Money Talks: Information and Seignorage*

Maxi Guennewig†

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Abstract

This paper analyses the consequences for monetary policy arising from private, centralised digital currencies (PCDC) such as Facebook's Diem. Firms introduce PCDC to generate seignorage revenues and information on consumers. In a benchmark model of imperfectly competing firms, information shapes the degree of currency competition: firms do not accept their competitors' currencies, which limits the seignorage base. Issuers of PCDC then optimally implement the Friedman rule to remove their seignorage income altogether. As a result, public currency is unable to compete unless the central bank follows suit, resulting in deflation. However, private currency market power breaks this benchmark: inflationary pressures arise if firms form currency consortia, but decision powers and seignorage claims are concentrated in the hands of one firm. The paper highlights scenarios in which information collection is inflationary, and offers an explanation for the Diem consortium's plan to issue stablecoins denominated in public currencies.

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†University of Bonn, Department of Economics, Institute for Finance and Statistics. Adenauerallee 24-26, 53113 Bonn, Germany. E-mail: *mguennewig@uni-bonn.de*. Web: *www.mguennewig.com*.

1 Introduction

The last decade has seen large-scale innovation in the realm of digital currencies. The most prominent example is Bitcoin, a private, decentralised digital currency (PDDC) for which transactions are verified using cryptographic technologies, and which has been issued in large amounts. Not yet in existence but already the subject of research and discussion are central bank digital currencies (CBDC), to be issued by monetary authorities and complementing banknotes, coins and bank reserves. This paper discusses a third type of digital currency: private, centralised digital currencies (PCDC) issued and operationally managed by firms or groups of firms that produce consumption goods. With the announcement of Libra—now renamed Diem and to be issued by a consortium of 100 firms lead by Facebook—they are perceived as a serious rival to central bank currency in the future. With over a billion users of Alibaba's Alipay and Tencent's WeChat Pay each, they are already much more important in number of transactions than Bitcoin.[1]

Unlike decentralised digital currencies, PCDC do not offer anonymity. The owner of the technology knows the identity of the consumer and verifies transactions centrally. The collected transaction data have great value in understanding consumer tastes, raising the profits of the seller. Therefore, introducing a centralised digital currency brings one benefit: it generates information rents. Unlike CBDC, PCDC generate further income that stays in private hands and is not rebated to the fiscal authorities. Issuing zero-interest currency backed by interest-paying assets, the firms obtain a second benefit: seignorage revenues. This paper studies how these two benefits affect the issuance of PCDC. It also studies their effect on monetary policy, both private and public, once the PCDC is widely used.

To this end, I develop a benchmark model in which the introduction of a PCDC endangers the central bank's policy autonomy. Building towards this benchmark, I first introduce a partial equilib-

[1]Diem is clearly designed as a currency as users will hold tokens in their digital wallets. These tokens can be exchanged for other currencies at the prevailing exchange rate. However, Alipay and WeChat Pay have features of both currencies and payment technologies which simply access bank accounts. On the one hand, users can hold tokens in their digital wallets. On the other hand, when making payments using the app, users can also debit their bank accounts. In this case, Alipay and WeChat Pay resemble payment technologies such as debit cards, rather than currencies. Furthermore, the value of Alipay and WeChat Pay tokens is pegged one-for-one to the RMB. The results of the model hold if two conditions are met: tokens are used as media of exchange when purchasing consumption goods; and the issuer of tokens obtains seignorage revenues. Both of these conditions are met for Alipay and WeChat Pay tokens. However, heavy government regulation—discussed in Section 6—affects the degree to which such technologies compete with government currency.

rium framework of imperfect competition with information frictions. Consumers value consumption heterogeneously, but their types and purchases are unobservable initially. Firms—best thought of as vertically-integrated platforms, conglomerates or firm consortia which supply the entire range of consumption goods—can only identify consumers after introducing a payment technology that generates transaction data. The model thus contains a notion of information based on past purchasing behaviour. Furthermore, firms have market power. Consumers are subject to search frictions and must engage in directed, sequential search for price offers in the spirit of Diamond (1971). As a result, firms charge prices above marginal costs and use information to attract the most profitable customers.

