A Model-based Commodity Risk Measure on Commodity and Stock Market Returns*

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Abstract

We show that the information produced by term structure models is useful in commodity market asset pricing. The term structure model based characteristic (TSMC) we develop has a natural interpretation of downside risk premium and outperforms other well-known characteristics in explaining the cross-section of commodity returns. None of the existing factors is able to explain the returns of the high minus low portfolio constructed from sorting TSMC. An aggregate index constructed from individual TSMCs predicts future stock market returns even after controlling for popular economic predictors, suggesting that it contains unique forecasting information.

Keywords: Commodities, Term structure models, Predictability, Cross-sectional asset pricing

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1 Introduction

Multi-factor term structure models have long been used to study the commodity futures. For example, Gibson and Schwartz (1990), Schwartz (1997), Casassus and Collin-Dufresne (2005), Trolle and Schwartz (2009), Liu and Tang (2011), and Chiang, Hughen, and Sagi (2015), among others. These studies help us understand the connection between the dynamics of commodity spot and futures on various tenors. The term structure models also provide tools to effectively transform raw term structure of futures prices into information with economic meanings, e.g., the convenience yield, and disentangle risk neutral and statistical expectations to extract risk premium measures. In the meantime, another stream of literature in the commodity markets focusing on explaining the cross-section of commodity returns has been developing in parallel using the well-established methodologies from the empirical asset pricing in the equity market. For example, Yang (2013), Gorton, Hayashi, and Rouwenhorst (2013), Szymanowska et al. (2014), and Daskalaki, Kostakis, and Skiadopoulos (2014) apply cross-sectional asset pricing tests to study factors and anomalies in commodities market.

Both areas are important for understanding and investing in the commodity markets. Considering that multi-factor term structure models economically and effectively extract deeper information beyond simple price-based characteristics used in traditional asset pricing tests, it is natural to ask if the information produced by a multi-factor term structure model could be useful in explaining the cross-section of commodity returns. To the best of our knowledge, this question remains unexplored in the current literature. The answer from our empirical results is *yes*.

Specifically, we bring the term structure modeling and option pricing into the commodity markets asset pricing by developing a term structure model based characteristic for individual commodities. The term structure model we adopt is a three-factor Gaussian-affine model of commodity spot price, convenience yield, and risk free interest rate. The model closely follows Casassus and Collin-Dufresne (2005)'s framework, which features parsimony and analytical solutions, and has been the standard Gaussian term structure model unify-ing many previous models for commodity futures pricing in the literature.[1] Thanks to the standard pricing results on affine term structure models (Dai and Singleton, 2000; Duffie, Pan, and Singleton, 2000), the model yields closed-form solutions for futures. We extend the model and derive analytical (up to a numerical integral) pricing formulas for the At The Money (ATM) binary put option, which can be interpreted as the $\mathbb{Q}$(risk neutral)-measure probability of the commodity spot return being negative in the next period.

We apply the *essentially* affine market price of risk specification (Duffee, 2002) to specify the $\mathbb{P}$(physical)-measure dynamics of the three factors. The futures model is estimated using Kalman Filter in conjunction with Maximum Likelihood Estimation (MLE), which is the standard estimation method for term structure models with latent factors (see, e.g., Babbs and Nowman, 1999), for 29 commodity futures in all four markets: Agriculture, Energy, Livestock, and Metals, using daily term structure data of futures prices and US Treasury bill yields (from maturities up to 12 months). The full sample period is from March 1990 to March 2021. To avoid the look-into-future bias, we re-estimate the model at each month using data only up to the estimation month in an expanding window.

Based on the estimated model for each commodity, we infer the prices for the ATM binary put option under both $\mathbb{Q}$ and $\mathbb{P}$ measures. Our term structure model based characteristic is the difference of between the option's prices under $\mathbb{Q}$-measure and $\mathbb{P}$-measure. The fact that we take the difference between $\mathbb{Q}$-measure and $\mathbb{P}$-measure expectations means this characteristic has a natural risk premium interpretation, in-line with the way premium is defined in the variance risk premium literature (see, e.g., Bollerslev, Tauchen, and Zhou, 2009; Bekaert and Hoerova, 2014; Cheng, 2019). Also, since the expectations are for put option payoffs, the implied premium is for the downside risk. By definition, the characteristic has a support within -1 to 1, making it comparable cross commodities. Our analytical results also reveal that the basis defined as the log difference between spot and futures prices is positively related to the ATM binary put option price, i.e., the $\mathbb{Q}$-measure probability of the next period return being negative. This means that the basis only partially

[1] The models nested in Casassus and Collin-Dufresne (2005)'s framework include: Gibson and Schwartz (1990), Brennan (1991), Schwartz (1997), Ross (1997), and Schwartz and Smith (2000).

captures the downside risk premium as it ignores the physical downside risk. Therefore, our characteristic is a theoretically better predictor than the basis for future commodity returns.

From the panel data of the characteristic values, the Principal Components Analysis (PCA) results show that the total variation in the values of the characteristic among all commodities cannot be simply explained by a small set of principal components. This observation is consistent with Daskalaki, Kostakis, and Skiadopoulos (2014) who find that the commodity markets are considerably heterogeneous and the cross-section of individual commodity returns can hardly be explained by a small set of common factors. We then proceed to portfolio sorting based asset pricing tests using the characteristic. Since this characteristic measures the downside risk premium of individual commodities, higher returns are expected for taking long (short) positions on commodities with more positive (negative) downside risk premium. Therefore, we expect the High minus Lower (H-L) portfolios would deliver significantly positive returns on average. Indeed, our empirical results provide strong evidence supporting this hypothesis. More concretely, our H-L portfolios deliver an average monthly return of 0.87% with t-statistic of 2.63. They significantly outperform the H-L portfolios based on sorting other commonly accepted benchmark measures, such as the basis and momentum (Szymanowska et al., 2014). The alpha coefficients of our H-L portfolios are positive and significant in all regressions controlled for a wide range of factors including: the basis (Szymanowska et al., 2014), momentum, commodity market (average returns of all commodities' front month futures), carry (commodity carry returns of Koijen et al., 2018), and Fama-French five factors. We also estimate Lettau, Maggiori, and Weber (2014) downside risk beta for our H-L portfolio returns and find it is not significant, indicating the returns cannot be explained by the market downside risk in the existing literature.

Recent studies predict that the integration and co-movement between commodity and stock markets should be strong due to the prevailing financialization of commodities.[2] In particular, the theoretical studies of Basak and Pavlova (2016) and Goldstein and Yang

[2] Institutional investors entering commodity futures markets is referred to as the financialization of commodities.

(2021) show that the large inflow of financial capital from other financial markets should lead to the segmented commodity futures markets becoming more integrated with financial markets. In a survey paper, Cheng and Xiong (2014) show that correlations of commodity prices with prices in other asset classes, especially stock markets, have noticeably increased after 2000 (see Figure 3 of Cheng and Xiong, 2014).

Despite these predictions, the existing evidence on the predictive power of *commodity returns* on stock index returns has been underwhelming and mixed (see, e.g., Huang, Masulis, and Stoll, 1996; Black et al., 2014; Jacobsen, Marshall, and Visaltanachoti, 2019). We use the Partial Least Square (PLS) method of (Kelly and Pruitt, 2013, 2015) as our primary aggregation method to construct an aggregate predictor from the time series of individual commodity characteristics (TSMCs), denoted as $TSMC^{PLS}$. It is reasonable to assume that the true predictor is unobservable, and each commodity characteristic is just a proxy of it. Statistically, our main target is to extract an aggregate predictor from its underlying proxies (TSMCs) related only to stock returns by removing all noises of the individual errors irrelevant to stock returns. Further, we also use a recently developed aggregation method named sPCA of Huang et al. (2022). By proposing sPCA, Huang et al. (2022) improve PCA by scaling each predictor according to its predictive power for future stock returns, e.g., assigning more weight to more important predictors in forecasting future returns. In addition, we also consider the simple (average) combination of the aggregate predictors constructed with PLS and sPCA. Thus, in addition to $TSMC^{PLS}$, we have two alternative aggregate predictors, $TSMC^{sPCA}$ and $TSMC^{PLS+sPCA}$.

For the out-of-sample analysis, we use both the R^2_{OS} metric of Campbell and Thompson (2008) and the mean squared forecasting errors (MSFE)-adjusted statistic of Clark and West (2007). By using a 20-year expanding estimation window, we show that, $TSMC^{PLS}$, $TSMC^{sPCA}$ and $TSMC^{PLS+sPCA}$ generate economically sizable out-of-sample R^2_{OS} across prediction horizons of up to two years, which are statistically significant in most cases. These results indicate that through the lens of the term structure model more forward looking information than the commodity returns per se can be extracted for effectively predicting stock index returns, confirming the theoretical predictions by Basak and Pavlova (2016) and Goldstein and Yang (2021) regarding the interconnection between commodity markets and stock market.

We also show that mean-variance investors can yield large investment gains from an asset allocation perspective by using our three aggregate TSMC predictors. For a risk aversion coefficient of one, the annualized certainty equivalent return (CER) gains by using $TSMC^{PLS}$, $TSMC^{sPCA}$ and $TSMCPLS + sPCA$ are 10.24%, 6.52%, and 10.25%, respectively, at the monthly horizon. The large investment gains maintain for longer investment horizons and alterative risk aversion coefficients, while they remain sizable after considering for transaction costs.

We further examine whether the out-of-sample forecasting performance exists by using alternative econometric and machine learning methods. By considering the combination ENet (C-ENet) of Dong et al. (2022), the simple (average) combination forecast of the individual univariate forecasts, and the Ridge shrinkage regression of Hoerl and Kennard (1970), we show that all these alternative methods produce economically sizable out-of-sample R^2_{OS}'s across prediction horizons, while all the R^2_{OS}'s generated by C-ENet and Ridge are statistically significant across prediction horizons according to the MSFE-adjusted statistics.

The uniqueness of our contributions lies in the fact that we contribute to the joint venue of three important strands of literature: a) Term structure of futures modeling and option pricing for commodities. In a pioneering study, Gibson and Schwartz (1990) develops and empirically tests a two-factor term structure model for pricing financial and real assets contingent on the price of oil. This two-factor model is subsequently enhanced in Schwartz (1997) by allowing mean-reverting in both spot price and convenience yield. Casassus and Collin-Dufresne (2005) unify the previous models into a three-factor Gaussian affine model with maximal flexibility in the sense of Dai and Singleton (2000). There are further works extending the framework to incorporate stochastic volatility, e.g., Trolle and Schwartz (2009), Liu and Tang (2011), and Chiang, Hughen, and Sagi (2015).

b) Asset pricing of cross-sectional commodity returns. This literature focuses on explaining the cross-section of commodity returns using the well-established methodologies from the empirical asset pricing in the equity market. For example, Szymanowska et al. (2014) and Yang (2013) find the commodity basis has pricing power in the cross-section of commodity portfolios. Daskalaki, Kostakis, and Skiadopoulos (2014) and Bakshi, Gao, and Rossi (2019) show momentum also has asset pricing implication in the commodity markets. Other factors such as commodity market average returns, inventory, and hedging pressure have also been proposed (see, e.g., Erb and Harvey, 2006; De Roon, Nijman, and Veld, 2000; Gorton, Hayashi, and Rouwenhorst, 2013).

c) Stock market return prediction. Welch and Goyal (2008) examine the performance of variables that have been suggested as good predictors of the equity premium and find that these variables have poor performance both in-sample and out-of-sample. Ferreira and Santa-Clara (2011) propose the sum-of-the-parts (SOP) method and use it to forecast stock market returns out of sample, they find that the SOP method produces statistically and economically significant gains and performs better than the historical mean. Huang et al. (2015) apply the partial least square method to traditional investor sentiment proxies and construct an aligned investor sentiment index that has strong predictive power on the aggregate stock market returns. Jiang et al. (2019) find a sentiment index based on the aggregated textual tone of corporate financial disclosures that is a strong negative predictor of future aggregate stock market returns. Jacobsen, Marshall, and Visaltanachoti (2019) empirically show that industrial metals such as copper and aluminium predict stock market returns. Using various shrinkage techniques, Dong et al. (2022) provide evidence on the link between long-short anomaly portfolio returns and the predictability of the aggregate market returns based on 100 representative anomalies from the literature.

We extend existing term structure modeling with option pricing components and develop a term structure model based characteristic for individual commodities, contributing to (a). We show that this characteristic has a natural interpretation of downside risk premium and strong explanatory power for the cross-section of individual commodity returns, contributing to (b). We also confirm that an aggregate commodity index constructed from the individual term structure model based characteristics has strong predictive power for the stock market returns that complement the role of other typical economic predictors, contributing to (c).

The remainder of the paper is organized as follows. Section 2 presents the model and develops the term structure model based characteristic. Section 3 describes the data, model estimation, and summary of the characteristic estimates. Sections 4 and 5 present the empirical results on the cross-sectional asset pricing of commodity returns and predicting stock market returns, respectively. Finally, section 6 concludes the paper. Appendices contain technical details and supplementary results.

