Disagreeing forever: a testable model with non-vanishing belief heterogeneity*

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Abstract

I develop an overlapping generations model where each generation is constituted of a continuum of agents with heterogeneous beliefs. This belief heterogeneity persists across successive generations, and I thus obtain empirically testable results. The model can equivalently be described as an economy with a sliding horizon where infinitely lived investors continuously revise their consumption plans, which eases the construction of the intertemporal representative agent. I further derive that the equilibrium stock mean return and volatility increase with belief dispersion. Using analyst forecasts from the IBES database, I show suggestive evidence that these positive relations are empirically verified and hold with data sampled at various frequencies when considering a market index.

Keywords: Heterogeneous beliefs, Overlapping generations, Continuum of agents, Asset pricing

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1 Introduction

It is well-established that there is a great belief heterogeneity among stock market participants. Using a recent survey administered to a panel of retail investors, Giglio et al. (2020) for instance show that beliefs are characterized by a large and persistent individual heterogeneity, and that investors are likely to exhibit a willingness to "agree to disagree". While early studies argued that traders with biased beliefs could be neglected, the recent literature has shown these views to be wrong, and has studied, both from empirical and theoretical perspectives, the implications of such belief dispersion. In particular, Jouini and Napp (2011) show theoretically that heterogeneity is important per se, as an economy with biased investors who are rational on average differs markedly from an economy with rational investors only. Moreover, surveys suggest that investors do not agree more on future stock market returns nowadays than they did in the past. Heterogeneous belief models should thus incorporate this disagreement persistence, which is unfortunately not often the case (see, e.g., Atmaz and Basak, 2018). Not only is this feature important for the sake of empirical relevance, but also because vanishing belief dispersion prevents the belief heterogeneity implications to be tested relevantly over long periods.

In this paper, I develop and empirically test a tractable model in which belief heterogeneity does not progressively vanish. More precisely, in a dynamic general equilibrium framework, I develop an overlapping heterogeneous generations model evolving in continuous time where each generation is constituted of a continuum of constant relative risk aversion (CRRA) investors who agree to disagree. Each member of a given generation gives birth to an investor who is endowed with the same beliefs. At the end of their life, they then consume part of their wealth and bequeath the remaining part to the next generation. The combination of the intra-family belief transmission assumption and of a dynamic family budget constraint ensures that the belief dispersion remains persistent across generations, as empirically observed. My main theoretical contribution is thus to derive stationary results with non-vanishing belief heterogeneity that confirm part of the existing results found in models where agents unrealistically tend towards rationality. It also allows to study if the model implications are verified empirically and if the belief heterogeneity impacts persist over longer horizons. Moreover, the model incorporates continuous effective consumption, which differs from Atmaz and Basak (2018) where there is a single consumption date when heterogeneity is resolved.

An important feature of the model is that it considers a large number of agents, which has two main advantages. First, it allows me to consider an unbounded investor type space, or, stated differently, to take into account every possible existing belief. In a model with bounded belief biases, the most biased agents (towards optimism or pessimism) would eventually control almost all of the wealth in the economy in the most extreme states (i.e. the very good or very bad ones), and belief heterogeneity would disappear. The choice of a continuum thus guarantees that belief heterogeneity persists not only for all dates but also in all states of the world. The second advantage of considering an infinite number of investors is that I can use a statistical distribution to describe their wealth shares. Similarly to, e.g., Cvitanic and Malamud (2011), I assume that the initial distribution is a Gaussian one with exogenously determined parameters and show that the normality of the wealth share distribution remains as time goes by. Thus, I only need to estimate two parameters, namely the average belief bias and the belief dispersion, in order to describe agents' beliefs, which eases the model empirical assessment. It also allows me to easily disentangle between the impacts of the first and second moments. This is interesting because, as underlined by Brandon and Wang (2020), most models focus on the effects of the latter.

Before discussing the theoretical implications, let me briefly talk about a dual approach of the model. It is based on an alternative model populated by a continuum of infinitely lived CRRA investors who continuously revise their consumption plans and, because they always consider a finite horizon economy of length T, continuously shift they consumption date, leading to a model with a sliding horizon (and no effective consumption). Roughly speaking, in this framework, each of the agents corresponds to an entire family in the overlapping heterogeneous generations model, and the successive plan revisions coincide with the different plans made by the successive generations. In fact, this alternative approach echoes the seminal work of Lindahl (1939), who observes that "the plans of the economic subjects at any given point of time are neither fully consistent with one another nor with the external conditions, and therefore they must be successively revised."[1] This is also in line with the more general temporary general equilibrium theory of Grandmont (1977, 2008). Importantly, this continuous plan revision feature ensures that the belief heterogeneity is persistent as these revisions prevent every investor to go extinct. This dual approach, which is fully equivalent to the main model, further helps constructing an intertemporal representative agent, defined as the fictitious agent who, if endowed with the total wealth of the economy, would have a marginal utility equal to the equilibrium price.

I now turn to the model's implications and see how they relate to the existing literature. Note that most of the theoretical results are similar to those in Atmaz and Basak (2018) evaluated at $t = 0$, i.e., when the heterogeneity has not started to vanish. Again, one of the theoretical contributions of my study is to show that simple mechanisms can prevent such belief dispersion vanishing to happen.

Looking at the stock price, I infer that it depends positively on the average belief bias, which is in line with the studies of Jouini and Napp (2007) and Kurz and Motolese (2011). I also derive that the impact of belief dispersion is positive for sufficiently good states of the world and negative for sufficiently bad ones. Note that the sign of this impact only depends on t through the current state of the world W_t, and not through the remaining time before

[1] Erik Lindahl, *Studies in the theory of money and capital*, 1939: p.38. Lindahl also indicates a general procedure to construct a solution consistent with such successive plan revisions:

> Starting from the plans and the external conditions valid at the initial point of time, we have first to deduce the development that will be the result of these data for a certain period forward during which no relevant changes in the plans are assumed to occur. Next we have to investigate how far the development during this first period—involving as it must various surprising for the economic subjects—will force them to revise their plans of action for the future, the principles for such a revision being assumed to be included in the data of the problem. And since on this basis the development during the second period is determined in the same manner as before, fresh deductions must be made concerning the plans for the third period, and so on.

the economy ending date as this is the case in a finite horizon setting. Finally, similarly to Atmaz and Basak (2018), I find a convex relation between the stock price and the cashflow news. This price convexity implies that the stock price reacts more to good news than to bad ones, and that the stock price reaction to any type of news is stronger in relatively good states. Basu (1997) and Nagel (2005) provide empirical evidence for the first prediction, and, consistent with the second one, Conrad et al. (2002) show that the market responds more strongly to bad news in good times than in bad times. Other theoretical studies derive this convex relation in a model with incomplete common information (Veronesi, 1999), or assuming short-sale constraints (Xu, 2007).

I also study the relation between belief heterogeneity and the stock mean return, and observe that the higher the heterogeneity is, the higher the expected returns are. The positive relation that I document is in line with the conjecture of Williams (1977) that more dispersion of opinion represents more risk, and therefore that agents should be more compensated for holding a riskier asset. Banerjee and Kremer (2010) confirm this predicted positive relation in a dynamic model in which investors disagree on the interpretation of public information, and Buraschi and Jiltsov (2006) derive a similar result linking heterogeneity in beliefs to option open interest. Conversely, another strand of the literature, based on the seminal work of Miller (1977), documents a negative link.[2] This negative relation critically depends on the presence of market frictions. For example, Chen et al. (2002) obtain this result by developing a model with differences of opinion and short-sales constraints. Alternatively, Atmaz and Basak (2018) theoretically derive that higher dispersion leads to higher returns when the view on the stock is sufficiently pessimistic, and to lower returns when the view is sufficiently optimistic. Coming back to my model's implications, I further obtain that the stock mean return unconditionally decreases with risk aversion. This is because, in a heterogeneous economy, more risk averse investors speculate

[2] In a model with short-sale constraints and differences of opinion, Miller (1977) argues that the stock is overpriced as it reflects the view of the optimistic agents. In fact, because of the short-sale constraints, pessimistic agents stay out of the market. The higher the differences of opinion, the higher this effect, and therefore the higher the stock overpricing, resulting in lower subsequent returns.

less aggressively, and thus earn lower returns.

Looking at the impacts on the stock volatility, I additionally derive that it monotonically increases with belief dispersion, and is higher than the production process volatility. As stated in Atmaz and Basak (2018), this is because higher fluctuations in the average belief bias translate to additional stock price fluctuations and therefore increase stock volatility. This monotonic positive relation between belief dispersion and stock volatility is well-documented in the theoretical literature (see, e.g., Shalen, 1993 in a two-period rational expectations model, Scheinkman and Xiong, 2003 in a model with short sale constraints, Buraschi and Jiltsov, 2006 in a model with rational agents with incomplete and heterogeneous information, Andrei et al., 2019 in a model with disagreement on the length of business cycles). I complement these findings by deriving a stationary formula where the heterogeneity effects on volatility remain persistent over time.

Because they are stationary, I then translate the main theoretical implications of the model into testable hypotheses, and, turning to the empirical part of the paper, see if they are verified using real data and running ordinary least squares (OLS) regressions. Note that I focus on market-wide implications because there is only one risky stock available in the model. More precisely, I study if a higher market belief dispersion predicts higher market returns and a higher market volatility. While most empirical studies in this literature focus on monthly data, I also ask whether these relations hold for data computed over longer horizons. These additional hypotheses are thereby directly supported by the non-vanishing belief heterogeneity feature of the model. Lastly, rolling window regressions with quarterly data complete the analysis and allow to more carefully study the time evolution of the belief dispersion predictive ability.

I use analyst monthly forecasts of the earnings-per-share (EPS) long-term growth rate (LTG) of individual stocks from the Institutional Brokers Estimate System (IBES) Unadjusted Summary database from January 1982 to December 2019 as a proxy for investors' beliefs. Building on Yu (2011), I aggregate them across assets and over various horizons (from one month to two years) to obtain market belief dispersion data, which is defined as the cross-sectional average of individual stock disagreements. I consider both value-and equally-weighted variables, and, as a robustness check, I also construct an alternative belief dispersion variable as the standard deviation of individual stock disagreements. I further use data on individual stock prices from the Center for Research in Security Prices (CRSP) database to construct my simple returns variables for various holding periods. Importantly, my empirical analysis differs from other existing ones because I specifically study the returns of an index—referred to as the market index—constituted of the individual stocks used in the construction of the belief dispersion variables. This allows me to more directly capture the link between the market characteristics and the investor beliefs. Similarly, I construct market volatility data from daily returns of this market index.

The empirical tests confirm the predicted positive relation between market disagreement and market returns for most specifications and horizons considered. Thus, considering a model grounded non-vanishing belief dispersion framework and using more tightly linked data, I show new results on the long-run impacts. They contribute to a large empirical debate, already discussed above from the theoretical point of view. Diether et al. (2002) for instance report that high dispersion stocks earn lower returns. Interestingly, Doukas et al. (2006) replicate their results, and find that the relation becomes positive when controlling for uncertainty in analysts' earnings forecasts. An empirical positive link is also found in, e.g., Anderson et al. (2005) or Banerjee (2011). Other studies derive mixed results or no relation. In particular, Buraschi et al. (2014) find that the relation is ambiguous and leverage dependent: it is positive and significant for high leverage firms, but can turn negative and non significant for moderately leveraged firms. Finally, Avramov et al. (2009) find that financial distress drives the negative dispersion effect, and show that it is a facet of non-investment grade firms which account for less than 5% of the total market capitalization, and that the effect is virtually non existent otherwise.

