Exploiting the European Volatility Index features: Anti-persistence, Skewness and Kurtosis Analysis

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Abstract: The use and study of volatility indices on stock markets arouses more and more interest. With this paper, through the empirical study of the classical statistical characteristics of the main volatility index of the European stock markets (VStoxx), it has been possible to find the confirmation of the anti-persistence and the "mean reversion" as characteristic features of the volatility market. The empirical observation of data has highlighted interesting connections and cause-effect relationships between Hurst exponent and the moments of distribution. The conclusions of this paper are oriented towards a phenomenon whereby the variations occurring in the Hurst exponent tend to correct the variations of the other moments of the distribution such as skewness and kurtosis. This is an important additional signal to classical valuation metrics to evaluate near-term potential market reversal.

Keywords: VStoxx; Hurst Exponent; Kurtosis; Skewness; cointegration; vector error correction model; Granger-causality

JEL Classification: G01; G12; G14; G20

1 Introduction

The volatility indices in recent years are arousing increasing interest among the various market players. Accurate modelling and forecasting of volatility is extremely crucial in financial markets. Although there exist different groups of volatility forecasting approaches, historical and implied volatility models are the most common exploited. *Siriopoulos et al.* (*2019*) test and document the information content of all publicly available implied volatility indices regarding both the realized volatility and the returns of the underlying asset. Their findings suggest that implied volatility includes information about future volatility beyond that contained in past volatility but, at the same time, they show that implied volatilities in commodities, bonds, currencies and volatility react differently to underlying price changes compared to equities. Implied volatility models bases on the Black-Scholes Model (BSM). The BSM is the standard model for valuing options. Volatility is one of not deterministic inputs available for immediate application in the formula, assumed constant over time and grounded on the assumption of normally distributed log returns. *Iqbal* (*2018*) shows a usual leptokurtic distribution in practice of the log returns of spot prices rather than a normal distribution, meaning that it exhibits excess kurtosis and more weight in the tails compared with the normal. In other terms, it is more likely that spot prices will remain unchanged than implied by the normal and, at the same time, that spot exhibits extreme moves than implied by the normal. The author shows also how this pricing feeds into the volatility smile.
Most papers have focused principally on VIX Index (or CBOE Volatility Index) predictive power for future stock market returns. *Giot* (*2005*) proves that high (low) levels of the VIX correspond to positive (negative) future returns. Also *Chow et al.* (2014) also show the existence of a positive relationship between market returns and the VIX Index. For that concerns the evaluation of the VIX features, *Fleming et al.* (*1995*) were the first to analyze the persistence (long-memory behavior) of this index. Poorly developed is the literature on the main European volatility indices (i.e. VStoxx and VDax). *Stanescu et al.* (*2013*) focus on linkages between Eurostoxx50, S&P500, VSTOXX, VIX and VSTOXX futures series that can be used by equity investors to generate alpha and protect their investments during turbulent times. *Fahling et al.* (*2019*) show that the best forecasting model for the one month VDAX is a GARCHX(1,1) model and an ARX(1) model for the one year VDAX, while for the VSTOXX, an ARX model is the best fit under each scenario for both strategies. However, in the writing of this paper, no one published and working paper could be found which is studying the statistical features and linkage between the moments of the VStoxx index distribution over time with a fractal approach.
Given this premise, the intention of this paper is a depth study of various aspects and facets. In particular, we tried to analyze:

1. the statistical and structural characteristics of the volatility indices through the study of the distribution of returns over time. As in other analysis papers (see *Bagato et al., 2018*), this study starts from the analysis of the basic characteristics of the distribution of logarithmic returns of the European volatility index Vstoxx. So we started directly from the moments of distribution, an important starting point and basis on which to empirically describe some significant evidence;
2. the confirmation of the anti-persistence and the mean reversion structurally present in the historical series through the study of the behavior of the Hurst exponent. The Hurst exponent is an index of fundamental importance in the analysis of the long-range dependence features of observable time-series (*Resta, 2012*). After detecting the empirical evidence of the moments of distribution, we tried to observe the Hurst exponent in the historical series observed relating to the European index of equity volatility. In this analysis of the Hurst exponent, we have empirically observed in the historical series of logarithmic returns that the exponent structurally assumes values between 0 and 0.5. This was important evidence to confirm the anti-persistence

