to fix the mortgage rate for a term exceeding that to maturity.

Another major finding is that an optimal breakdown into several contracts is associated with initial contracts being longer-termed than subsequent contracts, at least when interest rates equal their mean and the borrower expects rates neither to increase nor to decrease. For example,

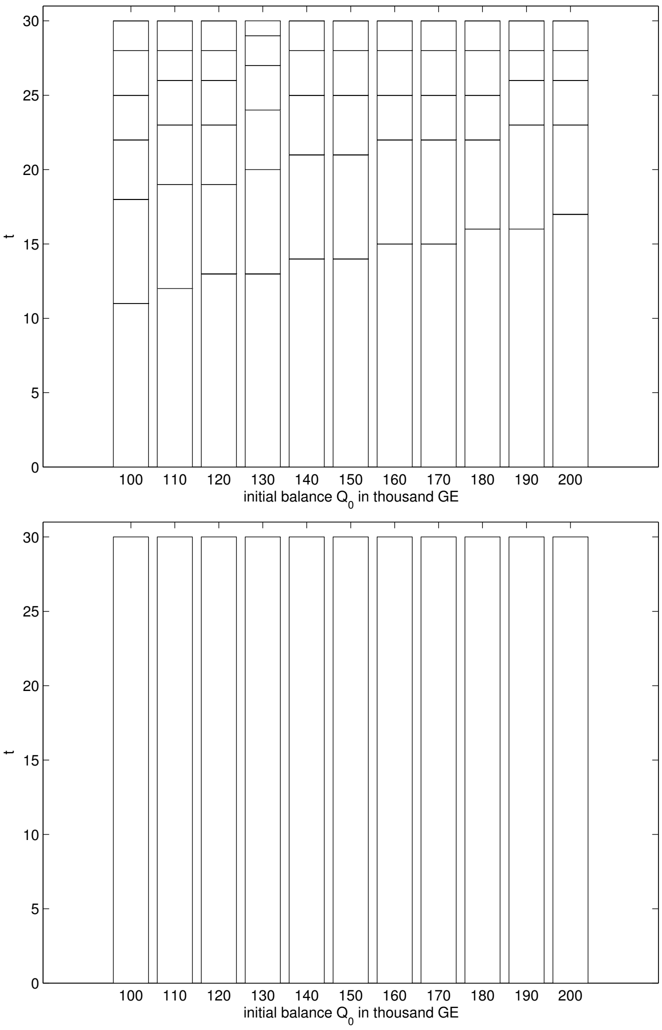

Figure 4: Optimal breakdown of a mortgage into single contracts for different initial balances Q_0. Other parameters are calibrated as follows: $T = 30$, $\gamma = 0.003$, $r_0 = \nu = 0.055$. The upper graph shows the results for an amortizing mortgage, the lower graph for a non-amortizing mortgage

Figure 4 shows that for a gradually amortized mortgage maturing in 30 years, short-term contracts become increasingly beneficial as the mortgage ages. The argument is that borrower vulnerability to interest rate shocks decreases gradually with the balance. In the first years after origination, the greatest portion of the mortgage payment goes towards interest payments, while amortization payments comprise an increasing portion of the payment as the mortgage matures. In other words, borrowers should fix the rate until the mortgage has been amortized significantly, because their interest rate risk exposure is then reduced. As the mortgage is amortized more rapidly in the later stages, the initial contract should be longer-term than subsequent contracts. Therefore, in the short run, the borrower is shielded from interest rate fluctuations by fixing the rate, and in the long run, by the decreasing balance. Consequently, it follows that mortgages with a high principal are best broken down into fewer, long-term contracts. Note that even for a low balance, non-amortizing mortgages should be designed as a fully fixed-rate mortgage, due to interest rate risk being substantial over a 30-year horizon.

