How is Credit Risk priced in the German Market for Structured Products?

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Abstract

Structured retail products are unsecured bonds and are subject to the bankruptcy risk of the issuer. We analyze the price-setting policy of issuers with respect to this credit risk. Using a long-term data set of discount certificates in the German market, we find that (i) quoted prices do depend on issuer credit risk, but (ii) this dependency is under-proportional. Hence, retail investors are only partly compensated for bearing issuer credit risk. A deeper analysis reveals that the pricing of credit risk is mainly driven by systematic factors, while issuer-specific credit risk plays only a minor role.

1 Introduction

The market for structured retail products (SRP) has become a common investment for private investors. Structured retail products are combinations of stocks and derivatives, creating packages of specific payoff profiles attractive to certain types of retail investors. For obtaining such desired payoff profiles, they are willing to pay a margin on top of the fair price. Additional to risks regarding the contractual payoff, the investor faces the risk of issuer bankruptcy before the product matures, because SRPs are unsecured products. The default of Lehman Brothers for example led to losses of about 80% of the investment amount.

Since issuers act as a market maker and price setter for their own products, quoted prices do not directly reflect supply and demand, but the pricing policy of the issuer. Because of lacking valuation skills of retail investors and the opaqueness of the market, they may incorporate a margin on top the fair value, which is not transparent in terms of size and composition to investors. In particular issuers may refrain from incorporating their own default risk into their prices.

A few studies analyze the pricing of default risk for SRPs. Baule et al. (2008) estimate the quantitative relation between default risk and margins to find the former to be a significant portion of the latter. It is not clear however whether prices actually react to credit spread changes. Arnold et al. (2021) analyzes this question with data from the Swiss primary market around the Lehman default, with SRPs traded over-the-counter directly between issuer and investor. They find that prices (respectively, margins) do show a reaction to credit spread changes only after the Lehman default, when attention to default of private investors was high.

This paper adds to this topic by quantifying if and how much of their credit spread is priced by issuers into their products on the German secondary market by analyzing a large data set of discount certificates. We use price data on a daily basis throughout a nearly ten year period, so that the focus can be put on short-termed pricing reactions to changes in credit risk. Our approach together with our large data set also provides a way to directly estimate the extent to which credit risk is priced by quantifying the proportion of credit risk which is actually reflected in the prices. This direct measure can be compared across issuers to judge the appropriateness of and variability in between their default risk pricing. Using a structural model for the pricing of credit risk in SRPs that incorporates the correlation between market risk and credit risk (Baule et al., 2008), we show that default risk is priced with percentages ranging from around 10% to 50% of the adequate amount across our sample issuers. Our results refine and extend the findings of Arnold et al. (2021), who have shown that default risk pricing exists in the

Swiss primary market (which is quite different from the German secondary market). As a further refinement, we distinguish between systematic credit risk and issuer-specific credit risk. We find that the major portion of credit risk pricing can be explained by systematic factors, while not all issuers additionally incorporate their specific risk. This finding is in line with the argumentation of Arnold et al. (2021), in the sense that systematic credit risk changes, as observed in a crisis, draw more investor attention than idiosyncratic risk.

Our results add to the literature regarding margin and price setting behavior for SRPs, especially Wilkens et al. (2003), Stoimenov and Wilkens (2005), Baule (2011), who investigate the margin setting over the product lifetime. They also relate to literature regarding information asymmetry between issuers and private investors. Henderson and Pearson (2011) for example state that issuers exploit the uninformedness of private investors by hiding certain information or adding complexity. Since the fair price should adequately consideration default risk, issuer effectively take higher margins in crisis periods when their quotes reflect credit risk under-proportionally. Entrop et al. (2016) argue that investor choices are more sensitive to irrational factors. Our findings suggest however that the investor behavior at least leads to an partial default risk pricing. This is in line with surveys done by the German derivatives association stating that default risk is an important feature for investors to select an issuer.

The paper is structured as follows: Section 2 will present the market for SRPs and discount certificates as well as the models used to evaluate them. Section 3 introduces margins as well as our hypothesis for issuer pricing and how they relate to margins. Section 4 explains our empirical approach as well as the data used for evaluation. Section 5 presents our results and robustness checks. Section 6 concludes.

2 SRP market in Germany and pricing of discount certificates

The market for SRPs has grown to about 74 billion Euros as of June 2020[1] making up for about 1% of household financial wealth in Germany. SRPs are unsecured products and hence investors face default risk in addition to traditional risks. They are traded in the 'primary market' over the counter from issuers to investors, typically at issuance, and are afterwards traded on an exchange referred to as the 'secondary market'. Here

[1]These numbers are taken from the monthly statistics released by the German derivative association

issuers are required to quote bid and ask spreads to ensure that investors are able to trade. Besides issuers themselves the market mostly consists of non-professional private investors, who often lack the ability to accurately price the products and whose trading decision often rely on irrational factors (see Entrop et al. (2016)). At the Stuttgart exchange discount certificate are the most traded type and make up for $45,86\%$ of the volume for all executed orders regarding investment product and about 6% of the market volume for all retail products as of 2020[2]. Discount certificates are a combination of an underlying together with a short position in a respective vanilla call option. Due to the short option the discount certificate can be viewed in two ways: It can either be interpreted as an investment in the underlying, which can be bought cheaper than usually in return of limiting the possible returns or be viewed as a zero bond with the issuer having the option of returning the underlying instead[3]. By arbitrage arguments the discount certificate can therefore be priced by an equivalent portfolio of an underlying and a short call or a risk free investment with a shorted put option. It is possible to derive margins by using values for the portfolio constituents from exchange traded underlying and option prices. This direct approach is often not viable, since there may not be options to perfectly match the discount certificates inherent parameters. Additionally the mid-day quotes of our data set do not align in time with prices of options, who are only available end-of-day to us. Instead we follow the literature and calculate fair values by applying a pricing formula with parameters derived from exchange traded assets and options.

For our first analysis we require a default free model to quantify the amount of credit spread that is priced. We use what is sometimes referred to as the 'practitioners Black Scholes model'. It takes the classical formulas from Black and Scholes (1973), but instead of constant volatility and risk free yield it uses a yield curve spanned by maturity and a volatility surface spanned by maturity and moneyness respectively. In analogy of Black Scholes we use the definition of moneyness as $\log \frac{Underlying}{Strike}$. The yield curve and especially the volatility surface allow for calculation close to exchange prices. To find a fair value for discount certificates, the pay off profile of discount certificates is replicated with a long position in the underlying and a short position in a vanilla call option with matching strike, whose value is calculated analytically. Additionally one has to adjust for the difference between actual maturity date t_{mat}, where the pay off is due, and a prior reference date t_{ref} at which the pay off is determined. This can be done with an additional discount factor appropriate to the time difference between those points. Since pay off occurs only a few days after the reference date, this discount is not of

[2] These numbers are taken from the yearly report of 2020 from the EUWAX exchange in Stuttgart

[3] This interpretation comes from replicating the discount certificate with a bond and a short put option.

large magnitude. Issuers often match the reference date exactly to an exchange traded call option with the same strike and underlying to allow for perfect hedges. Hence by inserting the Black-Scholes-formula for call options the price for a discount certificate at time t is

$$\begin{aligned} DC_t^{BS} &= e^{-r(t_{mat}-t_{ref})}(S_t - c_t) \\ &= e^{-r(t_{mat}-t_{ref})}\big(S_t - [S_t \cdot N(d_1) - K \cdot e^{-r(t_{ref}-t)} \cdot N(d_2)]\big) \\ &= e^{-r(t_{mat}-t_{ref})}\big(S_t \cdot N(-d_1) + K \cdot e^{-r(t_{ref}-t)} \cdot N(d_2)\big) \end{aligned}$$

Within this equation, S_t and c_t are the underlying and call price at time t respectively, $r = r(t, t_{ref})$ is the risk free yield and $N(\cdot)$ is the normal cumulative distribution function. With the strike K and underlying volatility $\sigma = \sigma(t, t_{ref}, \log \frac{S_t}{K})$ from the volatility surface the numbers d_1 and d_2 can be calculated according to

$$d_1 = \frac{\log \frac{S_t}{K} + (r + \frac{\sigma^2}{2}) \cdot (t_{ref} - t)}{\sigma \cdot \sqrt{t_{ref} - t}}$$

and

$$d_2 = d_1 - \sigma \cdot \sqrt{t_{ref} - t} \quad .$$

To quantify the appropriateness of default risk pricing we apply a structural model of Baule et al. (2008) in our second analysis. The model can be viewed as a combination of the Black Scholes model and the Merton model of Merton (1974). In this model, the asset process of the issuer is modelled via a Brownian motion with default triggering if this process falls below a certain barrier, which relates to the amount of debts the issuer faces. In the Klein model, the asset process and the underlying process are simply modelled as correlated Brownian motion instead of independent ones. This correlation captures the fact that default risk (via the asset process) and underlying are correlated and should not be priced as if they where independent. As a consequence, the Klein model includes the famous Hull-White model as a special case for no correlation. A problem regarding this model however is the difficulty of parameter estimation. Especially asset level, volatility, default barrier and recovery rate are not easily measurable or available on a regular timely basis. Additionally the calculation of multivariate normal cumulative distributions makes it computationally expensive for our large data set. We therefore use a approximation of Baule (2021) that addresses both of these issues. The formula uses a downside delta in analogy to a traditional delta approach given by

$$\tilde{\Delta} = \frac{f(S_t) - f(S^-)}{S_t - S^-} \tag{1}$$

where S_t is the current underlying value, $S^- := E[S_T|V_T = D]$ is the expected underlying value at time T if the asset process V_t reaches the default barrier D, and f is another, default free pricing formula depending on the current underlying value. In our case we use the Black Scholes approach from above. The expectancy under the assumptions of the Klein model takes on the form

$$S^- = S_t \cdot \exp\left((r - \rho^2\sigma^2)T - \rho\sigma\sqrt{T}N^{-1}(e^{-c \cdot T})\right) \tag{2}$$

with all parameters as in our Black Scholes approach and ρ and c being the underlying-asset-correlation and the credit spread respectively. The pay off profile is than approximated by a $\tilde{\Delta}$ amount in a vulnerable share in the underlying and the rest invested into vulnerable zero bond, so that the effective credit spread for the asset is a weighted average of the zero bond credit spread and the effective credit spread of the vulnerable share:

$$c_{eff} = \frac{\tilde{\Delta}S_t \cdot c_{eff,S} + (f(S_t) - \tilde{\Delta}S_t) \cdot c}{f(S_t)} \tag{3}$$

where $c_{eff,S}$ can be calculated under the Klein model assumptions via

$$c_{eff,S} = -\frac{1}{T} \ln N(N^{-1}(e^{-cT}) + \rho\sigma\sqrt{T}) \tag{4}$$

The effective credit spread is than capped below by 0 to avoid negative spreads and used to determine the price p of the certificate via

$$DC_t^{SM} = e^{-c_{eff} \cdot T} \cdot f(S_t) = e^{-c_{eff} \cdot T} \cdot DC_t^{BS} \tag{5}$$

