Hedge Fund Performance under Misspecified Models[☆]

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Abstract

We develop a formal approach for comparing performance across misspecified models—a common feature of models faced with the challenge of evaluating hedge funds. This comparison sharpens performance evaluation by identifying models less prone to misspecification. We show that the standard models include factors with limited ability to capture hedge fund returns. As a result, they produce the same positive alphas as a simple CAPM. Building on the recent literature, we then form a new model based on economically-motivated factors, including variance, carry, and time-series momentum. We find that this model achieves a substantial reduction in hedge fund performance.

Keywords: Hedge funds performance, alternative strategies, misspecification, model comparison

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1. Introduction

Over the past three decades, the growth of the hedge fund industry has been outstanding. Its total size has increased from $40 billion in 1990 to close to $3 trillion at the end of 2016 (Getmansky, Lee, and Lo, 2015). A common interpretation for this trend is that hedge fund managers are able to outperform the market because they are more sophisticated, less constrained, and more incentivized than mutual fund managers. Consistent with this interpretation, the empirical literature generally finds that hedge funds deliver strong and positive alphas.[1]

One caveat against this interpretation is model misspecification. Hedge funds follow a rich set of alternative strategies across multiple countries and asset classes (Lhabitant, 2007; Pedersen, 2015). It is therefore likely that any model used for benchmarking hedge funds is misspecified—that is, it omits factors that are relevant for tracking these alternative strategies. To the extent that hedge funds rely on alternative strategies to boost their returns (Carhart et al., 2014), omitted factors contaminate the fund alphas and artificially increase their values. Misspecification has therefore important implications for the allocation decisions of hedge fund investors and for the debate on the performance achieved by the active fund industry.

Previous studies on hedge fund performance routinely rely on standard multi-factor models such as the models of Carhart (1997) and Fung and Hsieh (2004). However, doubts remain regarding the ability of these models to mitigate the impact of misspecification—a point recently made by Joenväärä et al. (2021). There is no formal analysis showing that the standard models produce a performance evaluation that differs from a simple CAPM. In addition, they do not include factors recently uncovered by the asset pricing literature. For instance, Pedersen (2015) argues that factors such as liquidity and carry are important drivers of hedge fund returns.

To address misspecification, we provide the first formal comparison of hedge fund models. Our comparison approach evaluates the ability of any competing models to capture the alternative strategies followed by hedge funds. As such, it provides a framework for choosing models that are less prone to misspecification. In addition, it provides information about the importance of each factor in driving hedge fund returns. Applying this approach, we construct a new model that

[1] A non-exhaustive list of hedge fund papers documenting positive performance includes Ackermann, McEnally, and Ravenscraft (1999), Avramov et al. (2011), Avramov, Barras, and Kosowski (2013), Capocci and Hübner (2004), Buraschi, Kosowski, and Trojani (2014), Chen, Cliff, and Zhao (2017), Diez de los Rios and Garcia (2010), Duarte, Longstaff, and Yu (2006), Getmansky, Lee, and Lo (2015), Ibbotson, Chen, and Zhu (2011), Kosowski, Naik, and Teo (2007), Liang (1999), and Patton and Ramadorai (2013).

sharpens the evaluation of hedge fund performance. This model, which is both parsimonious and economically motivated, achieves a substantial reduction in alphas relative to the standard models.

Our comparison approach has two distinguishing features. First, it explicitly accounts for misspecification. Our theoretical analysis shows that when a model is misspecified, the sampling variation of the omitted factors affects all funds simultaneously. As a result, cross-sectional performance measures such as the average alpha remains noisy—even in a population of several thousand funds. Building on this insight, we design valid tests for comparing misspecified models, and show that they considerably raise the bar for detecting significant differences between models.

Second, our approach applies to the entire alpha distribution—that is, we do not simply compare the average alpha across models but the entire shape of the distribution. Extending the analysis beyond the average alpha is essential for investors because they only invest in a handful of hedge funds (*e.g.*, Bollen, Joenväärä, and Kauppila, 2021). The alpha distribution is therefore informative about the range of outcomes faced by these investors. For instance, it determines the proportion of funds with positive alpha and the performance achieved by the best funds.[2]

We construct our hedge fund sample by collecting monthly net returns from five databases (BarclayHedge, Eurekahedge, HFR, Morningstar, and TASS). This combined database provides extensive coverage of more than 20,000 hedge funds over the period 1994–2016. As such, it mitigates the selection bias that arises because hedge fund reporting is not mandatory (see Joenväärä et al., 2021). We then estimate the alpha distribution across individual funds for several competing models. To begin, we examine the standard models used in previous studies on performance evaluation. We focus on a representative set of five models: the three- and five-factor models of Fama and French (1992, 2015), the models of Carhart (1997) and Fung and Hsieh (2004), and the model of Asness, Moskowitz, and Pedersen (2013), which extends the traditional value and momentum strategies to multiple asset classes.

Our comparison of the standard models uncovers several new insights. We find that they all produce the same evaluation of hedge fund performance. This similarity does not only apply to the average alpha—it extends to the entire shape of the alpha distribution. As a result, the estimated

[2]Our approach is related to previous work on asset pricing comparison tests across misspecified models (*e.g.*, Kan and Robotti, 2009; Kan, Robotti, and Shanken, 2013). Our asymptotic analysis departs from these studies by allowing for a large number of funds in order to match the large fund population observed in the data. As we explain in Section 3, this difference has a strong impact on the asymptotic theory and the properties of the comparison tests.

performance of the hedge fund industry is economically large across all five models. The average alpha is close to 2.4% per year, and more than 65% of the funds deliver positive alphas to investors.

More striking is the strong similarity with the simplest model—the CAPM. Using the CAPM produces similar average and proportion values (2.6% per year and 68%). It also leaves the dispersion and tail properties of the alpha distribution largely unchanged. Our tests confirm these results by comparing the main characteristics of the alpha distribution (mean, standard deviation, proportion of positive-alpha funds, quantiles at 10% and 90%). Out of the 25 characteristic differences between the CAPM and the standard models, only three are statistically significant at the 5%-level.

These results arise because the standard factors struggle to track alternative hedge fund strategies. More than 80% of them contribute less than 0.2% per year to the average returns earned by hedge funds. Overall, our analysis calls for an examination of a broader set of factors, as suggested by Joenväärä et al. (2021) who argue for an updated model that reflects the post-2004 literature.

To address this issue, we construct a new hedge fund model. Key to our approach is the identification of a small number of factors that plausibly drive hedge fund returns. To illustrate, there is ample evidence that hedge funds load on variance risk using options, buy cheap assets with high carry, and follow market trends (Lhabitant, 2007; Pedersen, 2015). These alternative strategies should therefore be captured by variance, carry, and time-series (TS) momentum factors. Whereas there is no perfect way of forming hedge fund models, our factor selection favors economic intuition and mitigates data-mining concerns that arise when examining many factors.

Our new model adds to the CAPM the excess returns of five mechanical strategies: (i) the illiquidity factor of Pástor and Stambaugh (2003), (ii) betting-against-beta, which captures the return earned by hedge funds from exploiting their leverage capacity (Frazzini and Pedersen, 2014), (iii) the variance factor, which tracks the realized variance of the S&P500 (e.g., Carr and Wu (2009)), (iv) carry, which invests in assets with high carry (Koijen et al. (2018)), and (v) TS momentum, which invests in assets with high past returns (Moskowitz, Ooi, and Pedersen, 2012).

The new model largely reduces the performance of the hedge fund industry. We find that the average alpha turns negative at -0.2% per year, and the proportion of positive-alpha funds barely reaches 50%. The statistical significance of these results is strong—even after accounting for the estimation noise due to misspecification, the tests consistently reject the null hypothesis that the new model produces the same performance as the standard models. The performance reduction achieved by the new model is also a robust feature of the data. It holds across different investment categories and subperiods, and after accounting for the costs of trading the new factors.

Consistent with the above results, we find that the majority of funds load on the new factors. This finding supports the view that hedge funds follow alternative strategies to increase their returns (Carhart et al., 2014). Carry, TS momentum, and variance are particularly important drivers of hedge fund returns—their contributions to the average fund returns are equal to 1.13%, 0.84%, and 0.49% per year. The analysis of the fund loadings across investment categories is also consistent with economic intuition. For instance, trend-following funds are heavily exposed to TS momentum (Pedersen, 2015, ch. 12), while fixed-income funds are sensitive to variance risk through their option-based strategies (Duarte, Longstaff, and Yu, 2006).

Finally, the new model changes the characteristics of the worst and best funds. It uncovers the strong and negative alphas achieved by the worst funds. The 10%-quantile is equal to -11.3% per year, versus only -6.9% under the standard models. This sharp difference arises because the worst funds load heavily on the new factors—possibly to hide their lack of skills. The new model also changes the identity of the best funds—in the top decile, the overlap with the standard models is around 60%. These funds produce alphas above 9.7% per year and have higher levels of managerial incentive and discretion, consistent with past studies (*e.g.*, Agarwal, Daniel, and Naik, 2009).

Overall, these results have several implications for hedge fund investors. First, the sharp reduction in average alpha makes the hedge fund industry less appealing. It calls for a reduced allocation into hedge funds given the emergence of cheaper products that track alternative strategies (Jorion, 2021). Second, investors face substantial performance heterogeneity in their selection of individual funds—under the new model, the standard deviation of alpha reaches 11% per year. This cross-sectional dispersion highlights the importance of conducting proper due diligence to avoid the large number of funds with negative alpha.

Our paper also contributes to the debate on active management. Previous studies document a large performance gap between hedge funds and mutual funds. This gap seems puzzling because it implies that investors are able to extract a lot of value from hedge funds, but none from mutual funds. It is also at odds with Sharpe's arithmetic which implies that active funds cannot, on average, deliver positive alphas (Pedersen, 2018; Sharpe, 1991). Our results mitigate these issues by showing that the performance of the hedge fund industry is not as large as previously thought.

The remainder of the paper is as follows. Section 2 presents our framework for measuring hedge fund performance. Section 3 describes our comparison approach. Section 4 presents the hedge fund dataset and factors. Section 5 contains the empirical analysis, and Section 6 concludes. The appendix provides additional information on the methodology, the dataset, and the results.

2. Hedge Fund Performance Evaluation

2.1. Measuring Performance across Individual Funds

2.1.1. The Fund Alpha

We evaluate the performance of a population of n individual funds over T observations, where we denote each fund by the subscript i ($i = 1, \ldots, n$) and each observation by the subscript t ($t = 1, \ldots, T$). To measure the performance of each fund, we use the alpha defined as the average return difference between the fund and its benchmark:

$$\alpha_i^* = E[r_{i,t}] - E[r_{i,t}^B] = E[r_{i,t}] - E[\beta_i^{*\prime} f_t] = E[r_{i,t}] - \beta_i^{*\prime}\lambda\,, \tag{1}$$

where $r_{i,t}$ is the excess net-of-fee return of the fund, and $r_{i,t}^B$ is the excess return of the benchmark portfolio constructed from a set of trading factors whose excess return vector is denoted by f_t. The factors capture the mechanical strategies that hedge funds follow, including dynamic strategies in which the asset weights depend on public information.[3] The factor premia defined as $\lambda = E[f_t]$ can be the outcome of systematic risk compensation, limits to arbitrage, or imperfect risk sharing driven by segmentation or behavioral biases (see Pedersen, 2015, ch. 3, for a discussion). In this paper, we remain agnostic on this issue—our objective is simply to benchmark the fund against trading strategies that could be implemented by investors.

A positive alpha signals that the fund delivers superior returns to the investor (over the benchmark). Alternatively, the alpha can be interpreted based on utility theory. If the investor's stochastic discount factor m_t depends linearly on the trading strategies f_t, we can rewrite Equation (1) as $\alpha_i^* = E[m_t r_{i,t}]$ (Chen and Knez, 1996; Almeida, Ardison, and Garcia, 2020). A positive α_i^* implies that the investor can increase his overall utility by investing in the fund (see Ferson, 2013).

[3] To see how Equation (1) accommodates dynamic strategies, consider an equity fund whose weight on the market $\beta_{im,t}^*$ varies linearly with a (demeaned) public signal z_{t-1} that predicts the future market return $r_{m,t}$, *i.e.*, we have $\beta_{im,t}^* = \beta_{im,0}^* + \beta_{im,1}^* z_{t-1}$. In this case, the benchmark return is equal to $r_{i,t}^B = \beta_{im,t}^* r_{m,t}$ which can be rewritten as in Equation (1): $r_{i,t}^B = \beta_i^{*\prime} f_t$, where $\beta_i^* = (\beta_{i,m,0}^*, \beta_{i,m,1}^*)'$, and $f_t = (r_{m,t}, z_{t-1} r_{m,t})'$.

