On the Design of Bail-in-able Bonds from the Perspective of Non-Financial Firms

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Abstract

To prevent black swan events such as the financial tsunami or COVID-19 from crashing overall economic systems, governments provide bailouts to too-big-to-fail financial institutions and non-financial firms such as airline companies with tax payers' money. To avoid unfair bailouts, using bail-in-able bonds to absorb an issuing bank's losses and to fulfill capital requirements such as BASEL III has been widely studied. Without capital adequacy requirements, non-financial firms also issue bail-in-able bonds because the embedded loss-absorbing mechanism can cut debt repayments down to reduce bankruptcy risk and thus increase the overall benefits of these firms' claim holders. However, bond investors require higher rewards to compensate for their potential losses once this mechanism is triggered. In addition, unlike financial institutions, which are strictly regulated, such a mechanism may induce non-financial issuers to take on excessive risk to benefit themselves at the expense of their debt holders. To investigate how the issuances of various (non)-bail-in-able bonds influence the equity and the existing debt holders' benefits from a non-financial issuer's prospective, we develop novel quadrature pricing formulas based on the structural credit risk model. We find that while non-bail-in-able bonds can be wise choices for low-leverage issuers, bail-in-able bonds with benefit-sharing mechanisms allowing investors to share issuers' upside profits can efficiently suppress interest expenses and circumvent the asset substitution problem for high-leverage issuers. If the value of an issuer (such as an oil shale firm) mainly depends on the value of that asset (such as oil), issuing a bail-in-able bond referring to the asset could be a wise choice. Otherwise, issuing a bond referring to the issuer's stock price could provide timely fund injections and low interest expenses for a high-leverage public firm.

(JEL Classification: G12, G23)

Keywords: bail-in-able bond, contingent capital, asset substitution problem, structural credit risk model

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1 Introduction

The 2007–2009 financial crisis taught that the failures of important banks can bring the collapse of financial industry and spread credit crises to other industries, as it did with the automotive industry crisis (see Klier and Rubenstein, 2010, 2011). It is also well documented that bankruptcies of firms in concentrated industries spread credit risk towards up/down-stream industries and fund suppliers (see Hertzel et al., 2008; Hertzel and Officer, 2012; Kolay et al., 2016). To prevent credit contagion from bringing down entire economic systems and damaging social stability, governments may bailout systemically important financial institutions and socially critical non-financial firms. The emergency bailouts for Bear Sterns, American International Group (AIG), General Motors (GM), and Ford due to the financial crisis of 2007 to 2009 are well-known examples. Azgad-Tromer (2017) studies the examples of bailouts for hospital trusts in the U.K., exclusive providers of employment and socially amenities in Russian monotowns, and Pacific Gas & Electric Corporation (PG&E) in the U.S. The public press also reported big bailouts for commercial airline companies due to the COVID-19 outbreak (see Abate et al., 2020).

Although bailouts can decelerate the chain reactions of credit contagion, they increase moral hazard, since firm executives could adopt profitable but risky strategies to maximize their own benefits at the expense of taxpayers (see Poole, 2009). To circumvent such a dilemma, regulators and academics advocate bail-in strategies in place of public bailouts and focus on the development of prudential capital structure for banks with *contingent capital* (CC), a bail-in-able debt instrument with automatic triggers to absorb losses by compulsively writing down debt or by converting debt into stocks during financial distress. A reverse convertible bond (RC) is one prominent proposal for CC put forward by Flannery (2005); issuances of RCs by banks are widely recognized as contingent convertible bonds (CoCo bonds).[1] While the literature includes systematic studies on the pros and cons of different designs of CoCos issued by too-big-to-fail banks (see Sundaresan and Wang, 2015; Himmelberg and Tsyplakov, 2020; Lee and Park, 2020; Avdjiev et al., 2020),[2] few studies have addressed the same issues for too-critical-to-fail non-financial firms.

Indeed, there exist some prototypes of CC for non-financial firms in the capital market, because they also provide credit enhancement for issuers like CoCos. For example, Chidambaran et al. (2001) study the pros and cons of a gold-mining firm issuing a gold-denominated bond (GB) with increasing (declining) repayment when the gold price rises (falls). Ammann and Seiz (2006) analyze a mandatory convertible bond (MC) that can be compulsively converted into stocks once its issuer's stock price rises or falls to reach a prespecified upper or lower trigger. In contrast to bank-issued CoCos only with a "one-sided" loss-absorbing mechanism that favors issuers, we note that both GBs and MCs possess "two-sided" loss-absorbing and benefit-sharing mechanisms that favor either their issuers or investors. Although the rationale behind the inclusions of benefit-sharing mechanisms in non-bail-in-able bonds is well discussed (see Dutordoir et al., 2014), it is still an unfamiliar topic in terms of CC design. This paper will study the necessity of the two-sided design for bonds as CC for non-financial firms.

Although capital adequacy regulations provide banks with incentives and positive announcement effects to issue CoCos,[3] the use of similar bonds only with loss-absorbing mechanisms by non-financial

[1] Our later analyses will focus on RCs issued by non-financial firms.

[2] Studies of using CC are extended to broader financial industries. For example, Allen and Tang (2016) propose dual-trigger CoCo bonds for banks, broker-dealers, and insurance companies, and Lin et al. (2009) study the CC triggered by catastrophic risk for insurance companies.

[3] Issuing CoCos helps banks to meet capital adequacy requirements, as a high-trigger CoCo is classified as Additional Tier I Capital in BASEL III. Such issuances may have positive announcement effects (see Ammann et al., 2017) due to the increments in capital utilization rates and benefits for equity holders.

firms tends to emit negative signals due to the deteriorated financial status expected by market participants. It is thus important to design proper bail-in-able bonds such that the pros outweigh the cons from the perspective of non-financial firms, for which the incentive measures in BASEL III do not apply. Albul et al. (2015) suggest that the loss-absorbing mechanism reduces the default likelihood of a non-financial issuer and in consequence improves the overall benefits of equity and debt holders because of decreased bankruptcy costs and increased tax shield benefits according to trade-off theory. However, interest premiums required by investors increase significantly (see Szymanowska et al., 2009), as such credit enhancements are created at the expense of bail-in-able bond investors. In addition, since non-financial firms are typically not highly regulated, loss-absorbing mechanisms may lead the firms to adopt riskier strategies that benefit their equity holders at the expense of existing debt holders (i.e, the asset substitution problem) (see Berg and Kaserer, 2015). The presence of loss-absorbing mechanisms highlights the necessity of including additional benefit-sharing mechanisms for non-financial firms planning to issue bail-in-able bonds, because the two-sided design alleviates the aforementioned high interest premium and the asset substitution problems. By allowing bail-in-able bond investors to share the issuers' upside potential profits, investors accept lower returns, which means that corresponding interest expenses are lower. Furthermore, risk-shifting incentives are attenuated because the benefits of increasing firms' risk shift from equity holders to investors via the benefit-sharing mechanisms (see Green, 1984).

To analyze whether the presence of benefit-sharing mechanisms resolves the problems for non-financial firms issuing bail-in-able bonds, this paper analyzes four types of bonds that have been issued before: an `RC`, a reverse exchangeable bond (`RE`, see Benet et al., 2006), an `MC`, and a `GB`. `RC` and `RE` bonds merely possess loss-absorbing mechanisms (i.e., are one-sided bonds), whereas `MC` and `GB` possess both loss-absorbing and benefit-sharing mechanisms (i.e., are two-sided bonds).

We first introduce one-sided bonds as follows. An `RC` is converted into its issuer's stock when an indicator reflecting the deterioration of the issuer's financial healthiness is triggered (see Flannery, 2005). Triggering this loss-absorbing conversion deleverages the issuer and improves its financial status. These contract-specified indicators include the issuer's capital ratio (see Glasserman and Nouri, 2012), its stock price (see Sundaresan and Wang, 2015; Pennacchi and Tchistyi, 2019), and so on.[4] To fairly compare with an two-sided `MC`, this paper considers a non-perpetual `RC` with a stock price trigger. In view of this, we follow Sundaresan and Wang (2015), who accentuate the necessity of setting a proper trigger level avoiding wealth transfers between equity holders and `RC` investors during conversions to prevent malicious market manipulations. Though perpetuity characterizes most actual `CoCos` to avoid the wealth-transfer condition (see Pennacchi and Tchistyi, 2019), it seems that it is not a common

[4] Many studies have been conducted on designing a proper trigger for an `RC` to provide timely loss-absorbing conversions to rescue the issuer without incurring moral hazard. There are in summary four types of triggers in this strand of literature. Dickson (2010) proposes a trigger determined by regulatory discretion, but one potential weakness of this design is insufficient information from and/or ineffective monitoring by regulatory supervisors. Rather than activating conversion triggers from the outside of the issuer, Glasserman and Wang (2011) and Bolton and Samama (2012) suggest that the triggers can be decided by the bank management. However, this design results in delay or no conversion even when the firm is financially distressed, because prospective loss-absorbing conversions lead to significant dilutions that cause managers to finally turn to anticipate bailouts. The triggers can also rely on publicly-accessible information about the issuer. Squam Lake Working Group (2009) and Glasserman and Nouri (2012) propose indicators that depend on the issuer's accounting ratios, but Sundaresan and Wang (2015) argue that accounting-based criteria do not reflect the issuer's prospects and are prone to manipulation by managers. To address such concerns, McDonald (2013) and Sundaresan and Wang (2015) place triggers on market prices that rapidly reflect the firm's financial status; conversions are thus harder to manipulate. The former proposes a dual trigger based on issuer- and market-specific indicators such as the financial institution index (see McDonald, 2013) or the aggregate losses (see Gupta et al., 2021). As the name "dual" suggests, the conversion is triggered when both indicators satisfy prespecified conditions. The latter proposes a single trigger placed on the firm's stock price.

design for bonds issued by non-financial firms (see Custódio et al., 2013). We also do not consider RCs featuring principal write-down, as they in nature allow wealth transfers from the RC investors to the equity holders once the write-downs are triggered. On the other hand, REs are widely traded in U.S. structured product markets. In contrast to an RC offering its issuer emergency equity capital injections, an RE holder's terminal payoff is linked to the value of the reference stock specified in the RE's indenture. In particular, the principal received at maturity can be either the face value of the RE or a specified amount of the reference stock if the stock price falls below the predetermined level. An RE implicitly contains a loss-absorbing mechanism when the reference stock price is significantly and positively correlated with the issuer's financial status (see Benet et al., 2006). That is, an RE repayment generally decreases when the issuer's financial status deteriorates.

The two-sided bonds are introduced as follows. We focus on a special bond denominated in a contractually specified asset called the "reference asset" X; thus, the bond repayment is proportional to the value of X. If the value of X is significantly and positively correlated with the issuer's financial status, a bond holder generally receives more (less) when the issuer's financial status improves (deteriorates); this feature entails the inclusion of both benefit-sharing and loss-absorbing mechanisms. X could be oil and gas for an oil and gas provider, respectively, as discussed in Haushalter (2000). Chidambaran et al. (2001) study GBs issued by a gold-mining firm whose asset value is clearly closely related to the gold price. The following discussions will use a GB to refer to this type of bond. Instead of referring to the value of X, a MC can be converted into the issuer's stock either when the stock price rises to reach a predetermined upper level or when it falls to reach another lower level (see Ammann and Seiz, 2006).[5]

In addition to the presence or absence of benefit-sharing mechanisms, we classify four types of bail-in-able bonds by the assets their payoffs link to. The payoff of a RC or a MC is directly linked to the issuer's stock price. To confirm no-wealth-transfer conditions during conversions, we adopt the trigger settings proposed by Sundaresan and Wang (2015). This could also alleviate issuers' risk-shifting incentives (see Berg and Kaserer, 2015). The payoff of an RE or a GB, in turn, depends on its reference asset value. Therefore, we can analyze the impact of including benefit-sharing mechanisms in bail-in-able bonds when their payoffs depend on issuers' stock prices (by comparing RCs with MCs) or when they depend on reference assets (by comparing REs with GBs).

To study the pros and cons of issuing the types of bonds mentioned above, we develop a novel quadrature method to evaluate debt contracts with complex payoff functions linking to the values of reference assets for which closed-form solutions are unavailable. Based on the structural credit risk model pioneered by Merton (1974), our quadrature method treats equity, bail-in-able bonds, and other existent debts as contingent claims on the issuing firm's asset and the reference asset (if applicable). We then follow Moody's definition of debt default when the issuer misses a disbursement of a contractually-obligated interest or principal payment (see Ou et al., 2011), since acceleration clauses are rare in corporate bond indentures.

The evaluations of contingent claims before and after issuing (non)-bail-in-able bonds can be systematically solved via our methods as follows. According to the values of the issuer's asset and the reference asset, the payoff functions for the issuers' contingent claims can be divided into scenarios such as occurrences of loss-absorbing conversions, issuers' default, and so on. Each scenario reflects an area of the joint distribution plot of the issuers' asset and the reference asset values. Thus a contingent claim

[5] Similar to MCs, contingent conversion convertible bonds studied by Di Girolamo et al. (2012) also have both mechanisms, except that their benefit-sharing conversion grants holders the right to convert the bond at an arbitrary time before maturity.

value contributed by a scenario is evaluated by the integral of the joint density function multiplied by the payoff function over the area of that scenario. The contingent claim value is then the lump sum of the values contributed by all scenarios. The interest expense for a newly-issued bond is then represented by the coupon rate that makes the offering price equal to the bond face value. Claim dilution effects (the severity of asset substitution problems) are analyzed quantitatively by comparing the value changes in equity and existent debt due to issuances of new bonds (the occurrence of risk-shifting behaviors).

The impact of the presence and absence of loss-absorbing and benefit-sharing mechanisms is summarized as follows. First, since loss-absorbing mechanisms reduce default likelihood and hence bankruptcy costs for issuers, the advantages of issuing bail-in-able bonds tend to be more significant when issuers are more likely to default. Therefore, firms tend to issue bail-in-able bonds when they are close to distress. Specifically, even though a bail-in-able bond investor theoretically should require a higher return for bearing additional risks than a non-bail-in-able investor, the interest expense for issuing the former bond tends to be lower than that for issuing the latter one when the issuer's leverage ratio is high, the bond maturity is long, and the issuance amount is high. In addition, such issuances could simultaneously benefit both equity and other already existent debt holders in that the changes in their values due to issuances of new bonds are positive. On the other hand, the interest expenses for issuing non-bail-in-able bonds are generally lower than those for issuing bail-in-able bonds when the issuer is unlikely to default.

Second, the presence of benefit-sharing mechanisms provides upside potential profits for bail-in-able bond investors which motivates them to require lower returns. Thus the interest expense for issuing an `RC` (`RE`) is higher than that for issuing an otherwise identical `MC` (`GB`). In addition, allowing bond investors to share upside potential profits prevents the issuers from increasing their equity holders' benefits at the expense of other debt holders. We find that issuing `MC`s rather than `RC`s effectively alleviates asset substitution problems in that increments in equity values (or decrements in existent debt values) due to the issuers' risk-shifting behaviors after issuing `MC`s are much smaller than those after issuing `RC`s. On the other hand, although an `RE` or a `GB` issuer cannot directly control the reference asset prices, the issuer can unwind the loss-absorbing mechanism by longing forwards on the reference asset as discussed in Chidambaran et al. (2001) to increase equity holders' benefits. Our analyses confirm their findings by showing that the presence of benefit-sharing mechanisms in `GB`s attenuates such risk-shifting incentive.

Third, the correlation between the value of the reference asset and the issuer's asset strongly influences the merits of issuing `GB`s and `RE`s. Interest expenses for issuing these bonds increase with decrements in the correlation as loss-absorbing mechanisms are weakened. Our analyses show that the interest expense for issuing a `GB` is even lower than that for issuing a non-bail-in-able bond when the correlation is extremely high, regardless of the issuer's leverage ratio.

As summarized below, the above analyses constitute a guideline for non-financial firms for selecting proper bail-in-able bonds as fundraising tools. Two-sided bail-in-able bonds are better than one-sided bonds in terms of interest expenses and mitigation of agency problems. A `GB` is especially suitable for a non-financial firm if its business depends heavily on a certain asset value. This result echoes the studies of Haushalter (2000) and Chidambaran et al. (2001) which show that a gas provider or a gold mining company can issue a bond denominated in gas or gold. On the other hand, issuing an `MC` rather than a non-bail-in-able bond could be a better choice for a high-leverage public firm that seeks to raise long-term debt capital. This reflects the observations in Chemmanur et al. (2003) that a high-leverage firm chooses MCs over standard convertible bonds.[6]

The rest of the paper is organized as follows. In Section 2, we introduce our model settings and the corresponding baseline assumptions. In addition, we describe the designs of four bail-in-able bonds and the corresponding mathematical settings. In Section 3, we develop a novel quadrature evaluation framework based on the structural credit risk model. Section 4 illustrates our theoretical results with empirical implications, and Section 5 concludes the paper. Extensive sensitivity analyses and the corresponding empirical implications of our model are given in the Appendix.

2 Baseline Model Setup

2.1 Fundamental Settings of Structural Models

A structural model specifies the evolution of the market value of a bond-issuing firm's assets and the conditions of defaults. All securities issued by the firm are viewed as contingent claims on the firm's asset. Here we follow Merton (1974) by assuming that this issuer's asset value at an arbitrary time t, $V(t)$, follows a log-normal process under the risk-neutral probability measure:

$$\frac{dV(t)}{V(t)} = rdt + \sigma_1 dZ_1(t). \tag{1}$$

We follow Attaoui and Poncet (2013) by setting r to the average interest rate. σ_1 denotes the volatility and is regarded as a proxy for the bond issuing firm's business risk as in Merton (1974). We follow Fan and Sundaresan (2000) by setting σ_1 to a constant, since the firm manager cannot alter the business risk arbitrarily at any time due to restrictive covenants in outstanding bonds of the firm. Z_1 denotes a standard Brownian motion.

We follow the two-bond capital structure setting adopted in Attaoui and Poncet (2013) to formulate our framework as follows. Assume that the firm decides to raise D_2 debt capital by issuing a T-year subordinate bond at time 0. Its capital structure prior to the issuance consists of O shares of common stocks with market value $S^U(0)$ and one straight bond (SB) with par value D_1, T-year time to maturity, and annual coupon rate C_1. This paper analyzes the pros and cons of issuing either one of the four bail-in-able bonds mentioned above or an otherwise identical straight junior bond (JB). Each newly-issued bond is assumed to be issued at par so we can compare the interest expenses for issuing different kinds of bonds by directly comparing the corresponding coupon rates C_2 evaluated by our quadrature method. Using debt capital incurs two types of market friction: corporate income taxes and bankruptcy costs. As long as the firm is solvent, its coupon payments are tax-deductible at rate τ, $\tau \in (0, 1)$. We follow Moody's definition of debt default by assuming that the firm announces bankruptcy once its asset cannot fulfill disbursements of interest or principal payments (see Ou et al., 2011), since acceleration clauses are rare in corporate bond indentures. The firm is then liquidated immediately after filing for bankruptcy based on the Chapter 7 proceedings as in Hackbarth and Mauer (2011) and Kuehn and Schmid (2014). A constant fraction α, $\alpha \in (0, 1)$, of the firm's asset value is lost as bankruptcy costs (e.g., legal fees,see Leland, 1994) and the remaining asset value is distributed according to the absolute priority rule as reported empirically by Bris et al. (2006).[7]

The relation between the firm's levered value and the sum of its all contingent claim values imme-

[6]According to Chemmanur et al. (2003), 5 billion dollars of MCs were issued in 1996, accounting for 25% of the convertible market. The issuance amount then increased to 20 billion dollars in 2001.

[7]For simplicity, the reorganization procedures in the Chapter 11 proceedings such as grace periods and subsequent debt renegotiations are not considered in this paper.

diately before the new bond issuance (denoted by time 0^-), and after the issuance (denoted by time 0) can be expressed by the following equations based on trade-off theory. These equations not only sketch the changes of the issuer's capital structure but verify the correctness of the evaluation results of all contingent claims generated by our quadrature method. At an arbitrary time t, denote the value of the `SB` by $SB(t)$, the newly-issued `JB` by $JB(t)$, and the four otherwise identical bail-in-able bonds `RC`, `RE`, `GB`, and `MC` by $RC(t)$, $RE(t)$, $GB(t)$, and $MC(t)$, respectively. $TB(t)$ and $BC(t)$ indicate the present values of the sum of future tax shield benefits and bankruptcy costs occurring during the period $[t, T]$, respectively. Raising D_2 debt capital by issuing a subordinate bond yields the relation $V(0) = V(0^-) + D_2$. Before issuing a new subordinate bond, the trade-off theory suggests that

$$\begin{aligned} V^L(0^-) &= O \cdot S^U(0^-) + SB(0^-) \\ &= V(0^-) + TB(0^-) - BC(0^-), \end{aligned}$$

where V^L denotes the firm's levered value. After issuing a new subordinate bond, the equation changes to reflect the change of the issuer's capital structure as

$$\begin{aligned} V^L(0) &= O \cdot S^U(0) + SB(0) + X(0) \\ &= V(0) + TB(0) - BC(0), \end{aligned} \tag{2}$$

where $X(0)$ could be the value of a `JB` or one of the four bail-in-able bonds. Note that both the par value of the newly issued bond and $X(0)$ are equal to D_2 since the bond is assumed to be issued at par. The impact of issuing a new bond on an existent claim holder, such as the `SB` holder, can be estimated by comparing the changes of the `SB` value: $SB(0) - SB(0^-)$. We compare the value changes of the equity and the `SB` due to issuances of different (non)-bail-in-able bonds to study the pros and cons of a non-financial firm issuing a bail-in-able bond and the corresponding agency problems.