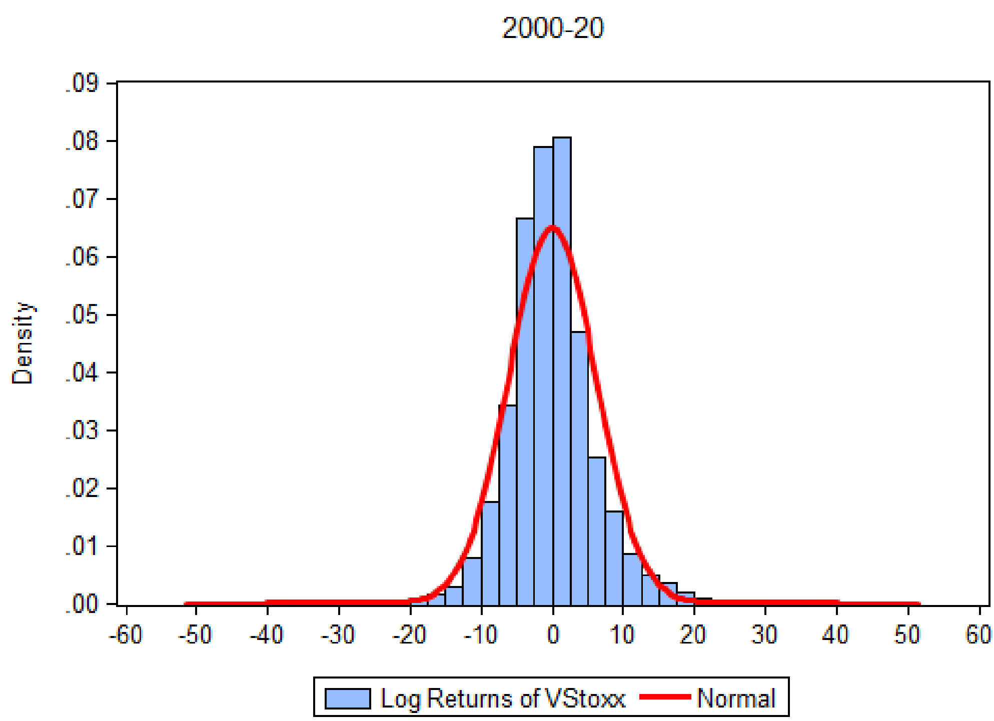
characteristic of the European volatility index Vstoxx. This value structurally lower than 0.5 of the Hurst exponent also confirmed the so-called mean reversion characteristic of the time series. In the literature, a Hurst exponent that structurally moves between 0 and 0.5 has led to the definition of volatility as "rough" (see *Gatheral et al., 2014*). This characteristic of anti-persistence of the historical series of logarithmic returns also brings with it the adoption of *Fractional Brownian Motion* models (see *Neuman et al., 2018*) used in the logic of analysis and pricing when the Hurst exponent assumes values that are structurally different from 0.5, typical value of the classic Brownian Motion;

3. the relationships that exist between the various moments of the distribution and the Hurst exponent. We have focused more on this last aspect even if our analysis is not exhaustive but opens the way for further analysis and in-depth analysis both in the field of risk management but also in terms of pricing of derivative instruments with non-linear payoffs linked to stock volatility indices. Nevertheless, we believe it is an additional useful tool for portfolio managers besides classical evaluation metrics (i.e. P/E, P/S, Dividend Yield, etc.) to signal a potential near-term market excess (increasing probability of cyclical market inversion point) when it is close to its interval extremes.

2 Model and Data description

In order to conduct the empirical analysis, we use daily VStoxx close prices for the interval 2000-20 (5423 observations) transformed in log returns (5422 observations). Logarithmic transformation allows time-series stationarity and a closer to normal distribution. Data come from *Bloomberg*. *Exhibit 1* reports the Vstoxx log returns over the entire time horizon.