The baseline specification in Equation (1) calls for several comments. First, α_i^* measures performance, not skill. Whereas the two notions are commonly used interchangeably, they differ in important ways (Berk and van Binsbergen, 2015). Skill is defined from the viewpoint of funds, *i.e.*, it determines whether hedge funds are able to extract value from capital markets. In contrast, performance is defined from the viewpoint of investors, *i.e.*, it determines whether the value created by the funds, if any, is shared with them.

Second, Equation (1) captures the fund's long-term performance. In other words, it does not model the short-term variations in alpha around its average α_i^*. For instance, performance could vary with the business cycle (*e.g.*, Avramov, Barras, and Kosowski, 2013), or with the levels of industry competition and aggregate mispricing (*e.g.*, Pástor, Stambaugh, and Taylor, 2015, 2017). Whereas modeling the short-term dynamics in alpha is certainly informative, correctly specifying all the variables that drive short-term performance remains a difficult task, especially when investment strategies among funds are heterogeneous.

Third, Equation (1) measures the factor returns f_t gross of trading costs. As a result, the alpha excludes the value of the diversification services provided by the fund in replicating the trading strategies (Berk and van Binsbergen, 2015). However, such services can be quite valuable for hedge fund investors (*e.g.*, Cochrane, 2013). To examine this issue, we also conduct a performance analysis that incorporates the costs of trading the factors (see Section 5.3).

2.1.2. The Cross-Sectional Alpha Distribution

Our performance evaluation focuses on the entire distribution of the fund alphas. Extending the analysis beyond the average alpha allows us to capture the heterogeneity in performance across funds. This information is particularly relevant for hedge fund allocation. Because of investment requirements and due diligence costs, hedge fund investors select only a handful of funds (Lhabitant, 2007; Bollen, Joenväärä, and Kauppila, 2021). Therefore, they want to know the probability of choosing a positive-alpha fund and the range of possible performance outcomes. In contrast, the average alpha only captures the performance of a highly-diversified hedge fund portfolio.

We can also use the alpha distribution to examine why hedge funds deliver non-zero alphas in equilibrium. To illustrate, investors can earn positive alpha if they hold bargaining power in the fee negociation. This is, for instance, the case if they can threaten the fund of expropriating its investment ideas (Glode and Green, 2011). From the estimated alpha distribution, we can quantify the level and dispersion in bargaining power required to match the data.

If we know the correct model in Equation (1), estimating the alpha distribution is straightforward. We simply compute the alpha of each fund using the following time-series regression:

$$r_{i,t} = \alpha_i^* + \beta_i^{*\prime} f_t + \varepsilon_{i,t}^* \,, \tag{2}$$

where $\varepsilon_{i,t}^*$ denotes the fund residual term. Equation (2) is interpreted as a random coefficient model (*e.g.*, Hsiao, 2003) in which the fund alpha α_i^* is not a fixed parameter, but a random realization from a continuum of funds. We can then invoke cross-sectional limits to estimate the alpha distribution using the estimated alphas $\hat{\alpha}_i^*$ ($i = 1, \ldots, n$)—an approach used by Barras, Gagliardini, and Scaillet (2022) to evaluate mutual fund performance.[4]

2.2. Model Misspecification

2.2.1. The Impact of Model Misspecification

A key requirement for measuring performance is that we use the correct model in Equation (2). This is, however, a difficult task for hedge funds because they follow a large number of investment strategies. Hedge funds invest across many asset classes in both developed and emerging markets (Lhabitant, 2007; Pedersen, 2015). They also dynamically change their asset allocation when they adjust their leverage ratio or respond to economic conditions (Ang, Gorovyy, and van Inwegen, 2011; Patton and Ramadorai, 2013). Finally, hedge funds deliver nonlinear payoffs that may not be fully captured by a limited set of option-based factors (Karehnke and de Roon, 2020).

This analysis implies that we are likely to use a misspecified model. To see the impact of model misspecification on performance evaluation, consider a model that only includes the factors $f_{I,t}$, but omits the factors $f_{O,t}$ (with $f_t = (f_{I,t}', f_{O,t}')'$):

$$r_{i,t} = \alpha_i + \beta_{i,I}' f_{I,t} + \varepsilon_{i,t} \,. \tag{3}$$

The fund alpha α_i obtained with the misspecified model in Equation (3) typically differs from the true alpha α_i^*. We can write the omitted factors as: $f_{O,t} = \alpha_O + \Psi_{O,I} f_{I,t} + u_{O,t}$, where $\Psi_{O,I}$ is

[4]Gagliardini, Ossola, and Scaillet (2016) use a similar sampling scheme for testing the arbitrage pricing theory in a large cross-section of assets (see also Gagliardini, Ossola, and Scaillet, 2020, for a review).

the matrix of slope coefficients, $u_{O,t}$ is the vector of factor residuals, and $\alpha_O = \lambda_O - \Psi_{O,I}\lambda_I$ is the vector of alphas of the omitted factors. Replacing λ with $(\lambda_I', \lambda_O')'$ and β_i^* with $(\beta_{i,I}^{*\prime}, \beta_{i,O}^{*\prime})'$ in Equation (2), we have $E[r_{i,t}] = \alpha_i^* + \beta_{i,I}^{*\prime}\lambda_I + \beta_{i,O}^{*\prime}\lambda_O = \alpha_i + \beta_{i,I}'\lambda_I$, where $\beta_{i,I} = \beta_{i,I}^* + \Psi_{O,I}'\beta_{i,O}^*$. We then obtain the following result:

$$\alpha_i = \alpha_i^* + \beta_{i,O}^{*\prime}(\lambda_O - \Psi_{O,I}\lambda_I) = \alpha_i^* + \beta_{i,O}^{*\prime}\alpha_O\,. \tag{4}$$

Equation (4) reveals that α_i is informative about α_i^*. However, this information is noisy because α_i is impacted by the omitted factor component $\beta_{i,O}^{*\prime}\alpha_O$.[5] We are therefore unable to perfectly infer the true cross-sectional alpha distribution.

Equation (4) resonates with the commonly-held view that hedge funds load on alternative factors, defined as trading strategies with positive premia and low correlations with traditional equity strategies (*e.g.*, Carhart et al., 2014). In this case, $\beta_{i,O}^*$ is positive on average, which implies that the misspecified model overestimates the true average alpha of the hedge fund industry (*i.e.*, we have $E[\alpha_i] > E[\alpha_i^*]$).

2.2.2. *The Benefits of Comparing Models*

To mitigate the impact of misspecification, we formally compare the alpha distribution across several models. Our comparison approach brings two main benefits. First, it sharpens the evaluation of hedge fund performance. Hedge fund models include multiple factors to track the returns of important alternative strategies. Some of them are likely to do a good job at capturing these strategies or, more formally, the omitted term $\beta_{i,O}^{*\prime}\alpha_O$ in Equation (4). In contrast, other models may produce the same performance as the CAPM in spite of their greater complexity. By examining how the estimated performance varies, we can detect models that are less prone to misspecification.[6]

Second, our approach measures the contribution of each factor in explaining the average fund

[5] If the factors are uncorrelated ($\Psi_{O,I} = 0$), Equation (4) simply becomes: $\alpha_i = \alpha_i^* + \beta_{i,O}^{*\prime}\lambda_O$, where the impact of each omitted factor is captured by its premium—a quantity that does not depend on the specific factors $f_{I,t}$ included in the misspecified model (contrary to α_O). This assumption is fairly in line with the data because most of the hedge funds factors examined in this paper are weakly correlated.

[6] A complementary approach to reduce misspecification is to pool all the models together (O'Doherty, Savin, and Tiwari, 2016). This approach allows for the inclusion of many factors without inducing large estimation errors (see Timmermann, 2006) for a literature review on pooling methods). However, the weight of each model in the pooled benchmark must be estimated and varies with each fund. As a result, it is difficult to compute fund performance and determine the level of misspecification of each model in the entire population.

returns. We can therefore isolate the most relevant factors, and examine whether their impact across funds and investment styles is in line with economic intuition. Understanding the importance of the factors is also useful for hedge fund investors. For instance, if investors discover that many funds load on a single alternative factor, they can look for financial products that provide the same factor exposure at a lower cost (Jorion, 2021; Carhart et al., 2014).

2.2.3. A Simple Illustrative Example

Before formalizing our approach, we illustrate the two benefits discussed above using a simple hypothetical example. We assume that the average return of each fund is given by $E[r_{i,t}] = \alpha_i^* + \beta_{i,m}^*\lambda_m + \beta_{i,1}^*\lambda + \beta_{i,2}^*\lambda + \beta_{i,3}^*\lambda$, where λ_m is the market equity premium, and λ denotes the risk premium on each of three uncorrelated, alternative strategies. For each fund, the true alpha α_i^* is drawn from a normal $N(\mu_\alpha^*, \sigma_\alpha^{*2})$, and its alternative beta $\beta_{i,j}^*$ from a normal $N(\mu_{\beta_j}^*, \sigma_\beta^{*2})$, where $\mu_{\beta_j}^*$ $(j = 1, 2, 3)$ is positive based on the premise that hedge funds load on alternative factors to increase their returns. We further assume that the first factor is a more prevalent strategy for hedge funds by setting $\mu_{\beta_1}^* = \mu_\beta^*$, $\mu_{\beta_2}^* = \mu_\beta^*/3$, and $\mu_{\beta_3}^* = \mu_\beta^*/3$.

To begin, suppose that we use the CAPM to evaluate hedge fund performance. Given the above assumptions, the cross-sectional alpha distribution is given by a normal:

$$\alpha_i^{CAPM} \sim N(\mu_\alpha^* + \lambda(\mu_{\beta_1}^* + \mu_{\beta_2}^* + \mu_{\beta_3}^*), \sigma_\alpha^{*2} + 3\lambda^2\sigma_\beta^{*2}) \,, \tag{5}$$

where the two moments are equal to the cross-sectional average and variance of Equation (4). Because the CAPM does not separate the alternative factors from the true alpha, it overestimates both the mean and the dispersion of the alpha distribution. To illustrate, Panel A of Figure I plots the alpha distribution using the following annual parameter values: $\mu_\alpha^* = 0$, $\sigma_\alpha^* = 1.5\%$, $\lambda = 7.5\%$, $\mu_\beta^* = 0.3$, and $\sigma_\beta^* = 0.4$.[7] We find that the average alpha is as large as 3.75% per year, and more than 76% of the funds achieve positive alphas as they load on alternative strategies.

Next, suppose that we compare the CAPM with a hedge fund (HF) model that includes the

[7] We calibrate μ_α^* and σ_α^* using Barras, Gagliardini, and Scaillet (2022, Table VI). We set λ equal to the average market return and μ_β, σ_β equal to the mean and volatility of the market beta in our sample of hedge funds.

market, as well as factors 1 and 2. In this case, we have:

$$\alpha_i^{HF} \sim N(\mu_\alpha^* + \lambda\mu_{\beta_3}^*, \sigma_\alpha^{*2} + \lambda^2\sigma_\beta^{*2}) \,. \tag{6}$$

The comparison between the CAPM and the HF model sharpens the evaluation of performance. By capturing the impact of two alternative factors, the HF model reduces the magnitude of alphas that seem implausibly large under the CAPM—the average alpha drops from 3.75% to 0.75% per year. This comparison also reveals the key role played by factor 1 in explaining hedge fund returns. For each fund, we measure the contributions of factors 1 and 2 in explaining the difference in alpha between the two models as $rp_{i,1} = \beta_{i,1}^*\lambda$ and $rp_{i,2} = \beta_{i,2}^*\lambda$. Examining the entire contribution distributions in Panel B uncovers the stronger impact of factor 1 on the cross-section of funds.

[Insert Figure I about here.]

3. Estimation Methodology under Misspecified Models

3.1. Overview of the Estimation Approach

We now describe our approach for estimating the alpha distribution across multiple models. A key aspect of our analysis is that we allow each model to be misspecified. Formally, we consider a set of K models, where each model is indexed by k ($k = 1, \ldots, K$). Each of these models is misspecified as it includes the factors $f_{I,t}^k$ but omits the factors $f_{O,t}^k$ (with $f_t = (f_{I,t}^{k\prime}, f_{O,t}^{k\prime})'$).