Recall that the payoff of a `GB` or an `RE` is determined by the value of the reference asset that does not directly link to the issuer's asset. We model the value of this reference asset at time t as $G(t)$ to obey the following process under the risk-neutral probability measure:

$$\frac{dG(t)}{G(t)} = r dt + \sigma_2 dZ_2(t), \tag{3}$$

where Z_2 denotes a standard Brownian motion. The correlation between Z_1 and Z_2 is denoted by ρ, where $\rho \in [-1, 1]$. By solving the differential forms described in **Equations (1)** and **(3)**, we derive the mathematical expression of $V(t)$ and $G(t)$ as

$$V(t) = V(0)e^{\sigma_1 B_1(t)}, \tag{4}$$

$$G(t) = G(0)e^{\sigma_2 B_2(t)}, \tag{5}$$

where

$$B_1(t) = \frac{1}{\sigma_1}\left(r - \frac{\sigma_1^2}{2}\right)t + Z_1(t), \quad B_2(t) = \frac{1}{\sigma_2}\left(r - \frac{\sigma_2^2}{2}\right)t + Z_2(t).$$

Note that the correlation between B_1 and B_2 is also ρ, and Liu et al. (2021) suggest that the joint density function of B_1 and B_2 is

$$f_{B_1,B_2}(b_1,b_2)=\frac{1}{2\pi t\sqrt{1-\rho^2}}\,e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{b_1-\frac{(r-0.5\sigma_1^2)t}{\sigma_1}}{\sqrt{t}}\right)^2-2\rho\left(\frac{b_1-\frac{(r-0.5\sigma_1^2)t}{\sigma_1}}{\sqrt{t}}\right)\left(\frac{b_2-\frac{(r-0.5\sigma_2^2)t}{\sigma_2}}{\sqrt{t}}\right)+\left(\frac{b_2-\frac{(r-0.5\sigma_2^2)t}{\sigma_2}}{\sqrt{t}}\right)^2\right]}. \quad (6)$$

2.2 Payoffs of Newly-Issued Bonds

For ease of illustration, we consider a (non)-bail-in-able bonds with a single repayment to highlight how the repayment and hence the issuer's default likelihood is changed due to the trigger of different loss-absorbing mechanisms. It also facilitates the illustration of how the bond investors receive more due to the triggers of benefit-sharing mechanisms (if applicable). Bonds with more than one coupon payment can be extended easily, and their evaluations can be handled by just summing all of the repayments as in Collin-Dufresne and Goldstein (2001). First, the payoff of a non-bail-in-able `JB` at time T is

$$JB(T)=\begin{cases} D_2+C_2D_2T & \text{if the issuer survives, i.e.,}\\ & V(T)\geq(D_1+D_2)+(1-\tau)(C_1D_1+C_2D_2)T,\\ \text{Max}(\ (1-\alpha)V(T)-(D_1+C_1D_1T)\ ,\ 0\ ) & \text{if the issuer files for bankruptcy, i.e.,}\\ & V(T)<(D_1+D_2)+(1-\tau)(C_1D_1+C_2D_2)T. \end{cases} \quad (7)$$

Note that the issuance of a `JB` increases the debt repayment by $D_2+(1-\tau)C_2D_2T$ as well as the default likelihood and hence results in a claim dilution effect on other already existent claim holders.

Next, we consider an `RC` that realizes its loss-absorbing mechanism by converting the `RC` into O_{cc} shares of the issuer's common stocks when the unconverted stock price $S^U(T)$ falls below the conversion trigger price S' specified in the contract. The payoff of an `RC` can be expressed as

$$RC(T)=\begin{cases} D_2+C_2D_2T & \text{if the issuer survives, and}\\ & \text{the RC is not converted, i.e., } S^U(T)>S';\\ O_{cc}S^C(T)+C_2D_2T & \text{if the issuer survives, and}\\ & \text{the RC is converted, i.e., } S^U(T)\leq S';\\ \text{Max}(\ (1-\alpha)V(T)-(D_1+C_1D_1T)\ ,\ 0\ ) & \text{if the issuer files for bankruptcy, i.e.,}\\ & V(T)<D_1+(1-\tau)(C_1D_1+C_2D_2)T. \end{cases} \quad (8)$$

The `RC` holder is assumed to receive the accrued interest C_2D_2T after converting the `RC` by following the setting for an ordinary convertible bond (see Bhattacharya, 2012). The conversion of the `RC` dilutes equity shares; we use $S^C(T)$ to denote the corresponding stock price after the conversion. To prevent stock price manipulation, we follow the no-wealth-transfer condition proposed in Sundaresan and Wang (2015) to establish the relation between O_{cc} and S' as discussed in Section 2.3. Our formula can be easily modified to consider principal write-down by setting O_{cc} to 0. Though the principal write-down could prevent equity dilution, it clearly induce the `RC` issuers to create net windfall gains for their equity holders at the expense of the `RC` investors. Thus our later analyses focus on the `RC`s with stock conversion terms, comparing them with two-sided `MC`s to elucidate the impact of including benefit-sharing mechanisms.

An `RE` is exchanged for the reference asset worth $D_2\frac{G(T)}{G'}$ at time T once the value of the reference asset $G(T)$ falls below the contract specified value G', that is, part of the principal repayment is written off. This property properly performs the function of a loss-absorbing mechanism if the correlation between $G(T)$ and $V(T)$, ρ, is close to 1. The payoff of an RE can be expressed as

$$RE(T) = \begin{cases} D_2 + C_2 D_2 T & \text{if the issuer survives, and} \\ & \text{the RE is not exchanged, i.e., } G(T) > G'; \\ D_2 \frac{G(T)}{G'} + C_2 D_2 T & \text{if the issuer survives, and} \\ & \text{the RE is exchanged, i.e., } G(T) \leq G'; \\ \text{Max}(\ (1-\alpha)V(T) - (D_1 + C_1 D_1 T)\ ,\ 0\ ) & \text{if the issuer files for bankruptcy, i.e.,} \\ & V(T) < (D_1 + D_2) \\ & \quad +(1-\tau)(C_1 D_1 + C_2 D_2)T,\ G(T) > G'; \\ & V(T) < (D_1 + \frac{G(T)}{G'} D_2) \\ & \quad +(1-\tau)(C_1 D_1 + C_2 D_2)T,\ G(T) \leq G'. \end{cases} \tag{9}$$

The payoff of a GB is denominated in the value of the reference asset $G(T)$ as

$$GB(T) = \begin{cases} \frac{G(T)}{G'}(D_2 + C_2 D_2 T) & \text{if the issuer survives, i.e.,} \\ & V(T) \geq (D_1 + \frac{G(T)}{G'} D_2) \\ & \quad + (1-\tau)(C_1 D_1 + \frac{G(T)}{G'} C_2 D_2)T \\ \text{Max}(\ (1-\alpha)V(T) - (D_1 + C_1 D_1 T)\ ,\ 0\ ) & \text{if the issuer files for bankruptcy otherwise.} \end{cases} \tag{10}$$

Both the loss-absorbing and benefit-sharing mechanisms are properly performed by a GB contract if ρ is close to 1. In particular, the loss-absorbing mechanism is realized in the way that the repayment of the GB decreases as $G(T)$ falls below G'; the benefit-sharing mechanism is realized in the opposite way. Note that a relatively low correlation between $G(T)$ and $V(T)$ damages the functionalities of both mechanisms and may introduce a greater repayment when the issuer is in a poor financial status.

An MC also possesses both loss-absorbing and benefit-sharing mechanisms. The former mechanism converts an MC into O_{cc} shares of the issuer stock as $S^U(T)$ falls below the predetermined level S'; the latter converts an MC into O_u shares of the issuer stock as $S^U(T)$ exceeds another predetermined price level $\lambda S'$, where the upward conversion threshold $\lambda > 1$. The payoff at time T is

$$MC(T) = \begin{cases} O_u S^C(T) + C_2 D_2 T & \text{if the issuer survives, and} \\ & \text{the MC is converted, i.e., } S^U(T) \geq \lambda S^{'}; \\ D_2 + C_2 D_2 T & \text{if the issuer survives, and} \\ & \text{the MC is not converted, i.e., } S^{'} < S^U(T) < \lambda S^{'}; \\ O_{cc} S^C(T) + C_2 D_2 T & \text{if the issuer survives, and} \\ & \text{the MC is converted mandatorily, i.e., } S^U(T) \leq S^{'}; \\ \text{Max}(\ (1-\alpha)V(T) - (D_1 + C_1 D_1 T)\ ,\ 0\ ) & \text{if the issuer files for bankruptcy, i.e.,} \\ & V(T) < D_1 + (1-\tau)(C_1 D_1 + C_2 D_2)T. \end{cases} \tag{11}$$

2.3 Market Price Triggers and Conversion Volumes

RCs and MCs can provide timely emergency bail-ins, since their loss-absorbing mechanisms are triggered according to their issuers' stock prices. However, such designs may motivate the equity holders and the bail-in-able bond investors to increase their own benefits by manipulating the issuers' stock prices. To eliminate the incentives, Sundaresan and Wang (2015) suggest a proper setting of the conversion volume O_{cc} given the conversion trigger $S^{'}$ to prevent wealth transfer between equity holders and bail-in-able bond investors when conversion occurs at $S^{'}$. In other words, converting an RC (or an MC) at $S^{'}$ should not change the stock price. Let V' be the firm's asset value that makes the stock price prior to conversion equal to S'; then we have

$$\begin{aligned} S' &= \frac{V' - [D_1 + D_2 + (1-\tau)(C_1 D_1 T + C_2 D_2 T)]}{O} \\ \Rightarrow \quad V' &= OS' + [D_1 + D_2 + (1-\tau)(C_1 D_1 T + C_2 D_2 T)]. \end{aligned} \tag{12}$$

Since the after-conversion stock price should not change due to the no-wealth-transfer requirement and the conversion does not change the asset value, we have

$$\begin{aligned} S' &= \frac{V' - [D_1 + (1-\tau)(C_1 D_1 T + C_2 D_2 T)]}{O + O_{cc}} \\ \Rightarrow \quad V' &= OS' + O_{cc} S' + [D_1 + (1-\tau)(C_1 D_1 T + C_2 D_2 T)]. \end{aligned} \tag{13}$$

By equating **Equations (12)** and **(13)**, we obtain the conversion volume ensuring the no-wealth transfer requirement

$$O_{cc} = \frac{D_2}{S'} \tag{14}$$

given a predetermined conversion trigger level S'. The corresponding V' can then be solved as

$$V' = D_1 + \left(1 + \frac{O}{O_{cc}}\right) D_2 + (1-\tau)(C_1 D_1 T + C_2 D_2 T). \tag{15}$$

Similarly, the conversion volume O_u in **Equation (11)** can be solved to be

$$O_u = \frac{D_2}{\lambda S'} \tag{16}$$

to prevent price manipulation, and it reveals that the level of equity dilution due to a trigger of the benefit-sharing conversion diminishes with increments in the trigger level. The firm's asset value V'' that makes the stock price equal to $\lambda S'$ is

$$V'' = D_1 + \left(1 + \frac{O}{O_u}\right) D_2 + (1-\tau)(C_1 D_1 T + C_2 D_2 T). \tag{17}$$

The conversion volumes specified in `RC`s and `MC`s adopt the above no-wealth-transfer argument to wipe out the incentives to manipulate their underlying stock prices. However, such argument may not applied to `RE`s and `GB`s as the prices of specified reference assets are relatively difficult for their issuers or investors to manipulate.

2.4 Avoidance of Asset Substitution Problems

The loss-absorbing mechanisms in bail-in-able bonds may motivate the issuers to take excessive risk for their equity holders to gain extra benefits at the expense of their `SB` holders. This paper will show that the presence of benefit-sharing mechanisms not only decreases the interest expense for raising debt capital but also attenuates risk-shifting incentives by considering two following comparisons. First, we compare two otherwise identical bonds, an `RC` and an `MC`, whose payoffs link to their issuers' stock prices. Issuers make riskier decisions to increase the volatility of their asset value and thus the stock prices. To demonstrate how the benefit-sharing mechanism in an `MC` alleviates the risk-shifting incentives, we compare the value changes of the equity and the `SB` due to the change of asset value volatility σ_1 defined in **Equation (1)** following the issuance of an `RC` or an otherwise identical `MC`. Similarly, we compare two otherwise identical bonds, an `RE` and a `GB`, whose payoffs link to another reference asset. Although it is difficult for their issuers to manipulate the volatility of the reference asset, they may long forward contracts on the reference asset at time $t = 0^+$ to unwind the loss-absorbing mechanism as shown in Chidambaran et al. (2001). We again compare the value changes of the equity and the `SB` due to longing forwards following the issuance of an `RE` or an otherwise identical `GB`. We will show that a two-sided bond issuer has smaller value changes (i.e, the agency problem is less severe) given otherwise identical conditions.

3 Proposed Evaluation Framework

In this section we illustrate a novel quadrature method for evaluating a bond issuer's contingent claims Since a `GB` can be viewed as an `RE` plus a benefit-shifting mechanism, we will discuss the general case `GB` in detail in Section 3.1 and brief the degenerate case `RE` in Appendix A. Similarly, we will discuss the general case `MC` in Section 3.2 and brief the degenerate case `RC` in Appendix B. To evaluate a contingent claim, we will first enumerate all possible scenarios, such as loss-absorbing conversions, benefit-sharing conversions, and defaults into non-overlap areas of a two-dimensional plane of the issuer's asset value $V(T)$ and the reference asset value $G(T)$ defined in **Equations (1)** and **(3)**, respectively. Each area is then converted into its corresponding area of another two-dimensional plane of jointly normal random variables $B_1(T)$ and $B_2(T)$. A contingent claim value contributed by a scenario is then evaluated as the integral of the joint density function multiplied by the claim's payoff over the area of that scenario. The overall claim value is the lump sum of the values contributed by all scenarios.

3.1 Evaluation of the Firm with a `GB` Outstanding

Recall that a `GB` or an `RE` issuer can unwind the loss-absorbing mechanism by longing forwards on the reference asset. Thus we first evaluate the value changes of its contingent claims due to the issuance of a `GB`. Then we discuss the agency problem by analyzing the impact of longing forwards to unwind the `GB`.

3.1.1 Impacts of Issuing a `GB`

Let a firm with a T-year `SB` decide to issue a T-year `GB` with the setting $G' = G(0)$ at time 0. Three possible scenarios at time T are expressed by the corresponding areas located on a two-dimensional plane of $V(T)$ and $G(T)$ as illustrated in **Figure 1**. The firm survives if its asset value exceeds the after-tax debt repayments of the `SB` ($\mathbf{A} \equiv D_1 + (1-\tau)C_1 D_1 T$) and the `GB` ($\frac{G(T)}{G(0)}(D_2 + (1-\tau)C_2 D_2 T)$) (see **Equation (10)**). We use scenario 1 (abbreviated as S1) to denote the survival scenario. Summing the aforementioned repayments yields a yellow-line boundary that separates the survival and bankrupt statuses defined as

$$
\begin{aligned}
V(T) &= \mathbf{A} + \frac{G(T)}{G(0)}\mathbf{B} \\
&= \left(D_1 + \frac{G(T)}{G(0)} D_2\right) + (1-\tau)\left(C_1 D_1 + \frac{G(T)}{G(0)} C_2 D_2\right) T,
\end{aligned}
\tag{18}
$$

where $\mathbf{B} = D_2 + (1-\tau)C_2 D_2 T$. Note that the positive slope of the default boundary reflects the fact that the debt repayment decreases with decrements in the reference asset price. If the correlation between the values of the reference asset and the issuer's asset is high, this property properly yields a loss-absorbing effect to decrease the issuer's default likelihood. On the other hand, the firm files for bankruptcy if it fails to repay debts. The asset after paying the bankruptcy cost is first repaid to the `SB` holder and the remaining value (if any) is then distributed to the `GB` investor. We use scenario 2 (abbreviated as S2) to denote the case in which the `SB` holder is fully repaid but the `GB` investor is repaid partially. Scenario 3 (S3) denotes the case in which the `SB` holder is repaid partially and the

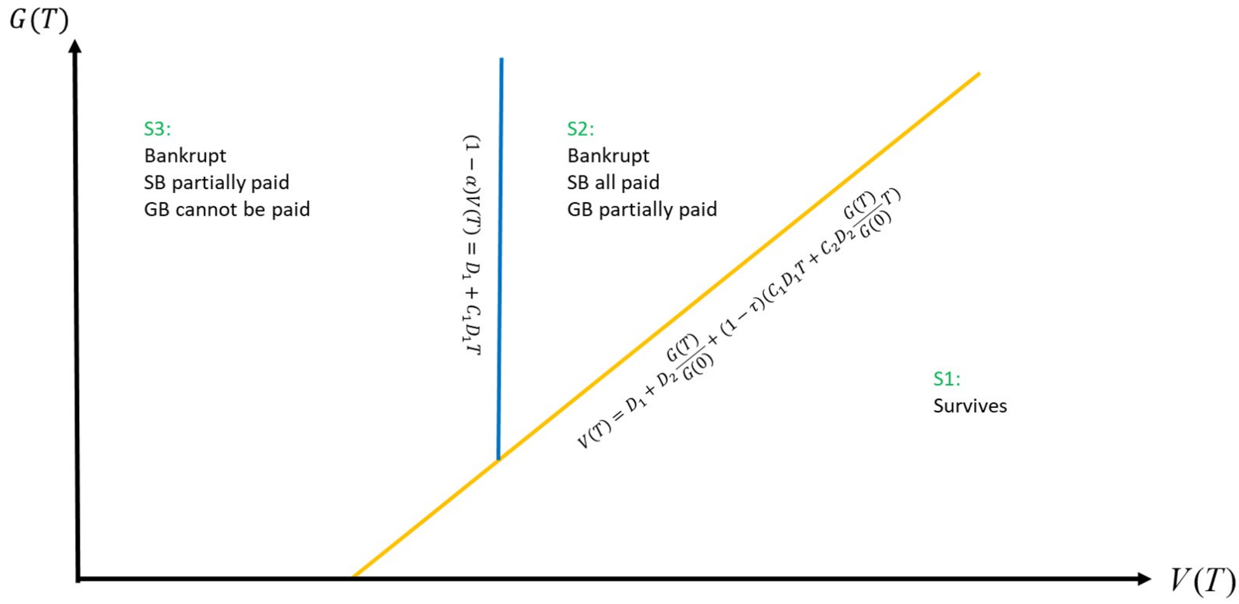


Figure 1: Possible Scenarios for Issuing a GB. The X- and Y-axes denote the values of the GB issuer's asset and the reference asset, respectively, at time T. The equations for the yellow and blue boundaries are listed next to the boundaries. The description for each scenario is given in boldface below the scenario.

GB investor receives nothing. We use the blue line boundary defined as

$$V(T) = \frac{D_1 + C_1 D_1 T}{1-\alpha} \tag{19}$$

to separate scenarios S2 and S3.