2 Term structure model for futures and option pricing

2.1 Commodity futures

We follow Casassus and Collin-Dufresne (2005)'s framework and set up a three-factor $\{r, \delta, X\}$ system to model the log spot commodity price, where r is the risk free interest rate, δ is the convenience yield, and X is the log spot commodity price. We start from the specification of the risk neutral $\mathbb{Q}$-measure dynamic.

$$dr_t = \kappa_r (\bar{r} - r_t)\, dt + \sigma_r dz^{\mathbb{Q}}_{r,t}, \tag{2.1}$$

$$d\delta^0_t = \kappa_\delta \left(\bar{\delta} - \delta^0_t\right) dt + \sigma_\delta dz^{\mathbb{Q}}_{\delta,t}, \tag{2.2}$$

$$dX_t = \left[r_t - \left(\delta^0_t + \alpha_r r_t + \alpha_X X_t\right) - \frac{1}{2}\sigma^2_X\right] dt + \sigma_X dz^{\mathbb{Q}}_{X,t}, \tag{2.3}$$

where $z^{\mathbb{Q}}_{r,t}$, $z^{\mathbb{Q}}_{\delta,t}$, and $z^{\mathbb{Q}}_{X,t}$ are three correlated Wiener processes under the $\mathbb{Q}$-measure. Under this specification, the convenience yield δ_t is a linear combination of δ^0_t, r_t and X_t, i.e., $\delta_t = \delta^0_t + \alpha_r r_t + \alpha_X X_t$. Under the $\mathbb{Q}$-measure, the drift term of X_t is $r_t - \delta_t - \frac{1}{2}\sigma^2_X$ which ensures the arbitrage-free pricing of the futures price. Therefore, we can price the time t futures maturing at T, $F_t(T)$, as the conditional expectation of $\exp(X_T)$ under the $\mathbb{Q}$-measure:

$$F_t(T) = \mathbb{E}^{\mathbb{Q}}_t(e^{X_T}).$$

For notational convenience, we rewrite the three-factor system in the matrix form. Denote Y_t as $[r_t, \delta_t, X_t]^\intercal$:

$$dY_t = (K_0 + K_1 Y_t)dt + \sqrt{\Sigma} dZ^{\mathbb{Q}}_t, \tag{2.4}$$

where $Z_t^{\mathbb{Q}}$ is an IID 3×1 Wiener processes vector under the $\mathbb{Q}$-measure, and

$$\underbrace{K_0}_{3\times 1} = \begin{bmatrix} \kappa_r \bar{r} \\ \kappa_\delta \bar{\delta} \\ -\frac{\sigma_X^2}{2} \end{bmatrix}, \underbrace{K_1}_{3\times 3} = \begin{bmatrix} -\kappa_r & 0 & 0 \\ 0 & -\kappa_\delta & 0 \\ 1-\alpha_r & -1 & -\alpha_X \end{bmatrix}, \underbrace{\Sigma}_{3\times 3} = \begin{bmatrix} \sigma_r^2 & \sigma_r\sigma_\delta\rho_{r\delta} & \sigma_X\sigma_r\rho_{Xr} \\ \sigma_r\sigma_\delta\rho_{r\delta} & \sigma_\delta^2 & \sigma_X\sigma_\delta\rho_{X\delta} \\ \sigma_X\sigma_r\rho_{Xr} & \sigma_X\sigma_\delta\rho_{X\delta} & \sigma_X^2 \end{bmatrix}.$$

The system of (2.4) is a multivariate Gaussian process, the affine techniques developed in Duffie, Pan, and Singleton (2000), Dai and Singleton (2000), and Casassus and Collin-Dufresne (2005) can be readily applied to solve the futures price, $\mathbb{E}_t^{\mathbb{Q}}\left(e^{X_{t+\tau}}\right)$. Specifically, we have:

$$\mathbb{E}_t^{\mathbb{Q}}\left(e^{X_{t+\tau}}\right) = \mathbb{E}_t^{\mathbb{Q}}\left(e^{\iota^\intercal Y_{t+\tau}}\right) = e^{A(\tau)+B(\tau)^\intercal Y_t},$$

and $A(\tau)$ and $B(\tau)$ satisfy the following system of ODEs

$$\frac{\partial B(\tau)}{\partial \tau} = K_1^\intercal B(\tau) \tag{2.5}$$

$$\frac{\partial A(\tau)}{\partial \tau} = K_0^\intercal B(\tau) + \frac{1}{2} B(\tau)^\intercal \Sigma B(\tau) \tag{2.6}$$

with boundary conditions $B(0) = \iota$ and $A(0) = 0$. From (2.5), we have:

$$B(\tau) = \exp(K_1^\intercal \tau)\iota.$$

Given $B(\tau)$, from (2.6) we obtain:

$$A(\tau) = \frac{\iota^\intercal \left[\int_0^\tau \exp(K_1 s)\, \Sigma \exp(K_1^\intercal s)\, ds\right] \iota}{2} + \iota^\intercal \left[\exp(K_1\tau) - I\right] K_1^{-1} K_0.$$

In summary, the futures prices can be written as:

$$F_t(t+\Delta t) = e^{\frac{\iota^\intercal \Omega(K_1,\Delta t)\iota}{2} + \iota^\intercal [\exp(K_1\Delta t)-I] K_1^{-1} K_0 + \iota^\intercal \exp(K_1 \Delta t) Y_t}, \tag{2.7}$$

where $\iota = [0,0,1]^\intercal$, I is the identity matrix, and

$$\Omega(K_1, \Delta t) = \int_0^{\Delta t} \exp(K_1 s)\, \Sigma \exp(K_1^\intercal s)\, ds.$$

2.2 Option pricing and TSMC

Under this setting, we consider the price of a τ-maturity At-The-Money (ATM) binary put option with payoff being one if $X_{t+\tau} < X_t$ and zero otherwise.[3] By Proposition 2 in Duffie, Pan, and Singleton (2000), we have:

$$\begin{aligned}
\mathbb{E}_t^{\mathbb{Q}}\left(\mathbb{1}_{\{X_{t+\tau}<X_t\}}\right) &= \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\exp\left(-\frac{v^2\iota^\intercal\Omega(K_1,\tau)\iota}{2}\right)\mathbf{Im}\left\{\exp\left[iv\begin{pmatrix}\iota^\intercal\left[\exp(K_1\tau)-I\right]K_1^{-1}K_0\\ +\iota^\intercal\exp(K_1\tau)Y_t - X_t\end{pmatrix}\right]\right\}}{v}dv \\
&= \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \exp\left(-\frac{v^2\iota^\intercal\Omega(K_1,\tau)\iota}{2}\right)\sin\left[v\iota^\intercal\begin{pmatrix}\left[\exp(K_1\tau)-I\right]K_1^{-1}K_0\\ +\exp(K_1\tau)Y_t - Y_t\end{pmatrix}\right]/v\,dv \\
&= \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \exp\left(-\frac{v^2\iota^\intercal\Omega(K_1,\tau)\iota}{2}\right)\sin\left[vG(K_0,K_1,\tau,Y_t)\right]/v\,dv \qquad (2.8)
\end{aligned}$$

where $\mathbf{Im}(c)$ denotes the imaginary part of c, and

$$G(K_0, K_1, \Delta t, Y_t) = \iota^\intercal\left[\exp(K_1\Delta t) - I\right]\left(K_1^{-1}K_0 + Y_t\right).$$

By definition, the price of the ATM binary put option $B_t^{\mathbb{Q}}(t+\Delta t) = \mathbb{E}_t^{\mathbb{Q}}\left(\mathbb{1}_{\{X_{t+\Delta t}<X_t\}}\right)$. When valued using information calibrated from the commodity futures, $B_t^{\mathbb{Q}}(t+\Delta t)$ has a natural economic interpretation: it measures the market assessment of the downside risk implicit in the term structure of the commodity futures prices, as (2.8) is the $\mathbb{Q}$ measure probability of the spot price at time $t+\Delta t$ being lower than the current spot price at time t.

We follow Duffee (2002)'s *essentially* affine market price of risk specification and specify

[3] Rigorously speaking the option's pricing should be $\mathbb{E}_t^{\mathbb{Q}}\left(e^{-\int_t^{t+\tau} r_s ds}\mathbb{1}_{\{X_{t+\tau}<X_t\}}\right)$. Since we only consider $\tau =$ one month in our empirical study, the difference is negligible.

the SDE of Y_t under the physical $\mathbb{P}$-measure as[4]

$$dY_t = (K_0^{\mathbb{P}} + K_1^{\mathbb{P}} Y_t)dt + \sqrt{\Sigma} dZ_t^{\mathbb{P}}. \tag{2.9}$$

Given (2.9), we define the $\mathbb{P}$ measure probability of the spot price at time $t + \Delta t$ being lower than the current spot price at time t as:

$$B_t^{\mathbb{P}}(t+\Delta t) = \mathbb{E}_t^{\mathbb{P}}\left(\mathbb{1}_{\{X_{t+\Delta t} < X_t\}}\right) \tag{2.10}$$

$$= \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\exp\left(-\frac{v^2 \iota^\intercal \Omega\left(K_1^{\mathbb{P}}, \Delta t\right)\iota}{2}\right)\sin\left[vG(K_0^{\mathbb{P}}, K_1^{\mathbb{P}}, \Delta t, Y_t)\right]}{v} dv. \tag{2.11}$$

By definition, $B_t^{\mathbb{P}}(t+\Delta t)$ measure the physical (statistical) downside risk implied by the historical dynamics of the commodity futures prices. Our model-based commodity characteristic, which we refer to as Term Structure Model based Characteristic (TSMC), is defined as the difference between $B_t^{\mathbb{Q}}(t+\Delta t)$ and $B_t^{\mathbb{P}}(t+\Delta t)$:

$$\text{TSMC}_t(\Delta t) = B_t^{\mathbb{Q}}(t+\Delta t) - B_t^{\mathbb{P}}(t+\Delta t). \tag{2.12}$$

$\text{TSMC}_t(\Delta t)$ has a natural risk premium interpretation. This is similar to the way premium is defined in the variance risk premium literature (see, e.g., Bollerslev, Tauchen, and Zhou, 2009). By definition, TSMC_t's support is within -1 to 1, which makes it naturally comparable cross commodities and an ideal measure to be used for index construction.

2.3 Downside risk interpretation of the basis

The basis in the commodity literature is often defined as the log difference between spot and near maturity futures prices (Yang, 2013; Daskalaki, Kostakis, and Skiadopoulos, 2014), i.e., $\text{Basis}_t = X_t - \log F_t(t+\Delta t)$.

[4] This specification imposes zero correlation across commodities on the correlation structure of the vector of Wiener processes under the $\mathbb{P}$-measure. This allows us to avoid an intractable joint estimation with all commodities and estimate the model for individual commodities separately. As shown in Casassus and Collin-Dufresne (2005), relaxing this restriction offers little improvement in the model estimation.

In the light of (2.7), (2.8) can be rewritten as a function of the basis:

$$B_t^{\mathbb{Q}}(t+\Delta t) = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} e^{-\frac{v^2 \iota^\intercal \Omega(K_1,\Delta t)\iota}{2}} \sin\left[v\left(\text{Basis}_t + \frac{\iota^\intercal \Omega\left(K_1, \Delta t\right)\iota}{2}\right)\right] / v\, dv. \quad (2.13)$$

From (2.13), we can see that the market assessment of the downside risk $B_t^{\mathbb{Q}}(t+\Delta t)$ clearly increases with the basis. This theoretical finding is in-line with those in Szymanowska et al. (2014) and Gorton, Hayashi, and Rouwenhorst (2013) who infer from their models that the basis contains information about the spot risk premium. As we shown above, the clean measure of risk premium is $\text{TSMC}_t(\Delta t)$ in (2.12). Therefore, even though the basis contains information about the risk premium, it only measures the total price ($\mathbb{Q}$-expectation) of downside risk that does not offset the $\mathbb{P}$-measure downside risk. Given the theoretical superior of our risk premium measure over the basis, we expect TSMC_t to have better performance in commodity asset pricing tests than the basis and other price based measures, e.g., the momentum.

3 Data and model estimation

3.1 Data

We use the daily data on 29 individual commodity futures contracts from Refinitiv. The names and types are summarized in Table 1. The futures maturities range from one month to 12 months (with a few exceptions up to 18 months in the early part of the sample). The data sample covers period from March 1990 to March 2021. We reserve the data from March 1990 to December 2000 for initial model estimation, and study the asset pricing implications using the data from January 2001 to March 2021 where all model outputs are out-of-sample estimates. We plot the cross-sectional distribution of the monthly total returns (start from January 2001 as one) of the front-month futures in Figure 1. The summary statistics of the front-month monthly returns during the same period are presented in Table 1. To estimate the parameters of the risk free interest rate model, we also use four-week, three-month, six-month, and one-year Treasury bill yields data downloaded from Federal Reserve Economic

Data at Federal Reserve Bank of St. Louis. The sample period of the Treasury bill yields is matched with that of the commodity futures data.

