Portfolio selection under ambiguity and the under-diversification puzzle

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Abstract

We show that well-known under-diversification puzzles are linked to ambiguity in the data. We develop a novel robust optimization model under data ambiguity and show that the optimal portfolios satisfy second-order stochastic dominance. We put the model to the data of 21 developed economies and 19 emerging markets, and find that it generates optimal international portfolios with allocations that match the observed home bias for reasonable ambiguity parameters and regardless of investor risk-aversion. This speaks to the home-bias puzzle. We also apply the model to individual investors and find that the under-diversification puzzle is also explained by the portfolio ambiguity.

Keywords: Ambiguity, home bias puzzle, individual under-diversification, international portfolio diversification, conditional Value-at-Risk.
JEL Classification: D81, G11, G15, G41, H31.

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Contents

List of Figures

List of Tables

1 Introduction

The equity home bias puzzle (French and Poterba, 1991) refers to the significant discrepancy between international investor's domestic equity holdings from what they should hold if they were guided by the market portfolio being the optimal risky portfolio with the highest diversification benefits (Adler and Dumas, 1983). Optimality of the market portfolio assumes that nominal security returns are Brownian with known means, standard deviations, and covariance. Albeit, these data are only known to the extent that they belong to an ambiguity set. We propose a novel robust Mean-to-Conditional Value-at-Risk (robust MtC) portfolio optimization model under data ambiguity, prove that it is second order stochastic dominance (SSD) consistent, and apply it to a universe of 21 developed economies and 19 emerging markets to find that it generates allocations that match those of international equity investors for ambiguity sets obtained from market data. The equity home allocations are not biased but, instead, are optimal for investors with increasing and concave utility functions given the data ambiguity.

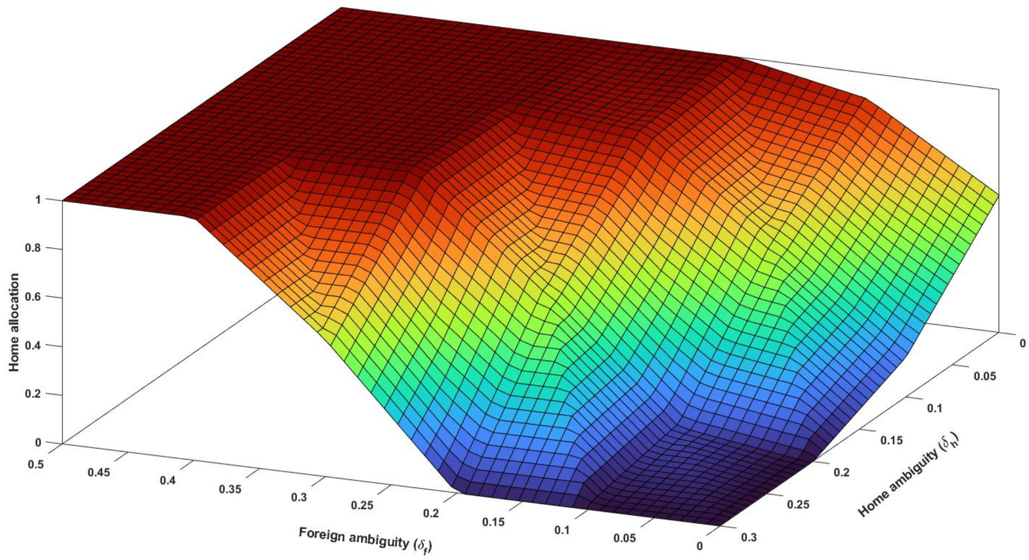
We test the implications of the model on the holdings of 60,000 US households, using data from a large discount brokerage house spanning 1991–1996 (Barber and Odean, 2000, 2001), and find that the least diversified portfolios are the least exposed to ambiguity in the distribution and mean returns of their portfolios. This suggests that the under-diversification puzzle can be explained by market data ambiguity.

Ambiguity is offered in current literature as an explanation of the home equity and under diversification puzzles, but our work differs in a fundamental way to contribute a new explanation. The earlier studies show that ambiguity averse investors diversify less than mean-variance based theory implies. This point is made in laboratory setups (Ahn et al., 2014; Bossaerts et al., 2010) and with theoretical models (Bossaerts et al., 2010; Cao et al., 2005; Dow et al., 1992; Easley and O'Hara, 2009; Epstein and Miao, 2003; Peijnenburg, 2018; Uppal and Wang, 2003). Empirical research confirms that investor characteristics relating to ambiguity aversion are strongly correlated with under-diversified portfolios (Bianchi and Tallon, 2019; Dimmock et al., 2016b, 2021; Mitton and Vorkink, 2007; Polkovnichenko, 2005), although they suggest different channels (behavioural probability weighting, preference for skewness, prospect theory preferences), and supporting evidence has not been found in all population samples (Dimmock et al., 2016a). In contrast to these studies we consider ambiguity in the market data to show, using a novel SSD consistent model, that it is optimal for investors to hold the portfolios they hold. This we show to be true for both international equity investors and US households.

Our contribution, beyond the methodological innovation of robust MtC, is twofold. First, we offer a more general explanation of the channels identified in the empirical studies by requiring only the mildest characteristic of a non-increasing convex utility

Figure 1: Ambiguity in mean returns and optimal home allocation

This figure illustrates the optimal allocation to home as a function of foreign and home mean return ambiguity parameters, δ_f and δ_h, respectively. The model optimizes robust MtC for a portfolio consisting of home (US) and foreign (Canada) equity market indices. The sample period spans 1 January 1999 to 31 December 2019.



investor. Second, and importantly, we fill a gap in the empirical setups. Namely, the fact that some investor characteristic correlates with under-diversified portfolios does not imply that the asset data justify this under-diversification nor that the held portfolios are optimal given the investor characteristic. Likewise, to the theoretical models we add ambiguity in the data instead of ambiguity aversion. Our approach leads to a data-driven explanation of the puzzles instead of a behavioural explanation.

The robust mean-to-Conditional Value-at-Risk portfolio model is optimizing a performance ratio for stable distributions (Martin et al., 2003) assuming ambiguity in distribution and mean returns, using the robust optimization methodology of Ben-tal and Nemirovski (1998); Ben-Tal et al. (2009). We construct the ambiguity sets with two different methods and two potential sources of ambiguity. One is based on ellipsoids constructed to include all parameter realisations obtained from market observed data, as is standard in robust optimization literature (Ben-Tal et al., 2009; Lotfi and Zenios, 2018), and the other using the economic policy uncertainty index (EPU of Baker et al. (2016) as a source of ambiguity. Figure 1 illustrates the robust MtC model optimal home allocation for US investors under mean return ambiguity in US and Canadian market indices. When there is no ambiguity in either market mean returns the optimal asset allocation is roughly 0.5/0.5, but when ambiguity is introduced the allocation can shift up to 100% towards home or towards foreign, depending on which market is more ambiguous.

We show that the optimal solution of the robust MtC model is also SSD consistent so that they are preferred by any investor with non-increasing convex utility functions. As a by-product of our result on robust MtC, we inform the literature about the characteristics of robust risk measures. Zhu and Fukushima (2009) show that if a risk measure is coherent, then its robust counterpart is also coherent. We show that this transitivity holds also for SSD consistency. We obtain analytically the solution of the two-securities model (home and foreign) and illustrate how the optimal portfolio choice can be biased toward domestic equity. The robust MtC model can be formulated as a second-order cone program and solved efficiently using the interior-point method for problems with many assets.

The recent literature models ambiguity-aversion in the context of expected utility maximization using approaches introduced in Gilboa and Schmeidler (1989) and Klibanoff et al. (2005). Gilboa and Schmeidler (1989) approach, known as the multiple-priors utility model (or max-min), maximizes the minimal expected utility given the decision-maker set of priors (see Bossaerts et al. (2010); Cao et al. (2005); Easley and O'Hara (2009); Epstein and Miao (2003); Peijnenburg (2018); Ui (2011); Uppal and Wang (2003)). Klibanoff et al. (2005) proposes a smooth ambiguity model that effectively weights all possible beliefs using a weighting function similar to expected utility. Ambiguity-aversion implies the concavity of the weighting function with the implication that worse beliefs will get more weight (see Asano and Osaki (2020); Gollier (2011); Maccheroni et al. (2013)). They are used to explain low market participation, equity home bias puzzle, and individuals portfolio under-diversification under specific utility functional form, assumptions about ambiguity-aversion and risk-aversion, and portfolio composition.

Ambiguity in the data does not enter any of the above models. Only Boyle et al. (2012) study the role of data ambiguity in determining portfolio under-diversification and his model can generate 'flight to familiarity" episodes using a mean-variance portfolio problem in which the asset mean returns belong to some confidence intervals. They find that an agent who views the stock market as ambiguous, relative to some limited number of familiar individual stocks, will invest in an under-diversified portfolio. This is the paper closest to ours. Our modeling advance is that we have a model that accounts for higher order moments and establish that it is SSD consistent. Importantly, however, we put the model to the data for the large data sets of the two under-diversification problems to show that i) the model matches observed home allocation weights with those observed empirically, ii) the level of ambiguity tested in the model is actually the one observed in the data, and iii) the puzzle ca be explained for reasonable ambiguity levels.

None of the papers cited above put their models to empirical data.[1] Cooper et al.

[1]For example, Uppal and Wang (2003) apply their model to a portfolio of three market indices, and find that the the optimal solutions generated under ambiguity do not match the observed allocation in

(2012) emphasize that the challenge in explaining the equity home bias puzzle is not only to generate allocations that have the same size as the observed allocations but also whether they are obtained using realistic parameter values. This is what we achieve in this paper. Our method uses ambiguity level implied by market data so we can determine whether the ambiguity parameters for which we observe the match between model-implied weights and actual home allocation are realistic and below the market-implied ambiguity parameters.

Another strand of literature investigates empirically the potential explanation of the home bias and household portfolio under-diversification puzzles. There are a few studies that investigate the association between the household portfolio characteristic and under-diversification. Dimmock et al. (2021) show that people display inverse-S-shaped probability weighting, i.e. overweighting low probability events, are less likely to own mutual funds, and are more likely to hold individual stock. Using the Survey of Consumer Finances data, Polkovnichenko (2005) shows that an investor with cumulative prospect theory preferences may take an under-diversified position in positively skewed security. Mitton and Vorkink (2007) test the preference for skewness in a sample of US households and show the household portfolio under-diversification is positively related to their preference for skewness. Dimmock et al. (2016b) find the ambiguity-aversion is significant in explaining the low market participation and home bias in a sample of US households. In contrast, Dimmock et al. (2016a) do not find ambiguity-aversion significant in explaining low equity market participation in a sample of Dutch households. However, it is notable and interesting that they find it significant when interacting with their "proxy" of ambiguity where this proxy itself is significant. This observation shows ambiguity on its own is a significant factor in explaining low market participation (household under-diversification). So there is no conclusive empirical evidence regarding the impact of ambiguity-aversion in explaining the low market participation puzzle. More recently, Bianchi and Tallon (2019) provide evidence that ambiguity affect households portfolio using a sample of French household portfolio. In particular, they show that ambiguity averse investors bear more risk due to a lack of diversification and are relatively more exposed to the French than to the international stock market. We add to this strand of literature by showing the household portfolio under-diversification is negatively related to the ambiguity level of the expected returns of the securities in the portfolio, and that is why ambiguity-averse investors hold less diversified portfolios.

Measurement of ambiguity is another angle that we contribute. Dow et al. (1992) mea-

home equity bias French and Poterba (1991) for reasonable ambiguity parameters. Using their model with more recent data spanning 1999-2019, we confirm that the model still does not generate optimal portfolio choices to match the observed ones. The direction of adjustment is unclear regarding when the joint distribution ambiguity ambiguity increases but the ambiguity may decrease. Indeed, using their model with more recent data, we observe this is the case (See Appendix Figure A1).

sure ambiguity-aversion by the sum of the non-additive probabilities. Baillon et al. (2018) extend this approach by measuring discrimination among different levels of likelihood. Baillon and Bleichrodt (2015) propose five ambiguity indices for ambiguity-aversion measurement using matching non-additive probabilities. These measures, however, cannot be computed from the data and depend on investor characteristic. Garlappi et al. (2007) and Boyle et al. (2012) measure data-driven ambiguity using a box or an ellipsoid around the data point mean return estimates while Ui (2011) measures ambiguity by the difference between the minimal possible mean and the true mean, which is very similar to the box ambiguity set used by Boyle et al. (2012). The interval ambiguity set ignore the correlation present across assets. In addition, none of the three aforementioned studies assumes ambiguity in the distribution of returns. Our ambiguity measurement is not only data-driven but also do not force any assumption on returns distribution and is able to capture the correlation among assets.

To the best of our knowledge, this is the first paper offering an explanation of the equity home bias and household portfolio under-diversification puzzles via ambiguity in the mean and distribution of returns, independently of a utility functional form. The robust MtC model is SSD consistent and the maximum robust MtC portfolios are preferred by the class of investors with concave and non-increasing utility functions. Our choice of base model (MtC) does use a normality assumption, which is prevalent the literature cited above and, furthermore, with an ellipsoidal ambiguity set we take into account the correlation between returns. Finally, we gauge implied ambiguity from market data, and we use it to evaluate model performance in resolving the equity home bias puzzle.

Robust optimization has been used extensively in portfolio optimization literature.[2] We contribute to this strand of literature the robust counterpart of MtC optimization model when distribution and means are ambiguous. The closet work in this respect is Goel et al. (2019) who developed robust counterpart for two variants of stable tail-adjusted return ratio (STARR), one with mixed conditional value-at-risk (MCVaR) and the other with deviation MCVaR, under joint ambiguity in the distribution modeled using copulas.

The broad groups of explanations to home equity bias puzzle are: (i) hedging real risk (real exchange rate risk and nontradable income risk), (ii) explicit costs and barrier for foreign investors in international financial markets (transaction cost, differences in tax treatments, in the legal framework and barriers for foreign investors), (iii) information asymmetry, (iv) trade, (v) governance and transparency, and (vi) behavioral bias (overconfidence, optimism, and familiarity). For a full description of the aforementioned explanations, we refer the readers to (Coeurdacier and Rey, 2013; Cooper et al., 2012)

[2]see (Ceria and Stubbs, 2006; Chen et al., 2014; El Ghaoui et al., 2003; Gao et al., 2017; Goldfarb and Iyengar, 2003; Lotfi and Zenios, 2018; Paç and Pınar, 2014; Tütüncü and Koenig, 2004; Ye et al., 2012)

and the references therein. Theoretical works build on macroeconomic modeling of an open economy in which they find portfolio choice in a two-country equilibrium macro model. The main motivation in this line of literature is the "*homogeneity*" assumption of the standard model of Lucas (1982). Various source of heterogeneity introduced in the model to explore the equity home bias puzzle (see e.g. Lewis (1999); Obstfeld and Rogoff (2000); Sercu (1980); Stulz (1981); Uppal and Wang (2003)) and has been tested empirically (see e.g. Coeurdacier and Guibaud (2011); Dahlquist et al. (2003); Errunza et al. (1999); Fidora et al. (2007); Glassman and Riddick (2001); Massa and Simonov (2006); Mishra and Ratti (2013)). So far, the empirical and theoretical literature on the equity home bias puzzle has the conclusion that the puzzle is a combination of all discovered channels, information asymmetries and economic openness channels shown to be empirically more significant, though (Cooper et al., 2012). We add to this strand of literature by introducing a theoretical model for considering market data ambiguity in the optimal portfolio choice selection and empirically testing it on two well-known puzzles.

The paper is organized as follow: First, we introduce the MtC portfolio optimization model and obtain its robust counterpart in Section 2. In Section 3 we describe our data and the application of the model to the equity home bias puzzle and individuals under-diversification problem. Finally, Section 4 concludes the paper.

2 Robust Mean-to-CVaR portfolios

2.1 Preliminaries

Portfolio return $\tilde{r}_p = \tilde{r}^\top x$ is a function of the vector of portfolio weights $x \in \mathbb{X} \subset \mathbb{R}^n_+$ and the random vector $\tilde{r} \in \mathbb{R}^n$ of asset returns, with expected value $\bar{r}$. $\mathbb{X}$ is the set of feasible portfolios assumed, for simplicity, to be linear. Given a risk-free asset with return r_f, the investment problem is to decide the allocation of wealth between this asset and the risky portfolio with random return $\tilde{r}_p$. If $y \geq 0$ is the proportion in the risky portfolio, the return of the *complete* portfolio is

$$\tilde{r}_c = y\tilde{r}_p + (1-y)r_f. \tag{1}$$

To account for higher-order moments we use CVaR as the risk criterion in portfolio selection. CVaR is coherent (Artzner, Delbaen, Eber, and Heath, 1999), and it can be minimized as a linear program for discrete distributions (Rockafellar and Uryasev, 2002). These clear advantages turned CVaR into a widely used risk measure, with Basel III shifting to CVaR to capture tail risk (Basel Committee, 2019), and with several financial and other applications.[3] Additional background is given in Appendix C.1.