The GB value at time 0, GB(0), can be evaluated as the discounted expected payoffs defined in **Equation (10)** by taking advantage of the risk-neutral valuation method as

$$GB(0) = \tilde{E}\left[\left[(D_2 + C_2 D_2 T)\frac{G(T)}{G(0)}\right] e^{-rT} \cdot \mathbf{I}_{\{S1\}}\right] + \tilde{E}\left[\text{Max}[(1-\alpha)V(T) - (D_1 + C_1 D_1 T), 0]e^{-rT} \cdot \mathbf{I}_{\{S2,S3\}}\right], \tag{20}$$

where $\tilde{E}$ denotes the expected value under the risk-neutral probability. Substituting **Equations (4)** and **(5)** for $V(T)$ and $G(T)$, respectively, yields

$$GB(0) \;=\; e^{-rT}\left[(D_2 + C_2 D_2 T)\tilde{E}\left[e^{\sigma_2 B_2(T)} \cdot \mathbf{I}_{\{S1\}}\right] + \;(1-\alpha)V(0)\tilde{E}\left[e^{\sigma_1 B_1(T)} \cdot \mathbf{I}_{\{S2\}}\right] - (D_1 + C_1 D_1 T)\tilde{E}\left[\mathbf{I}_{\{S2\}}\right]\right]. \tag{21}$$

The corresponding SB value can be expressed as the discounted expected payoffs contributed by three scenarios:

$$\begin{aligned} \text{SB}(0) &= \tilde{E}\left[(D_1 + C_1 D_1 T)e^{-rT} \cdot \mathbf{I}_{\{S1\}}\right] + \tilde{E}\left[\text{Min}[(1-\alpha)V(T), (D_1 + C_1 D_1 T)]e^{-rT} \cdot \mathbf{I}_{\{S2,S3\}}\right], \\ &= e^{-rT}\left[(D_1 + C_1 D_1 T)\left[\tilde{E}\left[\mathbf{I}_{\{S1\}}\right] + \tilde{E}\left[\mathbf{I}_{\{S2\}}\right]\right] + (1-\alpha)V(0)\tilde{E}\left[e^{\sigma_1 B_1(T)} \cdot \mathbf{I}_{\{S3\}}\right]\right]. \end{aligned} \tag{22}$$

The corresponding equity value can be evaluated as

$$\begin{aligned} \text{E}(0) &= \tilde{E}\left[\left[V(T) - \left[\mathbf{A} + \mathbf{B}\frac{G(T)}{G(0)}\right]\right]e^{-rT} \cdot \mathbf{I}_{\{S1\}}\right] \\ &= e^{-rT}\left[V(0)\tilde{E}\left[e^{\sigma_1 B_1(T)} \cdot \mathbf{I}_{\{S1\}}\right] - \mathbf{A}\tilde{E}\left[\mathbf{I}_{\{S1\}}\right] - \;\mathbf{B}\tilde{E}\left[e^{\sigma_2 B_2(T)} \cdot \mathbf{I}_{\{S1\}}\right]\right]. \end{aligned} \tag{23}$$

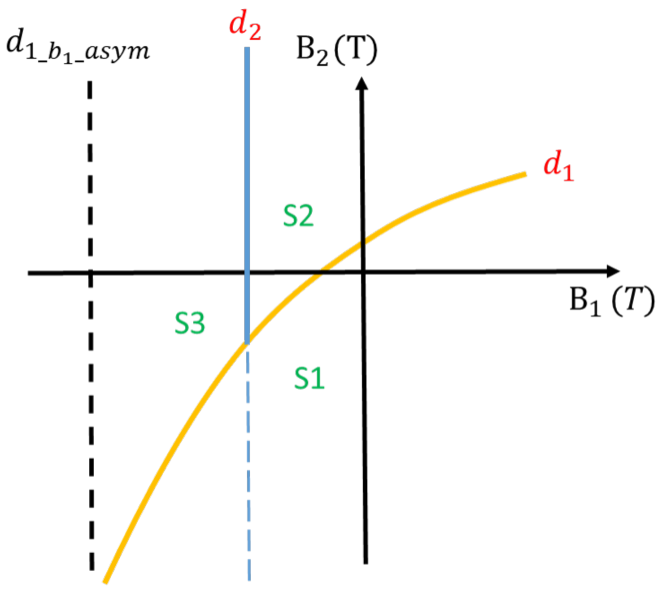


Figure 2: Transforming Scenario Areas from Figure 1 into the ($B_1(T)$, $B_2(T)$) Coordinate System. The yellow curve d_1 and the blue solid line d_2 are transformed from the yellow and the blue boundaries, respectively, in **Figure 1** to separate the overall plane into S1, S2, and S3. d_1 separates the survival scenario (S1) from the bankrupt ones (S2 and S3). Given the firm is bankrupt, the blue solid line separates the case in which the SB is fully repaid (S2) from the case in which the SB is partially repaid. The black dashed line $d_{1\_b_1\_asym}$ is the asymptote of d_1.

Note that GB(0), SB(0), and E(0) can be expressed as linear combinations of $\tilde{E}\left[g \cdot \mathbf{I}_{\{S\}}\right]$, where g can be 1, $e^{\sigma_1 B_1(T)}$, or $e^{\sigma_2 B_2(T)}$, and S denotes a scenario. The following analyses will focus on the evaluations of these expectations.

Now we transform the aforementioned three scenarios from the $(V(T), G(T))$ two-dimensional plane illustrated in **Figure 1** into the $(B_1(T), B_2(T))$ plane illustrated in **Figure 2**. The boundary to separate survival from bankrupt status (the yellow curve) is transformed from **Equation (18)** into the equation for curve d_1 illustrated in **Figure 2** as

$$B_2(T) = \frac{1}{\sigma_2} \ln \left[\frac{V(0)e^{\sigma_1 B_1(T)} - \mathbf{A}}{\mathbf{B}} \right]. \tag{24}$$

Note that $B_2(T) \to -\infty$ as $B_1(T) \to d^+_{1\_b_1\_asym}$, where

$$d_{1\_b_1\_asym} = \frac{1}{\sigma_1} \ln \left[\frac{\mathbf{A}}{V(0)} \right].$$

Similarly, the boundary separating scenario S2 from S3 in **Figure 1** (the blue line) is transformed from **Equation (19)** into the function of d_2 in **Figure 2** as

$$B_1(T) = \frac{1}{\sigma_1} \ln \left[\frac{D_1 + C_1 D_1 T}{(1-\alpha)V(0)} \right]. \tag{25}$$

The contingent claim values described in **Equations (21)**, **(22)**, and **(23)** are expressed by the numerical integrations with the joint probability distribution of B_1 and B_2 described in **Equation (6)** and the lower/upper limits determined by the boundaries analyzed above as

$$\begin{aligned}
\tilde{E}\Big[g \cdot \mathbf{I}_{\{S1\}}\Big] &= \int_{d_{1_b_1_asym}}^{\infty} \int_{-\infty}^{d_1} g f_{B_1,B_2}(b_1, b_2) db_2 db_1, \\
\tilde{E}\Big[g \cdot \mathbf{I}_{\{S2\}}\Big] &= \int_{d_2}^{\infty} \int_{d_1}^{\infty} g f_{B_1,B_2}(b_1, b_2) db_2 db_1, \\
\tilde{E}\Big[g \cdot \mathbf{I}_{\{S3\}}\Big] &= \int_{-\infty}^{d_{1_b_1_asym}} \int_{-\infty}^{\infty} g f_{B_1,B_2}(b_1, b_2) db_2 db_1 + \int_{d_{1_b_1_asym}}^{d_2} \int_{d_1}^{\infty} g f_{B_1,B_2}(b_1, b_2) db_2 db_1.
\end{aligned}$$

3.1.2 Unwinding Both Mechanisms by Longing Forwards

If the correlation between the values of the reference asset and the issuer's asset is high, the GB repayment in **Equation (10)** properly performs the loss-absorbing (benefit-sharing) mechanism by paying less (more) when the issuer's asset value is low (high) as illustrated by the positive yellow boundary in **Figure 1**. Although such arrangements could reduce the issuer's default likelihood and increase the GB investor's potential benefits, the issuer may increase its equity holders' benefits at the expenses of the SB holder by longing m units of forwards on the reference asset to unwind both mechanisms. For convenience, we assume that the repayment of the forwards is senior to those of the SB and the GB when the firm files for bankruptcy. Thus there are four scenarios if the firm longs m units of forwards as illustrated in **Figure 3**. The firm survives if its asset value plus the forward payment exceeds the after-tax SB and GB repayments. Thus the boundary separating the survival (i.e., S1) and bankrupt statuses can be modified from **Equation (18)** as

$$V(T) + m(G(T) - F) = \mathbf{A} + \mathbf{B}\frac{G(T)}{G(0)} \tag{26}$$

to reflect the case in which the issuer's value plus the forward's payoff just covers the SB and GB repayments. Both mechanisms are just offset if m is set to $(D_2 + (1-\tau)C_2D_2T)/G(0)$; this is illustrated via the change from the positively sloped yellow boundary in **Figure 1** to the vertical yellow boundary in **Figure 3**. On the other hand, the issuer files for bankruptcy if it fails to repay debts, and its asset after paying the bankruptcy cost is distributed according to the absolute priority rule; that is, the after-liquidation asset value is first used to fulfill the forward repayment, and the remaining asset value (if any) is then used to fulfill the SB repayment. The remaining asset value finally goes to the GB investor. Here we use scenario 4 (S4) to denote the case in which no residual payment go to the SB holder, scenario 3 (S3) to denote that the SB holder is partially repaid, and scenario 2 (S2) to denote that the GB investor is partially repaid. The red boundary that separates S3 and S4 is derived as

$$(1-\alpha)V(T) + m(G(T) - F) = 0 \tag{27}$$

to reflect the case in which the after-liquidation asset value covers only the forward repayment. The blue boundary that separates S2 and S3 is derived as

$$(1-\alpha)V(T) + m(G(T) - F) = D_1 + C_1D_1T \tag{28}$$

to reflect the case in which the after-liquidation asset value covers only the forward plus the SB repayments. The GB value at time 0 changes from **Equation (20)** to

$$\begin{aligned}
GB(0) \quad = \quad & \tilde{E}\left[\left[(D_2 + C_2D_2T)\frac{G(T)}{G(0)}\right] e^{-rT} \cdot \mathbf{I}_{\{S1\}}\right] \\
& + \tilde{E}\left[\text{Max}\left[(1-\alpha)V(T) + m(G(T) - F) - (D_1 + C_1D_1T), 0\right] e^{-rT} \cdot \mathbf{I}_{\{S2,S3,S4\}}\right]
\end{aligned} \tag{29}$$

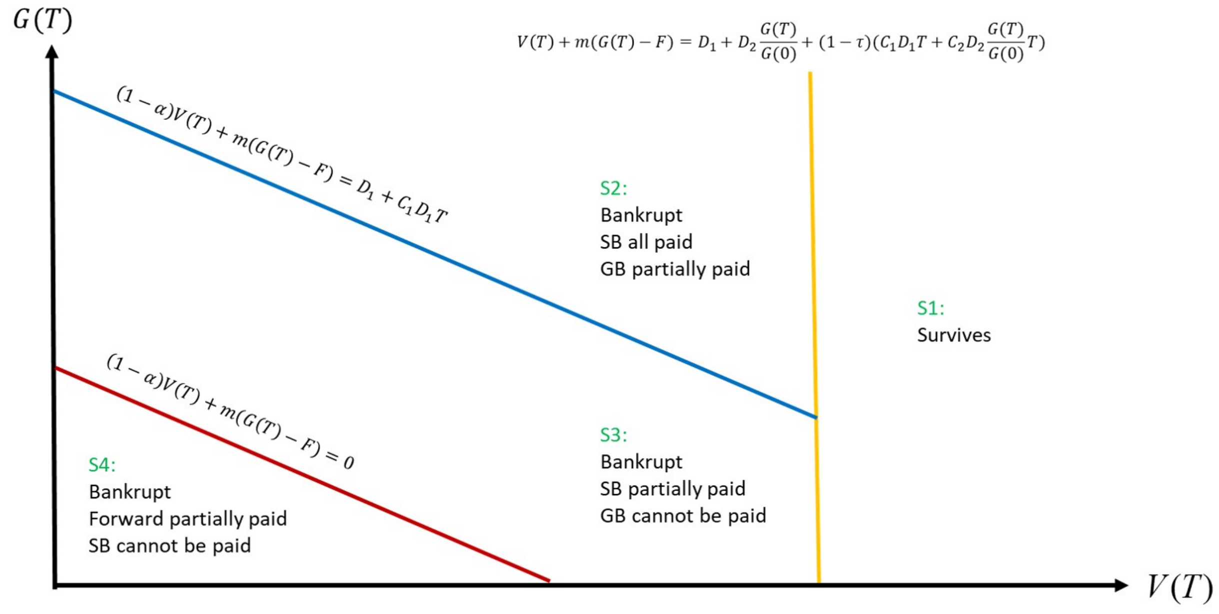


Figure 3: Possible Scenarios by Longing Forwards to "Hedge" the GB. The X- and Y-axes denote the values of the issuer's asset and the reference asset, respectively, at time T. The equations for the yellow, blue, and red boundaries are listed next to these boundaries, which separate the plane into four scenarios. Below each scenario is its description. This figure reflects case (a) of **Figure 4**.

to reflect the impact of longing forwards. The corresponding SB and equity values are then expressed as

$$\begin{aligned} SB(0) &= \tilde{E}\Big[(D_1 + C_1D_1T)e^{-rT} \cdot \mathbf{I}_{\{S1\}}\Big] \\ &\quad + \tilde{E}\Big[\text{Min}\Big[\text{Max}\left[(1-\alpha)V(T) + m(G(T) - F), 0\right], (D_1 + C_1D_1T)\Big]e^{-rT} \cdot \mathbf{I}_{\{S2,S3,S4\}}\Big], \end{aligned} \tag{30}$$

$$E(0) = \tilde{E}\Big[\Big[V(T) + m(G(T) - F) - \Big[\mathbf{A} + \mathbf{B}\frac{G(T)}{G(0)}\Big]\Big]e^{-rT} \cdot \mathbf{I}_{\{S1\}}\Big]. \tag{31}$$

By substituting **Equations (4)** and **(5)** for $V(T)$ and $G(T)$, respectively in **Equations (29)**, **(30)**, and **(31)**, $GB(0)$, $SB(0)$, and $E(0)$ can be expressed as linear combinations of $\tilde{E}\Big[g \cdot \mathbf{I}_{\{S\}}\Big]$, where g can be 1, $e^{\sigma_1 B_1(T)}$, or $e^{\sigma_2 B_2(T)}$, and S denotes a scenario.

Now we transform the aforementioned four scenarios from the $(V(T), G(T))$ two-dimensional plane displayed in **Figure 3** to the $(B_1(T), B_2(T))$ plane in **Figure 4**. The yellow boundary that separates survival from bankrupt statuses in **Figure 3** can be transformed from **Equation (26)** into curve d_1 in **Figure 4** as

$$B_1(T) = \frac{1}{\sigma_1}\ln\left[\frac{\mathbf{C} - \tau C_1D_1T}{V(0)}\right], \tag{32}$$

where $\mathbf{C} = D_1 + C_1D_1T + mF$. Similarly, the blue boundary separating S2 and S3 can be transformed from **Equation (28)** into the function of d_2 as

$$B_2(T) = \frac{1}{\sigma_2}\ln\left[\frac{\mathbf{C} - (1-\alpha)V(0)e^{\sigma_1 B_1(T)}}{mG(0)}\right]. \tag{33}$$

Note that $B_2(T) \to -\infty$ as $B_1(T) \to d^-_{2\_b_1\_asym}$, where

$$d_{2\_b_1\_asym} = \frac{1}{\sigma_1}\ln\left[\frac{\mathbf{C}}{(1-\alpha)V(0)}\right], \tag{34}$$

and $B_1(T) \to -\infty$ as $B_2(T) \to d^-_{2\_b_2\_asym}$, where

$$d_{2\_b_2\_asym} = \frac{1}{\sigma_2} \ln \left[\frac{\mathbf{C}}{mG(0)} \right]. \tag{35}$$

Finally, the red boundary separating `S3` and `S4` can be transformed from **Equation (27)** into the function of d_3 as

$$B_2(T) = \frac{1}{\sigma_2} \ln \left[\frac{mF - (1-\alpha)V(0)e^{\sigma_1 B_1(T)}}{mG(0)} \right]. \tag{36}$$

Note that $B_2(T) \to -\infty$ as $B_1(T) \to d^-_{3\_b_1\_asym}$, where

$$d_{3\_b_1\_asym} = \frac{1}{\sigma_1} \ln \left[\frac{mF}{(1-\alpha)V(0)} \right], \tag{37}$$

and $B_1(T) \to -\infty$ as $B_2(T) \to d^-_{3\_b_2\_asym}$, where

$$d_{3\_b_2\_asym} = \frac{1}{\sigma_2} \ln \left[\frac{F}{G(0)} \right]. \tag{38}$$

The size relationship among the boundary **Equation (32)** and the asymptotes **(34)** and **(37)** influences the layout of the boundaries as illustrated in cases (a) and (b) of **Figure 4**. Observe that $d_{3\_b_1\_asym} < d_{2\_b_1\_asym}$ and $d_1 < d_{2\_b_1\_asym}$ in both cases. The former inequality entails that d_3 does not intersect d_2 as $B_2(T) \to -\infty$, and the latter entails that d_2 intersects d_1. The size relationship between d_1 and $d_{3\_b_1\_asym}$ determines whether d_1 intersects d_3. Case (a) describes the scenario in which $d_{3\_b_1\_asym} < d_1$ (i.e., $mF - (1-\alpha)(\mathbf{C} - \tau C_1 D_1 T) \leq 0$) and the boundary d_1 does not intersect with d_3. Case (b) describes $d_1 < d_{3\_b_1\_asym}$ (i.e., $mF - (1-\alpha)(\mathbf{C} - \tau C_1 D_1 T) > 0$); d_1 and d_3 intersect. In addition, **Equations (35)** and **(38)** entail that $d_{3\_b_2\_asym} < d_{2\_b_2\_asym}$ and that d_2 and d_3 do not intersect as $B_1(T) \to -\infty$. The contingent claim values described in **Equations (29)**, **(30)**, and **(31)** can be expressed by numerical integrations with the joint probability distribution of B_1 and B_2 in **Equation (6)** and the lower/upper limits determined by the above boundaries as

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S1\}}\Big] = \int_{d_1}^{\infty} \int_{-\infty}^{\infty} g f_{B_1,B_2}(b_1,b_2) db_2 db_1,$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S2\}}\Big] = \int_{-\infty}^{d_1} \int_{d_2}^{\infty} g f_{B_1,B_2}(b_1,b_2) db_2 db_1,$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S3\}}\Big] = \begin{cases} \int_{-\infty}^{d_{3\_b_1\_asym}} \int_{d_3}^{d_2} g f_{B_1,B_2}(b_1,b_2) db_2 db_1 + \int_{d_{3\_b_1\_asym}}^{d_1} \int_{-\infty}^{d_2} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case (a);} \\ \int_{-\infty}^{d_1} \int_{d_3}^{d_2} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case (b).} \end{cases}$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S4\}}\Big] = \begin{cases} \int_{-\infty}^{d_{3\_b_1\_asym}} \int_{-\infty}^{d_3} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case (a);} \\ \int_{-\infty}^{d_1} \int_{-\infty}^{d_3} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case (b).} \end{cases}$$

3.2 Evaluation of the Firm with a `MC` Outstanding

Unlike a `GB` or an `RE` issuer, an `MC` issuer cannot trade its own stock or related derivatives on the stock to unwind the loss-absorbing and benefit-sharing mechanisms. Thus in this subsection we analyze the impact of issuing an `MC` without considering unwinding. Let a firm with a T-year `SB` decides to issue a T-year `MC` at time 0. Five possible scenarios at time T separated by four boundaries at a $(V(T), G(T))$ coordinate system are illustrated in **Figure 5**. These four boundaries are vertical as the `MC`'s payoff

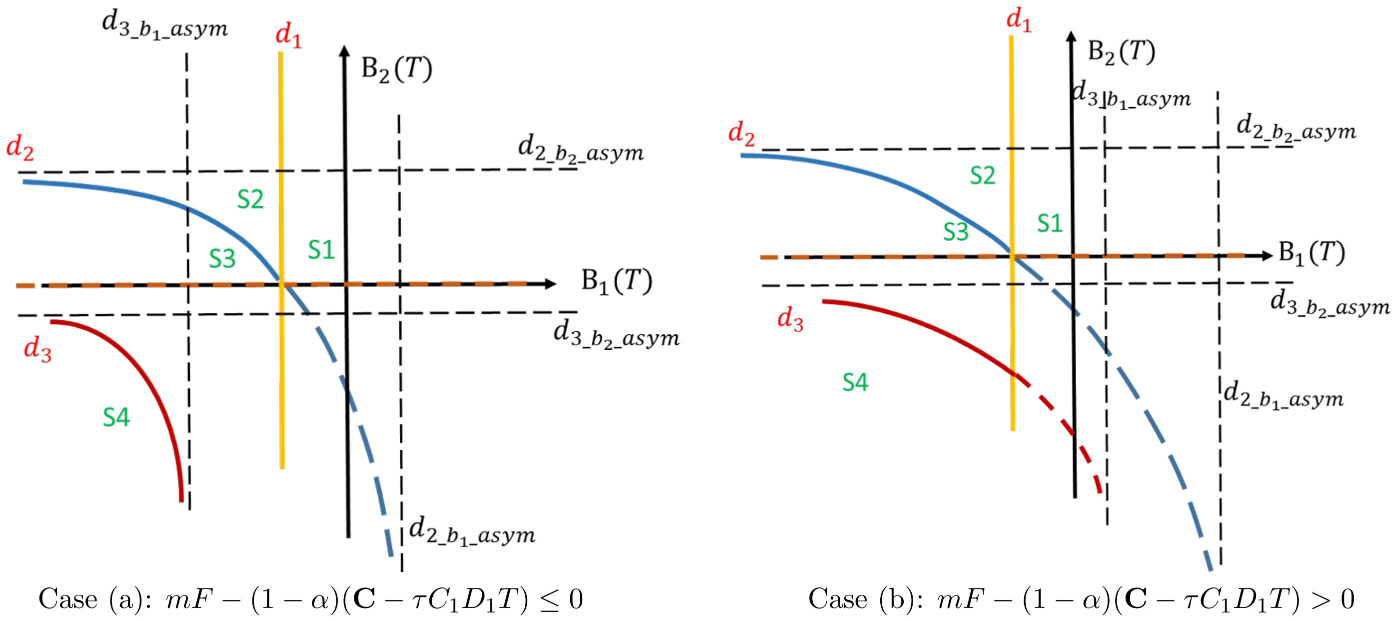


Figure 4: Transforming Scenario Areas from Figure 3 into the $(B_1(T),\ B_2(T))$ Coordinate System. The yellow line d_1, the solid blue curve d_2, and the solid red curve d_3 are transformed from the yellow, blue, and red boundaries, respectively, in **Figure 4** to separate the plane into scenarios S1, S2, S3, and S4. Case (a) and (b) denote the scenarios in which d_3 does not intersect or does intersect with d_1, respectively. d_1 separates the survival scenario (S1) from the bankrupt scenarios (S2, S3, and S4). Given the firm files for bankruptcy, d_2 (the blue solid curve) separates scenario S2 from S3, and d_3 (the red solid curve) separates scenario S3 from S4. The black dashed lines are the asymptote of d_2 and d_3.

listed in **Equation (11)** is irrespective of $G(T)$. The firm survives if its asset meets the after-tax `SB` and `MC` repayments as denoted by the yellow line that separates survival scenarios S1, S2, and S3 from the bankrupt scenarios S4 and S5. S1 indicates the scenario in which the issuer's stock price exceeds the upper contractually-specified price level that triggers the benefit-sharing conversion. On the other hand, in S3 the stock price falls below the lower price level that triggers the loss-absorbing conversion. In S2 the stock price lies between the lower and upper price levels, and the `MC` remains unconverted. The boundary that separates S1 and S2 (S2 and S3) is described in **Equation (17)** (**Equation (15)**). Bankrupt scenarios S4 and S5 are separated by the boundary as described in **Equation (19)**. In the former scenario, the `SB` holder is fully repaid, whereas in the latter, the `SB` holder is partially repaid and the `MC` investor obtains nothing.