We summarize the shape of the alpha distribution using its main characteristics: (i) the cross-sectional mean and standard deviation, denoted by M_1^k and M_2^k, (ii) the proportion of funds with alpha below a given value a, denoted by $P^k(a) = P[\alpha_i^k \leq a]$, and (iii) the quantile at a given percentile level u, denoted by $Q^k(u) = (P^k)^{-1}(u)$. We derive below the asymptotic distribution of each characteristic, which allows us to conduct proper statistical inference across models. In practice, we do not observe the fund alpha α_i^k, but only its estimated value $\hat{\alpha}_i^k$ (contrary to our simple example above in which we assume that α_i^k is known). Our main contribution is to show how the noise in estimated fund alphas affects the properties of the distribution characteristics.

To compute the estimated alphas for each model k, we proceed as follows. First, we run a time-series regression of the excess returns of each fund on the included factors (as per Equation (3)):

$$\hat{\alpha}_i^k = E_1'(\hat{Q}_{x,i}^k)^{-1}\frac{1}{T_i}\sum_t I_{i,t}x_t^k r_{i,t}\,, \tag{7}$$

where E_1 is a vector with one in the first position and zeros elsewhere, $I_{i,t}$ is an indicator variable equal to one if $r_{i,t}$ is observable, $T_i = \sum_t I_{i,t}$ (*i.e.*, the number of observations for fund i), $x_t^k = (1, f_{I,t}^{k\prime})'$, and $\hat{Q}_{x,i}^k = \frac{1}{T_i}\sum_t I_{i,t}x_t^k x_t^{k\prime}$.

Second, we account for the unbalanced nature of the hedge fund sample. To this end, we follow Gagliardini, Ossola, and Scaillet (2016) and introduce a formal selection rule $\mathbf{1}_i^\chi$ equal to one if the following conditions are met: $\mathbf{1}_i^\chi = \mathbf{1}\left\{CN_i \leq \chi_{1,T}, \tau_{i,T} \leq \chi_{2,T}\right\}$, where $CN_i = \sqrt{eig_{\max}\left(\hat{Q}_{x,i}^k\right)/eig_{\min}\left(\hat{Q}_{x,i}^k\right)}$ is the condition number of $\hat{Q}_{x,i}^k$, $\tau_{i,T} = T/T_i$, and $\chi_{1,T}$, $\chi_{2,T}$ denote the two threshold values. The first condition $CN_i \leq \chi_{1,T}$ excludes funds for which the time-series regression is subject to multicollinearity problems (*e.g.*, Belsley, Kuh, and Welsch, 2004). The second condition $\tau_{i,T} \leq \chi_{2,T}$ excludes funds for which the sample size is too small.[8] Applying this selection rule, we work with a vector of estimated alphas of size $n_\chi = \sum_{i=1}^n \mathbf{1}_i^\chi$.

3.2. Estimating the Alpha Distribution under a Misspecified Model

3.2.1. Inference on the Distribution Characteristics

To begin with, we derive the properties of the estimated alpha distribution under each model k. From the estimated fund alphas, we compute the distribution characteristics (mean, standard deviation, proportion, quantile) as $\hat{M}_1^k = \frac{1}{n_\chi}\sum_i \mathbf{1}_i^\chi \hat{\alpha}_i^k$, $\hat{M}_2^k = \left(\frac{1}{n_\chi}\sum_i \mathbf{1}_i^\chi (\hat{\alpha}_i^k)^2 - (\frac{1}{n_\chi}\sum_i \mathbf{1}_i^\chi \hat{\alpha}_i^k)^2\right)^{1/2}$, $\hat{P}^k(a) = \frac{1}{n_\chi}\sum_i \mathbf{1}_i^\chi \mathbf{1}\{\hat{\alpha}_i^k \leq a\}$, and $\hat{Q}^k(u) = (\hat{P}^k)^{-1}(u)$. The following proposition derives the asymptotic distributions of $\hat{M}_1^k$, $\hat{M}_2^k$, $\hat{P}^k(a)$, and $\hat{Q}^k(u)$ as the number of funds n and the number of observations T grow large. To capture the large cross-sectional dimension of the hedge fund population, we impose that n is larger than T.

Proposition 1. *As $n, T \to \infty$, such that $T/n = o(1)$, we have:*

$$\sqrt{T}\left(\hat{M}_s^k - M_s^k\right) \Rightarrow N(0, V[M_s^k])\,, \tag{8}$$

$$\sqrt{T}\left(\hat{P}^k(a) - P^k(a)\right) \Rightarrow N(0, V[P^k(a)])\,, \tag{9}$$

$$\sqrt{T}\left(\hat{Q}^k(u) - Q^k(u)\right) \Rightarrow N(0, V[Q^k(u)])\,, \tag{10}$$

[8] Both thresholds $\chi_{1,T}$ and $\chi_{2,T}$ increase with the sample size T—with more return observations, we estimate the fund coefficients with greater accuracy which allows for a less stringent selection rule.

where $s \in \{1, 2\}$. The variance terms are given by:

$$V[M_s^k] = \left(\eta_{M_s}^{k\prime} \otimes E_1'(Q_x^k)^{-1}\right) \Omega_{ux}^k \left(\eta_{M_s}^k \otimes (Q_x^k)^{-1} E_1\right), \tag{11}$$

$$V[P^k(a)] = \left(\eta_{P(a)}^{k\prime} \otimes E_1'(Q_x^k)^{-1}\right) \Omega_{ux}^k \left(\eta_{P(a)}^k \otimes (Q_x^k)^{-1} E_1\right), \tag{12}$$

$$V[Q^k(u)] = V[P^k(Q^k(u))]/\phi^k(Q^k(u))^2, \tag{13}$$

where $\eta_{M_s}^k = \left(\frac{\partial M_s^k}{\partial E[g^k]}\right)' E\left[\frac{\partial g^k}{\partial \alpha_i^k}\beta_{i,O}^{k*}\right]$, $E[g^k]$ is the vector of uncentered moments with $g^k = (\alpha_i^k, (\alpha_i^k)^2)'$, $Q_x^k = E[x_t^k x_t^{k\prime}]$, $\Omega_{ux}^k = \lim_{T\to\infty} V\left[\frac{1}{\sqrt{T}}\sum_t u_{O,t}^k \otimes x_t^k\right]$, $\eta_{P(a)}^k = E[\beta_{i,O}^{k*}|\alpha_i^k = a]\phi^k(a)$, $\beta_{i,O}^{k*}$ and $u_{O,t}^k$ respectively denote the vectors of betas and residuals associated with the omitted factors $f_{O,t}^k$, and $\phi^k(a)$ is the alpha density function evaluated at a.

***Proof**. See the appendix.*

3.2.2. Properties of the Distribution Characteristics

Proposition 1 reveals two important insights. First, the variance of the estimated characteristics is large because the convergence rate depends on T (and not on n). This result is a priori surprising because the characteristics are all computed as cross-sectional averages—we sum across funds, not over time. Second, the estimators converge towards their respective parameter values. In other words, we are able to estimate the alpha distribution under model k without bias, even though we use as inputs noisy versions of the fund alphas (*i.e.*, we use $\hat{\alpha}_i^k$ instead of α_i^k). Proposition 1 therefore provides a theoretical justification to the common practice of computing distribution summary statistics (*e.g.*, boxplots) based on estimated coefficients.

These properties depart significantly from those obtained if we were to use the correct model shown in Equation (2) (Barras, Gagliardini, and Scaillet, 2022). In this case, the characteristics are estimated with greater precision because the convergence rate is equal to $\sqrt{n}$ (instead of $\sqrt{T}$). They also require an adjustment for the error-in-variable (EIV) bias due to the noise contained in the estimated alphas—an adjustment that is not necessary when the model is misspecified.

These sharp differences stem from the properties of the fund residual terms. In the correctly-specified case (Equation (2)), the residuals $\varepsilon_{i,t}^*$ ($i = 1, \ldots, n$) are weakly cross-sectionally correlated because the common factors exhaust the cross-sectional dependence across funds. In the misspecified case (Equation (3)), the residuals $\varepsilon_{i,t}^k$ ($i = 1, \ldots, n$) are strongly cross-sectionally correlated because they include the omitted factors via the term $u_{O,t}^k$, *i.e.*, we have $\varepsilon_{i,t}^k = \varepsilon_{i,t}^* + \beta_{O,i}^{k*\prime} u_{O,t}^k$. Therefore, the estimation error on the fund alpha $\hat{\alpha}_i^k$ involves the time-series average

$\bar{\varepsilon}_i^k = \bar{\varepsilon}_i^* + \beta_{i,O}^{k*\prime}\bar{u}_O^k$, where $\bar{u}_O^k$ is of order $1/\sqrt{T}$ and does not vanish when we average across funds. In short, the common term $\bar{u}_O$ (i) produces a slow convergence rate of $\sqrt{T}$, and (ii) dwarfs the EIV bias in magnitude, which makes any bias adjustment unnecessary.[9]

3.3. Comparing Alpha Distributions across Misspecified Models

3.3.1. Inference on the Differences in Distribution Characteristics

Next, we explain how to compare the alpha distributions under two misspecified models. Among the set of K models under consideration, we select a pair of models k and l that can be nested or not.[10] For each pair, we compute the differences in characteristics as $\Delta\hat{M}_s = \hat{M}_s^k - \hat{M}_s^l$, $\Delta\hat{P}(a) = \hat{P}^k(a) - \hat{P}^l(a)$, and $\Delta\hat{Q}(u) = \hat{Q}^k(u) - \hat{Q}^l(u)$. The following proposition derives the asympotic distributions of $\Delta\hat{M}_s$, $\Delta\hat{P}(a)$, and $\Delta\hat{Q}(u)$ as the number of funds n and the number of observations T grow large.[11]

Proposition 2. *As $n, T \to \infty$ such that $T/n = o(1)$, we have:*

$$\sqrt{T}\left(\Delta\hat{M}_s - \Delta M_s\right) \Rightarrow N\left(0, V[\Delta M_s]\right), \tag{14}$$

$$\sqrt{T}\left(\Delta\hat{P}(a) - \Delta P(a)\right) \Rightarrow N\left(0, V[\Delta P(a)]\right), \tag{15}$$

$$\sqrt{T}\left(\Delta\hat{Q}(u) - \Delta Q(u)\right) \Rightarrow N\left(0, V[\Delta Q(u)]\right), \tag{16}$$

where $s \in \{1, 2\}$. The variance terms are given by:

$$V[\Delta M_s] = V[M_s^k] + V[M_s^l] - 2Cov[\hat{M}_s^k, \hat{M}_s^l], \tag{17}$$

$$V[\Delta P(a)] = V[P^k(a)] + V[P^l(a)] - 2Cov[\hat{P}^k(a), \hat{P}^l(a)], \tag{18}$$

$$V[\Delta Q(u)] = V[Q^k(u)] + V[Q^l(u)] - 2Cov[\hat{Q}^k(u), \hat{Q}^l(u)], \tag{19}$$

where the covariance terms are $Cov[\hat{M}_s^k, \hat{M}_s^l] = \left((\eta_{M_s}^{k\prime} \otimes E_1'(Q_x^k)^{-1}\right)\Omega_{ux}^{kl}\left(\eta_{M_s}^l \otimes (Q_x^l)^{-1}E_1\right)$, $Cov[\hat{P}^k(a), \hat{P}^l(a)] = \left(\eta_{P(a)}^{k\prime} \otimes E_1'(Q_x^k)^{-1}\right)\Omega_{ux}^{kl}\left(\eta_{P(a)}^l \otimes (Q_x^l)^{-1}E_1\right)$, *and* $Cov[\hat{Q}^k(u), \hat{Q}^l(u)] = \frac{Cov[\hat{P}^k(\hat{Q}^k(u)), \hat{P}^l(\hat{Q}^l(u))]}{\phi^k(\hat{Q}^k(u))\phi^l(\hat{Q}^l(u))}$, *and* $\Omega_{ux}^{kl} = \lim_{T\to\infty} Cov\left[\frac{1}{\sqrt{T}}\sum_t u_{O,t}^k \otimes x_t^k, \frac{1}{\sqrt{T}}\sum_t u_{O,t}^l \otimes x_t^l\right]$.