Exhibit 1 – VStoxx log returns density distribution series from January 2000 to December 2019



Source: authors' calculations in Eviews 10 based on Bloomberg data.

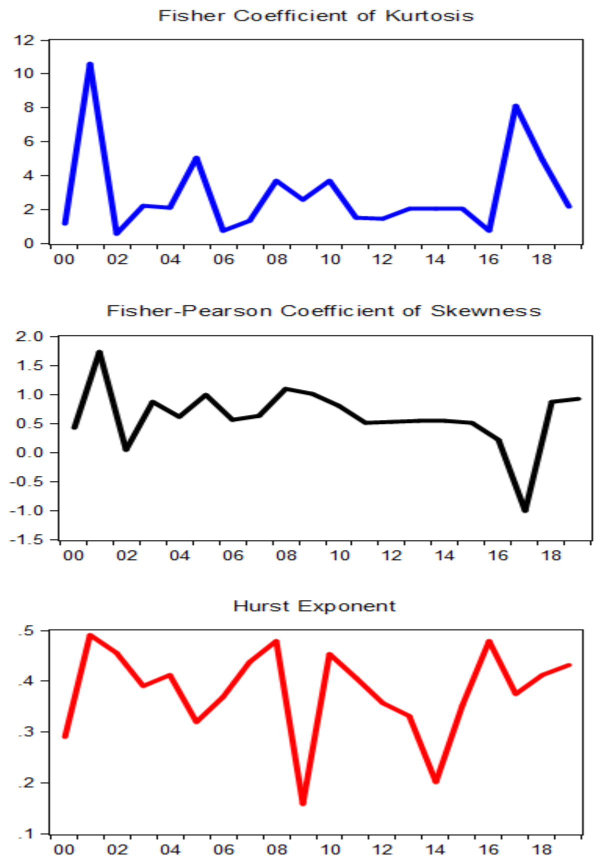
Exhibit 2 reports the descriptive statistics of VStoxx log returns series for the entire period.

Exhibit 2 - Descriptive statistics of data - January 2000 to December 2019 (*5422 daily obs.*)

Statistics	**VStoxx Log Returns** (%)
Mean	-0.005323
Median	-0.330572
Maximum	47.03052
Minimum	-43.47158
Std. Dev.	6.148458
Skewness	0.757710
Kurtosis	7.528615

Source: authors' calculations based on Bloomberg data.

Exhibit 3 – Fisher coefficient of kurtosis, Fisher-Pearson coefficient of skewness and Hurst exponent data derived from VStoxx log returns distribution from January 2000 to December 2019



Source: authors' calculations in Eviews 10 based on Bloomberg data.

Exhibit 4 reports the descriptive statistics of Fisher coefficient of kurtosis, Fisher-Pearson coefficient of skewness and Hurst exponent data derived from VStoxx log returns distribution.

Exhibit 4 - Descriptive statistics of Fisher coefficient of kurtosis, Fisher-Pearson coefficient of skewness and Hurst exponent - January 2000 to December 2019 (*20 annual obs.*)

Statistics	Fisher coefficient of kurtosis	Fisher-Pearson coefficient of skewness	Hurst Exponent
Mean	2.941126	0.618522	0.380551
Median	2.088555	0.582418	0.398457
Maximum	10.59942	1.730151	0.491704
Minimum	0.568985	-1.007011	0.158402
Std. Dev.	2.559573	0.524438	0.088324
Skewness	1.774520	-1.087760	-1.036014
Kurtosis	5.544337	6.415787	3.595575

Source: authors' calculations based on Bloomberg data.

Let FK_t (Fisher coefficient of kurtosis at time t) and FS_t (Fisher-Pearson coefficient of skewness at time t) define as:

$$FK_t = \frac{m_{4t}}{(m_2)^2{}_t} - 3 \quad (1)$$

$$FS_t = \frac{m_{3t}}{(m_2)^{3/2}{}_t} \quad (2)$$

where m_{2_t}, m_{3_t} and m_{4_t} are respectively the second, the third and the fourth central moment calculated for the time interval *t*.