The `MC` value at time 0, `MC(0)`, can be evaluated as the discounted expected payoff defined in **Equation (11)** as

$$
\begin{aligned}
MC(0) \quad = \quad & \tilde{E}\left[\left[\underbrace{O_u\,\frac{V(T)-\left[D_1+(1-\tau)(D_1C_1T+D_2C_2T)\right]}{O+O_u}}_{\mathbf{M}}+C_2D_2T\right]e^{-rT}\cdot \mathbf{I}_{\{S1\}}\right] \\
& +\tilde{E}\left[\left[D_2+C_2D_2T\right]e^{-rT}\cdot \mathbf{I}_{\{S2\}}\right] \\
& +\tilde{E}\left[\left[\underbrace{O_{cc}\,\frac{V(T)-\left[D_1+(1-\tau)(D_1C_1T+D_2C_2T)\right]}{O+O_{cc}}}_{\mathbf{N}}+C_2D_2T\right]e^{-rT}\cdot \mathbf{I}_{\{S3\}}\right], \\
& +\tilde{E}\left[\mathrm{Max}[(1-\alpha)V(T)-(D_1+C_1D_1T),0]e^{-rT}\cdot \mathbf{I}_{\{S4,S5\}}\right], \qquad (39)
\end{aligned}
$$

where parts M and N are equal to the after-conversion stock price $S^C(T)$. The corresponding `SB` and

Figure 5: Possible Scenarios for Issuing an `MC`. The X- and Y-axes denote the values of the `MC` issuer's asset and the reference asset, respectively, at time T. The equation for each boundary is listed next to the boundary. The description for each scenario is given in boldface below the scenario. The yellow line denotes the boundary separating the survival scenarios (S1, S2, and S3) from bankrupt scenarios (S4 and S5).

equity values are

$$
\begin{aligned}
SB(0) &= \tilde{E}\Big[(D_1 + C_1 D_1 T)e^{-rT}\cdot \mathbf{I}_{\{S1,S2,S3,S4\}}\Big] + \tilde{E}\Big[\mathrm{Max}[(1-\alpha)V(T), 0]e^{-rT}\cdot \mathbf{I}_{\{S5\}}\Big], \qquad (40)\\
E(0) &= \tilde{E}\Big[\Big[O\,\frac{V(T) - \big[D_1 + (1-\tau)(D_1C_1T + D_2C_2T)\big]}{O+O_u}\Big]e^{-rT}\cdot \mathbf{I}_{\{S1\}}\Big]\\
&\quad + \tilde{E}\Big[\Big[V(T) - \big[D_1 + D_2 + (1-\tau)(C_1D_1T + C_2D_2T)\big]\Big]e^{-rT}\cdot \mathbf{I}_{\{S2\}}\Big]\\
&\quad + \tilde{E}\Big[\Big[O\,\frac{V(T) - \big[D_1 + (1-\tau)(D_1C_1T + D_2C_2T)\big]}{O+O_{cc}}\Big]e^{-rT}\cdot \mathbf{I}_{\{S3\}}\Big]. \qquad (41)
\end{aligned}
$$

Similar to the approach described in Section 3.1, substituting **Equations (4)** and **(5)** for $V(T)$ and $G(T)$ in **Equations (39)**, **(40)**, and **(41)** makes these equations linear combinations of $\tilde{E}\Big[g\cdot \mathbf{I}_{\{S\}}\Big]$, where g can be 1, $e^{\sigma_1 B_1(T)}$, or $e^{\sigma_2 B_2(T)}$, and S denotes a scenario.

Now we transform the aforementioned boundaries and scenarios from the $(V(T), G(T))$ coordinate system in **Figure 5** to the $(B_1(T), B_2(T))$ system. The boundary that separates S1 and S2 can be derived as the vertical equation d_1 as

$$B_1(T) = \frac{1}{\sigma_1}\ln\left[\frac{D_1 + (1+\frac{O}{O_u})D_2 + (1-\tau)(C_1D_1T + C_2D_2T)}{V(0)}\right]. \qquad (42)$$

The boundary that separates S2 from S3 can be derived as the equation d_2:

$$B_1(T) = \frac{1}{\sigma_1}\ln\left[\frac{D_1 + (1+\frac{O}{O_{cc}})D_2 + (1-\tau)(C_1D_1T + C_2D_2T)}{V(0)}\right]. \qquad (43)$$

The boundary that separates S3 from S4 can be derived as d_3:

$$B_1(T) = \frac{1}{\sigma_1} \ln \left[\frac{D_1 + (1-\tau)(C_1 D_1 T + C_2 D_2 T)}{V(0)} \right]. \tag{44}$$

Finally, the equation $d4$ for the boundary separating S4 from S5 is **Equation (25)**. By comparing **Equations (42)–(44)**, we obtain $d_1 > d_2 > d_3$. If α is not small enough to make $d_3 < d_4$ (by comparing **Equations (44)** with **(25)**), scenario S4 does not exist. The contingent claim values for **Equations (39)**, **(40)**, and **(41)** can be expressed by the numerical integrations with the lower/upper limits determined by the above boundaries as

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S1\}}\Big] = \int_{-\infty}^{\infty} \int_{d_1}^{\infty} g f_{B_1,B_2}(b_1, b_2) db_2 db_1$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S2\}}\Big] = \int_{-\infty}^{\infty} \int_{d_2}^{d_1} g f_{B_1,B_2}(b_1, b_2) db_2 db_1$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S3\}}\Big] = \int_{-\infty}^{\infty} \int_{d_3}^{d_2} g f_{B_1,B_2}(b_1, b_2) db_2 db_1$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S4\}}\Big] = \begin{cases} \int_{-\infty}^{\infty} \int_{d_4}^{d_3} g f_{B_1,B_2}(b_1, b_2) db_2 db_1, & \text{if } d_4 < d_3; \\ 0, & \text{Otherwise.} \end{cases}$$

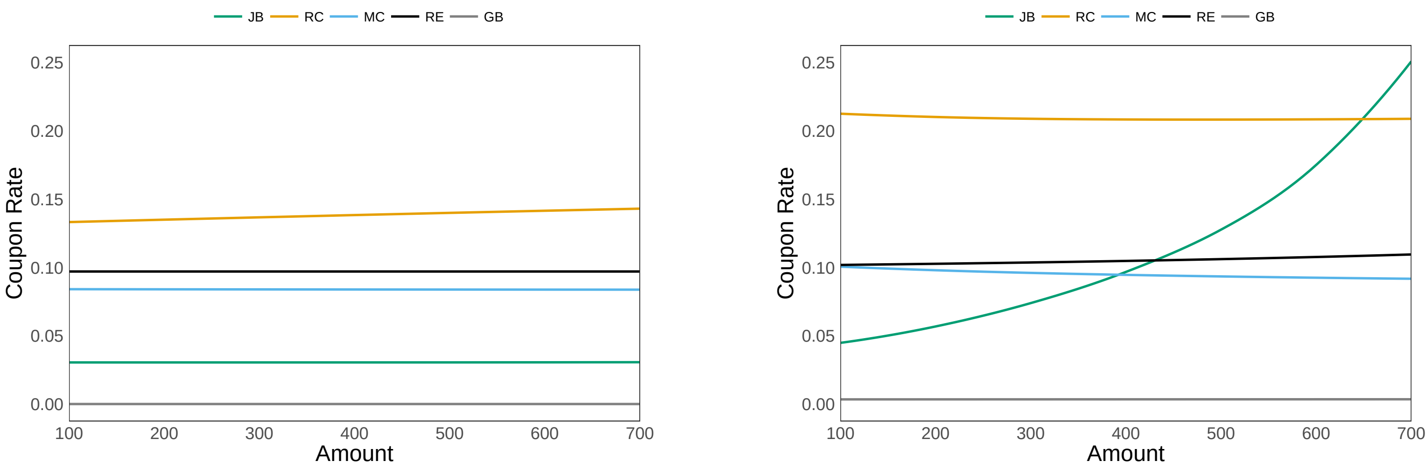
$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S5\}}\Big] = \begin{cases} \int_{-\infty}^{\infty} \int_{-\infty}^{d_4} g f_{B_1,B_2}(b_1, b_2) db_2 db_1, & \text{if } d_4 < d_3; \\ 0, & \text{Otherwise.} \end{cases}$$

4 Numerical Results with Empirical Implications

Although loss-absorbing mechanisms in bail-in-able bonds reduce their issuers' default likelihoods and increase the overall benefits of claim holders, bail-in-able bond investors could require high interest expenses to compensate for the risks of potential mandatory conversions or write-downs. In addition, issuers could adopt riskier strategies or unwind loss-absorbing mechanisms to increase the benefits of their equity holders at the expense of other claim holders. The high interest expenses and risk-shifting incentives could be alleviated by including benefit-sharing mechanisms. To elucidate the above arguments, we first compare the interest expense for issuing a non-bail-in-able `JB` with the expenses for issuing a one-sided `RE` or `RC` and a two-sided `GB` or `MC` in Section 4.1. Next, we will analyze how the credit enhancement effect benefits existent claim holders by examining the value changes of the equity and the `SB` once a new bond is issued with(out) a loss-absorbing mechanisms in Section 4.2. Finally, we show that benefit-sharing mechanisms alleviate the asset substitution problem by showing that the risk-shifting behaviors of issuers no longer significantly increase the values of their equity holders at the expense of the values of existent `SB` holders in Section 4.3.

The following analyses assume all newly-issued bonds are subordinated to the `SB` and are issued at par for consistency. Thus the coupon rate C_2 for each newly-issued bond represents the interest expense of that bond. In our baseline scenarios, the risk-free interest rate r is set to 2% to reflect the prevailing low interest rate environment. In addition, we set the face value of the previously-issued `SB` to 600 and the time 0^- asset value of the low- and high-leverage issuers to 1000 and 2000, respectively; thus the leverage ratios 600/1000 and 600/2000 are approximately equal to 59.5% and 30.7%, the average leverage ratios of BB- rated and AA+ rated sample firms in Kisgen (2006). The asset value increases by the face value of the newly-issued bond after issuance as in **Equation (2)**. The volatility of the issuer's asset value σ_1, the price volatility of the reference asset σ_2, the tax rate τ, and the bankruptcy cost α follow the settings in Leland (1994) and Liu et al. (2016). λ contracted in an MC (see **Equation (11)**) to control the upper conversion threshold is set to 1.2 as in Ammann and Seiz (2006). Comprehensive sensitivity analyses based on the baseline scenarios are presented in Appendix C to further confirm the robustness of our theoretical analyses.

4.1 Interest Expenses for Newly-Issued Bonds



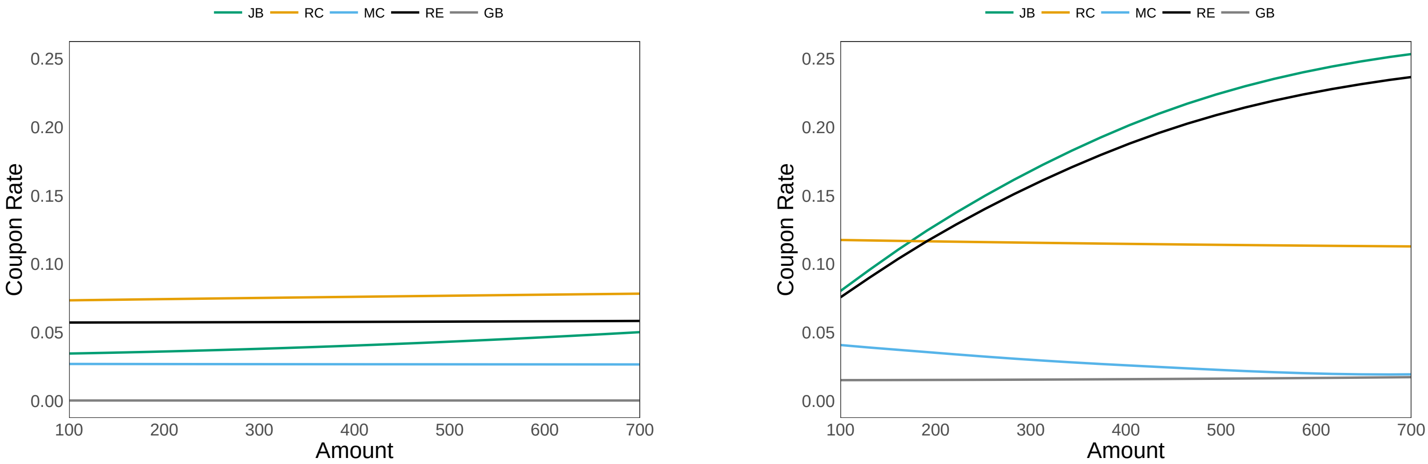
(a) Low-leverage firm

(b) High-leverage firm

Figure 6: Interest Expenses for Newly-Issued Short-Term (Non)-bail-in-able Bonds. Panels (a) and (b) display the coupon rates for newly-issued (non)-bail-in-able bonds given that the issuer has a low or high leverage ratio, respectively. The X- and Y-axes denote the issuance amount and the corresponding coupon rate, respectively. We use green, orange, blue, black, and gray curves to denote the coupon rates of the newly-issued JB, RC, MC, RE, and GB, respectively. The issuer's asset values prior to issuing a new bond are set to 1000 and 2000 for high-leverage and low-leverage scenarios, respectively. The asset value volatility σ_1 is set to 20%. The initial price $G(0)$ and the volatility σ_2 for the reference asset are set to 300 and 20%, respectively. λ contracted in the MC is set to 1.2 as in Ammann and Seiz (2006). The face value D_1 and the coupon rate C_1 of the SB are set to 600 and 3%, respectively. The time to maturity of a newly-issued bond T is set to 1 year. The risk-free interest rate r, the bankruptcy cost α, and the tax rate τ are set to 2%, 60%, and 35%, respectively. The correlation between the values of the issuer's asset and the reference asset ρ is set to 1 to eliminate correlation risk.

To examine whether the presence of loss-absorbing and benefit-sharing mechanisms influences interest expenses for debt raising, we compare the par rate C_2 required by a JB investor with the rates required by different bail-in-able bond investors. To focus the comparisons on bond indentures without the disturbances of the correlation risk described in Section 2.2, the correlation between the values of the issuer's asset and the reference asset, ρ, is set to 1 in **Figures 6** and **7**. The impact of correlation risk on the interest expense for issuing an RE or a GB is then analyzed in **Figure 8**.

The impact of incorporating the loss-absorbing mechanism is illustrated by comparing the difference in interest expenses between a JB and an RE (or an RC) under different scenarios. Since emergent repayment write-down (provided by an RE) and mandatory conversion (provided by an RC) reduce the issuer's default likelihood at the expense of the RE or the RC investor, it is no wonder that the par rate of an RE or an RC is higher than that of a JB. However, the issuer's default likelihood and hence the par rate of a JB significantly increase when the bond issuance amount, the issuer's leverage ratio, and the time to maturity of the newly-issued bond is high, as illustrated in **Figures 6(b)** and **7(b)**.



(a) Low-leverage firm (b) High-leverage firm

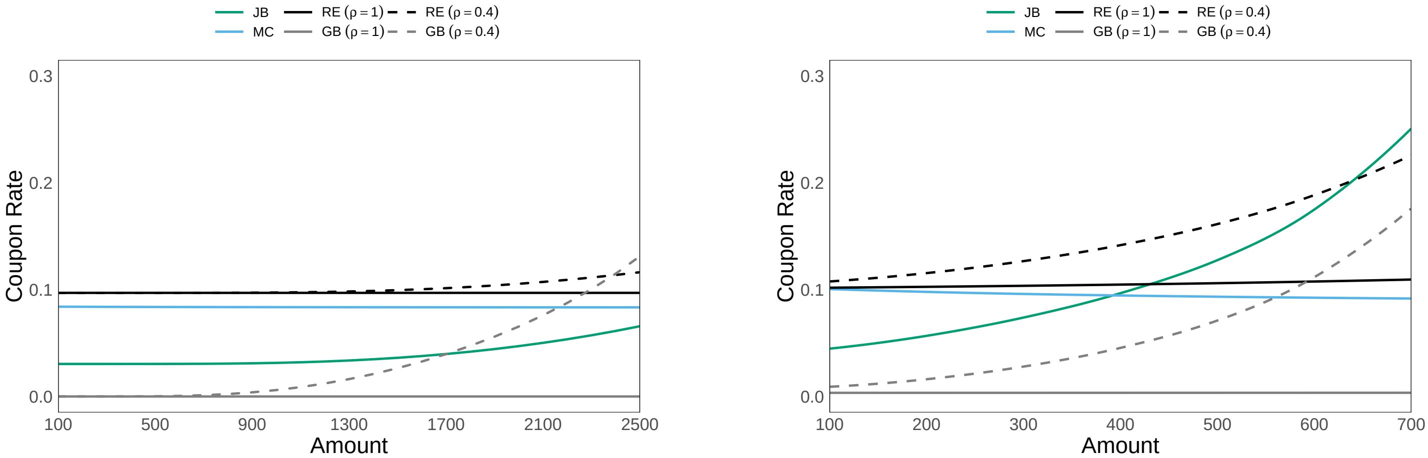
Figure 7: Interest Expenses for Newly-Issued Long-Term (Non)-bail-in-able Bonds. All settings are identical to those in **Figure 6** other than the time to maturity T, which is set to 5 years.

The credit enhancement effect provided by the loss-absorbing mechanism gains significance with increments in the issuer's default likelihood and thus brings the par rate of an RE or an RC below that of a JB under these scenarios. This pattern remains when the bankruptcy cost is high (see **Figure 15(b)** in Appendix C.1).[8] This suggests proper scenarios for issuing bonds with loss-absorbing mechanisms. On the other hand, the impact of incorporating the benefit-sharing mechanism is illustrated by comparing the difference in interest expenses between an RC and an MC (or an RE and a GB). Since this mechanism allows bail-in-able bond investors to share the issuer's upside potential profits, incorporating it can reduce the coupon rates regardless of the levels of issuance amounts, leverage ratios, and bond maturities as examined here and in a large sensitivity analysis in **Figure 15**. Specifically, the par rate of an RC (denoted by orange curves) is higher than the rate of an otherwise identical MC (blue curves); likewise, the par rate of an RE (black curves) is higher than the rate of a GB (gray curves).