The formulas show that only univariate normal distributions are used and the only additional parameters in excess of Black Scholes are credit spread and correlation. For discount certificates the structural model values and their approximations are close (see figure 2 in Baule (2021) for example). Furthermore the effective credit spread is usually somewhere between 0 and the credit spread c. The details of the approximation and structural model can be found in Baule (2021) and Baule et al. (2008) respectively.

3 Margins and Hypothesis

Margins measure the excess pricing above some model fair value. Therefore the choice of fair pricing has an influence on both the absolute height of margins as well as on their behaviour with regard to certain variables. In our case a way to explicitly define a margin for a discount certificate is via

$$m_{BS} = \log \frac{DC^{market}}{DC^{BS}} \tag{6}$$

or

$$m_{SM} = \log \frac{DC^{market}}{DC^{SM}} \tag{7}$$

depending on whether we consider Black Scholes or structural model prices as our fair value. In literature a 'percentage margin' like $\frac{DC^{market}}{DC^{BS}} - 1$ is chosen, but for our purpose regarding default risk, the above definition has theoretical benefits. Additionally 'log margins', like ours, and 'percentage margins' are close to each other for small values, which is usually the case for discount certificates.

Our two models reflect two ways in which issuers might price their default risk. The Black Scholes closely matches exchange values and hence can be considered a default free way of pricing. The structural model on the other hand does incorporate credit risk and even accounts for correlation effects and hence may be considered an 'appropriate way' to price default risk. As a consequence we can formulate the following two hypothesis:

H_0: Issuers determine a fair value via default free values and do not adjust their margins in response to default risk.

or

H_0^*: Issuers determine a fair value by correctly pricing their default risk and do not adjust their margins in response to default risk.

The additional restriction on any margins adjustment to credit spread is necessary. In an extreme case for example an issuer might price according to the structural model, but adjusts his margin setting in such a way to directly counter any compensation the model includes for default risk. In essence this would be equivalent to pricing in a default free way. The margin may however still depend on other characteristics, especially regarding the certificate, like maturity, moneyness or competition.

The above hypothesis directly translates into mathematical terms. In fact the requirement, that margins may not be correlated with default risk can be directly tested. Because we of log-margins and the way a credit spread is taken into the calculations via an exponential, the margin m_{BS} is linearly affected by the effective credit spread c_{eff} the issuer takes for his pricing. On the other hand m_{SM} is linearly affected by any excess credit spread above or below what is used by the structural model. This excess spread will be zero at all times if issuers price according to the model. One can write this as

$$m_{BS} = \log \frac{DC^{market}}{DC^{BS}} \approx m - c_{eff}T \tag{8}$$

where m is the margin the issuer adds on top of his fair value. The equivalent version of equation 8 for m_{SM} can be written by simply replacing c_{eff} with the excess credit spread c_{excess} with respect to the structural model. The linear relationship can be directly tested with a regression approach of m_{BS} or m_{SM} on the credit spread multiplied with time to maturity. In fact a non significance in the factor loadings would indicate that the issuer prices according to the respective model.

However some complications may arise when we would use equation 8 as the basis for our regression, because the value of m depends on the certificates characteristics. Since a discount certificate can be replicated by an underlying with a shorted call option, the margin is rationally bounded be the relative call option price to the underlying price. Otherwise an investor would simply invest into the underlying instead. Since relative call option prices increase with remaining time, it is only natural to lower the impact of maturity by analysing annualized versions of the above log-margins instead. The first empirical observation regarding this dependency are formulated in the 'life cycle hypothesis'. It roughly states that margins are big at the beginning of a products' life and tend to zero at maturity. It was first reported by Wilkens et al. (2003) and Stoimenov and Wilkens (2005) for the German market for SRPs and later linked to order flow by Baule (2011). It has since then been taken as a stylized fact by numerous studies. It can be expressed mathematical by assuming that m depends linearly on time to maturity T and has vanishing (or negligible) constant[4], so that

$$m \approx m^a \cdot T \tag{9}$$

with m^a being the annualized margin.

Applying this to equation 8 we arrive at

$$m_{BS} \approx [m^a - c_{eff}] \cdot T \tag{10}$$

$$\iff \quad m^a - c_{eff} \approx T^{-1} \cdot \log \frac{DC^{market}}{DC^{BS}} = \frac{m_{BS}}{T} =: m^a_{BS} \tag{11}$$

[4]It would be more appropriate to assume the Taylor expansion with respect to T has vanishing constant. One can then omit higher order terms to arrive at the same result.

The right hand side of 11 is the empirical annualized margin. Similarly for m_{SM} we get

$$m^a - c_{excess} \approx T^{-1} \cdot \log \frac{DC^{market}}{DC^{SM}} = \frac{m_{SM}}{T} =: m^a_{SM} \tag{12}$$

Falling back to annualized margins makes average values of margins more comparable over time, since otherwise they would natural decay under the life cycle hypothesis and then suddenly jump up whenever a new batch of certificates is issued. Furthermore annualized margins now linearly depend on the credit spreads instead of its product with time to maturity.
The annualized versions above will be at the center of our analysis. The expectation under our hypothesis H_0 or H_0^* are

- Under H_0 the values m^a_{BS} should be uncorrelated with the credit spread.
- Under H_0^* the values m^a_{SM} should be uncorrelated with the credit spread.

We suppressed one important detail in the above expectation regarding our hypothesis: Even if issuers would price according to the same model (either Black Scholes or structural), they would also need to use the exact same parameters as we did. One can show that m_{BS} also depends on the relative greeks $\frac{\frac{\delta DC_{BS}}{\delta x}}{DC_{BS}}$ with respect to the parameter vector x used for the Black Scholes model. In first order terms this effect can be written with the difference in parameters dx as

$$\log \frac{DC_{\text{market}}}{DC_{\text{BS}}} \approx m + \frac{\frac{\delta DC_{BS}}{\delta x}}{DC_{BS}} \cdot dx \tag{13}$$

where product of the greeks and dx is a scalar product. This fact is a natural source of error for our empirical analysis. To influence our estimation however this parameter difference dx would have to be systematically correlated with c_{eff}, which we do not consider likely. Additionally, some of these errors might be dealt with by the way we control in our regressions.

4 Empirical Approach

4.1 General Idea

In light of equation (11) and (12) we will utilize a two step approach: We will first estimate annualized margin (either m^a_{BS} or m^a_{SM}) at specific times in a cross sectional regression to obtain a time series. This will be done for both Black Scholes and structural modelling. In the second step we investigate whether credit spread has any explanatory value for these time series. Under H_0 or H_0^* this should not be the case for the respective model.

While the details in Section 3 hold for any specific certificate, we will estimate the annualized margins on a aggregate level. On a certificate level individual errors may be to large for the second step to not be influenced heavily be noise. It might be tempting to simply calculate annualized margins for each certificate individually and then taking some kind of average. This approach however has a certain number of problems: The annualized margins' dependency with regard to moneyness or competition means that the average is affected by the cross sectional composition of certificate characteristic. To find a sequence of comparable values, we will instead rely on the estimation of annualized margins via a cross sectional regression approach. This helps to control for the composition and has the additional advantage of not enlarging individual errors if margins are divided by small times to maturity.

The formula we will use for our cross sectional regressions is the following:

$$m_{model} = \log \frac{DC_{\text{market,i}}}{DC_{\text{model,i}}} \sim \widehat{m^a}_{model,t} \cdot T_i + \delta_t \cdot \text{Control}_i + \epsilon_i \quad , \tag{14}$$

where i is an index that runs over each certificate in the cross section of certificates at time t, Control_i is a vector of control variables and T_i is the time to maturity. 'model' can either be BS (Black Scholes) or SM (structural model) to estimate either m^a_{BS} or m^a_{SM}. We will repeat this for different times t and separately for each issuer. The control variables used for this regression to address the above problematic will be specified in a later section.

The resulting time series $\widehat{m^a}_{model,t}$ is then taken to be our estimate of the average value of m^a_{BS} or m^a_{SM} depending on time t and on which model was chosen. To test whether these these two time series are uncorrelated to credit spreads, we perform in our second step a time series regression of the respective sequence on the credit spread via

$$\widehat{m^a}_{model,t} \sim \alpha^{model} + \beta^{model}_{CS} \cdot \text{c}_t + \beta^{model}_{Control} \cdot \text{Control}_t + \epsilon_t \tag{15}$$

where c_t is the credit spread at time t, the vector Control_t consists of control variables and 'model' again can be either BS or SM.