Following Pagliardi et al. (2021), we define the *mean-to-CVaR ratio* as

$$\mathrm{MtC}_\alpha = \frac{\mathbb{E}(\tilde{r}_p - r_f)}{\mathrm{CVaR}_\alpha(\tilde{r}_p - r_f)}. \tag{2}$$

This is a reward ratio, like Sharpe, in the sense that it measures the expected excess return per unit of risk (see (Farinelli, Ferreira, Rossello, Thoeny, and Tibiletti, 2008)). The MC tangency portfolio is obtained by solving

$$\mathrm{MtC}^* = \max_{x \in \mathbb{X}} \frac{\mathbb{E}(\tilde{r}_p - r_f)}{\mathrm{CVaR}(\tilde{r}_p - r_f)}. \tag{3}$$

where $\mathbb{X}$ denotes the constraint set specifying feasible portfolios, and is defined as follows:

$$\mathbb{X} = \{x \in \mathbb{R}^n \mid x \geq 0, \sum_{i=1}^{n} x_i = 1\}. \tag{4}$$

From this tangency portfolio every other mean-CVaR (MC) efficient portfolio can be generated as a linear combination with the risk-free rate. Sharpe ratio is the slope of the tangency portfolio using variance as the risk measure, whereas MtC^* uses the coherent CVaR risk measure. CVaR under normality is given by $\mathrm{CVaR}_\alpha(\tilde{r}_p) = -\bar{r}_p + \kappa_{1-\alpha}\sigma_{\tilde{r}_p}$ where $\bar{r}_p$ and $\sigma_{\tilde{r}_p}$ are the mean and standard deviation of $\tilde{r}_p$, and $\kappa_{1-\alpha} = \frac{1}{1-\alpha}\phi(\Psi^{-1}(1-\alpha))$ with ϕ and Ψ the normal density and cumulative distribution functions, respectively. In this case the Sharpe ratio portfolio is a solution of model (3). Beyond the attractive properties of MtC optimization (coherence, SSD consistency, and a tractable linear programming model), Pagliardi et al. (2021) also demonstrate empirical advantages. Mean-CVaR model optimal weights are more robust than Mean-Variance model, and MtC portfolios are more positively skewed than Sharpe portfolios.

To understand the impact of ambiguity in the process of the portfolio optimization problem, we apply the *robust pptimization* approach, whose main building block is the ambiguity set. For example, when the distribution of return is ambiguous, that means you do not possess any information beforehand about neither probabilities nor the realizations, and they are only known to the extent that they belong to the so-called *ambiguity set*. The novelty of this approach is that for a given ambiguity set, it formulates the model and finds an optimal solution that assures the objective function is insensitive to

[3] Financial applications of CVaR optimization include, among others, Alexander and Baptista (2004); Alexander, Coleman, and Li (2006); Gotoh, Shinozaki, and Takeda (2013); Huang, Zhu, Fabozzi, and Fukushima (2008); Kibzun and Kuznetsov (2006); Mausser and Romanko (2018); Topaloglou, Vladimirou, and Zenios (2002); Xiong and Idzorek (2011). CVaR optimization also finds applications in areas such as the news vendor problem, radiation therapy treatment planning, carbon markets hedging, and water resources and energy management. See also (Zenios, 2007, ch. 5).

ambiguity and the constraints are satisfied with ambiguous data. This approach has a cost in the sense that optimal values are conservative.

The MtC model (3) depends on mean returns and the joint distributions of all securities in the portfolio.[4] To the extent this information is fully known, we have no ambiguity in the data and therefore there is no model mis-specification to address. Apparently, this is not the case. Practically speaking, the researchers adopt the following standard approach. They rely on discrete empirical distributions where the random variable of asset returns take discrete values from a finite set of equiprobable scenarios and obtain the corresponding formulation. Neither the scenarios nor the assigned equiprobable probabilities are presumed in reality.

In the next sub-section, we obtain the model under ambiguity by obtaining the robust MtC model and show that the model can be cast as a second-order cone program (SOCP) which can be solved efficiently using the interior-point methods. Furthermore, we show that the optimal solution of the model is SSD consistent.

2.2 Robust model formulation

In this section, we develop the robust counterpart of the MtC maximization model (3) under ambiguity in distribution and mean returns.[5] Doing so, we assume that the joint probability distribution of returns, π, is ambiguous and belong to the class of all distributions with means $\bar{r} \in \mathbb{R}^n$ and covariance matrix $\Sigma \in \mathbb{S}^n_+$.[6] Further, the mean values of returns, $\bar{r}$, belong to an ellipsoidal ambiguity set. The advantage of ellipsoidal ambiguity set over the interval one is the following. The ellipsoidal ambiguity set preserve the correlation information between the assets while the interval ambiguity set assumes the returns are independent. Below we present the formal definitions.

Definition 2.1 (Ambiguity in distribution)**.** *The random variable $\tilde{r}$ assumes a distribution from*

$$\mathbb{D} = \{\pi \mid \mathbb{E}_\pi[\tilde{r}] = \bar{r},\ Cov_\pi[\tilde{r}] = \Sigma \succ 0\},$$

where $\bar{r}$ and Σ are given and $\Sigma \succ 0$ indicates Σ is a positive definite matrix.

As it can be observed from definition (2.1), the returns distribution are ambiguous and specified only to the extent that their first and second central moments are known to be equal to $\bar{r}$ and Σ, respectively. We further extend the ambiguity to the mean returns next.

[4]From the fundamental minimization formula of CVaR in Appendix C.1, one can see that CVaR and, therefore MtC model depends on the conditional distribution of portfolio return that is linked to the joint distribution of returns.

[5]The model can be extended to the case where the joint distribution of returns are ambiguous, and means and covariance matrix belong to a box ambiguity set as described in Appendix D.

[6]$\mathbb{S}^n_+$ indicates the space of all positive semi-definite matrices.

Definition 2.2 (Ellipsoidal ambiguity for mean returns)**.** *Mean returns belong to the joint ellipsoidal set:*

$$U_\delta(\hat{r}) = \{\bar{r} \in \mathbb{R}^n \mid S(\bar{r} - \hat{r})^\top \hat{\Sigma}^{-1}(\bar{r} - \hat{r}) \le \delta^2\},$$

where $\hat{r}$ indicates the center of this ambiguity set, and parameter δ controls the size of the ambiguity set. he S is the number of scenarios used for estimation of means ($\hat{r}$) and covariance matrix ($\hat{\Sigma}$) entries.

The application of ellipsoidal ambiguity sets is prevalent in robust optimization literature since they result in less conservative optimal solutions than standard interval ambiguity sets. To specify an ellipsoidal ambiguity set, one would require to determine center $\hat{r}$ and parameter δ that controls the size of ambiguity set. Construction of the ambiguity set is an important step toward more parsimonious model as the size of the ambiguity set associates with conservativeness of optimal solutions. The larger the δ, the more conservative the robust optimal solutions are. To this end, we rely on the heuristic and algorithm developed by Lotfi and Zenios (2018).

Note that if we let $\delta = 0$, then the ambiguity set reduces to the single point of center $\hat{r}$. Intuitively, this suggests there is no ambiguity, and the mean returns are fully known. The ambiguity in mean returns is the most important source of ambiguity and model mis-specification since the optimal portfolios have a much higher sensitivity to the mean returns estimation errors compared to the covariance matrix estimation error (see e.g. (Chopra and Ziemba, 1993) for mean-variance model and Kaut et al. (2007) for mean-CVaR model). For this reason, we disregard the ambiguity in the covariance matrix. This helps to simplify the analyses and obtain solutions that are less conservative.[7]

For the purpose of the application of our model to home bias puzzle in Section 3, we develop our model where the joint distribution of assets returns is ambiguous like Uppal and Wang (2003) and additionally, the mean returns belong to an ellipsoidal ambiguity set.[8] We separate the ambiguity in the information of the last asset (henceforth home) from the rest of first $n-1$ assets (henceforth foreign assets). In particular, we consider two ambiguity sets, one containing the information of home mean return and the other set integrates the information of mean returns for foreign assets in the portfolio. In mathematical terms, we consider the following two ambiguity sets for home and foreign

[7]Many applications of robust optimization in portfolio optimization and risk management literature have adopted the same approach (see e.g. (Ceria and Stubbs, 2006; Garlappi et al., 2007; Paç and Pınar, 2014)).

[8]Note that ambiguity in joint distribution implies the marginal distribution of returns are also ambiguous, so we have the same generality as Uppal and Wang (2003), and in addition, we consider ambiguity in mean returns.

assets denoted by $U(\hat{r}_h, \delta_h)$ and $U(\hat{r}_f, \delta_f)$, respectively.

$$
\begin{aligned}
U(\hat{r}_h, \delta_h) &= \{\bar{r}_h \in \mathbb{R} \mid S(\frac{\bar{r}_h - \hat{r}_h}{\hat{\sigma}_h})^2 \leq \delta_h^2\}, \\
U(\hat{r}_f, \delta_f) &= \{\bar{r}_f \in \mathbb{R}^{n-1} \mid S(\bar{r}_f - \hat{r}_f)^\top \hat{\Sigma}_f^{-1}(\bar{r}_f - \hat{r}_f) \leq \delta_f^2\},
\end{aligned} \tag{5}
$$

This is the first study that does such separation while keeping the correlation information. To take into account the ambiguity in the joint distribution and mean returns when optimizing the MtC model (3), we consider the robust counterpart of MtC model (robust MtC) as follows:

$$
\max_{x \in \mathbb{X}} \min_{\substack{\bar{r}_f \in U(\hat{r}_f, \delta_f) \\ \bar{r}_h \in U(\hat{r}_f, \delta_f)}} \min_{\pi \in \mathbb{D}} \frac{\mathbb{E}(\tilde{r}_p - r_f)}{\mathrm{CVaR}_\alpha(\tilde{r}_p - r_f)}. \tag{6}
$$

The following theorem gives a second order cone programming (SOCP) formulation of the robust MtC model.

Theorem 2.1. *Assuming positive worst-case CVaR on excess returns of the optimal portfolio of robust MtC maximization model (6) with feasible set $\mathbb{X}$, then the robust MtC portfolio optimization model can be cast as follows:*

$$
\begin{aligned}
\max_{x'_f \in \mathbb{R}_+^{n-1}, x'_h \in \mathbb{R}_+} \quad & (\hat{r}_h - r_f e)x'_h + (\hat{r}_f - r_f e)^\top x'_f - \frac{\delta_h}{\sqrt{S}} x'_h \sigma_h - \frac{\delta_f}{\sqrt{S}} \sqrt{x'^\top_f \Sigma_f x'_f} \\
s.t. \quad & \\
& -(\hat{r}_h - r_f)x'_h - (\hat{r}_f - r_f e)^\top x'_f + \frac{\delta_h}{\sqrt{S}} x'_h \sigma_h + \frac{\delta_f}{\sqrt{S}} \sqrt{x'^\top_f \Sigma_f x'_f} \\
& + \frac{\sqrt{\alpha}}{\sqrt{1-\alpha}} \sqrt{x'^\top_f \Sigma_f x'_f + 2x_h \sigma_{hf}^\top x_f + x_h^2 \sigma_h^2} \leq 1 \\
& e^\top x' > 0.
\end{aligned} \tag{7}
$$

Given x'^*_h and x'^*_f, the optimal solution of (7), the optimal allocation to home asset in (6) would be $x^*_h = \frac{1}{e^\top(x'^*_h + x'^*_f)} x'^*_h$. Likewise, the optimal allocation to foreign asset can be obtained as $x^*_f = \frac{1}{e^\top(x'^*_h + x'^*_f)} x'^*_f$.

(For the proof see Appendix C.2.)

In the next section, we first illustrate the robust MtC model for the case of two assets where we can obtain the analytical solution. We compare our theoretical model implication with that of Uppal and Wang (2003) and find the same qualitative results. It is notable that in our set-up ambiguity parameter and consequently the optimal allocation under ambiguity are independent from risk-aversion because we select the tangency portfolio while in the model Uppal and Wang (2003) the optimal solutions that are adjusted for ambiguity are a function of ambiguity parameter and risk-aversion. Next, we show robust MtC portfolio is SSD consistent. This observation has an important implication as it elevates the application of robust risk measure further.

2.3 Second order stochastic dominance consistency

Zhu and Fukushima (2009) show that robustification preserve coherence property of Artzner et al. (1999) in the sense if the risk measure ρ is coherent, then its robust counterpart, ρ_w, defined as worst-case ρ over a distribution ambiguity set $\mathcal{P}$, is coherent too. Here we shed light on another aspect of robust risk measure i.e. SSD consistency. The stochastic dominance offers a way of analyzing risky decisions when a decision maker's utility function is known, to the extent that it belongs to a specific class of real-valued functions (see Appendix C.1 for definitions of stochastic dominance).

Proposition 2.1 (SSD consistency of robust risk measure)**.** *If risk measure associated with probability distribution π is SSD consistent, then the corresponding robust risk measure ρ_w associated with distribution ambiguity set $\mathcal{P}$ remains SSD consistent.*

Proof. Assume random variables $\tilde{X}$ and $\tilde{Y}$ are arbitrary given and $\tilde{X}$ dominates $\tilde{Y}$, or equivalently, $\tilde{X} \succeq_{SSD} \tilde{X}$. That is $\tilde{X}$ is preferred to $\tilde{Y}$ within all risk-averse preference models with an increasing and concave utility function. Since the risk measure ρ is SSD consistent then $\rho(\hat{X}) \leq \rho(\hat{Y})$, and therefore $\rho_w(\hat{X}) = \underset{\pi\in\mathcal{P}}{max}\rho(\hat{X}) \leq \underset{\pi\in\mathcal{P}}{max}\rho(\hat{Y}) = \rho_w(\hat{Y})$. That completes the proof.

$\square$

Theorem 2.2 (Second order stochastic dominance of robust MtC portfolios)**.** *Let $\mathbb{X}_+$ denote the space of all feasible portfolios that have positive worst-case mean excess return and worst-case CVaR associated with ambiguity sets $\mathbb{D}$, $U(\hat{r}_f, \delta_f)$, and $U(\hat{r}_h, \delta_h)$. Then robust MtC is SSD consistent for all portfolios in $\mathbb{X}_+$.*

Proof. Let us assume the portfolios x_1 and x_0 belong to $\mathbb{X}_+$, and x_1 dominates x_0. It means $\tilde{r}_{x_1} \succeq_{SSD} \tilde{r}_{x_0}$ or $\tilde{r}^e_{x_1} \succeq_{SSD} \tilde{r}^e_{x_0}$ where r^e denotes the excess returns over risk-free. This implies that $\mathbb{E}(\tilde{r}^e_{x_1}) \geq \mathbb{E}(\tilde{r}^e_{x_0}) > 0$ (Whang, 2019, Theorem 1.1.5), and, equivalently, $\mathbb{E}(\tilde{r}_{x_1} - r_f) \geq \mathbb{E}(\tilde{r}_{x_0} - r_f) > 0$. This implies worst-case mean excess returns of portfolios x_1 and x_0 satisfy the following inequality.

$$\min_{\substack{\tilde{r}_f\in U_\delta \\ \tilde{r}_h\in U_{\delta_h}}} \underset{\pi\in\mathcal{P}}{min}\,\mathbb{E}(\tilde{r}_{x_1} - r_f) \geq \min_{\substack{\tilde{r}_f\in U_\delta \\ \tilde{r}_h\in U_{\delta_h}}} \underset{\pi\in\mathcal{P}}{min}\,\mathbb{E}(\tilde{r}_{x_0} - r_f) > 0.$$

We also have that CVaR is SSD consistent (Ogryczak and Ruszczyński, 2002, Theorem 3.2). Applying Proposition 2.2 to CVaR implies that worst-case CVaR is SSD consistent or,

$$0 < \max_{\substack{\bar{r}_f \in U_\delta \\ \bar{r}_h \in U_{\delta_h}}} \underset{\pi \in \mathcal{P}}{max}\, \mathrm{CVaR}_\alpha(\tilde{r}_{x_1} - r_f) \leq \max_{\substack{\bar{r}_f \in U_\delta \\ \bar{r}_h \in U_{\delta_h}}} \underset{\pi \in \mathcal{P}}{max}\, \mathrm{CVaR}_\alpha(\tilde{r}_{x_0} - r_f).$$

Therefore the ratio of worst-case CVaR to worst-case mean return for portfolio x_1 is less than or equal to worst-case CVaR to worst-case mean return of portfolio x_0. That means the worst-case CVaR-to-mean ratio of portfolio x_1 is less than or equal to the worst-case CVaR-to-mean ratio of portfolio x_0. Hence, the inverse of worst-case MtC ratio is consistent with SSD. Since the assumptions made assures a positive worst-case MtC, one can easily see that the condition of $\rho(\tilde{X}) \leq \rho(\tilde{Y})$ is equivalent to $\frac{1}{\rho(\tilde{Y})} \leq \frac{1}{\rho(\tilde{X})}$ in Definition of risk measure consistency (see Appendix C.3), and therefore the robust MtC optimal portfolio satisfies that. Therefore robust MtC is SSD consistent.

□

2.4 Robust MtC optimal portfolios

To understand the robust MtC optimal portfolio choice characteristics, we consider the case that the portfolio consist of two risky assets indicated as x_h and x_f and find the analytical solution of robust MtC model (6). Let us denote the covariance matrix Σ as follows:

$$\Sigma = \begin{pmatrix} \sigma_f^2 & \sigma_{hf} \\ \sigma_{hf} & \sigma_h^2 \end{pmatrix}.$$

The following theorem finds analytical formulation of optimal allocations in the presence of ambiguity.

Theorem 2.3. *Let us define*

$$\begin{aligned} c_1 &= \sqrt{S}\left[\sigma_f^2(\hat{r}_h - r_f) - \sigma_{hf}(\hat{r}_f - r_f)\right] \\ c_2 &= \sqrt{S}\left[\sigma_h^2(\hat{r}_f - r_f) - \sigma_{hf}(\hat{r}_h - r_f)\right] \\ c_3 &= \sqrt{S}\left[(\sigma_f^2 - \sigma_{oh})(\hat{r}_h - r_f) - (\sigma_{hf} - \sigma_h^2)(\hat{r}_f - r_f)\right]. \end{aligned} \tag{8}$$

Assuming optimal solutions of robust MtC model (6), x_h^* and x_f^*, both being positive, then they can be obtained as a function of pair of (δ_h, δ_f) as follows:

$$\begin{aligned} x_h^*(\delta_h, \delta_f) &= \frac{c_1 - \delta_h \sigma_h \sigma_f^2 + \delta_f \sigma_f \sigma_{hf}}{c_3 - \delta_h(\sigma_h \sigma_f^2 - \sigma_h \sigma_{hf}) + \delta_f(\sigma_h \sigma_f^2 - \sigma_o \sigma_h^2)} \\ x_f^*(\delta_h, \delta_f) &= \frac{c_2 + \delta_h \sigma_h \sigma_{hf} - \delta_f \sigma_f \sigma_h^2}{c_3 - \delta_h(\sigma_h \sigma_f^2 - \sigma_h \sigma_{hf}) + \delta_f(\sigma_h \sigma_f^2 - \sigma_o \sigma_h^2)}. \end{aligned} \tag{9}$$

(For the proof see Appendix C.3.)