Proof. *See the appendix.*

[9] If we have weak factors in the observable factors only, the results of Proposition 1 hold. If we have weak factors in the omitted factors and at least one strong omitted factor, the results of Proposition 1 also hold. However, these results do not hold if we only have weak factors in the omitted factors. In this case, the convergence rate depends the rate of decay of $\beta_{i,O}^{k*}$ towards zero (which is unknown unless it is set a priori).

[10] Two models are nested if one model is included in the other (*i.e.*, it is obtained from the other by putting some of its factor loadings to zero).

[11] We assume in Proposition 2 that the two models k and l are misspecified. If one model is correctly specified, the convergence rate of the estimated moment equals $\sqrt{n}$ (see Barras, Gagliardini, and Scaillet, 2022), which is faster than the rate of $\sqrt{T}$ under the misspecified model. In this case, the asymptotic distributions of $\Delta\hat{M}_s$, $\Delta\hat{P}(a)$, and $\Delta\hat{Q}(u)$ are solely driven by the estimated characteristics under the misspecified model (*i.e.*, we can treat the estimated characteristic under the correctly-specified model as known).

3.3.2. Comparison Tests

Applying the results in Proposition 2, we can formally compare the alpha distributions under models k and l. To this end, we test the null hypothesis that a given characteristic (mean, standard deviation, proportion, quantile) is identical across the two models. For instance, the null hypothesis for the distribution quantile is $H_0 : \Delta Q(u) = Q^k(u) - Q^l(u) = 0$. The testing procedure is straightforward because all the characteristic differences are asymptotically normally distributed, which facilitates the computation of the rejection thresholds.

Importantly, the comparison tests inherit the high volatility of the estimated characteristics—as shown in Proposition 2, each characteristic difference converges at a slow rate equal to $\sqrt{T}$. Accounting for misspecification is crucial for valid comparison tests—in particular, it considerably raises the bar for finding statistically significant differences across models.

Our methodology departs from previous studies that develop asset pricing comparison tests for misspecified models (see, among others, Barillas and Shanken, 2017; Kan and Robotti, 2009; Kan, Robotti, and Shanken, 2013). First, we let n grow large to accommodate the large hedge fund population (double asymptotics with n and $T \to \infty$). In contrast, these studies consider a fixed cross-section of assets (simultaneous double asymptotics with n fixed and $T \to \infty$). Second, our comparison analysis examines the entire cross-sectional alpha distribution, instead of focusing on a single metric such as the Hansen-Jagannathan distance. Third, our procedure can be applied to any pair of models. This generality departs from previous comparison tests whose distributions typically depend on whether the models are nested or not.

3.4. Estimation of the Asymptotic Variance Terms

To apply the asymptotic results in Propositions 1 and 2, we need to estimate the asymptotic variances of the estimated characteristics and their differences. This estimation to get feasible inference procedures is not trivial because all variance terms depend on the omitted factor residuals $u^k_{O,t}$ and betas $\beta^{k*}_{i,O}$—quantities that are not observable. To address this issue, we derive a consistent variance estimator based on the observed fund residuals of each model $\hat{\varepsilon}^k_{i,t} = r_{i,t} - x'_t \hat{\gamma}^k_i$, where $\hat{\gamma}^k_i = (\hat{Q}^k_{x,i})^{-1} \frac{1}{T_i} \sum_t I_{i,t} x^k_t r_{i,t}$ is the OLS vector of coefficients of the time-series regression in Equation (3).[12]

[12]To simplify the exposition, we consider the case where $\varepsilon^k_{i,t}$ is independent over time. This assumption holds if the residual $\varepsilon^*_{i,t}$ and the omitted factors $f^k_{O,t}$ are independent over time. When this is not the case, we simply need to

We denote the generic estimated characteristic by $\hat{C}$ and its difference by $\Delta\hat{C}$, where $\hat{C} \in \{\hat{M}_s^k, \hat{P}^k(a), \hat{Q}^k(u)\}$ and $\Delta\hat{C} \in \{\Delta\hat{M}_s, \Delta\hat{P}(a), \Delta\hat{Q}(u)\}$. The estimators of the asymptotic variances of $\sqrt{T}(\hat{C} - C)$ and $\sqrt{T}(\Delta\hat{C} - \Delta C)$ are given by:

$$\hat{V}[\hat{C}] = \frac{1}{n^2 T} \sum_i \sum_j \sum_t \mathbf{1}_i^\chi \tau_{i,T} I_{i,t} \mathbf{1}_j^\chi \tau_{j,T} I_{j,t} \hat{a}_{i,t}(\hat{C}) \hat{a}_{j,t}(\hat{C})', \tag{20}$$

$$\hat{V}[\Delta\hat{C}] = \frac{1}{n^2 T} \sum_i \sum_j \sum_t \mathbf{1}_i^\chi \tau_{i,T} I_{i,t} \mathbf{1}_j^\chi \tau_{j,T} I_{j,t} \hat{a}_{i,t}^\Delta(\Delta\hat{C}) \hat{a}_{j,t}^\Delta(\Delta\hat{C})', \tag{21}$$

where the terms $\hat{a}_{i,t}(\hat{C})$ and $\hat{a}_{i,t}^\Delta(\Delta\hat{C})$ are functions of $\hat{C}$ and $\Delta\hat{C}$ (see the appendix). The following proposition shows that $\hat{V}[\hat{C}]$ and $\hat{V}[\Delta\hat{C}]$ are consistent variance estimators as the number of funds n and the number of observations T grow large.

Proposition 3. *As $n, T \to \infty$ such that $T/n = o(1)$, we have:*

$$\hat{V}[\hat{C}] \to V[\hat{C}], \tag{22}$$

$$\hat{V}[\Delta\hat{C}] \to V[\Delta\hat{C}], \tag{23}$$

where $\to$ denotes convergence in probability.

Proof. *See the appendix.*

4. Data Description

4.1. Hedge Fund Database

We evaluate hedge fund performance using monthly net-of-fee fund returns between January 1994 and December 2016 (276 observations). We construct our sample by combining five databases (BarclayHedge, Eurekahedge, HFR, Morningstar, and TASS). Using an aggregated data set mitigates the selection bias that arises from the voluntary nature of information disclosure by hedge funds. In particular, this aggregation offers an improved coverage of underperforming funds, which typically report to only one database. It also allows us to get n much larger than T as assumed in our asymptotic theory. The appendix provides more detail on the sample construction, which largely follows Joenväärä et al. (2021). To adjust for the backfill bias, we follow the standard approach of removing the first 12 months of data for each fund. In the appendix, we also use the more stringent approach of Joenväärä et al. (2021), which eliminates all the observations before

modify the variance estimator by including weighted cross-terms at different dates (Newey and West, 1987).

the fund listing date. Because the treatment of the backfill bias changes the fund alphas uniformly across all models, it has no impact on the model comparison presented below.

Table I reports summary statistics for the entire hedge fund dataset. In the entire population, the mean and volatility of the equally-weighted hedge fund portfolio are equal to 5.85% and 5.93% per year. We also consider three broad hedge fund categories: (i) equity funds, which rely on discretionary or quantitative analysis to detect mispriced stocks, (ii) macro funds, which invest in multiple asset classes and take directional bets using broad economic and financial indicators, and (iii) arbitrage funds, which exploit various sources of mispricing primarily in the debt market. Overall, the results are in line with those documented by Getmansky, Lee, and Lo (2015) over a similar time period (1996–2014).

To obtain reliable estimates of the alpha of each individual fund, we apply the selection rule of Section 3.1 and require that the condition number of $\hat{Q}_{x,i}^{k}$ is below 15 and the minimum number of return observations equals 36. Whereas the original sample includes all dead funds, this selection rule may introduce a survivorship bias if negative-alpha funds ($\alpha_i^* < 0$) disappear early. An offsetting effect is that it mitigates the reverse survivorship bias that arises when positive-alpha funds ($\alpha_i^* > 0$) disappear after unexpectedly low returns (Linnainmaa, 2013). In the appendix, we show that our main results are robust to changes in the selection rule.

[Insert Table I about here.]

4.2. Hedge Fund Factors

The hedge fund models examined here are formed from a total of 18 trading factors. We classify these factors into four groups. The first group (equity) includes the U.S. market, size, value, momentum, investment, profitability, illiquidity, and betting-against-beta factors. The second group (bond) contains the U.S. term and default factors. The third group (option) includes a set of option strategies that replicate short in the realized variance of the S&P 500 and in look-back straddles for bonds, commodities, and currencies.[13] The fourth group (global) contains strategies implemented across four asset classes (equities, bonds, commodities, and currencies). The global value and momentum factors are based on the traditional value and momentum strategies. The

[13]We define option strategies as short positions to guarantee that their premia λ are positive like the other factors. As shown, among others, by Bakshi and Madan (2000), long (delta-hedged) option positions deliver negative premia because they perform well in bad times when realized variance is high.

carry factor invests in assets with high carry, and the time-series (TS) momentum factor invests in assets with positive past returns. The appendix provides the data sources for all factors.

Table II reports the summary statistics for the excess return of each factor. Consistent with intuition, all but one factors (bond term) deliver positive average excess returns ($\lambda > 0$). We also find that the factor correlations are low—only 8 of the 153 pairwise correlations are above 0.5 (in absolute value). Therefore, the trading factors capture the returns of distinct hedge fund strategies.

[Insert Tables II about here.]

5. Empirical Results

5.1. Hedge Fund Performance under the Standard Models

5.1.1. The Cross-Sectional Alpha Distribution

We begin by evaluating hedge fund performance using five standard models. We consider the classic three-, four-, and five-factor models of Fama and French (1992, 2015) and Carhart (1997). We also examine the widely-used model of Fung and Hsieh (2004) (FH), which includes two equity factors (market, size), two bond factors (term and default), and three option straddles (bonds, commodities, currencies). Finally, we consider the model of Asness, Moskowitz, and Pedersen (2013), which adds to the CAPM two global value and momentum factors across asset classes. The set of standard models examined here is by no means exhaustive. However, it provides a good representation of the models commonly used in previous studies on fund performance evaluation (*e.g.*, Cremers, Petajisto, and Zitzewitz, 2013; Getmansky, Lee, and Lo, 2015).

For each standard model, we estimate the alpha distribution in the entire population of 8,665 individual funds. Applying the methodology presented in Section 3, we compute the annualized mean and standard deviation, the proportions of negative- and positive-alpha funds, and the quantiles at 10% and 90%. We then conduct a formal comparison of these characteristics with the simplest benchmark model—the CAPM. This analysis determines whether the factors in the standard models do a good job at capturing the alternative strategies followed by hedge funds.

Panel A of Table III reveals that the entire alpha distributions remain strikingly similar across the standard models. The values for the average annual alpha and the proportion of positive-alpha funds all cluster around 2.36% and 66.61%, respectively. Examining the standard deviation and quantiles, we also find that the dispersion and tail properties of the alpha distributions remain unchanged. The commonality across models is not an artifact of data aggregation—repeating the analysis for each investment category and database yields the same results (see the appendix).

Turning to the comparison with the CAPM, we find that the additional complexity of the standard models has no impact on performance evaluation. Out of the 35 characteristic differences reported in Panel B, only three are statistically significant at the 5% level. This result resonates with the recent findings of Joenväärä et al. (2021) and Getmansky, Lee, and Lo (2015) who document a strong similarity between the average alphas under the CAPM and the FH model. Here, we formally show that this similarity extends to the entire alpha distribution (not just the mean) and several models used in previous studies (not just the FH model).

Our comparison analysis highlights the importance of accounting for model misspecification. To elaborate, suppose that we mistakenly use the convergence rate of $\sqrt{n}$ that is only valid under correct specification (instead of $\sqrt{T}$). In this case, we find that all but two differences are statistically significant at the 1% level, which seems at odds with their small economic magnitude. Using proper statistical inference sets a higher bar for identifying differences across models and helps us to reconcile the statistical and economic significance of the results.

[Insert Table III about here.]

5.1.2. The Economic Importance of the Standard Factors

The strong performance in Table III suggests that the economic importance of the standard factors is small. To examine this issue, we measure for each standard model k ($k = 1, \ldots, 5$) the contribution of its factors in explaining the performance difference relative to the CAPM. Formally, we can use the same arguments as in Equation (4) to write the difference in fund alpha as

$$\alpha_i^{\text{CAPM}} - \alpha_i^k = \sum_j^{J_k} c_{i,j}^k = \sum_j^{J_k} \beta_{i,j}^k \alpha_{O,j}^{CAPM}\,, \tag{24}$$

where J_k is the number of factors included in model k (in addition to the market), and $c_{i,j}^k$ denotes the contribution associated with factor j ($j = 1, \ldots, J_k$), defined as the product between the fund beta on factor j and the CAPM alpha of factor j. We can then use the results of Proposition 1 to infer the cross-sectional distribution of contributions associated with factor j (see the appendix).