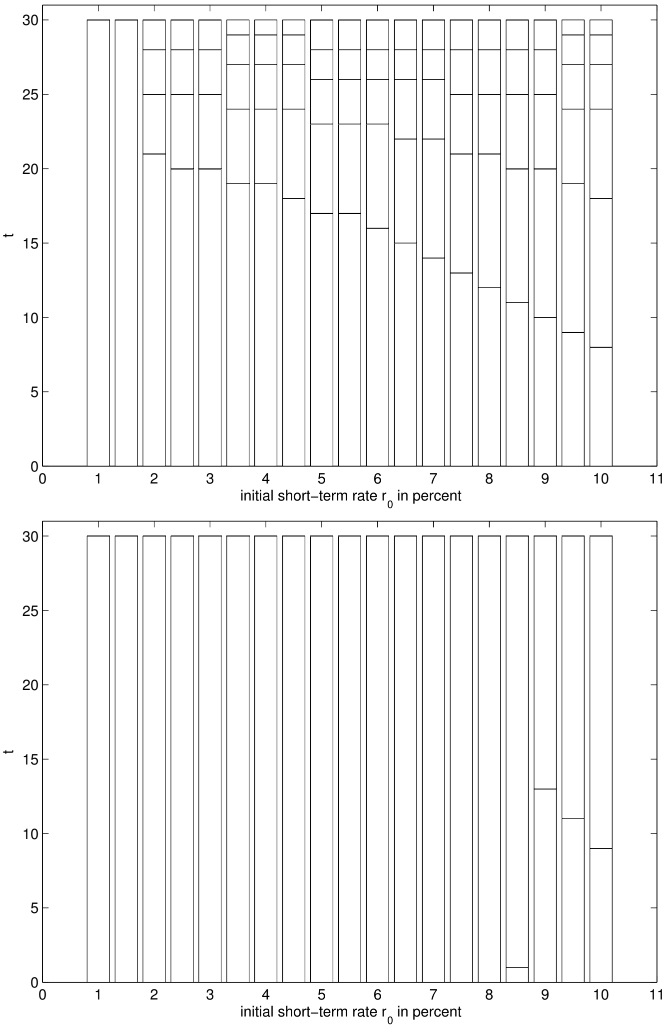

Figure 5: Optimal breakdown of a mortgage into single contracts for different initial short-term rates r_0. Other parameters are calibrated as follows: $T = 30$, $\gamma = 0.003$, $Q_0 = 200000$. The upper graph shows the results for an amortizing mortgage, the lower graph for a non-amortizing mortgage

Yet, the short-term rate r_0 has been assumed to equal its mean ν, implying that interest rates were expected to remain unchanged over the mortgage term. Figure 5 shows how different interest rate levels and therefore expectations of decreasing or increasing future interest rates influence the optimal contract term. Historically low rates, for example, should be locked in for the term to maturity, meaning that fully fixed-rate mortgages are superior, even if the borrower becomes less exposed to interest rate risk because the mortgage is amortized. Furthermore, rather than if the short-term rate equals the mean, there are scenarios in which, with an amortizing mortgage, the initial contract should be shorter-term than a subsequent contract. For example, for historically high rates, borrowers expect rates to decrease in the first years after origination and aim at benefiting by fixing the rate for a short term, ensuring that the mortgage rate adjusts to the potentially decreasing market rate in the near future. The subsequent contract may be taken out for a longer term than the initial one if, on the one hand, favorable interest rate movements became less probable, because the rate has reverted to and varies around its long-term mean and, on the other hand, the borrower is still vulnerable to interest rate shocks because the mortgage has been minimally amortized so far and still entails a substantial principal. As soon as borrowers are less exposed to interest rate fluctuations owing to a low outstanding balance, they are better off shortening the term of new contracts. In other words, expectations of decreasing rates result in short-term mortgages being favorable in the early stage of a mortgage, and decreasing mortgage balances result in short-term mortgages being favorable in the later stage of a mortgage. In the middle stage, neither of these impacts, which otherwise make short-term contracts more favorable, may be significant, so longer-term contracts may be preferable.

Expectations of future interest rates can also make the borrower better off breaking down a non-amortizing mortgage into at least two contracts, rather than fixing the rate for the term to maturity. Similarly to amortizing mortgages, the borrower closes the initial contract for a term shorter than that to maturity if rates are expected to decrease, in order to lock into a lower rate in a future period. Note that this strategy also requires the borrower to pay lower term spreads than in a fully fixed-rate mortgage. However, in contrast to amortizing mortgages, the first contract should always be shorter-term than the second. The argument is that breaking down a non-amortizing mortgage can be justified only by aiming to benefit from decreasing interest rates in the first years after origination. As mentioned, the borrower is exposed to substantial interest rate risk as the mortgage matures due to increasing interest rate volatility and a constant principal. However, the final contract, which need not be prolonged, is not associated with any interest rate risk. Therefore, borrower interest rate risk exposure is minimized if the term of the first contract is shorter than that of the second.