The econometric model implemented develops in *two sequential parts.* The first part is based on the methodology suggested by *Gonzalo* and *Granger* (*1995)*, each of which develops in *two stages*. The second and last part of the analysis is based on an OLS regression. Let us start with the description of the *first part*.

In the *first stage*, we verify whether the short-term deviations of these two series converge towards the long-term equilibrium through a «*cointegration analysis*» by performing the Augmented Dickey-Fuller test. The existence of a linear combination between these two series, indeed, supports the presence of a long-term equilibrium adjustment process, even if the series deviate one from the other in the short-term. In this case, series are cointegrated.

In the *second stage*, by using the *first stage* results[1], we will try to verify which variable is able to embody move more rapidly than the other one. In other terms, this allows us to evaluate the potential existence of a *leader* and *follower* variables, as well as halfway situations. To do so, we set a bivariate *Vector Error Correction Model* (*VECM*), as suggested by *Engle* and *Granger* (*1987*).

In order to perform the analysis, let us define the *Hurst exponent* variable. Following the intuition and the structure suggested by *Sang et al.* (*2001*), we estimates the Hurst Exponent using re-scaled range analysis (R/S) to provide some information about the statistical properties of European stocks volatility index time series. Re-scaled range analysis approach is robust to heavy tails (*Barunik et al., 2010*). Let us consider a VStoxx log return series Rt_n. Let τ denote the time span of the entire discrete series. The cumulative sum of the difference between return series and their mean, $X(n,\tau)$, is defined:

$$X(n,\tau) = \sum_{i=1}^{n}\left(Rt_i - \frac{1}{\tau}\sum_{n=1}^{\tau} Rt_n\right) \quad (3)$$

The range *R* denotes the difference between the maximum and minimum of $X(n,\tau)$, and *S* denotes standard deviation of the series. Therefore, *R* and *S* are defined as:

$$R(\tau) = \max X(n,\tau) - min\, X(n,\tau) \;\text{ for } n \in [1,\tau] \quad (4)$$

$$S(\tau) = \sqrt{\frac{1}{\tau}\sum_{n=1}^{\tau}\left(Rt_n - \frac{1}{\tau}\sum_{n=1}^{\tau} Rt_n\right)^2} \quad (5)$$

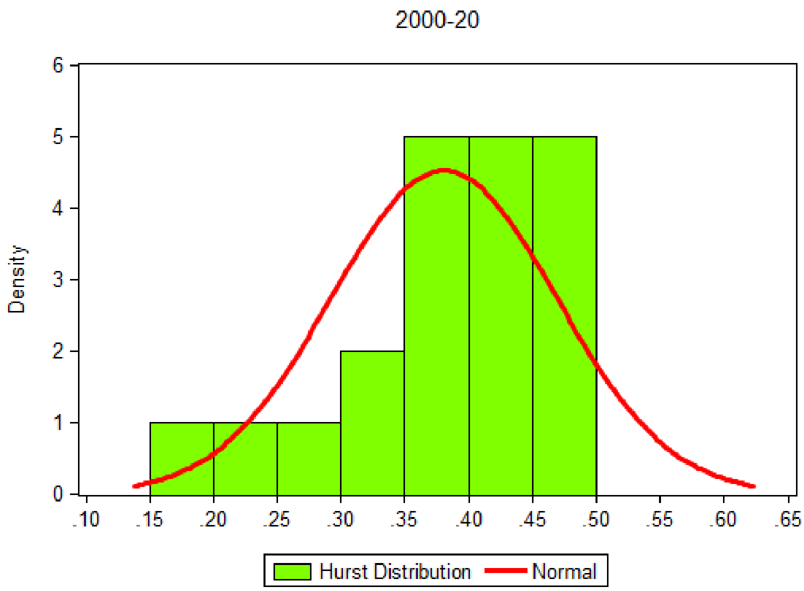
R and *S* are functions of τ. The R/S ratio is well described by the following empirical equation:

$$\frac{R}{S} = (c\tau)^H \quad (6)$$

where *c* is a constant and *H* the Hurst exponent. The Hurst exponent is a classical self-similarity parameter that measures the long-range dependence in a time series and provides measure of long-term nonlinearity (*Millen et al., 2003*).