Table IV confirms that the contribution of the standard factors to the average fund returns is small.[14] To illustrate, consider the FH model (Panel D), whose factors have a contribution close to zero on average (-0.01% per year). In addition, the factor contributions remain modest even among the funds with the highest factor loadings—the 90%-quantile equals 0.81% per year, on average. In short, the standard models produce the same alpha distribution as the CAPM because the economic importance of their factors is limited.

[Insert Table IV about here.]

5.1.3. True Performance versus Model Misspecification

Consistent with previous studies, our analysis of the standard model implies that hedge fund performance is strong.[15] However, several arguments suggest that the results in Table III are driven by the misspecification of the standard models. To begin, the positive hedge fund performance contrasts with the negative performance among mutual funds (*e.g.*, Barras, Scaillet, and Wermers, 2010; Harvey and Liu, 2018). Whereas these two types of funds are likely to differ along several dimensions, their difference in average alpha—around 2-3% per year—seems large. This difference is also at odds with the famous arithmetic of Sharpe (1991), which states that the average (gross-of-fee) alpha among active funds must be equal to zero.[16]

The performance similarity with the CAPM also provides compelling evidence that the standard models are misspecified. To complement this finding, we compute two misspecification statistics. First, we measure the average R^2 of the time-series regression in Equation (3). A high R^2 makes misspecification less likely because it requires that the omitted factors have implausibly high premia (see Cochrane, 2005, ch. 9). We find that the average R^2 across the standard models is only equal to 23.44%, leaving plenty of room for omitted factors.[17] Second, we use the formal

[14]By construction, the sum of the average contributions across all the factors included in a given model is equal to the difference in average alpha relative to the CAPM (reported in Table III, Panel B).

[15]A notable exception is the recent paper by Joenväärä et al. (2021), who find an average alpha close to zero. As shown in the appendix, this result stems from their data selection procedure, which eliminates all the observations before the fund listing date—in some cases, more than five years (Fung and Hsieh, 2009). Whereas this procedure provides a more stringent control of backfill bias, it also discards potentially important information about the fund performance (Aggarwal and Jorion, 2010).

[16]As noted by Pedersen (2018), Sharpe's arithmetic may not hold perfectly because it requires that passive investors do not trade and hold the entire market. In practice, passive investors regularly trade for various reasons (*e.g.*, lifecycle investing, index rebalancing) and do not hold the entire market as they focus on specific indices.

[17]The presence of omitted factors is consistent with the analysis of Bollen (2013) who notes that many hedge funds have R^2 close to zero (based on the Fung-Hsieh model), while their residual volatility cannot be diversified away.

criterion of Gagliardini, Ossola, and Scaillet (2019, GOS), which takes positive values when the model is misspecified (see the appendix for details). Our computations reveal that this criterion is positive for all five standard models.

5.2. *A New Hedge Fund Model*

5.2.1. *Construction of the New Model*

In this section, we construct a new hedge fund model and examine its ability to evaluate hedge fund performance. Whereas our parsimonious factor selection does not completely eliminate misspecification, it offers several advantages. First, it favors economically-motivated factors that plausibly capture several mechanical strategies followed by hedge funds. Second, it allows for a simple interpretation of the factors, which departs from a purely statistical factor identification based on PCA analysis (*e.g.*, Fung and Hsieh, 2001; Billio et al., 2012; Giglio, Liao, and Xiu, 2021). Third, using a small number of factors from past studies mitigates the data-mining concerns that arise when evaluating a large number of randomly-formed models.

We form an extended version of the CAPM that includes five factors recently uncovered by the asset pricing literature. The first factor is the marketwide illiquidity of Pástor and Stambaugh (2003). This factor captures the illiquid nature of hedge fund investments (see Pedersen, 2015, ch. 3, 15, 16). For instance, equity funds commonly invest in small-cap stocks, fixed-income funds take illiquid positions in the bond market (*e.g.*, convertible debt), and event-driven funds regularly accommodate selling pressure in the market (for instance, after merger announcements).

The second factor is betting-against-beta (BAB) (Frazzini and Pedersen, 2014), which tracks the returns earned by hedge funds from exploiting their leverage capacity. The intuition behind BAB is that traditional investors (mutual and pension funds) face leverage constraints and thus favor high-beta assets to increase their portfolio returns. The BAB strategy exploits the induced price distortions by taking long positions in low-beta assets and short positions in high-beta assets.

The third factor—variance—takes negative values when the realized variance of the S&P 500 is high (*e.g.*, Bondarenko, 2004; Carr and Wu, 2009). As such, it captures the funds' exposure to variance risk stemming from their option positions. The variance factor depends on the correlation structure between stocks (Driessen, Maenhout, and Vilkov, 2009). Therefore, it also captures unexpected rises in correlation, which lower the effectiveness of hedging strategies and signal crisis times when funding constraints get tighter (Buraschi, Kosowski, and Trojani, 2014).

The fourth factor is global carry (Koijen et al., 2018). This strategy invests in assets with high carry—that is, cheap assets with a low forward price relative to the spot price. As discussed by Pedersen (2015, ch. 9, 11, 14), a large number of hedge funds follow strategies similar to carry, including global macro funds, long-short equity funds, and fixed-income funds.

The fifth factor is global time-series (TS) momentum, which invests in assets with past positive returns (Moskowitz, Ooi, and Pedersen, 2012).[18] This factor potentially captures the trend-following strategies followed by hedge funds as they exploit trends in asset prices caused by behavioral biases, frictions, or slow-moving capital (Pedersen, 2015, ch. 12).[19]

5.2.2. Revisiting Hedge Fund Performance

We now estimate the alpha distribution under the new model. We compute the distribution characteristics for the entire fund population, and test whether they differ from the ones obtained with the standard models. We also repeat the analysis for each investment category (equity, macro, arbitrage). The results in Table V (Panels A to D) reveal two main insights.

First, the new model largely reduces the overall performance of the hedge fund industry. On average, the alpha equals -0.07% per year and is positive for only 52.12% of the funds. These numbers stand in sharp contrast with those reported for the standard models. To illustrate, Figure II plots the alpha distributions for the new model and the FH model. The differences in average annual alpha and proportion of positive-alpha funds are equal to -2.75% and -16.27%, respectively. We observe the same performance reduction for all three investment categories—in particular, among equity and macro funds.

Second, the new model produces a large performance heterogeneity—the standard deviation is equal to 11.40% per year (versus 9.25% for the standard models). This result is a priori surprising because the new factors should capture some of the cross-sectional variation in average fund returns. We find that this strong heterogeneity is driven by the left tail. The worst funds perform particularly poorly as the 10% quantile equals -11.31% per year (versus -6.86% for the standard

[18] As discussed by Moskowitz, Ooi, and Pedersen (2012), TS momentum departs from the classic momentum strategy, which invests in assets with positive past returns relative to the cross-sectional average return.

[19] Huang et al. (2020) recently argue that TS momentum is similar to a strategy that simply invests in assets based on their full-sample average returns. Whereas this result challenges the interpretation of TS momentum as a trend-following strategy, this factor can still be relevant for capturing the allocation of hedge funds within asset classes.

models). This result suggests that poorly performing funds load aggressively on the new factors—a point that we examine in more detail below.

The differences relative to the standard models are highly significant. In total, there are 96 estimated differences for the four characteristics mentioned above (mean, standard deviation, proportion, 10%-quantile) and the four hedge fund groups (all, equity, macro, arbitrage). We find that all of them are significant at conventional levels. This result is remarkable given the high estimation noise stemming from model misspecification discussed in Section 3.2.

[Insert Table V and Figure II about here.]

5.2.3. Implications of the Performance Analysis

The results in Table V have several implications for hedge fund investors. The sharp reduction in average alpha makes the hedge fund industry less appealing as a broad asset class. Investors can instead favor financial products that mimick the alternative strategies followed by hedge funds (Carhart et al., 2014; Jorion, 2021). Importantly, hedge fund investors only invest in a handful of funds (*e.g.*, Bollen, Joenväärä, and Kauppila, 2021). Therefore, the vast performance heterogeneity implies that uninformed investors have potentially much to lose—half of the funds produce negative alphas, which can be lower than -10% per year. It provides a strong rationale for implementing a due diligence process that allows investors to avoid the worst-performing funds.

Our analysis is relevant for the debate on active management. It reveals that hedge fund performance is similar to that of the mutual fund industry—the values for the average annual alpha and the proportion of positive-alpha funds are equal to -0.4% and 37.1% among US mutual funds (Barras, Gagliardini, and Scaillet, 2022). To the extent that hedge funds create more value than mutual funds through their superior skills, they do not share it with their investors. Our analysis also raises the bar for any theory explaining why hedge funds deliver different alphas in equilibrium. Some funds can underperform if they exploit unsophisticated investors or incur marketing expenses to attract investors with high search costs (Gruber, 1996; Roussanov, Ruan, and Wei, 2021), whereas others may outperform if their investors hold bargaining power (Glode and Green, 2011). Any of these explanations must produce large effects to match the heterogeneity under the model.

5.2.4. *The Economic Importance of the New Factors*

The impact of the new model on performance ultimately depends on its ability to capture hedge fund returns. To examine this issue, we measure the contribution of each new factor j ($j = 1, \ldots, 5$) in reducing alphas relative to the CAPM. Similar to Equation (24), we estimate the factor contribution for each fund as $c_{i,j}^{new} = \beta_{i,j}^{new} \alpha_{O,j}^{CAPM}$. We then report in Table VI the characteristics of the entire distribution of factor contributions.

Panel A provides support to the commonly-held view that hedge funds follow alternative strategies to boost their returns (*e.g.*, Carhart et al., 2014). On average, close to 60% of them have positive factor loadings. As a result, the contribution of the new factors is economically significant—it equals 0.54% per year on average, and is above 3.41% among the decile of funds with the highest loadings. Carry, TS momentum, and variance are particularly important because their contributions are equal 1.13%, 0.81%, and 0.36% per year in the entire population. Put differently, these factors play an crucial role in capturing the average returns earned by hedge funds.

The analysis of investment categories reveals several patterns in line with economic intuition (Panels B to D). We find that carry is important across all categories as hedge funds routinely use it for their allocation across asset classes. Equity and arbitrage funds perform poorly when variance rises, which captures the risk associated with their option positions and hedging strategies. Arbitrage funds are also sensitive to BAB as they rely extensively on leverage to magnify small price discrepancies in the bond market. Finally, macro funds are heavily exposed to TS momentum. This is consistent with the analysis of Pedersen (2015, ch. 12) who shows that the alpha of CTA indices turns negative after controlling for TS momentum.

[Insert Table VI about here.]

5.2.5. *Analysis of the Worst and Best Performing Funds*

The results so far show how the new model changes the shape of the alpha distribution. We now complement these results by focusing on the worst and best performing funds—those located in the bottom and top quantiles (at 25%, 10%, and 5%). Our analysis in Table VII is threefold. Panel A examines whether these funds differ from those identified by the standard models. Panel B measures the returns they earn from loading on the new factors. Finally, Panel C examines whether these funds have specific characteristics.

Panel A shows that the new model identifies a unique group of outperforming funds. In the top decile, the overlap with the standard models is only equal to 61%. Using the new model is therefore important to identify funds that deliver superior performance beyond the replication of mechanical alternative strategies. Interestingly, this result is not readily available from the comparison of the alpha distributions. As shown in Table V, the 90%-quantiles are similar across all models and thus do not reveal the large composition change among the best funds.

A key takeaway from Panel B is the high factor exposures among the worst funds. In the bottom 5%, the contributions of variance, carry, and TS momentum to average fund returns reach 4.96%, 3.98%, and 4.75% per year. In other words, poorly performing funds rely on the new factors to boost their returns—possibly to hide their lack of skills. This result explains why performance heterogeneity is larger under the new model. The standard models are too lenient in their evaluation of the worst funds because the estimated alphas are contaminated by the new factors.