The first implication of Theorem 2.3 is that $x_f^*(\delta_h, \delta_f)$ increases with δ_h and decreases with δ_f and since the sum of allocations adds up to one, that means optimal home allocation $x_h^*(\delta_h, \delta_f)$ decreases with δ_h and increases with δ_f. That is as the ambiguity in foreign asset increases, the allocation to home increases. This theoretical observation can explain the equity home equity bias puzzle where the investors allocate to domestic equities more than they should allocate according to International CAPM. It is consistent with the observation of Uppal and Wang (2003) that when the joint distribution of the two assets are fully ambiguous, and we know more on asset one, then allocations are biased toward asset one. Further, one can easily check that Theorem (2.3) implies

$$x_h^*(\delta_h, \delta_f) = \left(\frac{c_1 - \delta_h \sigma_h \sigma_f^2 + \delta_f \sigma_f \sigma_{hf}}{c_2 + \delta_h \sigma_h \sigma_{hf} - \delta_f \sigma_f \sigma_h^2} \right) x_f^*(\delta_h, \delta_f). \tag{10}$$

First, let us consider the case that there is ambiguity in the joint distribution of all assets, and additionally no ambiguity in mean return of home, but the foreign asset mean returns are ambiguous ($\delta_h = 0$, $\delta_f > 0$). The impact of ambiguity is two-folded. As it can be seen from equation (10), the numerator of slope coefficient increases with δ_f and simultaneously its denominator decreases with δ_f. Therefore, when there is no ambiguity in home asset, ambiguity on foreign asset can shift the allocations from foreign asset to home significantly. When both home and foreign assets are ambiguous and besides $\delta_h < delta_f$, then the slope adjustment depends on covariance matrix. The same holds when $\delta_h > delta_f$. Finally, when the ambiguity in home and foreign asset is the same, the optimal allocations of robust MtC and MtC coincide if and only if $(\hat{r}_h - r_f)(\sigma_f^3 \sigma_h^2 - \sigma_f \sigma_{hf}^2) - (\hat{r}_f - r_f)(\sigma_h \sigma_{hf}^2 - \sigma_h^3 \sigma_o^2) = 0$. Intuitively, this suggests the robust model is indifferent to the same level of ambiguity in the two assets if such a condition holds. For instance, if $\hat{r}_h = \hat{r}_f$ and $\sigma_h = \sigma_f$, the aforementioned condition is satisfied. So, if the means and standard deviation of the two assets are equal, we will get the same solution as in the case there is no ambiguity in means. It is noticeable that aforementioned results are independent of the investor's risk-aversion.

3 Application to under-diversification puzzles

In this section, we use our robust MtC model to examine the extent that our robust MtC model can explain home bias puzzles on equity and the household portfolio under-diversification problem.[9] First, we present the data we use in our analyses, explain

[9]We extend our model and obtain the robust MtC model where the joint distribution is ambiguous, and additionally, means and covariance matrix entries belong to an interval ambiguity set. This is a generalization of our initial ambiguity set. The details related to the definition of ambiguity set and the

how we measure ambiguity and discuss our empirical results for two instances of under-diversification puzzles.

3.1 Data

The first database we use to address the home equity bias puzzle includes information on equity market indices, market capitalization, and actual equity holdings across 21 developed and 19 emerging economies as per the MSCI classification, and it spans over January 1, 1999, to December 31, 2019.[10]

We use the MSCI Investable indices to avoid positive biases when ignoring investability frictions, such as illiquidity risk and index replicability that can be important in emerging markets. For investors in a specific country, country end-of-month index prices are calculated by multiplying the end-of-the-month country index price in USD currency by the corresponding contemporaneous spot rate. We then calculate the excess return using the risk-free rate chosen for investors as in Data Appendix Section. Data are from Datastream. The set of countries and descriptive statistics of their excess returns (calculated using index price in USD currency) during the sample period are in the Data Appendix Section. There are large differences in the return moments across countries, with most country indices being negatively skewed with considerable tail risk, especially in emerging markets.

Market capitalization (in USD) are from World Development Indicators of World Bank and World Federation of Exchanges databases.[11] IMF's Coordinated Portfolio Investment Surveys (CPIS), available from 2001-2019, provide holdings of equity across countries. Holdings are measured in USD. Even though the CPIS database suffers from measurement errors, it has been widely adopted in the international investment literature. Further Lane and Milesi-Ferretti (2008) showed the reporting of holdings by developed markets is of high quality.[12]

model development are gathered in Appendix D. The model with interval ambiguity sets is formulated as a semi-definite program (SDP) that are solvable using efficient interior-point methods. The advantage of interval ambiguity sets is that it considers the ambiguity in each security independently, giving more flexibility for ambiguity parameters. We can derive country-specific ambiguity parameters for each mean return and covariance matrix entry.

[10]The developed markets in our sample include Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and the US. The emerging markets in our sample include Brazil, Chile, China, Colombia, Czech Republic, Egypt, Hungary, India, Israel, Korea, Malaysia, Mexico, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, and Turkey.

[11]For Italy and Finland, we have missing values for which we use market capitalization (in EUR) from their Central Bank databases along with the foreign exchange rates from ECB. We fill in the missing values for Denmark and Sweden using market capitalization data from NASDAQ OMX and the corresponding foreign exchange rates from Thomson Reuters. The data are from Datastream.

[12]The first CPIS reported data for 1997 for 29 source countries, but some major investing nations did not participate, and therefore, we rely on data from the second CPIS, reporting holdings for 2001 and

To measure the magnitude of equity home bias, we use the standard measure in the literature (see Cooper et al. (2012); Mishra (2015)). Let EQ_i represent the holdings of investors in country i's stock market, TEQ_i, the value of the total equity holding for country i, and MC_i, the stock market capitalization of country i. Then actual holdings of domestic equities in the total equity portfolio of investors from country i, a_i is equal to EQ_i/TEQ_i.[13] Further, assuming ICAPM holds true, market-cap weight can be computed as $w_i = MC_i / \sum_{i=1}^{n} MC_i$, and equity home bias therefore is equal to:

$$EHB_i = \frac{a_i - w_i}{1 - w_i}. \tag{11}$$

When the EHB for country i is equal to one, there is full equity home bias while when it is equal to zero, the portfolio is optimally diversified.

To measure ambiguity in financial data, we apply the heuristic developed in Lotfi and Zenios (2018) where the covariance matrix is assumed un-ambiguous and is estimated using the whole sample. For robustness, we use an alternative measure of ambiguity for which we rely on Economic Policy Uncertainty indices which is available only for 23 countries in our primary sample.[14] The next sub-section describes ambiguity measurement in more details.

The second database is a record of monthly investor portfolio holdings at a major U.S. discount brokerage house. The database consists of the portfolios of 78,000 households and contains portfolio positions for the period of January 1991 through November 1996.[15] Similar to Mitton and Vorkink (2007), we aggregate all holdings to the household level. To measure the ambiguity of household portfolios, we obtain monthly return data from the Center for Research in Security Prices (CRSP). We exclude from our analysis household portfolios that have no investment in individual equities at any point during the sample period and additionally remove the households for which they have missing CUSIP or the equity price information from CRSP can not be found. Moreover, we exclude observations of monthly returns that are below -150% or higher than 150%. The descriptive statistics of portfolio value sorted by size indicate heterogeneity among household portfolios. For further information on this database, we refer the readers to Barber and Odean (2000, 2001, 2008).

onward.

[13]Note that the total equity holding comprises both foreign and domestic holdings. Domestic equity holding is the difference between the country's total market capitalization and foreign equity liabilities.

[14]The developed economies with available EPU index include Australia, Belgium, Canada, Denmark, France, Germany, Greece, Hong Kong, Italy, Japan, Netherlands, Spain, Sweden, United Kingdom, and the US. The emerging markets with available EPU index include Brazil, Chile, China, Colombia, India, Korea, Mexico, and Russia. The data are available at `http://www.policyuncertainty.com`.

[15]This database is subject to concern that the portfolios do not necessarily represent the entire investment holdings of the households in the database. Goetzmann and Kumar (2008) conclude that, by and large, the accounts in this database represent substantial portions of investor wealth.

3.2 Ambiguity measurement

In some cases, ambiguity can be defined as the confidence regions of the statistical estimators of the model parameters. For other cases like the applications we present, the ambiguity may be given via multiple estimates of model parameters, which raises the question of the appropriate ambiguity set. To measure ambiguity in financial data associated with the mean returns, we follow Lotfi and Zenios (2018) to compute the tightest ellipsoid that contains all potential realizations of model parameters. This method first finds the best possible center among all realizations where the "best" refers to the observation of the mean return vector with the minimum sum of distances from all others. Once the center of the ellipsoid is determined, the size of the ambiguity set (δ) is obtained by calculating the distance of each point in the ellipsoid from the center and letting δ be the maximum value. In this way we guarantee that all available estimates of the parameters are included in this ellipsoid. We use a rolling window of length L months to obtain instances of mean returns, and then using this information, we derive the center of the two ambiguity sets in (5) along with the corresponding parameter that controls the size of ambiguity set. Applying their method, we estimate the covariance matrix using whole data (data-based ellipsoidal ambiguity set hereafter).

We also measure ambiguity using the monthly Economic Policy uncertainty (EPU) index of Baker et al. (2016). To construct such an ambiguity set, we first construct a confidence interval for each mean return using the following formula:

$$\bar{\mu_i} - EPU\frac{F^{-1}(\beta)\sigma_i}{\sqrt{S}} \leq \mu \leq \bar{\mu_i} + EPU\frac{F^{-1}(\beta)\sigma_i}{\sqrt{S}},\ i = 1, \ldots, n, \tag{12}$$

where EPU is the maximum (or mean) value of the normalized EPU index (EPU indices are normalized by dividing the EPU index value by maximum EPU among all observations). The β is the confidence level, and F is the normal cumulative distribution function. S is the number of observations, and $\bar{\mu}$ and σ are the mean and standard deviation of returns obtained using all observations. The idea is that ambiguity can determine how the confidence interval is enlarged or tightened according to the EPU index. To capture the ambiguity effect implied by the EPU index, we select the highest confidence level, i.e. we set $\beta = 0.99$. This way, we can conclusively capture ambiguity implied by EPU variability, assuming there is no ambiguity in variance values.

The above formulation ensures a larger confidence interval for countries with high EPU indices, such as Greece. However, the interval ambiguity set is not an ideal selection for the type of ambiguity set. It ignores the correlation information among securities and robust optimal allocation are the most conservative. So we use simulation to generate the tightest ellipsoid as follows. First, we obtain the box (n-dimensional interval) implied by single interval ambiguity sets as (12) where EPU is set to be either maximum or mean value of EPU for each country. Next, we generate 5000 random observations within the box and find the ellipsoid containing these data points per the heuristic described above (EPU-based ellipsoidal ambiguity set hereafter). The ambiguity intervals obtained using maximum EPU are more compatible to those implied by the purely data-based ellipsoidal ambiguity set when compared to the ambiguity intervals obtained using mean EPU (see Online Appendix Figure A2).

3.3 Equity home bias puzzle

In this section, we first show the persistence of home equity bias using more recent data and illustrate the relevance of ambiguity to the equity home bias puzzle in the universe of 21 developed economies and 19 emerging markets and with recent data spanning over 1999-2019.

French and Poterba (1991) show the observed international investors' portfolios are under-diversified and heavily biased toward domestic equities. This observation goes against modern portfolio theory in which International CAPM is well-accepted and is well-known as "*equity home bias puzzle*". The market and observed weights in their paper implies an EHB index of 0.92 for Japan and 0.88 for the US as of 1989.[16] In our sample, we use more recent data from 2001 to 2018 and broaden the universe of countries by including 21 developed and 19 emerging equity market indices, including the universe of countries in (French and Poterba, 1991). The most recent data also illustrates the same picture with a smaller magnitude of the EHB index. In particular, we observe Japan and US have an average EHB index of 0.86 and 0.73 (see the last column of Table 1).[17] The same observation holds for all countries in our sample. In general, the HBI is much lower for developed markets than emerging markets (Average HBI among developed markets is 0.70 while the average HBI among the emerging markets is 0.94.). Also, there is huge variability among developed markets. In contrast, the HBI of emerging markets remains almost constant (HBI of developed markets has a standard deviation of 0.14 while for emerging markets, this value reaches as low as 0.06.).[18]

[16]The market capitalization data for Japan and US in French and Poterba (1991) are different from those reported in the World Bank database. In fact, market weights for Japan and the US using the same universe as French and Poterba (1991) are 44% and 35%, respectively, and therefore the calculated EHB index would be 0.96 and 0.91 for Japan and US, accordingly, indicating a much higher equity home bias.

[17]Note that the EHB index time series stops at 2018 because there are missing market capitalization values for Belgium, Canada, France, and the Netherlands for 2019, and this will bias the calculation of total market capitalization value and, therefore, home bias index, severely.

[18]Among European countries in our sample, Greece and Spain HBIs (0.96 and 0.91, respectively) are the highest while the Netherlands and Norway HBIs (0.39 and 0.38, respectively) are the lowest. Among non-European countries, Hong Kong and Japan HBIs (0.90 and 0.86, respectively) are the highest while New Zealand and United Kingdom HBIs (0.62 and 0.65, respectively) are the lowest (see Table 1). The lowest HBI among emerging markets belongs to Israel and Peru (0.82 and 0.84, respectively), while

[Insert Table 1 Near Here]

[Insert Table 2 Near Here]

We construct the ambiguity sets according to the formulation (5), and we rely on the heuristic developed in Lotfi and Zenios (2018) (see subsection 3.2 for details) for obtaining the ambiguity parameters. We define $m = \delta_h/\delta_f$ that measures the mean return ambiguity of home equity index relative to the foreign equity indices, and values of $m < 1$ reflect the cases where there is less ambiguity in the mean return of home index compared to the foreign equity indices. Obtained ambiguity parameters δ_f, δ_h, and m show reasonable amount of variations. The δ_f changes from a minimum of 8.21 for Belgium to a maximum value of 8.97 for the United Kingdom, with a standard deviation of 0.21. The m changes from minimum of 0.18 for Finland to maximum of 0.28 for Austria, with standard deviation of 0.03 (see Table 1). In the case of emerging markets, the δ_f changes from a minimum of 8 for the Philippines to a maximum value of 9.01 for Malaysia, with a standard deviation of 0.27. The m changes from 0.15 for Malaysia to 0.41 for Colombia and Russia, with standard deviation of 0.07 (see Table 2).[19] The data tabulated in Tables 1 and 2 shows there is a non-linear relationship between either HBI or Home weight and any of ambiguity parameters which are aligned with the observation of Figure 1 and optimal home allocation explicit formulation, equation (10).

Next, to assess the model performance, we run the robust MtC model (7) where we use the estimated center from the heuristic of Lotfi and Zenios (2018). However, instead of using the data-driven δ_f and m, we use a range of δ_f and m values below the data-driven estimated threshold and obtain the allocation to home country for each country's investors. The depicted results for selected sample of countries shows that robust MtC optimal home allocation increases with foreign ambiguity, and it is persistence for different values of m below the data-driven estimated m (see Figures 2, 3, and 4 for the sample of developed European, developed non-European, and emerging markets, respectively). Further, for a given δ_f, increasing home equity index ambiguity reduces home allocation. These observations are aligned with the theoretical implication of Theorem (2.3). However, the most important observation of this test is that the model can generate home allocations that match the actual home allocations for values of foreign and relative ambiguity far below data-implied ones.

[Insert Figure 2 Near Here]

[Insert Figure 3 Near Here]

[Insert Figure 4 Near Here]

Egypt, India, Philippines, Russia, and Turkey have the maximum home bias (see Table 2).

[19] The same qualitative observation can be found when we look at δ_h instead of m.

In particular, our test with selected European developed countries shows the model behaves in a way that we can achieve the actual home allocations for $\delta_f \leq 3$ and for $m \leq 0.2$ (The only exception is France that reaches it at $\delta_f = 3.1$ when $m = 0.2$). For investors in selected non-European developed economies, values of $\delta_f \leq 2.5$ and $m \leq 0.2$ can generate allocations to home as high as actual ones (The exceptions are Canada and Japan. When $m = 0.2$, our model reaches actual home allocation for Canada and Japan at $\delta_f = 2.7$ and 3, respectively).[20] Comparing these limits of δ_f and m with those obtained from data, we can gauge the credibility of our model in explaining the equity home bias puzzle. In particular, we report the approximate δ_f (hereafter $\delta_f^\dagger$) for which the robust MtC optimal home allocation matches with the actual home weight when the relative ambiguity parameter is fixed at $m = 0.2$ (see first column of Tables 1 and 2 for developed and emerging economies, respectively).[21]

In all case, the estimated $\delta_f^\dagger$ is much lower than the data-driven δ_f. The maximum $\delta_f^\dagger$ in developed economies sub-sample is obtained for Greece ($\delta_f^\dagger = 7.3$) and in emerging market sub-sample, it is obtained for Turkey ($\delta_f^\dagger = 8$) where the data-driven δ is equal to 8.87 and 8.99, respectively. The mean $\delta_f^\dagger$ for developed and emerging markets equals 2.86 and 2.66, with a high standard deviation of 1.28 and 1.69, respectively.

These observations establish that our model is capable of generating allocations comparable to observed ones and obtaining them for reasonable ambiguity parameters. This is an important step in claiming resolving puzzle as pointed out by Cooper et al. (2012). However, we are careful in interpreting our results for the case of emerging markets since we know most of these countries are bounded by other restrictions and that is why we select in our main illustration (Figure 4) only emerging markets with lowest EHB indices. The same applies to developed economies to a much less extent as these economies have less international investment barriers. Also, we do not claim the puzzle is only explained by ambiguity in financial data as we have not taken into account in our model other limitations such as transaction cost, international investment barrier, exchange risk, etc.