Limitations

This is the first research project to determine the optimal breakdown of a mortgage into several short-to-medium-term fixed-rate mortgages by examining the optimal trade-off between bearing interest rate risk and paying a term spread, so as to be protected against interest rate fluctuations. However, some issues remain open for future research and could be examined in greater detail and depth.

For example, this article assumes the yield curve to shift parallel. Actually, yield curves vary in their steepness, due to differing liquidity and risk premiums charged by lenders. Expectations of a steepening or flattening yield curve may influence the borrower's choice of the mortgage term. Assume that the yield curve is steep, but expected to flatten. The borrower may then choose short-term contracts in order to benefit from narrowing term spreads.

The model may also be extended by considering a stochastic borrower income. Precarious borrower income, which is correlated with interest rates, may counterbalance or exacerbate the risk of higher future payments. Previous research examining the optimal mortgage contract design when the borrower income is stochastic, focused on allocating the interest rate risk to the borrower, as opposed to the lender [see Edelstein and Urosevic, 2003] or examined the optimal mix of an adjustable and a fixed-rate mortgage in the presence of nominal and real shocks [see Szerb, 1996], rather than determining the breakdown of a mortgage into several contracts and taking into account the term spread to be paid in a fixed-rate mortgage.

Furthermore, in some markets, fixed-rate mortgages come with a prepayment option, allowing the borrowers to benefit from decreasing rates by prepaying their mortgage and applying for a new one. Future research could also consider that borrowers may have constant relative risk aversion and that there is a positive probability of a borrower default.

Last but not least, the model does not control for costs induced by taking out a mortgage contract. These costs may include not only fees charged by the lender for closing a new contract, but the borrower may also be opposed to frequently discussing the mortgage conditions. On the other hand, credit conditions may improve as the mortgage ages. Borrowers may be charged a lower margin when the mortgage is renegotiated, if the loan to value has decreased due to amortization payments.

Conclusion

This article examines both the optimal number of mortgage contracts which the borrower should take out until the mortgage matures, and the optimal term of these contracts. The borrower is generally better off breaking down the mortgage into several contracts, rather than taking out a fully adjustable or fixed-rate mortgage. The results coincide with the behavior of borrowers in various countries, in which the mortgage rate is usually fixed for terms between 5 and 10 years.

Fully fixed-rate mortgages are superior only for non-amortizing mortgages, which make the borrower vulnerable to interest rate shocks and if current interest rates are historically low and therefore expected to increase. The results also show that fixed-rate mortgages are preferable, if interest rate movements are less predictable and follow a random walk, rather than a mean-reverting process.

In none of the considered cases, then a fully adjustable-rate mortgage should be favored. However, in a series of contracts, periods with more frequent rate adjustments can be optimal. Shorter-term mortgages become more favorable for a low outstanding mortgage balance, making the borrower less vulnerable to interest rate shocks and less willing to pay a spread for protection against interest-rate fluctuations. For amortizing mortgages, this means that short-term contracts are beneficial in the later stage, when the mortgage has already been amortized substantially. Initial contracts should be comparatively short-term in a high interest rate environment, enabling the borrower to benefit from potentially decreasing rates.

A Utility

Consider $U_t(r_t) = -e^{-\gamma(-r_t \cdot Q_t)}$ with $r_t \sim N\left(\mu_t, \sigma_t^2\right)$

The expected utility function then equals

$$
\begin{aligned}
E_t[U_t(r_t)] &= \int_{-\infty}^{\infty} -e^{-\gamma(-r_t \cdot Q_t)} \frac{1}{\sigma_t \sqrt{2\pi}} e^{-\frac{(r_t-\mu_t)^2}{2\sigma_t^2}} dr_t \\
&= -\int_{-\infty}^{\infty} e^{\gamma r_t Q_t} f(r_t) dr_t
\end{aligned}
$$

From setting $x_t = -r_t Q_t$, $\phi_t = -\mu_t \cdot Q_t$ and $\delta_t = -\sigma_t \cdot Q_t$, it follows

$$
\frac{dr_t}{dx_t} = -\frac{1}{Q_t} \quad \Rightarrow dr_t = -\frac{1}{Q_t} dx_t
$$