Lastly, the results regarding the impacts on the stock volatility are more mixed. In fact, while I mostly derive positive coefficients (controlling for lagged volatility), they are not statistically significant. The rolling window regressions further show that the sign and intensity of the belief dispersion impact varies through time, which might explain the weakness of the results. Note that other empirical works study this relation either in the cross-section or using shorter time periods and confirm the positive relation (see, e.g., Ajinkya and Gift, 1985 using data over a 10-month period, Anderson et al., 2005 using monthly data over a 7-year period, Banerjee, 2011 in a cross-sectional analysis).

The paper is organized as follows. Section 2 presents the theoretical analysis and translates the main theoretical implications into testable hypotheses. Using empirical data, I then provide a test of these hypotheses in Section 3. Section 4 concludes. All proofs are reported in Appendix A, and Appendix B contains additional empirical results.

2 Theoretical part

This section presents the main overlapping heterogeneous generations model and derives its equilibrium. It also describes a dual approach based on a model with a sliding horizon, which allows to construct an intertemporal representative agent. Lastly, it further contains the theoretical results relative to the stock price, its mean return, and its volatility, and formulates testable hypotheses.

2.1 An overlapping heterogeneous generations model

Consider a pure-exchange security market economy evolving in continuous time with an infinite horizon. The economy is populated by overlapping generations of heterogeneous investors who maximize their expected utility from future endowment. They consume part of their wealth and bequeath the remaining part to the subsequent generation.

Uncertainty is modeled by a filtered probability space $(\Omega, F, (F_t), \mathbb{P})$, where Ω is the set of states of nature, F is the σ-algebra of observable events, (F_t) describes how information is revealed through time, and $\mathbb{P}$ is the true probability measure giving the likelihood of occurrence of the different events in F. I assume that there is a single source of risk, modeled by a $((F_t), \mathbb{P})$-Brownian motion W.

Let y denote the production process in the economy, and assume that, under the probability measure $\mathbb{P}$, it follows a geometric Brownian motion with drift μ and volatility σ.[3]

Each generation of investors is constituted of a large number of heterogeneous agents. More precisely, I assume that each generation is composed of a continuum of agents, endowed with a fraction of the production process and having heterogeneous beliefs, who are born at the same date and who all have a life of length T. These agents have standard CRRA preferences, characterized by $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$ with $\gamma > 0$ the coefficient of relative risk aversion, and both consume and bequeath their wealth according to some given proportions (common to all investors). Moreover, they disagree on the dynamic of the production process and are characterized by their own subjective beliefs, which give the subjective likelihood of occurrence of the different events in the economy. Formally, these subjective beliefs are indexed by $\delta \in \mathbb{R}$, and, for a given δ, the subjective beliefs are defined by the subjective probability measure $\mathbb{Q}^{\delta}$, which is assumed to be equivalent to the true probability measure $\mathbb{P}$. I call Agent-δ of a given generation the agent characterized by these beliefs. Concretely, in all generations, all agents agree on the volatility of the production process σ but disagree on its drift.[4] Instead of considering that it equals μ, Agent-δ believes that the drift of the production process is given by $\mu + \delta$. Thus, δ represents her belief bias, and she is relatively optimistic (resp. pessimistic) compared to an agent with true beliefs if her bias is positive (resp. negative). In fact, Agent-δ believes that the production process is given by

$$dy_t = (\mu + \delta)\, y_t dt + \sigma y_t dW_{\delta,t},$$

where W_δ is a standard unidimensional $\left((\mathcal{F}_t), \mathbb{Q}^{\delta}\right)$-Brownian motion, such that $W_{\delta,t} = W_t - \frac{\delta t}{\sigma}$.

As each generation is populated by a continuum of agents, I use probability density functions to describe their wealth share distribution, similarly to, e.g., Beddock and Jouini (2021). At time $t = 0$, I assume that the wealth share distribution of the investors of

[3] Note that this drift μ takes into account that part of the production process is consumed at each date as explained further. This is shown formally in the proof of Lemma 1.

[4] This assumption is in line with the literature which shows that the expected return is harder to estimate than the variance (see, e.g., Williams, 1977, Merton, 1980).

Generation-0 is given by a Gaussian distribution with parameters $\bar{\delta}$ and $\bar{\omega}$, which are given exogenously. I further show in Proposition 1 that, at any time $t \in \mathbb{R}^*$ and in state of the world W_t, the wealth share distribution of Generation-t—denoted by ν_{δ,t,W_t}—is still given by a normal probability density function with parameters $\bar{\delta}_{t,W_t}$ and $\bar{\omega}_{t,W_t}$, whose expression are endogenously determined.[5]

At time t, the investors of Generation-t are born and forecast that the production process will deliver a payoff y_{t+T} at time $t+T$. Because their common lifespan is T, they thus make their consumption and bequeath plans to consume and bequeath at the end of their life. More formally, Agent-δ of Generation-t plans to consume an amount $c_{\delta,t+T}$ of her endowment $y_{\delta,t+T}$ at time $t+T$ and to bequeath the remaining part $b_{\delta,t+T} = y_{\delta,t+T} - c_{\delta,t+T}$ to the next generation. Her expected utility is given by

$$\mathbb{E}_t \left(M_{\delta,t+T} \left(au \left(c_{\delta,t+T} \right) + u \left(b_{\delta,t+T} \right) \right) \right), \tag{2.1}$$

where $M_{\delta,t+T}$ is the Radon-Nikodym derivative of her subjective probability measure $\mathbb{Q}^\delta$ with respect to $\mathbb{P}$ and a is an exogenously given non-negative coefficient, common to all agents and all generations, that allows to determine the proportions of consumed and bequeathed wealth, as shown in Proposition 1. Setting $a = 1$ implies that the agents equally weight the utility they derive from consumption and bequeath, and thus allocate half of their wealth to each. For higher (resp. lower) values, it means that they prefer to consume (resp. bequeath), which reflects some selfishness (resp. altruism).

The Generation-t temporary equilibrium—defined by a continuum of consumption and bequeath plans, denoted by $(c_{\delta,t+T})_{\delta\in\mathbb{R}}$ and $(b_{\delta,t+T})_{\delta\in\mathbb{R}}$ respectively, to be realized at time $t+T$ and a positive density price p_{t+T}—is obtained when each agent of Generation-t maximizes her expected utility according to her beliefs such that both her static budget constraint and the market clearing condition are satisfied. Additionally, the sum of the consumption and the bequeath of a given agent must not exceed her endowment.

Let now more precisely study how this temporary equilibrium evolves from a generation

[5] Because she inherits later in her life, the time-t wealth share of a given agent of Generation-t more precisely describes this agent's expected share of total endowment at time $t+T$ seen from date t.

to another. I assume that Agent-δ of Generation-t gives birth to Agent-δ of Generation-$t+dt$ at time $t+dt$. These two agents are therefore part of Family-δ, and all members of the same family keep the same beliefs. Although the wealth share of each family is state-dependent and thus varies through time, this intra-family belief transmission is key because it allows belief heterogeneity to persist as time goes by. At time $t+dt$, agents of Generation-$t+dt$ plan to consume at time $t+dt+T$ a part of the bequeathed wealth of Generation-t that they will inherit at time $t+T$ and to bequeath the remaining part to the subsequent generation. Thus, it leads to the Generation-$t+dt$ temporary equilibrium, characterized by $(c_{\delta,t+dt+T})_{\delta\in\mathbb{R}}$, $(b_{\delta,t+dt+T})_{\delta\in\mathbb{R}}$, and p_{t+dt+T}. Applying the same reasoning to all subsequent generations results in a continuum of consumption and bequeath plans made by all successive generations. Because each generation is born at a different time and in a different state of the world, the successive plans of the members of a given family can differ markedly even if they share the same beliefs.

This intra-family belief transmission assumption implies an additional dynamic family budget constraint. More precisely, seen by Agent-δ of Generation-t, Family-δ's expected endowment should remain unchanged between $t+T$ and $t+dt+T$, which means that the expected value of the Family-δ time-$t+dt+T$ endowment should equal the expected value of its adjusted time-$t+T$ endowment evaluated at date t and at the time-$t+dt+T$ price.[6] Formally, for each time t and each Family-δ, this implies that

$$\mathbb{E}_t\left(p_{t+dt+T}y_{\delta,t+dt+T}\right)=\mathbb{E}_t\left(p_{t+dt+T}y_{\delta,t+T}\frac{y_{t+dt+T}}{y_{t+T}}\right). \tag{2.2}$$

Equation (2.2) simply states that the budget of each family evolves according to the evolution of the total production. Solving this equation together with the Generation-t temporary equilibrium equations delivers explicit solutions for the Generation-t wealth share distribution parameters $\bar{\delta}_{t,W_t}$ and $\bar{\omega}_{t,W_t}$. They are given in Proposition 1, where I also report the equilibrium features.

[6] Family-δ's adjusted time-$t+T$ endowment is given by $y_{\delta,t+T}\dfrac{y_{t+dt+T}}{y_{t+T}}$ and takes into account the growth of the production process between $t+T$ and $t+dt+T$.

Proposition 1 (Equilibrium and wealth share distribution)**.**
In equilibrium, at time t and in state of the world W_t:

1. *The Generation-t investors' consumption and bequeath plans for time $t+T$ are given by*

$$c_{\delta,t+T} = \frac{1}{1+a^{-\frac{1}{\gamma}}} y_{\delta,t+T}, \qquad \text{and} \qquad b_{\delta,t+T} = \frac{a^{-\frac{1}{\gamma}}}{1+a^{-\frac{1}{\gamma}}} y_{\delta,t+T},$$

with

$$y_{\delta,t+T} = y_{t+T} \left(\int \left(\lambda_{\delta,t,W_t} M_{\delta,t+T} \right)^{\frac{1}{\gamma}} d\delta \right)^{-1} \left(\lambda_{\delta,t,W_t} M_{\delta,t+T} \right)^{\frac{1}{\gamma}},$$

where $M_{\delta,t+T} = exp \left(\frac{\delta}{\sigma} (W_{t+T} - W_t) - \frac{1}{2} \frac{\delta^2}{\sigma^2} T \right)$ is the Generation-t Radon-Nikodym derivative of the subjective probability measure $\mathbb{Q}^\delta$ with respect to $\mathbb{P}$, and

$$\lambda_{\delta,t,W_t} = \frac{1}{\sqrt{2\pi}^{\gamma}} \frac{\sqrt{\sigma^2 + T\varphi\left(\bar{\omega}_{t,W_t}^2\right)}^{\gamma-1}}{\sqrt{\varphi\left(\bar{\omega}_{t,W_t}^2\right)\gamma}^{\gamma} \sqrt{\sigma^2}^{\gamma-1}} exp \left(- \frac{\left(\delta - \bar{\delta}_{t,W_t} + (1-\gamma) T\varphi\left(\bar{\omega}_{t,W_t}^2\right)\right)^2}{2\varphi\left(\bar{\omega}_{t,W_t}^2\right)} \right)$$

is the inverse of the Generation-t Lagrange multiplier with

$$\varphi(x) = \frac{x}{2} - \frac{\gamma\sigma^2}{2T} + \sqrt{\left(\frac{x}{2} - \frac{\gamma\sigma^2}{2T} \right)^2 + \frac{\sigma^2 x}{T}}.$$

2. *The state price density is given by*

$$p_{t+T} = y_{t+T}^{-\gamma} \left(\int \left(\lambda_{\delta,t,W_t} M_{\delta,t+T} \right)^{\frac{1}{\gamma}} d\delta \right)^{\gamma}.$$

3. *The Generation-t wealth share distribution is given by a normal probability density function with a standard deviation and a mean given respectively by*

$$\bar{\omega}_{t,W_t} = \bar{\omega}, \qquad \text{and} \qquad \bar{\delta}_{t,W_t} = \bar{\delta} + \frac{\varphi\left(\bar{\omega}^2\right) W_t}{\sigma},$$

where $\bar{\delta}$ and $\bar{\omega}$ are given constants describing the Generation-0 wealth share distribution.