[Insert Table 3 Near Here]

Therefore, the robust MtC model that we developed in sub-section 2.2 can (i) link the equity home bias puzzle to ambiguity in the financial data used for finding the optimal portfolio choice, (ii) show how overall ambiguity in joint distribution and relative ambiguity of home to the foreign market indices can bias the optimal portfolio choice toward the home, and (iii) generate optimal allocations to home equity index that matches the observed ones for reasonable values of ambiguity parameters.

[20] The same qualitative observation can be found with other European developed economies (see Appendix Figure A3).

[21] The only exception is Greece, in which we obtain $delta_f^\dagger$ when $m = 0.10$. Note that this is not against us as we still can find $delta_f^\dagger$ within the feasible range of values for selected feasible m.

Further, to complement our analyses, we use EPU-based ambiguity measurement described in sub-section 3.2 which has been used by Aït-Sahalia et al. (2021) as well. The sample of countries with available EPU indices is much less than the initial sample (15 developed and eight emerging economies). We first repeat the analysis using our ambiguity measurement (see the first four columns of Table 3). That is, we repeat the tests similar to those in Tables 1 and 2 for the sample of 23 countries. This is a robustness test, and we observe the results are qualitatively the same. All the $\delta_f^{\dagger}$ values for which the robust MtC optimal home allocation matches with actual ones are below the data-driven.[22]

We use the interval ambiguity sets defined as in equation (12) where we use maximum EPU for each country in our sample. Following the instruction in sub-section 3.2, we obtain the implied ellipsoidal ambiguity sets parameters (see Table 3 where columns containing the EPU-based ambiguity parameters are indicated with a subscript EPU). Comparing the home ambiguity parameters, EPU-implied ones are much smaller (average EPU-based home ambiguity value is almost half of pure data-driven one). On the other side, the foreign ambiguity parameters are much higher than those purely data-driven.

Most importantly, given the average $m_{EPU} = 0.09$, we set $m_{EPU} = 0.10$ and find $\delta_{f,EPU}$ for which that the robust MtC optimal home allocation matches with that of actual ones (denoted as $\delta^{\dagger}_{f,EPU}$). All $\delta^{\dagger}_{f,EPU}$s are below the EPU-based $\delta_{f,EPU}$. So our model performance in explaining the puzzle is robust with respect to ambiguity measurement.[23]

Moreover, when comparing the results obtained in Tables B1 with those in Tables 1 and 2, we observe the mean foreign ambiguity decreases sharply (from 8.76 to 6.94 and from 8.69 to 6.72 for developed and emerging economies, respectively). The data suggests the possibility of a negative relationship between ambiguity and number of assets in the portfolio. This interesting observation is consistent with our model prediction, as we will see in the following sub-section.

3.4 Household portfolio under-diversification

To compare the portfolios in terms of diversification, we use two different diversification measures. First diversification measure, D^2, is a Herfindahl index of the weights of each security in the household's portfolio defined as $D^2 = \sum_{i=1}^{n} w_i^2$ where w_i is the weight of security i in the household's portfolio. However, D^2 still does not capture covariance between securities in a household's portfolio. So we use the D^3 diversification measure

[22] In the case of Greece, we use the sub-sample that exclude the crisis period (2010-2015) and report values based on this sub-sample.

[23] We also use mean EPU and repeat the tests. The results are robust (see Appendix Table B1).

proposed by Mitton and Vorkink (2007) defined as:

$$D^3 = \sum_{i=1}^{n} w_i^2 + (1 - \sum_{i=1}^{n} w_i^2)\bar{\rho} \tag{13}$$

where $\bar{\rho} = \frac{\sum_{i,j=1}^{n} w_i w_j \rho_{ij}}{\sum_{i,j=1}^{n} w_i w_j}$. The ρ_{ij} indicates the correlation between security i and j. Higher values of D^2 and D^3 indicate lower levels of diversification.

To investigate the relationship between ambiguity and diversification, we use our model when there is only one ambiguity set that captures overall ambiguity in mean returns, and we show the corresponding parameter controlling the size of ambiguity set with δ. We use our robust MtC model to study the prediction of our model concerning the relationship between ambiguity and diversification.[24] We study the simple case of two assets where the random returns are generated from a multivariate distribution with annual mean and standard deviation of 3.6% and 3.1%, respectively, and with the correlation value varying between -0.30 and 0.90.[25] We also let the ambiguity parameter be exogenous and change between 0 and 6. For a given correlation and ambiguity parameter δ, we run the robust MtC model 500 times, find optimal weights, calculate the diversification measures D^2 and D^3, and report the average values of these measures (see Figure 5).

First, our results are in accordance with standard finance theory. Portfolios are less diversified as the correlation increase. Most importantly, the lower diversification measure D^2 and D^3 values correspond to optimal portfolios with lower ambiguity levels (see panel A and B of Figure 5, respectively). This observation is consistent with the finding of Dimmock et al. (2016a) where they show ambiguity-averse investors hold less diversified portfolios.[26] However, our novel model suggests a channel explaining that if you are ambiguity-averse, you hold less diversified portfolios because primarily these portfolios are less ambiguous.

[Insert Figure 5 Near Here]

We take a step further and empirically examine our channel credibility on actual household portfolio holdings data described in sub-section 3.1. We sort all household portfolios into quintiles according to diversification levels at three representative dates in our sample period (January 1991, 1993, and 1996).[27] We calculate returns of these

[24]This requires an adjustment and can be easily obtained from current model formulation (7) by setting $x_h = 0$.

[25]Note that we choose the portfolio of size two for the simplicity and better illustration reason.

[26]The D^3 measure figure does not offer a significant reduction as in the case of D^2. It is probably due to the chosen hypothetical correlation values that we use in our testing, and as we will see in the next test, there are significant reductions in D^3.

[27]The results when we use decile shows the same trend (see Appendix Table B2), although, around the middle deciles, the average δs are very similar, and that is why in a few cases, we observe diversification increases (D^2 and D^3 reduces) while δ decreases. However, when we use fewer quantiles, as we did with

portfolio holdings for nine subsequent years and use a rolling window of a length of 24 months to generate samples of mean returns for each security in the portfolio held. We use this information to construct the one unified ambiguity set according to the method described in sub-section 3.2 per each portfolio held. We then calculate the average ambiguity parameter, δ, of all portfolios within each quintile (as well as the average mean, standard deviation, skewness, and kurtosis).

[Insert Table 4 Near Here]

The portfolios sorted on D^2 and D^3 have reasonable variations, and the number of observations within each quintile is considerable. We observe that as D^2 and D^3 decrease, the estimated ambiguity parameter δ increases (see Table 4). That is, the less diversified portfolios held are those with low levels of ambiguity. Therefore, invests are ambiguity-averse and hold the less diversified portfolios that are less ambiguous. Moreover, we confirm the results of Mitton and Vorkink (2007) regarding the preference for skewness as both diversification measures show a negative relationship with skewness. The least diversified portfolios have the highest level of skewness. In addition to that, the results show the portfolios with higher D^2 and D^3 have higher kurtosis levels. Therefore, the investors prefer kurtosis as well, and the portfolios with fat-tail return distribution are preferred.

4 Conclusion

The equity home bias puzzle has been addressed in the literature extensively, both empirically and theoretically. However, there is not yet a conclusive answer to the puzzle. This study examines the impact of ambiguity on the optimal allocation in the international portfolio, applying it to equity home bias and household portfolio under-diversification. We show the robust MtC model that finds ambiguity-adjusted optimal allocation is able to (i) link the equity home bias puzzle to ambiguity in the financial data used for finding the optimal portfolio choice, (ii) show how overall ambiguity in joint distribution and relative ambiguity of home to the foreign market indices can bias the optimal portfolio choice toward the home, and (iii) generate optimal allocations to home equity index that matches the observed ones for reasonable values of ambiguity parameters. These results are robust with respect to sample specification and ambiguity measurement.

In addition, our theoretical model predicts a negative relationship between ambiguity and under-diversification. The empirical results from real household portfolio holding validate our model prediction and suggest the less diversified portfolios held by households are those with low levels of ambiguity. This is in line with literature findings regarding

quintile, we observe enough variations in δ.

the behavior of ambiguity-averse investors.

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Data Appendix

Descriptive statistics of market return indices

This table reports descriptive statistics for all countries in our sample, respectively, mean, standard deviation, skewness, excess kurtosis, Value-at-Risk and Conditional-Value-at-Risk for the monthly series of each country's excess returns, denominated in USD, over the US one-month T-Bill rate. "MtC" and "Sharpe" denote the mean-to-CVaR for each country's excess returns, and the Sharpe ratio. VaR, CVaR and MtC are computed at the 5% confidence level. The sample period spans January 1, 1999 to December 31, 2019. All statistics are reported at monthly frequency, except for the politics and policy variables that are semiannual. Mean, StdDev, VaR, and CVaR are in percentage points.

Country	Mean	StdDev	Skew	Kurt	VaR	CVaR	MtC	Sharpe
Australia	0.77	5.98	-0.54	1.99	8.35	13.77	0.06	0.13
Austria	0.60	6.81	-0.87	4.32	9.45	15.91	0.04	0.09
Belgium	0.35	6.00	-1.22	5.60	9.46	15.09	0.02	0.06
Brazil	1.38	10.55	-0.04	1.16	14.06	21.93	0.06	0.13
Canada	0.71	5.61	-0.53	2.62	8.39	12.09	0.06	0.13
Chile	0.67	6.26	-0.23	1.34	9.15	13.24	0.05	0.11
China	0.85	8.21	0.41	3.98	13.07	17.24	0.05	0.10
Colombia	1.15	8.20	-0.16	0.26	12.88	16.34	0.07	0.14
Czech Republic	1.02	7.43	-0.09	1.24	10.59	15.39	0.07	0.14
Denmark	0.87	5.70	-0.73	2.69	9.38	13.63	0.06	0.15
Egypt	0.79	8.93	0.07	2.14	13.41	18.50	0.04	0.09
Finland	0.60	8.11	0.10	2.07	13.42	18.13	0.03	0.07
France	0.49	5.80	-0.46	0.99	10.58	13.62	0.04	0.08
Germany	0.46	6.50	-0.37	1.64	10.25	15.48	0.03	0.07
Greece	-0.37	10.55	-0.23	0.68	18.01	24.24	-0.02	-0.03
Hong-Kong	0.70	6.04	-0.17	1.46	9.77	13.12	0.05	0.12
Hungary	0.88	9.16	-0.51	2.19	14.60	21.38	0.04	0.10
India	1.12	8.28	-0.02	2.04	13.22	17.38	0.06	0.13
Israel	0.62	6.26	-0.23	1.38	10.55	14.06	0.04	0.10
Italy	0.24	6.61	-0.22	0.58	11.20	14.70	0.02	0.04
Japan	0.32	4.77	-0.12	0.33	7.98	9.91	0.03	0.07
Malaysia	0.75	5.78	0.63	4.58	9.01	11.37	0.07	0.13
Mexico	0.80	6.67	-0.50	1.58	10.62	14.55	0.05	0.12
Netherlands	0.46	5.76	-0.71	1.94	9.65	14.05	0.03	0.08
New Zealand	0.93	5.74	-0.44	0.79	8.72	12.55	0.07	0.16
Norway	0.86	7.28	-0.65	2.79	9.39	16.38	0.05	0.12
Peru	1.19	7.64	-0.28	2.14	11.51	15.72	0.08	0.16
Philippines	0.57	6.95	-0.02	0.97	11.08	14.56	0.04	0.08
Poland	0.74	9.11	-0.10	0.79	13.16	18.98	0.04	0.08
Portugal	0.09	6.30	-0.33	0.82	10.03	13.97	0.01	0.01
Russia	1.91	10.59	0.55	3.44	15.09	20.26	0.09	0.18
South Africa	0.91	7.14	-0.31	0.10	10.62	14.36	0.06	0.13
South Korea	0.95	8.50	0.20	0.92	13.94	16.61	0.06	0.11
Spain	0.40	6.70	-0.14	1.04	10.08	14.31	0.03	0.06
Sweden	0.78	6.98	-0.15	1.93	11.70	16.00	0.05	0.11
Switzerland	0.51	4.43	-0.46	0.62	7.37	10.35	0.05	0.12
Thailand	1.07	8.47	-0.01	2.92	11.46	18.95	0.06	0.13
Turkey	1.18	13.51	0.53	3.12	17.10	27.07	0.04	0.09
UK	0.33	4.67	-0.38	1.45	7.22	10.17	0.03	0.07
US	0.52	4.33	-0.64	1.02	7.85	9.84	0.05	0.12

Risk-free source

This table reports the risk-free description for all countries in our sample. The data for all countries are obtained from Datastream except for Euro-area and USA that are obtained from Refinitiv and Kenneth French's website at `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Developed`.

Country	Description
Australia	One-month Australian Dollar deposit rate
Brazil	Brazil interbank deposit certificates rate
Canada	One-month Canada Treasury Bill rate
Chile	90-days Chile Disctb Promi Ssory Notes rate
China	One-month China Repo rate
Colombia	90-days Colombia certificate of deposit rate
Czech Republic	90-days Czech Republic inter-bank delayed rate
Denmark	One-month Denmark inter-bank delayed rate
Egypt	One-month Egypt inter-bank rate
Euro area	One-month Euribor rate
Hong Kong	One-month Hong Kong inter-bank rate
Hungary	One-month Hungary inter-bank rate
India	Overnight India deposit rate rate
Israel	One-month Tel Aviv inter-bank rate
Japan	30-days Japan domestic banks deposit rate
Korea	One-month South Korea inter-bank rate
Malaysia	One-month Malaysia inter-bank rate
Mexico	28-days Mexico Cetes closing rate
New Zealand	One-month New Zealand Dollar deposit rate
Norway	One-month Norway inter-bank delayed rate
Peru	Peru inter-bank rate
Philippine	30-60 days Philippine time deposit rate
Poland	One-month Polish Zloty deposit rate
Russia	30-days Russia inter-bank actual credit rate
South Africa	One-month South African JIBAR rate
Sweden	30-days Sweden Treasury Bill rate
Switzerland	One-month Swiss Franc deposit rate
Thailand	One-month Thailand inter-bank (Bangkok Bank) rate
Turkey	One-month Turkey deposit rate rate
United Kingdom	One-month United Kingdom Treasury Bill Tender rate
USA	One-month USA Treasury Bill rate

Descriptive statistics of household portfolio value by size

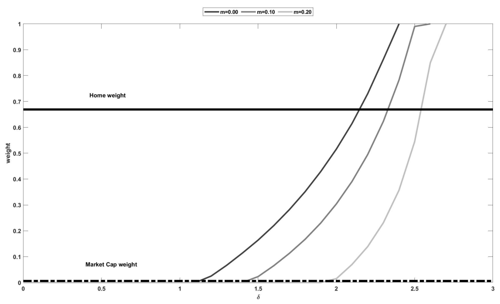
This table reports the descriptive statistics of household portfolios in the dataset. The dataset consists of the portfolio holding of 60000 investors at a large brokerage house in USA. The panels A, B, and C represents the portfolio value (in USD) statistics correspond to January 1991, 1993, and 1996, respectively over the sample period. The household portfolios are sorted based on the number of stocks held in the portfolio.

Portfolio Size	Number of Observation	Mean	25th quantile	Median	75th quantile	Standard deviation
Panel A: Portfolio as of January 1991						
1	15931	11724	1797	4700	10547	38833
2	9469	14333	3726	7125	13825	37986
3	6105	18941	5306	9700	18981	45434
4	4015	23745	6813	12531	23655	54879
5	2696	30604	8764	16144	32161	61555
6-9	5023	43486	12106	23594	46953	79655
10+	3134	138800	27568	61636	132462	541508
All	46373	27374	3982	9200	22074	151858
Panel B: Portfolio as of January 1993						
1	13704	16041	2313	6070	14325	52090
2	8875	21142	4985	9785	19508	91930
3	6158	25652	7413	13656	25844	53132
4	4309	32676	10128	17975	31631	66179
5	3279	39371	12206	21503	40394	77219
6-9	6482	64519	18189	31869	61784	346836
10+	4632	181512	41824	82888	181549	368124
All	47439	44147	6113	14688	35549	188386
Panel C: Portfolio as of January 1996						
1	6265	18605	1500	5844	15493	69342
2	4012	24457	5105	11487	24133	75819
3	2772	33524	8671	16920	33233	134177
4	2108	46735	12598	23069	44156	115110
5	1520	54535	16057	29326	52853	106611
6-9	3437	92919	23032	42356	78868	576715
10+	3126	271978	53491	116788	262775	709035
All	23240	71368	6850	19850	50945	360578

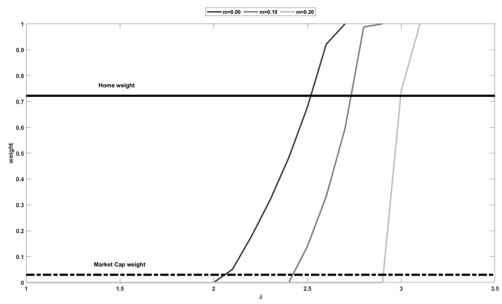
Figure 2: Optimal robust MtC weights for European developed markets

This figure illustrates the optimal allocation to home equity index for investors in developed markets as a function of foreign equity mean return ambiguity parameter δ_f when the relative ambiguity of mean return of home to foreign (m) is equal to 0, 0.1, and 0.2. The solid and dashed horizontal lines are average home and market capitalization weights, respectively. The model optimizes the robust MtC model over the sample of 40 equity market indices (21 developed and 19 emerging) and the sample period spans January 1, 1999 to December 31, 2019.