[*Insert Table 1 and Figure 1 about here*]

3.2 Kalman filter and maximum likelihood estimation

Consistent with Casassus and Collin-Dufresne (2005), we set:

$$K_0^{\mathbb{P}} = \begin{bmatrix} \alpha_r^{\mathbb{P}} \\ \alpha_\delta^{\mathbb{P}} \\ \alpha_X^{\mathbb{P}} \end{bmatrix} \text{ and } K_1^{\mathbb{P}} = \begin{bmatrix} \beta_r^{\mathbb{P}} & 0 & 0 \\ 0 & \beta_\delta^{\mathbb{P}} & 0 \\ \beta_{rX}^{\mathbb{P}} & \beta_{\delta X}^{\mathbb{P}} & \beta_X^{\mathbb{P}} \end{bmatrix}.$$

This setting ensures that the short rate follows an autonomous Ornstein-Uhlenbeck (OU) process under both $\mathbb{P}$ and $\mathbb{Q}$ measures, and the component of the convenience yield (δ_t^0) that is linearly independent of interest rate and spot price level under $\mathbb{Q}$ measures remains so under the $\mathbb{P}$ measure.

Since the log of the futures price is an affine function of the state variables, we can employ the Kalman filter (KF) in conjunction with MLE for the model estimation (see, e.g., Babbs and Nowman, 1999; De Jong, 2000). Details of the KF are given in Appendix A. We discretize and re-write the model in state space form, and use log of the futures price as the measurement equation. Specifically, the state space representation is:

$$Y_t = \left[\exp(K_1^{\mathbb{P}} h) - I\right] \left(K_1^{\mathbb{P}}\right)^{-1} K_0^{\mathbb{P}} + \exp(K_1^{\mathbb{P}} h) Y_{t-h} + \sqrt{\Omega\left(K_1^{\mathbb{P}}, h\right)}\epsilon_t, \quad \epsilon_t \sim \mathrm{IID}\boldsymbol{\mathcal{N}}\left(0, I_{3\times 3}\right)$$

Transition Equation

$$\log\left(\overline{F}_t^{\Delta t}\right) = \frac{\iota^\intercal \Omega\left(K_1, \Delta t\right)\iota}{2} + \iota^\intercal\left[\exp(K_1 \Delta t) - I\right] K_1^{-1} K_0 + \iota^\intercal \exp(K_1 \Delta t) Y_t + \xi_t, \quad \xi_t \sim \mathrm{IID}\boldsymbol{\mathcal{N}}\left(0, s I_{n_t \times n_t}\right),$$

Measurement Equation

where h is the time interval (one day), ξ_t is the measurement error on day t, and n_t is the number of futures on day t. The measurement $\log\left(\overline{F}_t^{\Delta t}\right)$ consists of log of all available futures (with various time to maturities) close prices observed on day t. Given the normally distributed measurement error, the distribution of $\log\left(\overline{F}_t^{\Delta t}\right)$ conditional on the information set $\boldsymbol{\mathcal{F}}_{t-h}$ is a multi-dimensional normal distribution with the mean $\log\left(\overline{F}_{t|t-h}^{\Delta t}\right)$ and covariance matrix $P_{\log\left(\overline{F}_t^{\Delta t}\right)}$. Thus, the transition density of $\log\left(\overline{F}_t^{\Delta t}\right)$ can be written as:

$$p_t = \left[(2\pi)^{\frac{n_t}{2}} \left| P_{\log\left(\overline{F}_t^{\Delta t}\right)} \right|^{\frac{1}{2}}\right]^{-1} \exp\left\{-\frac{1}{2}\left[\log\left(\overline{F}_t^{\Delta t}\right) - \log\left(\overline{F}_{t|t-h}^{\Delta t}\right)\right]^{\intercal} P^{-1}_{\log\left(\overline{F}_t^{\Delta t}\right)} \left[\log\left(\overline{F}_t^{\Delta t}\right) - \log\left(\overline{F}_{t|t-h}^{\Delta t}\right)\right]\right\},$$

where $\log\left(\overline{F}_{t|t-h}^{\Delta t}\right)$ and $P_{\log\left(\overline{F}_t^{\Delta t}\right)}$ are outputs from the Kalman filter update. Then the log-likelihood function is given by:

$$\ln \mathscr{L} \propto -\sum_{t=1}^{N} \log\left(p_t\right).$$

where N is the total number of days in the sample. Please note that the parameters of the risk-free interest rate r_t cannot be identified using only futures prices, they need to be estimated separately using the Treasury yields data (Casassus and Collin-Dufresne, 2005). Since the interest rate is modelled as a one-factor OU process, it is essentially the Vasicek (1977) model.[5] The same estimation method above can be applied to estimate the parameters of r_t using the Treasury yield data.

To make sure the TSMC estimates are ex-ante without look ahead bias, we conduct an expending window out-of-sample estimation. More concretely, we re-estimate the model each month after adding one month of daily data to the sample in each estimation. Each month we use the KF based on the previous month's model parameters to filter out state variables from the futures prices and compute the TSMCs. Since we use monthly returns in the asset pricing tests, we compute the TSMC with $\Delta t =$ one month. Hereafter, all referred TSMCs are associated with $\Delta t =$ one month unless stated otherwise explicitly.

3.3 Summary of the characteristics

In Figure 2, we plot the cross-sectional distribution of the TSMCs in the four types of commodities from 2001 to 2021. On average, the TSMCs are slightly negative after 2006,

[5]Details of the model can also be found in Casassus and Collin-Dufresne (2005, Appendix B).

especially so during the 2008 financial crisis. Looking at the aggregate level in the four types individually, the TSMCs in Agriculture (Energy) are more negative (positive), while those in Livestock and Metals are closer to zero with higher volatility found in Livestock than in Metals. Cross-sectionally, Agriculture and Energy exhibit more cross-sectional variation than Livestock and Metals over the years. It is worth noting that in Energy there is a shock to the cross-section of the TSMCs during 2014 to 2016, which corresponds to the steep plunge in the oil price during this period (Friedman, 2014). This large spike in the cross-sectional dispersion in the TSMCs reflects the unprecedented uncertainty shock. The differential cross-sectional variation among the commodities over time alongside its unified support within -1 and 1 make the individual TSMC a great characteristic to conduct asset pricing tests in the commodity markets.

[*Insert Figure 2 about here*]

3.4 Principal component analysis

In this subsection, we carry out the Principle Component Analysis (PCA) on the panel the TSMC estimates. The results show that the total variation in the TSMCs cannot be explained by a small set of common components.

Due to that several commodity index have many missing values before 2000, we perform the PCA based on the out-of-sample period, starting from January 2001 to March 2021.

[*Insert Figure 4 about here*]

Figure 4 plots the variation of the 29 TSMCs explained by the first 10 PCs. From the plot, we see that none of the PCs can catch up large variations in the panel of the TSMCs. The first, second, and thrid PCs only explain 36%, 14% and 8%, respectively, of the total variation. The remaining PCs hardly have explanatory power more than 5%. All 10 PCs together can only explain less than 90% of the total variation.

Next, we examine if there is any commonality in factor loadings of the TSMCs on these PCs. We run time series regressions (regressing all 29 TSMCs on the first five PCs) and collect the loading coefficients. A summary of these coefficients is presented in Table 2.

[*Insert Table 2 about here*]

From Table 2, we find that there is no clear pattern can be found in the loading coefficients' distribution, neither at individual level nor at sector level. All the loading estimates are small on average and with sizable cross-sectional standard deviations.

In general, we conclude that no small set of common factors can explain the overall variation in the panel of the TSMCs. This observation is consistent with Daskalaki, Kostakis, and Skiadopoulos (2014) who find that the commodity markets are considerably heterogeneous.

4 Commodity asset pricing

4.1 Portfolio sorting

Since the TSMC measures the downside risk premium of individual commodities, a positive TSMC indicates that investors are willing to pay more to avoid the downside risk. More concretely, the TSMC is the difference between the market value of the ATM binary put option and its actuarial value. The higher TSMC, the more investors are willing to pay for the put beyond what can be justified by its physical risk. Taking long positions in a commodity is somewhat analogous to writing put options for this commodity. Therefore, when a commodity's TSMC is high (low), positive returns are expected for taking long (short) positions on this commodity. Given this insight, we expect the High minus Lower (H-L) portfolio based on the TSMC would deliver significantly positive returns on average.

We form quartile portfolios using all 29 commodity front month futures returns based on the TSMCs at one month horizon. The formation period is one day before the start of the one month holding period. The portfolios are rebalanced at teh end of each month. The portfolio sorting results are presented in Table 3. For benchmarking purposes, we also form portfolios based on the basis (the difference between the log of front month futures and the log of second month futures on the formation period) and the momentum (the past one year cumulative return on the formation period). These two benchmarks have been commonly studied in the previous commodity asset pricing literature, e.g., Yang (2013), Szymanowska et al. (2014), and Daskalaki, Kostakis, and Skiadopoulos (2014).

[*Insert Table 3 about here*]

From Table 3, we can see that in the full sample (January 2001 to March 2021), only the TSMC H-L portfolios show significant returns: an average monthly return of 0.87% with t = 2.63, while Basis and Momentum produces insignificant returns: an average monthly return of 0.29% (t = 0.84) for Basis and 0.02% (t = 0.04) for Momentum. To further understand the difference between performance of the TSMC and the benchmarks, we also conduct subsample analysis. In the first half sample (January 2001 to May 2011), we find the TSMC and Basis portfolios have similar performance and they all deliver significant returns: an average monthly return of 0.93% (t = 1.98) for TSMC and 0.94% (t = 1.91) for Basis. Although insignificant, the Momentum portfolios also deliver a sizeable average monthly return of 0.76% (t = 1.36). The benchmark results from the first half sample confirm the findings documented in the previous studies using data before 2011. However, when looking at the second half of the sample, we see a different picture. Although less significant than the first half and the full sample returns, the TSMC portfolios produce a significant average monthly return of 0.82% (t = 1.73). In contrast, the average montly returns from both the Basis and Momentum portfolios turn negative and insignificant (-0.4% with t = -0.84 for Basis and -0.76% with t = -1.43 for Momentum). These observations show that the significant pricing power of Basis and Momentum documented in the previous literature disappears in recent 10 years of data, while the TSMC has robust performance in both samples. These observations become even clearer when we plot out the cumulative returns of the three portfolios over time. The time series are shown in Figure 3. Before 2015, the cumulative returns of all three portfolios have a clear upward trajectory. After 2015, the TSMC portfolio continues with its upward trajectory, while the two benchmarks diverge from the TSMC portfolio and develop into clear downward trajectories.

[*Insert Figure 3 about here*]

4.2 Potential explanations for the TSMC portfolio

By eyeballing Figure 3, we can also easily tell that the TSMC portfolio's returns can hardly be explained by the Basis or Momentum factor. We verify this conjecture by following the standard asset pricing practice and examining the alpha coefficients from regressing the TSMC portfolio returns on various other factors. These factors include: Basis, Basis High and Low portfolios, Momentum, commodity market portfolio, commodity carry factor (Koijen et al., 2018) and Fama-French five factors (Fama and French, 2015). Here, the Basis and Momentum are the High - Low portfolios. We also include the separate Basis High and Basis Low portfolios as another control considering the fact that Szymanowska et al. (2014) show that in additional to the Basis H-L's explanatory power on commodity futures' spot premia, the two separate portfolios can explain commodity futures' term premia. The results are shown in the left panel of Table 4. From the results, we can clearly see all alpha coefficients are significant on at least 5% level, indicating none of these factors is able to explain our TSMC portfolio's returns.

[*Insert Table 4 about here*]

Since the TSMC has a natural interpretation of downside risk premium, we explore whether common downside risk can explain the TSMC portfolio's returns. Lettau, Maggiori, and Weber (2014) (LMW) propose a downside risk CAPM in which expected returns are driven by the market beta and the market beta conditional on low returns. Following Koijen et al. (2018), we examine the TSMC portfolio's downside risk exposure by checking the significance of its LMW downside risk beta. The LMW downside risk beta is estimated by conditionally regressing the TSMC portfolio's returns on the stock market returns when the market returns are one standard deviation below their sample mean. For comparison, we also apply the same estimation to the Basis, Momentum, and commodity market portfolios. The results are reported in Table 5.

[*Insert Table 5 about here*]

The downside risk betas are significant for the Basis (at 10% level) and commodity market (at 1% level) portfolios, which is consistent with some of the results in Lettau, Maggiori, and Weber (2014) and Koijen et al. (2018). However, the downside risk beta is not significant for the TSMC portfolio which also has a highly significant alpha. These results indicate that despite the fact of the TSMC being a downside risk premium measure, the TSMC portfolio's returns cannot be explained by the market downside risk. Taken together, our results cannot be explained by, and are not subsumed by, a variety of well-established factors and risks in both commodity and equity markets.

4.3 Can TSMC returns explain other factors?

As shown above that the TSMC portfolio's returns are not spanned by existing factors, we now turn the tables and ask how much of existing factors can be explained by the TSMC portfolio. This is presented in the right panel of Table 4, where we report time-series regressions of various existing factors on the TSMC portfolio returns.