A first point to notice is that the higher a is, i.e., the more selfish each generation of investors is, the larger is the share of wealth they consume before they die and the smaller is the one they bequeath to the subsequent generation. On the other extreme, setting $a = 0$ means that the investors do not consume and bequeath all their wealth to the next generation.

Additionally, from the third item, I observe that the Generation-t wealth share distribution's standard deviation is constant and does not vanish as time goes by, which ensures that the agents differ persistently in their beliefs. This feature is in sharp contrast with Atmaz and Basak (2018) and other finite-horizon models with one terminal production (and consumption) date, where belief dispersion consistently decreases with time. Such a result is of great empirical importance as surveys show that beliefs are mostly characterized by large and persistent individual heterogeneity (see, e.g., Meeuwis et al., 2019, Giglio et al., 2020, Das et al., 2020). Moreover, as the mean of the wealth share distribution depends on W_t, the average belief bias fluctuates around the initial value $\bar{\delta}$. On average, a generation of investors is more optimistic when it is born in a good state of the world, and vice versa.[7]

2.2 A dual approach in a model with a sliding horizon

Alternatively, one can study an equivalent model which leads to a similar equilibrium and the same implications, and whose main feature is to consider a continuum of infinitely lived agents who continuously revise their plans in a sliding horizon framework. While it results in a model without effective consumption—as explained below the investors maximize utility derived from planned consumption but continuously postpone it so that they never consume—, the main advantage of this approach is to permit a natural construction of an intertemporal representative agent. In addition, the interpretation and the novelty of further derived results are easier to explain under this alternative view. In this section, I therefore briefly describe the settings of the model, before constructing the intertemporal

[7]More precisely, I say that a generation of investors is more optimistic if a larger share of the expected future total endowment is held by more optimistic agents.

representative agent.

This dual model shares most of its characteristics with the overlapping heterogeneous generations model presented above. Indeed, I still consider a pure-exchange economy, based on an expected production process similar to the process y, evolving in continuous time with an infinite horizon. Moreover, there is also a continuum of heterogeneous CRRA agents who maximize their expected utility from future (expected) consumption, and I use a normal probability density function with mean $\bar{\delta}$ and standard deviation $\bar{\omega}$ to characterize their initial wealth share distribution. A major difference, however, is that there is now a single generation of infinitely lived agents who revise their plans as time goes by. Roughly speaking, in this framework, a given Agent-δ, whose beliefs are defined as in Section 2.1, corresponds to the entire Family-δ, and her successive plan revisions coincide with the different plans made by the successive generations of this family.

The mechanism behind the successive plan revisions is the following. At time t and in state of the world W_t, the investors consider a finite horizon economy of horizon T and make their investment decisions. They forecast that the expected production process will deliver a payoff at time $t+T$, and agents thus plan to consume at this date. The parameter T can therefore be associated to the agents' prevision horizon or, more generally, to the agents' investment horizon. At time $t + dt$, the market reopens and new information W_{t+dt} comes in. Still considering a finite horizon economy of horizon T (because their prevision horizon is fixed), the agents thus shift the date of their actions and update them. They forecast that the expected production process will deliver a payoff at time $t+dt+T$ and plan to consume at time $t + dt + T$. As time goes by, this leads to a continuum of temporary equilibria—defined by a state price density and a continuum of expected consumption plans—with a sliding horizon where consumption plans are continuously revised and where effective consumption never occurs as it is continuously postponed.

The fact that the agents continuously revise their plans is a key feature of this model. In fact, it ensures to maintain a persistent belief heterogeneity. More precisely, because these revisions induce a continuous shift of the effective consumption date, the investors keep their beliefs and consider a similar maximization program at each date where consumption is supposed to happen after a period of length T (under an additional dynamic budget constraint similar to the one defined by (2.2)).

Let now properly state the equivalence of the two approaches in the following lemma.

Lemma 1. *The sliding horizon model is equivalent to the overlapping heterogeneous generations model under the assumption that the successive generations only bequeath and do not consume (i.e. $a = 0$). In case effective consumption is allowed, the two models still yield analogous implications up to a small adjustment in the drift of the underlying production processes.*

As stated before, the main interest of considering this alternative equivalent approach is that it allows for a natural definition of the intertemporal representative agent. Such an agent is constructed as the fictitious agent who, if endowed with the total wealth of the economy, would have a marginal utility equal to the equilibrium price. She has the same utility function as the other investors and is characterized by the Radon-Nykodym derivative of her subjective probability measure with respect to the true probability measure.

Proposition 2 (Representative agent)**.** *The intertemporal representative agent of the economy is the fictitious investor whose time-$t+T$ Radon-Nikodym derivative of the subjective probability measure $\mathbb{Q}^\delta$ with respect to $\mathbb{P}$ seen from date t is given by*

$$M_{RA,t+T} = \left(\int \left(\lambda_{\delta,t,W_t} M_{\delta,t+T}\right)^{\frac{1}{\gamma}} d\delta \right)^{\gamma},$$

where $M_{\delta,t+T}$ and λ_{δ,t,W_t} are defined as in Proposition 1.

On the technical side, this result, combined with the equivalence of the two models, is important because it allows to completely characterize the inverse of the Lagrange multipliers and thus to fully determine the models equilibrium.

2.3 The stock price and its dynamics

I now derive the stock price and its dynamics in the presence of belief heterogeneity. As stated in the previous section, the results are derived under the view of the alternative model with a sliding horizon which eases the interpretation and the comparison with the existing literature. For the sake of clarity and because both models are equivalent, I nevertheless use the notations defined in Section 2.1

I assume that a risky stock S is available for trading. The stock is in positive net supply of one unit and, at time t, is a claim to the payoff y_{t+T} expected to be paid at time $t+T$. Studying its properties leads to the following proposition.

Proposition 3 (Equilibrium stock price, mean return, and volatility)**.**
In equilibrium, at time t and in state of the world W_t:

1. *The stock price is given by*

$$S_t = \overline{S}_t exp\left(\bar{\delta}_{t,W_t} T - \frac{\varphi\left(\bar{\omega}^2\right) T^2}{2}\right),$$

2. *The mean stock return is given by*

$$\mu_{S_t} = \overline{\mu} + \sigma\left(\frac{\partial \bar{\delta}_{t,W_t}}{\partial W_t} T\right) + \frac{1}{2}\left(\frac{\partial \bar{\delta}_{t,W_t}}{\partial W_t} T\right)^2,$$

3. *The stock volatility is given by*

$$\sigma_{S_t} = \overline{\sigma} + \frac{\partial \bar{\delta}_{t,W_t}}{\partial W_t} T,$$

where $\overline{S}_t = y_t exp\left(\left(\mu - \gamma\sigma^2\right)T\right)$, $\overline{\mu} = \mu$, and $\overline{\sigma} = \sigma$ are the equivalent quantities obtained in a similar standard economy without belief heterogeneity.

The formulas share similarities with those in Atmaz and Basak (2018). However, the two models differ along one important dimension: unlike them, I design an economy where the effective consumption date is continuously postponed so that the time-t remaining time before consumption always equals T instead of $T-t$. As the time-t heterogeneity impact depends on the remaining time before consumption, I thus obtain a stationary model where the heterogeneity effects are not smoothed as time goes by. In particular, as implied by their framework, their time-T stock price formula reduces to $S_T^{AB} = y_T$. The heterogeneity effects that they observe thus completely vanish when approaching date T, and their model is not stationary. Conversely, my time-T stock price is still fully impacted by the time-T investors heterogeneity, characterized by $\bar{\delta}_{T,W_T}$ and $\bar{\omega}$.

The results stationarity and persistence are of first interest as they more accurately reflect the heterogeneous market participants reality. These features also allow to test the model empirically over a long period and to use data sampled at various frequencies. Before doing so in Section 3, I now discuss the properties of the market characteristics derived in Proposition 3. I first focus on the equilibrium mean stock return μ_{S_t}.

As stated before, one of the consequences of the sliding horizon methodology is that the time-t remaining time before expected effective consumption does not depend on t. Thus, the derivative of the stock price with respect to t differs markedly from the one obtained in Atmaz and Basak (2018), leading to different mean returns.

I find that a higher belief dispersion leads to a higher equilibrium mean stock return. More precisely, the belief dispersion has an impact on the sensitivity of the average belief bias to news: the higher the belief dispersion, the higher this sensitivity, and thus the higher the mean return. This is consistent with the recent work of Brandon and Wang (2020) who find that the average return on stocks with high sensitivity to earning belief shocks is 7.14% per year higher than that in stocks with low sensitivity. Conversely, Atmaz and Basak (2018) derive that the sign of the relation is state dependent and that higher dispersion leads to lower returns when the view is sufficiently optimistic. They further show that the relation between the mean return and the relative risk aversion coefficient depends on the level of optimism, while I derive an unambiguous negative relation. The intuition behind this result is simple: in an economy populated by heterogeneous agents, more risk averse agents speculate less aggressively, and thus earn lower returns. Lastly, I observe that the mean equilibrium stock return increases as the investment horizon T increases.

For the sake of completeness, I now briefly report the properties of the first and third items of Proposition 3, which are mostly similar to those in Atmaz and Basak (2018) except that the heterogeneity effects remain persistent. I refer the reader to their paper for more detailed explanations of the underlying mechanisms behind these results.

Specifically, for a given time t, I find analogous impacts of the belief distribution parameters on the stock price. First, the stock price depends positively on the time-t average belief bias $\bar{\delta}_{t,W_t}$. Second, the sign of the belief dispersion impact is state dependent: the impact is positive for sufficiently good states of the world and negative for sufficiently bad ones.[8] I also similarly derive that the stock price is convex in the time-t expected production level y_t. Finally, unlike the standard case, the impact of the coefficient of relative risk aversion γ is not always negative, but can be positive for sufficiently bad states of the world.[9]

The third item considers the stock volatility σ_{S_t}. Several observations are in order. First, in a heterogeneous economy, it is higher than the production process volatility σ, in line with empirical observations (see, e.g., Ajinkya and Gift, 1985, Anderson et al., 2005, and Banerjee, 2011). Recall that an important difference with Atmaz and Basak (2018) is that, although both formulas have the same shape, this excess volatility effect does not decrease as time goes by. This is because the investors' heterogeneity remains persistent. Second, all else equal, the higher the belief dispersion, the higher the state-sensitivity of the average belief bias, and thus the higher the excess volatility. In fact, a higher fluctuations in the average belief bias translate to additional stock price fluctuations and therefore increases stock volatility. Additionally, the coefficient of relative risk aversion has a negative impact on σ_{S_t}. Finally, a higher investment horizon T leads to a higher stock volatility.