Under H_0 the values for β^{BS}_{CS} should be non significant. Since excess credit spread linearly affects annualized margins, it is also directly interpretable as the actually percentage of priced credit spread. Therefore it is expected that this values lies somewhere between 0 (no pricing) and -1 (maximal pricing under Hull White).

Under H_0^* the factor β^{SM}_{CS} should show non significance. Otherwise the value quantifies the excess credit spread (or lack thereof) in percentage of the actual credit spread relative to what is applied by the structural model.

4.2 Data Basis

For our approach we use discount certificates because they are among the most traded SRPs in Germany and are fairly simple to price analytically. This means model error can be kept at a minimum. Additionally they feature smaller margins compared to more exotic products, so that a reaction to credit spread becomes more visible in comparison to background noise. Our data set consists of trade data from the secondary market for discount certificates on the DAX at the EUWAX stock exchange in Stuttgart. Each data point is obtained by taking the available information from the Stuttgart exchange (EUWAX), which is done once a day for every discount certificate that is currently registered there. Since we get daily quotes and discount certificates average at about one year to maturity at issuance, the data set holds about 25 million data points. The information taken includes the ISIN, cap, date of maturity, emission and reference date, as well as the last traded price together with an exact time stamp. In case no trade happened before the data was taken from the exchange, the price falls back onto the last bid price, which necessarily exists, since issuers have to quote one continuously. From 2008 to 2014 most of the time stamps are in the early morning or afternoon, in which case the price often coincides with the bid quote from the issuer. Afterwards the time stamp often falls into the late evening. Since the bid ask spread is generally between 1 and 2 cents for discount certificates on the DAX, the trade prices and bid quotes often coincide. The data is provided by the Vereinigte Wirtschaftsdienste GmbH.

There are two reasons we analyse the secondary market. Firstly, primary market data consisting of the first sell to costumers is hard to obtain in large quantities. Secondly, even if one would take primary data they would not necessarily cover a large portion of the time line, since emissions are typically done in batches. Data with frequent and consistent time steps is favourable to conduct a time series regression and analyse reactions to default risk.

For our analysis we select the 11 issuers with biggest data subset accumulating to about 100.000 certificates ranging from late 2008 to 2018. The smaller issuers are excluded because of gaps in the time line with no or a low number of data points. We additionally filter the data basis by omitting every discount certificate with time to maturity greater than 2 years. For long maturities call options are typically sparse and might include liquidity premiums, so price calculated from these option volatilities might include larger errors as well. Additionally the emission of longer maturity certificates is not consistent across issuer, so that these products are also omitted for comparability reasons. As a last filter, we only select those certificates that have margins above -5% and below 5%. This

excludes only a small fraction of certificates but allows to filter specific products with hidden features, that are not shown in our information. The total margin for discount certificates is found by Entrop et al. (2016) to be on average 0.58% per year before the financial crisis and about 1.24% during 2008 evaluated against the model of Hull and White (1995).
Table 1 shows the issuers in our sample as well as their number of data points and some descriptive statistics of remaining time and moneyness.

[Insert table 1 here]

4.3 Parameters for Black Scholes pricing

To calculate analytical prices, we need values for S_t and estimates for r and σ for each data point in our sample.
As the underlying, we use intraday DAX data matched on the exact time stamp of the data point. For discount certificates after 2014 these stamps often lie outside of the DAX trading window. Instead we use future prices on the DAX provided to us by the KKMDB[5]. This data consists of future prices for all intraday trades at the EUREX on Futures with the closest maturity $t_{f,mat}$ together with a time stamp t and the number of contracts traded. This means that maturity of the future approaches zero and then jumps to the next available maturity typically 3 months ahead. From this data we calculate a contract weighted average of future prices for each second between 2008 and 2018 where a trade has taken place. For seconds with no trade the last calculated value is kept constant. This future price average is then discounted with the last available 'implied risk free yield' calculated by comparing the last values of the DAX S_{close} with the matching future price average F_{close} via the formula

$$r = -\frac{1}{t_{f,mat} - t} \log \frac{F_{\text{close}}}{S_{\text{close}}} \quad . \tag{16}$$

For times outside of the DAX trading window this is roughly[6] equivalent to projecting the Future price movement onto the closing value of the DAX.
For the risk free rate $r(t, t_{ref})$ we use the Svensson-method from Svensson (1994) and calculate one yield curve for each trading day. All parameters are provided by the Deutsche Bundesbank [7] on a daily basis. The details can be found in Schich (1997). Since the

[5] Karlsruher Kapitalmarktdatenbank

[6] up to minor differences because of different discounting time.

[7] Time Series BBSIS.D.I.ZST.X.EUR.S1311.B.A604._Z.R.A.A._Z._Z.A where X can be either B0,B1,B2,B3,T1,T2

Bundesbank applies the approach to discrete yields, they need to be transformed into continuous yields, although the difference is small in most cases. We apply this yield curve to find a risk free rate matching the time to maturity for every data point in our sample.

For volatility we follow Baule (2011) and use end-of-day option prices for vanilla call and put options on the DAX traded at the Xetra exchange to calculate an implied volatility surface. The implied volatility makes use of the Black Scholes framework. Market imperfections however may make observed call prices unobtainable under Black Scholes if they are lower than their intrinsic values. An approach that works for these cases and also includes the information in put options is as follows:

We calculate an implied underlying value with the call and put prices together with the risk free yield from Svensson by solving the call-put-parity. For this implied underlying value both the put and call price are achievable under Black Scholes and the implied volatility is identical for both. Since certificates do not usually match the exact Maturity and strike of options available at the exchange, we need to do an interpolation. For any given certificate we first search the volatility surface for the next and previous available maturities. Since options are usually available for maturities on the third Friday for specific months this approach finds a number of options at these maturities. Afterwards we search these tranches for the strikes directly below and above the discount certificates cap. The four resulting volatility surface values are then interpolated linearly, first with respect to strike within their tranch and then with respect to their maturity to match both parameters of the discount certificate. Since we require discount certificate data to have remaining time under 2 years, this approach nearly always finds these four volatility values and hence an appropriate interpolated value. If it does not we omit the discount certificate from our data. This is mostly the case only a few days before maturity and only if the reference dates does not match a third Friday of the month.

Since implied volatilities are calculated with closing values of options, but discount certificates are time stamped intra day, this introduces additional volatility error. Different studies like Baule et al. (2018) and Lee and Ryu (2019) suggest that implied volatility decreases intraday. Additionally Lee and Ryu (2019) show that this effect is even more substantial on days with news announcement and away from the money. However the intraday decay of volatility would have to be correlated with credit spread movement to introduce an omitted variables effect, which we do not consider likely. We note however that even time exact volatilities would not overcome this problem since it is not obvious that issuer do not use 'old' volatilities for their price quotes.

4.4 Parameters for Default Risk

For our structural model, since we are using the approximation of Baule (2021), the only extra parameters in excess to the Black Scholes ones are a credit spread and a asset-underlying-correlation. Since our data set is filtered for maturities under 2 years and discount certificates are mostly emitted for 1-year-maturities, we use the spreads from 1-year-credit default swaps. They are available for almost all of our issuers except for Vontobel, for which we use the average of the other credit spreads. Although 5-year-credit default swaps are more liquid, the difference in spreads to the 1-year version might make them not appropriate. We address this in our robustness section. Additionally, since we only use larger issuers, the 1-year-CDS are relatively liquid as well, except for DZ Bank, for which the 5-year-version is no significant improvement. The credit default swaps are taken in their modified-modified version, which is the European standard for CDS. We can acquire the credit default swap spreads via Thomson Reuters, who calculate an average value of market spreads based on credit default swaps for different maturities from certain key sell-side banks in the CDS market. Picture 1 shows different issuers' 1-year-credit default swap spreads between 2008 and 2018 together with their average and a CDS index for the European bank sector.

[Insert Figure 1 here]

For correlation between asset and underlying process we follow Baule et al. (2008) and use equity correlations as proxies. They argue that, while empirical studies like Rösch (2003) show asset correlations to be lower then equity correlation, this is because these studies focus on small and medium sized companies. Since our banks can be considered large and diversified, asset correlation should be closer to equity correlation. A discussion is given in Düllmann and Scheule (2003) and evidence in Hahnenstein (2004). We estimate the correlation across 2008 to 2018 as the correlation between log-returns on the issuers share price and log-returns on the DAX. Table 2 shows the results:

[Insert Table 2 here]

For an analyses of the effect of correlation on discount certificate prices within the approximation see Baule (2021).

4.5 Control variables

In our two regressions the Control variables fulfil a slightly different purpose. In our cross sectional regressions (formula 14) we need to make sure, that we control for certificate individual characteristics. The literature uses two prominent ones: The first control variable is moneyness. It is easy to imagine issuers taking a 'safety margin' for away from the money certificates. Additionally, this control variables is able to control for the different moneyness composition of the cross section. Since the time to maturity composition is mostly dealt with by changing to annualized margins, this means that our time series is more comparable across time than if we would simply take average values. We follow the literature and use terms up to second order of moneyness. We assume the annualized margins to depend on moneyness, so that the controls enter our regression multiplied with the time to maturity T as factors $X \cdot T$ and $X^2 \cdot T$. This term also controls for model error due to implied volatility estimation. Hentschel (2003) show that these errors are large away from the money. The multiplication with T is an additional benefit here, since volatility error is larger for long running times and should approach zero close to maturity.