The paper then develops a general equilibrium framework which fully nests the partial equilibrium model. The payment technology is modelled as money: consumers face a cash-in-advance constraint following Lucas and Stokey (1987), forcing them to hold currency corresponding to their consumption expenditure. The government issues public currency. Firms pay a fixed cost to introduce a private currency which brings two important benefits. First, usage of their currency generates information rents. As in the partial equilibrium model, the issuer learns their customers' types and exploits this information to increase profits. Second, firms obtain seignorage revenues proportional to their issued money when investing the proceeds into interest-bearing bonds.[2] Finally, I endogenise the firms' currency acceptance. Thus, the money in circulation could be public, private, or both.

The benchmark model yields two important results. First, information shapes the degree of currency competition: firms do not accept competitor currencies. Information breaks the firms' indifference between the currencies in circulation since transaction data help to profitably allocate scarce advertising capacity. The model's prediction is consistent with empirical observations from the world of digital platforms and payment technologies. In China, Alipay and WeChat Pay dominate the market for digital payments. However, customers of Alibaba cannot use WeChat Pay to purchase goods. Similarly, Amazon does not accept Google Pay and PayPal, which until recently belonged to eBay. Given the model's prediction and above observations, I anticipate that firms such as Amazon, Apple and Google will not accept Facebook's Diem in the future.

This first result—the limited acceptance of PCDC due to information collection—is key to arrive

[2]Private currencies are thus modelled as stablecoins.

at the second main result, characterising the effect of optimal private monetary policy on public monetary policy. In models of money as the medium of exchange, seignorage revenues correspond to a tax on consumption. Holding money is costly: it does not pay interest and therefore yields a lower return than bonds. The nominal interest rate of the economy can be interpreted as a tax rate. Since consumers only hold money to purchase consumption, real consumption expenditure forms the tax base. By the first result, information limits private currency holdings—and thus the seignorage tax base—to consumption expenditure with the issuing firm. As the recipient of seignorage revenues, this firm perfectly internalises the effect of the seignorage consumption tax. To maximise profits, it removes the seignorage tax altogether and sets interest rates in the private currency to zero. Optimally, the firm does not levy a tax on top of its profit-maximising price.

In the model, profit-maximising private monetary policy restricts public monetary policy, and vice versa. Consumers do not hold a currency if it is associated with a higher total price of consumption goods, combining the product price and the currency's seignorage tax. Thus, whenever interest rates in the public currency are larger than zero, competitor firms which only accept this public currency are unable to compete. Consumers do not demand public currency, and the central bank is forced to implement a zero interest rate policy. While limiting its policy autonomy, the introduction of private currency disciplines the central bank: implementing the Friedman rule of zero nominal interest rates improves welfare, and only the firms' market power prevents efficiency. However, while zero interest rates are desirable in a model of money as medium of exchange, they are associated with deflation—an outcome that may be undesirable for reasons outside of this model.[3]

Breaking the benchmark. The third main result is that inflationary pressures arise if firms form currency consortia, but decision powers and seignorage claims are concentrated in the hands of one firm. I further show that the optimal seignorage tax rate is strictly positive whenever the seignorage tax base generated by consortium member firms is sufficiently large. In this context, information induces inflationary pressures if it increases the size of private currency transactions on which the consortium leader can levy a tax. Another interpretation of this result is that the central bank enjoys full policy autonomy as the private currency becomes widely used in the economy. In fact, I show scenarios in which the public currency disciplines the private currency.

[3]One example includes economies with nominal wage rigidities. The central bank may want to use inflation as a tool to reduce real wages in response to a negative productivity shock. See i.e. Uhlig and Xie (2021) for a model with nominal rigidities and multiple currencies.