The first column shows the factors under consideration. The column labeled "Mean" reports the mean of the factor returns over the 2001 to 2021 period. The remaining three columns report the regression intercept (Alpha), the loading on the TSMC H-L portfolio returns (Beta), and the regression R^2. We first examine the commodity related factors. They are Basis H-L returns, Basis High portfolio returns, Basis Low portfolio returns, Momentum H-L returns, Commodity Market returns, and Carry returns of Koijen et al. (2018). The results are reported in the first six rows. Among the six commodity related factors, only the Carry has a significant mean value of 56 basis points per month in our sample, but this drops to a statistically insignificant 22 basis points when we account for comovement the TSMC H-L returns. The regression R^2 for the Carry is as high as 16%, indicating our TSMC does a good job explaining the Carry returns. Although the mean values of other commodity related factors are statistically insignificant, they all load on the TSMC H-L return significantly, as shown by the significant beta coefficients in column labeled "Beta". The R^2's are also as high as 20% and 11% for the Basis and Basis High, respectively, again confirming the strong explanatory power of the TSMC on Basis/Carry premiums. Given the theoretical connection between the TSMC and Basis revealed in Section 2.3, and that TSMC amounts to a better risk premium measure than Basis, the results observed here are somewhat expected.

Next, we examine the equity market related factors. Here we consider the Fama-French five factors individually. They are FF5F-mkt (market factor, the excess return on the equity market), FF5F-smb (size factor, the average return on the nine small stock portfolios minus the average return on the nine big stock portfolios), FF5F-hml (value factor, the average return on the two value portfolios minus the average return on the two growth portfolios), FF5F-rmw (Robust Minus Weak factor, the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios), and FF5F-cma (Conservative Minus Aggressive, the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios). Although the R^2's are lower for the equity factors in general, the TSMC still manages to bring the mean values of FF5F-mkt and FF5F-smb from significant 65 basis points and 30 basis points down to insignificant 48 basis points and 22 basis points, respectively. It is worth mentioning that FF5F-mkt load highly significantly on the TSMC portfolio with a beta coefficient of 0.2. Considering the fact that the construction of TSMC does not use any equity market information directly, this explanatory power on the equity market factor, FF5F-mkt, is impressive. The TSMC does less well at explaining the other three FF5F factors, as reflected in lower R^2's, although FF5F-rmw and FF5F-cma still load significantly and negatively on the TSMC portfolio.

In sum, as expected the TSMC portfolio can explain the Basis/Carry factors of the commodity market well. We also find that the TSMC portfolio does a good job explaining equity factors, especially the equity market factor, FF5F-mkt. This points to an interesting interaction between TSMCs and the equity market returns, which we explore in more detail from a different perspective in the next section.

5 Predicting stock market returns using TSMCs

In previous sections, we show the TSMCs are a characteristic that explains the cross-sectional commodity returns. In this section, we show that they also have predictive power going beyond the commodity markets and can be used to forecast the future stock market returns. The last two decades have witnessed extensive financialization of commodities in which institutional investors enter commodity futures markets. As a result of this prevailing financialization, theoretical studies (see Basak and Pavlova, 2016; Goldstein and Yang, 2021) predict strong co-movement of commodity and stock markets. However, the existing evidence on the predictive power of *commodity returns* on stock index returns has been underwhelming and mixed (see, e.g., Huang, Masulis, and Stoll, 1996; Black et al., 2014; Jacobsen, Marshall, and Visaltanachoti, 2019). The strong evidence of the TSMC's predictive power on the stock market returns suggests that through the lens of the term structure model more forward looking information than the commodity returns per se can be extracted for effective prediction.

5.1 In-sample analysis and comparison with economic variables

We use the Partial Least Square (PLS) (PLS, see, e.g., Kelly and Pruitt, 2013, 2015) to collectively and efficiently construct aggregate predictors from the individual commodity indices by eliminating the negative effects of irrelevant to forecasting terms based on the future stock market excess returns. See **??** for technical details on the PLS. Further, we use a recently developed aggregation method named sPCA of Huang et al. (2022), and also consider the simple (average) combination of the aggregate predictors constructed with PLS and sPCA. Table 6 provides the pairwise correlations among the 24 individual TSMCs used in this section[6]. The correlation coefficients range from -0.69 to 0.90, suggesting that these 24 individual TSMCs capture both common and different aspects. Hence, aggregate TSMC predictors are necessary since using only individual TSMCs is unlikely to be complete in terms of the aggregate effect for forecasting the stock market.

[*Insert Table 6 about here*]

[6]We use 24 (out of the 29 in Table 1) individual TSMCs with full data history over the entire sample period (February 1994 to March 2021). Full history of data is necessary for the estimation of the various aggregation methods.

Specifically, we first run the following univariate predictive regression:

$$R_{t+1} = \alpha + \beta X_t + \epsilon_{t+1}, \tag{5.1}$$

where the stock market excess return at $t+1$ (R_{t+1}) is defined as the difference between the value-weight return of all CRSP firms in the US and the one-month T-bill rate. X_t is either one of the 14 economic variables in Welch and Goyal (2008), the investor sentiment index of Huang et al. (2015) denoted as S (available up to Dec 2020), the short interest index of Rapach, Ringgenberg, and Zhou (2016) denoted as SII, or an aggregate predictor of TSMCs constructed with PLS ($TSMC^{PLS}$), sPCA ($TSMC^{sPCA}$), and PLS+sPCA ($TSMC^{PLS+sPCA}$).

The in-sample forecasting ability of X_t is tested by estimating regression Equation (5.1) over the entire sample period (February 1994 to March 2021). The null hypothesis is that X_t has no predictive power ($\hat{\beta} = 0$) and in this case, regression equation (5.1) reduces to $R_{t+1} = \alpha + \varepsilon_t$. The alternative hypothesis is that β is different from zero, and hence X_t contains useful information for predicting R_{t+1}. We use the Newey and West (1987) standard error to compute the t-statistic for $\hat{\beta}$.

Table 7a shows that out of the 14 economic predictors, only dividend yield ratio (dy) displays significant positive predictive power for the market return at the 10% significance level, while its in-sample R^2 is larger than 1% (1.62% for dp and 1.90% for dy). All our three aggregate predictors $TSMC^{PLS}$, $TSMC^{sPCA}$, and $TSMC^{PLS+sPCA}$ have positive statistical significant slopes at 1%, and their in-sample R^2's are at 4.30%, 2.75%, and 4.28%, respectively. Hence, they outperform the 14 economic predictors, as well as the short interest and sentiment indices, in forecasting the excess stock market returns in-sample.

[*Insert Table 7 about here*]

We then investigate whether the predictive power of TSMC^{PLS} remains significant after controlling for economic predictors. We conduct the following bivariate predictive regressions:

$$R_{t+1} = \alpha + \beta \text{TSMC}_t^{PLS} + \psi Z_t^{\kappa} + \varepsilon_{t+1}, \ \kappa = 1, \ldots, 16 \tag{5.2}$$

where Z_t^{κ} is one of the 14 individual economic predictors, the sentiment index, or the short interest index. Table 7b shows that the estimates of the regression slopes of TSMC_t^{PLS} remain statistically significant after controlling for each of the individual economic variables, the sentiment index, or the short interest index, suggesting that economic fundamentals fail to explain the impact of TSMC^{PLS} on forecasting the excess stock market returns. Further, the magnitude of the β estimates of TSMC^{PLS} is quite large since they are greater than 0.82% in all cases, while all of the in-sample R^2's in equation (5.2) are greater than 4.30% and substantially larger than those in equation (5.1), pointing out the economic significance of TSMC^{PLS}. Similarly to $TSMC^{PLS}$, $TSMC^{sPCA}$, and $TSMC^{PLS+sPCA}$ also reach the same conclusions as shown in the Table 7c and Table 7d. Overall, Table 7 shows that TSMC^{PLS} has strong predictive ability for the excess stock market returns in-sample beyond economic predictors.

5.2 Out-of-sample analysis

Although an in-sample analysis provides more efficient estimates and precise forecasts by exploiting the entire sample period, Welch and Goyal (2008), among many others, argue that out-of-sample forecasting evaluations are more relevant in practice. We start with an initial estimation window to generate the first out-of-sample forecast return as follows:

$$\hat{R}_{t+h} = \hat{\alpha} + \hat{\beta} X_t \tag{5.3}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are the OLS estimates of the predictive regression: $R_{t+h} = \alpha + \beta X_t + \epsilon_{t+h}$, where R_{t+h} is the average stock market excess return over the prediction horizon h. We then use an expanding estimation window approach and recursive predictive regressions to generate the out-of-sample forecasts for the following periods until the end of the sample period.

We evaluate the out-of-sample forecasting performance by using the R^2_{OS} statistic of Campbell and Thompson (2008) that measures the proportional reduction in mean squared forecast error (MSFE) for the predictive regression relative to the historical average. The out-of-sample analysis is based on a 20-year expanding estimation window. This is in line with Rapach, Strauss, and Zhou (2010), Huang et al. (2015) and Chen et al. (2022), among others, who also use a relatively long initial estimation window so that the various parameters in the aggregation techniques used in this section are estimated with more precision In addition, we use the Clark and West (2007)'s MSFE-adjusted statistic for testing the hypothesis $H_0 : R^2_{OS} \leq 0$ against $H_A : R^2_{OS} > 0$ to uncover whether the predictive forecasts generate a statistically significant improvement in MSFE.

[*Insert Table 8 about here*]

Table 8 presents the out-of-sample forecasting results. We find that all three aggregate TSMC indices $TSMC^{PLS}$, $TSMC^{sPCA}$, and $TSMC^{PLS+sPCA}$ generate economically sizable out-of-sample R^2_{OS}'s across prediction horizons (up to two years, i.e., h=24). For example, the R^2_{OS} of $TSMC^{PLS}$ equals 1.07% at the monthly horizon, and increases to over 5% for prediction horizons of six months and above (h $\geq$6). More importantly, the R^2_{OS}'s of $TSMC^{PLS}$ are statistically significant across prediction horizons (except for h=3) according to the MSFE adjusted statistics, meaning that the out of sample MSFEs generated by $TSMC^{PLS}$ are significantly lower than that of the historical means. Similarly to $TSMC^{PLS}$, $TSMC^{sPCA}$ and $TSMC^{PLS+sPCA}$ also generate statistically significant R^2_{OS}'s in most of the prediction horizons we examine. Since a monthly out-of-sample R^2_{OS} of 0.5% can generate substantial economic value (Campbell and Thompson, 2008), in the following sub-section, we also conduct an asset allocation analysis to examine potential gains for mean-variance investors by using, $TSMC^{PLS}$, $TSMC^{sPCA}$, and $TSMC^{PLS+sPCA}$.

5.3 Economic value of predicting stock index

This sub-section evaluates the economic value of forecasting stock market returns with the aggregate predictor from the portfolio management perspective. Following Kandel and Stambaugh (1996), Campbell and Thompson (2008), Ferreira and Santa-Clara (2011), Huang et al. (2015), Jiang et al. (2019), Chen et al. (2022), among others, we use the certainty equivalent return (CER), which can be interpreted as the risk-free return that an investor would trade for a higher return associated with a given risk.

Suppose a mean-variance investor allocates his wealth between the stock market and the risk-free asset. Assume that he is maximizing the next one-month expected utility by investing a proportion of w_t to the stock market and a proportion of $1 - w_t$ to the risk-free asset at the start of each month:

$$U(R_{p,t+1}) = \mathbb{E}(R_{p,t+1}) - \frac{\lambda}{2}\mathbb{V}ar(R_{p,t+1}) \tag{5.4}$$

where $\mathbb{E}(R_{p,t+1})$ and $\mathbb{V}ar(R_{p,t+1})$ denote the mean and variance of the excess portfolio returns at $t+1$, and λ is the investor's risk aversion. The investor's portfolio return at the end of each month $(t+1)$ is given by:

$$R_{p,t+1} = w_t R_{t+1} + R_{f,t+1} \tag{5.5}$$

where R_{t+1} and $R_{f,t+1}$ are the excess stock market return and the risk-free rate, respectively, at $t+1$. With some simple algebra, one can easily compute the optimal portfolio weight to the stock market, w_t, at time t as follows:

$$w_t = \frac{1}{\lambda}\frac{\hat{R}_{t+1}}{\hat{\sigma}^2_{t+1}} \tag{5.6}$$

where the mean $(\hat{R}_{t+1})$ and variance $(\hat{\sigma}^2_{t+1})$ estimates of the market excess returns at $t+1$ used for computing w_t in Equation (5.6) are estimated based on information up to time t. We use a 60 month rolling window for estimating the variance of market excess returns, and follow Campbell and Thompson (2008) by ruling out short selling and allowing at most 50% leverage such that $w_t \geq 0$.

The CER of a portfolio is:

$$CER = \hat{\mu}_p - 0.5\lambda\hat{\sigma}^2_p \tag{5.7}$$

where $\hat{\mu}_p$ and $\hat{\sigma}^2_p$ are the mean and variance of portfolio excess returns over the entire out-of-sample period. The difference between the CERs (and SRs) by using TSMC^{AGG} and the historical means is a measure of the predictability's economic value.