The second control in our cross sectional regression is the competition a specific certificate faces. This has been done by authors like Baule (2011), Entrop et al. (2016), Arnold et al. (2021), Schertler (2021) and usually takes on a form like $1 - \frac{1}{1+n}$, where n is the number of similar[8] certificates. We follow this by defining n as the number of certificates from other issues with maturity not more than 5 days away and a cap not farther than 100 DAX points. This definition assumes that the issuers in our sample are representative for the market, which is justified given that these issuer make up for about 80% of the market[9]. One difference to the previous literature is that we assume again that annualized margin instead of the gross margin depend on competitions. Under the life cycle hypothesis it would not be consistent that short running certificates react in the same way to competition as long running ones are. Therefore competition C enters as $C \cdot T$ as a control into our regression.

Our cross sectional regression is hence fully expressed as

$$m_{model} \sim \widehat{m^a}_{model,t} \cdot T_i + \beta_{Comp,t} \cdot \text{Comp}_i \cdot T_i + \beta_{X,t} \cdot X \cdot T_i + \beta_{x^2,t} \cdot X^2 \cdot T_i + \epsilon_i \quad (17)$$

where 'model' can either be 'BS' or 'SM'. In line with the life cycle hypothesis we do not include an α in our regression. We cover this by adding lower order terms X, X^2, C in our robustness section.

In our time series regression (formula 15) we mainly control for influences on margin setting behaviour from issuers. Therefore, as a first control variable, we include the market share for discount certificates of the issuer in the German market. A low market

[8]for an appropriate notion of 'similar'

[9]These numbers are taken from the DDV.

share may incentivize issuers to lower their margins to attract investors, while on the other hand higher market shares might be a result of lower margins. We therefore have no definitive expectation of the sign for the regression results concerning market share. We obtain the data from statistics published by the DDV[10] on a quarterly basis. As a second control variable we include the Sentix private index for the DAX. It is a sentiment index created by questioning private investors about their short term[11] expected DAX movement. It is bounded by -1 and 1 respectively and available to us on a monthly basis via Thomson Reuters. Sentiment might influence margin setting, since issuer can adjust margins when investors tend to buy or sell back more respectively. It also ties to the literature investigating order flow, especially Baule (2011).
For our time series regression formulas we arrive at the form

$$\widehat{m^a}_{model,t} \sim \alpha^{model} + \beta^{model}_{CS} \cdot \mathrm{c}_t + \beta^{model}_{SEN} \cdot SEN_t + \beta^{model}_{MS} \cdot MS_t + \epsilon_t \tag{18}$$

with c_t being the 1-year-credit-swap-spread, SEN_t being the last available Sentix, MS_t the last available market share for the issuer and *model* can be either 'BS' or 'SM'. In all equations above we suppressed the issuer index for clarity reasons although the whole approach is done for every one separately.

5 Results

5.1 Main Results

For our empirical estimation of the aggregate annualized margins $\widehat{m^a}_{BS,t}$ and $\widehat{m^a}_{SM,t}$ we choose to pool certificates on a weekly basis. In this way we lower the issue of daily idiosyncratic error by averaging over short times and allow lag between credit spread and pricing reaction to still be captured. Figure 2 shows the estimations for each issuer separately together with the respective credit spread. While the estimation is more volatile for issuers with a lower number of certificates there is still a distinct negative correlation visible for most issuers. Especially from mid 2011 to end of 2012 the euro crisis had an strong impact on margins in Germany. It is notable that margin $\widehat{m^a}_{BS}$ even become negative in the period from 2009 to 2013. However values of $\widehat{m^a}_{SM}$ always stay positive, indicating that credit risk is a source of additional profit for issuers. This relates to the literature of Baule et al. (2008), who find similar results and Entrop et al. (2016), who see a substantial increase in margins around the financial crisis. Afterwards from

[10]Deutsche Derivate Verband, Umbrella union for German Derivatives

[11]6 months for this particular Sentix Index

2013 to end of 2018 margins are positive with values around 0.3% in agreement with the literature surrounding discount certificates (e.g. see Entrop et al. (2016), Baule (2011)). Furthermore we see across issuers a slight downward trend beginning in 2016, which is not related to credit spread events. Table 3 shows descriptive statistics about the values separated by issuer.

[Insert Table 3 here]

The descriptive statistics agree with our graphs. The average sample size denotes the number of data points, so that the number also captures that one ISIN might be quoted multiple times per week (once per trading day). The number of estimates varies across issuers, because some data is available for them only at a later point than 2009. Table 4 shows results for the annualized margins $\widehat{m^a}_{SM,t}$ against the structural model. Here we see the opposite with mostly higher margins in the crisis period than afterwards. Relative to the Black Scholes margins, these statistics are higher since the structural model always results in a lower price (and therefore higher margins) than the Black Scholes model.
For our time series regression, we need to rely on robust estimates of the aggregate annualized margin. We therefore omit every entry in $m^a_{BS,t}$ or $m^a_{SM,t}$ that had a basis of less than 100 data points. Due to the weekly pooling, this is almost always the case. Table 5 shows results from our time series regression (equation (18)) separated by issuer.

[Insert Table 5 here]

The values for β^{BS}_{CS} range from about -0.06 to -0.53 with significant for 7 out of 11 issuer on a 0.1% level and 1 additional issuer (Deutsche Bank) at a 1% level. This is in line with our expectation, that these values should lie in the range between 0 and -1. In light of our hypothesis H_0, that issuers price default free, we can reject H_0 in for the 8 issuer with significance and can only accept it for Citigroup, UBS Bank and Unicredit. This relate to Arnold et al. (2021), in that credit risk is priced by most issuers, but extends their results in that issuer pricing is not consistent across issuers. The estimations for α^{BS} can be interpreted as the average annualized margins if credit spread would be zero. Their values are in the same range as those from the literature, although we see a wide variation across issuers. The control variables show some significance, but not consistently across issuers.
In Contrast the table 6 shows results from our time series regression (equation (18)) separated by issuer.

[Insert Table 6 here]

For our structural model margins m^a_{SM} we see estimates of β^{SM}_{CS} in the range of 0.14 to 0.75 with 8 of 11 issuers significant on a 0.1% and one each for 1%, 5% level and no significance respectively. In light of our hypothesis H^*_0, that issuers price default risk correctly, we can therefore reject it for most issuers. The values indicate, how much excess credit spread is priced by the structural model. For example the value of Citigroup indicates that they are pricing 75% less of their credit spread than would be correct[12]. This results has major implications for investors, who should under these results refrain from investing when Credit risk is high. The Citigroup reached a credit spread of around 200 basis points in the euro crisis indicating. The 75% missing credit spread indicate that margins where around 150 basis points higher in this period. On the other hand values for α, β^{SM}_{SEN} and β^{SM}_{MS} are similar to results from m^a_{BS}. This is expected, since the structural model and Black Scholes mainly differ by their default risk pricing, so the other variables should not be affected much. Our findings indicate that we must both reject H_0 and H^*_0 on average. Neither do issuers price no credit spread at all, but they also do not price it correctly. Indeed they are on average across issuers they are missing about 40% to 50% of their credit spread. Our quantification shows that the reaction to default risk only makes up for about 30% of credit spread.

5.2 Systematic Credit Risk

In the above results we see huge variation in between issuers. The question arises whether issuer do even react to their individual credit spreads or whether a market wide effect has better explanatory power. To investigate this we add the CDS Index of picture 1 into our time series regression and replace the issuer specific credit spread with the orthogonalized version with respect to the index. This means that we first regress the issuer specific credit spread $c_{issuer,t}$ on the CDS Index $c_{index,t}$ and via

$$c_{issuer,t} \sim \alpha + \beta \cdot c_{index,t} + \epsilon_t \tag{19}$$

and define the orthogonalized version of the issuer credit spread $c_{resid,t}$ with respect to the CDS Index as the residuals ϵ_t from the above regression. This time series can be interpreted as the part of the issuer credit spread that is not correlated with the CDS Index and captures the idiosyncratic, issuer specific deviation from the sector wide default risk. We investigate this question for the Black Scholes margins m^a_{BS}. The regression

[12] as indicated by the structural model

formula therefore changes to

$$\widehat{m^a}_{BS,t} \sim \alpha_{ort}^{BS} + \beta_{CS,resid}^{BS} \cdot c_{resid,t} + \beta_{CS,index}^{BS} \cdot c_{index,t} + \beta_{SEN,ort}^{BS} \cdot SEN_t + \beta_{MS,ort}^{BS} \cdot MS_t + \epsilon_t \tag{20}$$

Table 7 shows the results. We find results for $\beta_{CS,index}^{BS}$ that are similar across all issuers with values from about -10% to -20% and 9 of 11 issuers with p-values of less than 0.1%. Only one issuer is not significant, but has a similar values. On the other hand the orthogonal part of the issuer specific credit spread looses explanatory power. Only 5 of 11 issuers show about 40% reaction to their idiosyncratic default risk. The results suggest that the market wide, systematic default risk is more important for pricing than issuer individual spreads. This is especially visible in our figure 2 for Deutsche Bank, where the huge spread movements in 2016 and 2018 have no distinct effect like the movement around the euro crisis.

5.3 Robustness

First Differences

To analyse whether our analysis is driven by time series properties we repeat our time series regression as a first difference estimation. Instead of equation 18 the formula is changed to

$$\Delta\widehat{m^a}_{model,t} \sim \alpha^{model} + \beta_{CS}^{model} \cdot \Delta \mathrm{c}_t + \beta_{SEN}^{model} \cdot \Delta SEN_t + \beta_{MS}^{model} \cdot \Delta MS_t + \epsilon_t \tag{21}$$

by replacing the variables with their first differences, where $\Delta\mathrm{Variable}_t = \mathrm{Variable}_t - \mathrm{Variable}_{t-1}$. We also repeat the approach with monthly pooling instead of weekly to investigate short versus long term changes. Table 8 shows the results for both the Black Scholes margins m_{BS} and structural model margins m_{SM}.

