Get Out or Get Down: Rival Options in a Declining Market

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Highlights:

Model Mutually Exclusive Options for Duopoly Divest and Switch

Comparison of Separate and Joint Formulations

Analytical Solutions for Some Real Rival Options

Evaluate Importance of Rival Options as Revenue Changes

Interesting Sensitivities for Rival Options

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Abstract

We formulate a multi-factor real option duopoly game model to determine the optimal times to divest the incumbent technology or to switch to a new smaller-scale and lower operating cost technology, with an uncertain output price, and declining output. The formulation takes two alternative forms: (i) the divest and switch options are treated separately (separate) and (ii) the two options are mutually-exclusive (joint). Although the first-mover has a salvage value advantage, the second-mover has a temporary market share advantage. The alternative forms yield significantly different outcomes: the thresholds are all lower under the separate formulation, and hysteresis is greater. When getting out (divest) or getting down, watch the competition and sometimes consider the options jointly.

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Introduction

We evaluate a real option game in the context of three significant changes: (i) market share changes arising from the individual player actions, (ii) revenue changes due to a declining market size and a stochastically evolving price, and (iii) net revenue changes arising from investing in an alternative technology having a more appropriate cost structure[2].

In markets where output prices are uncertain and output demand is declining, firms often consider simultaneously the option to switch to a new smaller-scale technology, benefitting thereafter from lower operating costs, and the option to divest. In addition, it is assumed that at the moment the firm evaluates the switch/divest problem, the random state variable (e.g., revenue) is above the switch threshold, otherwise it would trigger an immediate switch. However, Décamps et al. (2006) asked an interesting question: how should firms behave if suddenly the state variable is between the divest and switch thresholds? They show that if the switch and the divest thresholds are derived separately, the former threshold is lower than the latter, and if (by chance) the state variable is in this middle region, both the option to switch and the option to divest must coexist. In this scenario, if the state variable increases sufficiently, it will trigger the switch, whereas if it decreases enough, it will trigger the divest.

We note that downscaling during pandemics, change in fashion or technology, or conventional usage patterns, may well inspire first movers to switch technology. But who wants to be first, adapting to temporary client inertia regarding lower-cost operations (online vs on-campus education)? Other contexts are firms, industry or countries facing stagnation or revenue decline, due to natural factors such as in petroleum production, and economic or structural factors, where possible new alternative technologies may validate delaying exit-abandonment, or switching to lower cost production. Due to both pollution concerns and competition from natural gas, coal almost everywhere is being shut down, possibly awaiting cheaper emission control. Book shops and shopping malls in the US (Borders, and Barnes & Noble) are being closed, or converted to alternative uses (cafes and reading rooms, rather than book selling). Ceramics and textiles in developed countries faced closure or downsizing. Taxis,

[2] Many other configurations of market shares, salvage values, and revenue and operating cost changes can be designed, some suitable for our specific model, others requiring model redesign, appropriate for future research.

accommodation, and universities are experiencing competition from mobile-digital technologies.

Dias (2004) first raised the mutually exclusive option problem, and provided solutions using finite differences. Décamps et al. (2006) study irreversible investments in alternative projects and show that when firms hold the option to switch from a smaller scale to a larger scale project, a hysteresis region between the investment region can persist even if the uncertainty of the output price increases. Bobtcheff and Villeneuve (2010) examine investments in two mutually exclusive projects with two sources of uncertainty, and conclude that when these uncertainties hold simultaneously, the project payoffs are not sufficient criteria for deciding on the investment timing. Kwon (2010) looks at a declining profit stream following an arithmetic Brownian motion process, so the exit threshold decreases as volatility increases. Siddiqui and Fleten (2010) implement the Décamps et al. (2006) model for mutually exclusive projects with an unusual solution.

Adkins and Paxson (2011) investigate optimal capital replacement and abandonment decisions considering that both revenues and costs are uncertain and their value declines over time. Chronopoulos and Siddiqui (2015) study the timing of the replacement of an incumbent technology, assuming that there is technological uncertainty, and the ex-post revenues which the adoption of the new technology generate are uncertain. This investment analysis is examined under three different strategies, compulsive, laggard, and leapfrog. Their results reveal that, under the compulsive strategy, technological uncertainty has a non-monotonic impact on the optimal investment decision. There are applications of the theory of mutually exclusive options, such as Bakke et al. (2016), and of real competitive strategies, such as Comincioll et al. (2020), but apparently not joint competitive strategies.

Hagspiel et al. (2016) look at investment decisions in a new technology under uncertainty in profit declining markets, where firms hold the option to invest in a new technology with which they produce a new product, holding the option to exit the market and considering that the firms also decide on the capacity size. Among other findings, they show that a higher potential profitability of the new product market accelerates the investment timing, but the capacity choice can alter this result, reversing the above intuitive result, if the choice of the investment capacity is smaller.

Støre et al. (2018) study an irreversible switch from oil to gas production, with both oil and gas production declining over time. They provide analytical solutions for the switching threshold and the real option value of the switching opportunity. Huberts et al. (2019) show that entry may be deterred, possibly in a war of attrition or pre-emption, following interesting strategies. Adkins and Paxson (2019) study the appropriate rescaling for a monopoly from an incumbent large-scale technology assuming that market revenue is declining. They also consider the case of abandonment and treat the two investments both separately and jointly, showing different implications for government policies.

Several authors focus on the uncertainty of new technologies, which should provide interesting extensions of our current approach. Farzin et al. (1998) assume both the speed of arrival and degree of improvement of future technologies are uncertain. Doraszelski (2004) allows for future technologies with improvements. Hagspiel et al. (2015) also consider changing arrival rates for new technologies.

We have set up a context where the first-mover advantage is small, dependent on only obtaining full salvage value, so some of the option values and thresholds are very sensitive to small changes in the ex-post "market share". These market sharing assumptions constitute quasi-pre-emptive games, where the second-mover is not immediately motivated to adopt the cost reduction technology in the second stage (or perhaps not motivated because of the alternative temporary larger market share, maybe a management delusion). Eventually, the second-mover is allowed to adopt the new technology (but with an equal market share). Lieberman and Montgomery (1988) focus on technological leadership (which we adopt), pre-emption of scarce assets, and customer switching costs. Joaquin and Butler (2000) consider the first mover advantage of lower operating costs. Tsekrekos (2003) suggests both temporary and pre-emptive permanent market share advantages for the leader in a sequential investment pattern. Paxson and Pinto (2003) model a leader with an initial market share advantage, which then evolves as new customers arrive (birth) and existing customers depart (death). Paxson and Melmane (2009) provide a two-factor model where the leader starts with a larger market share, applied to show that (by foresight) Google was likely to be undervalued compared to Yahoo at the Google IPO. Bobtcheff and Mariotti (2010) consider a pre-emptive game of two innovative competitors, whose existence may be revealed only by first mover investment. See Azevedo and Paxson (2014) for a review of the literature on developing real option games.

The rest of the paper is organized as follows. Section 2 presents both the divestment and the switching models for a separate formulation and a sensitivity analysis. Section 3 derives the divestment and the switching models for a joint formulation, and presents a sensitivity analysis. Section 4 compares the separate and joint values, and option characteristics. Section 5 concludes the work and provides some suggestions for further research.

Section 2

We consider a duopoly market with two active and ex-ante symmetric rationale firms [holding the same parameter values] operating with an incumbent high operating cost technology, referred to as policy X, producing the same product output with a market price $p(t)$ subject to uncertainty and facing a declining market volume $q(t)$. Due to the inevitable erosion of viability, each firm holds the option to abandon production and receive a salvage value from the incumbent X stage[3]. A first-mover divestment advantage exists such that first-mover receives the full amount Z while the second-mover receives only the partial amount λZ where $0 \le \lambda < 1$. Once the divestment option is exercised, the firm exits the market which is referred to as policy O. Alternatively, each firm can switch while operating X to a more appropriate lower operating cost technology referred to as policy Y, but incurs a positive investment cost denoted by K. Since Y is the more cost efficient, if we denote the full-market operating cost by $f_J, J \in \{X, Y\}$ then $f_X \gg f_Y$. The eroding viability inevitably motivates the two players to adopt at some future time one of the two policies, O or Y. Either they will have to "get-out" by exiting the business completely, or to "get-down" by switching to policy Y with its more efficient cost structure.