[*Insert Table 9 about here*]

Table 9 reports the annualized CER (%) gains from asset allocation of a mean-variance investor with $\lambda = 1, 3$, and 5 for predicting future market excess returns by using, $TSMC^{PLS}$, $TSMC^{sPCA}$, and $TSMC^{PLS+sPCA}$, and relative to historical mean returns. We observe that the aggregate TSMC predictors generate economically sizable investment profits across prediction horizons, except for $TSMC^{sPCA}$, at the annual horizon when considering transaction costs. When there is no transaction cost, the CER gains of $TSMC^{PLS}$, for $\lambda = 1, 3$, and 5 are 10.24%, 3.41%, and 2.05%, respectively, at the monthly horizon, implying that an investor would be willing to pay an annual fee of up to 1024 ($\lambda = 1$), 314 ($\lambda = 3$), and 205 ($\lambda = 5$) basis points (bps) to access the predictive forecasts of $TSMC^{PLS}$. These large investment gains also maintain at longer investment horizons and remain sizable when there is a transaction cost of 50 bps. Similarly to $TSMC^{PLS}$, $TSMC^{sPCA}$ and $TSMC^{PLS+sPCA}$ also generate substantial investment profits. The CER gains of $TSMC^{sPCA}$ (or $TSMC^{PLS+sPCA}$) for $\lambda = 1, 3$, and 5 are 6.52% (10.25%), 2.17% (3.42%), and 1.30% (2.05%), respectively, at the monthly horizon. These gains also exist at longer investment horizons and remain sizable after considering for transaction costs.

In summary, there are potentially large economic gains in the asset allocation based on aggregate TSMC predictors, suggesting substantial economic values for mean-variance investors. This analysis then emphasizes the crucial role of aggregate predictors constructed with TSMCs on the stock market from an investment management perspective.

5.4 Out-of-sample analysis with alternative methods

In the previous sub-sections, we have shown that market returns can be significantly predicted by, $TSMC^{PLS}$, $TSMC^{sPCA}$ and $TSMC^{PLS+sPCA}$. We now examine whether our conclusions are robust to alternative econometric and machine learning methods. Particularly, we consider the combination ENet (C-ENet) of Dong et al. (2022), the simple (average) combination forecast of the individual univariate forecasts (Ave), and the Ridge shrinkage regression of Hoerl and Kennard (1970). These methods are introduced in detail in the Supplementary (Online) Appendix.

[*Insert Table 10 about here*]

Table 10 presents the out-of-sample results. There are many observations. First, all the three alternative methods generate economically sizable out-of-sample R^2_{OS}'s across prediction horizons (up to two years, i.e., h=24). Second, all the R^2_{OS}'s generated by C-ENet and Ridge are statistically significant across prediction horizons according to the MSFE-adjusted statistics. Finally, while these there alternative methods work well for predicting market returns, they generally underperform the PLS method especially for longer prediction horizons. This finding is in accordance toKelly and Pruitt (2015)'s conclusion, e.g., the PLS forecast is asymptotically consistent and generates the minimum MSFE as long as the consistency condition is satisfied.

6 Conclusion

Building on the term structure modeling and option pricing literature, we develop a term structure model based characteristic for individual commodities. This characteristic measures the downside risk premium implicit in the term structure of individual commodities' futures. We find that this characteristic has strong explanatory power for the cross-section of commodity returns. None of the well-known commodity market and equity market factors are able to explain the returns of the H - L portfolio constructed from sorting this characteristic. The returns also do not load on the market downside risk. This characteristic not only explains the cross-section of commodity returns, but also predicts the stock market returns. In an predictive exercise, we show that an aggregate commodity index constructed from the individual characteristics using the PLS has significant predictive power for the stock market returns that goes beyond the role of other typical economic predictors.

Our results point to new directions for future research in the commodity market. On the one hand, the advance in the term structure modeling literature has proven fruitful in commodity markets' cross-sectional asset pricing. Nevertheless, more work can be done in extracting more information useful for asset pricing from futures prices using term structure models. On the other hand, accurate measures of downside risk premium in individual commodities seem to be rather heterogeneous. It remains an open question whether the heterogeneity is due to financial market structure or industrial/production structure.

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Figure 1: Front month futures total return dynamics, January 2001 - March 2021

This figure plots the dynamics of total returns across the front month futures of all 29 commodities from January 2001 to March 2021. The total returns start from one for all commodities. The solid line is the cross-sectional mean, the grey area is the cross-sectional 90 and 10 percentiles.

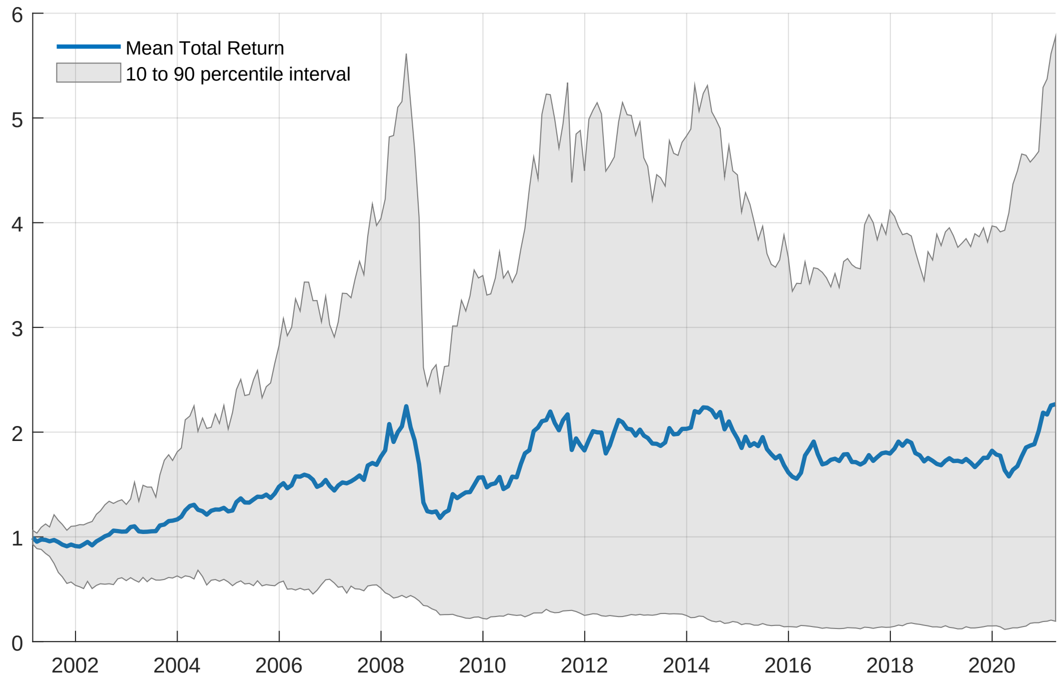

Figure 2: Dynamics of cross sectional risk premiums: January 2001 - March 2021

The five panels in this figure plot the dynamics of the cross-section TSMCs from January 2001 to July 2020 for all types (All), Agriculture, Energy, Livestock, and Metals, respectively. The solid line is the cross-sectional mean, the grey area is the cross-sectional maximum and minimum.

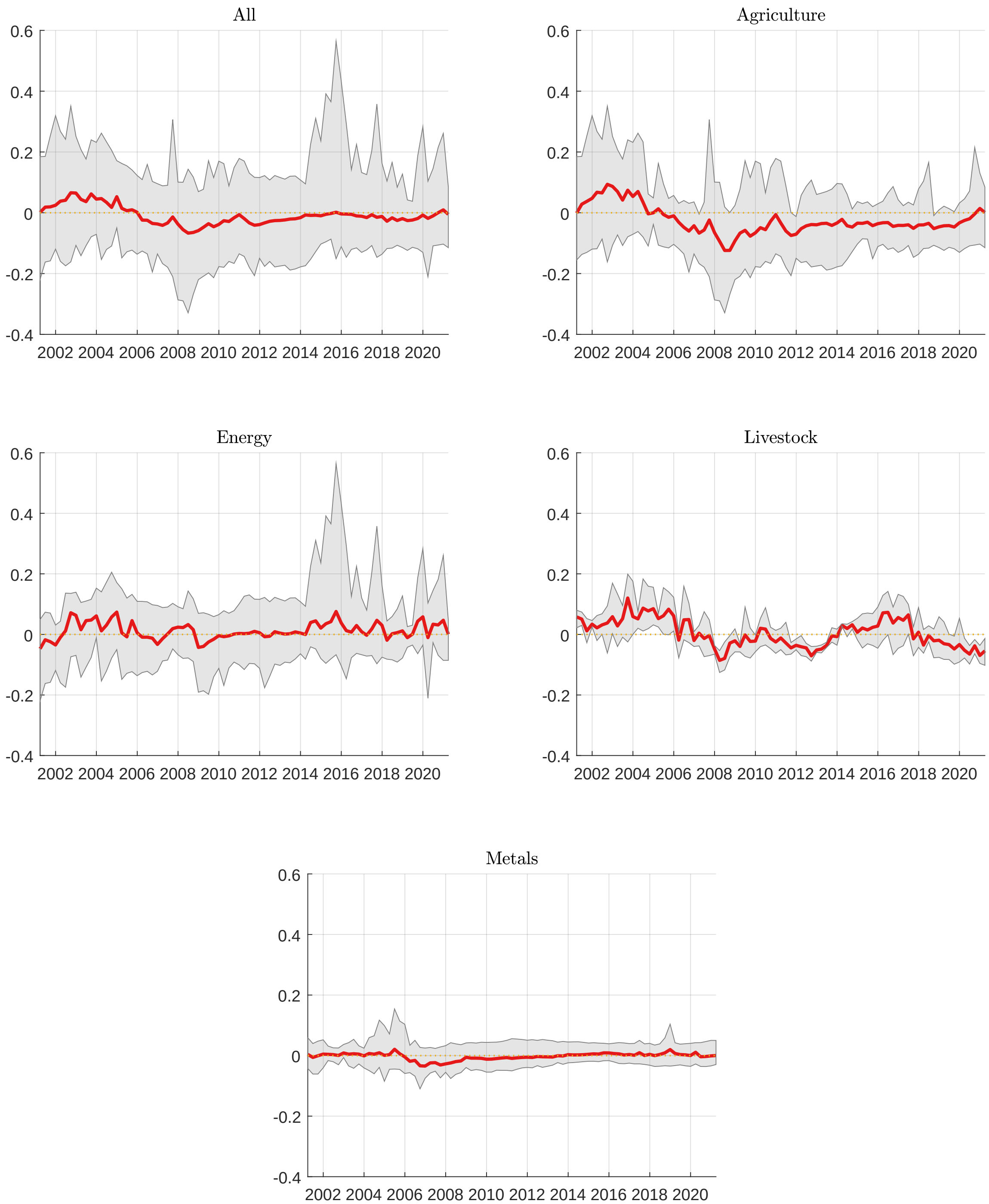

Figure 3: H-L Portfolios cumulative returns

This figure plots the time series of the cumulative returns from high minus low portfolios sorted by the TSMC (solid line), Basis (dashed line) and Momentum (dotted line). The sample period is January 2001 - March 2021.

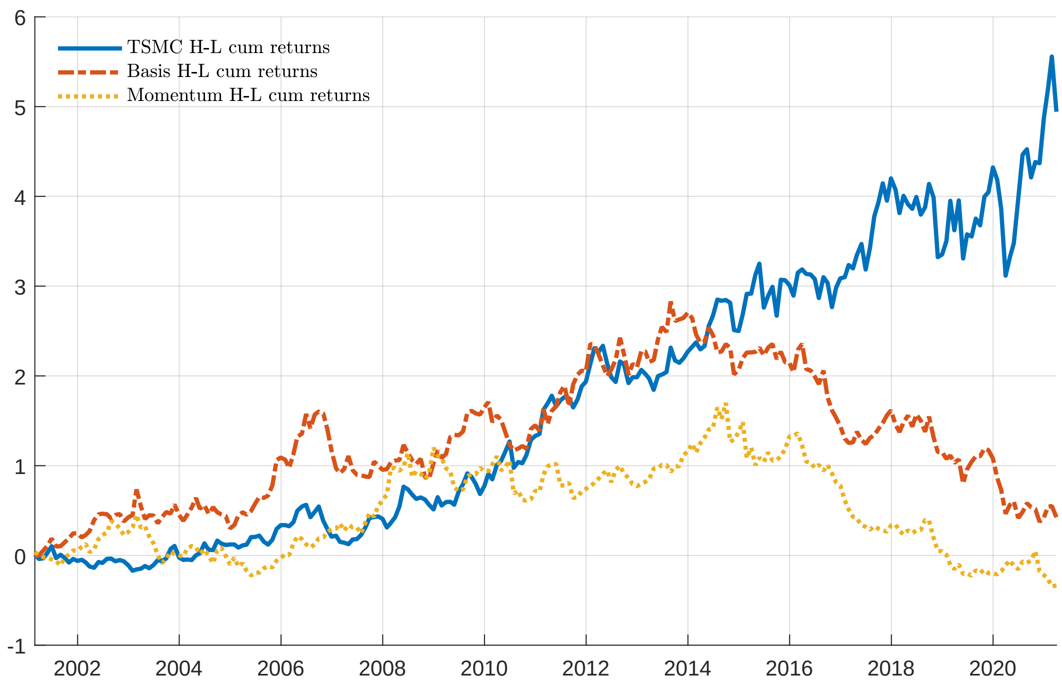

Figure 4: Total variations explained by PCs

This figure plots the total variation in the panel of TSMCs explained by the first 10 PCs. The left Y-axis measures the explained percentage by individual PCs, and the right Y-axis measures the explained percentage cumulatively.