Then

$$
\begin{aligned}
& -\int_{-\infty}^{\infty} e^{\gamma r_t Q_t} f(r_t) dr_t \\
=& -\frac{1}{\sigma_t \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\gamma r_t Q_t - \frac{(r_t-\mu_t)^2}{2\sigma_t^2}} dr_t \\
=& -\frac{1}{\sigma_t \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\gamma x_t - \frac{(-\frac{x_t}{Q_t}-\mu_t)^2}{2\sigma_t^2}} \left(-\frac{1}{Q_t}\right) dx_t \\
=& -\frac{1}{\delta_t \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\gamma x_t - \frac{(-x_t-\mu_t Q_t)^2}{2Q_t^2\sigma_t^2}} dx_t \\
=& -\frac{1}{\delta_t \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\gamma x_t - \frac{(x_t-\phi_t)^2}{2\delta_t^2}} dx_t \\
=& -\frac{1}{\delta_t \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-2\delta_t^2\gamma x_t - x_t^2 + 2x_t\phi_t - \phi_t^2}{2\delta_t^2}} dx_t \\
=& -\frac{1}{\delta_t \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-\left[x_t^2 - 2\left(\phi_t - \delta_t^2\gamma\right)x_t + \phi_t^2\right]}{2\delta_t^2}} dx_t \\
=& -e^{\frac{\phi_t^2 - 2\phi_t\delta_t^2\gamma + \delta_t^4\gamma^2 - \phi_t^2}{2\delta_t^2}} \frac{1}{\delta_t \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{-\left[x_t - \left(\phi_t - \delta_t^2\gamma\right)\right]^2}{2\delta_t^2}} dx_t \\
=& -e^{-\phi_t\gamma + 0.5\delta_t^2\gamma^2}
\end{aligned}
$$

Re-substitution in the expected utility function leads to

$$
\begin{aligned}
\Rightarrow \quad & E_t[U_t(r_t)] = -e^{-\phi_t\gamma + 0.5\delta_t^2\gamma^2} \\
= \quad & e^{-\gamma\left[+\phi_t - 0.5\delta_t^2\gamma\right]} \\
= \quad & e^{-\gamma\left[-\mu_t Q_t - 0.5\sigma_t^2 Q_t^2\right]}
\end{aligned}
$$

B Spread

The borrower takes out a fully fixed-rate mortgage, rather than a fully adjustable-rate mortgage, if $U_{F,0} > U_{A,0}$ By assuming $A_t = 0$, $Q_t = Q$, this is equivalent to

$$
\begin{aligned}
& \sum_{t=1}^{T} \left[-e^{-\gamma[-Q(r_0+\Delta(T))]}\right] \geq \sum_{t=1}^{T} \left[-e^{-\gamma\left[-Q\left(\mu_t+0.5\gamma\sigma_t^2 Q\right)\right]}\right] \\
\Leftrightarrow \quad & \sum_{t=1}^{T} -e^{\gamma Q(r_0+\Delta(T))} \geq \sum_{t=1}^{T} -e^{\gamma Q\left(\mu_t+0.5\gamma\sigma_t^2 Q\right)} \\
\Leftrightarrow \quad & T \cdot \left[-e^{\gamma Q(r_0+\Delta(T))}\right] \geq \sum_{t=1}^{T} -e^{\gamma Q\left(\mu_t+0.5\gamma\sigma_t^2 Q\right)} \\
\Leftrightarrow \quad & 0 \leq \tfrac{1}{\gamma Q} \log\left[\tfrac{1}{T} \sum_{t=1}^{T} e^{\gamma Q\left(\mu_t+0.5\cdot\gamma\sigma_t^2 Q\right)}\right] - r_0 - \Delta(T)
\end{aligned}
$$

For $T = 1$, we obtain

$$0 \leq \tfrac{1}{\gamma Q}\gamma Q\left(\mu_1 + 0.5 \cdot \gamma\sigma_1^2 Q\right) - r_0 - \Delta(1)$$

Substituting equations 2 and 3 leads to:

$$
\begin{aligned}
& \left[e^{-\alpha} r_0 + \nu\left(1 - e^{-\alpha}\right)\right] + 0.5 Q\gamma \tfrac{\theta^2}{2\alpha}\left(1 - e^{-2\alpha}\right) - r_0 - \Delta(1) \geq 0 \\
\Leftrightarrow \quad & (\nu - r_0)\left(1 - e^{-\alpha}\right) + 0.5 Q\gamma \tfrac{\theta^2}{2\alpha}\left(1 - e^{-2\alpha}\right) - \Delta(1) \geq 0
\end{aligned}
$$

Footnotes

[1]Past research describing the optimal strategy as an interest-risk-sharing rule that is close to an adjustable-rate mortgage with time-varying caps and floors, includes Arvan and Brueckner [1996] and Dokko and Edelstein [1991].