The two players in the duopoly game are designated the leader and the follower, referred to as L and F, respectively. This implies that the leader is always first to enact a policy change from X to either O or Y, and that the follower always enacts the identical policy change as the leader but subsequently. The net revenue for each player is determined from their respective market share, which is denoted by $D_{I|J_1,J_2}$ with $I \in \{L, F\}$ and $J_1, J_2 \in \{O, X, Y\}$ where J_1

[3] Salvage values from spending K (or K1, K2, etc.), and multiple stages, complicate this simple analysis, and are not considered.

represents the current policy pursued by the leader and J_2 that by the follower. So, $D_{F|Y,X}$ is to be interpreted as the market share of the follower given that the leader is pursuing policy Y and the follower policy X. Since their market shares sum to one, $D_{L|J_1,J_2} + D_{F|J_1,J_2} = 1$, for all J_1 and all J_2. Clearly, if the leader exits the market, then $D_{L|O,X} = 0$. We treat the two firms as being ex-ante symmetric, which implies that each firm has 50% of the market provided that the two firms are pursuing identical policies, so:

$$D_{L|X,X} = D_{F|X,X} = D_{L|Y,Y} = D_{F|Y,Y} = 0.5 .$$

Further, the net revenue for the firms is given by:

$$\begin{aligned} &D_{I|J_1,J_2} \times P(t)q(t) - D_{I|J_1,J_2} \times f_{J_1} \quad \text{if } I = L, \\ &D_{I|J_1,J_2} \times P(t)q(t) - D_{I|J_1,J_2} \times f_{J_2} \quad \text{if } I = F. \end{aligned}$$

If the two firms are pursuing the same policy, then their net revenues are identical. The net revenue for a firm is zero once it has exited the market.

We assume the market price p follows a geometric Brownian motion (gBm) process described by:

$$dp = \alpha p dt + \sigma p \mathrm{d}W ,$$

where α is the constant instantaneous conditional expected price change per unit of time, σ is its constant instantaneous conditional standard deviation per unit of time, and $\mathrm{d}W$ is the increment of a standard Wiener process. For convergence purposes $\delta = r - \alpha > 0$, where r is the riskless interest rate and δ the convenience yield. The market volume flow q is described by:

$$dq = -\theta q dt$$

where $\theta > 0$ denotes a known constant market depletion rate. Under risk-neutrality and using Ito's lemma, the firm value G satisfies the differential equation:

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 G}{\partial p^2} + (r - \delta) p \frac{\partial G}{\partial p} - \theta q \frac{\partial G}{\partial q} + D(pq - f) - rG = 0, \tag{1}$$

where for convenience we have ignored the various subscripts; subsequently we particularise the various solutions to (1) when we consider the individual cases. Because of the similarity principle, Paxson and Pinto (2005), we can replace the composite term for revenue by the single variable $v = pq$, yielding the solution to (1) as:

$$G(v) = A_1 v^{\beta_1} + A_2 v^{\beta_2} + \frac{Dv}{\delta+\theta} - \frac{Df}{r} \quad \text{for } 0 \le v \le v(0), \tag{2}$$

where:

$$\beta_{1,2} = \left(\frac{1}{2} - \frac{r-\delta-\theta}{\sigma^2}\right) \pm \sqrt{\left(\frac{1}{2} - \frac{r-\delta-\theta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \quad \beta_1 > 1, \beta_2 < 0, \tag{3}$$

$A_1, A_2 \ge 0$ are two unknowns to be determined from the context, and at $t = 0$ the prevailing revenue is defined as $v(0)$. In the absence of any optionality or value change due to the other player's action, $G(v) = Dv/(\delta+\theta) - Df/r$.

We examine in the following sections the two comparative aspects of the duopoly game. First, we consider the aspect that treats the firm decision of selecting between policy O and Y as being separate and independent by adapting the formulation proposed by Dixit (1993). This is referred to as the "separate" formulation; variables and functions associated with this formulation are labelled with the subscript I. Since this formulation investigates the impact of declining values in revenue on the policy values and the decision thresholds for making a policy change, only declining revenues are relevant in this analysis so in (2) A_2 is non-negative but A_1 is set equal to zero. We then consider the second aspect that treats the selection as being "joint" and dependent by adapting the formulation proposed by Décamps et al. (2006). This is referred to as the "joint" formulation; variables and functions associated with this formulation are labelled with the subscript II. Since this formulation investigates the impact of both increasing and decreasing revenue values on the policy values and the decision thresholds for making a policy change, both negative and positive revenue changes are relevant so in (2) both $A_1 A_2$ are non-negative. By comparing these two aspects of the duopoly game, we can investigate their key similarities and differences and explain the way they shape policy selection for a firm in a declining market.

The separate and joint formulations are assessed by comparing their analytical findings. When this is not possible, we use numerical evaluations using base case parameter values that are exhibited in Table 1. The values of β_1, β_2 for the base case are $1.667, -1.333$, respectively.

Table 1: Base Case Parameter Values

DEFINITION	NOTATION	VALUE
Risk-free rate	r	0.10
Convenience yield	δ	0.03
Market depletion rate	θ	0.04
Market price volatility	σ	0.30
Follower's divestment proportion	λ	0.40
Unadjusted periodic operating cost for policy X	f_X	10.0
Unadjusted periodic operating cost for policy Y	f_Y	1.0
Divestment value	Z	25.0
Switching investment cost to policy Y	K	32.0
Leader's market share given both leader and follower pursue policy X	$D_{L\|X,X}$	0.50
Leader's market share given both leader and follower pursue policy Y	$D_{L\|Y,Y}$	0.50
Leader's market share given leader pursues policy Y and follower policy X	$D_{L\|Y,X}$	0.40
Leader's market share given leader exits and follower pursues policy X	$D_{L\|O,X}$	0.00

Note: The follower's market shares for the various policy assortments are obtainable from the leader's market share.

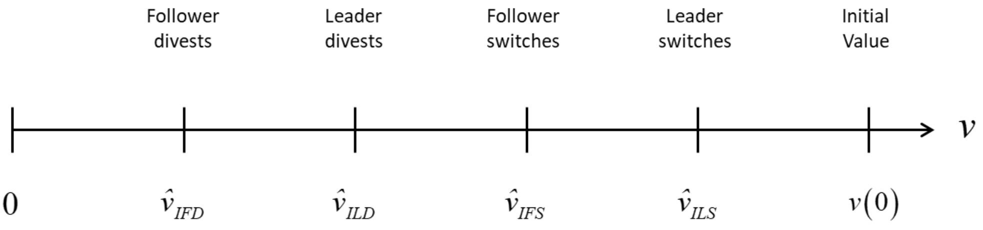
Separate Formulation

In our duopoly game, we implicitly assume that the switching threshold for each player is greater than the divestment threshold, and that whichever of the two strategies is selected, the leader always moves before the follower. If we denote by $\hat{v}_{ILS}, \hat{v}_{IFS}$ the switching thresholds for the leader and follower, respectively, and by $\hat{v}_{ILD}, \hat{v}_{IFD}$ the divestment thresholds for the leader and follower, respectively, then the assumption on thresholds can be characterized as:

$$\hat{v}_{IFS} < \hat{v}_{ILS} \le v(0), \hat{v}_{IFD} < \hat{v}_{ILD} \le v(0). \tag{4}$$

This is illustrated in Figure 1, which shows that as revenue declines the leader switches and divests ahead of the follower.

Figure 1
Leader and Follower Thresholds for a Randomly Declining Revenue v
Under the Separate Formulation



Divestment

The value function for the leader deliberating divestment is denoted by $V_{ILD}(v)$ and is derived from (2):

$$V_{ILD}(v)=\begin{cases} D_{L|X,X}\dfrac{v}{\delta+\theta}-D_{L|X,X}\dfrac{f_X}{r}+A_{2ILD}v^{\beta_2} & \text{for } \hat{v}_{ILD}<v\le v(0), \\ Z \quad \text{for } v\le \hat{v}_{ILD}. \end{cases} \tag{5}$$

In (5), the first line represents the expected present value of leader's net revenue plus the option value to divest, $A_{2ILD}v^{\beta_2}$ with $A_{2ILD}>0$; the second line represents the full divestment value. The two unknowns, $\hat{v}_{ILD}, A_{2ILD}$ are obtained from the value-matching relationship and associated smooth-pasting condition to yield[4]:

$$\left.\begin{aligned} \hat{v}_{ILD} &= \frac{\beta_2}{\beta_2-1}\frac{\delta+\theta}{r}\frac{D_{L|X,X}f_X+rZ}{D_{L|X,X}}, \\ A_{2ILD} &= -\frac{D_{L|X,X}\hat{v}_{ILD}^{1-\beta_2}}{\beta_2(\delta+\theta)}. \end{aligned}\right\} \tag{6}$$

The value function for the follower deliberating divestment is denoted by $V_{IFD}(v)$ and is derived from (2):

[4] Full derivations for the solutions for the separate formulation are available in the Supplementary Appendix A.