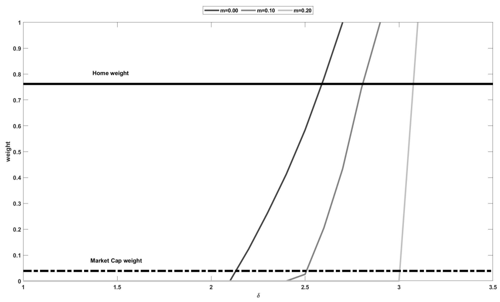
(a) Belgium



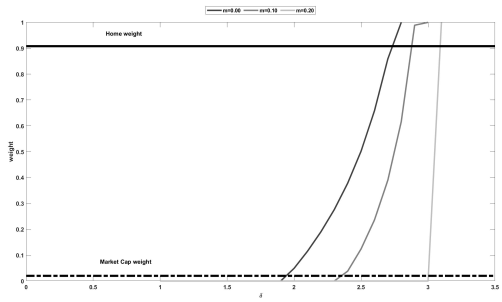
(b) Germany



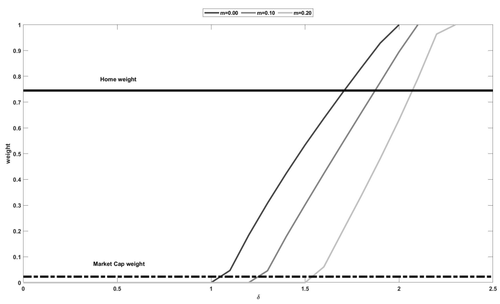
(c) France



(d) Spain



(e) Switzerland



(f) United Kingdom

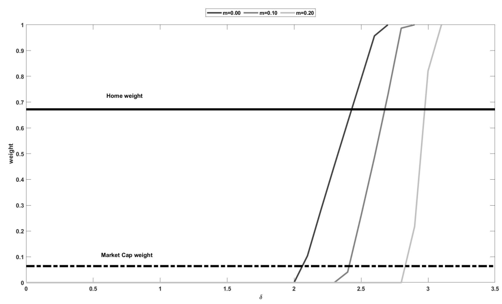
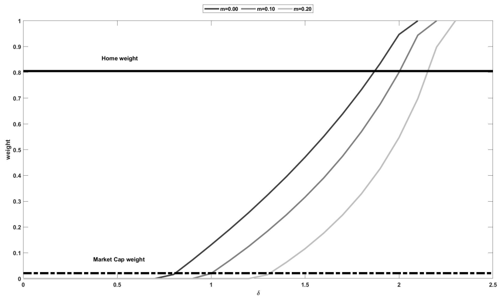


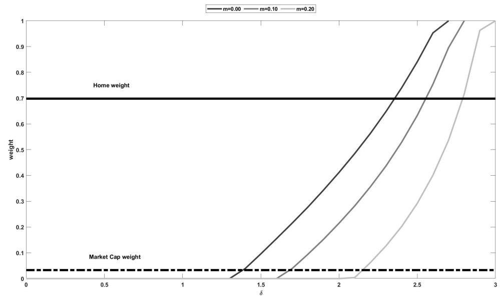
Figure 3: Optimal robust MtC weights for non-European developed markets

This figure illustrates the optimal allocation to home equity index for investors in developed markets as a function of foreign equity mean return ambiguity parameter δ_f when the relative ambiguity of mean return of home to foreign (m) is equal to 0, 0.1, and 0.2. The solid and dashed horizontal lines are average home and market capitalization weights, respectively. The model optimizes the robust MtC model over the sample of 40 equity market indices (21 developed and 19 emerging) and the sample period spans January 1, 1999 to December 31, 2019.

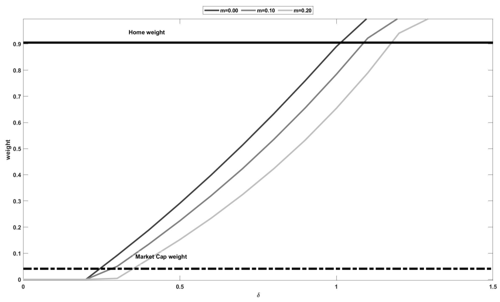
(a) Australia



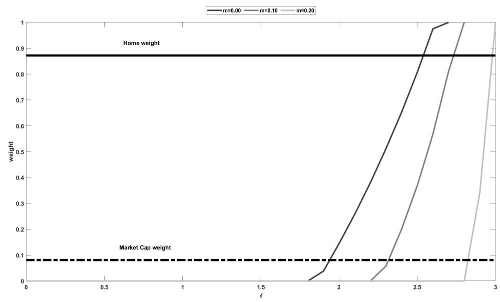
(b) Canada



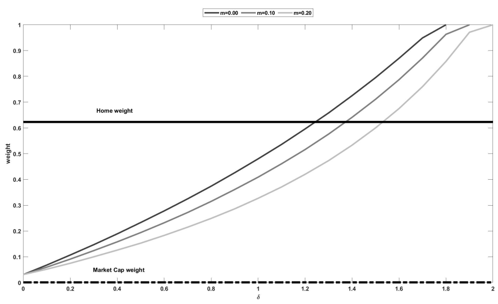
(c) Hong Kong



(d) Japan



(e) New Zealand



(f) USA

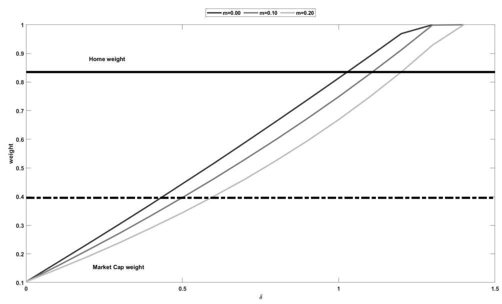
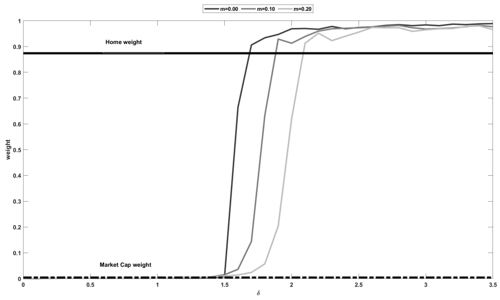


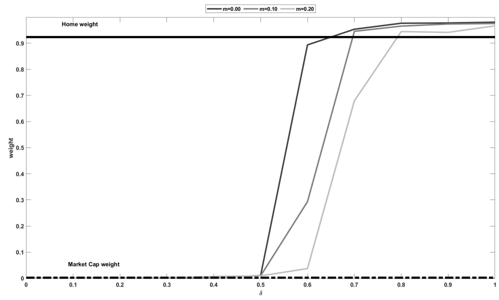
Figure 4: Optimal robust MtC weights for emerging markets

This figure illustrates the optimal allocation to home equity index for investors in emerging markets as a function of foreign equity mean return ambiguity parameter δ_f when the relative ambiguity of mean return of home to foreign parameter m is equal to 0, 0.1, and 0.2. The solid and dashed horizontal lines are average home and market cap weights, respectively. The model optimizes the robust MtC model over the sample of 40 equity market indices (21 developed and 19 emerging) and the sample period spans January 1, 1999 to December 31, 2019.

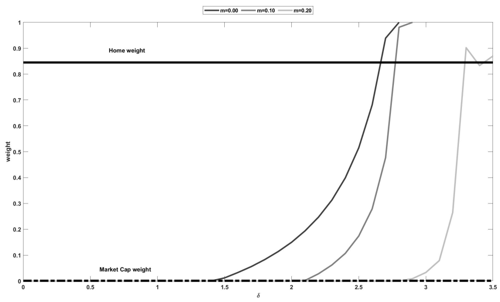
(a) Chile



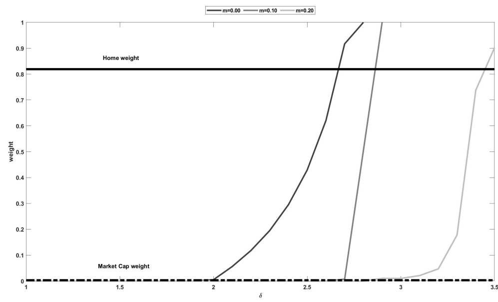
(b) Colombia



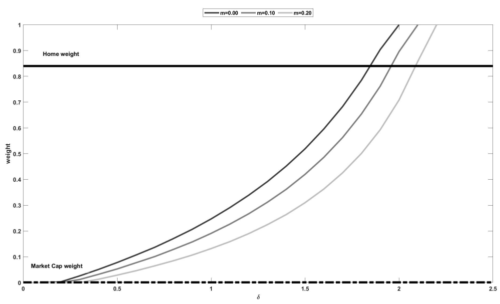
(c) Hungary



(d) Israel



(e) Peru



(f) South Africa

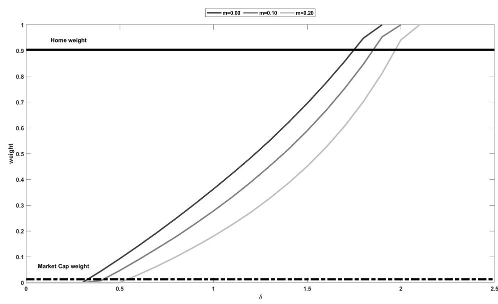
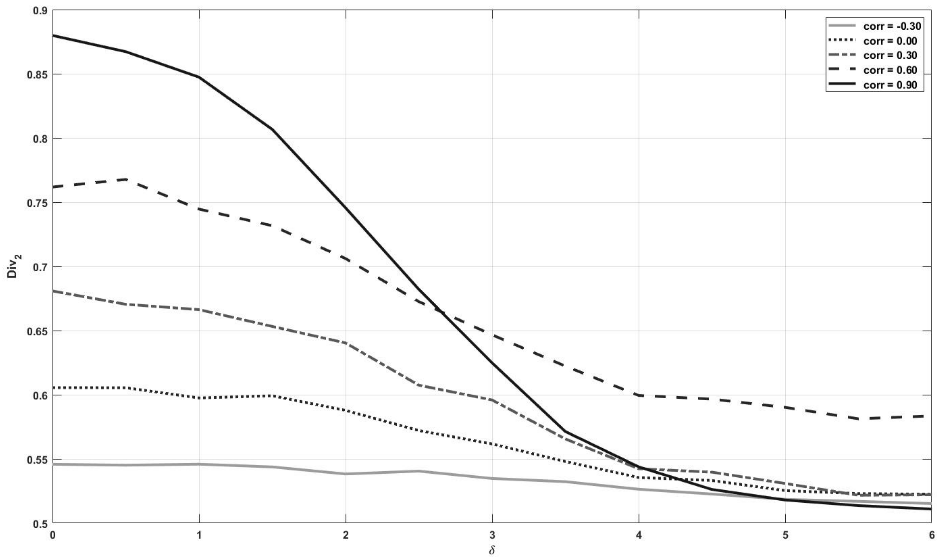


Figure 5: Diversification and ambiguity

This figure illustrates the results with simulation that estimate diversification measures and ambiguity parameter δ in a portfolio of two assets. Panel A and B shows the results for diversification measures D^2 and D^3, respectively. The model optimizes the robust MtC model over the sample of two assets where the returns are generated from a multivariate normal distribution with annual mean and standard deviation of 3.6% and 3.1%, respectively, and the correlation values vary between -0.30 and 0.90.

(a) D^2



(b) D^3

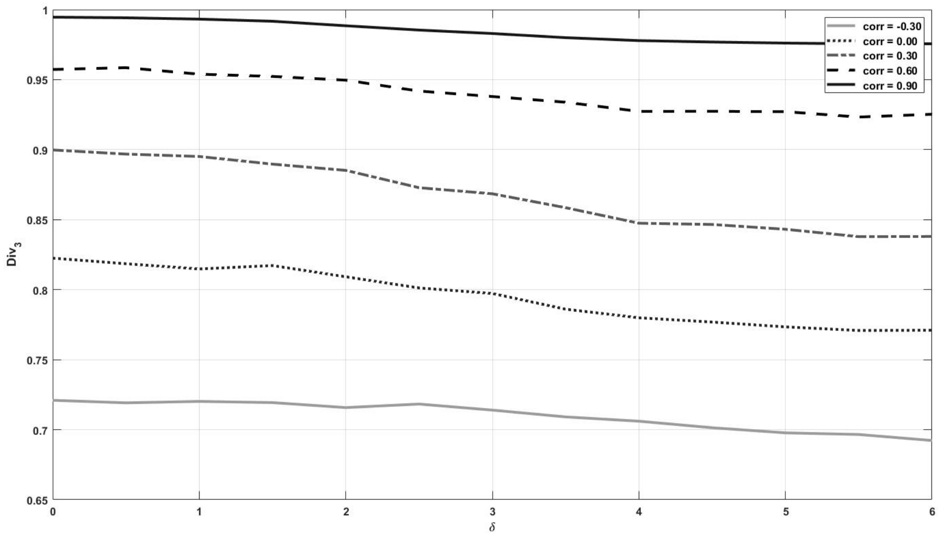


Table 1: Ambiguity in financial data and the home bias puzzle in developed markets

This table reports the estimated ambiguity parameters, average market and actual weights, and home bias index (HBI) for the sample of developed markets in our sample. The δ_f and δ_h are the parameters controlling the size of ambiguity sets of foreign and home equity mean returns obtained using the heuristic developed in Lotfi and Zenios (2018) and m indicates the relative ambiguity of home to foreign equity mean returns. The $\delta_f^\dagger$ corresponds to level of δ_f for which the optimal allocation to home country coincide the average actual weight when $m = 0.2$ (exception is Greece where we set $m = 0.10$). The robust MtC model is solved for a portfolio of 40 equity market indices (21 developed and 19 emerging markets) and the sample period spans January 1, 1999 to December 31, 2019.

Country	$\delta_f^\dagger$	δ_f	δ_h	m	Market cap	Home weight	HBI
Australia	2.14	8.65	2.19	0.25	0.022	0.81	0.80
Austria	2.87	8.93	2.47	0.28	0.002	0.59	0.59
Belgium	2.53	8.21	2.15	0.26	0.006	0.67	0.67
Canada	2.77	8.88	2.07	0.23	0.034	0.70	0.69
Denmark	2.12	8.93	1.65	0.19	0.005	0.58	0.58
Finland	3.92	8.93	1.58	0.18	0.004	0.66	0.65
France	3.06	8.79	1.74	0.20	0.038	0.76	0.75
Germany	3.00	8.85	1.75	0.20	0.029	0.72	0.71
Greece	7.30	8.87	2.32	0.26	0.002	0.96	0.96
Hong Kong	1.15	8.49	1.78	0.21	0.045	0.91	0.90
Italy	4.04	8.93	2.26	0.25	0.013	0.75	0.75
Japan	2.93	8.83	2.37	0.27	0.080	0.87	0.86
Netherlands	3.02	8.92	1.74	0.20	0.013	0.40	0.39
New Zealand	1.47	8.61	2.36	0.27	0.001	0.62	0.62
Norway	2.12	8.83	2.42	0.27	0.004	0.38	0.38
Portugal	4.08	8.89	2.39	0.27	0.001	0.77	0.77
Spain	3.07	8.93	2.07	0.23	0.020	0.91	0.91
Sweden	2.36	8.57	1.90	0.22	0.011	0.65	0.64
Switzerland	2.05	8.80	1.97	0.22	0.023	0.75	0.74
United Kingdom	2.98	8.97	1.76	0.20	0.061	0.67	0.65
USA	1.15	8.27	2.17	0.26	0.381	0.84	0.73
Mean	2.86	8.76	2.05	0.23	0.038	0.71	0.70
StdDev	1.28	0.21	0.28	0.03	0.079	0.15	0.14

Table 2: Ambiguity in financial data and the home bias puzzle in emerging market

This table reports the estimated ambiguity parameters, average market and actual weights, and home bias index (HBI) for the sample of emerging markets in our sample. The δ_f and δ_h are the parameters controlling the size of ambiguity sets of foreign and home equity mean returns obtained using the heuristic developed in Lotfi and Zenios (2018) and m indicates the relative ambiguity of home to foreign equity mean returns. The $\delta_f^\dagger$ corresponds to level of δ_f for which the optimal allocation to home country coincide the average actual weight when $m = 0.2$. The robust MtC model is solved for a portfolio of 40 equity market indices (21 developed and 19 emerging markets) and the sample period spans January 1, 1999 to December 31, 2019.

Country	$\delta_f^\dagger$	δ_f	δ_h	m	Market cap	Home weight	HBI
Brazil	3.00	8.88	2.56	0.29	0.017	0.99	0.99
Chile	2.06	8.73	2.52	0.29	0.004	0.87	0.87
China	0.99	8.40	2.39	0.28	0.085	0.96	0.95
Colombia	0.74	8.80	3.58	0.41	0.002	0.92	0.92
Czech Republic	2.86	8.98	2.81	0.31	0.001	0.89	0.89
Egypt	1.52	8.68	3.02	0.35	0.001	1.00	1.00
Hungary	3.29	8.86	1.90	0.22	0.001	0.85	0.85
India	1.55	8.49	2.21	0.26	0.023	1.00	1.00
Israel	3.35	8.42	1.63	0.19	0.003	0.82	0.82
Korea	2.69	8.75	1.50	0.17	0.019	0.93	0.93
Malaysia	2.52	9.01	1.38	0.15	0.006	0.96	0.96
Mexico	2.54	8.90	2.55	0.29	0.007	0.99	0.99
Peru	2.06	8.83	2.60	0.30	0.001	0.84	0.84
Philippines	2.33	8.00	2.34	0.29	0.003	1.00	1.00
Poland	6.00	8.91	2.22	0.25	0.003	0.97	0.97
Russia	2.11	8.29	3.40	0.41	0.012	1.00	1.00
South Africa	1.94	8.73	1.95	0.22	0.015	0.90	0.90
Thailand	1.02	8.49	1.49	0.18	0.005	0.99	0.99
Turkey	8.00	8.99	2.13	0.24	0.004	1.00	1.00
Mean	2.66	8.69	2.33	0.27	0.011	0.94	0.94
StdDev	1.69	0.27	0.60	0.07	0.019	0.06	0.06

Table 3: Ambiguity measures and the home bias puzzle

This table reports the estimated ambiguity parameters using alternative methods, average market and actual weights and home bias index (HBI) for the sample of 23 countries that have EPU index data available. The δ_f and δ_h are the parameters controlling the size of ambiguity sets of foreign and home equity mean returns obtained using the heuristic developed in Lotfi and Zenios (2018) and m indicates the relative ambiguity of home to foreign equity mean returns. The $\delta_f^{\dagger}$ corresponds to level of δ_f for which the optimal allocation to home country coincide the average actual weight when $m = 0.2$. The $\delta_{EPU,f}$ and $\delta_{EPU,h}$ are the parameters controlling the size of ambiguity sets of foreign and home equity mean returns obtained using maximum EPU index in equation 12 and m_{EPU} indicates the relative ambiguity of home to foreign equity mean returns. The $\delta^{\dagger}_{EPU,f}$ corresponds to level of $\delta_{EPU,f}$ for which the optimal allocation to home country coincide the average actual weight when $m_{EPU} = 0.1$. The robust MtC model is solved for a portfolio of 23 equity market indices (15 developed and 8 emerging markets) and the sample period spans January 1, 1999 to December 31, 2019.