Policies to escape the benchmark. In the extensions, I analyse whether particular policies allow the government to escape the privately-enforced zero interest rate environment. First, I consider interest-bearing CBDC, and find that the central bank indeed regains full autonomy as long as digital public currency yields the same return as bonds. I then evaluate a frequently made argument for the success of public currencies: governments can force firms to pay taxes in the public currency. In doing so, the government may indeed create demand for its currency and escape the zero interest rate environment. Interestingly, such a policy induces high levels of private currency inflation associated with welfare improvements. Firms deflate the value of their currency to obtain capital gains on public currency holdings while simultaneously compensating consumers with product discounts.

Contribution to the literature. This paper is the first to formally discuss private digital currencies as an information-generating device. It is also the first to discuss private, profit-maximising monetary policy conducted by producers of consumption goods and the arising consequences for monetary policy. The paper therefore primarily speaks to the literature on the digitalisation of money and currency competition.

Brunnermeier et al. (2019) discuss various aspects of the digitalisation of money. They describe, albeit without a model, how the introduction of digital currency areas helps promote platform cohesion and information collection. Central bank policy autonomy may be under pressure if public currencies lose their role as medium of exchange. In this work, I develop a formal model to endogenise the issuance of PCDC in order to achieve information and seignorage rents. I find that the introduction of one private currency already limits central bank autonomy. However, this result does not hold if the private currency is widely used in exchange.

The monetary policy consequences described in this analysis resonate with the findings of Benigno et al. (2019). In their paper, a global private currency is used in two countries together with the local public currencies. By a no-arbitrage argument, all exchange rates are fixed in equilibrium, and monetary policies become synchronised. The authors cover the special case of Diem as interest-bearing currency: central banks must set interest rates on bonds to zero for public currency to be able to compete. This result is obtained more generally when considering the nature of the issuer of currency, even if Diem does not pay interest. It also establishes that Diem-issuers have no incentive to pay interest if the currency is widely used.

Many more papers discuss digital currencies. However, they either analyse the effect of other types of digital currency on public monetary policy, or they analyse PCDC but not the consequences for monetary policy. In Fernández-Villaverde and Sanches (2019), entrepreneurs issue PDDC such as Bitcoin which compete with public currency. The entrepreneurs obtain seignorage revenues by expanding the money supply and may frustrate the government's attempts to implement the Friedman rule. Skeie (2019) discusses digital currency runs when a PDDC competes with public currency experiencing high inflation rates. Fernández-Villaverde et al. (2020) analyse an economy in which the central bank issues CBDC and invests in real assets. As a result, it may face a trade-off between price stability and excessive liquidation of its investments.

In Chiu and Wong (2020), a digital platform faces the choice between introducing private token money and accepting government currency. Issuing tokens is costly but generates transaction fees. However, the monopolist platform is fixed in size. Thus, it does not fully compete with other platforms or non-platform firms that accept government currency. I endogenise platform size. Private currency competes with government currency through the prices faced by consumers in each denomination. Gans and Halaburda (2015) discuss PCDC as a customer retention device in a partial equilibrium setting. Li and Mann (2018), Catalini and Gans (2018), Garratt and Van Oordt (2019), Prat et al. (2019), Rogoff and You (2020) and Cong et al. (2020), among others, analyse PCDC with a focus on optimal design to finance at low interest rates, rather than to compete with government fiat money. Keister and Monnet (2020) discuss how CBDC can generate information for central banks over the quality of banks' balance sheets. While sharing the notion of information and digital currency, I discuss how PCDC generates information for firms over consumer tastes.[4] Lastly, Garratt and van Oordt (2021) and Garratt and Lee (2021) discuss how payment data collection leads to welfare losses due to price discrimination and monopoly formation. CBDC preserve privacy and help increase consumer welfare. However, the technology through which firms collect information—PCDC in this analysis—is not directly modelled, and these papers do not analyse the consequences for monetary policy.