[1] If the two series are not cointegrated then the VECM cannot be implemented because it is not more valid.

Exhibit 5 – Hurst exponent distribution from January 2000 to December 2019



Source: authors' calculations in Eviews 10 based on Bloomberg data.

The formal specification of the model is defined by the following equations:

$$\Delta H_t = \beta_{10} + \sum_{t=1}^{l} \beta_{1t} \Delta H_{t-1} + \sum_{t=1}^{l} \alpha_{1t} \Delta FK_{t-1} + \lambda_1 ECT_{t-1} + \varepsilon_{1t} \quad (7)$$

$$\Delta FK_t = \beta_{20} + \sum_{t=1}^{l} \beta_{2t} \Delta H_{t-1} + \sum_{t=1}^{l} \alpha_{2t} \Delta FK_{t-1} + \lambda_2 ECT_{t-1} + \varepsilon_{2t} \quad (8)$$

where:

- ΔH_t and ΔFK_t are, respectively, the *first differences* for the *Hurst exponent* and the *Fisher coefficient of kurtosis* series;
- β_{10} and β_{20} are, respectively, the *constant terms* of the equation (7) and (8);
- ΔH_{t-1} and ΔFK_{t-1} are, respectively, the *delayed first differences* for the *Hurst exponent* and the *Fisher coefficient of kurtosis* series;
- l is the number of *lags*;
- ECT_{t-1} is the *Error Correction Term* (*ECT*). It is defined as $ECT_{t-1} = H_{t-1} - \alpha - \gamma FK_{t-1}$. In simple terms, it measures the deviations between the *Hurst exponent* and *Fisher coefficient of kurtosis* at time (*t-1*) with respect to the theoretical long period equilibrium. γ is the cointegrating coefficient and α is the intercept of the cointegrating term;
- λ_1 and λ_2 are the *adjustment coefficients*. They describe the speed of adjustment back to the long period equilibrium, that is they measure the proportion of correction of the series deviations from the long-run relationship;
- ε_{1t} and ε_{2t} are, respectively, the *error terms* of the equation (7) and (8).

It is intuitive that, for the aim of the analysis, the evaluation of the *sign*[2] and the *statistically significance* of the *adjustment coefficients* (λ_1 and λ_2) allows us to know which market contributes to

[2] We should expect the negative sign for λ_1 and the positive sign for λ_2 in order to favor the process of adjustment.

the adjustment process toward the long period equilibrium and which variable is able to move more rapidly than the other one. Hence, we should distinguish four cases:

1. if λ_1 is *statistically significant* and *negative* then it implicitly means that the Fisher coefficient of kurtosis adjusts more rapidly than the Hurst exponent. This means that the Hurst exponent is trying to restore the long-run equilibrium;
2. if λ_2 is *statistically significant* and *positive* then it implicitly means that the Hurst exponent moves more rapidly than the Fisher coefficient of kurtosis adjustment. This means that the Fisher coefficient of kurtosis adjustment is trying to restore the long-run equilibrium;
3. if λ_1 is *statistically significant* and *negative* and λ_2 is *statistically significant* and *positive* then both variables contribute to the adjustment process towards the long-run equilibrium. In this case, by following *Gonzalo-Granger* (*1995*), in order to evaluate the effective contribution of each market in the adjustment process, we follow the concept of *Market Share* e (*MS*)[3]. According to how the *MS* formula has been defined, we distinguish between three sub cases:

   a) if $MS \approx 1$ then the Hurst exponent is the *leading* variable and the Fisher coefficient of kurtosis is the *lagging* variable;

   b) if $MS \approx 0$ then the Fisher coefficient of kurtosis is the *leading* variable and the Hurst exponent is the *lagging* variable;

   c) if $MS \approx 0.5$ then both variables contribute in the same way;

4. if only one of the adjustment coefficients is statistically significant and it present the correct sign then only that variable contributes to the adjustment process towards the equilibrium.