Country	$\delta_f^{\dagger}$	δ_f	δ_h	m	$\delta^{\dagger}_{EPU,f}$	$\delta_{EPU,f}$	$\delta_{EPU,h}$	m_{EPU}	Market cap	Home weight	HBI
					Panel A: Developed economies						
Australia	1.50	6.73	2.19	0.33	1.20	10.11	0.90	0.09	0.024	0.82	0.81
Belgium	2.32	7.03	2.15	0.31	2.10	11.20	0.67	0.06	0.006	0.69	0.69
Canada	2.03	7.06	2.07	0.29	1.87	10.54	1.33	0.13	0.037	0.71	0.70
Denmark	1.38	7.15	1.65	0.23	1.31	10.78	0.71	0.07	0.005	0.62	0.62
France	2.50	6.96	1.74	0.25	2.26	8.79	1.54	0.18	0.041	0.79	0.78
Germany	2.43	7.03	1.75	0.25	2.20	10.24	1.22	0.12	0.032	0.77	0.76
Greece	3.50	5.75	1.87	0.33	3.00	12.33	0.92	0.08	0.003	0.96	0.96
Hong Kong	2.10	7.22	1.78	0.25	1.93	12.28	1.14	0.09	0.049	0.91	0.90
Italy	4.00	7.13	2.26	0.32	2.79	10.99	0.65	0.06	0.014	0.78	0.77
Japan	2.80	6.91	2.37	0.34	2.21	13.96	0.64	0.05	0.086	0.88	0.87
Netherlands	2.44	7.12	1.74	0.25	2.15	11.12	0.63	0.06	0.014	0.44	0.43
Spain	3.00	7.11	2.07	0.29	2.33	11.07	1.09	0.10	0.022	0.91	0.91
Sweden	1.70	6.77	1.90	0.28	1.58	10.75	0.42	0.04	0.012	0.71	0.70
United Kingdom	2.42	7.06	1.76	0.25	2.17	10.99	1.50	0.14	0.067	0.71	0.69
USA	2.01	7.10	2.17	0.31	1.85	13.01	0.76	0.06	0.413	0.86	0.75
Mean	2.41	6.94	1.97	0.28	2.06	11.21	0.94	0.09	0.055	0.77	0.76
StdDev	0.68	0.35	0.22	0.04	0.46	1.22	0.33	0.04	0.098	0.13	0.13
					Panel B: Emerging economies						
Brazil	3.50	6.80	2.56	0.38	1.21	15.40	1.81	0.12	0.019	0.99	0.99
Chile	1.80	6.62	2.52	0.38	1.32	11.78	0.76	0.07	0.004	0.88	0.88
China	3.30	7.12	2.39	0.34	1.82	12.45	2.60	0.21	0.092	0.96	0.95
Colombia	Any	6.45	3.58	0.56	Any	11.52	0.87	0.08	0.003	0.93	0.93
India	1.80	7.12	2.21	0.31	1.68	11.63	0.76	0.07	0.025	1.00	1.00
Korea	2.03	6.83	1.50	0.22	1.92	11.04	1.44	0.13	0.020	0.93	0.93
Mexico	2.30	6.93	2.55	0.37	1.81	11.03	1.15	0.10	0.008	0.99	0.99
Russia	1.80	5.93	3.40	0.57	1.63	12.41	1.16	0.09	0.013	1.00	1.00
Mean	2.36	6.72	2.59	0.39	1.63	12.16	1.32	0.11	0.023	0.96	0.96
StdDev	0.64	0.35	0.58	0.11	0.23	1.25	0.56	0.04	0.026	0.04	0.04

Table 4: Ambiguity in household portfolio and under-diversification

This table reports the estimated ambiguity parameters δ and return statistics of household portfolio for nine years subsequent to portfolio formation. Panel A and B correspond to the portfolio sorted into quintiles corresponding to their level of diversification as measure by D^2 and D^3, respectively. Returns are reported subsequent to three representative months during the sample period (January 1991, January 1993, and January 1996). N is the number of portfolios in the quintile. The Means, StdDev, Skew, and Kurt are the average of monthly returns mean, standard deviation, skewness and kurtosis over all the portfolios in the quintile.

Panel A: Portfolio sorted on D^2							
January 91							
Quintile	N	D^2	δ	Mean	StdDev	Skew	Kurt
Low div.	2469	0.80	2.96	1.79	0.86	0.24	0.88
2	2531	0.57	2.95	1.75	0.69	0.23	0.89
3	2530	0.49	3.06	1.69	0.59	0.22	0.80
4	2531	0.36	3.44	1.60	0.41	0.16	0.64
5	2530	0.21	4.13	1.52	0.26	0.14	0.40
January 93							
Low div.	2532	0.79	2.66	1.54	0.98	0.17	1.01
2	2584	0.56	2.64	1.41	0.77	0.16	0.95
3	2584	0.46	2.81	1.42	0.62	0.12	0.74
4	2584	0.33	3.12	1.41	0.42	0.02	0.52
5	2584	0.19	3.72	1.37	0.24	-0.12	0.25
January 96							
Low div.	754	0.82	2.74	1.65	1.39	0.23	0.90
2	755	0.58	2.87	1.53	1.13	0.18	0.80
3	756	0.50	2.85	1.48	0.97	0.15	0.78
4	755	0.37	3.24	1.64	0.81	0.07	0.58
5	756	0.22	3.81	1.78	0.54	-0.14	0.45

Panel B: Portfolio sorted on D^3							
January 91							
Quintile	N	D^3	δ	Mean	StdDev	Skew	Kurt
Low div.	2529	0.96	2.93	1.78	0.87	0.24	0.88
2	2531	0.86	2.94	1.76	0.69	0.21	0.89
3	2530	0.79	3.06	1.68	0.58	0.21	0.78
4	2531	0.67	3.47	1.62	0.42	0.16	0.63
5	2530	0.50	4.15	1.52	0.28	0.17	0.43
January 93							
Low div.	2583	0.96	2.62	1.53	0.97	0.17	1.03
2	2584	0.85	2.62	1.46	0.78	0.14	0.91
3	2584	0.76	2.83	1.41	0.61	0.12	0.76
4	2584	0.64	3.12	1.40	0.43	0.02	0.52
5	2584	0.46	3.75	1.36	0.26	-0.08	0.27
January 96							
Low div.	754	0.97	2.73	1.62	1.41	0.24	0.95
2	755	0.88	2.80	1.60	1.17	0.19	0.78
3	756	0.80	2.90	1.52	0.96	0.14	0.75
4	755	0.69	3.31	1.63	0.79	0.07	0.60
5	756	0.52	3.76	1.71	0.50	-0.14	0.45

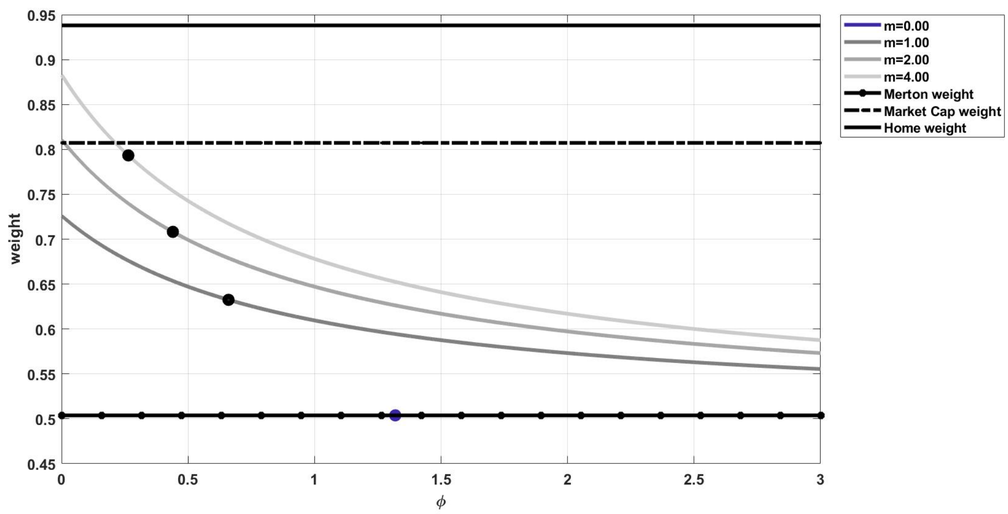
Online Appendix

A Figures

Figure A1: Portfolio weights adjusted for ambiguity as Uppal and Wang (2003)

This figure illustrates the dynamic of Uppal and Wang (2003) model's home allocation for US investor obtained in the universe of 3 countries and when there is ambiguity in joint and marginal distribution of return. Panel (A) illustrates the replication of Figure 1 in Uppal and Wang (2003) which uses the data points from French and Poterba (1991) (see Uppal and Wang (2003) for details). Panel (B) illustrate their model replication for the universe of Europe, Japan, and US where Europe index is constructed using equally-weighted portfolio of EMU constituent country indices. The filled circles show the model-based threshold of ϕ. The sample period spans January 1, 1999 to December 31, 2019.

(a) French and Poterba (1991) data



(b) 1999-2019 data

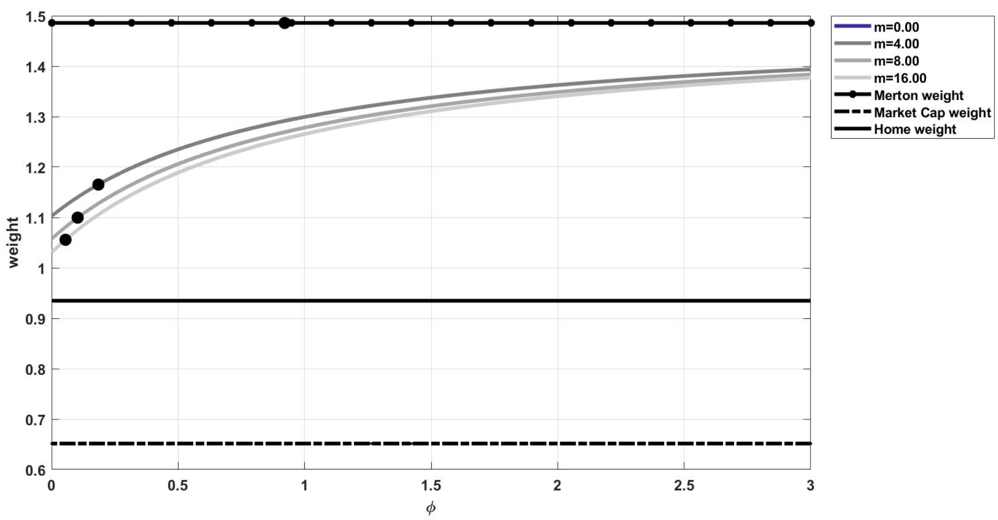
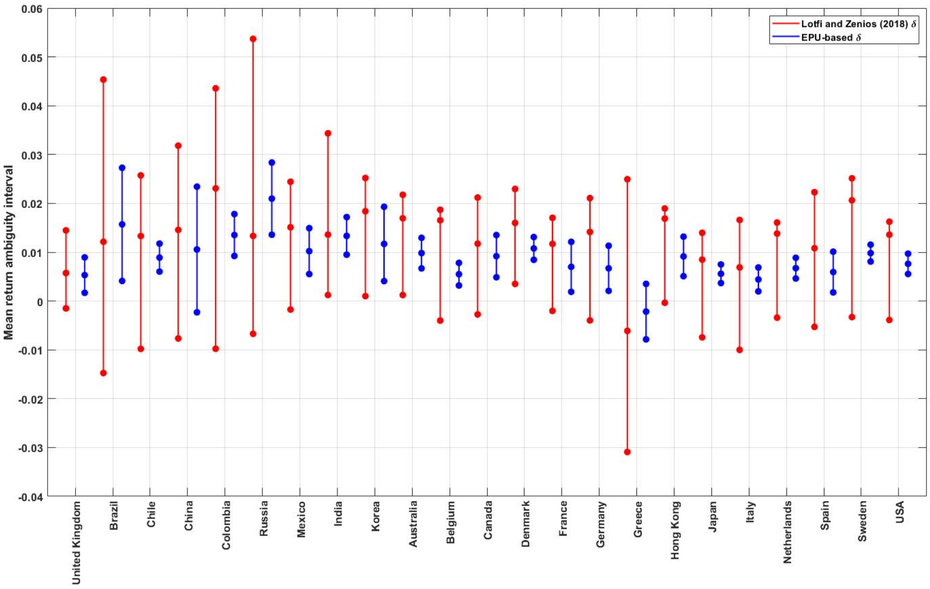


Figure A2: Difference in EPU-based confidence interval

This figure illustrates the interval centers and boundaries implied by different methods. Panel A and B illustrate the intervals implied when we use maximum and mean EPU for finding the mean returns confidence interval, respectively. In both panels we additionally illustrates the intervals implied by financial data using Lotfi and Zenios (2018) heuristic. The sample covers 23 countries that have EPU index data available and the sample period spans January 1, 1999 to December 31, 2019.

(a) Max EPU



(b) Mean EPU

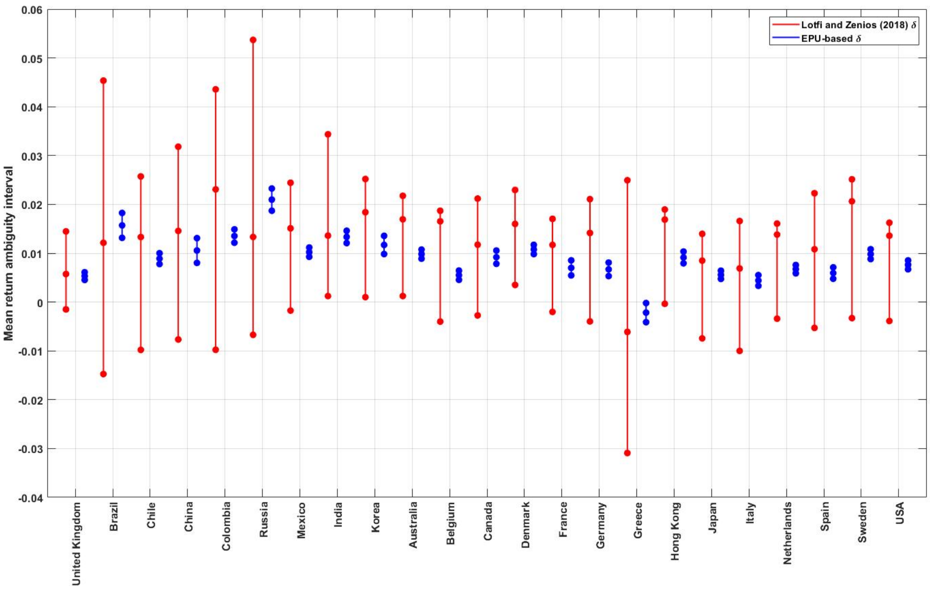
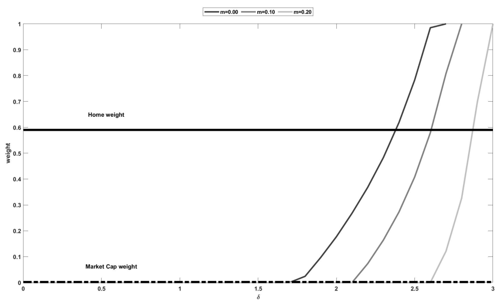


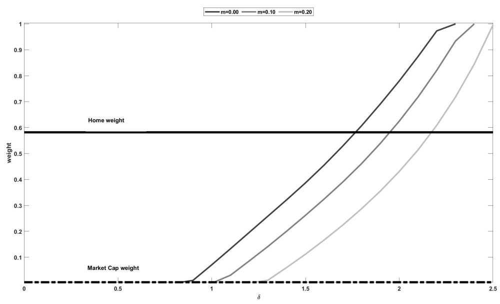
Figure A3: Optimal robust MtC weights for Other European developed markets

This figure illustrates the optimal allocation to home equity index for investors in developed markets as a function of foreign equity mean return ambiguity parameter δ_f when the relative ambiguity of mean return of home to foreign (m) is equal to 0, 0.1, and 0.2. The solid and dashed horizontal lines are average home and market capitalization weights, respectively. The model optimizes the robust MtC model over the sample of 40 equity market indices (21 developed and 19 emerging) and the sample period spans January 1, 1999 to December 31, 2019.