[8] Two effects are at play. On the one hand, as the function φ is increasing in belief dispersion, there is a direct negative effect. On the other hand, there is an indirect effect of the belief dispersion trough the average belief bias. For bad states, this indirect effect is negative and reinforces the first effect. For good ones, this is the opposite and the overall effect can even be positive in case of sufficiently good states. Formally, as the average belief bias is state dependent, I derive that the stock price increases in belief dispersion when $\bar{\delta}_{t,W_t} > \bar{\delta} + \frac{\varphi\left(\bar{\omega}^2\right)T}{2}$.

[9] Formally, the stock price increases in risk aversion when the following condition holds

$$\bar{\delta}_{t,W_t} < \bar{\delta} + \frac{\varphi\left(\bar{\omega}^2\right)T}{2} - \sqrt{\left(\gamma\sigma^2 - T\bar{\omega}^2\right)^2 + 4T\sigma^2\bar{\omega}^2}.$$

Overall, a rich set of predictions can be derived from Proposition 3. In particular, it allows to consider the impact of belief heterogeneity on the mean stock return and on the stock volatility. Moreover, as observed above, the impact of the belief bias on the model characteristics is indirect and depends on the belief dispersion which therefore appears to be the relevant predictor to consider. The persistence of belief dispersion that I obtain—which is not a feature of other existing models—also allows to use data sampled at various frequencies, leading to new testable implications. In sum, these theoretical results can be converted into testable hypotheses stated below that are empirically tested in the next section.

Hypothesis. *The main testable implications of the model are the following:*

H1. A higher belief dispersion predicts higher mean stock returns.

H2. The ability of belief dispersion to predict mean stock returns remains over long horizons.

H3. A higher belief dispersion predicts a higher stock volatility.

H4. The ability of belief dispersion to predict the stock volatility remains over long horizons.

3 Empirical test of the model

This section provides an empirical test of the hypotheses *H1-H4*. As I consider an economy where there is only one risky stock available for trading, the model is better suited to test an index—referred to as the market index and whose definition is given below—rather than to test individual stocks. I thus focus on studying the empirical ability of market belief dispersion to predict future market index returns and volatility over time using variables sampled at various frequencies (from one month to two years) from January 1982 to December 2019.

3.1 Market belief dispersion data

Let me first explain how the data on market belief dispersion, expressed in percentages, is constructed. I use analyst monthly forecasts of individual stocks as a proxy for investors' beliefs, and I aggregate them using different methodologies (described below) to obtain market belief dispersion variables. As underlined by Yu (2011), this bottom-up approach has the advantage of taking into account hundreds of forecasts at any given time and thus likely has a good signal-to-noise ratio. Using forecasts of individual stocks also allows to define and study the returns and volatility of a market index based on these stocks, which ensures a direct link between beliefs and market characteristics.

The data comes from two databases: I use the analyst monthly forecasts of the EPS LTG of individual stocks from the IBES database,[10] and the CRSP database to obtain monthly market capitalizations. The IBES data is winsorized at the 1% and 99% levels to account for potential outliers or data errors. I also winsorize the prices at the 99% level. Furthermore, I exclude stocks whose price is below five dollars at portfolio formation to avoid that extreme returns on penny stocks drive the results and stocks for which less than two analysts provide EPS LTG forecasts during the month to focus on stocks that exhibit some forecast dispersion. The data is available from January 1982 to December 2019. Throughout the sample, the average number of stocks used to compute the variable is 965 and each stock at any given time is followed on average by five to six analysts.[11] This large number of stocks alleviates the possibility that idiosyncratic firm disagreement drives the variations of the market belief dispersion.

[10] I thus use earnings data to measure cashflows rather than dividends data. This choice is motivated by Da (2009) who argues that potential problems of working with dividends could arise because of the dividend payout policy of some firms. Campbell (2000) further highlights other empirical difficulties. On the theoretical side, using the accounting clean surplus identity, Vuolteenaho (1999) shows that if one looks at the infinite horizon, cash flow and earnings contain the same information. Thus, earnings are both theoretically equivalent and empirically better behaved than dividends.

[11] The average number of analysts who provide an EPS LTG forecast for any given stock at any given time is 5.33.

For each common stock i listed on the NYSE/Amex/Nasdaq in each month t that meets the above mentioned requirements, I obtain the standard deviation of the analyst forecasts—that I refer to as the stock disagreement and that I denote $\tilde{\omega}_{i,t}$—from the IBES Unadjusted Summary database.[12] Additionally, I obtain the market capitalization of each of these stocks at the end of each month—that I denote $MKTCAP_{i,t}$—using the closing price and the number of shares outstanding of the stock considered from CRSP.

With this data in hand, I construct my first value-weighted measure of monthly market belief dispersion $\bar{\omega}^{VW}_{mean,1M}$ which is similar to the one defined in Yu (2011). For a given month t, it is defined as the cross-sectional (value-weighted) average of individual stock disagreements

$$\bar{\omega}^{VW}_{mean,1M,t} = \frac{\sum_i MKTCAP_{i,t} \times \tilde{\omega}_{i,t}}{\sum_i MKTCAP_{i,t}}.$$

To further rule out the possibility that the market belief dispersion is driven by idiosyncratic firm disagreement, I also consider an alternative value-weighted monthly measure: the cross-sectional value-weighted standard deviation of individual stock disagreements $\bar{\omega}^{VW}_{std,1M}$. A larger dispersion of individual stock disagreements indeed reflects a higher market disagreement among investors. Letting N_t denote the number of stocks that meet the requirements in month t, it is obtained with the following formula

$$\bar{\omega}^{VW}_{std,1M,t} = \frac{\sum_i MKTCAP_{i,t} \times \left(\tilde{\omega}_{i,t} - \bar{\omega}^{VW}_{mean,1M,t}\right)^2}{\frac{N_t-1}{N_t}\sum_i MKTCAP_{i,t}}.$$

Lastly, for the sake of robustness, I construct similar market belief dispersion variables, denoted by $\bar{\omega}^{EW}_{mean,1M}$ and $\bar{\omega}^{EW}_{std,1M}$ respectively, whose only difference with $\bar{\omega}^{VW}_{mean,1M}$ and $\bar{\omega}^{VW}_{std,1M}$ is to use equal-weighting rather than value-weighting.

The variables for longer horizons (one quarter ($3M$), six months ($6M$), one year ($12M$), and two years ($24M$)) are then obtained by averaging the monthly values over the period of interest. In the remainder of the analysis, I therefore use those market belief dispersion variables defined over various horizons as the predictors, and examine different specifica-

[12] Similarly to Buraschi et al. (2014), I use unadjusted data to circumvent the problem of using stock-split adjusted data.

tions depending on the horizon considered and the type of weighting used in the dependent variable.

Insert Figure 1 here.

To give an example of how the market belief dispersion variables vary as time goes by, Figure 1 shows the time series of the quarterly variables. I also report their summary statistics over the full sample in Table 1.

Insert Table 1 here.

3.2 Predicting market returns

In this section, I test the hypotheses *H1-H2*. Formally, I therefore use empirical data to see if a higher market belief dispersion leads to higher market returns and if this positive model implied relation holds for data sampled at longer frequencies.

Let first properly define the value-weighted market index whose returns are used in the analysis. Each month, it is constituted of all individual stocks, weighted by their market capitalization, whose price is above five dollars and for which at least two monthly EPS LTG forecasts are provided in the IBES database. In other words, the assets that constitute this market index are those used to construct the market belief dispersion variables, which allows to more precisely capture the link between the investor beliefs and the market characteristics. In addition, I construct a similar market index using equal weights. I then compute the (raw) simple returns for various holding periods of both indices, and report their summary statistics, expressed in percentages, in Table 2, where the subscript indicates the holding period considered and the superscript the type of weighting.

Insert Table 2 here.

I can now test the model implied hypotheses *H1-H2*. More specifically, *H1* implies that a higher market belief dispersion in a given period should result in higher market returns in the subsequent period. If *H2* is verified, this positive relation should hold no matter the horizon considered (from one month to two years). In order to check these hypotheses, I thus run the following standard OLS regression

$$RET^k_{i,t} = \gamma^k_{j,i} + \theta^k_{j,i}\bar{\omega}^k_{j,i,t-1} + \xi^k_{j,i,t}, \tag{3.1}$$

where t refers to the period t, $i = \{1M, 3M, 6M, 12M, 24M\}$, $j = \{mean, std\}$, and $k = \{VW, EW\}$. Inference is based on autocorrelation- and heteroskedasticity-robust standard errors (Newey and West, 1987), and all variables are standardized prior to estimation. Moreover, I consider non-overlapping returns for horizons longer than a month to avoid econometric issues. This leads to a total of 20 specifications, whose results are reported in Table 3.

Insert Table 3 here.

Several observations are in order. First, all coefficients are positive. More interestingly, most of them are statistically different from zero.[13] Thus, *H1* seems to be verified in the data. This complements the mixed results regarding the impact of belief dispersion on returns found in the literature. While some studies also derive a positive relation (see, e.g., Doukas et al., 2006), others find that a higher belief dispersion predicts lower returns (see, e.g., Diether et al., 2002). The novelty in my empirical study is that I focus on the returns of an index (either value- or equally-weighted) that only contains stocks for which there is some belief dispersion and that are thus used in the construction of the market belief dispersion data.

Second, I observe that the model implied positive relation holds for all horizons, meaning that the effects of belief dispersion do not vanish over longer periods. In other words, the hypothesis *H2* is verified and the persistence of belief dispersion that I document in my model is an important feature to be taken into account.

Lastly, note that since the model abstracts from interest rate issues (both under the overlapping generations view or the sliding horizon one), and thus does not allow to accurately define the risk premium, it is mostly suitable to study the ability of market belief

[13] Note that because the model implies a positive relation, I consider one-sided tests where the alternative hypothesis is that the coefficient is positive.

dispersion to predict the market index raw returns. Although the link is less direct, I nevertheless run a similar analysis using market index excess returns in Appendix B.1, which confirms the previous results.

3.3 Predicting market volatility

I now focus on the hypotheses *H3-H4* and study if they are verified empirically. Hence, I test if market belief dispersion positively predicts future market volatility and if this relation holds for various horizons.

As commonly done in the literature (see, e.g., French et al., 1987, Schwert, 1989), I exploit daily stock returns—taken from the CRSP database—to obtain my market volatility data. More precisely, to obtain value- and equally-weighted variables for a given horizon, I consider the two market indices defined in Section 3.2, compute the sum of their squared daily returns over the period of interest (discounted by the average daily market index returns of the period), and thereby obtain the value- and equally-weighted market variances for this horizon. I then easily convert them into the annualized market volatilities that I denote VOL^{VW} and VOL^{EW} respectively. As in the previous part, I further add subscripts to these variables to indicate the horizon considered.