Finally, Panel C shows that the best funds differ from the worst funds along several dimensions. They are generally represented by a higher (lower) proportion of arbitrage (macro) funds. They have stonger managerial incentives, as captured by fees and high water-mark provisions. They also have more managerial flexibility (longer lockup and notice periods), which potentially allows them to invest in illiquid assets and exploit arbitrage opportunities that take time to be profitable. Overall, these results are consistent with previous studies (*e.g.*, Agarwal, Daniel, and Naik, 2009), which suggests that managerial characteristics are useful for detecting the best funds.

[Insert Table VII about here.]

5.3. Additional Results

5.3.1. The Impact of Factor Trading Costs

Our baseline analysis does not include the costs of trading the new factors. Therefore, the estimated alpha may underestimate the diversification services provided by the fund in replicating these factors (Berk and van Binsbergen, 2015). In other words, hedge funds may attract flows because they provide exposures to several alternative strategies (*e.g.*, Cochrane, 2013).

To address this issue, we evaluate performance using factor returns net of trading costs. We estimate monthly trading costs using various sources (discussed in the appendix).[20] The costs of

[20]An alternative approach to infer trading costs is to apply the two-pass regression on two separate cross-sections

trading illiquidity, carry, and TS momentum are modest because these strategies are rebalanced annually or implemented in futures markets. For illiquidity, we use a value of 5.5 bps equal to the cost estimate for size and value (Novy-Marx and Velikov, 2016). For carry and TS momentum, we set the costs equal to 9.7 bps, which equals the average costs of rolling futures positions as estimated by Bollerslev et al. (2018). In contrast, the costs associated with BAB and variance are significantly higher. For BAB, we take the value computed by Novy-Marx and Velikov (2022) equal to 60 bps. For variance, we use a value of 75 bps, which corresponds to the costs of trading variance swaps (Dew-Becker et al., 2017). The results of this analysis are shown in Table VIII.

Consistent with intuition, we find that accounting for trading costs produces higher fund alphas. However, the change is modest—in the entire population (Panel A), the average alpha under the new model is equal to 0.36% per year (versus -0.17% without trading costs). Our analysis of the differences in average alpha relative to the standard model is conservative because we do not include the trading costs of the standard factors. Even in this case, we find that the performance reduction achieved by the new model remains strong. Our main conclusions are therefore robust to adding the costs of trading the new factors.

[Insert Table VIII about here.]

5.3.2. Subperiod Analyis

We now summarize additional results reported in the appendix. To begin with, we examine whether our main results hold once we divide the entire sample into two subperiods of 138 monthly observations each (from January 1994 to June 2005 and from July 2005 to December 2016).

Overall, the comparison analysis across models remains largely unchanged. In both periods, the new model produces a reduction in performance that is both economically and statistically significant. We also observe a general downward trend in performance over time. Across the standard and the new models, the average alpha drops by more than 2% per year during the second subperiod. This finding is consistent with Bollen, Joenväärä, and Kauppila (2021), who relate this performance reduction to increased regulation and central bank stimulus activity.

of assets and funds as proposed by Patton and Weller (2020) for mutual funds. However, this approach is problematic for hedge funds because it requires to use a correctly specified model.

5.3.3. Performance of Multi-Strategy Funds and Funds of Funds

Next, we examine two additional hedge fund categories—multi-strategy funds and funds of funds. These two categories may present a bigger challenge for the new model because their investment strategies are more diverse than those followed by equity, macro, and arbitrage funds.

We still find that replacing the standard models with the new model strongly reduces performance. For multi-strategy funds, the estimated differences are equal to -2.77% per year for the average alpha and 18.90% for the proportion of positive-alpha funds. Among funds of funds, the average alpha drops to -3.48% per year, and only 24.75% of the funds deliver positive alpha. Whereas it is well known that the performance of funds of funds is hampered by their additional fees (*e.g.*, Agarwal, Mullaly, and Naik, 2015), our results reveal that the degree of underperformance is worse than previously documented.

5.3.4. Performance under the Model of Joenväärä et al. (2021)

Finally, we measure performance using the model of Joenväärä et al. (2021, JKKT). In their detailed review of hedge fund databases, JKKT also suggest to add new factors for evaluating performance. Their model includes four standard factors (market, size, value, and momentum) and three new factors (illiquidity, BAB, and TS momentum).

Similar to the new model, the JKKT model leads to a lower evaluation of hedge fund performance evaluation. This result makes sense because the JKKT model and the new model have three new factors in common, including TS momentum for which the contribution to average returns is large . At the same time, the JKKT model does not include variance and carry. As discussed in Section 5.2, these two factors rest on solid economic intuition and are empirically important in explaining hedge fund returns. Consistent with this analysis, we find that the performance reduction under the JKKT model (average alpha, proportion) is always smaller than the one obtained with the new model. Furthermore, it is not statistically significant among arbitrage funds.

6. Conclusion

Measuring the performance of hedge funds is challenging because of the large number of strategies they follow. As a result, any model used for evaluating performance is likely to be misspecified as it omits relevant factors that drive hedge fund returns. In this context, comparing models is essential to identify the economic importance of alternative trading factors and sharpen the evaluation of performance. In this paper, we develop a novel approach to conduct such performance comparisons. A key feature of our approach is to explicitly account for misspecification. Our testing procedure relies on a theoretical asymptotic analysis and allows us to conduct proper statistical inference in a large set of funds. Another distinguishing feature of our approach is to compare the entire alpha distributions of fund alphas. Extending the analysis beyond the average alpha captures the large performance heterogeneity across funds.

Our empirical analysis yields several insights. We find that the standard models used in previous work produce the same alpha distribution as the CAPM. This similarity arises because the standard factors struggle to track the alternative strategies followed by hedge funds to boost their returns. As a result, the standard models all find that hedge fund performance is strong. Building on the recent asset pricing literature, we then construct a new model, which includes five factors that plausibly capture hedge fund strategies—illiquidity, BAB, variance, carry, and TS momentum. The formal comparison test reveal that the new model achieves a sizeable reduction in hedge fund performance relative to the standard models. These results have implications for the investors' allocation decisions, and for the debate on the performance gap between mutual and hedge funds.

Our comparison approach is quite general and can be applied in future research on performance evaluation. In particular, it can be used to improve the ability of the new model to capture hedge fund returns. One possibility is to consider style-specific factors. Whereas equity and arbitrage funds primarily invest in stocks and bonds, macro funds favor commodities and currencies. We could therefore use versions of carry and TS momentum tailored to each asset class and measure their economic relevance within investment categories.

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TABLE I. Summary Statistics for the Hedge Fund Dataset

This table provides summary statistics for all funds in the population, as well as equity, macro, and arbitrage funds. It reports the mean (annualized), standard deviation (annualized), skewness, kurtosis, and quantiles at 10% and 90% of the excess return of an equal-weighted portfolio of all existing funds at the start of each month. The statistics are computed using monthly data between January 1994 and December 2016.

	Moments				Quantiles	
	Mean(Ann.)	Std(Ann.)	Skewness	Kurtosis	10%	90%
All Funds	5.85	5.93	-0.27	4.05	-1.63	2.50
Equity	7.02	8.76	-0.44	4.90	-2.42	3.47
Long-Short	7.39	9.58	-0.42	4.90	-2.64	3.71
Market Neutral	4.09	3.33	-0.42	5.40	-0.67	1.54
Macro	4.77	6.37	0.49	3.45	-1.74	2.63
Global Macro	5.02	5.61	0.27	3.23	-1.63	2.43
CTA/Managed Futures	4.72	7.17	0.57	3.60	-1.97	3.03
Arbitrage	5.84	5.19	-1.96	13.55	-1.11	1.95
Relative Value	5.23	4.71	-2.30	17.90	-0.95	1.66
Event-Driven	6.73	6.21	-1.49	8.96	-1.42	2.42

TABLE II. Summary Statistics for the Trading Factors

This table provides summary statistics for all trading factors in the equity, bond, option, and global groups. It reports the mean (annualized), standard deviation (annualized), skewness, kurtosis, and quantiles at 10% and 90% of the excess return of each factor. The statistics are computed using monthly data between January 1994 and December 2016.

	Moments				Quantiles	
Panel A: Equity Strategies	Mean(Ann.)	Std(Ann.)	Skewness	Kurtosis	10%	90%
Market	7.56	15.13	-0.71	4.17	-5.11	6.00
Size	2.19	10.97	0.46	7.85	-3.56	3.65
Value	2.91	10.70	0.14	5.57	-2.95	3.62
Momentum	4.99	17.63	-1.49	13.26	-5.14	5.39
Investment	3.35	7.37	0.64	5.45	-1.85	2.98
Profitability	4.15	9.70	-0.43	12.10	-2.04	3.21
Iliquidity	6.31	12.43	-0.13	4.11	-3.84	4.98
Betting Against Beta	8.58	13.52	-0.52	5.59	-3.56	4.73
Panel B: Bond Strategies	Mean(Ann.)	Std(Ann.)	Skewness	Kurtosis	10%	90%
Term	-0.14	0.78	-0.18	4.68	-0.27	0.27
Default	0.02	0.64	1.22	19.18	-0.15	0.17
Panel C: Option Strategies	Mean(Ann.)	Std(Ann.)	Skewness	Kurtosis	10%	90%
Variance	333.61	267.92	-5.45	40.67	-15.90	74.31
Bond Straddle	20.09	52.77	-1.31	5.24	-18.99	17.73
Commodity Straddle	6.56	49.48	-1.06	4.53	-20.02	15.98
Currency Straddle	10.24	67.47	-1.36	5.51	-23.31	20.30
Panel D: Global Strategies	Mean(Ann.)	Std(Ann.)	Skewness	Kurtosis	10%	90%
Value (Global)	2.33	6.22	-0.70	13.40	-1.54	1.79
Momentum (Global)	3.81	7.82	-0.32	5.33	-2.31	2.74
Carry (Global)	7.61	4.88	-0.00	3.83	-1.01	2.34
TS Momentum (Global)	13.64	12.70	0.09	3.12	-3.54	5.80

TABLE III. Hedge fund Performance under the Standard Models
Panel A reports the main characteristics of the cross-sectional distribution of alphas under the five standard models (Fama-French three- and five-factor models, Carhart model, Fung-Hsieh model, and Asness-Moskowitz-Pedersen model (AMP)). Reported are the mean (annualized), standard deviation (annualized), the proportions (%) of funds with negative and positive alphas, and the quantiles (annualized) at 10% and 90%. Panel B reports the differences in characteristics between the CAPM and the five standard models. Figures in parentheses denote the estimated standard deviation of the estimated characteristics and their differences. ***, **, * indicate that the null hypothesis of equal characteristics is rejected at the 1%, 5%, and 10% levels.

	Panel A: Characteristics of the Distribution					
	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
Three-Factor	2.25 (0.72)	9.13 (0.49)	33.63 (3.78)	66.37 (3.78)	-7.03 (1.11)	10.72 (1.11)
Five-Factor	2.47 (0.76)	9.40 (0.46)	33.21 (3.90)	66.79 (3.90)	-6.90 (1.19)	11.16 (1.19)
Carhart	2.17 (0.71)	9.00 (0.43)	34.08 (3.74)	65.92 (3.74)	-6.75 (1.02)	10.46 (1.02)
Fung-Hsieh	2.68 (0.68)	9.39 (0.44)	31.61 (3.42)	68.39 (3.42)	-6.64 (1.31)	11.18 (0.66)
AMP	2.25 (0.76)	9.30 (0.45)	34.41 (4.14)	65.59 (4.14)	-6.98 (1.26)	10.87 (0.63)
Average	2.36	9.25	33.39	66.61	-6.86	10.88

	Panel B: Comparison with the CAPM					
	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
CAPM	2.62 (0.77)	9.19 (0.56)	31.66 (3.87)	68.34 (3.87)	-6.44 (1.37)	11.08 (1.37)
Difference						
vs Three-Factor	0.38 (0.26)	0.05 (0.22)	-1.97 (1.21)	1.97 (1.21)	0.59^{***} (0.00)	0.36^{*} (0.19)
vs Five-Factor	0.16 (0.36)	-0.21 (0.28)	-1.56 (1.80)	1.56 (1.80)	0.45^{*} (0.24)	-0.08 (0.48)
vs Carhart	0.46 (0.30)	0.19 (0.30)	-2.42^{*} (1.34)	2.42^{*} (1.34)	0.31 (0.29)	0.62^{**} (0.29)
vs Fung-Hsieh	-0.06 (0.39)	-0.20 (0.37)	0.05 (1.95)	-0.05 (1.95)	0.20 (0.22)	-0.10 (0.22)
vs AMP	0.37 (0.32)	-0.11 (0.35)	-2.76^{*} (1.66)	2.76^{*} (1.66)	0.53^{***} (0.20)	0.22 (0.20)
Average	0.26	-0.05	-1.73	1.73	0.42	0.21

TABLE IV. Contributions of the Standard Factors in Reducing CAPM Alphas

Panel A reports the main characteristics of the cross-sectional distribution of contributions for each factor included in the Fama-French three-factor model. We define the contribution of each factor as its contribution in explaining the difference in fund alphas between the CAPM and the three-factor model. Reported are the mean (annualized), standard deviation (annualized), the proportions (%) of funds with negative and positive contributions, and the quantiles (annualized) at 10% and 90%. Panel B to E repeat the analysis for the four remaining standard models (Fama-French five-factor model, Carhart model, Fung-Hsieh model, and Assess-Moskowitz-Pedersen model). Figures in parentheses denote the estimated standard deviation of the estimated characteristics.