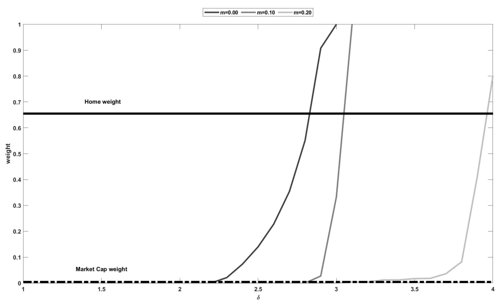
(a) Austria



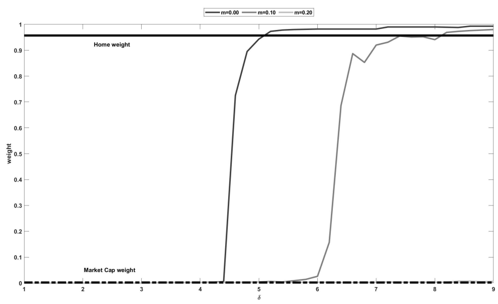
(b) Denmark



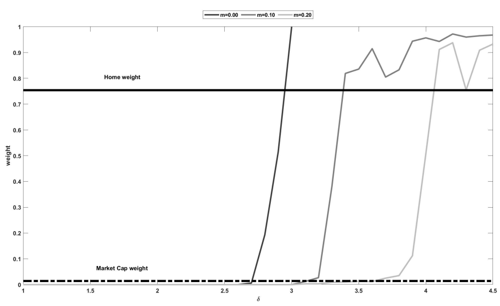
(c) Finland



(d) Greece



(e) Italy



(f) Netherlands

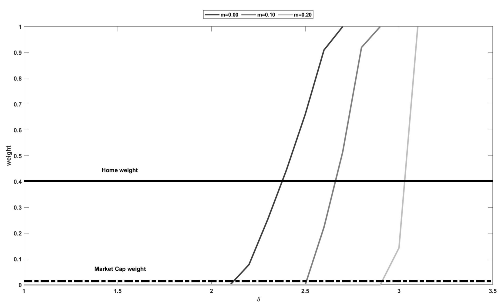
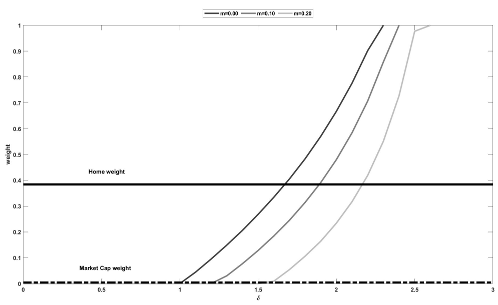
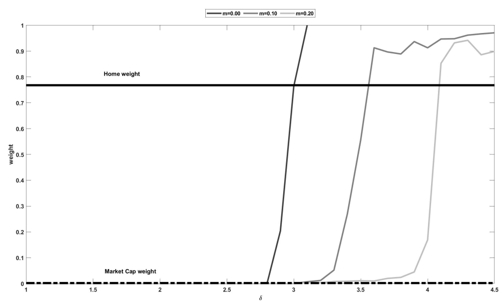


Figure A3: (continued)

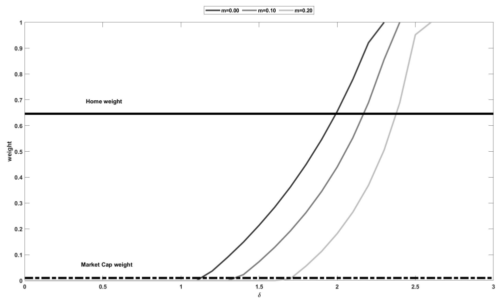
(a) Norway



(b) Portugal



(c) Sweden



B Tables

Table B1: Alternative ambiguity measures and the home bias puzzle

This table reports the estimated ambiguity parameters using alternative methods, average market and actual weights and home bias index (HBI) for the sample of 23 countries that have EPU index data available. The δ_f and δ_h are the parameters controlling the size of ambiguity sets of foreign and home equity mean returns obtained using the heuristic developed in Lotfi and Zenios (2018) and m indicates the relative ambiguity of home to foreign equity mean returns. The $\delta_f^{\dagger}$ corresponds to level of δ_f for which the optimal allocation to home country coincide the average actual weight when $m = 0.2$. The $\delta_{EPU,f}$ and $\delta_{EPU,h}$ are the parameters controlling the size of ambiguity sets of foreign and home equity mean returns obtained using mean EPU index in equation 12 and m_{EPU} indicates the relative ambiguity of home to foreign equity mean returns. The $\delta_{EPU,f}^{\dagger}$ corresponds to level of $\delta_{EPU,f}$ for which the optimal allocation to home country coincide the average actual weight when $m_{EPU} = 0.1$. The robust MtC model is solved for a portfolio of 23 equity market indices (15 developed and 8 emerging) and the sample period spans January 1, 1999 to December 31, 2019.

Country	$\delta_f^{\dagger}$	δ_f	δ_h	m	$\delta_{EPU,f}^{\dagger}$	$\delta_{EPU,f}$	$\delta_{EPU,h}$	m_{EPU}	Market cap	Home weight	HBI
Panel A: Developed economies											
Australia	1.50	6.73	2.19	0.33	1.20	3.09	0.27	0.09	0.024	0.82	0.81
Belgium	2.32	7.03	2.15	0.31	2.10	3.26	0.28	0.09	0.006	0.69	0.69
Canada	2.03	7.06	2.07	0.29	1.87	3.16	0.42	0.13	0.037	0.71	0.70
Denmark	1.38	7.15	1.65	0.23	1.31	3.29	0.29	0.09	0.005	0.62	0.62
France	2.50	6.96	1.74	0.25	2.26	2.54	0.46	0.18	0.041	0.79	0.78
Germany	2.43	7.03	1.75	0.25	2.20	3.03	0.36	0.12	0.032	0.77	0.76
Greece	3.50	5.75	1.87	0.33	12.33	3.00	0.92	0.08	0.003	0.96	0.96
Hong Kong	2.10	7.22	1.78	0.25	1.93	3.75	0.34	0.09	0.049	0.91	0.90
Italy	4.00	7.13	2.26	0.32	2.79	3.36	0.29	0.09	0.014	0.78	0.77
Japan	2.80	6.91	2.37	0.34	2.21	4.18	0.28	0.07	0.086	0.88	0.87
Netherlands	2.44	7.12	1.74	0.25	2.15	3.17	0.25	0.08	0.014	0.44	0.43
Spain	3.00	7.11	2.07	0.29	2.33	3.29	0.31	0.09	0.022	0.91	0.91
Sweden	1.70	6.77	1.90	0.28	1.58	3.39	0.25	0.07	0.012	0.71	0.70
United Kingdom	2.42	7.06	1.76	0.25	2.17	3.44	0.33	0.10	0.067	0.71	0.69
USA	2.01	7.10	2.17	0.31	1.85	3.74	0.33	0.09	0.413	0.86	0.75
Mean	2.44	6.93	1.95	0.28	2.74	3.28	0.36	0.10	0.029	0.76	0.76
StdDev	0.68	0.35	0.22	0.04	2.61	0.37	0.16	0.03	0.098	0.13	0.13
Panel B: Emerging economies											
Brazil	3.50	6.80	2.56	0.38	1.21	4.60	0.40	0.09	0.02	0.99	0.99
Chile	1.80	6.62	2.52	0.38	1.32	3.42	0.29	0.09	0.00	0.88	0.88
China	3.30	7.12	2.39	0.34	1.82	3.69	0.51	0.14	0.09	0.96	0.95
Colombia	Any	6.45	3.58	0.56	Any	3.48	0.28	0.08	0.00	0.93	0.93
India	1.80	7.12	2.21	0.31	1.68	3.56	0.25	0.07	0.03	1.00	1.00
Korea	2.03	6.83	1.50	0.22	1.92	3.29	0.35	0.11	0.02	0.93	0.93
Mexico	2.30	6.93	2.55	0.37	1.81	3.18	0.23	0.07	0.01	0.99	0.99
Russia	1.80	5.93	3.40	0.57	1.63	3.77	0.36	0.09	0.01	1.00	1.00
Mean	2.36	6.72	2.59	0.39	1.63	3.62	0.33	0.09	0.023	0.96	0.96
StdDev	0.68	0.37	0.61	0.11	0.25	0.41	0.09	0.02	0.027	0.04	0.04

Table B2: Ambiguity in household portfolio and under-diversification: decile-based

This table reports the estimated ambiguity parameters δ and return statistics of household portfolio for nine years subsequent to portfolio formation. Panel A and B correspond to the portfolio sorted into decile corresponding to their level of diversification as measure by D^2 and D^3, respectively. Returns are reported subsequent to three representative months during the sample period (January 1991, January 1993, and January 1996). N is the number of portfolios in the decile. The Means, StdDev, Skew, and Kurt are the average of monthly returns mean, standard deviation, skewness and kurtosis over all the portfolios in the decile.

Panel A: Portfolio sorted on D^2							
January 91							
Decile	N	D^2	δ	Mean	StdDev	Skew	Kurt
Low div.	1204	0.90	2.96	1.81	0.92	0.25	0.90
2	1265	0.71	2.96	1.77	0.80	0.23	0.87
3	1266	0.60	2.99	1.75	0.72	0.22	0.83
4	1265	0.54	2.91	1.75	0.66	0.23	0.96
5	1265	0.51	2.87	1.66	0.64	0.24	0.91
6	1265	0.47	3.25	1.72	0.55	0.19	0.69
7	1265	0.39	3.38	1.61	0.44	0.18	0.66
8	1266	0.33	3.51	1.60	0.38	0.14	0.62
9	1265	0.26	3.78	1.57	0.30	0.12	0.44
10	1265	0.16	4.47	1.46	0.21	0.17	0.35
January 93							
Low div.	1240	0.88	2.63	1.56	1.05	0.19	1.09
2	1292	0.70	2.68	1.52	0.92	0.16	0.94
3	1292	0.59	2.63	1.42	0.80	0.16	0.92
4	1292	0.53	2.65	1.41	0.74	0.16	0.99
5	1292	0.50	2.61	1.43	0.72	0.16	0.84
6	1292	0.42	3.01	1.41	0.53	0.07	0.65
7	1292	0.36	3.01	1.42	0.46	0.04	0.56
8	1292	0.30	3.22	1.41	0.39	0.01	0.49
9	1292	0.24	3.47	1.39	0.29	-0.06	0.37
10	1292	0.15	3.96	1.35	0.19	-0.17	0.14
January 96							
Low div.	376	0.91	2.78	1.74	1.48	0.25	0.92
2	378	0.73	2.71	1.55	1.31	0.21	0.89
3	377	0.62	2.84	1.56	1.14	0.19	0.82
4	378	0.55	2.89	1.49	1.13	0.17	0.78
5	378	0.51	2.75	1.39	1.02	0.16	0.86
6	378	0.48	2.96	1.57	0.93	0.15	0.71
7	378	0.40	3.25	1.62	0.81	0.06	0.59
8	377	0.34	3.22	1.67	0.81	0.08	0.57
9	378	0.27	3.57	1.71	0.58	-0.07	0.44
10	378	0.18	4.06	1.85	0.50	-0.22	0.47

Panel B: Portfolio sorted on D^3							
January 91							
Decile	N	D^3	δ	Mean	StdDev	Skew	Kurt
Low div.	1264	0.99	2.92	1.83	0.98	0.26	0.96
2	1265	0.94	2.93	1.73	0.76	0.21	0.80
3	1266	0.88	2.88	1.76	0.70	0.20	0.83
4	1265	0.84	3.00	1.76	0.69	0.22	0.96
5	1265	0.81	2.97	1.71	0.62	0.22	0.84
6	1265	0.76	3.15	1.65	0.54	0.21	0.73
7	1265	0.71	3.38	1.64	0.45	0.16	0.67
8	1266	0.64	3.56	1.60	0.38	0.15	0.59
9	1265	0.56	3.80	1.56	0.32	0.16	0.49
10	1265	0.43	4.50	1.49	0.23	0.19	0.38
January 93							
Low div.	1291	0.99	2.61	1.57	1.08	0.20	1.11
2	1292	0.93	2.63	1.49	0.87	0.14	0.95
3	1292	0.87	2.56	1.45	0.82	0.14	0.95
4	1292	0.83	2.67	1.46	0.74	0.14	0.88
5	1292	0.79	2.72	1.41	0.65	0.14	0.80
6	1292	0.73	2.93	1.41	0.56	0.10	0.71
7	1292	0.67	3.00	1.42	0.48	0.03	0.56
8	1292	0.60	3.24	1.38	0.38	0.01	0.48
9	1292	0.52	3.49	1.37	0.31	-0.05	0.40
10	1292	0.40	4.00	1.35	0.22	-0.12	0.14
January 96							
Low div.	376	0.99	2.76	1.74	1.46	0.24	0.92
2	378	0.95	2.69	1.49	1.37	0.23	0.98
3	377	0.90	2.76	1.62	1.17	0.17	0.73
4	378	0.85	2.85	1.58	1.18	0.20	0.82
5	378	0.82	2.89	1.39	1.03	0.15	0.87
6	378	0.78	2.91	1.64	0.90	0.13	0.63
7	378	0.73	3.24	1.49	0.84	0.08	0.58
8	377	0.66	3.39	1.77	0.74	0.06	0.62
9	378	0.58	3.47	1.68	0.58	-0.05	0.36
10	378	0.46	4.05	1.74	0.43	-0.24	0.53

C Background results and proofs

C.1 Definitions and background

Definition C.1 (Conditional Value-at-Risk, CVaR)**.** *The conditional Value-at-Risk at confidence level $\alpha \in (0, 1)$, for a continuously distributed random variable $\tilde{r}_p$ is*

$$\text{CVaR}_\alpha(\tilde{r}_p) = -\mathbb{E}[\tilde{r}_p \mid \tilde{r}_p \leq \zeta], \tag{14}$$

where $\mathbb{E}$ is the expectation operator, and $\zeta_{1-\alpha} \in \mathbb{R}$ is the Value-at-Risk*.*

The astute reader should note that Rockafellar and Uryasev (2002) develop their model for a loss random variable $\tilde{z}$ and not for returns. Their CVaR of losses is the expected value *above* a threshold ζ, whereas we take the CVaR of return to be the negative of expected value of returns *below* the $1 - \alpha$ probability threshold ζ. We use their results with $\tilde{z} = -\tilde{r}_p$ to develop our model in returns.

Value-at-Risk is the highest γ such that $\tilde{r}_p$ will not exceed γ with probability $1 - \alpha$,

$$\text{VaR}_{1-\alpha}(\tilde{r}_p) \doteq \zeta_{1-\alpha} = \max\{\gamma \in \mathbb{R} \mid \text{Prob}(\tilde{r}_p \leq \gamma) \leq 1 - \alpha\}. \tag{15}$$

By definition, ζ is the $(1 - \alpha)$-quantile of the random variable $\tilde{r}_p$. It depends on the portfolio x and the confidence level α, and so does CVaR.

Theorem C.1 (Fundamental minimization formula)**.** (Rockafellar and Uryasev, 2002) *As a function of $\gamma \in \mathbb{R}$, the auxiliary function*

$$F_\alpha(\tilde{r}_p, \gamma) = \gamma + \frac{1}{1-\alpha}\mathbb{E}\big[\max\{-\tilde{r}_p - \gamma, 0\}\big]$$

is finite and convex, with

$$\text{CVaR}_\alpha(\tilde{r}_p) = \min_{\gamma \in \mathbb{R}} F_\alpha(\tilde{r}_p, \gamma).$$

Definition C.2 (Stochastic dominance)**.** *Random variable $\tilde{X}$ dominates random variable $\tilde{Y}$ under first order stochastic dominance (FSD, $\tilde{X} \succeq_{FSD} \tilde{Y}$) if $\mathbb{E}(U(\tilde{X})) \geq \mathbb{E}(U(\tilde{Y}))$ for all increasing utility functions U. Similarly, $\tilde{X}$ dominates random variable $\tilde{Y}$ under second order stochastic dominance (SSD, $\tilde{X} \succeq_{SSD} \tilde{Y}$) if $\mathbb{E}(U(\tilde{X})) \geq \mathbb{E}(U(\tilde{Y}))$ for all increasing concave utility functions U.*

Definition C.3 (Risk measure consistency)**.** *Given a stochastic order $\succeq_{SSD}$ we say that a risk measure ρ is SSD consistent if $\tilde{X} \succeq_{SSD} \tilde{Y}$ implies $\rho(\tilde{X}) \leq \rho(\tilde{Y})$.*

.