[Insert Table 8 here]

The first difference results show that significance shifts towards the structural model. Most issuers loose significance on a weekly level, but some is retained on a monthly pooling. This indicates that the pricing may not necessarily be driven by the credit spread movements directly. It is also a strong indicator that investment in a crisis period is unfavourable for investors, with close around 90% of credit spread missing within pricing for some issuers.

Constant in Regression
As a further robustness check we test whether the addition of an α into our cross sectional regression in disregard of the life cycle hypothesis influences our results. The results are shown in table 9. We see a similar picture to the exclusion of α with slightly varying values.

5-year-Credit-Spreads
Next, we test whether the choice of the 1-year-credit-default-swap is relevant and replace it with the more liquid 5-year-credit-default-swap. All issuers remain significant and negative for the Black Scholes margins. In comparison to the 1-year version Citigroup, Deutsche Bank and UBS bank get more significant. Table 10 shows the results. They again differ in values to our main approach but show the same overall picture.

Different Controls
Lastly, we test different controlling regarding moneyness. Because discount certificates are an underlying with a short call option or a zero bond with a shorted put option, the margin can rationally never be greater than the relative call price to the underlying or the relative put price to the zero bond.[13] In these cases a private investor would choose to rather invest in the underlying or zero bond instead of the discount certificate. Another view is that issuer can 'hide' more margins in discount certificates, where the respective option price is high. We therefore replace our moneyness controls with the minimum of the relative option prices V to the underlying or bond prices respectively. This can be expressed via

$$V := \min\left(\frac{Call}{Underlying}, \frac{Put}{K \cdot e^{-r \cdot T}}\right)$$

Since the put option part effectively controls margins for positive moneyness and the call option part for negative moneyness, the margins are in fact bound only by the relative time value of the options instead of the intrinsic value. The results are shown in table 11 with 6 out of 11 issuers retainining significance and the rest with values close to zero. One has to interpret the results slightly differently than our main results however. Here the series $\widehat{m^a}_{BS,t}$ measures the difference of annualized margins with respect to a margin setting behaviour that simply adds a fraction of the option price as a margin, while our main results looks at the average annualized margins while controlling for moneyness.

[13] This can be derived by starting with the relation $e^m \cdot (S_t - \text{Call}_t) \leq S_t$ or $e^m \cdot (K \cdot e^{-r \cdot T} - \text{Put}_t) \leq K \cdot e^{-r \cdot T}$, solving for the margin m and using first order approximation of the logarithm.

6 Conclusion

In this paper we have investigated how much credit risk is priced by issuers of SRPs. To this end, we have analyzed a large data set of discount certificates with price quotes from the EUWAX exchange in Stuttgart from 2009 to 2018. We tested two hypotheses that (i) issuers price no default risk at all and (ii) they price it correctly. For our analysis we used a two-step approach, first estimating a time series of aggregate annualized margins with respect to a theoretical model via cross sectional regressions and then testing whether this series shows correlation with credit spread changes via a time series regression. By using theoretical models (i) without credit risk and (ii) with credit risk, we could effectively test the two hypotheses.

Both hypothesis had to be rejected for most issuers. Accordingly, credit risk is priced, but it is not priced correctly. The actually priced credit risk is only a fraction (about 40% to 50% for most issuers) of the true price for credit risk.

Additionally we found that systematic, market-wide credit risk has more explanatory power for this pricing than idiosyncratic, issuer-specific risk. This might be an indicator that the pricing is not done directly, but via indirect channels by adjusting margins. In light of the literature (e.g. Baule (2011), Entrop et al. (2016)) this could be a reaction of issuers to changes in order flow and trading behaviour of private investors. It also relates to Arnold et al. (2021) because investor attention to default risk might be more distinct in a crisis period. The fact that no clear reaction to the high credit spread surges in 2016 for Deutsche Bank is shown, even though it received media attention in Germany, shows that attention can not be the only factor for default risk pricing.

Since inappropriate pricing is equivalent to increased margins, our results confirm Baule et al. (2008) in the sense that default risk is a significant part of profit for issuers. The systematic inability of private investors to proper estimate fair prices for products (Entrop et al. (2016)) makes it possible for issuers to gain additional profits from investors in times of crisis. This suggests that information asymmetry and issuer domination on the SRP market might entail negative consequences for private investors. The recent study of Schertler (2021) supports this view by concluding that new listings on EUREX forces issuers to lower margins by reducing the information asymmetry between issuer and investor regarding the fair value of a product.

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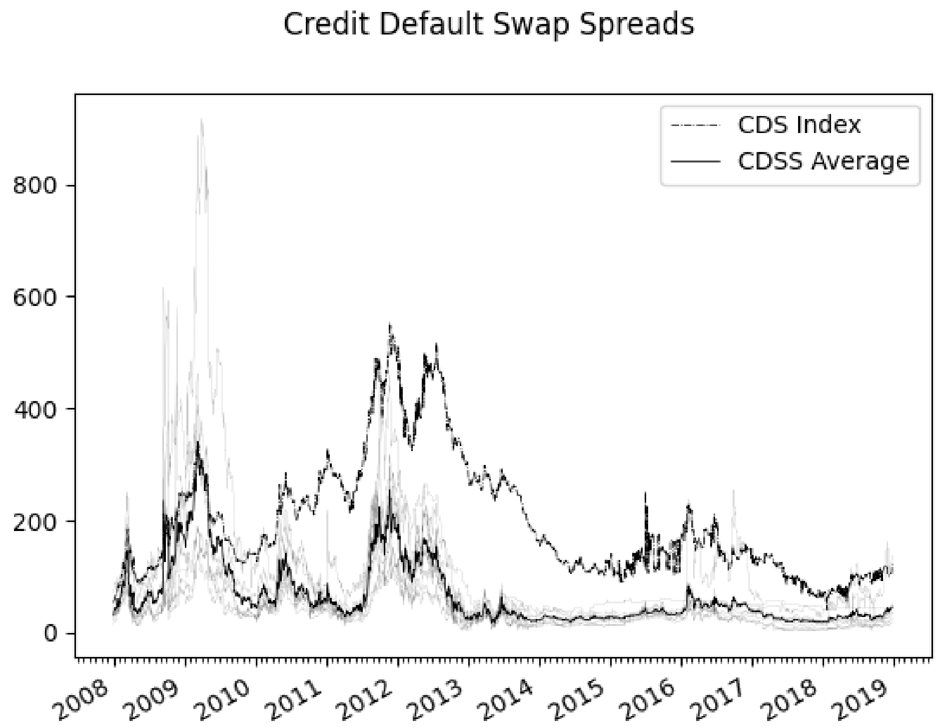

Figure 1: Graph of Credits Spread Index and Credit Default Swap Spread Averages
The figure shows credit default swap spreads as provided by Thomson Reuters of different issuers from 2008 to 2018. The darker dashed line is a credit spread index for the bank sector and the dark undashed line is the average of the issuer credit default swap spreads. The issuers are: Deutsche Bank, Goldman Sachs, BNP Paribas, UBS Bank, Commerzbank, Société Générale, Citigroup, HSBC Bank, DZ Bank, Unicredit Bank.