We then compare the impact of incorporating a loss-absorbing trigger placed on the issuer's stock price, such as an RC, with the trigger placed on a reference asset value, such as an RE. The stock price conversion trigger is quite sensitive to variations in the issuer's asset value, since the issuer's stock price is treated as a call option on the issuer's asset from the perspective of a structural credit risk model. That makes an RC's conversion timely as discussed in Sundaresan and Wang (2015). On the other hand, an RE's loss-absorbing exchange is less timely, because the exchange is triggered according to the relation between the issuer's asset and the reference asset, which is less sensitive to variations in the issuer's asset value than the issuer's stock price is. The timely loss-absorbing mechanism reflects a higher risk for a bail-in-able bond investor; hence, the par rate of an RC (or an MC) tends to be higher than the rate of an RE (or a GB). However, this timely conversion also reduces default risk significantly when the issuance amount, the issuer's leverage ratio, and the bond maturity are high. Thus the par rate of an RC can be lower than that of an RE as illustrated in **Figure 7(b)**.

From the perspective of lowering interest expenses, a GB seems better than other (non)-bail-in-able bonds if the correlation risk is not considered, as illustrated in **Figures 6** and **7**. However,

[8] According to the report in Davydenko et al. (2012), the cost of default ranges from about 10% to 80% of the market value of a firm's asset. Jankowitsch et al. (2014) on the other hand study the recovery rates of defaulted bonds in the U.S. corporate bond market. They show that the observed recovery rates of defaulted subordinated bonds range from 0.1% to 98.71%.



(a) Low-leverage firm

(b) High-leverage firm

Figure 8: Impact of Correlation Risk on Interest Expenses for Newly-Issued `RE`s and `GB`s. All settings are identical to those in **Figure 6** except for the correlation between the values of the reference asset and the issuer's asset, which is 1 (the solid curves) or 0.4 (dashed curves). The black and gray curves denote the coupon rates of `RE`s and `GB`s, respectively. For the low-leverage scenario in panel (a), these coupon rates are compared with the coupon rate of a `JB` (green curve). For the high-leverage scenario in panel (b), these coupon rates are compared with the coupon rate of an `MC` (blue curve).

incorporating the correlation risk (i.e., decreasing ρ) introduces other risk irrelevant to the issuer's business. That weakens the loss-absorbing mechanism and hence increases the par rates of an `RE` and a `GB`. This phenomenon becomes significant when the issuance amount is large, as illustrated in **Figure 8**. In particular, the rate of increment for the par rate of a `GB` is greater than that of an `RE`, since the correlation risk also weakens the benefit-sharing mechanism. This may even make the par rate of a `GB` in the low-leverage (high-leverage) scenario higher than the rates of a `JB` and an `MC` (the rate of an `MC`) when the issuance amount is large.

In sum, the interest expense for a non-bail-in-able `JB` is generally lower than other bail-in-able bonds when the issuer's leverage ratio is low and the new bond issuance amount is small. When the issuer's default likelihood becomes significant due to a high leverage ratio, long bond maturity, and a large bond issuance amount, the par rate of a `JB` becomes high, as illustrated by the green curves in **Figures 6(b)** and **7(b)**. Issuing bonds with loss-absorbing mechanisms is thus advantageous under these scenarios. In addition, bail-in-able bonds with benefit-sharing mechanisms further reduce interest expenses. Issuing a `GB` seems to be the dominant choice when the correlation risk is low. Thus it is especially suitable for an issuer whose asset value mainly depends on an asset, such as oil and gas producers (see Haushalter, 2000), regardless of the issuance amount and the leverage ratio. Otherwise, issuing an `MC` is suitable for raising a large-amount long-term bond for a high-leverage public firm whose value depends on complex risk factors (see Chemmanur et al., 2003; Ammann and Seiz, 2006). More comprehensive analyses to confirm the robustness of our arguments in this section are given in Appendix C.1.

4.2 Claim Dilution and Credit Enhancement Effects

Although issuing new bonds may dilute the values of existing claim holders, loss-absorbing mechanisms enhance the issuer's creditworthiness and hence its leveraged value by increasing tax-shield benefits and decreasing bankruptcy costs as illustrated in **Equation (2)**. This section first examines how the issuance of a JB dilutes the values of the issuer's equity and existing SB. Next, we investigate how the loss-absorbing mechanisms in an RC and an RE offset the claim dilution effects. Finally, we examine the impact of incorporating the benefit-sharing mechanisms in an MC and a GB. The value changes in the equity and the SB due to the issuance of a (non)-bail-in-able bond are plotted in **Figures 9(a)** and **(b)**, respectively. To avoid disturbances of correlation risk, we again assume that the price of an RE's (GB's) reference asset is perfectly correlated with the issuer's asset value.

The pure claim dilution effect can be reflected by the negative value changes in the equity and the SB due to the issuance of a JB, as illustrated by green curves. Such issuance increases the issuer's default likelihood and generally decreases the values of existing claim holders.[9] The impact of incorporating a loss-absorbing mechanism is measured by the increments in value changes from issuing a JB (denoted by green curves) to an RE (by black curves) or an RC (orange curves). Since this mechanism provides emergent repayment write-down or debt-to-equity conversion to reduce the issuer's default likelihood, increments in the issuer's leveraged value contributed by increments in tax benefits and decrements in bankruptcy costs goes to the equity and the SB holders. Furthermore, an RC provides a more timely loss-absorbing mechanism than an RE, as mentioned in Section 4.1. Thus an RC provides stronger credit enhancement and hence larger increments in both the equity and the SB values than an RE.

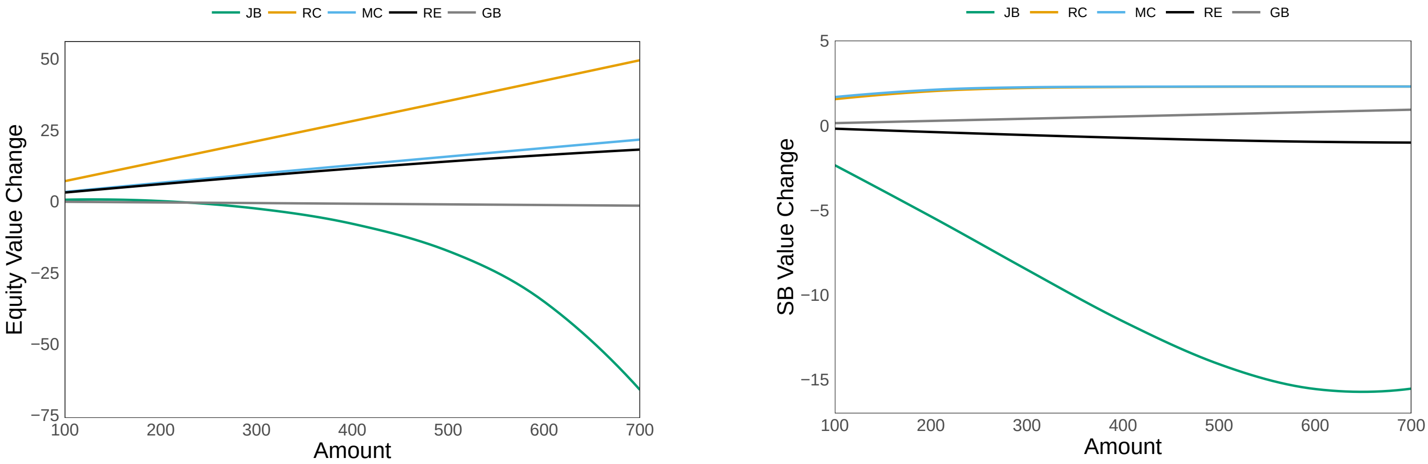
The impact of incorporating a benefit-sharing mechanism can be measured by comparing the differences in value changes in the equity and the SB between issuing an RE and issuing a GB (between issuing an RC and issuing an MC). This mechanism reduces the interest expenses for issuing a GB (an MC), as illustrated in Section 4.1, since the bond holders share the issuers' upside potential profits. This sharing reduces equity values as illustrated in **Figure 9(a)**. On the other hand, lower interest expenses due to the presence of benefit-sharing mechanisms reduce issuers' default likelihood; thus the SB value changes are higher if the newly-issued bonds contain mechanisms such as those illustrated in **Figure 9(b)**. More comprehensive analyses to confirm the robustness of our arguments in this section are given in Appendix C.2.

4.3 Asset Substitution Problem

Bonds with loss-absorbing mechanisms may induce their issuers to take excessive risk if they are not highly regulated (see Berg and Kaserer, 2015; Azgad-Tromer, 2017). Risk-shifting behavior such as longing forwards on the reference asset or raising the issuer's asset value volatility, as discussed in Section 2.4, increase the value of equity holders at the expense of the SB holders as illustrated in **Figures 10(a)** and **(b)**, respectively. Note that issuers seeking to pursue the best benefits for their equity holders may have greater incentives to take such actions if they increase the equity value. However, such actions increase issuers' risk and thus decrease the SB value. The severity of this agency problem following the issuances of different bail-in-able bonds can thus be measured by comparing the value changes in the equity and the SB described as follows.

Figure 10(a) plots the value changes in the equity and the SB due to longing forwards on reference assets to unwind the mechanism(s) embedded in an RE (a GB). The impact of unwinding a loss-absorbing mechanism on existing claim holders is illustrated by the RE-issuance scenario (denoted by solid curves). Clearly, the increments (decrements) in the equity (SB) value are significant, especially when the issuance amount is large. This suggests that an RE issuer may have strong risk-shifting

[9] When the issuance amount is small, the change in the equity value can be positive to reflect the fact that the issuer can raise debt capital to approach its optimal leverage ratio.



(a) Equity value change

(b) `SB` value change

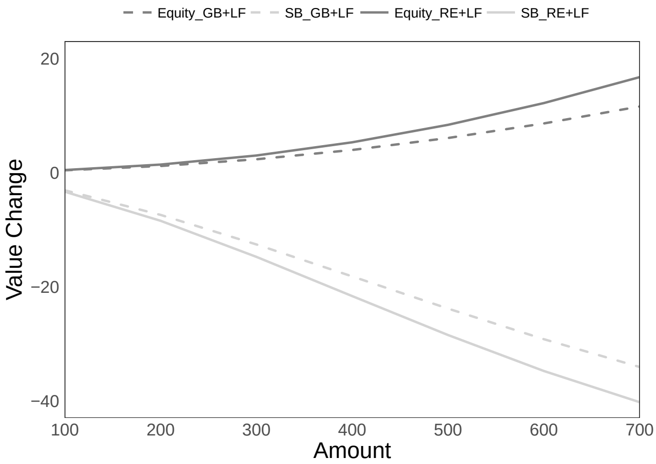
Figure 9: Value Changes in Equity and `SB` due to Issuances of (Non)-bail-in-able Bonds. Panels (a) and (b) display the value changes in equity and `SB`, respectively, due to issuances of (non)-bail-in-able bonds. All settings are identical to those in **Figure 6(b)**. The `SB` value change due to the issuance of an `MC` is slightly larger than that due to the issuance of an otherwise identical `RC`.

incentives and the agency problem is severe. On the other hand, the `GB`-issuance scenario (denoted by dashed curves) illustrates how the presence of a benefit-sharing mechanism in a bail-in-able bond alleviates risk-shifting incentives. It can be observed that increments in the equity value decreases from the dark gray solid curve to the dashed one, because the equity holders' benefits gained from longing forwards shifts partly toward the `GB` investor via the benefit-sharing mechanism. In addition, the decrement in the `SB` value decreases from the light gray solid curve to the dashed one to indicate that the `SB` holder suffers less harm from longing forwards. Thus we conclude that incorporating benefit-sharing mechanisms alleviates the asset substitution problem following the issuances of bail-in-able bonds referring to assets other than the issuers' stocks.

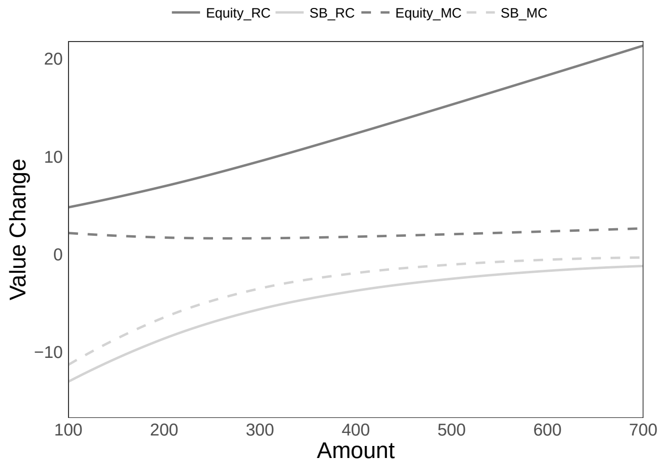
Figure 10(b) examines the same agency problem following the issuances of bail-in-able bonds with triggers placed on issuers' stock prices. The impact of increasing the issuer's asset volatility is illustrated by the `RC`-issuance scenario (denoted by solid curves). On the other hand, the `MC`-issuance scenario (denoted by dashed curves) illustrates how the presence of a benefit-sharing mechanism alleviates risk-shifting incentives. It can also be observed that increments (decrements) in the equity (`SB`) value in the `RC`-issuance scenario are larger than that in the `MC`-issuance scenario. Thus we can also conclude that a benefit-sharing mechanism alleviates the agency problem. Furthermore, by comparing panel (b) with (a), we find that decrements in the `SB` value decrease with increments in the `RC` or the `MC` issuance amount. This is because once the loss-absorbing mechanism is triggered, the debt principal of an `RC` or an `MC` is entirely converted into stocks, significantly reducing the payment risk and thus enhancing the issuer's credit, whereas the principal of an `RE` or a `GB` is only partly written down. More comprehensive analyses to confirm the robustness of our arguments in this section are given in Appendix C.3.

5 Conclusion

This paper studies the pros and cons of a non-financial firm issuing a bail-in-able bond with different designs. We develop a novel quadrature pricing method for evaluating a newly-issued (non)-bail-in-able



(a) Long forwards on reference assets



(b) Raising issuer's asset value volatility

Figure 10: Impact of Risk-Shifting Behavior on Value Changes of Equity and `SB`. The Y-axis denotes value changes in equity (dark gray) and `SB` (light gray) due to risk-shifting behavior. Panel (a) illustrates the value changes from longing forwards on reference assets to unwind mechanism(s) in `GB`s (dashed curves) or `RE`s (solid curves) as discussed in Section 2.4. Panel (b) illustrates value changes from raising issuers' asset value volatilities σ_1 from 0.2 to 0.3 after issuing `MC`s (dashed curves) or `RC`s (solid curves). All other settings are identical to those in **Figure 6(b)**.

bond and the corresponding value changes in the issuer's equity and existent debt due to the issuance. Although the interest expense for issuing a non-bail-in-able bond is low when the issuer's leverage ratio is low, we find that issuing a bond with a loss-absorbing mechanism significantly reduces default likelihood and hence the interest expense when the issuer's leverage ratio is high, the bond issuance amount is large, and (or) the maturity of the newly-issued bond is long. In addition, incorporating a benefit-sharing mechanism further reduces the interest expense by allowing the bond investor to share the bond issuer's upside potential profits. Credit enhancement effects provided by bail-in-able bonds referring to different assets are also compared. We show that bonds with triggers placed on their issuers' stock prices provide more timely loss-absorbing mechanisms than those with triggers placed on other asset prices. Therefore, *ceteris paribus*, the interest expenses for issuing the former bonds are generally higher, but the increments in the issuers' equity and existing debt values are also higher once the bonds are issued. The interest expenses for issuing the latter bonds are lower only when the correlations between the reference assets and the issuers' assets are high enough. Finally, we examine an issuer's risk-shifting incentive following the issuance of a bail-in-able bond and elucidate how this incentive is mitigated when the newly-issued bond contains a benefit-sharing mechanism. Thus we conclude that two-sided bonds such as `GB`s and `MC`s are proper bail-in-able debt instruments for non-financial firms, since they suppress interest expenses and risk-shifting incentives. `GB`s are especially suitable for issuers whose values depend mainly on assets such as gold. `MC`s are suitable for high-leverage public firms with complex business operations.

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Appendix A Evaluation of a Firm with an RE Outstanding

A.1 Impact of Issuing an RE

Consider that a firm with a T-year SB seeks to issue a T-year RE. There are four possible scenarios located in the $(V(T), G(T))$ coordinate system as illustrated in **Figure 11**. The issuer survives if its asset value exceeds the after-tax SB and RE repayments given that the RE is exchanged or not. The kinked red and yellow line separates the bankrupt scenarios from Scenario 2 (abbreviated as S2) and Scenario 1 (S1) given that the RE is exchanged (i.e., the reference asset value at time T, $G(T)$, falls below the contractually-specified value $G' = G(0)$ denoted by the gray line) or not (i.e., $G(T) > G(0)$), respectively. Summing the SB and RE repayments yields two linear equations for the yellow and the red boundaries listed at the top or the bottom of the boundaries, respectively. Note that the positive slope for the red default boundary reflects the fact that the debt repayment decreases with decrements in the reference asset value, which properly performs the loss-absorbing mechanism if the correlation between the values of the reference asset and the issuer's asset is high. On the other hand, failing to repay results in bankruptcy. The asset after paying the bankruptcy cost is first repaid to the SB holder, and the remaining value (if any) is then distributed to the RE investor. Scenario 3 (S3) denotes that the SB holder is fully repaid but the RE investor is partially repaid; Scenario 4 (S4) denotes that only the SB holder is repaid partially. The equation of the blue boundary that separates S3 from S4 is listed above the boundary in **Figure 11**.

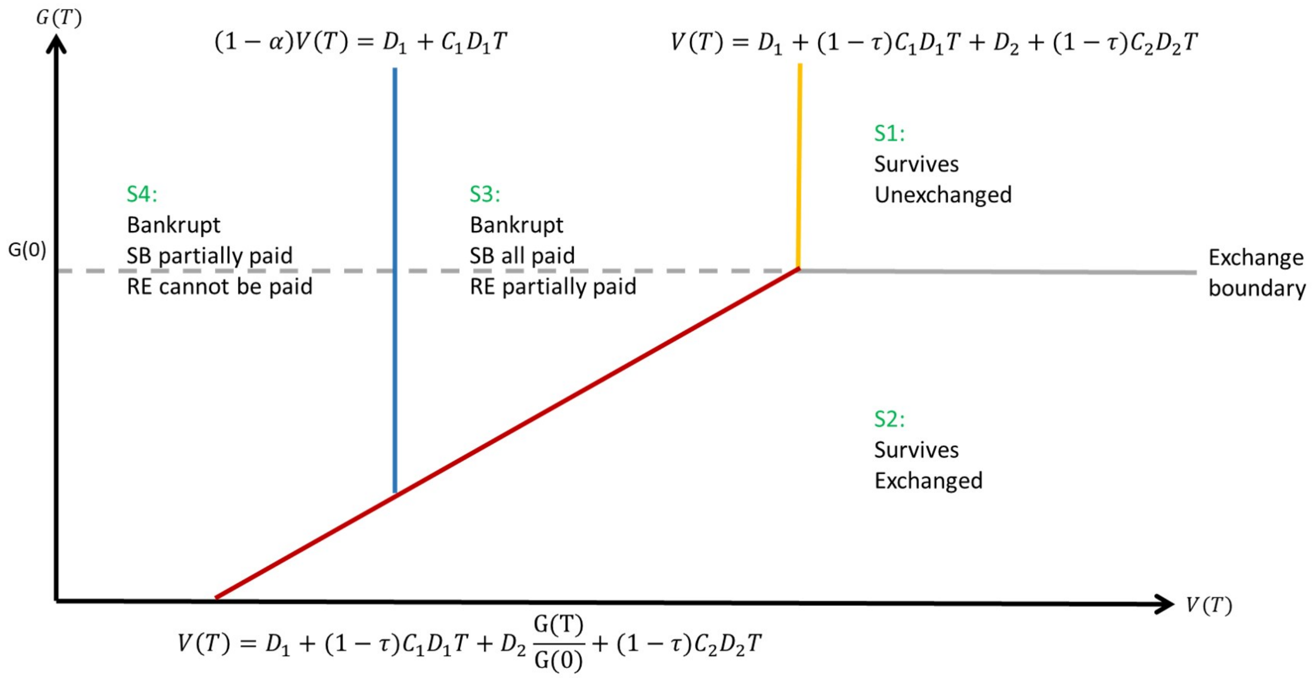


Figure 11: Scenarios for RE Issuance. The X- and Y-axes denote the values of the RE issuer's asset and the reference asset, respectively, at time T. Two equations for the yellow and red kinked boundary are listed at the top (given that the RE is not exchanged) and the bottom (the RE is exchanged) of the boundary. The equation for the blue boundary is listed above the boundary. The horizontal gray solid line refers to the boundary to trigger RE's exchange. The RE is not exchanged when the reference asset price is above this boundary. The description for each scenario is provided below the scenario. This figure reflects Case 2 of **Figure 12**.