Panel A: Thee-Factor Model						
	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
Value	0.19	1.89	41.02	58.98	-0.83	1.49
Size	0.19	1.55	40.68	59.32	-0.65	1.21
Average	0.19	1.72	40.85	59.15	-0.74	1.35
Panel B: Five-Factor Model						
	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
Value	0.15	2.10	44.64	55.36	-1.03	1.51
Size	0.20	1.61	39.35	60.65	-0.67	1.28
Investment	0.06	2.44	51.60	48.40	-1.23	1.30
Profitability	-0.25	2.53	54.44	45.56	-2.24	1.70
Average	0.04	2.17	47.51	52.49	-1.29	1.45
Panel C: Carhart Model						
	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
Value	0.24	1.93	40.65	59.35	-0.76	1.55
Size	0.16	1.58	41.42	58.58	-0.66	1.19
Momentum	0.06	1.81	51.33	48.67	-1.21	1.32
Average	0.15	1.77	44.47	55.53	-0.88	1.35
Panel D: Fung-Hsieh Model						
	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
Size	0.09	1.67	45.61	54.39	-0.78	1.08
Term	0.17	1.65	42.85	57.15	-0.88	1.36
Default	-0.28	1.53	63.49	36.51	-1.29	0.59
Bond Straddle	-0.04	1.71	47.48	52.52	-1.17	1.02
Currency Straddle	-0.02	0.80	49.32	50.68	-0.44	0.38
Commodity Straddle	0.02	0.92	49.51	50.49	-0.37	0.41
Average	-0.01	1.38	49.71	50.29	-0.82	0.81
Panel E: Asness-Moskowitz-Pedersen Model						
	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
Value (Global)	-0.10	2.11	50.17	49.83	-1.47	1.35
Momentum (Global)	0.47	2.82	40.75	59.25	-1.34	2.66
Average	0.19	2.47	45.46	54.54	-1.41	2.00

TABLE V. Hedge fund Performance under the New Model

Panel A reports the main characteristics of the cross-sectional distribution of alphas under the new model. The new model adds to the CAPM the returns of five alternative strategies (illiquidity, BAB, variance, carry, and TS momentum). Reported are the mean (annualized), standard deviation (annualized), the proportions (%) of funds with negative and positive alphas, and the quantiles (annualized) at 10% and 90%. Panel A also reports the differences in characteristics between the new model and the five standard models (Fama-French three- and five-factor models, Carhart model, Fung-Hsieh model, and Asness-Moskowitz-Pedersen model (AMP)). Panel B to D repeat the analysis for equity, macro, and arbitrage funds. Figures in parentheses denote the estimated standard deviation of the estimated characteristics and their differences. ***, **, * indicate that the null hypothesis of equal characteristics is rejected at the 1%, 5%, and 10% levels.

Panel A: All Funds						
	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
New Model	-0.17 (0.72)	10.97 (0.45)	48.36 (3.30)	51.64 (3.30)	-11.01 (1.00)	9.71 (1.00)
Difference						
vs Three-Factor	-2.42*** (0.53)	1.84*** (0.48)	14.73*** (2.74)	-14.73*** (2.74)	-3.98*** (0.92)	-1.01 (0.62)
vs Five-Factor	-2.64*** (0.58)	1.57*** (0.44)	15.14*** (2.96)	-15.14*** (2.96)	-4.11*** (0.93)	-1.45*** (0.31)
vs Carhart	-2.34*** (0.53)	1.98*** (0.42)	14.28*** (2.74)	-14.28*** (2.74)	-4.26*** (0.89)	-0.75** (0.30)
vs Fung-Hsieh	-2.86*** (0.59)	1.59*** (0.48)	16.75*** (2.92)	-16.75*** (2.92)	-4.37*** (0.77)	-1.47*** (0.39)
vs AMP	-2.43*** (0.55)	1.68*** (0.46)	13.94*** (2.95)	-13.94*** (2.95)	-4.04*** (1.04)	-1.16* (0.69)
Average	-2.54	1.73	14.97	-14.97	-4.15	-1.17

Panel B: Equity Funds						
	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
New Model	-0.48 (0.89)	9.94 (0.52)	50.75 (4.15)	49.25 (4.15)	-10.64 (1.78)	9.16 (1.07)
Difference						
vs Three-Factor	-2.16*** (0.61)	1.82*** (0.55)	14.75*** (3.20)	-14.75*** (3.20)	-3.66*** (1.13)	-0.46 (0.84)
vs Five-Factor	-2.67*** (0.69)	1.60*** (0.54)	16.20*** (3.51)	-16.20*** (3.51)	-4.30*** (1.00)	-1.44* (0.75)
vs Carhart	-2.10*** (0.60)	2.06*** (0.45)	14.33*** (3.11)	-14.33*** (3.11)	-3.98*** (0.86)	-0.12 (0.65)
vs Fung-Hsieh	-2.62*** (0.67)	1.77*** (0.50)	17.09*** (3.44)	-17.09*** (3.44)	-3.94*** (1.14)	-1.01* (0.57)
vs AMP	-2.42*** (0.65)	1.59*** (0.58)	14.51*** (3.35)	-14.51*** (3.35)	-3.60*** (1.11)	-1.02* (0.56)
Average	-2.39	1.77	15.38	-15.38	-3.89	-0.81

Panel C: Macro Funds						
	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
New Model	-1.03 (1.05)	13.81 (0.80)	54.59 (4.04)	45.41 (4.04)	-14.39 (1.97)	11.32 (1.47)
Difference						
vs Three-Factor	-3.93*** (0.90)	2.34*** (0.86)	20.38*** (3.68)	-20.38*** (3.68)	-6.08*** (1.54)	-2.07* (1.23)
vs Five-Factor	-3.57*** (0.93)	1.93** (0.79)	18.10*** (4.15)	-18.10*** (4.15)	-5.53*** (1.50)	-1.92 (1.20)
vs Carhart	-3.58*** (0.91)	2.43*** (0.79)	18.47*** (3.82)	-18.47*** (3.82)	-6.23*** (1.15)	-1.74** (0.86)
vs Fung-Hsieh	-4.35*** (0.94)	1.64* (0.91)	22.34*** (3.93)	-22.34*** (3.93)	-6.55*** (1.54)	-2.53** (1.16)
vs AMP	-3.38*** (0.82)	2.00** (0.79)	17.61*** (3.94)	-17.61*** (3.94)	-6.05*** (1.04)	-1.82* (1.04)
Average	-3.76	2.07	19.38	-19.38	-6.09	-2.02

Panel D: Arbitrage Funds						
	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
New Model	1.18 (0.71)	8.96 (0.67)	38.26 (3.92)	61.74 (3.92)	-7.92 (1.63)	9.11 (0.65)
Difference						
vs Three-Factor	-1.30** (0.55)	1.14** (0.50)	8.94*** (3.22)	-8.94*** (3.22)	-1.68** (0.73)	-0.83 (0.54)
vs Five-Factor	-1.65*** (0.58)	0.95** (0.45)	10.48*** (3.23)	-10.48*** (3.23)	-2.49*** (0.75)	-1.19*** (0.37)
vs Carhart	-1.46*** (0.55)	1.19** (0.47)	9.94*** (3.23)	-9.94*** (3.23)	-2.43*** (0.75)	-0.73* (0.37)
vs Fung-Hsieh	-1.71*** (0.61)	1.23** (0.58)	10.52*** (3.54)	-10.52*** (3.54)	-2.58*** (0.87)	-0.93 (0.65)
vs AMP	-1.47** (0.62)	1.32** (0.57)	9.32*** (3.48)	-9.32*** (3.48)	-2.40*** (0.81)	-0.96 (0.60)
Average	-1.52	1.17	9.84	-9.84	-2.31	-0.93

TABLE VI. Contributions of the New Factors in Reducing CAPM Alphas

Panel A reports the main characteristics of the cross-sectional distribution of contributions for each factor included in the new model (illiquidity, betting againts beta, variance, carry, and TS momentum). We define the contribution of each factor as its contribution in explaining the difference in fund alphas between the CAPM and the new model. Reported are the mean (annualized), standard deviation (annualized), the proportions (%) of funds with negative and positive contributions, and the quantiles (annualized) at 10% and 90%. Panel B to D repeat the analysis for equity, macro, and arbitrage funds. Figures in parentheses denote the estimated standard deviation of the estimated characteristics.

	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
Panel A: All Funds						
Variance	0.49	4.31	42.07	57.93	-2.38	3.45
Iliquidity	0.16	1.62	42.72	57.28	-0.79	1.26
Betting Against Beta	0.18	3.95	44.13	55.87	-2.59	2.91
Carry (Global)	0.84	3.65	38.65	61.35	-2.07	4.56
TS Momentum (Global)	1.13	4.83	41.20	58.80	-2.18	5.23
Average	0.56	3.67	41.75	58.25	-2.00	3.48
Panel B: Equity Funds						
Variance	0.76	4.23	39.90	60.10	-2.13	3.80
Iliquidity	0.24	1.64	40.19	59.81	-0.84	1.57
Betting Against Beta	0.14	4.02	44.27	55.73	-2.92	3.31
Carry (Global)	0.89	3.80	41.09	58.91	-2.29	5.07
TS Momentum (Global)	0.51	3.60	40.45	59.55	-2.23	3.56
Average	0.51	3.46	41.18	58.82	-2.08	3.46
Panel C: Macro Funds						
Variance	-0.26	5.35	56.50	43.50	-3.76	2.75
Iliquidity	0.10	1.94	45.94	54.06	-0.87	1.24
Betting Against Beta	0.14	4.86	47.04	52.96	-2.77	3.08
Carry (Global)	0.86	3.84	37.59	62.41	-2.50	4.81
TS Momentum (Global)	3.51	7.03	27.27	72.73	-1.61	12.36
Average	0.87	4.60	42.87	57.13	-2.30	4.85
Panel D: Arbitrage Funds						
Variance	0.83	2.93	30.81	69.19	-1.21	3.38
Iliquidity	0.10	1.14	43.44	56.56	-0.61	0.85
Betting Against Beta	0.28	2.56	40.95	59.05	-1.77	2.31
Carry (Global)	0.73	3.21	35.90	64.10	-1.31	3.52
TS Momentum (Global)	-0.33	2.18	56.52	43.48	-2.50	1.57
Average	0.32	2.41	41.52	58.48	-1.48	2.32

TABLE VII. Analysis of the Worst and Best Funds under the New Model
This table examines the properties of the worst and best performing funds identified by the new model, *i.e.*, those located in the bottom and top quantiles at 25%, 10%, and 5%. Panel A reports the proportion of funds that are included in a given quantile for both the new model and each standard model (Fama-French three- and five-factor models, Carhart model, Fung-Hsieh model, and Assess-Moskowitz-Pedersen model ((AMP))). Panel B shows, for each quantile, the average contribution of each new factor (illiquidity, betting against beta, variance, carry, and TS momentum) in explaining the difference in fund alphas between the CAPM and the new model. Panel C reports, for each quantile, the investment style composition (equity, macro, arbitrage), three measures of managerial incentives (management and performance fees, proportion of funds with high-mark provisions), and two measures of managerial discretion (lockup and notice periods).