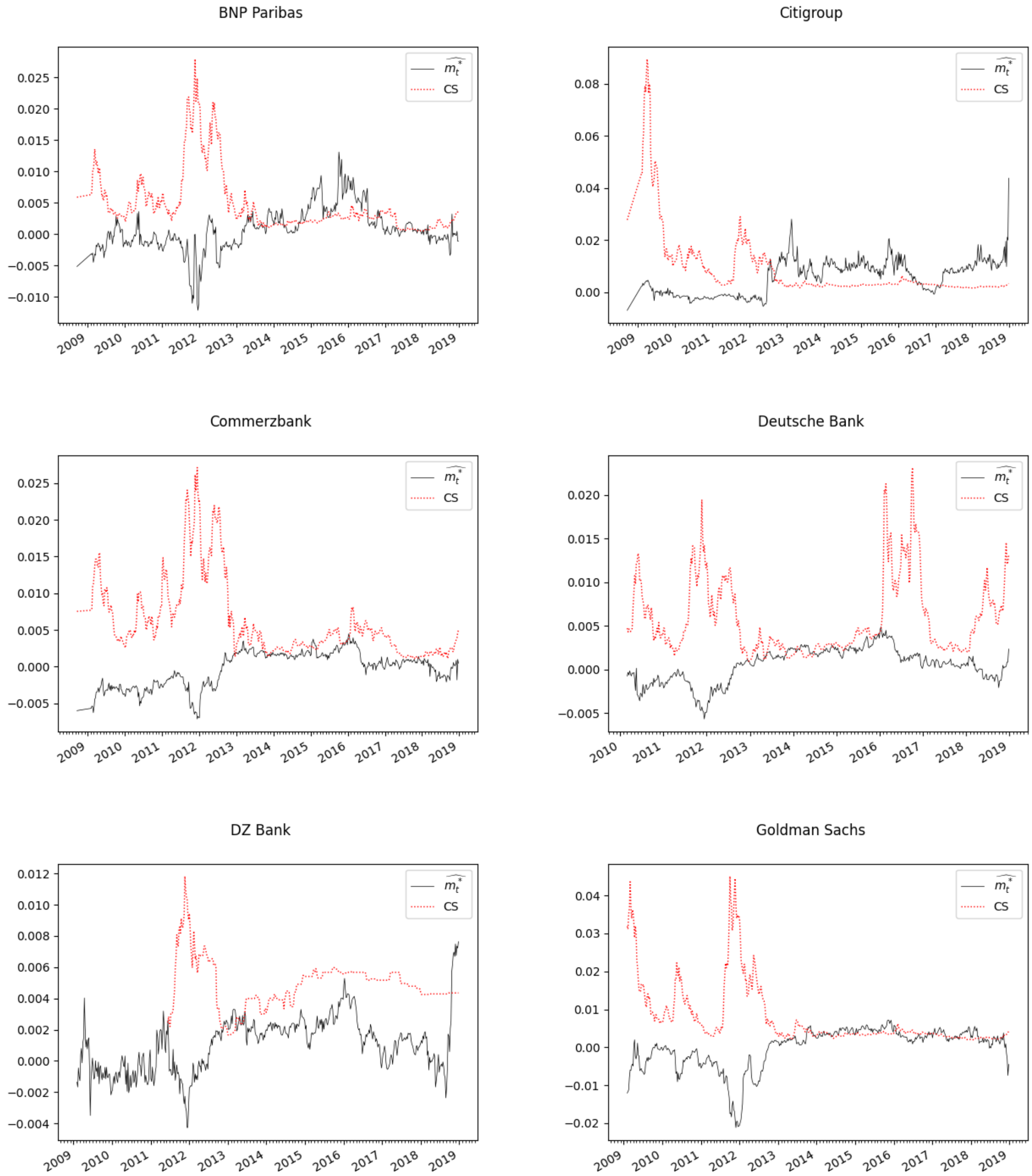

Figure 2: Graphs for Estimates of $\widehat{m^a}_{BS,t}$
The figure shows graphs for the weekly estimates of $\widehat{m^a}_{BS,t}$ values separately for each issuer together with the respective credit spread. The estimation was performed with a regression approach.

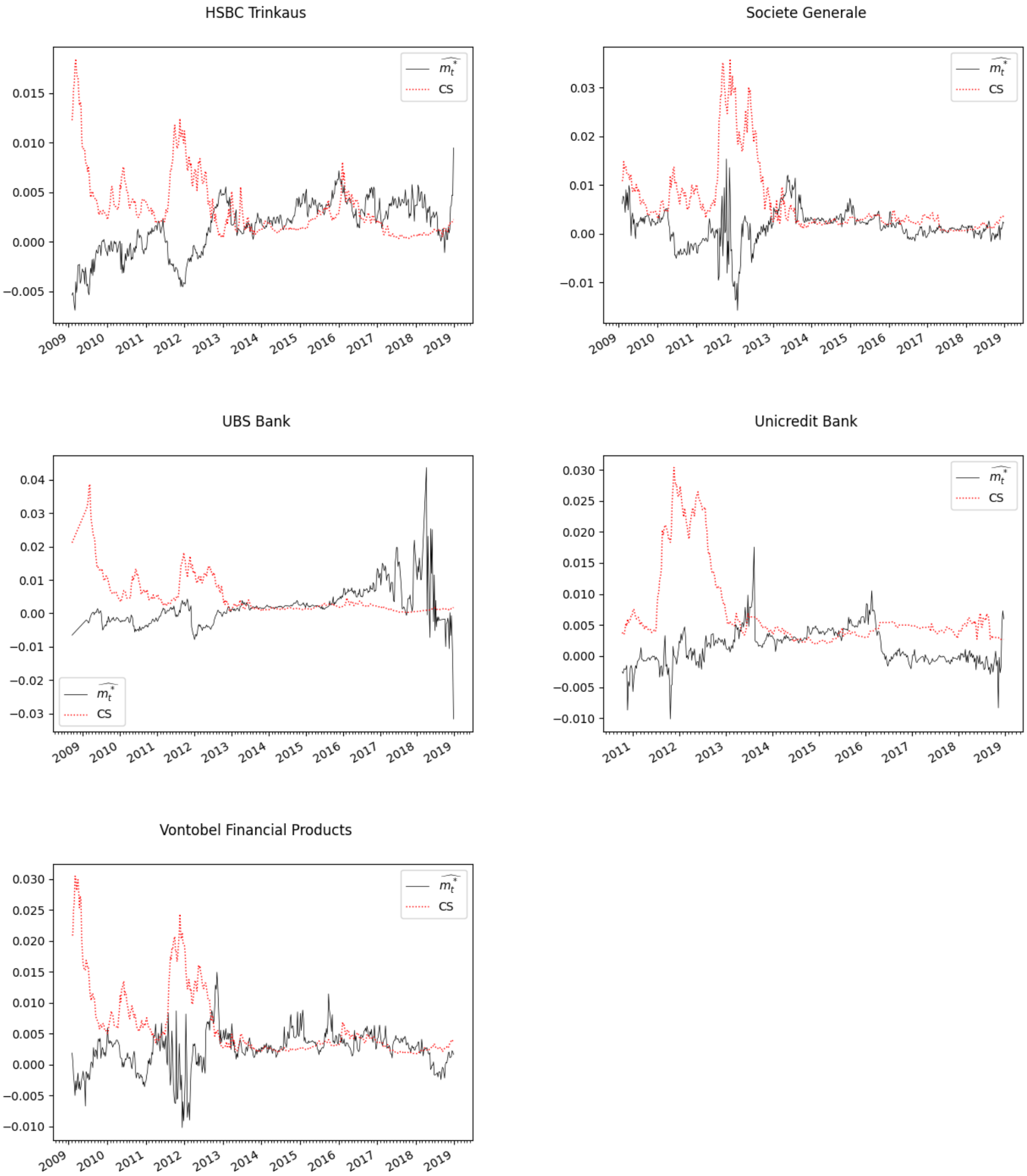

Figure 2: Graphs for Estimates of $\widehat{m^a}_{BS,t}$ (continued)
The figure shows graphs for the weekly estimates of $\widehat{m^a}_{BS,t}$ values separately for each issuer together with the respective credit spread. The estimation was performed with a regression approach.

			Averages		
issuer	certificates	observations	T	moneyness	discount
BNP Paribas	13050	2588823	0.9	0.05	-12.3%
Citigroup	5237	1427488	1.18	0.07	-14.5%
Commerzbank	16499	3749009	1.15	0.14	-19.3%
Deutsche Bank	16772	4397020	1.29	0.08	-15.5%
DZ Bank	8282	2359508	1.28	0.13	-17.3%
Goldman Sachs	10458	1863865	0.89	0.12	-16.9%
HSBC Trinkaus	5814	2014369	1.5	0.15	-19.1%
Societe Generale	7232	1735091	1.2	0.17	-19.1%
UBS Bank	7391	2297632	1.43	0.1	-19.0%
Unicredit Bank	3184	629190	0.94	0.03	-10.6%
Vontobel Financial Products	9554	2196247	1.04	0.06	-11.8%
total	103473	25258242	1.15	0.1	-16.15%

Table 1: Descriptive Statistics for our Data Set
The table shows descriptive statistics for our dataset of discount certificates split by issuer. The columns show the number of different ISINs in our time period, the number of overall observations and averages of time to maturity (T), moneyness and the average discount relative to buying the underlying directly for the first listing day on the exchange. The moneyness is positive on average with the underlying above the cap of the discount certificate, so that the inner value is already at its maximum and the investor only faces downside risks.

issuer	correlation to DAX
Goldman Sachs	0.5
Deutsche Bank	0.62
BNP Paribas	0.7
Citigroup	0.49
Commerzbank	0.62
HSBC Trinkaus	0.59
Societe Generale	0.66
UBS Bank	0.52
Unicredit Bank	0.63

Table 2: Correlation between Issuer Asset and Underlying Process
The table shows estimations of correlation between the issuers equity and the DAX. The estimates are used as proxies for asset to DAX correlations and are calculated by taking the empirical correlation of log returns from closing values of the stock prices and the DAX between the year 2008 and 2018.

			$\widehat{m^a}_{BS,t}$-statistics					
			crisis (09.2008-31.12.2012)		postcrisis (01.01.2013-31.12.2018)		full (09.2008-31.12.2018)	
issuer	estimates	cross sectional size	mean	std	mean	std	mean	std
BNP Paribas	518	5718	-0.0017	0.0026	0.0024	0.0028	0.0007	0.0034
Citigroup	518	3092	0.0001	0.0044	0.0095	0.0048	0.0057	0.0065
Commerzbank	518	8473	-0.0026	0.0018	0.0012	0.0012	-0.0003	0.0024
Deutsche Bank	461	10217	-0.0016	0.0015	0.0015	0.0012	0.0005	0.002
DZ Bank	517	5180	-0.0003	0.0014	0.0019	0.0015	0.0011	0.0018
Goldman Sachs	517	4191	-0.0052	0.0053	0.0032	0.0017	-0.0001	0.0054
HSBC Trinkaus	517	4253	-0.0009	0.0022	0.0031	0.0015	0.0015	0.0027
Societe Generale	517	3889	-0.0	0.0045	0.0023	0.0025	0.0014	0.0036
UBS Bank	503	4901	-0.0016	0.0024	0.0044	0.006	0.0019	0.0057
Unicredit Bank	429	1748	-0.0002	0.0025	0.0021	0.0029	0.0015	0.003
Vontobel Financial Products	517	5071	0.0011	0.0041	0.0032	0.0019	0.0024	0.0032

Table 3: Descriptive Statistics for Cross Sectional Estimations of $\widehat{m^a}_{BS,t}$
The table shows descriptive statistics about the cross sectional estimations of $\widehat{m^a}_{BS,t}$ split by issuer. The 'estimates' column indicates how often the regression was performed and 'cross sectional size' is the average number of discount certificates used in these regressions. The mean in the crisis period is negative for all issuers and positive afterwards with relatively high standard deviations ('std').