Computing their descriptive statistics shows that they are highly positively skewed and leptokurtic.[14] Similarly to Paye (2012), I thus define annualized market log volatility variables $LVOL^{VW}$ and $LVOL^{EW}$ as the natural logarithms of VOL^{VW} and VOL^{EW} respectively, whose distribution are approximately Gaussian (Andersen et al., 2001). Because the empirical analysis relies on linear models estimated by OLS, this latter property is of first importance, and I therefore use $LVOL^{VW}$ and $LVOL^{EW}$ as the independent variables in the subsequent empirical analysis. I report their summary statistics in Table 4.

Insert Table 4 here.

Volatility processes are known to exhibit a high degree of persistence when the data is sampled over short periods of time. To take this persistence into account and to ensure

[14]For instance, skewness $\left(VOL_{3M}^{VW}\right) = 2.89$, and kurtosis $\left(VOL_{3M}^{VW}\right) = 15.43$.

that a potential predictor contains valuable information, it is thus important to control for past (log) volatility when trying to predict future (log) volatility. Paye (2012) for instance includes six lagged values of (log) volatility in his monthly specification and two lagged values in his quarterly one. I thus adopt a similar approach when testing if a higher market belief dispersion in a given period predicts a higher market (log) volatility in the subsequent one. More precisely, I use six lags when dealing with monthly data, two lags when dealing with quarterly data, and one lag when dealing with data sampled every six months. For the longest horizons considered (one and two years), I do not control for past (log) volatility because the data is sampled over relatively long periods of time and is not strongly persistent. Formally, for, e.g., the quarterly specifications, I therefore consider the following regression for market (log) volatility

$$LVOL^k_{3M,t} = \alpha^k_{0,j,3M} + \alpha^k_{1,j,3M} LVOL^k_{3M,t-1} + \alpha^k_{2,j,3M} LVOL^k_{3M,t-2} + \beta^k_{j,3M} \bar{\omega}^k_{j,3M,t-1} + \epsilon^k_{j,3M,t}, \tag{3.2}$$

where t refers to quarter t, $j = \{mean, std\}$, and $k = \{VW, EW\}$.

The main interest of (3.2) and of other similar OLS regressions studying other horizons is to test the hypothesis $H_0 : \beta = 0$ against the alternative $H_1 : \beta > 0$: rejecting the null indeed implies that, when controlling for past (log) volatility, belief dispersion (positively) predicts future market (log) volatility. Similarly to the previous part, I consider one sided tests because the predicted sign of the coefficient is supported by the theoretical implications of the model. Again, inference is based on autocorrelation- and heteroskedasticity-robust standard errors, and all variables are standardized prior to estimation. The results are reported in Table 5.

Insert Table 5 here.

While most of the coefficients are positive, only those related to the equally-weighted specification for variables sampled at a yearly frequency are statistically significant. Thus, over the full sample, neither *H3* nor *H4* seem to be strongly validated by the data.

One of the potential explanations for this lack of statistical significance could be that the sign of the relation varies through the sample period considered. To further investigate this issue, I run rolling window regressions of Equation (3.2) using subsamples of 15 years.[15] For the sake of concision, I focus on data sampled at the quarterly frequency, which leads to a total of 91 regressions for each specification.[16] I therefore obtain time series of the estimated values of $\beta^{VW}_{mean,3M}$, $\beta^{VW}_{std,3M}$, $\beta^{EW}_{mean,3M}$, and $\beta^{EW}_{std,3M}$, and report them in Figure 2. In each panel, the horizontal axis shows the end date of the subsamples, and thicker rounds indicate statistically significant positive values at the 10% level.

Insert Figure 2 here.

As reported in Panel A of Table 6, between 52 and 83 of the 91 estimated β's are positive depending on the market belief dispersion variable considered. More interestingly, around a third of these estimated values are statistically significant at the 10% level. Moreover, none of the negative coefficients obtained from the value-weighted variables is significantly different from zero. The evidence therefore points towards the approval of the model implied positive effect of belief dispersion on the market volatility, and, while most of the significantly positive β's are obtained for subsamples ending between 2000 and 2010, the weak results found in Table 5 most likely result from the fact that negative coefficients are obtained from the most recent subsamples.

Insert Table 6 here.

As robustness checks, Panel B and Panel C of Table 6 further show results of alternative specifications that use subsamples of 10 and 20 years respectively. Overall, they confirm the results found in the main specification of the rolling window analysis.

[15]Conrad and Glas (2018) provide a similar analysis to test if macroeconomic variables predict volatility in the cross-section of industry portfolios. Note that, for the sake of completeness, I also provide in Appendix B.2 an analogous rolling window analysis concerning the market index raw returns to see how the ability of market belief dispersion to predict them evolves over time.

[16]Because I need to have the market (log) volatility data of the two previous quarters, my first subsample starts in the third quarter of 1982 and ends in the second quarter of 1997.

4 Conclusion

I define an infinite horizon economy populated by overlapping generations of investors who differ in their beliefs. For any generation, the wealth share distribution of its heterogeneous members can be described by a Gaussian distribution with a state-dependent mean and a constant standard deviation, implying persistent belief heterogeneity across time. I compute the model equilibrium, and, using a fully equivalent alternative approach based on continuous plan revisions of infinitely lived agents and a sliding horizon, construct an intertemporal representative agent. I then study the implications of the belief heterogeneity on various quantities of interest, namely the stock price, its mean return, and its volatility. In particular, I derive that both the stock mean return and volatility monotonically increase with belief dispersion. While the derived formulas share similarities with those in Atmaz and Basak (2018), a major difference is that the theoretical framework that I use leads to stationary results with non-vanishing heterogeneity. The contribution of such a modeling is twofold. First, the heterogeneity persistence is empirically observed, and consequently the model results are more in line with reality (see, e.g., Giglio et al., 2020). Second, it allows me to test the model empirically over long periods. Using analyst forecasts from the IBES database, I show evidence that the documented positive relations between the stock equilibrium mean return and volatility and the belief dispersion are verified in the data when considering a market index. This is true for data sampled at various frequencies (from one month to two years) and the persistence of belief dispersion that I document in my model is thus an important feature to be taken into account.

Lastly, note that the model only considers a single stock in the economy. It would thus be interesting to extend it to the case of a multi-stocks economy to derive testable cross-sectional relations. I leave this for future research.

A Proofs

It is more convenient to formally compute the model equilibrium and the representative agent under the dual approach based on the model presented in Section 2.2. Thus, in this appendix, I first prove Lemma 1, which states the equivalence of the two models. I then state and prove Theorem 1 (which is mostly an analogous version of Proposition 1 in the context of the sliding horizon model). Proposition 1 and Proposition 2 are then corollaries of this theorem. Lastly, I provide a proof of Proposition 3.

Proof of Lemma 1 The equivalence of the two models comes from the fact that they share similar settings, an analogous maximization program, and the same constraints. To see this, let formally present Agent-δ's maximization program in the context of both models at time t and in state of the world W_t.

Let start with the overlapping heterogeneous generations model. Using (2.1), the maximization program of Agent-δ at time t and in state of the world W_t is given by

$$\max_{c_{\delta,t+T},b_{\delta,t+T}} \mathbb{E}_t\left(M_{\delta,t+T}\left(au\left(c_{\delta,t+T}\right)+u\left(b_{\delta,t+T}\right)\right)\right), \tag{A.1}$$

where a is a given non-negative coefficient, common to all agents and all generations, that represents the agents' degree of selfishness, and $M_{\delta,t+T}$ is the Radon-Nikodym derivative of her subjective probability measure $\mathbb{Q}^\delta$ with respect to $\mathbb{P}$, such that

$$M_{\delta,t+T} = \exp\left(\frac{\delta}{\sigma}\left(W_{t+T}-W_t\right)-\frac{1}{2}\frac{\delta^2}{\sigma^2}T\right). \tag{A.2}$$

Denoting by $y_{\delta,t+T}$ the endowment (at time $t+T$) of Agent-δ of Generation-t, solving (A.1) under the condition that the sum of her consumption and her bequeath equals her endowment ($c_{\delta,t+T}+b_{\delta,t+T}=y_{\delta,t+T}$) leads to

$$c_{\delta,t+T} = \frac{1}{1+a^{-\frac{1}{\gamma}}}y_{\delta,t+T}, \qquad \text{and} \qquad b_{\delta,t+T} = \frac{a^{-\frac{1}{\gamma}}}{1+a^{-\frac{1}{\gamma}}}y_{\delta,t+T}. \tag{A.3}$$

Because $au\left(c_{\delta,t+T}\right)+u\left(b_{\delta,t+T}\right) = a\left(1+a^{-\frac{1}{\gamma}}\right)^{\gamma}u\left(y_{\delta,t+T}\right)$, maximizing (A.1) is therefore equivalent to solving

$$\max_{y_{\delta,t+T}} \mathbb{E}_t\left(M_{\delta,t+T}u\left(y_{\delta,t+T}\right)\right), \tag{A.4}$$

under the same market clearing condition and budget constraint, before dividing it into consumption and bequeath according to the above mentioned proportions.

The market clearing condition and the budget constraint that both (A.1) and (A.4) need to satisfy are respectively given by

$$y_{t+T} = \int y_{\delta,t+T} d\delta, \tag{A.5}$$

and

$$\left(\nu_{\delta,t,W_t} y_{t+T} - y_{\delta,t+T}\right) p_{t+T} \geq 0. \tag{A.6}$$

Additionally, (2.2) need to be verified for all families.

Let now more formally present the model with a sliding horizon. I denote by $\tilde{y}$ the expected production process of this model. It follows a geometric Brownian motion with the same volatility σ as the process y and a drift denoted by $\tilde{\mu}$. I also define $\tilde{\nu}_{\delta,t,W_t}$ which stands for the time-t wealth share distribution of the infinity lived agents. Recall that the initial investors' wealth share distributions are the same in the two models. Recall also that, as stated in Section 2.2, the way heterogeneous beliefs are defined are identical in both models: $M_{\delta,t+T}$ is thus defined as in (A.2).

At time t and in state of the world W_t, Agent-δ's maximization program is given by

$$\max_{\tilde{y}_{\delta,t+T}} \ \mathbb{E}_t \left(M_{\delta,t+T} u\left(\tilde{y}_{\delta,t+T}\right)\right), \tag{A.7}$$

under the following constraints:

- the market clearing condition: $\tilde{y}_{t+T} = \displaystyle\int \tilde{y}_{\delta,t+T} d\delta$.
- the time-t static budget constraint: $\left(\tilde{\nu}_{\delta,t,W_t} \tilde{y}_{t+T} - \tilde{y}_{\delta,t+T}\right) \tilde{p}_{t+T} \geq 0$, where $\tilde{p}_{t+T}$ is the time-$t+T$ equilibrium state price density.
- the time-t dynamic budget constraint: seen from date t, and before trading and consumption reallocation, Agent-δ's expected consumption share should remain unchanged between $t+T$ and $t+T+dt$ because the endowment of each agent is fixed. Formally, it implies that $\mathbb{E}_t \left(\tilde{p}_{t+T+dt} \tilde{y}_{\delta,t+T+dt}\right) = \mathbb{E}_t \left(\tilde{p}_{t+T+dt} \tilde{y}_{\delta,t+T} \dfrac{\tilde{y}_{t+T+dt}}{\tilde{y}_{t+T}}\right)$.