At the RE issuance date (time 0), the present value of the RE is evaluated as the discounted expected payoffs defined in **Equation (9)** as

$$\begin{aligned} RE(0) = \tilde{E}\left[(D_2 + C_2D_2T)e^{-rT}\cdot\mathbf{I}_{\{S1\}}\right] + \tilde{E}\left[(D_2\frac{G(T)}{G(0)} + C_2D_2T)e^{-rT}\cdot\mathbf{I}_{\{S2\}}\right] \\ + \tilde{E}\left[\mathrm{Max}[(1-\alpha)\,V\,(T) - (D_1 + C_1D_1T), 0]e^{-rT}\cdot\mathbf{I}_{\{S3,S4\}}\right], \end{aligned} \tag{45}$$

where the first, second, and third expectations denote the values contributed by S1, S2, and other bankrupt scenarios, respectively. The corresponding present value of the `SB` and the equity can be expressed as

$$SB(0) = \tilde{E}\left[\ (D_1 + C_1 D_1 T)\, e^{-rT} \cdot \mathbf{I}_{\{S1,S2\}}\right] + \tilde{E}\left[\mathrm{Min}[D_1 + C_1 D_1 T,\ (1-\alpha)\, V\,(T)]e^{-rT} \cdot \mathbf{I}_{\{S3,S4\}}\right]. \tag{46}$$

$$\begin{aligned} E(0) = &\ \tilde{E}\left[\left[V\,(T) - \left[D_1 + D_2 + (1-\tau)(C_1 D_1 T + C_2 D_2 T)\right]\right]e^{-rT} \cdot \mathbf{I}_{\{S1\}}\right] \\ &+ \tilde{E}\left[\left[V\,(T) - \left[D_1 + \frac{G\,(T)}{G\,(0)} D_2 + (1-\tau)(C_1 D_1 T + C_2 D_2 T)\right]\right]e^{-rT} \cdot \mathbf{I}_{\{S2\}}\right]. \end{aligned} \tag{47}$$

By substituting **Equations (4)** and **(5)** for $V(T)$ and $G(T)$, respectively, the pricing formulae for $RE(0)$, $SB(0)$, and $E(0)$ mentioned above can be expressed as linear combinations of $\tilde{E}\left[g \cdot \mathbf{I}_{\{S\}}\right]$, where g can be 1, $e^{\sigma_1 B_1(T)}$, or $e^{\sigma_2 B_2(T)}$, and S denotes a scenario.

Now we transform the aforementioned four scenarios from the $(V(T), G(T))$ two-dimensional plane illustrated in **Figure 11** to the $(B_1(T), B_2(T))$ plane illustrated in **Figure 12**. The yellow, red, blue, and gray boundaries in the former figure can be transformed to boundaries d_1, d_2, d_3, and d_4 in the latter figure. The yellow and red kinked boundary that separates the survival scenarios from the bankrupt ones can be transformed into d_1 and d_2, where the equation of d_1 reflects the fact that the `RE` is not exchanged:

$$B_1(T) = \frac{1}{\sigma_1} \ln\left[\frac{D_1 + D_2 + (1-\tau)(C_1 D_1 T + C_2 D_2 T)}{V(0)}\right],$$

and the equation of d_2 reflects the fact that the `RE` *is* exchanged:

$$B_1(T) = \frac{1}{\sigma_1} \ln\left[\frac{D_1 + D_2 e^{\sigma_2 B_2(T)} + (1-\tau)(C_1 D_1 T + C_2 D_2 T)}{V(0)}\right].$$

When $B_2(T) \to -\infty$, $B_1(T) \to d^+_{2\_b_1\_asym}$, whose value is equal to the right-hand side of **Equation (44)**. The equation of boundary d_3 separating S3 and S4 is the same as **Equation (25)**. The equation of boundary d_4 separating the exchanged scenario S2 from the non-exchanged one S1 is

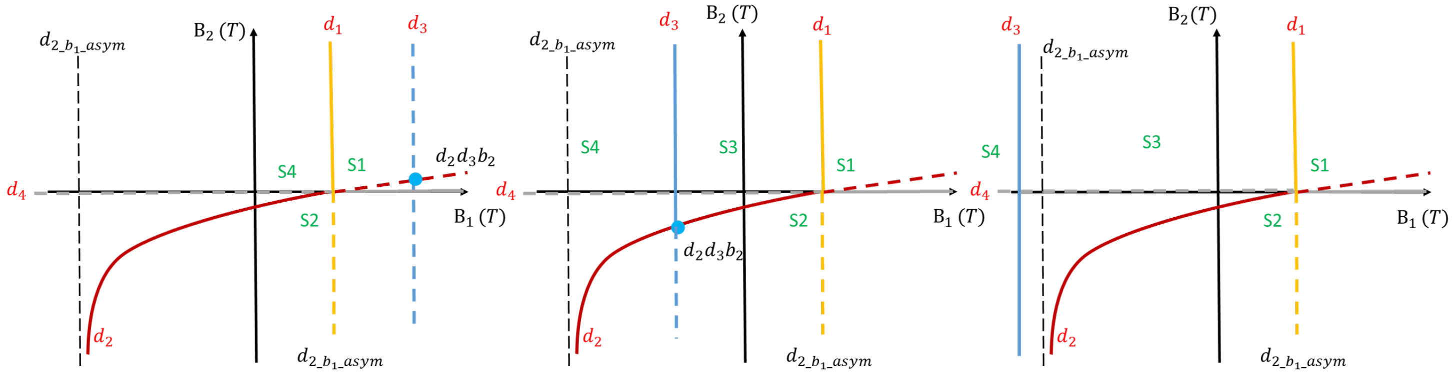
$$B_2(T) = 0.$$

By comparing the size relation among the y coordinates of vertical lines d_1, d_3, and $d_{2\_b_1\_asym}$, we obtain the three different cases illustrated as Cases 1, 2, and 3 of **Figure 12**.[10] The d_2 and d_3 intersects in Case 2, and the y coordinate of the intersection $d_2 d_3 b_2$ can be evaluated as

$$\frac{1}{\sigma_2} \ln\left[\frac{D_1 + C_1 D_1 T - (1-\alpha)[D_1 + (1-\tau)(C_1 D_1 T + C_2 D_2 T)]}{(1-\alpha) D_2}\right]. \tag{48}$$

The contingent claim values described in **Equations (45)**, **(46)**, and **(47)** can be expressed by the integrations with the joint probability distribution of B_1 and B_2 in **Equation (6)** and the lower/upper limits determined by the aforementioned boundaries analyses as

[10] Note that the y coordinate of d_1 is always larger than that of $d_{2\_b_1\_asym}$.



Case 1: $d_1 < d_3$ Case 2: $d_{2\_b_1\_asym} < d_3 < d_1$ Case 3: $d_3 < d_{2\_b_1\_asym}$

Figure 12: Transforming Scenario Areas from Figure 11 to $(B_1(T),\ B_2(T))$ Coordinate System. The transforms can be categorized into three cases by the size relation among d_1, d_3, and $d_{2\_b_1\_asym}$, where the last one in the black dashed line denotes the asymptote of d_2. The yellow line d_1 and the red curve d_2 separating the survival scenarios (S1 and S2) from the bankrupt ones (S3 and S4) are transformed from the kinked yellow and red default boundary in **Figure 11**, and d_3 and d_4 are transformed from the blue and the gray boundaries, respectively. Solid and dashed patterns denote whether the boundaries take effect. For example, the boundary d_3 separating the two bankrupt scenarios S_3 and S_4 becomes nonexistent if it goes through the survival regions as in Cases 1 and 2. $d_2d_3b_2$ denotes the y coordinate of the intersection point of d_2 and d_3.

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S1\}}\Big] = \int_0^\infty \int_{d_1}^\infty g f_{B_1,B_2}(b_1,b_2)db_1db_2;$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S2\}}\Big] = \int_{-\infty}^0 \int_{d_2}^\infty g f_{B_1,B_2}(b_1,b_2)db_2db_1;$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S3\}}\Big] = \begin{cases} 0, & \text{Case 1;} \\ \int_{d_2d_3b_2}^0 \int_{d_3}^{d_2} g f_{B_1,B_2}(b_1,b_2)db_2db_1 + \int_0^\infty \int_{d_3}^{d_1} g f_{B_1,B_2}(b_1,b_2)db_2db_1, & \text{Case 2;} \\ \int_{-\infty}^0 \int_{d_3}^{d_2} g f_{B_1,B_2}(b_1,b_2)db_2db_1 + \int_0^\infty \int_{d_3}^{d_1} g f_{B_1,B_2}(b_1,b_2)db_2db_1, & \text{Case 3.} \end{cases}$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S4\}}\Big] = \begin{cases} \int_{-\infty}^0 \int_{-\infty}^{d_2} g f_{B_1,B_2}(b_1,b_2)db_2db_1 + \int_0^\infty \int_{-\infty}^{d_1} g f_{B_1,B_2}(b_1,b_2)db_2db_1, & \text{Case 1;} \\ \int_{-\infty}^{d_2d_3b_2} \int_{-\infty}^{d_2} g f_{B_1,B_2}(b_1,b_2)db_2db_1 + \int_{d_2d_3b_2}^\infty \int_{-\infty}^{d_3} g f_{B_1,B_2}(b_1,b_2)db_2db_1, & \text{Case 2;} \\ \int_{-\infty}^\infty \int_{-\infty}^{d_3} g f_{B_1,B_2}(b_1,b_2)db_2db_1, & \text{Case 3.} \end{cases}$$

A.2 Unwinding the Loss-Absorbing Mechanism by Longing Forwards

The issuer may unwind the loss-absorbing mechanism in an RE by longing m units of forwards on the reference asset to benefit the equity holders at the expense of the SB holder by mimicking the risk-shifting behavior mentioned in **Section 3.1.2**. Here we set m to $(D_2 + (1-\tau)C_2D_2T)/G(0)$ to just unwind the mechanism: the slope of the red default boundary changes from positive in **Figure 11** to vertical in **Figure 13**. The corresponding values of the RE, the SB, and the equity are evaluated as

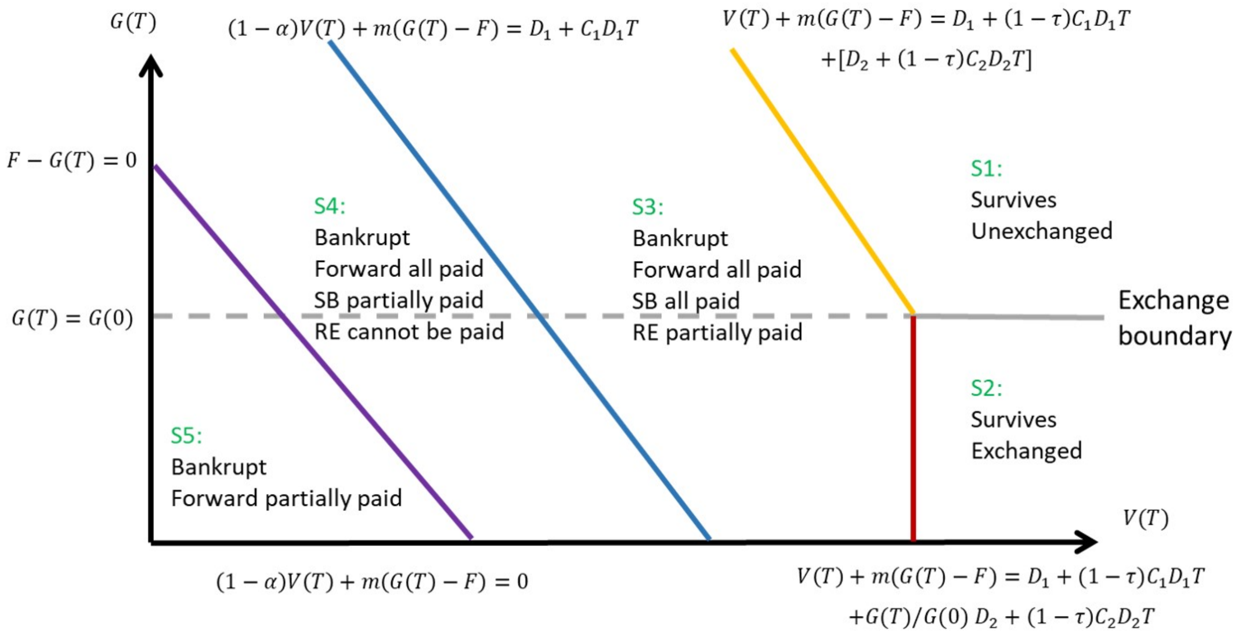


Figure 13: Possible Scenarios by Longing Forwards to "Hedge" the RE. The X- and Y-axes denote the values of the issuer's asset and the reference asset at time T, respectively. The two equations for the kinked yellow and red boundary are listed at the top (given that RE is not exchanged) and the bottom (RE is exchanged) of the boundary. The equation of each boundary is listed next to the boundary. The four boundaries separate the plane into the five following scenarios. S1 and S2 denote that the issuer survives and the RE is not exchanged or exchanged, respectively. The issuer announces default in the following scenarios. S3 denotes that both the forward seller and the SB holder are fully repaid. S4 denotes that only the forward seller is fully repaid; the SB holder is partially repaid. S5 denotes that only the forward seller is repaid; other bond holders get nothing.

the discounted expected payoffs contributed by the five scenarios illustrated in **Figure 13**:

$$\begin{aligned}
RE(0) &= \tilde{E}\Big[(D_2+C_2D_2T)e^{-rT}\cdot\mathbf{I}_{\{S1\}}\Big]+\tilde{E}\Big[(D_2\frac{G(T)}{G(0)}+C_2D_2T)e^{-rT}\cdot\mathbf{I}_{\{S2\}}\Big]\\
&\quad+\tilde{E}\Big[\mathrm{Max}\Big[(1-\alpha)V(T)+m(G(T)-F)-(D_1+C_1D_1T),0\Big]e^{-rT}\cdot\mathbf{I}_{\{S3,S4,S5\}}\Big] \qquad (49)\\
SB(0) &= \tilde{E}\Big[(D_1+C_1D_1T)e^{-rT}I_{\{S1,S2\}}\Big]\\
&\quad+\tilde{E}\Big[\mathrm{Min}\big[D_1+C_1D_1T,\mathrm{Max}[(1-\alpha)V(T)+m(G(T)-F),0]\big]e^{-rT}\cdot\mathbf{I}_{\{S3,S4,S5\}}\Big], \qquad (50)\\
E(0) &= \tilde{E}\Big[\big[V(T)+m(G(T)-F)-(D_1+D_2+(1-\tau)(C_1D_1T+C_2D_2T))\big]e^{-rT}\cdot\mathbf{I}_{\{S1\}}\Big]\\
&\quad+\tilde{E}\Big[\Big[V(T)+m(G(T)-F)-\Big(D_1+\frac{G(T)}{G(0)}D_2+(1-\tau)(C_1D_1T+C_2D_2T)\Big)\Big]e^{-rT}\cdot\mathbf{I}_{\{S2\}}\Big]. \qquad (51)
\end{aligned}$$

Again, by substituting **Equations (4)** and **(5)** for $V(T)$ and $G(T)$, respectively, the above pricing formulae can be expressed as linear combinations of $\tilde{E}\Big[g\cdot\mathbf{I}_{\{S\}}\Big]$, where g can be 1, $e^{\sigma_1 B_1(T)}$, or $e^{\sigma_2 B_2(T)}$, and S denotes a scenario.

By transforming the yellow, red, blue, and purple boundaries from the $(V(T),G(T))$ plane to the $(B_1(T),B_2(T))$ plane, we obtain equations of boundaries d_1, d_2, d_3, and d_4, respectively, as follows. First, d_1 separates survival scenarios from bankrupt ones given that the RE is not exchanged; its equation is

$$B_2(T)=\frac{1}{\sigma_2}\ln\left[\frac{\mathbf{D}-V(0)e^{\sigma_1 B_1(T)}}{mG(0)}\right], \qquad (52)$$

where $\mathbf{D}=D_1+D_2+(1-\tau)(C_1D_1T+C_2D_2T)+mF$. Note that d_1 contains the two asymptotes.

As $B_2(T) \to -\infty$, we have

$$B_1(T) \to d_{1\_b_1\_asym} = \frac{1}{\sigma_1} \ln \left[\frac{\mathbf{D}}{V(0)} \right]. \tag{53}$$

As $B_1(T) \to -\infty$, we have

$$B_2(T) \to d_{1\_b_2\_asym} = \frac{1}{\sigma_2} \ln \left[\frac{\mathbf{D}}{mG(0)} \right]. \tag{54}$$

Second, d_2 is the default boundary given that the `RE` is exchanged; its equation is

$$B_1\,(T) = \frac{1}{\sigma_1} \ln \left[\frac{\mathbf{D} - D_2}{V(0)} \right]. \tag{55}$$

Third, the equation of d_3 is the same as **Equation (33)**. d_3 has asymptotes $d_{3\_b_1\_asym}$ and $d_{3\_b_2\_asym}$, which are the same as the right-hand side of **Equations (34)** and **(35)**, respectively. Finally, the equation of d_4 is the same as **Equation (36)**. Its asymptotes $d_{4\_b_1\_asym}$ and $d_{4\_b_2\_asym}$ are equal to the right-hand sides of **Equations (37)** and **(38)**, respectively. By comparing the size relation among the y coordinates of the vertical boundary d_2 and asymptotes $d_{4\_b_1\_asym}$, $d_{3\_b_1\_asym}$, and $d_{1\_b_1\_asym}$, there are 14 cases in the $(B_1(T), B_2(T))$ coordinate system summarized in **Table 1**. The contingent claim values described in **Equations (49)**, **(50)**, and **(51)** are expressed by the joint probability distribution of $B_1(T)$ and $B_2(T)$ in **Equation (6)** and the analyses summarized in **Table 1** as

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S1\}}\Big] = \int_{-\infty}^{d_2} \int_{d_1}^{\infty} g f_{B_1,B_2}(b_1,b_2) db_2 db_1 + \int_{0}^{\infty} \int_{d_2}^{\infty} g f_{B_1,B_2}(b_1,b_2) db_2 db_1$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S2\}}\Big] = \int_{-\infty}^{0} \int_{d_2}^{\infty} g f_{B_1,B_2}(b_1,b_2) db_2 db_1$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S3\}}\Big] = \begin{cases} \int_{-\infty}^{d_2} \int_{d_3}^{d_1} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 1,3,7,11,13;} \\ \int_{-\infty}^{d_1 d_3 b_1} \int_{d_3}^{d_1} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 2,5,6,10;} \\ 0, & \text{Case 4,8,9,12;} \\ \int_{-\infty}^{d_{3\_b_1\_asym}} \int_{d_3}^{d_1} g f_{B_1,B_2}(b_1,b_2) db_2 db_1 + \int_{d_{3\_b_1\_asym}}^{d_2} \int_{-\infty}^{d_1} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 14;} \end{cases}$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S4\}}\Big] = \begin{cases} \int_{-\infty}^{d_2} \int_{d_4}^{d_3} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 1, 3, 7;} \\ \int_{-\infty}^{d_1 d_3 b_1} \int_{d_4}^{d_3} g f_{B_1,B_2}(b_1,b_2) db_2 db_1 + \int_{d_1 d_3 b_1}^{d_2} \int_{d_4}^{d_1} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 2, 6;} \\ \int_{-\infty}^{d_2} \int_{d_4}^{d_1} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 4, 9;} \\ \int_{-\infty}^{d_1 d_3 b_1} \int_{d_4}^{d_3} g f_{B_1,B_2}(b_1,b_2) db_2 db_1 + \int_{d_1 d_3 b_1}^{d_1 d_4 b_1} \int_{d_4}^{d_1} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 5;} \\ \int_{-\infty}^{d_1 d_4 d_1} \int_{d_4}^{d_1} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 8;} \\ \int_{-\infty}^{d_{4\_b_1\_asym}} \int_{d_4}^{d_3} g f_{B_1,B_2}(b_1,b_2) db_2 db_1 + \int_{d_{4\_b_1\_asym}}^{d_1 d_3 b_1} \int_{-\infty}^{d_3} g f_{B_1,B_2}(b_1,b_2) db_2 db_1 & \\ + \int_{d_1 d_3 b_1}^{d_2} \int_{-\infty}^{d_1} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 10;} \\ \int_{-\infty}^{d_{4\_b_1\_asym}} \int_{d_4}^{d_3} g f_{B_1,B_2}(b_1,b_2) db_2 db_1 + \int_{d_{4\_b_1\_asym}}^{d_2} \int_{-\infty}^{d_3} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 11,13;} \\ \int_{-\infty}^{d_{4\_b_1\_asym}} \int_{d_4}^{d_1} g f_{B_1,B_2}(b_1,b_2) db_2 db_1 + \int_{d_{4\_b_1\_asym}}^{d_2} \int_{-\infty}^{d_1} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 12;} \\ \int_{-\infty}^{d_{4\_b_1\_asym}} \int_{d_4}^{d_3} g f_{B_1,B_2}(b_1,b_2) db_2 db_1 + \int_{d_{4\_b_1\_asym}}^{d_{3\_b_1\_asym}} \int_{-\infty}^{d_3} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 14;} \end{cases}$$

$$\tilde{E}\Big[g \cdot \mathbf{I}_{\{S5\}}\Big] = \begin{cases} \int_{-\infty}^{d_2} \int_{-\infty}^{d_4} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 1,2,3,4,6,7,9;} \\ \int_{-\infty}^{d_1 d_4 b_1} \int_{-\infty}^{d_4} g f_{B_1,B_2}(b_1,b_2) db_2 db_1 + \int_{d_1 d_4 b_1}^{d_2} \int_{-\infty}^{d_1} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 5,8;} \\ \int_{-\infty}^{d_{4\_b_1\_asym}} \int_{-\infty}^{d_4} g f_{B_1,B_2}(b_1,b_2) db_2 db_1, & \text{Case 10,11,12,13,14.} \end{cases}$$

Note that $d_1 d_3 b_1$ and $d_1 d_4 b_1$ in these equations denote the x coordinate of the intersection point of d_1 and d_3 and the intersection point of d_1 and d_4, respectively.