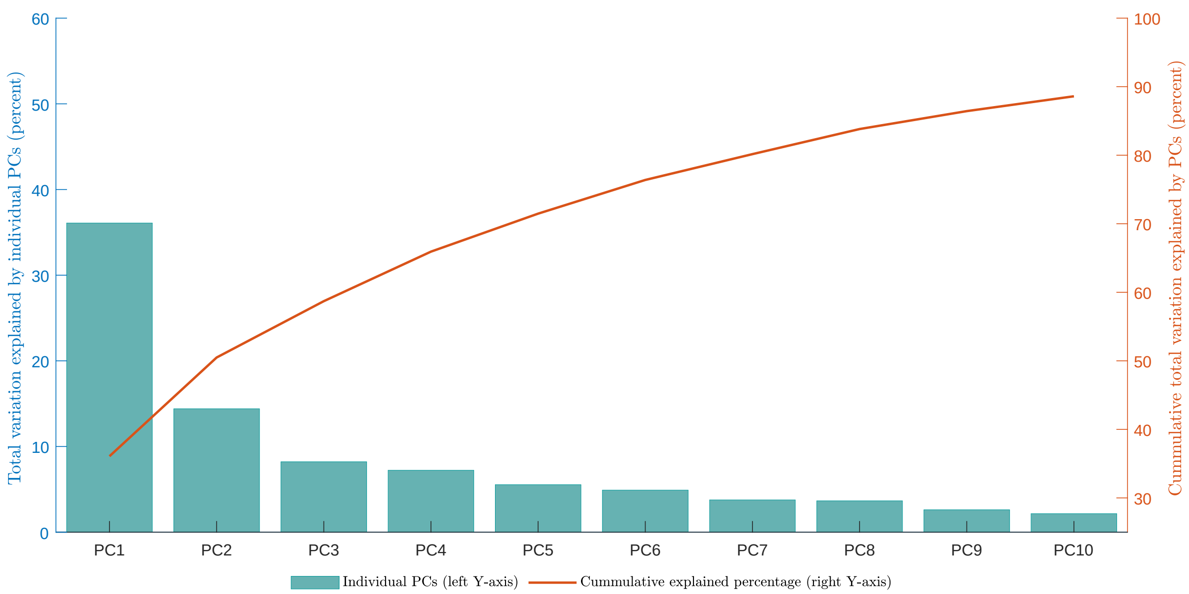

Table 1: All commodity names and monthly return summary statistics

The left three columns of this table lists names, types, and Reuters Instrument Code (RIC) of all 29 commodity futures in four types (seven Energy commodities, 14 Agriculture commodities, three Livestock commodities, and five Metals commodities) included in our sample. The right five columns present the summary statistics of the monthly returns of the front month futures for all 29 commodities. The summaries include Mean, Standard deviation (Std), Minimum (Min), Median, and Maximum (Max). All number are shown in the raw value.

Commodity Type	Name	RIC	Mean	Std	Min	Median	Max
Energy	Crude oil	LCO	0.006	0.095	-0.469	0.013	0.337
	Heating oil	LHO	-0.001	0.089	-0.320	0.001	0.253
	Natural gas	NG	-0.020	0.133	-0.320	-0.021	0.516
	RBOB gasoline	RB	0.007	0.105	-0.599	0.013	0.335
	Propane	A7E	0.012	0.108	-0.256	0.026	0.302
	Natural gas (ICE)	NGLNQ	-0.011	0.102	-0.316	-0.016	0.450
	Gas oil	LGO	0.007	0.094	-0.334	0.009	0.271
Agriculture	Corn	C	-0.001	0.081	-0.228	-0.009	0.283
	Kansas wheat	KW	-0.001	0.086	-0.241	-0.008	0.360
	Oats	O	0.012	0.100	-0.267	0.005	0.350
	Soybean meal	SM	0.015	0.086	-0.272	0.009	0.301
	Soybean oil	BO	0.005	0.075	-0.252	0.002	0.269
	Soybeans	S	0.010	0.074	-0.234	0.009	0.196
	Wheat	W	-0.004	0.088	-0.252	-0.008	0.377
	Cocoa	CC	0.009	0.091	-0.250	0.008	0.332
	Coffee	KC	-0.002	0.091	-0.236	-0.011	0.436
	Cotton	CT	0.000	0.083	-0.226	0.004	0.247
	Sugar	SB	0.002	0.090	-0.297	-0.003	0.311
	Rough rice	RR	-0.003	0.075	-0.228	-0.001	0.222
	Orange juice	OJ	0.001	0.089	-0.210	-0.007	0.276
	Lumber	LB	0.004	0.110	-0.322	-0.009	0.584
Livestock	Feeder cattle	FC	0.002	0.046	-0.206	0.002	0.128
	Lean hogs	LH	-0.001	0.091	-0.260	-0.002	0.385
	Live cattle	LC	0.002	0.046	-0.231	0.003	0.160
Metals	Copper	HG	0.010	0.077	-0.360	0.007	0.354
	Gold	GC	0.007	0.048	-0.183	0.004	0.136
	Palladium	PA	0.018	0.081	-0.222	0.028	0.249
	Platinum	PL	-0.001	0.063	-0.181	-0.001	0.140
	Silver	SI	0.009	0.091	-0.280	0.003	0.301

Table 2: Summary of the TSMCs' factor loadings on the first five PCs

This table reports a summary of the factor loading coefficients. The loading coefficients from time series regressions in which all 29 TSMCs are regressed on the first five PCs. Cross-sectional averages alongside standard deviations (in parentheses) are reported here. In the 'All' row, the loading coefficients for all 29 TSMCs are included in the calculation, in other sector-specific rows, only loading coefficients for the TSMCs in the corresponding sector are included in the calculation.

Sector	PC1	PC2	PC3	PC4	PC5
All	0.102	0.059	0.045	0.042	0.052
	(0.158)	(0.179)	(0.183)	(0.184)	(0.182)
Energy	0.005	-0.034	0.130	0.077	0.082
	(0.169)	(0.221)	(0.168)	(0.283)	(0.119)
Agriculture	0.177	0.119	0.057	-0.012	0.050
	(0.156)	(0.185)	(0.218)	(0.139)	(0.247)
Metal	0.015	0.053	-0.018	0.020	0.009
	(0.074)	(0.067)	(0.050)	(0.028)	(0.059)
Live Stock	0.126	0.010	-0.108	0.245	0.058
	(0.032)	(0.113)	(0.033)	(0.139)	(0.084)

Table 3: Portfolio sorting with individual TSMC, Basis and Momentum

This table presents the average (equally weighted) monthly returns (in percentage) of the portfolios sorted by individual TSMC, Basis and Momentum. The High (Low) portfolio includes commodities in the quartile with the highest (lowest) values of a characteristic, i.e., TSMC, Basis, or Momentum. The H-L is the difference between High and Low. The t-statics of these average monthly returns are shown in the parentheses. The full sample period covers January 2001 to March 2021, the first half is January 2001 to May 2011, and the second half is June 2011 to March 2021.

Sample	Portfolio	TSMC	Basis	Mom.
Full	High	0.69	0.47	0.24
		(1.93)	(1.33)	(0.70)
	Low	-0.19	0.18	0.22
		(-0.58)	(0.59)	(0.59)
	H-L	0.87	0.29	0.02
		(2.63)	(0.84)	(0.04)
1st Half	High	0.99	0.99	0.86
		(1.95)	(1.95)	(1.61)
	Low	0.04	0.05	0.10
		(0.08)	(0.12)	(0.20)
	H-L	0.93	0.94	0.76
		(1.98)	(1.91)	(1.36)
2nd Half	High	0.38	-0.08	-0.42
		(0.75)	(-0.17)	(-1.06)
	Low	-0.44	0.32	0.34
		(-1.08)	(0.73)	(0.63)
	H-L	0.82	-0.40	-0.76
		(1.73)	(-0.84)	(-1.43)

Table 4: Alphas of the TSMC H-L returns and other factors

In this table, the two columns under TSMC ∼ factors present the full sample alpha (monthly) coefficients and R^2 from regressing the TSMC H-L returns on various factors. The controlling factors include: Basis (Basis H-L returns), BasisHnL (separate Basis High portfolio and Basis Low portfolio), Momentum (Momentum H-L returns), Commodity Market (average returns of all commodities' front month futures), Carry (commodity carry returns of Koijen et al., 2018), FF5F (returns of Fama-French 5 factors), and All (all factors mentioned above). The four columns under Factors ∼ TSMC present results from time-series regressions of various factor returns on the TSMC H-L returns. The factors include: Basis (Basis H-L returns), BasisH (Basis High portfolio) BasisL (Basis Low portfolio), Momentum (Momentum H-L returns), Commodity Market, Carry, FF5F-mkt (fama french market factor, the excess return on the equity market), FF5F-smb (fama french size factor, the average return on the nine small stock portfolios minus the average return on the nine big stock portfolios), FF5F-hml (fama french value factor, the average return on the two value portfolios minus the average return on the two growth portfolios), FF5F-rmw (fama french Robust Minus Weak factor, the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios), and FF5F-cma (fama french Conservative Minus Aggressive, the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios). The column labeled "Mean" is the mean value of the factor return. Alpha is the regression intercept from regressing the factor return on the TSMC H-L return. Beta is the loading of the factor return on the TSMC H-L return. The final column reports the R^2 of the regression. The full sample period covers January 2001 to March 2021. The t-statistics based on Newey-West standard errors are reported in the parentheses. Both Mean and Alpha and their standard errors are in percentage. ***, **, and * indicate 1%, 5%, and 10% significance levels, respectively.

	TSMC ∼ factors		Factors ∼ TSMC			
Factors	Alpha	R^2	Mean	Alpha	Beta	R^2
Basis	0.75***	0.20	0.29	−0.11	0.46***	0.20
	(2.72)		(0.84)	(−0.37)	(7.58)	
BasisHnL	0.72***	0.21				
	(2.72)					
BasisH			0.47	0.16	0.35***	0.11
			(1.33)	(0.43)	(4.42)	
BasisL			0.18	0.27	−0.11*	0.01
			(0.59)	(0.80)	(−1.74)	
Momentum	0.87***	0.03	0.02	0.24	0.10**	0.02
	(2.95)		(0.04)	(−0.35)	(2.17)	
Commodity Market	0.82***	0.02	0.32	0.22	0.39*	0.16
	(2.94)		(1.24)	(0.75)	(1.65)	
Carry	0.65**	0.16	0.56*	0.48	0.20***	0.05
	(2.15)		(1.72)	(0.66)	(6.23)	
FF5F	0.73**	0.08				
	(2.40)					
FF5F-mkt			0.65**	0.48	0.20***	0.05
			(2.25)	(1.54)	(3.11)	
FF5F-smb			0.30*	0.22	0.09**	0.03
			(1.73)	(1.27)	(2.30)	
FF5F-hml			0.01	0.02	−0.01	0.00
			(0.03)	(0.07)	(−0.29)	
FF5F-rmw			0.34**	0.39**	−0.06**	0.02
			(2.40)	(2.44)	(−2.18)	
FF5F-cma			0.16	0.20	−0.05***	0.02
			(1.36)	(1.56)	(−2.78)	
All	0.63**	0.28				
	(2.24)					

Table 5: Exposures to market downside risk

The table presents Lettau, Maggiori, and Weber (2014)'s two beta estimates and their implied alpha for the four portfolios' returns: TSMC, Basis, Momentum, and Commodity Market. $\beta_{LMW,mkt}$ is the full sample market beta and $\beta_{LMW,down}$ is the sub-sample market beta where the excess market return is one standard deviation below its sample mean. The implied alpha is the coefficient from regressing $\hat{y}$ on the vector of ones (without an intercept), where

$$\hat{y} = r_{\text{Portfolio}} - \beta_{LMW,mkt}\left(r_{mkt} - r_{down}\right) - \beta_{LMW,down}\left(r_{down}\right)$$

and $r_{down} = 0$ if $r_{mkt} > \bar{r}_{mkt} - \sigma_{r_{mkt}}$ and $r_{down} = r_{mkt}$ otherwise, $\bar{r}_{mkt}$ and $\sigma_{r_{mkt}}$ are the sample mean and sample standard deviation of the market return, respectively. The t-statistics based on Newey-West standard errors are reported in the parentheses. ***, **, and * indicate 1%, 5%, and 10% significance levels, respectively. The implied alphas are monthly and in percentage.