			$\widehat{m^a}_{SM,t}$-statistics					
			crisis (09.2008-31.12.2012)		postcrisis (01.01.2013-31.12.2018)		full (09.2008-31.12.2018)	
issuer	estimates	average n	mean	std	mean	std	mean	std
BNP Paribas	518	5718	0.0041	0.0037	0.0039	0.0031	0.004	0.0034
Citigroup	518	3092	0.0143	0.0156	0.0116	0.0048	0.0127	0.0106
Commerzbank	518	8473	0.0052	0.0045	0.0036	0.0019	0.0043	0.0032
Deutsche Bank	461	10217	0.0034	0.0019	0.0056	0.0037	0.0049	0.0034
DZ Bank	394	5747	0.0044	0.0012	0.0057	0.0016	0.0054	0.0016
Goldman Sachs	517	4191	0.0064	0.0056	0.0059	0.0018	0.0061	0.0038
HSBC Trinkaus	517	4253	0.003	0.0015	0.0045	0.0019	0.0039	0.0019
Societe Generale	517	3889	0.0086	0.0076	0.0041	0.003	0.0059	0.0058
UBS Bank	503	4901	0.0055	0.0053	0.0057	0.0061	0.0056	0.0058
Unicredit Bank	429	1748	0.0103	0.007	0.0054	0.0029	0.0067	0.0049
Vontobel Financial Products	517	5071	0.0097	0.0043	0.0057	0.0021	0.0073	0.0037

Table 4: Descriptive Statistics for Cross Sectional Estimations of $\widehat{m^a}_{SM,t}$
The table shows descriptive statistics about the cross sectional estimations of $\widehat{m^a}_{SM,t}$ split by issuer. The 'estimates' column indicates how often the regression was performed and 'cross sectional size' is the average number of discount certificates used in these regressions. The mean in the crisis period is negative for all issuers and positive afterwards with relatively high standard deviations ('std').

issuer	α^{BS}	β_{CS}^{BS}	β_{SEN}^{BS}	β_{MS}^{BS}
BNP Paribas	0.0017***	−0.353***	0.0019**	0.0092**
	(0.0005)	(0.0609)	(0.0007)	(0.0036)
Citigroup	0.0052***	−0.0649	0.0046*	0.1231***
	(0.001)	(0.0578)	(0.0021)	(0.0295)
Commerzbank	0.0023***	−0.2666***	0.004***	−0.0026
	(0.0006)	(0.0295)	(0.0005)	(0.0032)
Deutsche Bank	0.0052***	−0.1544**	0.0009	−0.0188***
	(0.0008)	(0.0545)	(0.0005)	(0.0036)
DZ Bank	0.0014	−0.4798***	−0.0023	0.0127
	(0.0016)	(0.0821)	(0.0016)	(0.0076)
Goldman Sachs	0.0062***	−0.4388***	−0.0018*	−0.0871***
	(0.0004)	(0.0502)	(0.0009)	(0.0127)
HSBC Trinkaus	0.0002	−0.5229***	0.001*	0.0249***
	(0.0008)	(0.064)	(0.0005)	(0.0054)
Societe Generale	0.0042***	−0.2304***	0.0009	−0.0395***
	(0.0005)	(0.0675)	(0.0011)	(0.0062)
UBS Bank	0.0086***	−0.1403	0.0033**	−0.0938**
	(0.0021)	(0.0901)	(0.001)	(0.0322)
Unicredit Bank	0.0037***	−0.1589***	0.0	−0.8855***
	(0.0005)	(0.0341)	(0.0025)	(0.1732)
Vontobel Financial Products	0.0049***	−0.3216***	0.0006	−0.0124
	(0.0005)	(0.0442)	(0.0006)	(0.0072)

Table 5: Results for Time Series Regression of $\widehat{m^a}_{BS,t}$
The table shows the results for α^{BS}, β_{CS}^{BS}, β_{SEN}^{BS} and β_{MS}^{BS} from our time series regression of $\widehat{m^a}_{BS,t}$. The period is from 2008 to 2018 for all issuers except DZ Bank and Deutsche Bank, who start in mid 2011 and early 2010 respectively. The estimates are performed with the classic OLS estimators and standard errors are in brackets below and estimated with an HAC-estimator of Newey and West (1987) with lags equal to $\sqrt[4]{T}$. The number of stars indicate an 0.05, 0.01 and 0.001 p-value respectively.

issuer	α^{SM}	β^{SM}_{CS}	β^{SM}_{SEN}	β^{SM}_{MS}
BNP Paribas	0.0018***	0.335***	0.0023***	0.0089*
	(0.0005)	(0.0653)	(0.0007)	(0.0036)
Citigroup	0.005***	0.7536***	0.0042	0.1269***
	(0.001)	(0.063)	(0.0022)	(0.0302)
Commerzbank	0.0023***	0.4738***	0.0046***	−0.0017
	(0.0007)	(0.0337)	(0.0005)	(0.0037)
Deutsche Bank	0.006***	0.5893***	0.0007	−0.0225***
	(0.0009)	(0.0586)	(0.0006)	(0.0038)
DZ Bank	0.0015	0.2573**	−0.0027	0.0136
	(0.0016)	(0.0843)	(0.0016)	(0.0076)
Goldman Sachs	0.0063***	0.3479***	−0.0016	−0.0849***
	(0.0004)	(0.0452)	(0.0008)	(0.0124)
HSBC Trinkaus	0.0002	0.1514*	0.001*	0.0256***
	(0.0008)	(0.0603)	(0.0005)	(0.0053)
Societe Generale	0.004***	0.5177***	0.0019	−0.0363***
	(0.0006)	(0.0789)	(0.0011)	(0.0063)
UBS Bank	0.0088***	0.6049***	0.0033**	−0.0979**
	(0.0021)	(0.0895)	(0.001)	(0.0321)
Unicredit Bank	0.004***	0.5458***	−0.0006	−0.8866***
	(0.0005)	(0.0394)	(0.0027)	(0.1777)
Vontobel Financial Products	0.0051***	0.4294***	0.0006	−0.0132
	(0.0005)	(0.0421)	(0.0006)	(0.0071)

Table 6: Results for Time Series Regression of $\widehat{m^a}_{SM,t}$
The table shows the results for α^{SM}, β^{SM}_{CS}, β^{SM}_{SEN} and β^{SM}_{MS} from our time series regression of $\widehat{m^a}_{SM,t}$. The period is from 2008 to 2018 for all issuers except DZ Bank and Deutsche Bank, who start in mid 2011 and early 2010 respectively. The estimates are performed with the classic OLS estimators and standard errors are in brackets below and estimated with an HAC-estimator of Newey and West (1987) with lags equal to $\sqrt[4]{T}$. The number of stars indicate an 0.05, 0.01 and 0.001 p-value respectively.

issuer	α_{ort}^{BS}	$\beta_{CS,resid}^{BS}$	$\beta_{CS,index}^{BS}$	$\beta_{SEN,ort}^{BS}$	$\beta_{MS,ort}^{BS}$
BNP Paribas	0.0034***	−0.0899	−0.1694***	0.0027***	0.0101***
	(0.0006)	(0.0943)	(0.0234)	(0.0007)	(0.003)
Citigroup	0.0109***	−0.0007	−0.2262***	0.0081***	0.0421
	(0.0016)	(0.0524)	(0.0544)	(0.0021)	(0.0329)
Commerzbank	0.003***	−0.3344***	−0.1058***	0.0038***	−0.0034
	(0.0006)	(0.0421)	(0.0148)	(0.0005)	(0.0035)
Deutsche Bank	0.0031***	−0.0322	−0.1047***	0.0019**	−0.0013
	(0.0007)	(0.056)	(0.0179)	(0.0006)	(0.0044)
DZ Bank	0.0008	−0.4077***	−0.0435***	−0.0021	0.0077
	(0.0016)	(0.064)	(0.0118)	(0.0015)	(0.0082)
Goldman Sachs	0.008***	−0.3719***	−0.2876***	−0.0007	−0.0643***
	(0.0005)	(0.0552)	(0.0356)	(0.0009)	(0.0153)
HSBC Trinkaus	0.0019*	−0.3602***	−0.1261***	0.0022***	0.0189***
	(0.0008)	(0.0878)	(0.0119)	(0.0005)	(0.0053)
Societe Generale	0.0056***	−0.2012*	−0.1375***	0.001	−0.0407***
	(0.0007)	(0.0997)	(0.036)	(0.0011)	(0.0054)
UBS Bank	0.0103***	0.0876	−0.1599***	0.0063***	−0.0707*
	(0.0021)	(0.0788)	(0.035)	(0.0013)	(0.0326)
Unicredit Bank	0.0042***	−0.1499	−0.0763***	0.0	−0.8983***
	(0.0005)	(0.0912)	(0.0205)	(0.0026)	(0.1568)
Vontobel Financial Products	0.0047***	−0.3521***	−0.0961**	0.0002	−0.0092
	(0.0008)	(0.0721)	(0.0313)	(0.0008)	(0.0077)

Table 7: Results for orthogonalized Credit Spread in the Time Series Regression
The table shows results from the time series regression, where a CDS-index is used in addition to the control variables, and the issuer specific credit spread is replaced with an orthogonalized version with respect to the index. The results where performed for Black Scholes margins $\widehat{m^a}_{BS,t}$ and the period is from 01.09.2008 to 31.12.2018 for all issuers except DZ Bank and Deutsche Bank, who start in mid 2011 and early 2010 respectively. A classical OLS estimators was used and standard errors are in brackets below and calculated with the HAC-estimator of Newey and West (1987) with lags equal to $\sqrt[4]{T}$. The number of stars indicate an 0.05, 0.01 and 0.001 p-value respectively.