$$V_{IFD}(v)=\begin{cases} D_{F|X,X}\dfrac{v}{\delta+\theta}-D_{F|X,X}\dfrac{f_X}{r}+A_{2IFDD}v^{\beta_2} \quad \text{for } \hat{v}_{ILD}<v\le v(0), \\ D_{F|O,X}\dfrac{v}{\delta+\theta}-D_{F|O,X}\dfrac{f_X}{r}+A_{2IFD}v^{\beta_2} \quad \text{for } \hat{v}_{IFD}<v\le \hat{v}_{ILD}, \\ \lambda Z \quad \text{for } v\le \hat{v}_{IFD}. \end{cases} \tag{7}$$

In (7), the first line represents the expected present value of follower's net revenue plus the present value accruing to the follower when the leader exits the market, denoted by $A_{2IFDD}v^{\beta_2}$ (note that this present value includes both the net revenue change and the divest option value)[5]; the second line represents the expected present value of follower's net revenue plus the option value to divest, $A_{2IFD}v^{\beta_2}$ with $A_{2IFD}>0$; the third line represents the follower's partial divestment value. The two unknowns, $\hat{v}_{IFD}, A_{2IFD}$ are obtained from the value-matching relationship evaluated at $v=\hat{v}_{IFD}$ and associated smooth-pasting condition, and the unknown A_{2IFDD} from the value-matching relationship evaluated at $v=\hat{v}_{ILD}$, to yield:

$$\left.\begin{aligned} \hat{v}_{IFD} &= \frac{\beta_2}{\beta_2-1}\frac{\delta+\theta}{r}\frac{D_{F|O,X}f_X+r\lambda Z}{D_{F|O,X}}, \\ A_{2IFD} &= -\frac{D_{F|O,X}\hat{v}_{IFD}^{1-\beta_2}}{\beta_2(\delta+\theta)}, \\ A_{2IFDD} &= A_{2IFD}+\left(D_{F|O,X}-D_{F|X,X}\right)\left(\frac{\hat{v}_{ILD}}{\delta+\theta}-\frac{f_X}{r}\right)\hat{v}_{ILD}^{-\beta_2}. \end{aligned}\right\} \tag{8}$$

Since the follower captures the full market as soon as the leader exits, $D_{F|O,X}>D_{L|X,X}$, so $A_{2IFDD}>0$ and the follower gains value at $v=\hat{v}_{ILD}$. It can be shown that by comparing (6) with (8) the leader is always first to divest because $\hat{v}_{ILD}>\hat{v}_{IFD}$, since $D_{F|O,X}>D_{L|X,X}$ and $\lambda<1$.

[5] Since the A_{2IFDD} is exercised by the rival follower, we also refer to this as a Rival Option RO FDD, similarly for the other Rival Options.

Switching

The value function for the leader deliberating switching is denoted by $V_{ILS}(v)$ and is derived from (2):

$$V_{ILS}(v)=\begin{cases} D_{L|X,X}\dfrac{v}{\delta+\theta}-D_{L|X,X}\dfrac{f_X}{r}+A_{2ILS}v^{\beta_2} & \text{for } \hat{v}_{ILS}<v\le v(0), \\ D_{L|Y,X}\dfrac{v}{\delta+\theta}-D_{L|Y,X}\dfrac{f_Y}{r}+A_{2ILSS}v^{\beta_2} & \text{for } \hat{v}_{IFS}<v\le \hat{v}_{ILS}, \\ D_{L|Y,Y}\dfrac{v}{\delta+\theta}-D_{L|Y,Y}\dfrac{f_Y}{r} & \text{for } v\le \hat{v}_{IFS}. \end{cases} \quad (9)$$

In (9), the first line represents the expected present value of leader's net revenue plus the option value to switch, $A_{2ILS}v^{\beta_2}$ with $A_{2ILS}>0$; the second line represents the expected present value of leader's net revenue plus the present value accruing to the leader when the follower switches, denoted by $A_{2ILSS}v^{\beta_2}$; the third line represents the expected present value of leader's net revenue once the follower has switched.

The value function $V_{IFS}(v)$ for the follower deliberating switching is:

$$V_{IFS}(v)=\begin{cases} D_{F|X,X}\dfrac{v}{\delta+\theta}-D_{F|X,X}\dfrac{f_X}{r}+A_{2IFSS}v^{\beta_2} & \text{for } \hat{v}_{ILS}<v\le v(0), \\ D_{F|Y,X}\dfrac{v}{\delta+\theta}-D_{F|Y,X}\dfrac{f_X}{r}+A_{2IFS}v^{\beta_2} & \text{for } \hat{v}_{IFS}<v\le \hat{v}_{ILS}, \\ D_{F|Y,Y}\dfrac{v}{\delta+\theta}-D_{F|Y,Y}\dfrac{f_Y}{r} & \text{for } v\le \hat{v}_{IFS}. \end{cases} \quad (10)$$

In (10), the first line derived from (2) represents the expected present value of follower's net revenue plus the present value accruing to the follower when the leader switches, denoted by $A_{2IFSS}v^{\beta_2}$ (note that this present value includes both the net revenue change and the switch option value); the second line derived from (2) represents the expected present value of follower's net revenue plus the option value to switch, $A_{2IFS}v^{\beta_2}$ with $A_{2IFS}>0$; the third line represents expected present value of follower's net revenue once both players have switched.

We first consider from (10) the follower's value-matching relationship at the follower's switching threshold, $v=\hat{v}_{IFS}$:

$$D_{F|Y,X}\frac{v}{\delta+\theta}-D_{F|Y,X}\frac{f_X}{r}+A_{2IFS}v^{\beta_2}=D_{F|Y,Y}\frac{v}{\delta+\theta}-D_{F|Y,Y}\frac{f_Y}{r}-(K-\lambda Z). \quad (11)$$

In (11), although paying the full investment cost the follower only receives a partial divestment receipt because of the second-mover disadvantage. The two unknowns, $\hat{v}_{IFS}, A_{2IFS}$, can be obtained from (11) and its associated smooth-pasting condition to yield:

$$\left.\begin{aligned} \hat{v}_{IFS} &= \frac{\beta_2}{\beta_2 - 1}\frac{\delta+\theta}{r}\frac{D_{F|Y,X}\, f_X - D_{F|Y,Y}\, f_Y - rK + r\lambda Z}{D_{F|Y,X} - D_{F|Y,Y}}, \\ A_{2IFS} &= -\frac{\left(D_{F|Y,X} - D_{F|Y,Y}\right)\hat{v}_{IFS}^{1-\beta_2}}{\beta_2\left(\delta+\theta\right)}. \end{aligned}\right\} \tag{12}$$

In (12), the follower's switching threshold, $\hat{v}_{IFS}$, adopts a standard form and is determined from the net gain arising from adopting policy Y by surrendering X, the difference between the present value of the fixed cost differential and the net investment cost. The threshold also imposes the conditions on the follower's market shares $D_{F|Y,X} > D_{F|Y,Y}$, and on the relative fixed cost values, switching cost and divestment value.

Next, we consider from (9) the leader's value-matching relationship at the follower's switching threshold, $v = \hat{v}_{IFS}$:
$$D_{L|Y,X}\frac{\hat{v}_{IFS}}{\delta+\theta} - D_{L|Y,X}\frac{f_Y}{r} + A_{2ILSS}\,\hat{v}_{IFS}^{\beta_2} = D_{L|Y,Y}\hat{v}_{IFS} - D_{L|Y,Y}\frac{f_Y}{r}, \tag{13}$$

to derive the coefficient A_{2ILSS}:

$$A_{2ILSS} = \left(D_{L|Y,Y} - D_{L|Y,X}\right)\left(\frac{\hat{v}_{IFS}}{\delta+\theta} - \frac{f_Y}{r}\right)\hat{v}_{IFS}^{-\beta_2}. \tag{14}$$

From (9), the switching threshold $\hat{v}_{ILS}$ and option coefficient A_{2ILS} is obtained from the leader's value-matching relationship at the leader's switching threshold:

$$D_{L|X,X}\frac{v}{\delta+\theta} - D_{L|X,X}\frac{f_X}{r} + A_{2ILS}\,v^{\beta_2} = D_{L|Y,X}\frac{v}{\delta+\theta} - D_{L|Y,X}\frac{f_Y}{r} + A_{2ILSS}\,v^{\beta_2} - \left(K - Z\right), \tag{15}$$

and its associated smooth-pasting condition to yield:

$$\left.\begin{aligned} \hat{v}_{ILS} &= \frac{\beta_2}{\beta_2 - 1}\frac{\delta+\theta}{r}\frac{D_{L|X,X}\, f_X - D_{L|Y,X}\, f_Y - rK + rZ}{D_{L|X,X} - D_{L|Y,X}}, \\ A_{2ILS} &= A_{2ILSS} - \frac{\hat{v}_{ILS}\left(D_{L|X,X} - D_{L|Y,X}\right)}{\beta_2\left(\delta+\theta\right)}\hat{v}_{ILS}^{-\beta 2}. \end{aligned}\right\} \tag{16}$$

In (16), the leader's switching threshold, $\hat{v}_{ILS}$, similarly adopts the standard form and is determined from the net gain arising from adopting policy Y by surrendering X, the difference between the present value of the fixed operating cost differential and the net investment cost. The threshold also imposes the conditions on the leader's market shares $D_{L|X,X} > D_{L|Y,X}$, and on the relative fixed cost values, switching cost and divestment value. Also, the leader's switching option coefficient, A_{2ILS}, is the difference between two positive elements: the first relates to the present value accruing to the leader when the follower exercises their switching option, the second relates to the leader's switching option.