It is then clear that (A.4) and (A.7) are similar. Moreover, the three above mentioned constraints are similar to (A.5), (A.6), and (2.2) respectively. This thus leads to equivalent implications for the two models. Note however that the alternative model with a sliding horizon does not allow for (effective) consumption. A small adjustment thus needs to be made in the drift of the production process $\tilde{y}$ to obtain a full equivalence.

Let come back to the overlapping heterogeneous generations model. At time $t+dt+T$, the total endowment of the economy is given by $\frac{a^{-\frac{1}{\gamma}}}{1+a^{-\frac{1}{\gamma}}} y_{t+dt+T}$ (because all the agents of Generation-t have consumed a fraction $\frac{1}{1+a^{-\frac{1}{\gamma}}}$ of their individual time-$t+T$ endowments, given that a is common to all investors and all generations). Denoting by $\widehat{\mu}$ the drift of the production process in this model if there was no consumption and defining $\frac{1}{1+a^{-\frac{1}{\gamma}}} = \alpha dt$, the overall endowment process of the model is therefore given by $dy_t = (\widehat{\mu} - \alpha)\, y_t dt + \sigma y_t dW_t$. As stated in footnote 3, I thus define $\mu = \widehat{\mu} - \alpha$ to take this consumption into account.

From there, it is clear that setting $\tilde{\mu} = \widehat{\mu} - \alpha$ allows for a full equivalence of the two models. Finally, note that the case $a = 0$ (meaning that agents only bequeath) implies $\alpha = 0$ and thus $\tilde{\mu} = \mu = \widehat{\mu}$. ■

Theorem 1 (Equilibrium and representative agent in the model with a sliding horizon)**.** *In equilibrium, at time t and in state of the world W_t:*

1. *The investors' consumption plans and the state price density are given by*

$$\begin{aligned} \tilde{y}_{\delta,t+T} &= \tilde{y}_{t+T} \left(\int \left(\lambda_{\delta,t,W_t} M_{\delta,t+T}\right)^{\frac{1}{\gamma}} d\delta \right)^{-1} \left(\lambda_{\delta,t,W_t} M_{\delta,t+T}\right)^{\frac{1}{\gamma}}, \\ \tilde{p}_{t+T} &= \tilde{y}_{t+T}^{-\gamma} \left(\int \left(\lambda_{\delta,t,W_t} M_{\delta,t+T}\right)^{\frac{1}{\gamma}} d\delta \right)^{\gamma}, \end{aligned}$$

where $M_{\delta,t+T} = exp\left(\frac{\delta}{\sigma}\left(W_{t+T} - W_t\right) - \frac{1}{2}\frac{\delta^2}{\sigma^2}T\right)$ is the time-$t+T$ Radon-Nikodym derivative of the subjective probability measure $\mathbb{Q}^\delta$ with respect to $\mathbb{P}$ (seen from date t), and

$$\lambda_{\delta,t,W_t} = \frac{1}{\sqrt{2\pi}^{\gamma}} \frac{\sqrt{\sigma^2 + T\varphi\left(\bar{\omega}^2_{t,W_t}\right)}^{\gamma-1}}{\sqrt{\varphi\left(\bar{\omega}^2_{t,W_t}\right)\gamma}^{\gamma} \sqrt{\sigma^2}^{\gamma-1}} exp\left(- \frac{\left(\delta - \bar{\delta}_{t,W_t} + (1-\gamma)\, T\varphi\left(\bar{\omega}^2_{t,W_t}\right)\right)^2}{2\varphi\left(\bar{\omega}^2_{t,W_t}\right)} \right)$$

is the inverse of the time-t Lagrange multiplier with

$$\varphi(x) = \frac{x}{2} - \frac{\gamma\sigma^2}{2T} + \sqrt{\left(\frac{x}{2} - \frac{\gamma\sigma^2}{2T}\right)^2 + \frac{\sigma^2 x}{T}}.$$

2. *The intertemporal representative agent of the economy is the fictitious investor whose time-$t+T$ Radon-Nikodym derivative of the subjective probability measure $\mathbb{Q}^\delta$ with respect to $\mathbb{P}$ (seen from date t) is given by*

$$M_{RA,t+T} = \left(\int \left(\lambda_{\delta,t,W_t} M_{\delta,t+T}\right)^{\frac{1}{\gamma}} d\delta\right)^{\gamma}.$$

3. *The wealth share distribution of the continuum of investors is given by a normal probability density function with a standard deviation and a mean given respectively by*

$$\bar{\omega}_{t,W_t} = \bar{\omega}, \qquad \text{and} \qquad \bar{\delta}_{t,W_t} = \bar{\delta} + \frac{\varphi\left(\bar{\omega}^2\right) W_t}{\sigma},$$

where $\bar{\delta}$ and $\bar{\omega}$ are given constants describing the time-0 wealth share distribution.

Proof of Theorem 1 The three items of the proposition are solved altogether in the following proof.

The maximization program of Agent-δ at time t and in state of the world t is given by (A.7) in the proof of Lemma 1, which also explicitly states the constraints that must be satisfied.

The first order conditions directly give

$$\begin{cases} \tilde{y}_{\delta,t+T} & = \tilde{y}_{t+T} \left(\int \left(\lambda_{\delta,t,W_t} M_{\delta,t+T}\right)^{\frac{1}{\gamma}} d\delta\right)^{-1} \left(\lambda_{\delta,t,W_t} M_{\delta,t+T}\right)^{\frac{1}{\gamma}}, \\ \tilde{p}_{t+T} & = \tilde{y}_{t+T}^{-\gamma} \left(\int \left(\lambda_{\delta,t,W_t} M_{\delta,t+T}\right)^{\frac{1}{\gamma}} d\delta\right)^{\gamma}, \end{cases}$$

where λ_{δ,t,W_t} is the inverse of the time-t Lagrange multipliers of the form

$$\lambda_{\delta,t,W_t} = K_{t,W_t} \exp\left(-\frac{1}{2a_{t,W_t}^2}\delta^2 - b_{t,W_t}\delta\right). \tag{A.8}$$

To further identify a_{t,W_t} and b_{t,W_t}, notice that the budget constraint of the maximization program can be equivalently formulated as

$$\tilde{\nu}_{\delta,t,W_t} = \frac{\mathbb{E}_t\left(\tilde{p}_{t+T}\tilde{y}_{\delta,t+T}\right)}{\mathbb{E}_t\left(\tilde{p}_{t+T}y_{t+T}\right)}. \tag{A.9}$$

Let then look for a solution of the form $\tilde{\nu}_{\delta,t,W_t} = \frac{1}{\sqrt{2\pi}\bar{\omega}_{t,W_t}}\exp\left(-\frac{\left(\delta-\bar{\delta}_{t,W_t}\right)^2}{2\bar{\omega}_{t,W_t}^2}\right)$, and see if it verifies (A.9).

Explicit computations show that it is indeed the case, which thus proves that the time-t wealth share distribution is given by a normal probability density function. Formally, by identification, simple algebra leads to

$$\begin{cases} a_{t,W_t}^2 &= \frac{\bar{\omega}_{t,W_t}^2}{2} - \frac{\gamma\sigma^2}{2T} + \sqrt{\left(\frac{\bar{\omega}_{t,W_t}^2}{2} - \frac{\gamma\sigma^2}{2T}\right)^2 + \frac{\sigma^2\bar{\omega}_{t,W_t}^2}{T}}, \\ b_{t,W_t} &= (1-\gamma)\,T - \frac{\bar{\delta}_{t,W_t}}{a_{t,W_t}^2}. \end{cases}$$

Note also that using a reasoning similar to the one in the proof of Theorem 4 in Bianchi et al. (2021), based on the homogeneity property of the CRRA utility function, one can show the uniqueness of this solution (see also Dana, 1995).

To complete the identification of λ_{δ,t,W_t}, let determine K_{t,W_t}, which is obtained via the construction of the intertemporal representative agent.

By construction, this agent is the fictitious investor who, if endowed with the total wealth of the economy, would have a marginal utility equal to the equilibrium price. Formally, because her utility function is given by $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, it translates into $\tilde{p}_{t+T} = \tilde{y}_{t+T}^{-\gamma}M_{AR,t+T}$, which directly leads to

$$M_{AR,t+T} = \left(\int\left(\lambda_{\delta,t,W_t}M_{\delta,t+T}\right)^{\frac{1}{\gamma}}d\delta\right)^{\gamma}.$$

Moreover, for the intertemporal representative agent beliefs to be well-defined, one needs to ensure that $\mathbb{E}_t\left(M_{AR,t+T}\right) = 1$. Easy computations then result in

$$K_{t,W_t} = \frac{1}{\sqrt{2\pi}^{\gamma}}\frac{\sqrt{\sigma^2+Ta_{t,W_t}^2}^{\gamma-1}}{\sqrt{a_{t,W_t}^2\gamma}^{\gamma}\sqrt{\sigma^2}^{\gamma-1}}\exp\left(-\frac{a_{t,W_t}^2b_{t,W_t}^2}{2}\right).$$

Defining the function φ as given in Theorem 1 and plugging the expressions of a_{t,W_t}, b_{t,W_t}, and K_{t,W_t} into (A.8), before rearranging the terms, yield the results.

Lastly, I need to ensure that the time-t dynamic budget constraint is verified. To do so, let define $\mu_{\tilde{p}_{t+T}}$ and $\sigma_{\tilde{p}_{t+T}}$ (that depend on $\bar{\delta}_{t,W_t}$ and $\bar{\omega}_{t,W_t}$) such that $d\tilde{p}_{t+T} = \mu_{\tilde{p}_{t+T}}\tilde{p}_{t+T}dt + \sigma_{\tilde{p}_{t+T}}\tilde{p}_{t+T}dW_t$. I further define $\mu_{\tilde{y}_{\delta,t+T}}$ and $\sigma_{\tilde{y}_{\delta,t+T}}$ (that also depend on the parameters of the Generation-t wealth share distribution) similarly.

Using Ito's lemma, this budget constraint leads to

$$\mathbb{E}_t\left(\mu_{\tilde{y}_{\delta,t+T}} + \sigma_{\tilde{p}_{t+T}}\sigma_{\tilde{y}_{\delta,t+T}} - \tilde{\mu} - \sigma\sigma_{\tilde{p}_{t+T}}\right) = 0. \tag{A.10}$$

Direct computations allow to rewrite the left hand-side of (A.10) as a polynomial function of δ of degree two. As (A.10) must be verified for all agents, each coefficient of the polynomial form must equal zero. By identification, this leads to the expressions of $\bar{\delta}_{t,W_t}$ and $\bar{\omega}_{t,W_t}$ derived in the theorem. ∎

Proof of Proposition 1 The proof of Proposition 1 directly follows from the equivalence of the two models given in Lemma 1 and from the proof of the first and third items in Theorem 1. One simply needs to replace $\tilde{y}_{t+T}$ by y_{t+T}, $\tilde{y}_{\delta,t+T}$ by $y_{\delta,t+T}$, $\tilde{p}_{t+T}$ by p_{t+T} and $\tilde{\nu}_{\delta,t,W_t}$ by ν_{δ,t,W_t}. Moreover, at time t, the agents of the sliding horizon model correspond to the Generation-t of the overlapping generations model.