Table 1: Fourteen Cases on the $(B_1(T),B_2(T))$ Plane Transformed from Figure 13

	Asymptote on $B_1(T) - axis$	Asymptote on $B_2(T) - axis$	d_1, d_3	d_1, d_4	d_2, d_3	d_2, d_4
Case 1	$d_2 < d_{4\_b_{1\_asym}} < d_{3\_b_{1\_asym}} < d_{1\_b_{1\_asym}}$	**D**-**C**>0	X	X	O	O
Case 2		**D**-**C**>0	Left of d_2			
Case 3	$d_2 < d_{4\_b_{1\_asym}} < d_{1\_b_{1\_asym}} < d_{3\_b_{1\_asym}}$	**D**-**C**>0	Right of d_2	X	O	O
Case 4		**D**-**C**≤0	X			
Case 5		**D**-**C**>0	Left of d_2	Left of d_2		
Case 6		**D**-**C**>0	Left of d_2	Right of d_2		
Case 7	$d_2 < d_{1\_b_{1\_asym}} < d_{4\_b_{1\_asym}} < d_{3\_b_{1\_asym}}$	**D**-**C**>0	Right of d_2	Right of d_2	O	O
Case 8		**D**-**C**≤0	X	Left of d_2		
Case 9		**D**-**C**≤0	X	Right of d_2		
Case 10		**D**-**C**>0	Left of d_2			
Case 11	$d_{4\_b_{1\_asym}} < d_2 < d_{1\_b_{1\_asym}} < d_{3\_b_{1\_asym}}$	**D**-**C**>0	Right of d_2	X	O	X
Case 12		**D**-**C**≤0	X			
Case 13	$d_{4\_b_{1\_asym}} < d_2 < d_{3\_b_{1\_asym}} < d_{1\_b_{1\_asym}}$	**D**-**C**>0	X	X	O	X
Case 14	$d_{4\_b_{1\_asym}} < d_{3\_b_{1\_asym}} < d_2 < d_{1\_b_{1\_asym}}$	**D**-**C**>0	X	X	X	X

[1] "O" and "X" represent whether the crossing point of the given two boundaries exists or not, respectively.
[2] **D** and **C** are defined in **Equations (52)** and **(32)**, respectively.

Appendix B Evaluation of a Firm with an `RC` Outstanding

Similar to an `MC` issuer analyzed in Section 3.2, an `RC` issuer cannot trade its own stock or related derivatives on the stock to unwind the loss-absorbing mechanism. Thus in this subsection we analyze the impact of issuing an `RC` without considering unwinding. Consider that a firm with a T-year `SB` decides to issue a T-year `RC` at time 0. Four possible scenarios separated by three boundaries at the $(V(T),\, G(T))$ coordinate system are illustrated in **Figure 14**. If the issuer files for bankruptcy, the `SB` holder is repaid first and the remaining asset (if any) is then distributed to the `RC` investor. If the issuer's stock price falls below the predetermined conversion trigger level as defined in **Equation (8)**, the `RC` is converted into stocks as in S2, S3, and S4. The present values of the `RC`, the `SB`, and the

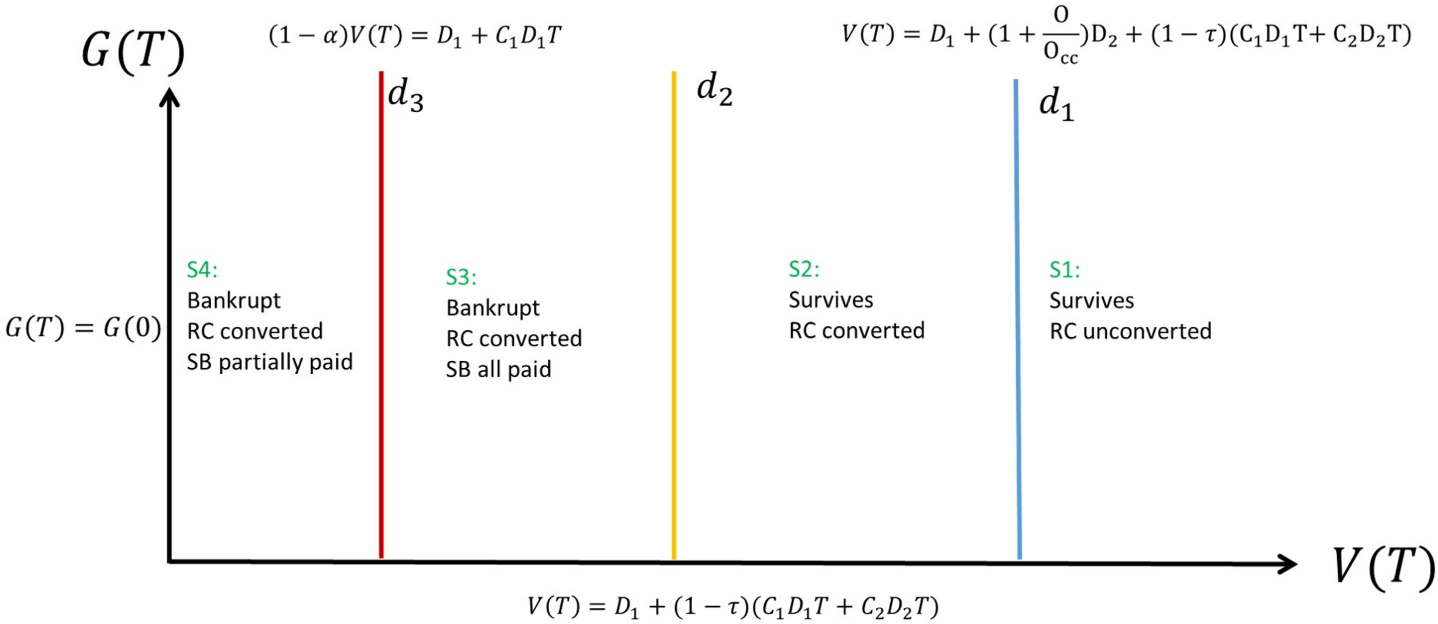


Figure 14: Possible Scenarios for Issuing an `RC`. The X- and Y-axes denote the values of the `RC` issuer's asset and reference asset, respectively. The equation for each boundary is listed next to the boundary. The description for each scenario is listed below the scenario. The yellow boundary d_2 separates survival scenarios (S1 and S2) from bankrupt ones (S3 and S4). Boundary d_1 separates S1 from S2, which denotes that the `RC` is not converted or converted, respectively. Boundary d_3 separates S3 from S4, which denotes that the `SB` holder is fully repaid or not, respectively.

equity can be evaluated as the discounted payoff defined in **Equation (8)** as

$$
\begin{aligned}
RC(0) = {} & \tilde{E}\Big[[D_2 + C_2D_2T]e^{-rT} \cdot \mathbf{I}_{\{S1\}}\Big] \\
& + \tilde{E}\Bigg[\Big[O_{cc}\underbrace{\frac{V(T) - [D_1 + (1-\tau)(C_1D_1T + C_2D_2T)]}{O + O_{cc}}}_{\mathbf{R}} + C_2D_2T\Big]e^{-rT} \cdot \mathbf{I}_{\{S2\}}\Bigg] \\
& + \tilde{E}\left[\mathrm{Max}\left[(1-\alpha)\,V(T) - (D_1 + C_1D_1T)\,, 0\right]e^{-rT} \cdot \mathbf{I}_{\{S3,S4\}}\right];
\end{aligned}
$$

$$
SB(0) = \tilde{E}\Big[(D_1 + C_1D_1T)\,e^{-rT} \cdot \mathbf{I}_{\{S1,S2,S3\}}\Big] + \tilde{E}\Big[\mathrm{Min}[D_1 + C_1D_1T,\ (1-\alpha)\,V(T)]e^{-rT} \cdot \mathbf{I}_{\{S4\}}\Big];
$$

$$
\begin{aligned}
E(0) = {} & \tilde{E}\Big[\Big[V(T) - [D_1 + D_2 + (1-\tau)(C_1D_1T + C_2D_2T)]\Big]e^{-rT} \cdot \mathbf{I}_{\{S1\}}\Big] \\
& + \tilde{E}\left[\left[O\underbrace{\frac{V(T) - [D_1 + (1-\tau)\,(C_1D_1T + C_2D_2T)]}{O + O_{cc}}}_{\mathbf{R}}\right]e^{-rT} \cdot \mathbf{I}_{\{S2\}}\right],
\end{aligned}
$$

where the **R** part is equal to the after-conversion stock price $S^C(T)$. By substituting **Equations (4)** and **(5)** for $V(T)$ and $G(T)$, respectively, the above pricing formulae can be expressed as linear combinations of $\tilde{E}\left[g \cdot \mathbf{I}_{\{S\}}\right]$, where g can be 1, $e^{\sigma_1 B_1(T)}$, or $e^{\sigma_2 B_2(T)}$ and S denotes a scenario.

By transforming the boundaries from the $(V(T), G(T))$ coordinate system illustrated in **Figure 14** to the $(B_1(T), B_2(T))$ system, the equations of d_1, d_2, and d_3 in the latter coordinate system are **Equations (43)**, **(44)**, and **(25)**, respectively. These equations entail that $d_1 > d_2$ and $d_1 > d_3$. $\tilde{E}\left[g \cdot \mathbf{I}_{\{S\}}\right]$ can be expressed in terms of the joint probability distribution of B_1 and B_2 in **Equation (6)** as

$$\tilde{E}\left[g \cdot \mathbf{I}_{\{S1\}}\right] = \int_{-\infty}^{\infty} \int_{d_1}^{\infty} g f_{B_1,B_2}(b_1, b_2) db_2 db_1$$

$$\tilde{E}\left[g \cdot \mathbf{I}_{\{S2\}}\right] = \int_{-\infty}^{\infty} \int_{d_2}^{d_1} g f_{B_1,B_2}(b_1, b_2) db_2 db_1$$

$$\tilde{E}\left[g \cdot \mathbf{I}_{\{S3\}}\right] = \begin{cases} \int_{-\infty}^{\infty} \int_{d_3}^{d_2} g f_{B_1,B_2}(b_1, b_2) db_2 db_1, & \text{if } d_3 < d_2; \\ 0, & \text{otherwise.} \end{cases}$$

$$\tilde{E}\left[g \cdot \mathbf{I}_{\{S4\}}\right] = \begin{cases} \int_{-\infty}^{\infty} \int_{-\infty}^{d_3} g f_{B_1,B_2}(b_1, b_2) db_2 db_1, & \text{if } d_3 < d_2; \\ \int_{-\infty}^{\infty} \int_{-\infty}^{d_2} g f_{B_1,B_2}(b_1, b_2) db_2 db_1, & \text{otherwise.} \end{cases}$$

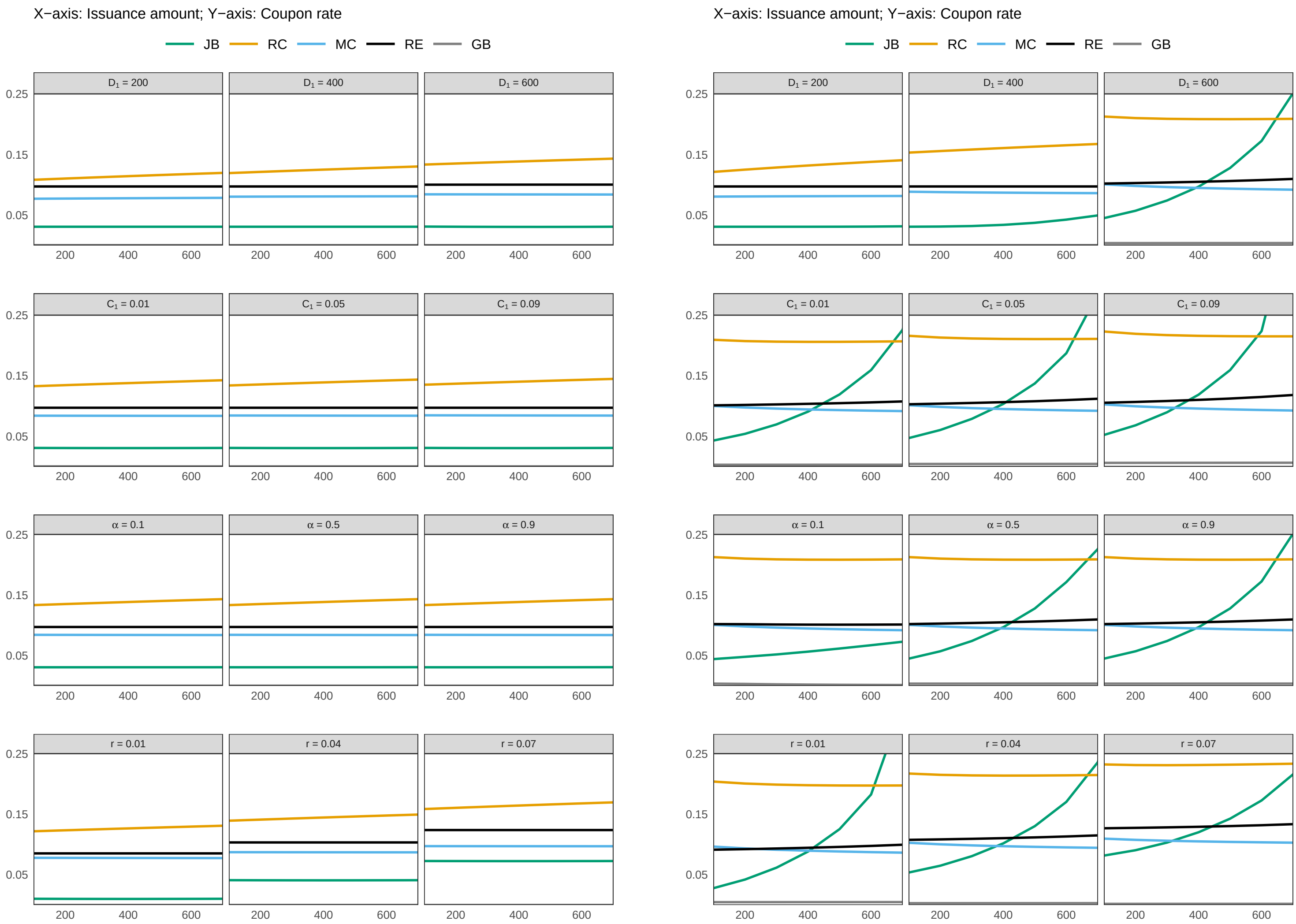
Appendix C Extensive Analyses and Robustness Check

In this section, our arguments for the pros and cons of a non-financial firm issuing a bail-in-able bond with different designs in Section 4 are consolidated via comprehensive sensitivity analyses. Section C.1 examines how the issuer's leverage ratio, the bond types, and other related variables influence the interest expenses for issuing (non)-bail-in-able bonds. All analyses in this subsection are made under low-leverage (proxied by setting $V(0^-)$ to 2000) and high-leverage ($V(0^-)$ to 1000) scenarios. We show that issuing bail-in-able bonds is especially beneficial for a high-leverage issuer. Thus, the experiments in the following two subsections will focus on the high-leverage scenario. We do not examine the impact of changing corporate income tax rates, since their values are quite clustered at 0.35 according to the *John Graham Corporate Tax Database*. Section C.2 analyzes the claim dilution and credit enhancement effects for a high-leverage issuer by examining the value changes in its equity and existent SB once a (non)-bail-in-able bond is issued. Section C.3 revisits the risk-shifting problem following the issuances of different bail-in-able bonds, and the severity of this agency problem is measured by comparing the value changes in the equity and the SB due to risk-shifting behavior. Specifically, for the case of issuing an RE or a GB, the behavior is proxied by longing forwards on the reference asset to unwind the loss-absorbing and the benefit-sharing mechanism (if applicable). For the case of issuing an RC or an MC, the behavior is proxied by increasing the volatility of the issuer's asset value σ_1.

C.1 Interest Expenses for Newly-Issued Bonds

The impact of changing the SB's face value D_1, coupon rate C_1, bankruptcy cost α, and the risk-free interest rate r on the par rates C_2 of different (non)-bail-in-able bonds are illustrated in **Figure 15**. This can be viewed as the extensive sensitivity analyses of **Figure 6** in Section 4.1. The correlation ρ is set to 1 to eliminate concerns about correlation risk. We first examine the impact of incorporating loss-absorbing mechanisms by comparing the differences in par rates between a JB and an RE (or an RC). In the low-leverage scenario (panel (a)), an RE (or an RC) investor requires a higher par rate than a JB investor for bearing the risk to absorb the issuer's loss regardless of changes to D_1, C_1, α, and r. In the high-leverage scenario (panel (b)), a lower D_1 ($= 200$ or 400) or a higher recovery rate (due to lower bankruptcy cost α ($= 0.1$)) still makes the JB's par rate lower than the RE's (or the RC's). However, as D_1 ($= 600$) and α ($= 0.6$) are high, the JB's par rate becomes greater than the RE's (or the RC's) when the issuance amount is large enough regardless of changes to C_1 and r. These comprehensive analyses confirm our argument that the credit enhancement effect provided by loss-absorbing mechanisms gains significance with increments in the issuer's default likelihood, and can make the par rate of an RE or an RC lower than that of an otherwise identical JB. We then examine the impact of incorporating benefit-sharing mechanisms by comparing the differences in par rates between an RE and a GB (or an RC and an MC). It can be observed in all subfigures of **Figure 15** that allowing a bond investor to share the issuer's upside potential profits reduces the investor's required return regardless of D_1, C_1, α, and r. Such a reduction is more salient for a high-leverage issuer.