Finally, we derive the follower's coefficient A_{2FSS} by considering the follower's value-matching relationships at the leader's switching threshold $\hat{v}_{ILS}$. From (10):

$$D_{F|X,X}\frac{\hat{v}_{ILS}}{\delta+\theta} - D_{F|X,X}\frac{f_X}{r} + A_{2IFSS}\,\hat{v}_{ILS}^{\beta_2} = D_{F|Y,X}\frac{\hat{v}_{ILS}}{\delta+\theta} - D_{F|Y,X}\frac{f_X}{r} + A_{2IFS}\,\hat{v}_{ILS}^{\beta_2},$$

so: $$A_{2IFSS} = A_{2IFS} + \left(D_{F|Y,X} - D_{F|X,X}\right)\left(\frac{\hat{v}_{ILS}}{\delta+\theta} - \frac{f_X}{r}\right)\hat{v}_{ILS}^{-\beta_2}. \tag{17}$$

In (17), the follower's coefficient is composed of two positive elements: the first is the follower's option switching coefficient and the second relates to the present value accruing to the follower arising from a market share increase when the leader exercises its switching option[6].

Numerical Evaluations

Using the base case values in Table 1, we present the numerical solutions for the leader's and follower's various thresholds and coefficients in Table 2. This reveals that the results meet our assumptions underpinning the definitions of the leader and follower as well as the requirements as stipulated by Dixit (1993). The switch threshold exceeds the divest threshold for both leader and follower, $\hat{v}_{ILS} > \hat{v}_{ILD}$, $\hat{v}_{IFS} > \hat{v}_{IFD}$, respectively, the switch option coefficient exceeds the divest option coefficient for both leader and follower, $A_{2ILS} > A_{2ILD}$, $A_{2IFS} > A_{IFD}$, respectively, the switch and divest thresholds for the leader exceed those for the follower, $\hat{v}_{ILS} > \hat{v}_{IFS}$, $\hat{v}_{ILD} > \hat{v}_{IFD}$, respectively, and the switch and divest option coefficients for the leader exceed those for the follower, $A_{2ILS} > A_{2IFSS} > A_{2IFS}$, $A_{2ILD} > A_{IFD}$. So, in the context of a randomly

[6] All of these value functions, and solutions are consistent with the conventional stochastic discount function approach, as shown in the Supplementary Appendix C.

declining revenue, the first mover is the leader and switching is the preferred strategy. Table 2 also reveals that while the coefficients, $A_{2ILSS}, A_{2IFSS}, A_{2IFDD}$, are all positive, $A_{2IFDD} < A_{2IFS}$ that indicates that the follower's net revenue change is negative when the leader divests since the leader's net revenue at divestment is negative. The leader's and follower's net value (defined as the difference between the respective value function and the investment cost if relevant) profiles are presented in Figure 2. NPV indicates the net present value thresholds if no options are considered.

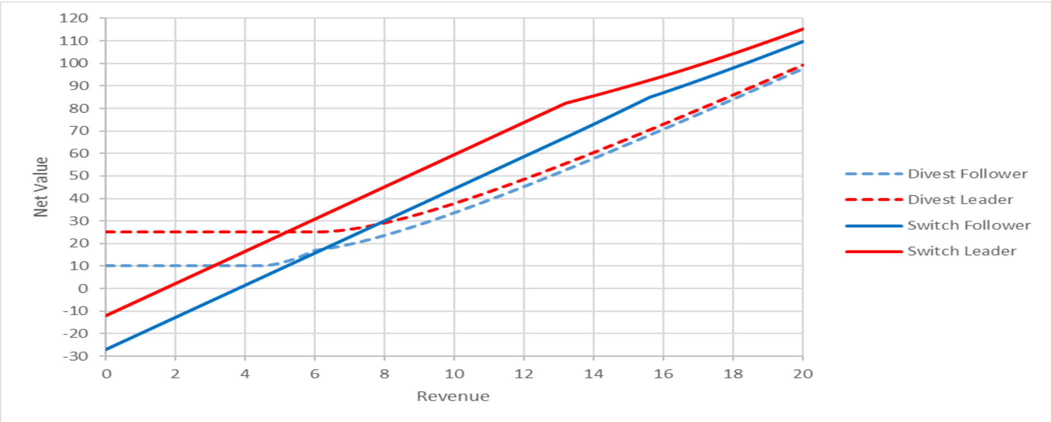
Table 2

Values for the Various Thresholds and Coefficients[7]
Under the Separate Formulation

<u>**Leader**</u>		<u>**Follower**</u>	
Divest			
$\hat{v}_{ILD}$	6.000	$\hat{v}_{IFD}$	4.400
A_{2ILD}	350.445	A_{2IFD}	339.901
NPV	10.5	A_{2IFDD}	262.025
		NPV	7.7
Switch			
$\hat{v}_{ILS}$	15.600	$\hat{v}_{IFS}$	13.200
A_{2ILS}	1208.580	A_{2IFS}	441.200
A_{2ILSS}	557.071	A_{2IFSS}	920.087
NPV	27.3	NPV	23.1

Figure 2

Values for the Follower and Leader as a Function of Revenue (v)



[7] As shown in the Supplementary Appendix Figure E3, A_{2IFD} is important in the lower v range, while A_{2IFDD} is significant around the medium v range. Other significant option values are noted in Figures E4-E5-E6, and E1-E2 for the joint formulation.

Sensitivity Analysis

In Table 3 we present the percentage change in the thresholds and coefficients for the separate model due to a 1% increase in the various parameter values. The results are generally as expected for the usual divest/switch options. Notable points are regarding λ and σ. Changes in λ only affect the follower's divestment value and not the leader's divestment decision nor their switching threshold, but positive increments have positive influence on the leader's switching coefficients because of the follower switching earlier. A 1% volatility increase postpones both the switch and divest for both the leader and the follower owing to the fall in their respective thresholds while making the incumbent appear to be more desirable. While all of the option coefficients decline with increased volatility, the divest option values increase and the switch option values decreases.

The sensitivity analysis for the players' market shares naturally excludes that for the follower after the leader exits the business. When considering the analysis for the other market shares, an absolute increase in the leader's market share automatically entails an equivalent decrease in the market share of the follower. A market share increase for the leader when both the leader and follower are pursing the incumbent policy $D_{L|X,X}$ makes the leader's current strategy more valuable, the follower's less valuable.

Table 3 reveals that a 1% increase in the leader's market share $D_{L|X,X}$ produces a divestment deferral and makes the divestment option more expensive due to the leader's more attractive market share. Although it reduces the present value accruing to the follower when the leader exits the market because the market share change is less, it has no effect on the follower's divestment threshold and option value, since these are determined by the follower's market share after the leader exits the market. Further, it yields for the leader a switch investment deferral and a less expensive switch option because of the greater market share loss. Again, it reduces the present value accruing to the follower when the leader switches to policy Y and has no effect on the follower's switch threshold and option value, since these are determined by the follower's market share after the leader switches. The divestment thresholds and option values are not affected by the market shares once the leader switches. An increase in the market share for the leader after switching, $D_{L|Y,X}$, is valuable not only for the leader but also for the follower. The leader gains from the market share increase reflected in an advanced switching threshold and greater option value, while the follower shares this gain but to a lesser extent through an earlier switching exercise and greater option value. Finally, a 1% increase in market share for the leader after the follower switches, $D_{L|Y,X}$, is unfavourable to the follower because of the loss of market share, which is reflected in the follower's deferred switching and a lower option value, but also to the leader to a lesser extent since the market share gain is not attained until the follower switches.

These parameter values are not subject to change without limit. As an illustration, the value of λ is constrained to ensure the leader is the first-mover and the follower the second-mover. This implies that $\hat{v}_{ILS} \geq \hat{v}_{IFS}$. From (12) and (16), the inequality condition simplifies to:

$$\lambda \leq 1 - \frac{\left(D_{F|Y,X} + D_{F|X,X} - 1\right)\left(f_X - f_Y\right)}{rZ} < 1,$$

or alternatively as:

$$D_{F|X,X} + D_{F|Y,X} \leq 1 + \frac{rZ\left(1-\lambda\right)}{f_X - f_Y}, \quad D_{L|X,X} + D_{L|Y,X} \geq 1 - \frac{rZ\left(1-\lambda\right)}{f_X - f_Y}.$$

Based on Table 1 values and the first inequality, λ can be no greater than 0.64. To make it greater requires a reduction in the follower's market share after the leader switches, $D_{F|Y,X}$, a smaller differential between the incumbent and alternative technologies fixed cost, $f_X - f_Y$, or increases in the risk-free rate, r, or divestment value, Z.