[2]Campell and Cocco [2003] examined the benefits of a mortgage rate which is linked to inflation. They found that a nominal fixed-rate mortgage has a risky real capital value, while an inflation-indexed fixed-rate mortgage removes this wealth risk, without incurring the income risk of an adjustable-rate mortgage.

[3]In some markets, the overall mortgage amount is also split into several loans with varying terms, which are collateralized by the same piece of real estate.

[4]The typical mortgage contract design, however, is not static. Scanlon et al. [2008], for example, demonstrated mortgage product design trends for 13 developed countries.

[5]Koijen et al. [2009] studied the link between the term structure of interest rates and mortgage choice. They showed that the long-term bond premium is also a theoretical determinant of mortgage choice, which is distinct from the yield spread.

[6]Previous research showing substantial differences between borrowers who self-select between adjustable or fixed-rate mortgages include Cunningham and Capone [1990], Deng et al. [2003], Hakim and Haddad [1999], Phillips et al. [1996], Posey and Yavas [2001], Calhoun and Deng [2002] and Ben-Shahar [2006].

[7]Mortgage rates in Germany are usually fixed for 5 or 10 years. Mortgages with terms of more than 10 years are usually not offered. The argument is that mortgages in Germany do not include a prepayment option, but can be paid off after a term of 10 years by law, without requiring the borrower to make good the loss accruing to the lender caused by the borrower breaking the mortgage contract.

[8]In this context, Arvan and Brueckner [1996] stated that an efficient contract between a lender and a borrower includes an interest-risk-sharing rule for variable-rate contracts. Dokko and Edelstein [1991] also explored the appropriate allocation of interest rate risk between a borrower and a lender through varying interest payments.

[9]Mori et al. [see 2009] previously found that borrowers believe that interest rates are mean-reverting.

[10]The use of a Vasicek process can be questioned, because it allows for negative values, while interest rates are always positive. However, it also has several positive characteristics. The process is easy to use and yields closed-form solutions. It also limits interest volatility and ensures that interest rates do not rise indefinitely in the long run.

[11]For $\alpha = 0$, interest rates follow a random walk.

[12]No time preferences are set, meaning that consumption in the late stage of the mortgage is valued as much as in the early stage. The argument is that in a mortgage contract, the borrowers may build up house equity, increasing the incentive to sustain mortgage payments in order to avoid a default. They may be willing to spend a similar or even higher portion of the income for mortgage payments at later points in time, in order to protect housing wealth against being eroded by foreclosure costs.

[13]See Appendix A.

[14]See Appendix B.

[15]The results for the non-amortizing mortgage are calculated by setting $M = 1000$, which effectively leads to insignificant amortization up to $T = 30$.

References

Alm, J. and J. R. Follain (1984). Alternative Mortgage Instruments, the Tilt Problem, and Consumer Welfare, Journal of Financial and Quantitative Analysis, 19, 113-1126.

Arvan, L. and J. K. Brueckner (1986). Efficient Contracts in Credit Markets Subject to Interest Rate Risk: An Application of Raviv's Insurance Model, American Economic Review, 76(1), 259-263.

Barakova, I., R. W. Bostic, P. S. Calem and S. M. Wachter (2003). Does Credit Quality Matter for Homeownership?, Journal of Housing Economics, 12, 318-336.

Ben-Shahar, D. (2006). Screening Mortgage Default Risk: A Unified Theoretical Framework, Journal of Real Estate Research, 28, 215-239.

Brueckner, J. K. (1992). Borrower Mobility, Self-Selection, and the Relative Prices of Fixed- and Adjustable-Rate Mortgages, Journal of Financial Intermediation, 2, 401-421.

Brueckner, J. K. (1993). Why do We Have ARMs?, Journal of American Real Estate and Urban Economics Association, 21, 333-345.

Calhoun, C. A. and Yongheng Deng (2002). A Dynamic Analysis of Fixed- and Adjustable-Rate Mortgage Terminations, Journal of Real Estate Finance and Economics, 24(1-2), 9-33.

Campbell, J. Y. and Joao F. Cocco (2003). Household Risk Management and Optimal Mortgage Choice, Quarterly Journal of Economics, 118(4), 1449-1494.