Information and privacy in the digital economy are being tackled from many other directions; see Goldfarb and Tucker (2019) and Bergemann and Bonatti (2019) for extensive surveys. Customer recognition by imperfectly competing firms has been addressed in Villas-Boas (1999) and Fudenberg

[4]For further papers discussing pricing of cryptocurrencies, see, among others, Athey et al. (2016), Chiu and Koeppl (2017), Biais et al. (2018), Prat and Walter (2018), Budish (2018), Schilling and Uhlig (2019), Sockin and Xiong (2020) and Choi and Rocheteau (2020).

and Tirole (2000).[5] In more recent work, Bonatti and Cisternas (2020) investigate price discrimination by short-lived monopolists based on a consumer score that aggregates information on past purchases. Modelling information in a general equilibrium framework, I endogenise the introduction and acceptance of digital currencies by firms in order to discuss consequences for monetary policy.

Finally, this paper is related but does not belong to the literature on search-theoretic models of money. In the seminal work by Lagos and Wright (2005), anonymous agents search for trade partners in a decentralised market. Money facilitates transactions since anonymity precludes credit. In this analysis, I take it as given that the Diem consortium—a group of private companies lead by data firm Facebook which is in the business of collecting information on consumers, effectively overcoming anonymity—are issuing currency. I then set out to discuss profit-maximising behaviour and characterise the consequences for monetary policy. In the process, I develop a variant of the Diamond sequential search model in order to create a tractable framework, incorporating notions of imperfect competition and information based on past purchase behaviour into a monetary model. Search frictions are nothing but a useful modelling tool: the model yields closed form expressions for equilibrium prices, profits and thus the equilibrium value of information.[6,7]

Organisation of this paper. Section 2 presents two very simple monetary models capturing the paper's main results intuitively. The partial equilibrium model containing notions of imperfect competition and information based on past purchasing behaviour is developed in Section 3. Section 4 combines this building block with a general equilibrium framework in which firms issue PCDC, and discusses consequences for monetary policy. The effects of currency consortium ownership structures on inflation outcomes are developed in Section 5. Section 6 discusses interest-bearing CBDC and policies that force firms to hold public currency. Section 7 concludes.

[5] See also Acquisti and Varian (2005), and Fudenberg and Villas-Boas (2012) for a survey of this literature.

[6] In reality, the search process leads to noise in the equilibrium price: see Burdett and Judd (1983), as well as Burdett et al. (2017) and Bethune et al. (2020) in a monetary context.

[7] In a similar vein, I do not offer a comprehensive theory of advertising and product pricing. Advertising serves as a tool to exploit information. With a focus on the search process and pricing of consumption goods, advertising for attention has been explored by, among others, Armstrong and Zhou (2011) and Haan and Moraga-González (2011).

2 Two very stylised monetary models

This paper develops three building blocks to derive its main results. It is therefore useful to present two very stylised monetary models—fully microfounded and generalised in Sections 3 to 5—which capture the complete framework's intuition.

2.1 A very stylised benchmark model

First, consider an economy in which two firms $i \in \{f, g\}$ produce an identical consumption good and charge the same price p. Let the consumer demand at firm i be denoted by C^i. The economy features two competing monies: the government supplies a strictly positive amount of public currency, and firm f supplies a strictly positive amount of private currency called *Diem*. Suppose that the demand for Diem, $m^{\approxeq}$, and the consumption demand with firm f's are tightly linked; suppose further that the same is true for the demand for public money, $m^{\$}$, and for consumption with firm g:

$$\begin{aligned} m^{\approxeq} &= p\,C^f \qquad \text{with} \qquad C^f = \begin{cases} C^f\big(p(1+\tau^{\approxeq})\big) > 0 & \text{if } \tau^{\approxeq} \leq \tau^{\$}, \\ 0 & \text{otherwise.} \end{cases} \\ m^{\$} &= p\,C^g \qquad \text{with} \qquad C^g = \begin{cases} C^g\big(p(1+\tau^{\$})\big) > 0 & \text{if } \tau^{\$} \leq \tau^{\approxeq}, \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{1}$$