Table 3
Percentage Change in the Thresholds and Coefficients
due to a 1% Parameter Value Increase for the Separate Formulation
Panel A exhibits the % changes for the Divestment Opportunity
and Panel B for the Switch Opportunity

Panel A	Leader		Follower		
	$\hat{v}_{ILD}$	A_{2ILD}	$\hat{v}_{IFD}$	A_{2IFD}	A_{2IFDD}
r	-0.11%	1.56%	-0.35%	0.45%	1.60%
δ	0.33%	0.04%	0.33%	0.13%	0.03%

θ	0.44%	0.05%	0.44%	0.17%	0.03%
σ	-0.66%	-3.60%	-0.66%	-2.98%	-3.68%
f_X	0.67%	1.56%	0.91%	2.13%	1.60%
Z	0.33%	0.78%	0.09%	0.21%	0.74%
λ	0.00%	0.00%	0.09%	0.21%	0.28%
$D_{L\|X,X}$	-0.33%	0.22%	0.00%	0.00%	-0.76%

Note: Changes in f_Y, K, $D_{L|Y,X}$ and $D_{L|Y,Y}$ have no impact on the divestment decision.

Panel B	Leader			Follower		
	$\hat{v}_{ILS}$	A_{2ILS}	A_{2ILSS}	$\hat{v}_{IFS}$	A_{2IFS}	A_{2IFSS}
r	-0.62%	1.95%	1.86%	-1.11%	0.56%	2.17%
δ	0.33%	-0.33%	-0.42%	0.33%	-0.19%	-0.38%
θ	0.44%	-0.43%	-0.56%	0.44%	-0.26%	-0.50%
σ	-0.66%	-6.00%	-6.63%	-0.66%	-5.14%	-6.33%
f_X	1.28%	3.65%	4.40%	1.82%	4.29%	3.75%
f_Y	-0.10%	-0.32%	-0.42%	-0.15%	-0.35%	-0.34%
Z	0.64%	1.14%	0.73%	0.30%	0.71%	1.40%
K	-0.82%	-2.09%	-2.30%	-0.97%	-2.25%	-2.41%
λ	0.00%	0.33%	0.73%	0.30%	0.71%	0.34%
$D_{L\|X,X}$	-3.54%	-1.87%	0.00%	0.00%	0.00%	-3.31%
$D_{L\|Y,X}$	4.06%	4.17%	2.79%	2.90%	2.63%	5.78%
$D_{L\|Y,Y}$	0.00%	-2.87%	-6.22%	-4.62%	-5.97%	-2.86%

Section 3 Joint Formulation

A key assumption underpinning the separate formulation described in §2, adapted from the model proposed by Dixit (1993), is that the continuation region lies between the initial revenue value $v(0)$ and the first encountered policy change threshold. This continuation region, which characterizes pursuance of the incumbent policy X, is sometimes referred to as the inaction region. Dias (2004), Décamps et al. (2006) develop the innovative idea of a second inaction region lying between a firm's thresholds to switch to policy Y and to divest with policy O. Normally, we would expect a revenue decline reaching the switching threshold to trigger a policy change from X to Y. Here, we are assuming that the revenue has declined below the switching threshold either because policy Y was not known or not available at the relevant time.

The nature of the duopoly game is that the leader always commits to a policy change ahead of the follower. Further, for the current context, the switch threshold is always greater than the divestment threshold, Décamps et al. (2006). Under the joint formulation, we denote the switching thresholds for the leader and follower by $\hat{v}_{IILS}, \hat{v}_{IIFS}$, respectively, the divestment thresholds for the leader and follower by $\hat{v}_{IILD}, \hat{v}_{IIFD}$, respectively, then the thresholds' order of magnitude is accordingly:

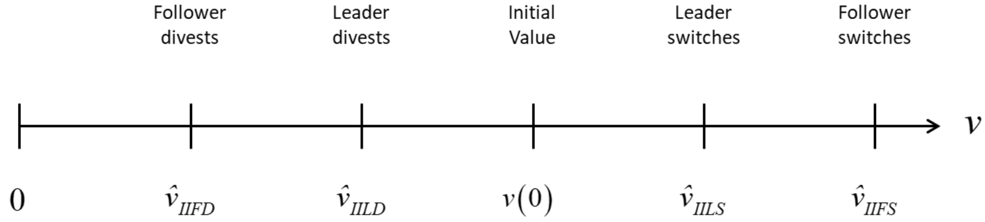
$$\hat{v}_{IIFD} < \hat{v}_{IILD} < \hat{v}_{IILS} < \hat{v}_{IIFS}.$$

Since the leader enacts a policy change ahead of the follower, the inaction region lies between the leader's thresholds to change policy from X to Y and O. Since the initial revenue value $v(0)$ lies within this inaction region, then:

$$\hat{v}_{IIFD} < \hat{v}_{IILD} < v(0) < \hat{v}_{IILS} < \hat{v}_{IIFS}. \tag{18}$$

This is illustrated in Figure 3, which shows that sufficiently high increases and decreases in revenue inevitably results in the leader exercising the option to switch to policy Y and to divest in policy O, respectively.

Figure 3
Leader and Follower Thresholds for a Randomly Declining Revenue v
Under the Joint Formulation



The value function under the joint formulation for the leader is denoted by $V_{IIL}(v)$ and is derived from (2):

$$V_{IIL}(v)=\begin{cases} D_{L|Y,Y}\dfrac{v}{\delta+\theta}-D_{L|Y,Y}\dfrac{f_Y}{r} \quad \text{for } v\geq\hat{v}_{IIFS} \\ D_{L|Y,X}\dfrac{v}{\delta+\theta}-D_{L|Y,X}\dfrac{f_Y}{r}+A_{1IILSS}v^{\beta_1} \quad \text{for } \hat{v}_{IILS}\leq v<\hat{v}_{IIFS}, \\ D_{L|X,X}\dfrac{v}{\delta+\theta}-D_{L|X,X}\dfrac{f_X}{r}+A_{1IILS}v^{\beta_1}+A_{2IILD}v^{\beta_2} \quad \text{for } \hat{v}_{IILD}<v<\hat{v}_{IILS}, \\ Z \quad \text{for } v\leq\hat{v}_{IILD}. \end{cases} \tag{19}$$

In (19), the first line represents the expected present value of leader's net revenue once the follower has switched; the second line represents the expected present value of leader's net revenue plus the present value accruing to the leader when the follower switches, denoted by $A_{1IILSS}v^{\beta_2}$; the third line represents the expected present value of leader's net revenue plus the option values to switch, $A_{1IILS}v^{\beta_1}$ with $A_{1IILS}>0$ and to divest, $A_{2IILD}v^{\beta_2}$ with $A_{2IILD}>0$; the fourth line represents the leader's receipt from divesting the incumbent policy.

The value function under the joint formulation for the follower is denoted by $V_{IIF}(v)$ and is derived from (2):

$$V_{IIF}(v)=\begin{cases} D_{F|Y,Y}\dfrac{v}{\delta+\theta}-D_{F|Y,Y}\dfrac{f_Y}{r} \quad \text{for } v\geq\hat{v}_{IIFS} \\ D_{F|Y,X}\dfrac{v}{\delta+\theta}-D_{F|Y,X}\dfrac{f_X}{r}+A_{1IIFS}v^{\beta_1}+A_{2IIFD}v^{\beta_2} \quad \text{for } \hat{v}_{IILS}\leq v<\hat{v}_{IIFS}, \\ D_{F|X,X}\dfrac{v}{\delta+\theta}-D_{F|X,X}\dfrac{f_X}{r}+A_{1IIFS}v^{\beta_1}+A_{2IIFD}v^{\beta_2} \\ \qquad +A_{1IIFSS}v^{\beta_1}+A_{2IIFDD}v^{\beta_2} \quad \text{for } \hat{v}_{IILD}\leq v<\hat{v}_{IILS}, \\ D_{F|O,X}\dfrac{v}{\delta+\theta}-D_{F|O,X}\dfrac{f_X}{r}+A_{1IIFS}v^{\beta_1}+A_{2IIFD}v^{\beta_2} \quad \text{for } \hat{v}_{IIFD}\leq v<\hat{v}_{IILD}, \\ \lambda Z \quad \text{for } v<\hat{v}_{IIFD}. \end{cases} \tag{20}$$

In (20), the first line represents the expected present value of follower's net revenue once the follower has switched; the second line represents the expected present value of follower's net revenue plus the sum of the option values to switch, $A_{1IIFS}v^{\beta_1}$ with $A_{1IIFS}>0$, and to divest, $A_{2IIFD}v^{\beta_2}$ with $A_{2IIFD}>0$; the third line represents the expected present value of follower's net revenue plus the sum of the option values to switch, $A_{1IIFS}v^{\beta_1}$, and to divest, $A_{2IIFD}v^{\beta_2}$, and the sum of the present values (gains or losses) accruing to the follower when the leader switches, $A_{1IILSS}\, v^{\beta_1}$, and when the leader divests, $A_{2IIFDD}\, v^{\beta_2}$ (note that we have separated the follower's option value from the present value accruing to the follower to add clarity when discussing the results, but of course the two terms could be composited as a single term); the fourth line represents the expected present value of follower's net revenue plus the sum of the option values to switch, $A_{1IIFS}\, v^{\beta_1}$, and to divest, $A_{2IIFD}\, v^{\beta_2}$; the fifth line represents the follower's value on divestment.