This concludes the *first part* of the whole analysis. Let us introduce the *second* (and last) *part*.

The log returns of spot prices shows a usual leptokurtic distribution in practice rather than a normal distribution (*Iqbal*, *2018*), meaning that it exhibits excess kurtosis and more weight in the tails compared with the normal. The evolution of the excess of kurtosis (which represent a classical feature of the volatility distribution) variations, therefore, should be evaluated as the main determinant of the other fundamental aspect of the volatility distribution, that is the skewness progression. In this simple way, we can roughly estimate the "gamma effect"[4] of the excess of kurtosis on the distribution skewness. Let us define, as conclusive part of the whole analysis, the following OLS regression:

$$\Delta FS_t = c + \beta \Delta FK_t + \varepsilon_t \tag{9}$$

where:

- ΔFS_t is the *first difference* of the log variation of the Fisher-Pearson coefficient of skewness at time *t*;
- ΔFK_t is the *first difference* of the log variation of the Fisher coefficient of kurtosis at time *t*;
- c is the constant term;
- β is the regressor's coefficient at time *t*;
- ε_t is the error term at time *t*.

[3] The formula suggested by *Gonzalo* and *Granger* (*1995*) is the following: $MS = \frac{\lambda_2}{\lambda_2 - \lambda_1}$
[4] This expression is just an analogy with the option instruments terminology. "Gamma", indeed, is the rate of change in an option's delta per 1-point move in the underlying asset's price.

Data employed to run the above OLS regression have been extracted from annual distributions. In other terms, we have 16 annual observations (after adjustment[5]) for regression (9). The number of observations matter for inference, particularly in presence of non-normal distributed residuals. *Jenkins et al.* (*2020*) investigate on the process to clearly identify a minimum N (number of observations) needed for a study. Authors recommend a minimum N = 8 for a tight data pattern (i.e., very low variance) and a minimum N ≈ 25 to clearly match a model to the data pattern with high variance. Their findings support our OLS model performance.

3 Results

All preliminary and complementary tests on time series and further statistical tests validating the acceptance of OLS assumptions are not reported here. With regard to these latter tests, they confirm the presence of heteroskedasticity, no serial correlation and non-normal distributed residuals. To limit the problem of the heteroskedasticity, we calculate robust estimates by using the *Huber-White* procedure. To deal with non-normal distributed residuals, we follow *Jenkins et al.* (*2020*).
As suggested by *Liew* (*2004*), we use *Akaike's information criterion* (*AIC*) as lag length selection criteria in determining the autoregressive lag length. Author shows its superiority than the other criteria under study in the case of small sample (60 observations and below).

3.1 ***<u>First Part</u>* - VECM: Hurst exponent and Fisher coefficient of kurtosis**

According to the *first stage* of the analysis, we evaluate the existence of cointegration between the two series through the *Augmented Dickey-Fuller Test*. The latter is reported in *Exhibit 6*.

Exhibit 6 - *Augmented Dickey-Fuller Test:* period *2000-20*

Augmented Dickey-Fuller Test		
Residuals		
Period	*t-Statistic*	*Prob.**
2000-20	-4.142184	0.0056

Source: authors' own calculations in Eviews 10 based on Bloomberg data.

As suggested by the test, the two series are cointegrated. Therefore, it is possible to realize the second stage of the analysis and estimate the *VECM* in order to assess which market contributes to the adjustment process toward the long-term equilibrium. The Akaike's information criterion suggests three lags as optimal lag length structure. The *VECM* estimation outputs are reported in the following *Exhibit 7* (*a*) and *Exhibit 7* (*b*).

[5] The Fisher-Pearson coefficient of skewness is negative in 2017.