where τ^z denotes the cost of holding currency $z \in \{\approxeq, \$\}$. It captures the rate at which currency, a liability of the issuer, loses real value. Let me call the total loss in a currency's real value, given by $\tau^z m^z$, the *seignorage* revenues of the issuer.[8] The above demand schedules imply that firm f sells strictly positive amounts of the consumption good whenever the private currency seignorage rate is weakly less than public currency seignorage rate; the relationship is reversed for firm g. Equation (1) further assumes that the cost of holding money scales up firm prices in perfect analogy to a tax rate. It thus affects money demand through the demand for consumption goods. Finally, to complete the set-up, suppose that competition among firms is imperfect.[9] In particular, assume both firms charge the price that a monopolist would charge in order to maximise product profits in

[8] Section 4 establishes a direct link between the rates of seignorage, nominal interest and inflation.

[9] Section 3 provides a microfoundation using search frictions in the spirit of Diamond (1971).

the absence of any seignorage. I thus refer to p as the *profit-maximising price*:

$$p \;=\; arg\,\max_{p^i} \;\; \left[p^i - mc\right]\; C^i\left(p^i\right) \qquad \text{for} \;\; i \in \{f, g\} \tag{2}$$

where mc denotes the constant marginal cost of producing one unit of the consumption good.

In this environment, it must be that private and public currency feature the same cost in equilibrium: given the strictly positive supply of both currencies and the money demand schedules of Equation (1), money markets only clear if

$$\tau \;=\; \tau^{\$} \;=\; \tau^{\approx} \tag{3}$$

Turning to private monetary policy, firm f implements the Diem seignorage rate that maximises the sum of product profits and seignorage revenues. Optimally, seignorage revenues are fully removed:

$$0 \;\;=\;\; arg\,\max_{\tau} \;\; \left[p(1+\tau) - mc\right]\; C^f\left(p(1+\tau)\right) \tag{4}$$

As the recipient of seignorage revenues, firm f internalises their effect—equivalent to the effect of a tax—on product profits. Since firm f already charges the profit-maximising price, they do not levy a tax on top of it.[10] Importantly, the money demand functions imply that private monetary policy fully restricts public monetary policy: no consumers demand public money if the government does attempt to raise seignorage revenues.

This simple economy is microfounded and generalised in Sections 3 and 4. Consumers require money in order to purchase consumption goods. Besides seignorage, usage of Diem generates information which firm f utilises to increase their profits at the expense of firm g. The money demand schedules of Equation (1) are then the endogenous outcomes of firm g's decision not to accept Diem; and of firm f's decision not to generate seignorage revenues, which makes Diem cheaper than the public currency when purchasing consumption goods from firm f whenever $\tau^{\$} > 0$. Information crucially limits the Diem seignorage base to the product revenues of the Diem-issuing firm. In other words, information breaks the money portfolio indeterminacy typical of models of currency competition—originally described by Kareken and Wallace (1981), and recently rediscovered in pa-

[10] This result is generalised in Section 4. Firms could also generate seignorage income but would optimally provide compensating product discounts.

pers such as Schilling and Uhlig (2019), Benigno et al. (2019) and Fernández-Villaverde and Sanches (2019).

2.2 A very stylised model breaking the benchmark

Second, consider an economy with three firms selling two different goods: firm f operates as a monopolist in one market, and firms f^* and g engage in imperfect competition in another market. All firms charge the profit-maximising price p. Firm f again issues Diem and conducts private monetary policy by choosing $\tau^{\approxeq}$. Suppose demand for Diem is now given by:

$$m^{\approxeq} \;=\; \begin{cases} p\,C^f\big(p(1+\tau^{\approxeq})\big) \;+\; p\,C^{f^*}\big(p(1+\tau^{\approxeq})\big) & \text{if } \;\tau^{\approxeq} \;\leq\; \tau^{\$}, \\ p\,C^f\big(p(1+\tau^{\approxeq})\big) & \text{otherwise.} \end{cases} \tag{5}$$