In (19) and (20), the generic term Av^{β} has two interpretations. First, it can refer to the value of an option, which can be exercised specifically by a single player to change policy from X to Y or O. Second, it can refer to the present value accruing to one player when the other exercises their option to change the incumbent policy. There are three occasions when this happens: (i) when the follower switches to Y, the leader benefits from a gain in market share; (ii) when the leader switches to Y, and (iii) when the leader switches to O, the follower experiences a gain in market share. These three occasions mirror those for the separate formulation.

Together, (19) and (20) create a set of equations from which the solutions to the unknown thresholds and coefficients are obtainable. There are four unknown thresholds signalling the leader's and follower's switching and divesting policies, $\hat{v}_{IILS}, \hat{v}_{IIFS}, \hat{v}_{IILD}, \hat{v}_{IIFD}$, respectively, four unknown option coefficients associated with the leader's and follower's switching and divesting policies, $A_{1IILS}, A_{2IILD}, A_{1IIFS}, A_{2IIFD}$, respectively, and three unknown coefficients associated with the leader's present value accruing when the follower switches, A_{1IILSS}, with the follower's present value accruing when the leader switches, A_{1IIFSS}, and divests, A_{2IIFDD}, making a total of eleven in all. There are eleven equations for solving the unknowns. There are four value-matching relationships occurring when the leader and the follower switch and divest, respectively, the four associated smooth-pasting conditions reflecting optimality, the value-matching relationship for the leader occurring when the follower switches, and the two for the follower occurring when the leader switches and divests[8].

There exist two value matching relationships for the follower, occurring at the switching and divesting thresholds, $\hat{v}_{IIFS}, \hat{v}_{IIFD}$, respectively. So, from (20) we have:

$$\left.\begin{aligned} D_{F|Y,X}\frac{\hat{v}_{IIFS}}{\delta+\theta}-D_{F|Y,X}\frac{f_X}{r}+A_{1IIFS}\,\hat{v}_{IIFS}^{\beta_1}+A_{2IIFD}\,\hat{v}_{IIFS}^{\beta_2} \\ =D_{F|Y,Y}\frac{\hat{v}_{IIFS}}{\delta+\theta}-D_{F|Y,Y}\frac{f_Y}{r}-\left(K-\lambda Z\right), \\ D_{F|O,X}\frac{\hat{v}_{IIFD}}{\delta+\theta}-D_{F|O,X}\frac{f_X}{r}+A_{1IIFS}\,\hat{v}_{IIFD}^{\beta_1}+A_{2IIFD}\,\hat{v}_{IIFD}^{\beta_2}=\lambda Z. \end{aligned}\right\} \tag{21}$$

From (21) and its associated smooth-pasting condition, we can obtain[9] solutions for the follower's two thresholds $\hat{v}_{IIFS}, \hat{v}_{IIFD}$ from the non-linear simultaneous equations:

$$\left.\begin{aligned} \hat{v}_{IIFD}^{\beta_2}\left(\hat{v}_{IIFS}\frac{D_{F|Y,Y}-D_{F|Y,X}}{\delta+\theta}\frac{\beta_1-1}{\beta_1}-\frac{D_{F|Y,Y}f_Y-D_{F|Y,X}f_X}{r}-\left(K-\lambda Z\right)\right) \\ =\hat{v}_{IIFS}^{\beta_2}\left(\lambda Z-\frac{D_{F|O,X}\hat{v}_{IIFD}}{\delta+\theta}\frac{\beta_1-1}{\beta_1}+\frac{D_{F|O,X}f_X}{r}\right), \\ \hat{v}_{IIFD}^{\beta_1}\left(\hat{v}_{IIFS}\frac{D_{F|Y,Y}-D_{F|Y,X}}{\delta+\theta}\frac{\beta_2-1}{\beta_2}-\frac{D_{F|Y,Y}f_Y-D_{F|Y,X}f_X}{r}-\left(K-\lambda Z\right)\right) \\ =\hat{v}_{IIFS}^{\beta_1}\left(\lambda Z-\frac{D_{F|O,X}\hat{v}_{IIFD}}{\delta+\theta}\frac{\beta_2-1}{\beta_2}+\frac{D_{F|O,X}f_X}{r}\right). \end{aligned}\right\} \tag{22}$$

The follower's switching and divestment option coefficients are, respectively:

$$\left.\begin{aligned} A_{1IIFS}&=\frac{1}{\beta_1\Delta_F}\left(\hat{v}_{IIFS}\frac{D_{F|Y,Y}-D_{F|Y,X}}{\delta+\theta}\hat{v}_{IIFD}^{\beta_2}+\hat{v}_{IIFD}\frac{D_{F|O,X}}{\delta+\theta}\hat{v}_{IIFS}^{\beta_2}\right), \\ A_{2IIFD}&=\frac{1}{\beta_2\Delta_F}\left(-\hat{v}_{IIFS}\frac{D_{F|Y,Y}-D_{F|Y,X}}{\delta+\theta}\hat{v}_{IIFD}^{\beta_1}-\hat{v}_{IIFD}\frac{D_{F|O,X}}{\delta+\theta}\hat{v}_{IIFS}^{\beta_1}\right), \end{aligned}\right\} \tag{23}$$

where $\Delta_F=\hat{v}_{IIFS}^{\beta_1}\hat{v}_{IIFD}^{\beta_2}-\hat{v}_{IIFS}^{\beta_2}\hat{v}_{IIFD}^{\beta_1}$. From (23), we expect A_{1IIFS}, A_{2IIFD} to be both positive.

[8] These joint value functions and the associated solutions can be represented in the conventional SDF approach as shown in the Supplementary Appendix C.
[9] Full derivations for the solutions for the joint formulation are available in the Supplementary Appendix B.

Now that the follower's switching threshold $v=\hat{v}_{IIFS}$ is known, we can deduce the impact of the follower switching on the leader's net return of operating policy Y, as denoted by $A_{1IILSS}\,v^{\beta_2}$. From (19), the leader's value-matching relationship defined at $v=\hat{v}_{IIFS}$ is:

$$D_{L|Y,X}\frac{\hat{v}_{IIFS}}{\delta+\theta}-D_{L|Y,X}\frac{f_Y}{r}+A_{1IILSS}\,\hat{v}_{IIFS}^{\beta_1}=D_{L|Y,Y}\frac{\hat{v}_{IIFS}}{\delta+\theta}-D_{L|Y,Y}\frac{f_Y}{r}.$$

This yields the unknown coefficient A_{1IILSS}:

$$A_{1IILSS}=\left(\frac{\hat{v}_{IIFS}}{\delta+\theta}-\frac{f_Y}{r}\right)\left(D_{L|Y,Y}-D_{L|Y,X}\right)\hat{v}_{IIFS}^{-\beta_1}. \tag{24}$$

From (24), we expect A_{1IILSS} to be positive.