	$\beta^{BS}_{CS,FD}$			
	weekly		monthly	
issuer	Black Scholes	Structural	Black Scholes	Structural
BNP Paribas	0.1681	0.8268***	−0.011	0.663***
	(0.1061)	(0.1115)	(0.1497)	(0.1415)
Citigroup	0.0887	0.9361***	0.104**	0.9842***
	(0.0465)	(0.1051)	(0.0353)	(0.0617)
Commerzbank	−0.0614	0.6905***	−0.1118*	0.6354***
	(0.0333)	(0.0325)	(0.0471)	(0.0498)
Deutsche Bank	−0.0037	0.7432***	−0.0323	0.7073***
	(0.0188)	(0.0269)	(0.0289)	(0.037)
DZ Bank	0.0041	0.7541***	−0.3365**	0.4084**
	(0.068)	(0.0675)	(0.1229)	(0.1311)
Goldman Sachs	−0.1245*	0.6685***	−0.2678**	0.5201***
	(0.0578)	(0.0635)	(0.0947)	(0.092)
HSBC Trinkaus	−0.1203*	0.5473***	−0.2556**	0.436***
	(0.0594)	(0.0618)	(0.097)	(0.0941)
Societe Generale	0.0669	0.9185**	0.016	0.8479***
	(0.1729)	(0.3468)	(0.074)	(0.0809)
UBS Bank	−0.0218	0.7623***	0.0522	0.8106***
	(0.0786)	(0.0869)	(0.0745)	(0.0771)
Unicredit Bank	−0.0279	0.6473***	0.0456	0.811***
	(0.1188)	(0.112)	(0.1253)	(0.1658)
Vontobel Financial Products	−0.0533	0.7417***	−0.207	0.5581***
	(0.1625)	(0.1637)	(0.1085)	(0.0773)

Table 8: Results for First Difference Estimation of the Time Series Regression
The tables shows results for the time series regression for $\widehat{m^a}_{BS,t}$ and $\widehat{m^a}_{SM,t}$ performed as a first differences regression. The cross sectional estimation where pooled on a weekly or monthly basis to investigate long versus short time lags.

	Black-Scholes-Model		Structural model (Baule Approximation)	
issuer	α^{BS}	β^{BS}_{CS}	α^{SM}	β^{SM}_{CS}
BNP Paribas	0.0017**	−0.3759***	0.0018**	0.2609***
	(0.0006)	(0.0632)	(0.0006)	(0.0651)
Citigroup	0.0045***	−0.0449	0.0042***	0.7547***
	(0.0009)	(0.0526)	(0.001)	(0.0579)
Commerzbank	0.0033***	−0.1827***	0.0032***	0.5095***
	(0.0006)	(0.0276)	(0.0007)	(0.0312)
Deutsche Bank	0.0054***	−0.082	0.0066***	0.6202***
	(0.0008)	(0.0433)	(0.0008)	(0.0482)
DZ Bank	0.0015	−0.3347***	0.0015	0.3699***
	(0.0017)	(0.0958)	(0.0018)	(0.0987)
Goldman Sachs	0.0057***	−0.4811***	0.0058***	0.2765***
	(0.0004)	(0.0561)	(0.0004)	(0.0495)
HSBC Trinkaus	−0.0016*	−0.417***	−0.0017*	0.228***
	(0.0008)	(0.0618)	(0.0008)	(0.0586)
Societe Generale	0.0047***	−0.323***	0.0045***	0.3762***
	(0.0006)	(0.07)	(0.0006)	(0.0761)
UBS Bank	0.0109***	−0.0556	0.0112***	0.6562***
	(0.0027)	(0.1151)	(0.0027)	(0.1157)
Unicredit Bank	0.0039***	−0.1098*	0.004***	0.5772***
	(0.0008)	(0.0498)	(0.0008)	(0.0537)
Vontobel Financial Products	0.0047***	−0.5039***	0.0049***	0.2231***
	(0.0006)	(0.0633)	(0.0006)	(0.0642)

Table 9: Results for added constant in the Time Series Regression
The table shows the results for β^{BS}_{CS}, β^{SM}_{CS} and alphas from our robustness check changing the original regression formulas by adding a constant into the cross sectional regressions. The period for the time series regression is from 2008 to 2018 except DZ Bank and Deutsche Bank, who start in mid 2011 and early 2010 respectively. The estimates are performed with the classic OLS estimators and standard errors are in brackets below and estimated with an HAC-estimator of Newey and West (1987) with lags equal to $\sqrt[4]{T}$. The number of stars indicate an 0.05, 0.01 and 0.001 p-value respectively.

issuer	α^{BS}	β_{CS}^{BS}	α^{SM}	β_{CS}^{SM}
BNP Paribas	0.0027***	−0.2636***	0.0007	0.2582***
	(0.0007)	(0.0526)	(0.0006)	(0.0528)
Citigroup	0.0076***	−0.1913*	−0.0035	0.9437***
	(0.0016)	(0.0945)	(0.0027)	(0.1718)
Commerzbank	0.0031***	−0.1876***	0.0	0.4339***
	(0.0007)	(0.0334)	(0.0006)	(0.0243)
Deutsche Bank	0.0061***	−0.1778***	0.0052***	0.4609***
	(0.0008)	(0.0393)	(0.0011)	(0.0592)
DZ Bank	0.0046*	−0.3577***	0.0028	0.0172
	(0.0022)	(0.0866)	(0.0022)	(0.0725)
Goldman Sachs	0.0089***	−0.5405***	0.004***	0.4479***
	(0.0006)	(0.074)	(0.0005)	(0.0628)
HSBC Trinkaus	0.0022	−0.3644***	−0.001	0.1471**
	(0.0012)	(0.0629)	(0.001)	(0.0527)
Societe Generale	0.005***	−0.1791**	0.0005	0.5171***
	(0.0007)	(0.0585)	(0.0009)	(0.0808)
UBS Bank	0.0085***	−0.3239***	0.0058*	0.3643**
	(0.0023)	(0.0952)	(0.0027)	(0.1246)
Unicredit Bank	0.0035***	−0.0802*	0.0006	0.514***
	(0.0007)	(0.0338)	(0.0007)	(0.0371)
Vontobel Financial Products	0.0059***	−0.2696***	0.0017*	0.5199***
	(0.0008)	(0.0745)	(0.0008)	(0.0635)

Table 10: Results for 5-year-CDS in the Time Series Regression
The table shows the results for β_{CS}^{BS}, β_{CS}^{SM} and alphas from our robustness check replacing 1-year-CDS-spreads with 5-year-CDS-spreads in our time series regression. The period for the time series regression is from 2008 to 2018 except DZ Bank and Deutsche Bank, who start in mid 2011 and early 2010 respectively. The estimates are performed with the classic OLS estimators and standard errors are in brackets below and estimated with an HAC-estimator of Newey and West (1987) with lags equal to $\sqrt[4]{T}$. The number of stars indicate an 0.05, 0.01 and 0.001 p-value respectively.

issuer	α^{BS}	β_{CS}^{BS}	β_{SEN}^{BS}	β_{MS}^{BS}
BNP Paribas	-0.0002	-0.3214^{***}	0.0013^{*}	0.0059
	(0.0005)	(0.0523)	(0.0005)	(0.0033)
Citigroup	0.0034^{***}	-0.0349	0.0015	-0.0198
	(0.0009)	(0.0441)	(0.0019)	(0.0278)
Commerzbank	-0.0005	-0.1215^{***}	0.0023^{***}	-0.0025
	(0.0007)	(0.0227)	(0.0004)	(0.0031)
Deutsche Bank	0.0002	0.0281	0.0016^{***}	-0.0055
	(0.0007)	(0.0475)	(0.0005)	(0.0031)
DZ Bank	-0.0076^{***}	-0.0973	0.0014	0.036^{***}
	(0.0021)	(0.1036)	(0.0014)	(0.0106)
Goldman Sachs	0.0028^{***}	-0.3374^{***}	-0.0026^{**}	-0.0543^{***}
	(0.0004)	(0.0534)	(0.0008)	(0.0128)
HSBC Trinkaus	-0.0009	-0.0848	0.0016^{***}	-0.0188^{*}
	(0.001)	(0.0534)	(0.0005)	(0.0073)
Societe Generale	0.0021^{***}	-0.5434^{***}	0.0002	-0.0297^{**}
	(0.0006)	(0.0839)	(0.0008)	(0.0094)
UBS Bank	0.0048^{*}	0.031	0.0051^{***}	-0.0748^{*}
	(0.0022)	(0.0655)	(0.0009)	(0.0333)
Unicredit Bank	0.0015^{*}	-0.1444^{**}	-0.0024	-1.1853^{***}
	(0.0007)	(0.0464)	(0.0022)	(0.1929)
Vontobel Financial Products	0.0023^{***}	-0.314^{***}	-0.0002	-0.0423^{***}
	(0.0006)	(0.0668)	(0.0008)	(0.0063)

Table 11: Results for different Moneyness Controls in the Time Series Regression
The table shows the results for the time series regression of $\widehat{m_{BS,t}^{a}}$, where cross sectional estimation is performed with a different moneyness control. The time series regression was unchanged. The period for the time series regression is from 2008 to 2018 except DZ Bank and Deutsche Bank, who start in mid 2011 and early 2010 respectively. The estimates are performed with the classic OLS estimators and standard errors are in brackets below and estimated with an HAC-estimator of Newey and West (1987) with lags equal to $\sqrt[4]{T}$. The number of stars indicate an 0.05, 0.01 and 0.001 p-value respectively.