Firm f's total profits are then given by:

$$\Pi^f \;=\; \big[p(1+\;\tau^{\approxeq}) - mc\big]\; C^f\big(p(1+\tau^{\approxeq})\big) \;+\; \mathbf{1}_{\{\tau^{\approxeq}\,\leq\,\tau^{\$}\}}\tau^{\approxeq}\,p\,C^{f^*}\big(p(1+\tau^{\approxeq})\big) \tag{6}$$

Firm f trades off maximising its own product profits ($\tau^{\approxeq} = 0$) against generating seignorage revenues from firm f^*. For some very mild assumptions on the functional forms of the consumption demand schedules, firm f optimally implements a strictly positive seignorage tax rate whenever $\tau^{\$} > 0$:

$$arg\,\underset{\tau^{\approxeq}}{max}\;\; \Pi^f \;\;>\;\; 0 \tag{7}$$

Whenever Diem seignorage rates are positive, the central bank is also able to implement positive seignorage rates and thus regains policy autonomy.

The second economy is microfounded and analysed in much generalised form in Section 5. Firms f and f^* form a currency consortium and jointly issue Diem, but monetary policy is controlled by firm f. Firm g again only accepts public currency due to the information rents. Section 5 provides a more formal characterisation of private monetary policy for different distributions of seignorage dividend shares and for different sizes of currency consortia. It also highlights scenarios in which information increases the seignorage base available to firm f, and thus becomes inflationary.

3 Partial equilibrium model

This section introduces a two-period partial equilibrium model with notions of imperfect competition and information based on past purchasing behaviour. Firms do not know their customers and cannot observe their types until they introduce a private payment technology that reveals consumer purchases. The paper employs search frictions in the spirit of Diamond (1971) as a useful modelling device. In particular, they allow for an analytical characterisation of equilibrium prices and thus the value of information. Given this characterisation, the model endogenises the introduction and general acceptance of firms' payment technologies. Importantly, the partial equilibrium model is flexible enough to serve as a building block for the general equilibrium framework presented in Section 4, in which the payment technology is modelled as money.

3.1 Environment

3.1.1 Consumers

There is a continuum of consumers j on the unit interval. Consumers differ in their taste of the consumption good. Their type θ_j is both private information and constant over time. There are two periods: $t \in \{0, 1\}$. Consumer j's period utility derived from consumption is given by:

$$u_j(c_t) = \theta_j^{1-\alpha} c_t^{\alpha} - p_t c_t \qquad \alpha \in (0,1) \tag{8}$$

where p_t denotes the price of the consumption good. At the beginning of the game, each consumer draws their type from a publicly known, common binary distribution: $\theta_j \in \{\theta_L, \theta_H\}$, $P[\theta_j = \theta_H] = q$, with $\theta_H > \theta_L \geq 0$.[11]

Firms, introduced below, cannot transmit any information to consumers, neither about their own price nor their competitor's price. Consumers thus have to search for price offers. I assume directed, sequential search with perfect recall. Each period features two sub-periods, day and night. There is no discounting between sub-periods. During the day, firms set their prices for the period, and consumers obtain an initial price quote by visiting one firm. Having learnt one price, consumers

[11]The exponent on θ_j is included for exposition purposes and without loss of generality. As a result, consumer demand schedules and equilibrium firm profits are linear in consumer types.

have the option to visit a second firm at night, and learn about their price, before making their final consumption decision. The first search is free, the second search costs $S > 0$.[12]