Cunningham, D. F. and C. A. Capone (1990). The Relative Termination Experience of Adjustable to Fixed-Rate Mortgages, Journal of Finance, 45(5), 1687-1703.

Demyanyk, Y. and O. van Hemert (2011). Understanding the Subprime Mortgage Crisis, Review of Financial Studies, 24(6), 1848-1880.

Deng, Y. and J. M. Quigley and R. Van Order (2000). Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options, Econometrica, 68(2), 275-308.

Dhillon, U. S. and J. D. Shilling and C. F. Sirmans (1987). Choosing between Fixed and Adjustable Rate Mortgages: A Note, Journal of Money, Credit and Banking, 19(2), 260-267.

Dokko, Y. and R. H. Edelstein (1991). Interest Rate Risk and Optimal Design of Mortgage Instruments, Journal of Real Estate Finance and Economics, 4(1), 59-68.

Edelstein, R. and B, Urosevic (2003). Optimal Loan Interest Rate Contract Design, Journal of Real Estate Finance and Economics, 26(2-3), 127-156.

European Mortgage Federation (2006). Study on Interest Rate Variability in Europe, European Mortgage Federation.

Foote, C. L., K. Gerardi, L. Goette and P. S. Willen (2008). Just the Facts: An Initial Analysis of Subprime's Role in the Housing Crisis, Journal of Housing Economics, 17, 291-305.

Hakim, S. and M. Haddad (1999). Borrower Attributes and the Risk of Default of Conventional Mortgages, Atlantic Economic Journal, 27(2), 210-220.

Koijen, R. S. J., O. Van Hemert and S. Van Nieuwerburgh (2009). Mortgage Timing, Journal of Financial Economics, 93, 292-324.

Lea, M. (2010). International Comparison of Mortgage Product Offerings, Mortgage Bankers Association, Research Institute for Housing America Research.

Linneman P. and S. Wachter (1989). The Impacts of Borrowing Constraints on Homeownership, AREUEA Journal, 17, 389-402.

Maddaloni, A. and J.-L. Peydro (2011). Bank Risk-Taking, Securitization, Supervision, and Low Interest Rates: Evicence from the Euro-Area and the U.S. Lending Standards, Review of Financial Studies, 24(6), 2121-2165.

Mori, M., J. III Diaz and A. J. Ziobrowski (2009). Why do Borrowers Choose Adjustable-Rate Mortgages over Fixed-Rate Mortgages?: A Behavioral Investigation, International Real Estate Review, 12, 98-120.

Pavlov, A. and S. M. Wachter (2006). The Inevitability of Marketwide Underpricing of Mortgage Default Risk, Real Estate Economics, 34(4), 479-496.

Phillips, R. A., E. Rosenblatt and J. H. VanderHoff (1996). The Probability of Fixed- and Adjustable-Rate Mortgage Termination, Journal of Real Estate Finance and Economics, 13(2), 95-104.

Posey, L. L. and A. Yavas (2001). Adjustable and Fixed Rate Mortgages as a Screening Mechanism for Default Risk, Journal of Urban Economics, 49(1), 54-79.

Scanlon, K. and C. Whitehead (2004). International Trends in Housing Tenure and Mortgage Finance, Council of Mortgage Lenders, London.

Scanlon, K., J. Lunde and C. Whitehead (2008). Mortgage Product Innovation in Advanced Economies: More Choice, More Risk, European Journal of Housing Policy, 8, 109-131.

Smith, D. J. (1987). The Borrower's Choice between Fixed and Adjustable Rate Loan Contracts, AREUEA Journal, 15, 110-116.

Szerb, L. (1996). The Borrower's Choice of Fixed and Adjustable Rate Mortgages in the Presence of Nominal and Real Shocks, Real Estate Economics, 24, 43-54.

Templeton, W. K., R. S. Main and J. B. Orris (1996). A Simulation Approach to the Choice between Fixed and Adjustable Rate Mortgages, Financial Services Review, 5(2), 101-117.

Vandell, K. D. (1978). Default Risk under Alternative Mortgage Instruments, Journal of Finance, 33, 1279-1296

VanderHoff, J. (1996). Adjustable and Fixed Rate Mortgage Termination, Option Values and Local Market Conditions: An Empirical Analysis, Real Estate Economics, 24, 379-406.

Vasicek, O. (1977). An Equilibrium Characterization of the Term Structure, Journal of Financial Economics, 5(2), 177-188.