Exhibit 7 (a) - *VECM:* dependent variable ΔH - period *2000-20*

	Coefficient	Std. Error	t-Statistic	Prob.
β_{10}	0.024691	0.022269	1.108756	0.2998
β_{11}	1.143627	0.881665	1.297121	0.2307
β_{12}	0.632662	0.582242	1.086597	0.3089
β_{13}	0.124346	0.260562	0.477221	0.6460
α_{11}	-0.088861	0.040669	-2.184990	0.0604
α_{12}	-0.051683	0.028474	-1.815051	0.1071
α_{13}	-0.012478	0.015004	-0.831633	0.4297
λ_1	-2.684862*	1.215836	-2.208245	0.0582

Note: *** signals parameter significance at 1%. *Source*: authors' calculations in Eviews 10 based on Bloomberg data.

Exhibit 7 (b) - *VECM*: dependent variable ΔFK - period *2000-20*

	Coefficient	Std. Error	t-Statistic	Prob.
β_{20}	0.070415	0.704636	0.099931	0.9229
β_{21}	-3.355230	20.28667	-0.165391	0.8727
β_{22}	4.119106	14.91121	0.276242	0.7894
β_{23}	0.726912	4.973915	0.146145	0.8874
α_{21}	-0.459197	1.034419	-0.443918	0.6689
α_{22}	-0.359277	0.764539	-0.469926	0.6510
α_{23}	-0.274273	0.391455	-0.700650	0.5034
λ_2	9.620120	29.54178	0.325645	0.7530

Note: *** signals parameter significance at 1%. *Source*: authors' calculations in Eviews 10 based on Bloomberg data.

As we can see from *Exhibit 7* (*a*) and *Exhibit 7* (*b*), only λ_1 is *statistically significant* and *negative* while λ_2 is *positive* but *not statistically significant*. This means that the Fisher coefficient of kurtosis adjusts more rapidly than the Hurst exponent. This latter moves in the direction to restore the long-run equilibrium relationship. This proves that the Hurst exponent is a proxy measure of the degree of volatility mean reversion.

3.2 ***Second part* – OLS regression**

Exhibit 8 reports the augmented Dickey–Fuller test.

Exhibit 8 - *Augmented Dickey-Fuller Test*: period *2000-20*

Augmented Dickey-Fuller Test		
Residuals		
Period	*t-Statistic*	*Prob.**
2000-20	-8.755012	0.0000

Source: authors' own calculations in Eviews 10 based on Bloomberg data.

The augmented Dickey–Fuller tests show that series is stationary. *Exhibit 9* shows the estimated coefficients for the equation (9).

Exhibit 9 – *OLS regression:* dependent variable ΔFS - period *2000-20* (*16 annual obs.*)[6]

	Coefficient	**Std. Error**	**t-Statistic**	**Prob.**
c	0.009524	0.305148	0.031212	0.9755
β	0.982585***	0.198585	4.947931	0.0002

Note: *** signals parameter significance at 1%. *Source*: authors' calculations in Eviews 10 based on Bloomberg data.

In this period, the estimated determination coefficient R^2 is equal to 0.78 for the equation (9). The relationship between the Fisher-Pearson coefficient of skewness elasticity (*dependent variable*) and the Fisher coefficient of kurtosis elasticity (*independent variable*) is positive as expected, reporting a coefficient of about 0.98. This means that an increase/decrease of 1 percentage point of the regressor corresponds to an increase/decrease of approximately 98 basis points of the distribution skewness elasticity. This last result would enhance the underlying idea that, by making different hypothesis on the Hurst exponent within its habitat, it could further improve the simulation scenarios for the optimization of volatility option pricing procedure. In practical terms, however, these dynamics further confirm us how the Hurst exponent information can be used to evaluate potential near-term market excess in combination with the classical equity metrics (such as P/E, P/S, Dividend Yield, etc.).