Finally, note that the proportions of consumed and bequeathed wealth are explicitly given by (A.3) in the proof of Lemma 1. ∎

Proof of Proposition 2 The proof of Proposition 2 directly follows from the proof of the second item in Theorem 1. ∎

Proof of Proposition 3

1. By no arbitrage, the time-t stock price is given by

$$S_t = \frac{\mathbb{E}_t\left(p_{t+T}y_{t+T}\right)}{\mathbb{E}_t\left(p_{t+T}\right)}.$$

Computing the numerator and the denominator and rearranging the terms lead to the formula in Proposition 3.

To determine the benchmark economy stock price $\overline{S}_t$, I set $\bar{\delta}$ and $\bar{\omega}$ to zero, and substitute them into the stock price formula.

2. 3. Applying Ito's lemma to the time-t stock price formula yields the results.

Similarly to the first item, $\overline{\mu}$ and $\overline{\sigma}$ are defined by setting $\bar{\delta}$ and $\bar{\omega}$ to zero into the formulas.

■

B Additional empirical results

B.1 Predicting market excess returns

In this appendix, I run a similar analysis as the one presented in Section 3.2. The only difference is that the dependent variable (either value- or equally-weighted) is now given by the excess returns of the market index instead of its raw returns. The results are reported in Table 7 and further confirm the model implied positive relation: the coefficients are all positive and statistically significant. These results thus provide additional support for the validation of *H1-H2*.

Insert Table 7 here.

B.2 Rolling window analysis of the raw simple market index returns at the quarterly horizon

In order to gain deeper insights of the predicted positive relation between market belief dispersion and market index raw returns, I run rolling window regressions of Equation (3.1) using quarterly data and subsamples of 15 years, i.e. with 60 quarterly observations. This is the same methodology as the one in Section 3.3, and the results are reported in Figure 3.

Insert Figure 3 here.

The framework leads to a total of 92 regressions for each specification, and I therefore obtain time series of the estimated values of θ.[17] The horizontal axis shows the end date of the subsamples, while the vertical axis gives the estimated value. Thicker rounds (resp. crosses) indicate statistically significant positive (resp. negative) values at the 10% level. The graphs show that most of the subsamples ending between 2011 and 2019 yield positive estimated values, while subsamples ending during the previous decade lead to negative ones. Additionally, Table 8 reports the number of positive estimated coefficients for all specifications, along with the number of significantly positive (second column) and negative (third column) ones. Panel A presents the results for rolling windows of 60 quarterly observations, and Panels B and C use different window lengths of 40 and 80 quarterly observations respectively.

Insert Table 8 here.

Overall, apart from those obtained with the variable $\theta^{VW}_{std,3M}$, the results mostly confirm that the model implication seems to be verified empirically.

[17]The first subsample starts in the second quarter of 1982 and ends in the first quarter of 1997.

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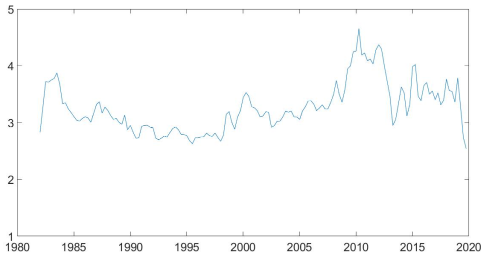
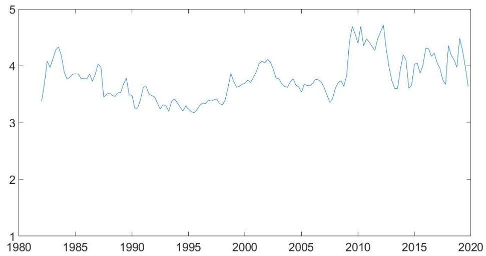
Figure 1: Time series of the quarterly market belief dispersion variables

Panel A: cross-sectional average of individual stock disagreements

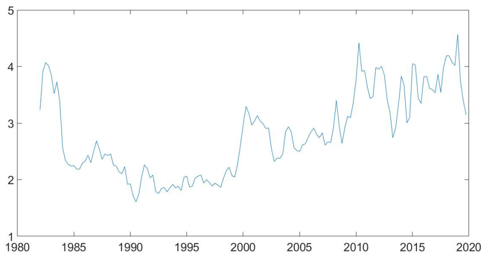


$\bar{\omega}^{VW}_{mean,3M}$

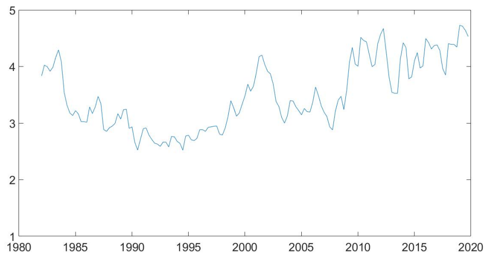


$\bar{\omega}^{EW}_{mean,3M}$

Panel B: cross-sectional standard deviation of individual stock disagreements



$\bar{\omega}^{VW}_{std,3M}$

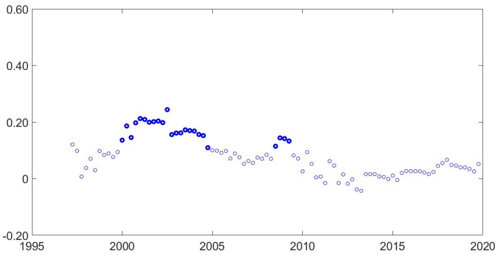


$\bar{\omega}^{EW}_{std,3M}$

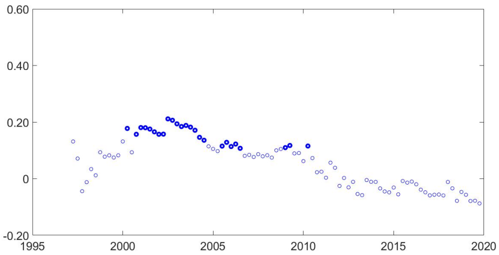
The figure presents the time evolution of the quarterly market belief dispersion variables expressed in percentages. Panel A (resp. Panel B) shows the variables computing as the cross-sectional average (resp. standard deviation) of individual stock disagreements. The left column shows the value-weighted variables and the right column the equally-weighted ones. The sample goes from the first quarter of 1982 to the fourth quarter of 2019.

Figure 2: Time series of rolling window estimates of β

Panel A: Value-weighted specifications

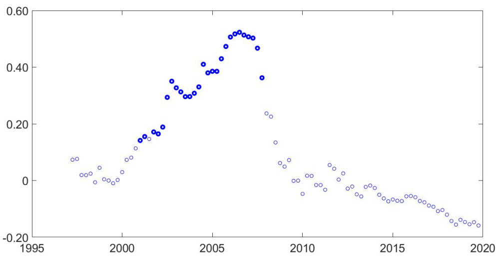


$\hat{\beta}^{VW}_{mean,3M}$

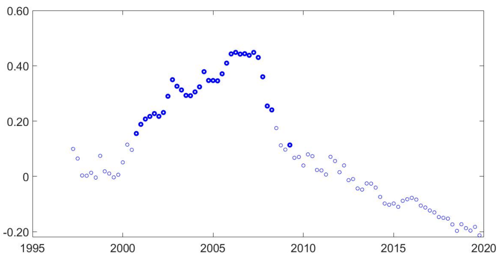


$\hat{\beta}^{VW}_{std,3M}$

Panel B: Equally-weighted specifications



$\hat{\beta}^{EW}_{mean,3M}$

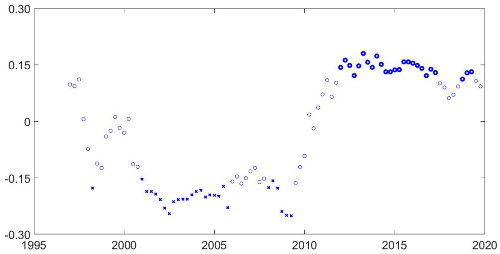


$\hat{\beta}^{EW}_{std,3M}$

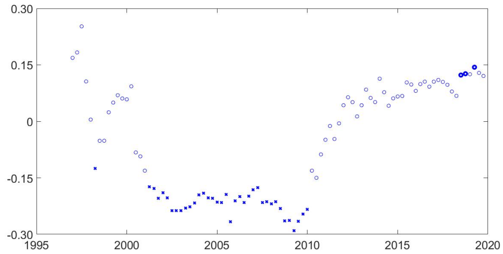
The figure presents the estimated values of $\beta^{VW}_{mean,3M}$, $\beta^{VW}_{std,3M}$, $\beta^{EW}_{mean,3M}$, and $\beta^{EW}_{std,3M}$ obtained from rolling window regressions with 60 quarterly observations. The horizontal axis shows the end date of the subsamples (the first one ends in the second quarter of 1997). Thicker rounds indicate statistically significant positive values at the 10% *level (using one-sided tests).*

Figure 3: Time series of rolling window estimates of θ

Panel A: Value-weighted specifications

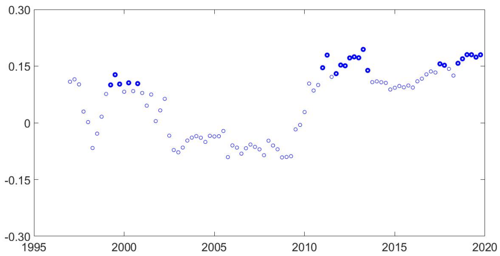


$\hat{\theta}^{VW}_{mean,3M}$

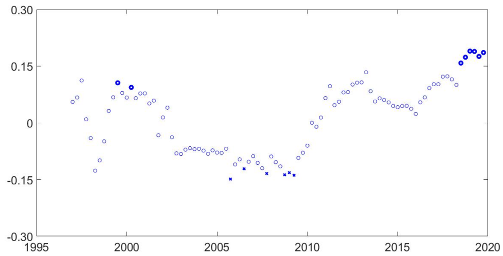


$\hat{\theta}^{VW}_{std,3M}$

Panel B: Equally-weighted specifications



$\hat{\theta}^{EW}_{mean,3M}$



$\hat{\theta}^{EW}_{std,3M}$

The figure presents the estimated values of $\theta^{VW}_{mean,3M}$, $\theta^{VW}_{std,3M}$, $\theta^{EW}_{mean,3M}$, and $\theta^{EW}_{std,3M}$ obtained from rolling window regressions with 60 quarterly observations. The horizontal axis shows the end date of the subsamples (the first one ends in the first quarter of 1997). Thicker rounds (resp. crosses) indicate statistically significant positive (resp. negative) values at the 10% *level (using one-sided tests).*

Table 1: Descriptive statistics of the quarterly market belief dispersion variables

	Mean	St. Dev.	Skewness	Kurtosis	ρ_1	ρ_2
Panel A: value-weighted specification						
$\bar{\omega}^{VW}_{mean,3M}$	3.26	0.43	0.84	3.28	0.89	0.77
$\bar{\omega}^{VW}_{std,3M}$	2.80	0.74	0.39	2.01	0.94	0.85
Panel B: equally-weighted specification						
$\bar{\omega}^{EW}_{mean,3M}$	3.78	0.37	0.53	2.63	0.88	0.74
$\bar{\omega}^{EW}_{std,3M}$	3.48	0.61	0.34	1.87	0.94	0.84

The table contains descriptive statistics of the quarterly belief dispersion variables, expressed in percentages. The mean, standard deviation, skewness, and kurtosis are reported for each variable, as well as the first- and second-order sample autocorrelations (ρ_1 and ρ_2). The sample goes from the first quarter of 1982 to the fourth quarter of 2019.