3.1.2 Firms

The model features two firms, $i \in \{f, g\}$, which supply a homogeneous non-durable good. They set their period prices during the day. Firms seek to maximise profits and produce at constant marginal costs mc. In the second period, firms have the ability to send an advertising message to a limited fraction $\xi < 1$ of consumers, at zero cost.[13] They cannot advertise to any consumers beyond this fraction.[14,15] Advertising is the mechanism through which firms exploit information on consumer types. In particular, I make a behavioural assumption affecting the initial search decision: consumers visit the more heavily advertising firm first, as long as they expect this firm to charge a weakly lower price than its competitor. In equilibrium, all firms charge the same price to all consumers, regardless of type and time period. Unless firms can direct the consumers' initial search towards their firm, they have no means of making use of information. Let firm i's decision to send an advertising message to consumer j be denoted by $a^{i,j} \in \{0, 1\}$. Importantly, firms can neither observe consumer types nor competitor strategies, and thus have to form beliefs. Denote the set of firm i's strategies and beliefs by

$$\sigma_0^i = p_0^i, \quad \sigma_1^i = \left(p_1^i, a^i\right), \qquad \text{and} \qquad \mu_t^i\left(\sigma_t^{-i}, \theta\right), \quad t \in \{0, 1\} \tag{9}$$

3.1.3 Payment technologies

At the beginning of the first period, firms may pay a fixed cost to introduce a private payment technology that reveals otherwise unobservable first period purchases. This generates information, as firms identify their customers and back out their types θ. In the absence of a private payment

[12] As is common in the literature, the first search is free in order to prevent the market from breaking down. For sufficiently small yet strictly positive initial search costs, all consumers enter the market due to the concavity of the utility function.

[13] I assume that $\xi > q/2$. If firms split the market equally, a firm sells to a measure $q/2$ of high valuation consumers. This assumption implies that their advertising capacity exceeds the number of their high valuation customers.

[14] I implicitly assume that reaching the first fraction ξ of consumers is costless, reaching any consumers beyond the initial fraction ξ is infinitely costly. This is an extreme version of the cost function considered in Grossman and Shapiro (1984) and Tirole (1988).

[15] I could in principle allow firms to advertise in the first period but—without any information on consumer types—they cannot do better than randomising.

technology, I assume that an unmodelled neutral payment technology is used.[16] In this case, firms do not learn consumer identities or types. I also assume that—once a firm has introduced their private payment technology—all of their transactions are conducted using this technology.[17] Firms decide whether to accept the competitor's technology. For a given transaction, only one payment technology can be used: if a consumer of firm i uses the payment technology of firm $-i$, then the information accrues to firm $-i$.

Let firm i's strategy set be denoted by Γ^i, consisting of the binary introduction and acceptance decisions (γ^i, $\gamma^{i,-i}$). Firm strategies Γ are fully observable by all agents in the economy.

3.2 Consumer strategies in the final period

Consider the consumer's decision problem at time-1. In the final period of the game, decisions do not affect any future payoffs. Omitting time subscripts for readability, consumer j's demand schedule conditional on purchasing from firm i charging price p^i is given by

$$c(p^i, \theta_j) \; = \; arg \max_c \left(\theta_j^{1-\alpha} c^{\alpha} - p^i c \right) \; = \; \theta_j \left(\frac{\alpha}{p^i} \right)^{\frac{1}{1-\alpha}} \tag{10}$$

Given price p^i, consumers simply demand quantities that maximise utility. Their types θ shift their demand schedules: the higher the consumer's valuation, the larger the quantities that they purchase. Clearly, consumers prefer to be charged low prices, but they cannot observe a firm's pricing strategy until they obtain a price offer. Since firms cannot transmit any information, consumers have to form beliefs. Let $\mu^j(p)$ denote such beliefs of consumer j, and let $\psi^j(\mu^j(p), a^j)$ denote their initial search strategy.[18] Consumers first visit the firm which they expect to charge the lower prices. If they expect both firms to charge the same price, the relative advertising intensity determines the first search given the behavioural assumption. Importantly, advertising breaks the consumers' indifference in equilibria in which all firms charge the same price. Only if consumers expect both firms to charge the same price, and both firms have advertised with the same intensity, consumers randomise.