C.2 Proof of Theorem 2.1

We start by rewriting the MtC model as follows. Given the assumption that worst-case CVaR of optimal solution is positive, we can find a neighborhood for which $\mathrm{CVaR}(\tilde{r}_p - r_f) > 0$ for all $\pi \in \mathbb{D}$, $\bar{r}_h \in U_{\delta_h}$, and $\bar{r}_f \in U_{\delta_f}$. Defining $\xi = \mathrm{CVaR}(\tilde{r}_p - r_f) > 0$, we can write the maximum MtC ratio maximization model as:

$$\begin{aligned} \max_{x\in\mathbb{X},\,\xi\in\mathbb{R}} \quad & \frac{1}{\xi}(\mathbb{E}(\tilde{r}) - r_f e)^\top x \\ s.t. \quad & \\ & \mathrm{CVaR}_\alpha((\bar{r} - r_f e)^\top x) \leq \xi \\ & \xi > 0. \end{aligned} \tag{16}$$

Setting $x' = \frac{x}{\xi}$ and $\nu = \frac{1}{\xi}$ and using the positive homogeneity property of coherent risk measure we can re-write the above as:

$$\begin{aligned} \max_{x'\in\mathbb{R}^n_+} \quad & (\mathbb{E}(\tilde{r}) - r_f e)^\top x' \\ s.t. \quad & \\ & \mathrm{CVaR}_\alpha((\bar{r} - r_f e)^\top x') \leq 1 \\ & e^\top x' > 0. \end{aligned} \tag{17}$$

Therefore, the robust counterpart of MtC model is as follows:

$$\begin{aligned} \max_{x'\in\mathbb{R}^n_+} \quad & \max_{\substack{\bar{r}_f\in U_{\delta_f} \\ \bar{r}_h\in U_{\delta_h}}} \max_{\pi\in\mathbb{D}} \; (\mathbb{E}(\tilde{r}) - r_f e)^\top x' \\ s.t. \quad & \\ & \max_{\substack{\bar{r}_f\in U_{\delta_f} \\ \bar{r}_h\in U_{\delta_h}}} \max_{\pi\in\mathbb{D}} \; \mathrm{CVaR}_\alpha((\bar{r} - r_f e)^\top x') \leq 1 \\ & e^\top x' > 0. \end{aligned} \tag{18}$$

Using the fundamental minimization formula of CVaR in Online Appendix C.1, one can write the above as:

$$
\begin{aligned}
\max_{x' \in \mathbb{R}^n_+} \quad & \max_{\substack{\bar{r}_f \in U_\delta \\ \bar{r}_h \in U_{\delta_h}}} \max_{\pi \in \mathbb{D}} \; (\mathbb{E}(\tilde{r}) - r_f e)^\top x' \\
s.t. \quad & \\
& \max_{\substack{\bar{r}_f \in U_\delta \\ \bar{r}_h \in U_{\delta_h}}} \max_{\pi \in \mathbb{D}} \; F_\alpha((\tilde{r} - r_f e)^\top x', \gamma) \leq 1 \\
& e^\top x' > 0.
\end{aligned} \tag{19}
$$

Obviously, $\max_{\pi \in \mathbb{D}} (\mathbb{E}(\tilde{r}) - r_f e)^\top x' = (\bar{r} - r_f e)^\top x'$. Further, the innermost maximization in the first constraint with respect to ambiguity in distribution can be obtained using Proposition (1) in Lotfi and Zenios (2018). Therefore the above formulation can be written as follows:

$$
\begin{aligned}
\max_{x' \in \mathbb{R}^n_+} \quad & \max_{\substack{\bar{r}_f \in U_{\delta_f} \\ \bar{r}_h \in U_{\delta_h}}} (\bar{r} - r_f e)^\top x' \\
s.t. \quad & \\
& \max_{\substack{\bar{r}_f \in U_{\delta_f} \\ \bar{r}_h \in U_{\delta_h}}} - (\bar{r}_h - r_f e)^\top x' + \frac{\sqrt{\alpha}}{\sqrt{1-\alpha}} \sqrt{x'^\top \Sigma x'} \leq 1 \\
& e^\top x' > 0.
\end{aligned} \tag{20}
$$

Let us consider $x = \begin{pmatrix} x_f \\ x_h \end{pmatrix}$ where $x_f \in \mathbb{R}^{n-1}_+$ is the first $n-1$ elements of x, indicating the allocation to foreign assets rather than home and $x_h \in \mathbb{R}_+$ indicates the allocation to home. Thus, we re-write the above formulation as

$$
\begin{aligned}
\max_{x' \in \mathbb{R}^n_+} \quad & \max_{\bar{r}_h \in U_{\delta_h}} \max_{\bar{r}_f \in U_{\delta_f}} (\bar{r}_f - r_f e)^\top x'_f + (\bar{r}_h - r_f) x'_h \\
s.t. \quad & \\
& \max_{\bar{r}_h \in U_{\delta_h}} \max_{\bar{r}_f \in U_{\delta_f}} - (\bar{r}_f - r_f e)^\top x'_f - (\bar{r}_h - r_f) x'_h + \\
& \frac{\sqrt{\alpha}}{\sqrt{1-\alpha}} \sqrt{x'^\top_f \Sigma_f x'_f + 2 x_h \sigma^\top_{hf} x_f + x_h^2 \sigma_h^2} \leq 1 \\
& e^\top x' > 0.
\end{aligned} \tag{21}
$$

First we obtain the robust model with respect to $\bar{r}_f \in U_{\delta_f}$. Following the technique used in Lotfi and Zenios (2018), one can obtain the following formulation

$$
\begin{aligned}
\max_{x' \in \mathbb{R}^n_+} \quad & \left[\max_{\bar{r}_h \in U_{\delta_h}} \ (\bar{r}_h - r_f) x'_h\right] + (\hat{r}_f - r_f e)^\top x'_f - \frac{\delta_f}{\sqrt{S}} \sqrt{{x'}_f^\top \Sigma_f x'_f} \\
s.t. \quad & \\
& \left[- \min_{\bar{r}_h \in U_{\delta_h}} \ (\bar{r}_h - r_f) x'_h\right] - (\hat{r}_f - r_f e)^\top x'_f + \frac{\delta_f}{\sqrt{S}} \sqrt{{x'}_f^\top \Sigma_f x'_f} \\
& \frac{\sqrt{\alpha}}{\sqrt{1-\alpha}} \sqrt{{x'}_f^\top \Sigma_f x'_f + 2 x_h \sigma_{hf}^\top x_f + x_h^2 \sigma_h^2} \leq 1 \\
& e^\top x' > 0.
\end{aligned}
\tag{22}
$$

(see Appendix A.1 of Lotfi and Zenios (2018) for more details). Finally one can easily check that

$$
\begin{aligned}
\max_{\bar{r}_h \in U_{\delta_h}} (\bar{r}_h - r_f) x'_h &= (\hat{r}_h - r_f) x'_h - \frac{\delta_h}{\sqrt{S}} x'_h \sigma_h \\
\min_{\bar{r}_h \in U_{\delta_h}} (\bar{r}_h - r_f) x'_h &= (\hat{r}_h - r_f) x'_h + \frac{\delta_h}{\sqrt{S}} x'_h \sigma_h.
\end{aligned}
\tag{23}
$$

This completes the proof.

C.3 Proof of Theorem 2.3

The assumption that the optimal solution of robust MtC model (6) are positive implies that the optimal solutions of (7), i.e. $x_h^{'*}$ and $x_f^{'*}$, are also positive. The robust MtC model (7) is convex and satisfy Slater regularity condition, therefore the Karush-Kuhn-Tucker (KKT) optimality conditions are the necessary and sufficient condition for optimality. Let us define $\sigma_p = {x_h^{'*}}^2\sigma_h^2 + {x_f^{'*}}^2\sigma_f^2 + 2x_h^{'*}x_f^{'*}\sigma_{hf}$. Therefore, KKT optimality conditions reduces to:

$$\hat{r}_h - r_f - \frac{\delta_h\sigma_h}{\sqrt{S}} + \lambda^*\left(-\hat{r}_h + r_f + \frac{\delta_h\sigma_h}{\sqrt{S}} + \frac{\sqrt{\alpha}}{\sigma_p\sqrt{1-\alpha}}(\sigma_h^2x_h^{'*} + \sigma_{hf}x_f^{'*})\right) = 0$$
$$\hat{r}_f - r_f - \frac{\delta_f\sigma_f}{\sqrt{S}} + \lambda^*\left(-\hat{r}_f + r_f + \frac{\delta_f\sigma_f}{\sqrt{S}} + \frac{\sqrt{\alpha}}{\sigma_p\sqrt{1-\alpha}}(\sigma_f^2x_f^{'*} + \sigma_{hf}x_h^{'*})\right) = 0,$$

where $\lambda^* > 0$ is the Lagrange multiplier. Rewriting the condition above, one can easily check

$$\frac{\lambda^*-1}{\lambda^*} = \frac{\sqrt{\alpha}}{\sigma_p\sqrt{1-\alpha}}\left(\frac{\sigma_h^2x'_h + \sigma_{hf}x'_f}{\hat{r}_h - r_f - \frac{\delta_h\sigma_h}{\sqrt{S}}}\right)$$
$$\frac{\lambda^*-1}{\lambda^*} = \frac{\sqrt{\alpha}}{\sigma_p\sqrt{1-\alpha}}\left(\frac{\sigma_f^2x'_f + \sigma_{hf}x'_h}{\hat{r}_f - r_f - \frac{\delta_f\sigma_f}{\sqrt{S}}}\right).$$

Therefore,

$$\frac{\sigma_h^2x_h^{'*} + \sigma_{hf}x_f^{'*}}{\hat{r}_h - r_f - \frac{\delta_h\sigma_h}{\sqrt{S}}} = \frac{\sigma_f^2x_f^{'*} + \sigma_{hf}x_h^{'*}}{\hat{r}_f - r_f - \frac{\delta_f\sigma_f}{\sqrt{S}}},$$

or equivalently,

$$(\hat{r}_f - r_f - \frac{\delta_f\sigma_f}{\sqrt{S}})(\sigma_h^2x_h^{'*} + \sigma_{hf}x_f^{'*}) - (\hat{r}_h - r_f - \frac{\delta_h\sigma_h}{\sqrt{S}})(\sigma_f^2x_f^{'*} + \sigma_{hf}x_h^{'*}) = 0.$$

Multiplying the above by $\sqrt{S}$ and dividing by $x_h^{'*} + x'*_f > 0$

$$(\sqrt{S}(\hat{r}_f - r_f) - \delta_f\sigma_f)(\sigma_h^2x^*{}_h + \sigma_{hf}x_f^*) - (\sqrt{S}(\hat{r}_h - r_f) - \delta_h\sigma_h)(\sigma_f^2x^*{}_f + \sigma_{hf}x_h^*) = 0.$$

where x_h^* and x_f^* now are the optimal solutions of robust MtC model (6). Replacing x_h^* with $1 - x_f^*$, and re-arranging the terms we get

$$(\sqrt{S}(\hat{r}_h - r_f) - \delta_h\sigma_h)(\sigma_f^2 - \sigma_{hf}) - (\sqrt{S}(\hat{r}_0 - r_f) - \delta_f\sigma_f)(\sigma_{hf} - \sigma_h^2)x_f^* =$$
$$\sigma_h^2(\sqrt{S}(\hat{r}_0 - r_f) - \delta_f\sigma_f) - \sigma_{hf}(\sqrt{S}(\hat{r}_h - r_f) - \delta_h\sigma_h).$$

Finally,

$$x_f^* = \frac{\sigma_h^2(\sqrt{S}(\hat{r}_0 - r_f) - \delta_f\sigma_f) - \sigma_{hf}(\sqrt{S}(\hat{r}_h - r_f) - \delta_h\sigma_h)}{(\sqrt{S}(\hat{r}_h - r_f) - \delta_h\sigma_h)(\sigma_f^2 x_h - \sigma_{hf}) - (\sqrt{S}(\hat{r}_0 - r_f) - \delta_f\sigma_f)(\sigma_{hf} - \sigma_h^2)}.$$

Defining

$$c_1 = \sqrt{S}\left[\sigma_f^2(\hat{r}_h - r_f) - \sigma_{hf}(\hat{r}_f - r_f)\right],$$

$$c_2 = \sqrt{S}\left[\sigma_h^2(\hat{r}_f - r_f) - \sigma_{hf}(\hat{r}_h - r_f)\right],$$

and

$$c_3 = \sqrt{S}\left[(\sigma_f^2 - \sigma_{oh})(\hat{r}_h - r_f) - (\sigma_{hf} - \sigma_h^2)(\hat{r}_f - r_f)\right],$$

one can easily see that

$$x_f^* = \frac{c_2 + \delta_h\sigma_h\sigma_{hf} - \delta_f\sigma_f\sigma_h^2}{c_3 + \delta_f(\sigma_{oh}\sigma_f - \sigma_o\sigma_h^2) - \delta_h(\sigma_h\sigma_f^2 - \sigma_h\sigma_{hf})}.$$

and

$$x_h^* = 1 - x_f^* = \frac{c_1 + \delta_f\sigma_f\sigma_{hf} - \delta_h\sigma_h\sigma_f^2}{c_3 + \delta_f(\sigma_h\sigma_f^2 - \sigma_o\sigma_h^2) - \delta_h(\sigma_h\sigma_f^2 - \sigma_h\sigma_{hf})}.$$

This completes the proof.

D Interval ambiguity extension

In this section, we develop the robust MtC model under ambiguity in distribution, means, and covariance matrix. We first construct interval ambiguity sets as in El Ghaoui et al. (2003) and derive the robust counterpart of MtC maximization model (3). In particular, we assume that the probability distribution of returns are ambiguous, and further the mean values and the covariance matrix of returns are only known within some interval ambiguity sets.

Definition D.1 (Interval ambiguity for mean returns and covariance matrix). (El Ghaoui et al., 2003)
Mean returns and covariance matrix belong to the following interval set:

$$U_I = \{(\bar{r}, \Sigma) \in \mathbb{R}^n \times \mathbb{S}^n \mid \bar{r}_- \leq \bar{r} \leq \bar{r}_+,\, \Sigma_- \leq \Sigma \leq \Sigma_+\},$$

where $\bar{r}_-$, $\bar{r}_+$, Σ_-, Σ_+ *are given vectors and matrices, and the inequalities are component-wise, and* $\mathbb{S}^n$ *denotes the cone of positive semi-definite matrices. We assume there is at least one* $(\bar{r}, \Sigma) \in U_I$ *for which* $\Sigma \succeq 0$.

We consider the robust counterpart of MtC model with respect to ambiguity set U_I as follows:

$$\max_{x \in \mathbb{X}} \min_{(\bar{r}, \Sigma) \in U_I} \frac{\mathbb{E}(\tilde{r}_p - r_f)}{\mathrm{CVaR}_\alpha(\tilde{r}_p - r_f)}. \tag{24}$$

The following theorem obtains the SOCP formulation of robust MtC model.

Theorem D.0. *Assuming positive worst-case CVaR on excess returns of the optimal portfolio of robust MtC maximization model (24) with feasible set* $\mathbb{X}$ *is positive, then the robust MtC portfolio optimization model can be cast as follows:*

$$\begin{aligned}
\max_{v'_+, v'_- \in \mathbb{R}^n,\, v \in \mathbb{R},\, \Lambda, \Lambda_+, \Lambda_- \in \mathbb{S}^n} \quad & (\bar{r}_- - r_f e)^\top v'_- - (\bar{r}_+ - r_f e)^\top v'_+ \qquad (25)\\
s.t. \quad & \\
& (\bar{r}_+ - r_f e)^\top v'_+ - (\bar{r}_- - r_f e)^\top v'_- + \\
& \frac{\alpha}{1-\alpha} v + tr(\Lambda_+ \Sigma_+) - tr(\Lambda_- \Sigma_-) + \leq 1 \\
& \begin{bmatrix} \Lambda & \frac{v'_- - v'_+}{2} \\ \frac{(v'_- - v'_+)}{2}^\top & v \end{bmatrix} \succeq 0 \\
& \Lambda \preceq \Lambda_+ - \Lambda_- \\
& e^\top (v'_- - v'_+) > 0 \\
& v'_+, v'_- \geq 0,\, \Lambda,\, \Lambda_+,\, \Lambda_- \succeq 0,
\end{aligned}$$

where tr *and* $\bar{r}$ *shows the trace operator (defined as the sum of diagonal elements of* *a matrix) and vector of risky assets mean returns, respectively. Given $v_+'^*$ and $v_-'^*$, the optimal solutions of (24), the optimal solution of robust MtC portfolio optimization can be obtained as $x^* = \frac{1}{e^\top v'*} v'*$ where $v'* = v_-'^* - v_+'^*$.*

Proof. Similar to the proof of Theorem (2.1) in Online Appendix C.2, we can write the maximum MtC ratio maximization model as:

$$\begin{aligned}
\max_{x' \in \mathbb{R}^n_+} \quad & (\mathbb{E}(\tilde{r}) - r_f e)^\top x' \\
s.t. \quad & \\
& \mathrm{CVaR}_\alpha((\bar{r} - r_f e)^\top x') \leq 1 \\
& e^\top x' > 0,
\end{aligned} \tag{26}$$

where $x' = \frac{x}{\xi}$ and $\nu = \frac{1}{\xi}$. Therefore the robust counterpart of maximum MtC is as follows:

$$\begin{aligned}
\max_{x' \in \mathbb{R}^n, \nu \in \mathbb{R}} \quad & \min_{(\bar{r}, \Sigma) \in U_I,\, \pi \in \mathbb{D}} (\bar{r} - r_f e)^\top x' \\
s.t. \quad & \\
& \max_{(\bar{r}, \Sigma) \in U_I,\, \pi \in \mathbb{D}} \mathrm{CVaR}_\alpha((\bar{r} - r_f e)^\top x') \leq 1 \\
& e^\top x' > 0 \\
& x' \geq 0, .
\end{aligned} \tag{27}$$

Using the representation of CVaR *in Theorem C.1 of Online Appendix C.1, one can write the above as:*

$$\begin{aligned}
\max_{x' \in \mathbb{R}^n, \nu, \gamma \in \mathbb{R}} \quad & \min_{(\bar{r}, \Sigma) \in U_I,\, \pi \in \mathbb{D}} (\bar{r} - r_f e)^\top x' \\
s.t. \quad & \\
& \max_{(\bar{r}, \Sigma) \in U_I,\, \pi \in \mathbb{D}} F_\alpha((\tilde{r} - r_f e)^\top x', \gamma) \leq 1 \\
& e^\top x' > 0 \\
& x' \geq 0, .
\end{aligned} \tag{28}$$

The above is equivalent to:

$$\begin{aligned}
\max_{x' \in \mathbb{R}^n,\, \nu \in \mathbb{R}} \quad & \min_{(\bar{r}, \Sigma) \in U_I} (\bar{r} - r_f e)^\top x' \\
s.t. \quad & \\
& \max_{(\bar{r}, \Sigma) \in U_I} -(\bar{r} - r_f e)^\top x' + \frac{\sqrt{\alpha}}{\sqrt{1-\alpha}} \sqrt{x'^\top \Sigma x'} \leq 1 \\
& e^\top x' > 0 \\
& x' \geq 0, .
\end{aligned} \tag{29}$$

(see Proposition 1 of (Lotfi and Zenios, 2018)). Appendix A4. of the same paper allow us to reformulate the inner minimization in the objective function and the maximization appeared in the first constraint and get the following formulation.

$$
\begin{aligned}
\max_{v'_+, v'_- \in \mathbb{R}^n,\, \upsilon, \nu \in \mathbb{R},\, \Lambda, \Lambda_+, \Lambda_- \in \mathbb{S}^n} \quad & (\bar{r}_- - r_f e)^\top v'_- - (\bar{r}_+ - r_f e)^\top v'_+ \\
s.t. \quad & \\
& (\bar{r}_+ - r_f e)^\top v'_+ - (\bar{r}_- - r_f e)^\top v'_- + \\
& \frac{\alpha}{1-\alpha} \upsilon + tr(\Lambda_+ \Sigma_+) - tr(\Lambda_- \Sigma_-) + \leq 1 \\
& \begin{bmatrix} \Lambda & \frac{v'_- - v'_+}{2} \\ \frac{(v'_- - v'_+)}{2}^\top & \upsilon \end{bmatrix} \succeq 0 \\
& \Lambda \preceq \Lambda_+ - \Lambda_- \\
& e^\top (v'_- - v'_+) > 0 \\
& v'_+, v'_- \geq 0,\, \Lambda,\, \Lambda_+,\, \Lambda_- \succeq 0.
\end{aligned}
\tag{30}
$$

This completes the proof.