Portfolio	Implied alpha	$\beta_{LMW,mkt}$	$\beta_{LMW,down}$
TSMC	0.79^{***}	0.26^{***}	0.35
	(2.84)	(3.17)	(0.86)
Basis	0.67^{*}	0.16^{*}	0.65^{*}
	(1.92)	(1.80)	(1.91)
Momentum	0.22	-0.18^{*}	-0.10
	(0.55)	(-1.75)	(-0.27)
Commodity Market	0.72^{***}	0.42^{***}	1.10^{***}
	(2.58)	(5.13)	(2.92)

Table 6: Correlations of individual TSMCs

This table shows the pairwise correlations of the 24 TSMCs used in Section 5. We standardize all predictors to have zero mean and unit variance. The sample spans the period from Feb 1994 to Mar 2021.

	LGO	***C***	***KW***	***O***	***SM***	***BO***	***S***	***W***	***CC***	***KC***	***CT***	***SB***	***RR***	***OJ***	***LB***	***FC***	***LH***	***LC***	***HG***	***GC***	***PA***	***PL***	***SI***
LCO	0,29	0,30	0,31	0,32	0,20	0,03	0,26	0,40	0,15	-0,06	0,17	0,39	0,18	0,51	0,12	0,06	0,28	0,23	0,11	0,41	-0,05	0,45	-0,38
LGO		-0,13	0,31	0,06	0,40	0,14	0,47	0,54	0,03	0,23	-0,32	0,40	0,37	0,56	-0,01	0,35	0,38	0,26	0,48	0,22	-0,57	0,30	-0,57
C			0,50	0,34	-0,01	0,19	0,09	0,20	0,00	0,12	0,30	0,02	-0,41	-0,07	0,11	-0,11	-0,13	0,02	0,14	0,02	0,05	-0,12	0,04
KW				0,40	0,03	0,22	0,12	0,67	0,05	0,13	0,11	0,25	-0,18	0,10	-0,22	0,38	0,23	0,38	0,58	0,06	-0,43	-0,04	-0,52
O					0,25	0,33	0,37	0,60	0,55	0,10	0,19	0,33	0,36	0,18	0,17	0,00	0,08	0,33	0,19	0,38	-0,14	0,36	-0,22
SM						0,29	0,90	0,44	0,39	0,34	-0,19	0,35	0,51	0,38	0,22	-0,09	0,09	0,00	-0,08	0,47	-0,23	0,34	-0,04
BO							0,56	0,33	0,29	0,02	0,29	0,18	0,22	0,09	0,22	0,29	0,03	0,42	0,19	0,30	-0,02	0,19	-0,11
S								0,54	0,42	0,30	-0,08	0,42	0,56	0,45	0,25	0,02	0,09	0,17	0,00	0,56	-0,23	0,42	-0,11
W									0,50	0,23	-0,03	0,58	0,44	0,50	-0,03	0,36	0,32	0,46	0,47	0,49	-0,53	0,44	-0,67
CC										0,03	0,04	0,20	0,46	0,27	0,33	-0,13	-0,02	0,18	0,04	0,37	-0,18	0,34	-0,10
KC											-0,24	0,21	-0,05	0,18	0,08	-0,14	0,27	-0,18	0,10	0,37	0,07	0,18	-0,12
CT												0,19	-0,03	-0,08	0,05	0,24	0,03	0,33	0,16	0,10	0,16	0,04	0,01
SB													0,54	0,43	-0,05	0,29	0,31	0,41	0,29	0,50	-0,30	0,48	-0,50
RR														0,50	0,17	0,16	0,13	0,25	-0,03	0,43	-0,29	0,55	-0,24
OJ															0,22	0,15	0,48	0,19	0,28	0,60	-0,20	0,71	-0,56
LB																-0,29	-0,11	-0,07	-0,04	0,15	0,11	0,16	0,16
FC																	0,38	0,56	0,55	0,15	-0,31	0,14	-0,60
LH																		0,11	0,45	0,47	-0,03	0,57	-0,69
LC																			0,41	0,31	-0,28	0,19	-0,47
HG																				0,07	-0,39	0,14	-0,67
GC																					0,09	0,72	-0,37
PA																						0,10	0,40
PL																							-0,49

Table 7: In sample results: univariate and bivariate

Panel A reports the results (slopes, Newey-West t-values and in-sample $R^2s(\%)$) of a univariate predictive regression for predicting the market excess returns. The regression is: $R_{t+1} = \alpha + \psi Z_t + \epsilon_{t+1}$, where Z_t is either one of the 14 economic variables in Welch and Goyal (2008), the investor sentiment index of Huang et al. (2015) denoted as S (available up to Dec 2020), the short interest index of Rapach, Strauss, and Zhou (2010) denoted as SII, or an aggregate predictor of TSMCs constructed with PLS ($TSMC^{PLS}$), sPCA ($TSMC^{sPCA}$), and PLS+sPCA ($TSMC^{PLS+sPCA}$). Panel B reports the results of a bivariate regression for forecasting the excess market returns with the $TSMC^{PLS}$ and economic predictors. The regression is: $R_{t+1} = \alpha + \beta X_t + \psi Z_t + \epsilon_{t+1}$, where X_t denotes the $TSMC^{PLS}$. Panel C reports the results of a bivariate regression for forecasting the excess market returns with the $TSMC^{sPCA}$ and economic predictors. The regression is: $R_{t+1} = \alpha + \beta X_t + \psi Z_t + \epsilon_{t+1}$, where X_t denotes the $TSMC^{sPCA}$. Panel D reports the results of a bivariate regression for forecasting the excess market returns with the $TSMC^{PLS+sPCA}$ and economic predictors. The regression is: $R_{t+1} = \alpha + \beta X_t + \psi Z_t + \epsilon_{t+1}$, where X_t denotes the $TSMC^{PLS+sPCA}$. ***, **, and * indicate significance at 1%, 5% and 10% levels, respectively. The sample spans the period from Feb 1994 to Mar 2021. We standardize all predictors to have zero mean and unit variance.

(a) Univariate - GW and Indices

Variable	ψ (%)	R^2 (%)
dp	0.56	1.55
dy	0.60**	1.83
ep	0.15	0.11
de	0.16	0.13
svar	0.07	0.02
b/m	0.34	0.60
ntis	0.25	0.30
tbl	-0.26	0.34
lty	-0.40	0.81
ltr	0.24	0.29
tms	-0.12	0.07
dfy	-0.11	0.06
dfr	0.23	0.28
infl	0.19	0.17
SII	-0.54**	1.46
S	-0.57***	1.60
TSMC^{PLS}	0.93***	4.30
TSMC^{sPCA}	0.74***	2.75
TSMC$^{PLS+sPCA}$	0.92***	4.28

(b) Bivariate - GW and Indices: TSMC^{PLS}

Variable	β (%)	ψ (%)	R^2 (%)
dp	0.84**	0.22	4.51
dy	0.82**	0.24	4.54
ep	1.02***	-0.25	4.56
de	0.97***	0.30	4.73
svar	0.97***	0.25	4.60
b/m	0.93***	0.00	4.30
ntis	0.92***	0.03	4.30
tbl	0.91***	-0.05	4.31
lty	0.88***	-0.23	4.56
ltr	0.94***	0.28	4.69
tms	0.95***	-0.23	4.55
dfy	0.97***	0.16	4.42
dfr	0.91***	0.16	4.43
infl	0.94***	0.24	4.59
SII	0.86***	-0.15	4.38
S	0.83***	-0.24	4.45

(c) Bivariate - GW and Indices: $TSMC^{sPCA}$

Variable	β (%)	ψ (%)	R^2 (%)
dp	0.65*	0.42	3.59
dy	0.63*	0.45	3.70
ep	0.79**	-0.13	2.83
de	0.81***	0.33	3.28
svar	0.79***	0.23	3.02
b/m	0.70**	0.15	2.85
ntis	0.73***	0.02	2.75
tbl	0.72**	-0.13	2.83
lty	0.70**	-0.31	3.23
ltr	0.75***	0.27	3.12
tms	0.76***	-0.21	2.97
dfy	0.79***	0.16	2.86
dfr	0.73***	0.17	2.90
infl	0.74**	0.20	2.95
SII	0.63**	-0.22	2.93
S	0.66**	-0.44**	3.69

(d) Bivariate - GW and Indices: $TSMC^{PLS+sPCA}$

Variable	β (%)	ψ (%)	R^2 (%)
dp	0.83**	0.23	4.50
dy	0.82**	0.25	4.53
ep	1.02***	-0.25	4.54
de	0.97***	0.30	4.72
svar	0.97***	0.25	4.59
b/m	0.92***	0.00	4.28
ntis	0.92***	0.02	4.28
tbl	0.91***	-0.05	4.29
lty	0.88***	-0.23	4.54
ltr	0.94***	0.28	4.67
tms	0.95***	-0.23	4.54
dfy	0.97***	0.17	4.41
dfr	0.91***	0.16	4.41
infl	0.94***	0.24	4.57
SII	0.86***	-0.15	4.36
S	0.82***	-0.24	4.45

Table 8: Out of sample results: univariate

This table reports the out-of-sample $R^2_{OS}s$ and MSFE-adjusted statistics for predicting the average excess stock market returns over the prediction horizon h by using the aggregate predictor based on TSMCs constructed with PLS (Panel A), sPCA (Panel B), and the simple (average) combination of PLS and sPCA: PLS+sPCA (Panel C). h=1 month, 3, 6, 9 ,12 , 18, and 24 months. Statistical significance for the R^2_{OS}'s is based on the p-value of Clark and West (2007) MSFE-adjusted statistic. The out-of-sample analysis is based on a 20-year expanding estimation window. The sample spans the period from Feb 1994 to Mar 2021. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Panel A: TSMC^{PLS}		Panel B: TSMC^{sPCA}		Panel C: TSMC$^{PLS+sPCA}$	
Horizon	R^2_{OS} (%)	MSFE-adjusted	R^2_{OS} (%)	MSFE-adjusted	R^2_{OS} (%)	MSFE-adjusted
h=1	1.07*	1.31	1.34	1.24	1.09*	1.31
h=3	1.99	1.07	3.32	1.22	2.07	1.08
h=6	6.19**	2.17	7.41*	1.39	6.30**	2.13
h=9	5.35**	1.66	7.42*	1.35	5.47**	1.67
h=12	8.19*	1.59	6.94*	1.32	8.18*	1.58
h=18	13.16*	1.39	12.15*	1.58	13.17*	1.40
h=24	21.67*	1.48	20.83**	1.94	21.79*	1.49

Table 9: Economic value results

This table reports the annualized CER gains (%) for a mean-variance investor with $\lambda = 1, 3$ & 5 for predicting future market excess returns by using PLS, sPCA, and PLS+sPCA relative to historical mean returns across prediction horizons. We consider two cases: zero transaction cost and a proportional transaction cost of 50 basis points per transaction. The sample spans the period from Feb 1994 to Mar 2021.

(a) $CER(\%)\ Gain - \text{TSMC}^{PLS}$

TCs=0 bps	$\lambda = 1$	$\lambda = 3$	$\lambda = 5$	TCs=50 bps	$\lambda = 1$	$\lambda = 3$	$\lambda = 5$
h=1	10.24	3.41	2.05	h=1	6.62	2.21	1.32
h=3	9.49	3.16	1.90	h=3	5.89	1.96	1.18
h=6	8.27	2.76	1.65	h=6	5.09	1.70	1.02
h=9	6.80	2.27	1.36	h=9	3.93	1.31	0.79
h=12	3.16	1.05	0.63	h=12	0.49	0.16	0.10
h=18	6.61	2.20	1.32	h=18	4.61	1.54	0.92
h=24	5.03	1.68	1.01	h=24	3.41	1.14	0.68

(b) $CER(\%)\ Gain - \text{TSMC}^{sPCA}$

TCs=0 bps	$\lambda = 1$	$\lambda = 3$	$\lambda = 5$	TCs=50 bps	$\lambda = 1$	$\lambda = 3$	$\lambda = 5$
h=1	6.52	2.17	1.30	h=1	3.39	1.13	0.68
h=3	5.53	1.84	1.11	h=3	2.70	0.90	0.54
h=6	3.20	1.07	0.64	h=6	0.58	0.19	0.12
h=9	3.17	1.06	0.63	h=9	0.81	0.27	0.16
h=12	0.43	0.14	0.09	h=12	-1.65	-0.55	-0.33
h=18	4.64	1.55	0.93	h=18	3.23	1.08	0.65
h=24	2.32	0.77	0.46	h=24	1.15	0.38	0.23

(c) $CER(\%)\ Gain - \text{TSMC}^{PLS+sPCA}$

TCs=0 bps	$\lambda = 1$	$\lambda = 3$	$\lambda = 5$	TCs=50 bps	$\lambda = 1$	$\lambda = 3$	$\lambda = 5$
h=1	10.25	3.42	2.05	h=1	6.63	2.21	1.33
h=3	9.48	3.16	1.90	h=3	5.91	1.97	1.18
h=6	8.20	2.73	1.64	h=6	5.04	1.68	1.01
h=9	6.77	2.26	1.35	h=9	3.91	1.30	0.78
h=12	3.09	1.03	0.62	h=12	0.44	0.15	0.09
h=18	6.63	2.21	1.33	h=18	4.65	1.55	0.93
h=24	4.99	1.66	1.00	h=24	3.38	1.13	0.68