Panel A: Proportion of Overlapping Funds

	Percentile						
	5th	10th	25th	75th	90th	95th	All
Three-Factor	47.81	53.35	65.10	65.42	60.51	57.97	100.00
Five-Factor	45.27	53.12	64.50	65.37	60.74	57.97	100.00
Carhart	52.66	58.20	67.54	68.65	65.36	60.51	100.00
Fung-Hsieh	39.26	50.23	61.87	62.00	56.81	52.42	100.00
AMP	49.65	53.93	65.37	66.85	62.36	57.04	100.00

Panel B: Average Factor Contribution

	Percentile						
	5th	10th	25th	75th	90th	95th	All
Variance	4.49	3.19	1.91	-0.72	-1.42	-2.19	0.49
Illiquidity	0.69	0.50	0.38	-0.03	-0.12	-0.28	0.16
Betting Against Beta	2.69	1.58	0.91	-0.54	-1.27	-1.85	0.18
Carry (Global)	4.45	3.76	2.67	-0.50	-0.91	-1.49	0.84
TS Momentum (Global)	3.90	3.15	2.35	0.31	0.10	-0.06	1.13

Panel C: Average Fund Characteristic

	Percentile						
	5th	10th	25th	75th	90th	95th	All
Investment Style							
Arbitrage (%)	15.94	16.74	18.19	30.56	24.48	17.78	26.48
Equity (%)	39.49	41.80	45.75	41.74	40.53	41.57	41.36
Macro (%)	44.57	41.45	36.06	27.70	34.99	40.65	32.16
Management Incentives							
Management Fees (% per year)	2.99	2.73	2.50	2.71	2.90	3.01	2.69
Performance Fees (% per year)	18.31	17.57	17.09	19.20	19.74	20.08	17.78
Managerial Discretion							
High Water Mark (%)	64.32	65.45	67.26	78.32	75.85	73.24	69.66
Lockup (months)	60.19	52.31	50.53	128.49	161.23	206.37	74.57
Notice (months)	19.78	20.70	20.89	35.28	30.56	29.89	26.16

TABLE VIII. Impact of the Costs of Trading the New Factors

Panel A reports the main characteristics of the cross-sectional distribution of alphas under the new model after accounting for the costs of trading the five alternative strategies (illiquidity, betting against beta, variance, carry and TS momentum). Reported are the mean (annualized), standard deviation (annualized), the proportions (%) of funds with negative and positive alphas, and the quantiles (annualized) at 10% and 90%. Panel A also reports the differences in characteristics between the new model and the five standard models (Fama-French three- and five-factor models, Carhart model, Fung-Hsieh model, and Asness-Moskowitz-Pedersen model (AMP)). Panel B to D repeat the analysis for equity, macro, and arbitrage funds. Figures in parentheses denote the estimated standard deviation of the estimated characteristics and their differences. ***, **, * indicate that the null hypothesis of equal characteristics is rejected at the 1%, 5%, and 10% levels.

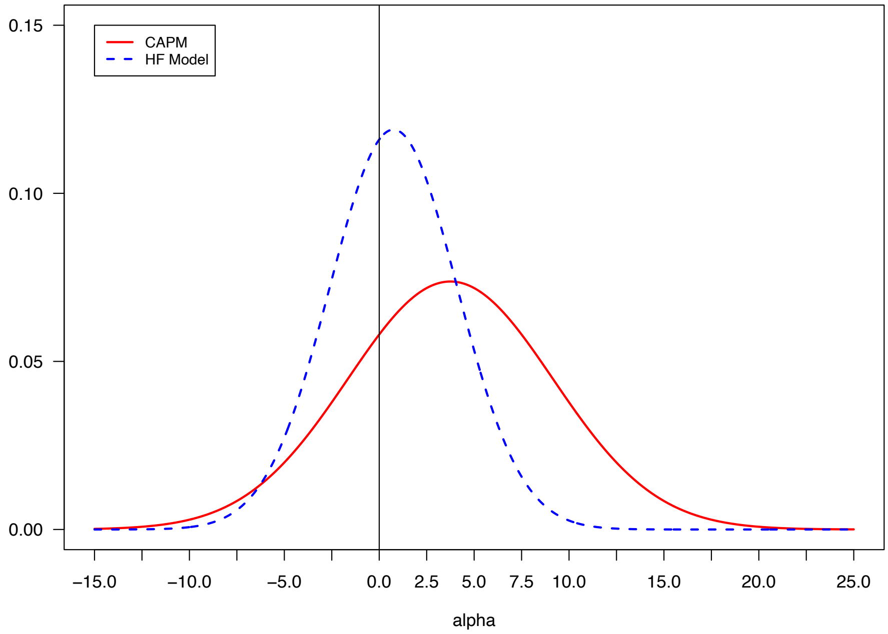
Panel A: All Funds	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
New Model	0.36 (0.74)	10.55 (0.43)	45.49 (3.46)	54.51 (3.46)	-10.05 (1.61)	9.83 (1.08)
Difference						
vs Three-Factor	-1.89*** (0.52)	1.41*** (0.44)	11.86*** (2.69)	-11.86*** (2.69)	-3.01*** (0.87)	-0.89 (0.58)
vs Five-Factor	-2.11*** (0.56)	1.15*** (0.41)	12.28*** (2.93)	-12.28*** (2.93)	-3.15*** (0.80)	-1.34** (0.53)
vs Carhart	-1.81*** (0.53)	1.55*** (0.38)	11.41*** (2.72)	-11.41*** (2.72)	-3.29*** (0.81)	-0.63** (0.27)
vs Fung-Hsieh	-2.32*** (0.57)	1.16*** (0.44)	13.88*** (2.83)	-13.88*** (2.83)	-3.41*** (0.96)	-1.35** (0.64)
vs AMP	-1.89*** (0.51)	1.25*** (0.42)	11.08*** (2.79)	-11.08*** (2.79)	-3.07*** (0.94)	-1.04* (0.63)
Average	-2.00	1.30	12.10	-12.10	-3.19	-1.05

Panel B: Equity Funds	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
New Model	0.01 (0.90)	9.54 (0.55)	47.17 (4.36)	52.83 (4.36)	-9.95 (1.76)	9.26 (0.71)
Difference						
vs Three-Factor	-1.66*** (0.60)	1.42*** (0.55)	11.17*** (3.04)	-11.17*** (3.04)	-2.97*** (1.01)	-0.36 (0.75)
vs Five-Factor	-2.18*** (0.66)	1.20** (0.54)	12.62*** (3.39)	-12.62*** (3.39)	-3.61*** (1.16)	-1.33*** (0.46)
vs Carhart	-1.60*** (0.59)	1.66*** (0.47)	10.75*** (3.07)	-10.75*** (3.07)	-3.29*** (0.70)	-0.02 (0.47)
vs Fung-Hsieh	-2.13*** (0.65)	1.37*** (0.49)	13.51*** (3.24)	-13.51*** (3.24)	-3.25*** (1.04)	-0.91* (0.52)
vs AMP	-1.93*** (0.62)	1.19* (0.60)	10.93*** (3.09)	-10.93*** (3.09)	-2.91*** (1.07)	-0.92* (0.54)
Average	-1.90	1.37	11.79	-11.79	-3.21	-0.71

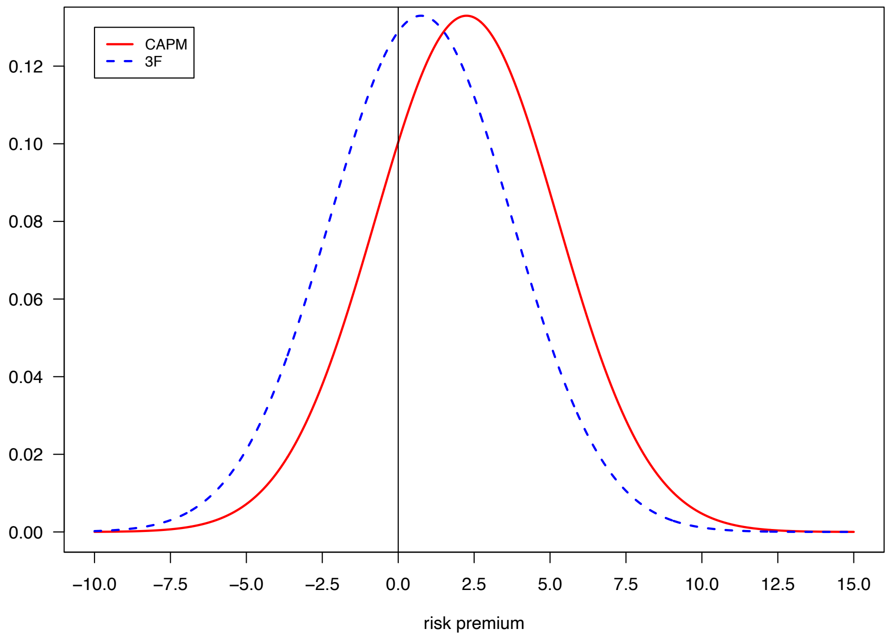
Panel C: Macro Funds	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
New Model	-0.42 (1.06)	13.24 (0.82)	52.51 (4.33)	47.49 (4.33)	-12.96 (1.61)	11.32 (1.08)
Difference						
vs Three-Factor	-3.32*** (0.90)	1.77** (0.80)	18.30*** (3.82)	-18.30*** (3.82)	-4.65*** (1.33)	-2.08 (1.33)
vs Five-Factor	-2.96*** (0.93)	1.36* (0.73)	16.02*** (4.26)	-16.02*** (4.26)	-4.11*** (1.22)	-1.93** (0.97)
vs Carhart	-2.97*** (0.92)	1.86** (0.73)	16.39*** (3.94)	-16.39*** (3.94)	-4.80*** (1.05)	-1.74** (0.79)
vs Fung-Hsieh	-3.74*** (0.94)	1.07 (0.87)	20.26*** (4.05)	-20.26*** (4.05)	-5.12*** (1.27)	-2.54*** (0.95)
vs AMP	-2.77*** (0.79)	1.43** (0.70)	15.53*** (3.92)	-15.53*** (3.92)	-4.62*** (0.93)	-1.83** (0.93)
Average	-3.15	1.49	17.30	-17.30	-4.66	-2.02

Panel D: Arbitrage Funds	Moments		Proportions		Quantiles	
	Mean(Ann.)	Std(Ann.)	Negative	Positive	10%(Ann.)	90%(Ann.)
New Model	1.70 (0.68)	8.68 (0.58)	35.73 (3.76)	64.27 (3.76)	-7.05 (1.43)	9.35 (0.57)
Difference						
vs Three-Factor	-0.78 (0.51)	0.86** (0.42)	6.42** (3.00)	-6.42** (3.00)	-0.81 (0.69)	-0.59* (0.35)
vs Five-Factor	-1.13** (0.54)	0.67 (0.44)	7.95*** (3.04)	-7.95*** (3.04)	-1.61** (0.71)	-0.95*** (0.36)
vs Carhart	-0.94* (0.52)	0.92** (0.41)	7.41** (3.04)	-7.41** (3.04)	-1.55*** (0.54)	-0.49 (0.36)
vs Fung-Hsieh	-1.19** (0.57)	0.96* (0.50)	7.99** (3.35)	-7.99** (3.35)	-1.71** (0.73)	-0.69* (0.37)
vs AMP	-0.95 (0.58)	1.05** (0.49)	6.79** (3.29)	-6.79** (3.29)	-1.52* (0.80)	-0.72* (0.40)
Average	-1.00	0.89	7.31	-7.31	-1.44	-0.69

Figure I. Comparing Performance across Misspecified Models – A Simple Example
We consider two misspecified models (CAPM and hedge fund (HF) model) to evaluate the performance of a hypothetical population of hedge funds. Whereas the CAPM omits three relevant factors, the HF model only omits one factor (*i.e.*, it includes factors 1 and 2). The top figure plots the cross-sectional alpha distributions (annualized) under the two models. The bottom figure plots the cross-sectional contribution distributions for the two factors 1 and 2, which are only included in the HF model. For each fund, the factor contributions measure the importance of factors 1 and 2 in explaining the difference in fund alpha between the CAPM and the HF model.



(a) A cross-sectional distribution of alphas



(b) Cross-sectional distributions of factor contributions

Figure II. Cross-sectional Distribution of Alphas
This figure plots the alpha distributions (annualized) for all funds in the population under the new model and Fung-Hsieh model. The new model adds to the CAPM the excess returns of five alternative strategies (illiquidity, betting against beta, variance, carry, and TS momentum).