There exist two value matching relationships for the leader, occurring at the switching and divesting thresholds, $\hat{v}_{IILS},\hat{v}_{IILD}$, respectively. So, from (19) we have:

$$\left.\begin{aligned}
&D_{L|X,X}\frac{\hat{v}_{IILS}}{\delta+\theta}-D_{L|X,X}\frac{f_X}{r}+A_{1IIFS}\,\hat{v}_{IILS}^{\beta_1}+A_{2IILD}\,\hat{v}_{IIFS}^{\beta_2}\\
&\qquad=D_{L|Y,X}\frac{\hat{v}_{IIFS}}{\delta+\theta}-D_{L|Y,X}\frac{f_Y}{r}+A_{1IILSS}\,\hat{v}_{IILS}^{\beta_1}-\left(K-Z\right),\\
&D_{L|X,X}\frac{\hat{v}_{IILD}}{\delta+\theta}-D_{L|X,X}\frac{f_X}{r}+A_{1IILS}\,\hat{v}_{IILD}^{\beta_1}+A_{2IILD}\,\hat{v}_{IILD}^{\beta_2}=Z.
\end{aligned}\right\} \tag{25}$$

From (25) and its associated smooth-pasting condition, we can obtain solutions for the leader's two thresholds $\hat{v}_{IILS},\hat{v}_{IILD}$ from the non-linear simultaneous equations:

$$\left.\begin{aligned}
&\hat{v}_{IILD}^{\beta_2}\left(\hat{v}_{IILS}\frac{D_{L|Y,X}-D_{L|X,X}}{\delta+\theta}\frac{\beta_1-1}{\beta_1}-\frac{D_{L|Y,X}f_Y-D_{L|X,X}f_X}{r}-\left(K-Z\right)\right)\\
&\qquad=\hat{v}_{IILS}^{\beta_2}\left(Z-\frac{D_{L|X,X}\hat{v}_{IILD}}{\delta+\theta}\frac{\beta_1-1}{\beta_1}+\frac{D_{L|X,X}f_X}{r}\right),\\
&\hat{v}_{IILD}^{\beta_1}\Big(\hat{v}_{IILS}\frac{D_{L|Y,X}-D_{L|X,X}}{\delta+\theta}\frac{\beta_2-1}{\beta_2}-\frac{D_{L|Y,X}f_Y-D_{L|X,X}f_X}{r}\\
&\quad+A_{1IILSS}\,\hat{v}_{IILS}^{\beta_1}\frac{\beta_2-\beta_1}{\beta_2}-\left(K-Z\right)\Big)\\
&\qquad=\hat{v}_{IILS}^{\beta_1}\left(Z-\frac{D_{L|X,X}\hat{v}_{IILD}}{\delta+\theta}\frac{\beta_2-1}{\beta_2}+\frac{D_{L|X,X}f_X}{r}\right).
\end{aligned}\right\} \tag{26}$$

The leader's switching and divestment option coefficients are, respectively:

$$\left.\begin{aligned} A_{1IILS} &= \frac{1}{\beta_1\Delta_L}\left(\left(\hat{v}_{IILS}\frac{D_{L|Y,X}-D_{L|X,X}}{\delta+\theta}+\beta_1 A_{1IILSS}\,\hat{v}_{IILS}^{\beta_1}\right)\hat{v}_{IILD}^{\beta_2}+\hat{v}_{IILD}\frac{D_{L|X,X}}{\delta+\theta}\hat{v}_{IILS}^{\beta_2}\right),\\ A_{2IILD} &= \frac{1}{\beta_2\Delta_L}\left(-\left(\hat{v}_{IILS}\frac{D_{L|Y,X}-D_{L|X,X}}{\delta+\theta}+\beta_1 A_{1IILSS}\,\hat{v}_{IILS}^{\beta_1}\right)\hat{v}_{IILD}^{\beta_1}-\hat{v}_{IILD}\frac{D_{L|X,X}}{\delta+\theta}\hat{v}_{IILS}^{\beta_1}\right),\end{aligned}\right\} \tag{27}$$

where $\Delta_L = \hat{v}_{IILS}^{\beta_1}\hat{v}_{IILD}^{\beta_2}-\hat{v}_{IILS}^{\beta_2}\hat{v}_{IILD}^{\beta_1}$. From (27), we expect A_{1IILS}, A_{2IILD} to be both positive.

Finally, the coefficients A_{1IIFSS}, A_{2IIFDD} of $A_{1IIFSS}\,v^{\beta_1}, A_{2IIFDD}\,v^{\beta_2}$ representing the present value accruing to the follower when the leader changes policy from X to Y and O, respectively, are obtainable from the follower's two value-matching relationships (20) defined at $v=\hat{v}_{IILS}$ and $v=\hat{v}_{IILD}$, respectively:

$$\left.\begin{aligned} D_{F|Y,X}\frac{\hat{v}_{IILS}}{\delta+\theta}-D_{F|Y,X}\frac{f_X}{r} &= D_{F|X,X}\frac{\hat{v}_{IILS}}{\delta+\theta}-D_{F|X,X}\frac{f_X}{r}+A_{1IIFSS}\,\hat{v}_{IILS}^{\beta_1}+A_{2IIFDD}\,\hat{v}_{IILS}^{\beta_2},\\ D_{F|O,X}\frac{\hat{v}_{IILD}}{\delta+\theta}-D_{F|O,X}\frac{f_X}{r} &= D_{F|X,X}\frac{\hat{v}_{IILD}}{\delta+\theta}-D_{F|X,X}\frac{f_X}{r}+A_{1IIFSS}\,\hat{v}_{IILD}^{\beta_1}+A_{2IIFDD}\,\hat{v}_{IILD}^{\beta_2}.\end{aligned}\right\} \tag{28}$$

The solutions are:

$$\left.\begin{aligned} A_{1IIFSS} &= \left(D_{F|Y,X}-D_{F|X,X}\right)\left(\frac{\hat{v}_{IILS}}{\delta+\theta}-\frac{f_X}{r}\right)\frac{\hat{v}_{IILD}^{\beta_2}}{\Delta_L}\\ &\quad -\left(D_{F|O,X}-D_{F|X,X}\right)\left(\frac{\hat{v}_{IILD}}{\delta+\theta}-\frac{f_X}{r}\right)\frac{\hat{v}_{IILS}^{\beta_2}}{\Delta_L},\\ A_{2IIFDD} &= -\left(D_{F|Y,X}-D_{F|X,X}\right)\left(\frac{\hat{v}_{IILS}}{\delta+\theta}-\frac{f_X}{r}\right)\frac{\hat{v}_{IILD}^{\beta_1}}{\Delta_L}\\ &\quad +\left(D_{F|O,X}-D_{F|X,X}\right)\left(\frac{\hat{v}_{IILD}}{\delta+\theta}-\frac{f_X}{r}\right)\frac{\hat{v}_{IILS}^{\beta_1}}{\Delta_L}.\end{aligned}\right\} \tag{29}$$

From (29), it is not possible to demonstrate definitively the signs of A_{1IIFSS}, A_{2IIFDD}.

Numerical Evaluations

Using the base case values in Table 1, we present the numerical solutions for the leader's and follower's various thresholds and coefficients in Table 4. This reveals that our results for the duopoly model meet the conditions as prescribed by Décamps et al. (2006) for the monopolistic context. The thresholds yielded by the joint formulation are always less than those under the separate formulation, $\hat{v}_{IILD} < \hat{v}_{ILD}$, $\hat{v}_{IILS} < \hat{v}_{ILS}$, $\hat{v}_{IIFD} < \hat{v}_{IFD}$, $\hat{v}_{IIFS} < \hat{v}_{IFS}$. Also, the leader is the first-mover since $\hat{v}_{IIFD} < \hat{v}_{IILD} < \hat{v}_{IILS} < \hat{v}_{IIFS}$. Over the range $\hat{v}_{IILD} < v \le \hat{v}_{IILS}$, the respective coefficients for v^{β_1} and v^{β_2} are greater for the leader than for the follower since $A_{2IILS} > A_{2IIFS} + A_{2IIFSS}$ and $A_{2IILD} > A_{IIFD} + A_{2IIFDD}$. This implies that the leader is the first-mover owing not only to the relative threshold sizes but also to the leader having the greater value. This is revealed in Figure 4, which profiles the leader's and follower's net value (defined as the difference between the respective value function and the investment cost if relevant) and shows the leader to have a greater net value the follower over the specified range. Finally, we observe that while A_{2IILSS} and A_{2IIFSS} are both positive, A_{2IIFDD} is negative. This indicates that while the leader gains when the follower switches and the follower gains when the leader switches, the follower loses when the leader divests. This arises because the leader is experiencing a negative net revenue at their divestment threshold, so the follower is accepting an additional loss due to their market share gain. This echoes the finding for the separate formulation.