Table 2: Descriptive statistics of the market index returns for various holding periods

	Mean	St. Dev.	Skewness	Kurtosis
Panel A: value-weighted specification				
RET_{1M}^{VW}	0.29	3.79	-0.55	5.26
RET_{3M}^{VW}	0.86	6.23	-0.25	4.47
RET_{6M}^{VW}	1.74	9.36	0.23	5.29
RET_{12M}^{VW}	3.25	12.71	-0.20	2.39
RET_{24M}^{VW}	6.30	14.94	-0.48	2.80
Panel B: equally-weighted specification				
RET_{1M}^{EW}	0.35	4.07	-0.87	7.00
RET_{3M}^{EW}	1.04	7.01	-0.53	4.49
RET_{6M}^{EW}	2.03	9.85	-0.03	4.47
RET_{12M}^{EW}	3.92	13.66	0.03	2.95
RET_{24M}^{EW}	7.39	16.58	0.80	3.07

The table contains descriptive statistics of the raw market index simple returns for various holding periods, expressed in percentages. The mean, standard deviation, skewness, and kurtosis are reported for each variable. The subscript indicates the holding period considered and the superscript the type of weighting. The sample goes from January 1982 to December 2019.

Table 3: Market belief dispersion and market index raw returns

	$1M$	$3M$	$6M$	$12M$	$24M$
Panel A: value-weighted specification					
$\theta^{VW}_{mean,\cdot}$	0.03	0.09*	0.14*	0.10	0.28**
	(0.81)	(1.39)	(1.60)	(0.75)	(2.03)
$\theta^{VW}_{std,\cdot}$	0.05	0.10*	0.14*	0.13	0.31**
	(1.18)	(1.41)	(1.47)	(1.15)	(1.92)
Panel B: equally-weighted specification					
$\theta^{EW}_{mean,\cdot}$	0.08**	0.13**	0.20***	0.25*	0.42***
	(1.78)	(1.98)	(2.42)	(1.67)	(2.58)
$\theta^{EW}_{std,\cdot\cdot}$	0.07*	0.10*	0.17**	0.25**	0.48***
	(1.62)	(1.58)	(2.01)	(1.84)	(3.01)

The table contains the results of regression (3.1). *Inference is based on autocorrelation- and heteroskedasticity-robust standard errors (Newey and West, 1987), and all variables are standardized prior to estimation. For frequencies longer than a month, I use non-overlapping returns. I consider one sided tests where the null is* $H_0 : \theta = 0$ *against the alternative* $H_1 : \theta > 0$. *The sample goes from January 1982 to December 2019.*

Table 4: Descriptive statistics of the annualized market (log) volatility for various horizons

	Mean	St. Dev.	Skewness	Kurtosis
Panel A: value-weighted specification				
$LVOL_{1M}^{VW}$	-2.06	0.46	0.62	3.91
$LVOL_{3M}^{VW}$	-2.00	0.41	0.81	4.12
$LVOL_{6M}^{VW}$	-1.97	0.39	0.75	3.85
$LVOL_{12M}^{VW}$	-1.93	0.38	0.39	2.97
$LVOL_{24M}^{VW}$	-1.90	0.34	0.46	2.65
Panel B: equally-weighted specification				
$LVOL_{1M}^{EW}$	-2.09	0.49	0.70	3.95
$LVOL_{3M}^{EW}$	-2.01	0.43	0.92	4.29
$LVOL_{6M}^{EW}$	-1.98	0.41	0.94	4.28
$LVOL_{12M}^{EW}$	-1.94	0.39	0.73	3.31
$LVOL_{24M}^{EW}$	-1.91	0.37	0.80	3.56

The table contains descriptive statistics of the annualized market log volatility variables for various horizons. The mean, standard deviation, skewness, and kurtosis are reported for each variable. The subscript indicates the horizon considered and the superscript the type of weighting. The sample goes from January 1982 to December 2019.

Table 5: Market belief dispersion and annualized market (log) volatility

	$1M$	$3M$	$6M$	$12M$	$24M$
Panel A: value-weighted specification					
$\beta^{VW}_{mean,.}$	0.02	0.04	0.04	0.12	0.01
	(0.60)	(0.78)	(0.52)	(0.70)	(0.04)
$\beta^{VW}_{std,.}$	-0.00	0.01	0.03	0.07	-0.07
	(-0.09)	(0.26)	(0.43)	(0.37)	(-0.29)
Panel B: equally-weighted specification					
$\beta^{EW}_{mean,.}$	0.01	0.01	0.09	0.26*	0.18
	(0.18)	(0.15)	(0.71)	(1.56)	(0.86)
$\beta^{EW}_{std,..}$	0.01	0.04	0.10	0.26*	0.11
	(0.34)	(0.55)	(0.95)	(1.41)	(0.69)

The table contains the results of regression (3.2) *and other similar regressions that use (log) volatility as the dependent variable and market belief dispersion as the predictor (controlling for past (log) volatility). Inference is based on autocorrelation- and heteroskedasticity-robust standard errors (Newey and West, 1987), and all variables are standardized prior to estimation. For frequencies longer than a month, I use non-overlapping data. I consider one sided tests where the null is* $H_0 : \beta = 0$ *against the alternative* $H_1 : \beta > 0$*. The sample goes from January 1982 to December 2019.*

Table 6: *Statistics of rolling window estimates of* β

	$\#\hat{\beta} > 0$	$\#\hat{\beta}^* > 0$	$\#\hat{\beta}^* < 0$
	Panel A: 15 year rolling window (91 samples)		
$\hat{\beta}^{VW}_{mean,3M}$	83	24	0
$\hat{\beta}^{VW}_{std,3M}$	58	25	0
$\hat{\beta}^{EW}_{mean,3M}$	52	27	7
$\hat{\beta}^{EW}_{std,3M}$	59	32	14
	Panel B: 10 year rolling window (111 samples)		
$\hat{\beta}^{VW}_{mean,3M}$	92	36	0
$\hat{\beta}^{VW}_{std,3M}$	73	22	10
$\hat{\beta}^{EW}_{mean,3M}$	60	29	10
$\hat{\beta}^{EW}_{std,3M}$	59	31	22
	Panel C: 20 year rolling window (71 samples)		
$\hat{\beta}^{VW}_{mean,3M}$	53	24	0
$\hat{\beta}^{VW}_{std,3M}$	49	27	4
$\hat{\beta}^{EW}_{mean,3M}$	52	18	4
$\hat{\beta}^{EW}_{std,3M}$	50	40	10

The table contains some statistics on the estimated values of $\beta^k_{j,3M}$ *(*$j = \{mean, std\}$ *and* $k = \{VW, EW\}$*) obtained from rolling window regressions for several settings. Panel A refers to the specification defined by Equation* (3.2) *and uses 15 year rolling windows (60 quarterly data). Panel B and C show results with 10 year rolling windows (40 quarterly data) and 20 year rolling windows (80 quarterly data) respectively. The first column reports the number of positive coefficients obtained from these regressions. The second (resp. third) column report the number of significantly positive (resp. negative) coefficients at the* 10% *level obtained from these regressions (using one-sided tests). The overall sample goes from the first quarter of 1982 to the fourth quarter of 2019.*

Table 7: Market belief dispersion and market index excess returns

	$1M$	$3M$	$6M$	$12M$	$24M$
	Panel A: value-weighted specification				
$\theta^{VW}_{mean,\cdot}$	0.06*	0.14**	0.20***	0.19*	0.43***
	(1.45)	(2.16)	(2.45)	(1.41)	(2.67)
$\theta^{VW}_{std,\cdot}$	0.07**	0.15**	0.20***	0.23**	0.46**
	(1.92)	(2.33)	(2.42)	(1.92)	(2.29)
	Panel B: equally-weighted specification				
$\theta^{EW}_{mean,\cdot}$	0.10**	0.17***	0.25***	0.32**	0.50***
	(2.27)	(2.53)	(2.99)	(2.02)	(2.62)
$\theta^{EW}_{std,\cdot\cdot}$	0.09**	0.14**	0.23***	0.33**	0.56***
	(2.20)	(2.25)	(2.61)	(2.23)	(2.95)

The table contains the results of regression (3.1) *when the dependent variable is given by the market index excess returns. Inference is based on autocorrelation- and heteroskedasticity-robust standard errors (Newey and West, 1987), and all variables are standardized prior to estimation. For frequencies longer than a month, I use non-overlapping returns. I consider one sided tests where the null is* $H_0 : \theta = 0$ *against the alternative* $H_1 : \theta > 0$. *The sample goes from January 1982 to December 2019.*

Table 8: Statistics of rolling window estimates of θ

	$\#\hat{\theta} > 0$	$\#\hat{\theta}^* > 0$	$\#\hat{\theta}^* < 0$
	Panel A: 15 year rolling window (92 samples)		
$\hat{\theta}^{VW}_{mean,3M}$	44	25	27
$\hat{\theta}^{VW}_{std,3M}$	43	3	37
$\hat{\theta}^{EW}_{mean,3M}$	60	23	0
$\hat{\theta}^{EW}_{std,3M}$	55	8	6
	Panel B: 10 year rolling window (112 samples)		
$\hat{\theta}^{VW}_{mean,3M}$	55	8	24
$\hat{\theta}^{VW}_{std,3M}$	61	16	36
$\hat{\theta}^{EW}_{mean,3M}$	75	34	2
$\hat{\theta}^{EW}_{std,3M}$	64	24	18
	Panel C: 20 year rolling window (72 samples)		
$\hat{\theta}^{VW}_{mean,3M}$	40	13	12
$\hat{\theta}^{VW}_{std,3M}$	22	8	19
$\hat{\theta}^{EW}_{mean,3M}$	59	22	0
$\hat{\theta}^{EW}_{std,3M}$	40	2	2

The table contains some statistics on the estimated values of $\theta^k_{j,3M}$ ($j = \{mean, std\}$ and $k = \{VW, EW\}$) obtained from rolling window regressions defined similarly as Equation (3.1)*. Panel A uses 15 year rolling windows (60 quarterly data). Panel B and C show results with 10 year rolling windows (40 quarterly data) and 20 year rolling windows (80 quarterly data) respectively. The first column reports the number of positive coefficients obtained from these regressions. The second (resp. third) column report the number of significantly positive (resp. negative) coefficients at the* 10% *level obtained from these regressions (using one-sided tests). The overall sample goes from the first quarter of 1982 to the fourth quarter of 2019.*