Table 10: Out of sample results using alternative aggregation approaches

This table reports the out-of-sample $R^2_{OS}s$ and MSFE-adjusted statistics for predicting the average excess stock market returns over the prediction horizon h by using the aggregate predictor based on TSMCs constructed with C-ENet (Panel A), simple average combination (Panel B), and Ridge (Panel C). h=1 month, 3, 9, 12, 18, and 24 months. Statistical significance for the $R^2_{OS}s$ is based on the p-value of Clark and West (2007) MSFE-adjusted statistic. The out-of-sample analysis is based on a 20-year expanding estimation window. The sample spans the period from Feb 1994 to Mar 2021. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	Panel A: TSMC^{C-ENet}		Panel B: TSMC^{Ave}		Panel C: TSMC^{Ridge}	
Horizon	R^2_{OS} (%)	MSFE-adjusted	R^2_{OS} (%)	MSFE-adjusted	R^2_{OS} (%)	MSFE-adjusted
h=1	1.41*	1.44	0.14	0.81	0.06*	1.53
h=3	4.39*	1.48	0.33	0.64	0.20*	1.52
h=6	7.71*	1.33	1.15*	1.40	0.50*	1.55
h=9	9.12*	1.43	1.08	1.22	0.68*	1.56
h=12	3.02*	1.28	1.38	1.09	0.88*	1.54
h=18	1.55*	1.57	2.85*	1.31	1.50*	1.61
h=24	3.39**	1.92	4.71**	1.68	2.35**	2.19

Supplementary (Online) Appendix

for “A Model-based Commodity Risk Measure on Commodity and Stock Market Returns” by Ai Jun Hou, Emmanouil Platanakis, Xiaoxia Ye, & Guofu Zhou

A Kalman filter

We present a bespoke KF procedure designed for our estimation method outlined in Section 3.2. For more general KF references, please see Harvey (1989, Chapter 3) and Hamilton (1994, Chapter 13). We start the KF by choosing the initial values of the state variables and their covariance matrix as their steady state values $Y_{0|0} = [r_0, \delta_0, X_0]^\intercal$, and $P_{Y,0|0} = 0_{3\times3}$, where r_0 is the Treasury yield with the shortest maturity on the first day of the sample, X_0 is the log of the front month futures price on the first day of the sample, δ_0 is a free parameter to be estimated alongside other parameters. Given $Y_{t-h|t-h}$ and $P_{Y,t-h|t-h}$, the *ex ante* prediction of the state variables and their covariance matrix are given by

$$Y_{t|t-h} = e^{K_1^{\mathbb{P}} h} Y_{t-h|t-h} \text{ and } P_{Y,t|t-h} = e^{K_1^{\mathbb{P}} h} P_{Y,t-h|t-h} \left(e^{K_1^{\mathbb{P}} h}\right)^\intercal + \Omega(K_1^{\mathbb{P}}, h),$$

Given $Y_{t|t-h}$ and $P_{Y,t|t-h}$, the *ex ante* predictions of the measurement and the associated covariance become

$$\begin{aligned}
\log\left(\overline{F}^{\Delta t}_{t|t-h}\right) &= \frac{\iota^\intercal \Omega\left(K_1, \Delta t\right) \iota}{2} + \iota^\intercal \left[\exp(K_1 \Delta t) - I\right] K_1^{-1} K_0 + \iota^\intercal \exp(K_1 \Delta t) Y_{t|t-h} \\
P_{\log\left(\overline{F}^{\Delta t}_t\right)} &= M P_{Y,t|t-1} M^\intercal + \xi I_{n_t \times n_t},
\end{aligned}$$

where M is $n_t \times 3$ and its ith row is $\iota^\intercal \exp(K_1 \Delta t_i)$ with Δt_i being the time to maturity of the ith futures on day t and $i = 1, 2, \ldots, n_t$. Finally, the *ex post* updates on the filtered state variables are given by

$$Y_{t|t} = Y_{t|t-h} + \Xi_t \left(\log\left(\overline{F}^{\Delta t}_t\right) - \log\left(\overline{F}^{\Delta t}_{t|t-h}\right)\right) \text{ and } P_{Y,t|t} = P_{Y,t|t-h} - \Xi_t P_{\log\left(\overline{F}^{\Delta t}_t\right)} \Xi_t^\intercal,$$

where $\Xi_t = P_{Y,t|t-1} M P^{-1}_{\log\left(\overline{F}^{\Delta t}_t\right)}$ is the Kalman gain. $Y_{t|t}$ is used to compute the RP_t on day t.

B Technical details on forecasting stock index

We consider the following one-period forecasting model:

$$R_{t+1} = \alpha + \beta \text{TSMC}_t^* + \epsilon_{t+1} \tag{B.1}$$

where R_{t+1} denotes the realized excess stock market return at $t+1$. TSMC_t^* represents the true but unobservable aggregate TSMC at t that matters for forecasting R_{t+1} and ϵ_{t+1} is a noise term unrelated to TSMC_t^*. Let $Ind_{\text{TSMC},t} = (Ind_{\text{TSMC}_1,t}, \ldots, Ind_{\text{TSMC}_N,t})^\intercal$ denotes an $N \times 1$ vector of individual TSMCs at t. Assume the following factor structure for $Ind_{\text{TSMC}_i,t}$ $(i = 1, 2, ...N)$,

$$Ind_{\text{TSMC}_i,t} = n_{i,0} + n_{i,1}\text{TSMC}_t^* + n_{i,2}\epsilon_t + e_{i,t} \tag{B.2}$$

where $n_{i,1}$ is the regression slope that captures $Ind_{\text{TSMC}_i,t}$'s sensitivity to TSMC_t^*. ϵ_t is the common approximation error component of all individual TSMCs that is irrelevant to stock returns, and $e_{i,t}$ represents idiosyncratic noise.

Our aim here is to efficiently estimate the relevant for forecasting, but unobservable, aggregate index TSMC_t^* by imposing a factor structure equation (B.2) on $Ind_{\text{TSMC}_i,t}$ and filtering out the irrelevant components ϵ_t and e_{it} when estimating TSMC_t^*. Hence, we consider a popular information aggregating method, the Partial Least Squares (PLS), as well as a shifting technique of PLS and $1/N$. The latter is suitable when constructing and evaluating aggregate indices in the out-of-sample. PLS is an efficient technique for constructing aggregate predictors as shown in Kelly and Pruitt (2013, 2015), and Light, Maslov, and Rytchkov (2017), among others.

To extract TSMC_t^*, PLS exploits the covariance between TSMC_t^* and future stock market returns, and uses a linear combination of individual TSMCs ($Ind_{\text{TSMC}_i,t}$) for predicting stock returns. PLS follows a two-step process involving a time-series regression in the first step and a cross-sectional regression in the second step. In the first step, the time-series regression of each $Ind_{\text{TSMC}_{i},t}$ on a constant and future realized excess stock return R_{t+1} is:

$$Ind_{\text{TSMC}_{i},t} = \pi_0 + \pi_i R_{t+1} \mu_{i,t}, \text{ for } i = 1, \ldots, N \tag{B.3}$$

where the regression slope π_i captures the sensitivity of each individual TSMC to the unobservable aggregate index TSMC_t^*. Since the latter drives the future stock market returns as shown in equation (B.1), each $Ind_{\text{TSMC}_{i},t}$ is unrelated with any unforecastable errors, and hence the slope $\pi_{i,1}$ is a good approximation on how each $Ind_{\text{TSMC}_{i},t}$ depends on the unobservable aggregate commodity index TSMC_t^*.

In the second step, the cross-sectional regression is as follows:

$$Ind_{\text{TSMC}_{i},t} = c_0 + \text{TSMC}_t^{PLS} \hat{\pi}_i + v_{i,t}, \text{ for } i = 1, \ldots, N \tag{B.4}$$

where the independent variable $\hat{\pi}_i$ in regression equation (B.3) has been estimated during the first step of the PLS method in regression equation (B.3). The aggregate PLS commodity index TSMC^{PLS} is the slope in regression equation (B.4) to be estimated.

Mathematically, the two-step PLS algorithm can be expressed as a one-step liner combination of $Ind_{\text{TSMC}_{i},t}$ where the weight on each $Ind_{\text{TSMC}_{i},t}$ in TSMC^{PLS} depends on their covariance with the future realized excess stock return.

$$\text{TSMC}^{PLS} = (\text{TSMC}_1^{PLS}, \text{TSMC}_2^{PLS}, ..., \text{TSMC}_T^{PLS})^\intercal$$

is computed as follows:

$$\text{TSMC}^{PLS} = Ind_{\text{TSMC}} \cdot J_N \cdot Ind_{\text{TSMC}}^\intercal \cdot J_T \cdot R \cdot K \cdot R^\intercal \cdot J_T \cdot R \tag{B.5}$$

where $K = (R^\intercal \cdot J_T \cdot Ind_{\text{TSMC}} \cdot J_T \cdot Ind_{\text{TSMC}}^\intercal \cdot J_T \cdot R)^{-1}$; $R = (R_2, \ldots R_{T+2})^\intercal$ denotes the $T \times 1$ vector of excess stock returns, and $Ind_{\text{TSMC}} = (Ind_{\text{TSMC}_1}^\intercal, \ldots, Ind_{\text{TSMC}_T}^\intercal)^\intercal$ is the $T \times N$ matrix of individual TSMCs. The matrices $J_T = I_T - \frac{1}{T}\ell_T \cdot \ell_T^\intercal$ and $J_N = I_N - \frac{1}{N}\ell_N \cdot \ell_N^\intercal$ are present in equation (B.5) because the regressions in each step of the algorithm are run with a constant; I_X is the X-dimensional identity matrix, and ℓ_X is a $X \times 1$ vector of ones. PLS exploits the factor nature of the joint system in equation (B.1) and equation (B.2) to infer the relevant for forecasting aggregate index TSMC^{PLS} by using future stock returns to extract TSMC^{*} and eliminating the negative effects of common and idiosyncratic components that are not relevant for predicting.

C Additional results on forecasting stock index

[*Insert Table A1 about here*]

Table A1: Univariate-individual TSMCs

This table reports the in-sample sample R^2's(%) from the regression: $R_{t+h} = \alpha + \beta IndCom_{i,t} + \epsilon_{t+1}$, where $IndCom_{i,t}$ is the univariate-individual TSMC. The regression slopes and the Newey-West t-values are reported in each Panel. ***, **, and * indicate significance at 1%, 5% and 10% levels, respectively. The sample spans the period from Feb 1994 to Mar 2021. We standardize all predictors to have zero mean and unit variance.

	Panel A: $h = 1$		Panel B: $h = 6$		Panel C: $h = 12$	
Variable	ψ (%)	R^2 (%)	ψ (%)	R^2 (%)	ψ (%)	R^2 (%)
$TSMC^{PLS}$	0.93***	4.30	0.89***	21.71	0.81***	34.12
$TSMC^{sPCA}$	0.74***	2.75	0.69**	13.28	0.57**	17.21
$TSMC^{PLS+sPCA}$	0.92***	4.28	0.89**	21.60	0.81***	33.79
LCO	0.00	0.00	-0.02	0.02	-0.03	0.06
LGO	-0.29	0.41	-0.32	2.91	-0.35*	6.06
C	0.47	1.10	0.48**	6.33	0.52***	14.05
KW	0.41*	0.84	0.37*	3.89	0.29*	4.62
O	0.24	0.30	0.14	0.55	0.04	0.08
SM	-0.11	0.07	-0.21	1.31	-0.25	3.43
BO	0.44*	0.95	0.42*	4.81	0.38*	7.40
S	-0.13	0.08	-0.09	0.23	-0.10	0.55
W	-0.11	0.07	-0.12	0.42	-0.12	0.76
CC	-0.15	0.12	-0.26	1.85	-0.22	2.43
KC	0.37*	0.71	0.27	2.12	0.21	2.46
CT	0.65**	2.13	0.58*	9.37	0.42*	9.58
SB	0.11	0.06	0.04	0.05	-0.01	0.01
RR	-0.38	0.71	-0.37	3.98	-0.43*	9.98
OJ	-0.13	0.09	-0.13	0.45	-0.15	1.16
LB	0.18	0.16	0.06	0.11	0.06	0.19
FC	0.06	0.02	0.07	0.14	0.11	0.65
LH	0.01	0.00	0.05	0.07	0.04	0.07
LC	0.19	0.18	0.27	2.05	0.27	3.91
HG	0.24	0.29	0.25	1.83	0.24	3.09
GC	0.08	0.03	0.11	0.36	0.12	0.80
PA	0.12	0.07	0.24	1.71	0.33*	5.78
PL	-0.07	0.02	-0.13	0.49	-0.19	1.94
SI	0.13	0.09	0.04	0.04	0.03	0.05