The impact of changing the initial value and the volatility of the reference asset, $G(0)$ and σ_2, on the par rates of an RE and a GB is illustrated in **Figure 16**. Again, the RE's par rate can be lower than an otherwise identical JB's when the issuance amount is large enough in the high-leverage scenario. The GB's par rate is lower than that of the RE due to the presence of the benefit-sharing mechanism. Based on the relation above, it can be observed that the par rates of both the RE and the GB are insensitive to changes in $G(0)$, since the payoffs of both the RE and the GB depend on the



(a) Low-leverage firm

(b) High-leverage firm

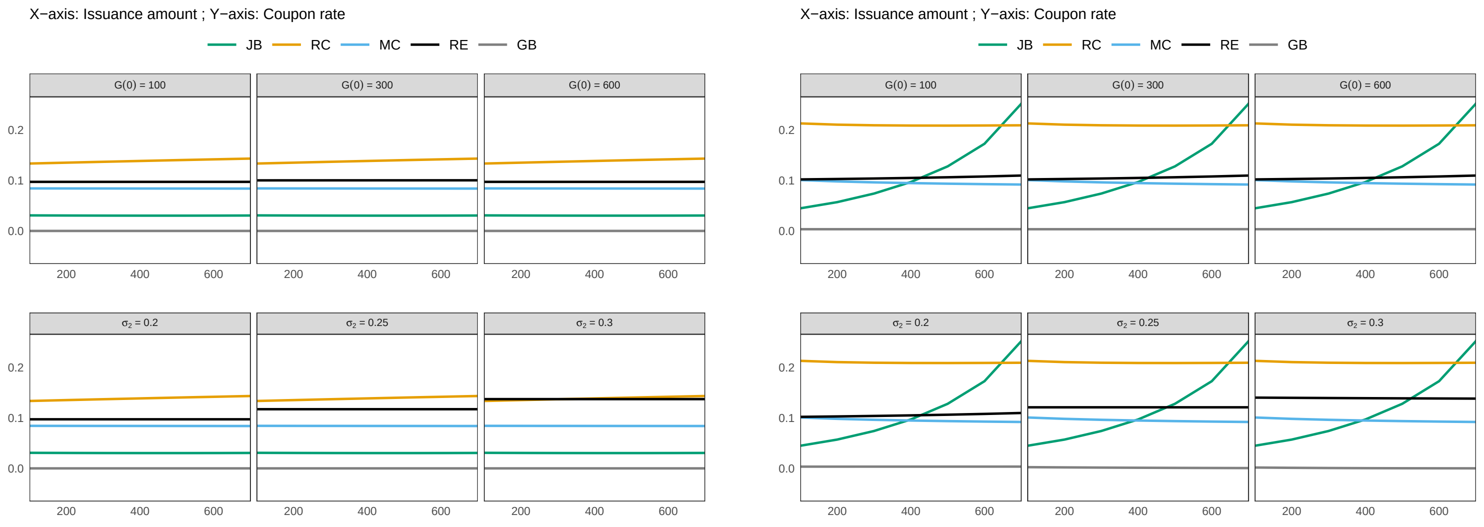
Figure 15: Interest Expenses for Newly-Issued Short-Term (Non)-bail-in-able Bonds. Panels (a) and (b) display the par rates C_2 of different newly-issued (non)-bail-in-able bonds given that the issuer's leverage ratio is low (by setting $V(0^-) = 2000$) and high ($V(0^-) = 1000$), respectively. The X- and Y-axes denote the issuance amount D_2 and the corresponding par rates C_2 of the newly-issued bonds, respectively. All settings are identical to those in **Figure 6** except the parameters described in the upper part of each subfigure described as follows. In the first row, the SB's face value D_1 is set to 200, 400, or 600. In the second row, the SB's coupon rate C_1 is set to 1%, 5%, or 9%. In the third row, the bankruptcy cost α is set to 10%, 50%, or 90%. In the fourth row, the risk-free interest rate r is set to 1%, 4%, or 7%.

ratio of $G(T)/G(0)$ according to **Equations (9)** and **(10)** rather than solely on $G(0)$.[11] On the other hand, σ_2 determines the volatility of $G(T)/G(0)$ as defined in **Equation (3)**. A higher σ_2 increases the probability of triggering the loss-absorbing mechanism, resulting in a higher par rate required by an RE investor. However, the par rate required by a GB investor is quite stable regardless of changes in σ_2, because a higher σ_2 also increases the probability of triggering the benefit-sharing mechanism.

Finally, the impact of changing the upper conversion threshold λ on the par rate of the MC is examined in **Figure 17**. The par rate of the MC is lower than that of an otherwise identical RC due to the presence of the benefit-sharing mechanism. However, the probability of triggering the mechanism

[11]Note that the G' contracted in an RE and a GB is set to $G(0)$ in Section 4.

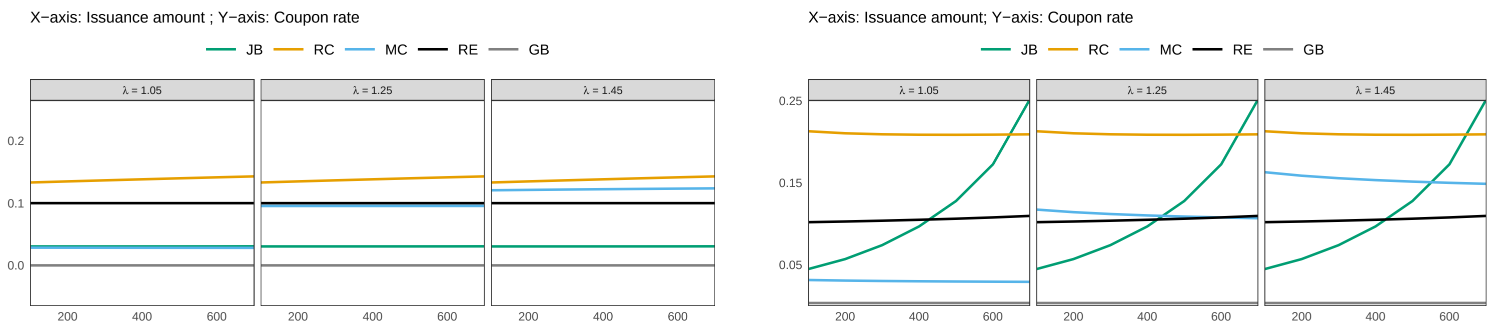
decreases with increments in λ. That makes the par rate of the MC par rate converge to that of the RC.



(a) Low-leverage firm

(b) High-leverage firm

Figure 16: Impact of Changing the Initial Values and Volatilities of Reference Assets on Interest Expenses for Newly-Issued REs and GBs. All settings are identical to those in **Figure 6** other than the initial value and the volatility of the reference asset, $G(0)$ and σ_2, described as follows. $G(0)$ is set to 100, 300, and 600 given that $\sigma_2 = 20\%$. σ_2 is set to 20%, 25%, and 30% given that $G(0) = 600$.



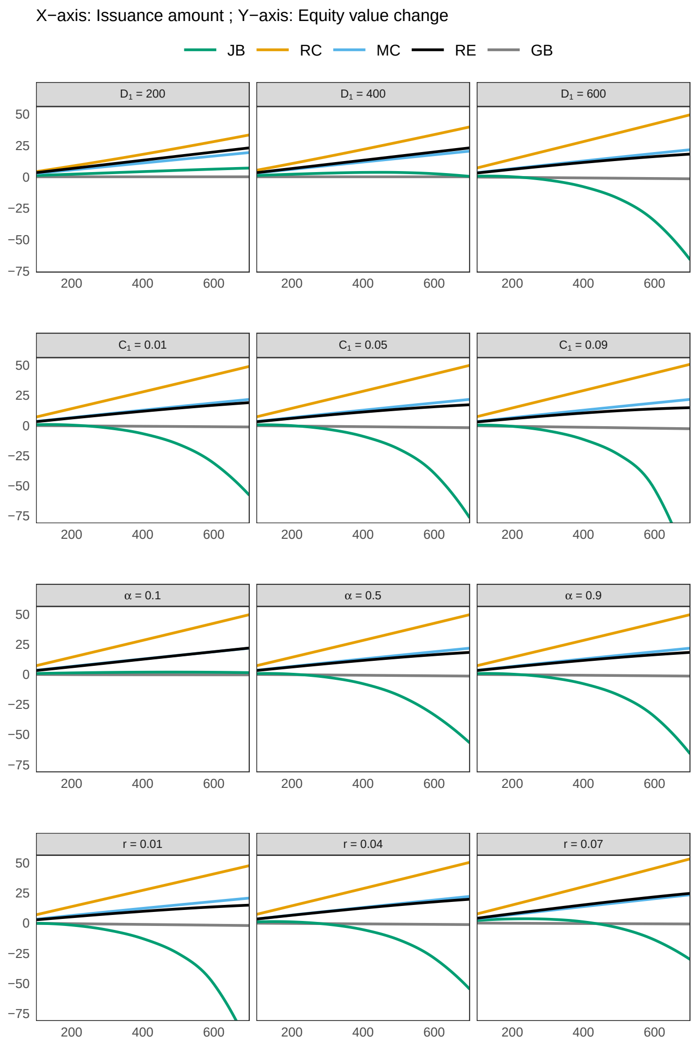
(a) Low-leverage firm

(b) High-leverage firm

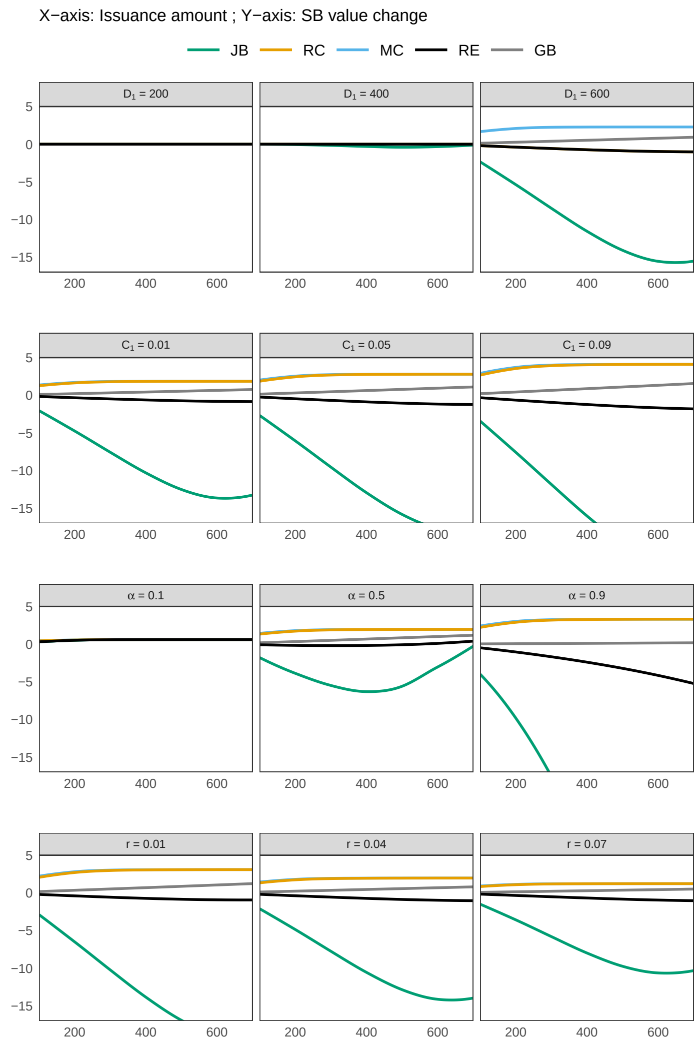
Figure 17: Impact of Changing Upper Conversion Thresholds on Interest Expenses for Newly-Issued MCs. All settings are identical to those in **Figure 6** except for the upper conversion threshold. The threshold is set to 1.05, 1.25 and 1.45.

C.2 Claim Dilution and Credit Enhancement Effects

This section examines claim dilution and credit enhancement effects in **Figures 18** and **19**. They can be regarded as the extensive sensitivity analyses of **Figure 9** in Section 4.2. First, the pure claim dilution effect is reflected by the negative value changes in the equity and the SB once a JB is issued (denoted by green curves). It can be observed in **Figure 18** that this effect is greater when the SB's face value D_1, the SB's coupon rate C_1, or the bankruptcy cost α is higher, or when the interest rate r is lower. It can also be observed in panel (a) that the equity value changes can be positive when the JB issuance amount is small. This shows that the issuer can raise debt capital to reach



(a) Equity value change

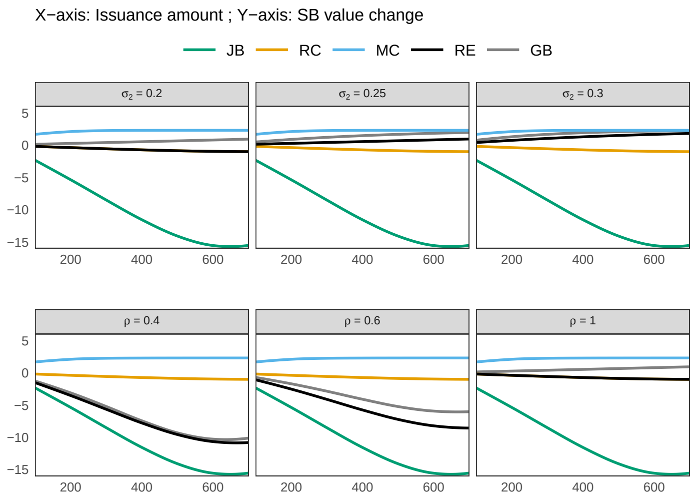


(b) SB value change

Figure 18: Value Changes in Equity and SB due to Issuances of (Non)-bail-in-able Bonds. The X- and the Y-axes denote the issuance amount of newly-issued bonds D_2 and the corresponding value changes in equity and SB, respectively. All settings are identical to those in **Figure 9** except the parameters described in the upper part of each subfigure described as follows. In the first row, the SB's face value D_1 is set to 200, 400, and 600. In the second row, the SB's coupon rate C_1 is set to 1%, 5%, and 9%. In the third row, the bankruptcy cost α is set to 10%, 50%, and 90%. In the fourth row, the risk-free interest rate r is set to 1%, 4%, and 7%. The SB value change due to the issuance of an MC is slightly larger than that due to the issuance of an otherwise identical RC.

its optimal leverage ratio. The credit enhancement effect provided by a loss-absorbing mechanism is measured by the increments in value changes from issuing a JB to an RE (black curves) or to an RC (orange curves); it shares the same pattern as those in **Figure 18** regardless of changes in D_1, C_1, α, and r. In addition, the orange curves are always higher than the black curves, showing that an RC provides stronger credit enhancement than an RE. The impact of incorporating a benefit-sharing mechanism is measured by comparing the differences in value changes in the equity and the SB between issuing an RE and issuing a GB (gray curves) or between issuing an RC and issuing an MC (blue curves). Incorporating such a mechanism reduces the equity value but increases the SB value as illustrated in **Figure 18**. This confirms the argument in **Figure 9** that the issuances of two-sided bonds lead to smaller (greater) equity (SB) value changes than the issuances of the otherwise identical one-sided bonds. This is because the interest expenses for issuing two-sided bonds are suppressed at the expense of the equity holders via the benefit-sharing mechanisms to alleviate the issuers' payment risk and hence the default likelihood.

(a) Equity value change



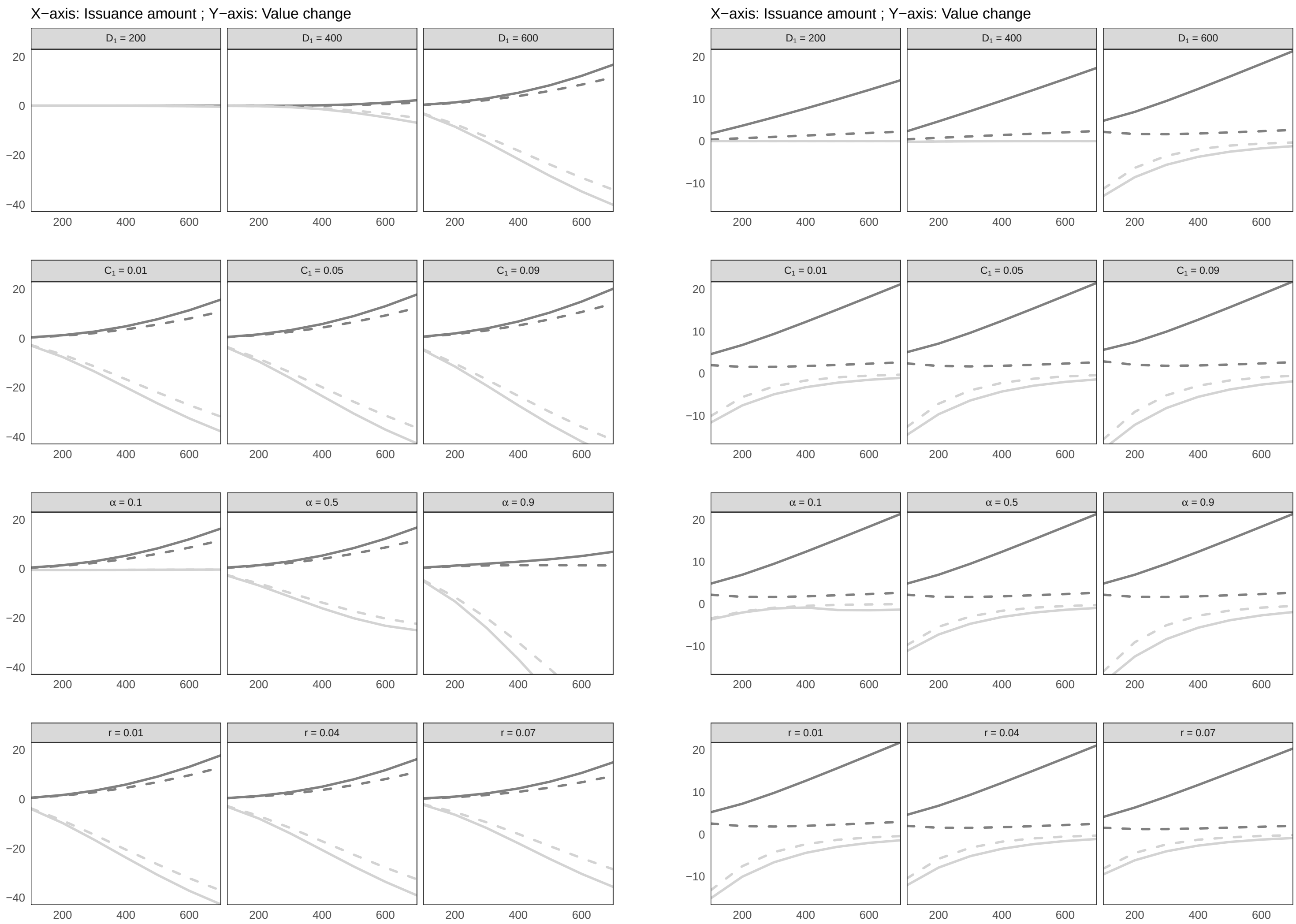
(b) SB value change

Figure 19: Value Changes of Equity and SB due to Issuances of REs and GBs when σ_2 and ρ are Different. All settings are identical to those in **Figure 9** except σ_2 and ρ specified in the upper part of each subfigure. In the first row, σ_2 is set to 20%, 25%, and 30%. In the second row, ρ is set to 0.4, 0.6, and 1.

The impact of changing the volatility of the reference asset, σ_2, and the correlation between the values of the issuer's asset and reference asset, ρ, on our arguments in Section 4.2 is examined in **Figure 19**. The increments from the green curves to the black curves illustrate that the credit enhancement effect provided by the RE continues regardless of changes in σ_2 and ρ. In particular, the credit enhancement increases as σ_2 increases in that the increments from the green curves to the black curves become more significant as σ_2 increases. This is because the probability of triggering the loss-absorbing mechanism increases with increments in σ_2. However, it becomes weaker as ρ decreases in that the differences between the green and the black curves become less significant as ρ decreases. This is because the correlation risk weakens the loss-absorbing mechanism. On the other hand, the impact of including a benefit-sharing mechanism also becomes stronger as σ_2 increases in that in the first row of panel (a), the decrements from the black curves to the gray curves gain significance with increments in σ_2. However, the credit enhancement effect provided by the GB deteriorates more rapidly as ρ decreases in that the differences between the green and the gray curves become less salient more rapidly with the decrement of ρ. This is because the correlation risk also weakens the benefit-sharing mechanism.

C.3 Asset Substitution Problem

The asset substitution problem following the issuances of different bail-in-able bonds is examined in **Figure 20**. It can be regarded as the extensive sensitivity analyses of **Figure 10** in Section 4.3. In particular, an issuer's risk-shifting behavior is proxied by longing forwards on the reference assets in the RE- and GB-issuance scenarios, and is proxied by raising the issuer's asset value volatility in the RC- and MC-issuance scenarios. The severity of this agency problem is measured by comparing the



(a) Longing forwards on reference assets

(b) Raising the issuer's asset value volatility

Figure 20: Impact of Risk-Shifting Behavior on Value Changes of Equity and SB. The Y-axis denotes the value changes in equity (dark gray) and SB (light gray) due to risk-shifting behavior. Panel (a) illustrates the value changes from longing forwards on reference assets to unwind the mechanism(s) in GBs (dashed curves) or REs (solid curves). Panel (b) illustrates the value changes from raising the issuer's asset value volatility σ_1 from 0.2 to 0.3 after issuing MCs (dashed curves) or RCs (solid curves). All settings are identical to those in **Figure 10** except the parameters described in the upper part of each subfigure.

value changes of the SB and the equity due to the risk-shifting behavior. The more significant the difference in the value changes in the SB and the equity, the severer the agency problem. It can be observed in panels (a) and (b) that, *ceteris paribus*, the problem is gains severity as D_1, C_1, and α increase or as r decreases. Both panels also confirm the argument that incorporating benefit-sharing mechanisms in bail-in-able bonds alleviates the risk-shifting incentives in that increments (decrements) in the equity (SB) value in the GB-issuance (MC-issuance) scenario are smaller than those in the RE-issuance (RC-issuance) scenario regardless of changes in D_1, C_1, α, and r. This is because the presence of a benefit-sharing mechanism in a GB (MC) shifts part of the equity holders' benefits gained from the risk-shifting behavior to the GB (MC) investor. Panel (b) also confirms that decrements in the SB value becomes less significant with increments in the RC or the MC issuance amount, since the debt principal of the RC or the MC can be entirely converted to equity capital to significantly reduce the payment risk and thus enhance the issuer's credit.