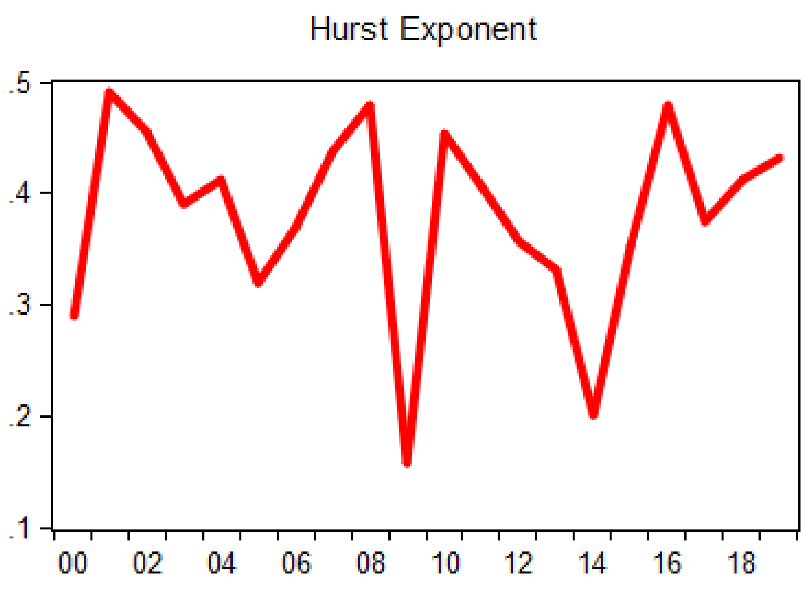
4 Economic discussion and Conclusions

From an empirical point of view, this paper has achieved some important goals. Important confirmations have been found regarding the anti-persistence and mean-reversion characteristics of the historical price series of the volatility indices. These confirmations were the basis for empirically investigating the behavior of the European volatility indices. In particular, with the aid of statistical analysis it has been possible to ascertain the existence of relationships between the variables relating to persistence and the moments of distribution of the historical series relating to the main European volatility index (VStoxx). In this regard, it has been interesting to underline that the empirical results highlight causal links between the trend of anti-persistence and kurtosis on the one hand, the excess of

[6] Let us restate that the Fisher-Pearson coefficient of skewness is negative in 2017.

kurtosis and skewness elasticity on the other. In fact, it has been possible to observe that, on the one hand, in the historical series relating to the most important European volatility index VStoxx, the variations of the Hurst Exponent somehow are linked to the variations of the kurtosis. On the other hand, in the observed time series, it has been gauged a strict relationship between the excess of kurtosis and skewness elasticities. While it could potentially improve the simulation on the volatility index distribution by making hypothesis on the Hurst Exponent within its "habitat" for the option pricing procedure, we found it is surely has an active role in signaling potential near-term market inversion point. This latter claim is mostly true if compared with information provided by the classical equity metrics (i.e. P/E, P/S, Dividend Yield, etc.). *Exhibit 10* graphically proves what we are claiming.

Exhibit 10 – *Eurostoxx 50* and *Hurst Exponent*: historical chart since 2000s



Source: macrotrends.net (*Eurostoxx 50*), authors' calculations in Eviews 10 based on Bloomberg data (*Hurst exponent*).

How it is possible to observe from *Exhibit 10*, extreme high values of the Hurst exponent signaled important inversion points as in the early 2000s, mid-2000s and the local major top in 2017. Of course, it does not mean it represents the Holy Grail for anticipating major market top, but it can be useful to exploit information from implied volatility structure and put together with the classical fundamental and technical metrics to have a further confirmation about the market trend progression.

The theoretical and empirical structure of this work does not have the ambition to be exhaustive but rather opens the door to various insights and themes of investigation in various directions. In fact, the evidence of the structural anti-persistence of the historical series of the returns of the VStoxx (as main European volatility index) and the links between the Hurst exponent and the moments of distribution (in particular, skewness and kurtosis) leaves room for further study and fields of investigation. These empirical evidences could enter at a certain level in the pricing models of derivatives (in particular in the pricing of options) related on equity volatility indices.

Acknowledgements: None. No funding to declare.

Author Contributions: Conceptualization and writing, M. Anelli and A. Gioia; methodology, software, data curation, M. Anelli; validation, M. Anelli, A. Gioia, M. Patanè, S. Zedda. All authors have read and agreed to the published version of the manuscript.

Conflict of Interest: All authors have no conflict of interest to report.

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