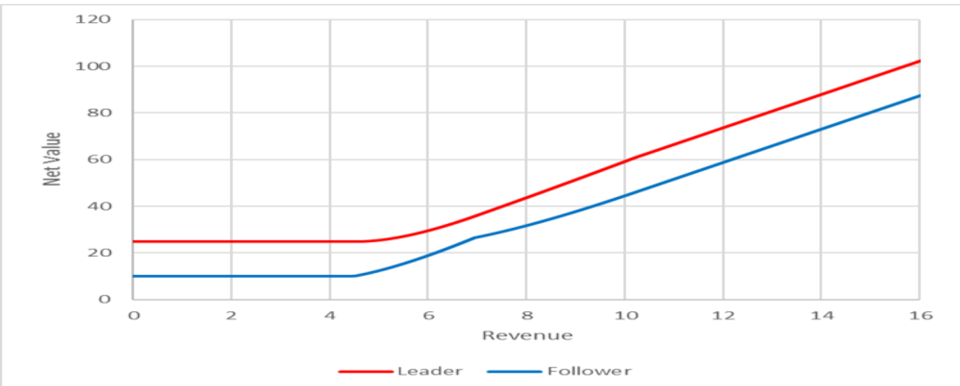
Table 4

Values for the Various Thresholds and Coefficients
Under the Joint Formulation

Leader		**Follower**	
Divest			
$\hat{v}_{IILD}$	4.524	$\hat{v}_{IIFD}$	4.328
A_{2IILD}	258.016	A_{2IIFD}	334.144
		A_{2IIFDD}	-182.405
Switch			
$\hat{v}_{IILS}$	6.948	$\hat{v}_{IIFS}$	10.206
A_{1IILS}	0.6628	A_{1IIFS}	0.0693
A_{1IILSS}	0.2828	A_{1IIFSS}	0.5409

Figure 4

Follower and Leader Net Values as a Function of Revenue (v)



Sensitivity Analysis

In Table 5 we present the percentage change in the thresholds and coefficients for the joint model due to a 1% increase in the various parameter values. Overall, the results are as expected. A 1% volatility increase produces deferrals in exercising both the switch and divestment options for both the leader and the follower, together with falls in the usual divest and switch option coefficients, but the rival switching option coefficients increase with volatility, in contrast to the separate formulation.

Changes in λ should seemingly only affect the follower, with a positive change being seen as attractive. A 1% increase leads to advancing both the divestment and switching decisions for the follower and rises in the respective option values. For the leader, an increase in λ postpones the divestment decision and lowers the respective option value but advances the switch decision, and raises the respective option value by a very small amount.

Increases in the leader's market share and the consequential decrease in the follower's market share could be interpreted as being attractive for the leader at the detriment to the follower. Table 5 shows that an increase in the leader's market share when pursuing policy X, $D_{L|X,X}$, makes the divestment opportunity more attractive for the leader and leads to an earlier exercise, while making the switch opportunity less attractive but also leading to an earlier exercise. It has, though, no impact on the follower's strategy since the divestment and switch opportunities only become available after the leader has divested and switched, respectively, except for the positive change in the follower's present accrued value when the leader divests because of the greater gain in market share.

If the market share after switching for the leader, $D_{L|Y,X}$, increases, then policy Y for the leader becomes more attractive. This is revealed in a less attractive divestment opportunity for the leader with a later exercise and a more attractive switch opportunity with an earlier exercise. Further, there is a fall in the present value accruing to the leader when the follower switches because the leader's market share gain is less. The switching opportunity for follower also becomes more attractive with a deferred threshold because the loss in the follower's market share becomes less. Also, there is an increase in the present value accruing to the follower when the leader switches because of the gain in the follower's market share.

Once both parties have switched, an increase in the leader's market share, $D_{L|Y,Y}$, makes the leader's switching opportunity more attractive with an earlier exercise and their divesting opportunity less attractive with a later exercise. Also, the increase in market share produces an increased present value accruing to the leader when the follower switches. In contrast, the follower's switch opportunity becomes significantly less desirable with a deferred exercise owing to the greater decrease in market share, but the follower's divest opportunity becomes more desirable with an earlier exercise. Finally, there is an increase in the present value accruing to the follower when the leader switches.

Table 5
Percentage Change in the Thresholds and Coefficients
due to a 1% Parameter Value Increase for the Joint Formulation
Panel A exhibits the % changes for the Divestment Opportunity
and Panel B for the Switch Opportunity

Panel A	Leader		Follower		
	$\hat{v}_{IILD}$	A_{2IILD}	$\hat{v}_{IIFD}$	A_{2IIFD}	A_{2IIFDD}
r	-0.12%	1.06%	-0.35%	0.42%	0.00%
δ	0.58%	0.34%	0.48%	0.22%	0.06%
θ	0.43%	0.25%	0.36%	0.16%	0.04%
σ	-0.31%	-2.76%	-0.53%	-2.82%	-3.20%
f_X	0.18%	1.01%	0.75%	1.97%	2.99%
f_Y	0.09%	0.10%	0.03%	0.03%	-0.10%
Z	0.08%	0.50%	0.05%	0.17%	0.21%
K	0.64%	0.72%	0.17%	0.17%	-0.75%
λ	-0.01%	-0.01%	0.05%	0.17%	0.00%
$D_{L\|X,X}$	0.09%	0.70%	0.00%	0.00%	1.16%
$D_{L\|Y,X}$	-0.09%	-0.10%	-0.10%	-0.10%	0.27%
$D_{L\|Y,Y}$	-0.53%	-0.60%	0.36%	0.37%	0.08%

Panel B	Leader			Follower		
	$\hat{v}_{IILS}$	A_{1IILS}	A_{1IILLS}	$\hat{v}_{IIFS}$	A_{1IIFS}	A_{1IIFSS}
r	-0.27%	1.21%	1.73%	-0.85%	0.86%	-1.05%
δ	0.68%	-2.41%	-2.19%	0.76%	-3.74%	-2.08%
θ	0.51%	-1.81%	-1.64%	0.57%	-2.82%	-1.56%
σ	0.50%	-0.11%	1.04%	1.16%	-7.01%	1.02%
f_X	-0.34%	1.87%	0.13%	-0.21%	9.12%	3.08%
f_Y	0.18%	-0.44%	-0.19%	0.19%	-1.62%	-0.50%
Z	-0.18%	0.96%	0.10%	-0.17%	2.22%	0.61%
K	1.36%	-3.02%	-0.72%	1.23%	-10.30%	-3.73%
λ	-0.01%	0.06%	0.10%	-0.17%	2.22%	0.03%
$D_{L\|X,X}$	-0.13%	-0.79%	0.00%	0.00%	0.00%	1.44%
$D_{L\|Y,X}$	-0.36%	0.43%	-4.26%	0.46%	5.98%	1.13%
$D_{L\|Y,Y}$	-0.49%	2.53%	4.29%	1.15%	-22.00%	1.21%

Section 4 Comparing the Separate and Joint Solutions

The separate and joint formulations of §2 and §3 share common features, representing a duopoly game involving a leader and a follower facing an inevitable revenue decline and deliberating between divesting and switching to a more appropriate technology in place of the incumbent high operating cost strategy, each having 11 equations and 11 unknowns[10]. Further, we assume that switching is the preferred choice, and the leader is always the first-mover. But the two formulations have a significant difference in the location of the initial revenue value. For the separate formulation, this creates a recursively solved model involving only put-style options, but a more complex model structure for the joint formulation involving both call- and put-style options, with significantly different findings.

[10] As shown in the Supplementary Appendix D, the joint solution is economically reduced to just solving 4 equations for the 4 thresholds.

In the standard model, a volatility increase is known to make both the investment opportunity and the divestment opportunity more attractive with a deferred exercise. By being composed of only put options, the effect of a volatility increase for the separate formulation is straightforward and predictable. In contrast, the combination of call and put options implies the solution to the joint formulation cannot be derived recursively, and the effect of a volatility increase is possibly less straightforward and less predictable[11].

Section 5 Conclusion

Is a joint formation model feasible, with a solution using joint option coefficients, which should be consider in mutually-exclusive option contexts? Are the results using a joint formulation model significant in capital budgeting, that is are the thresholds justifying immediate action (divestment, or switching in our case) different from using the separate formulation? Can the joint formulation be extended to a duopoly with first mover advantages? What is the role of the Rival's Options used herein?

We conclude that: i) the joint formulation with option coefficients and thresholds is feasible and perhaps should be considered in many other contexts; ii) the action thresholds for capital budgeting for mutually-exclusive opportunities are 1/2 or 3/4 of those determined using conventional real option theory, while the separate thresholds are 57% of the NPV, except for the follower divest ; iii) extending the joint formulation to a duopoly is feasible and interesting, and introduces market share sensitivities for both option coefficients and thresholds; and iv) the Rival Options are a novel concept with interesting implications, offering new perspectives on capital budgeting and management.

There are extensions for the partial derivatives of all of the formulation thresholds and option coefficients, with plausible illustrations (and indicated hedging opportunities). This joint formulation approach might be used for many other mutually-exclusive option contexts, with different configurations of switch opportunities and costs, market shares and sequential real option games. The topic of rival options seems to us a fertile area for future research.

[11] The supplementary Appendix F shows the complexity of the effect of increasing volatilities on the option coefficients and the option values at some specific v levels, above and between the thresholds, for the separate and joint formulation models.

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Supplementary Appendix

A Derivation of Separate Solutions

B Derivation of Joint Solutions

C Representation Using the Stochastic Discount Function

D Excel Spreadsheet Solutions

E Decomposition of Value Functions

F Volatility Implication for the Separate and Joint Formulations