Having obtained one price offer p^i, consumers need to decide whether to search again. Let the continuation search strategy be denoted by $\omega^j(p^i, \mu^j(p^{-i}))$. Consumers trade off the expected gains

[16]From Section 4 onwards, public currency plays the role of this neutral payment technology.
[17]This is the endogenous outcome of the monetary framework presented in Section 4.
[18]A formal description of the consumer's search strategies is provided in Appendix C.1.

due to a lower price offer against the fixed cost of searching $S > 0$. This gives rise to a cut-off rule: consumers are only willing to pay the fixed cost if the other firm's expected price $\mathbb{E}\left[p^{-i}|\mu^j\right]$ is sufficiently lower than the first price offer p^i.

3.3 Characterising the monopoly price

Given the consumer demand schedule, profits selling to a consumer of type θ are given by

$$\Pi(p,\theta) \;=\; (p - mc)\, c(p,\theta) \tag{11}$$

The profit function $\Pi(p,\theta)$ is continuous and concave in p for all $p > 0$, and there exists a unique profit-maximising price $p(\theta)$ given by:

$$p(\theta) \;=\; arg\ \max_{p}\ \Pi(p,\theta) \;=\; \frac{mc}{\alpha} \;\equiv\; \tilde{p} \tag{12}$$

The profit-maximising monopoly price is given by a constant mark-up over marginal costs and is thus independent of the consumer's type θ. Two assumptions yield this result: θ does not affect the price elasticity of consumption; and marginal costs are constant. It follows that monopoly profits are linear in the consumer type:

$$\Pi(\theta) \;=\; \kappa\theta \tag{13}$$

where κ is a constant. The profit function $\Pi(p,\theta)$ is strictly increasing in p for all $p < \tilde{p}$, and strictly decreasing in p for all $p > \tilde{p}$.

3.4 Equilibrium definition and prices

I now proceed to define the equilibrium and to solve for equilibrium strategies. First, I show that firms charge the monopoly price $\tilde{p}$ in equilibrium.[19] Equilibrium profits on a given consumer increase

[19] I focus on equilibria in which $p^i_t \leq \tilde{p}$ for all i, t. It is easy to see that one firm charging a price $p^i = \tilde{p}$ and the other firm charging a price $p^{-i} > \tilde{p}$ constitutes an equilibrium, with consumer beliefs formed rationally and thus consistent with above prices. All consumers visit firm i. Neither firm has an incentive to deviate from above prices. Firm i charges the profit-maximising monopoly price and has no incentive to increase the price. Lowering the price does not attract any additional consumers. Firm $-i$ cannot attract any consumers by lowering their price, given consumer beliefs.

linearly in type θ, and advertising thus aims to maximise the average type of a firm's customer base.

Equilibrium definition. *The Perfect Bayesian Equilibrium (PBE) of this game is given by the set of firm strategies* $\left(\Gamma^i, \sigma_t^i\right)_{t\in\{0,1\},\, i\in\{f,g\}}$ *that solve the firms' profit maximisation problems, given beliefs* $\mu_t^i(\sigma_t^{-i}, \theta)$*; and the set of consumer strategies* $\left(\psi_t^j, \omega_t^j, c_t^j\right)_{t\in\{0,1\},\, j\in[0,1]}$ *that solve the utility maximisation problem, given beliefs* $\mu_t^j(p_t)$*. Beliefs are formed rationally and updated according to Bayes' Law.*

Lemma 1 (Dynamic Diamond Paradox)**.** *For any* $S > 0$*, both firms charge the monopoly price in both time periods in equilibrium:*

$$p_t^i(\theta) \;=\; \tilde{p} \quad \textit{for all} \;\; i, t \tag{14}$$

The proof is provided in Appendix C.2. Intuitively, having obtained a price offer, consumers operate a cut-off rule in the final period. They compare the additional gains from searching—potentially obtaining a lower price offer—against the fixed cost of searching. This gives market power to firms. Conditional on being visit by a consumer, they face a profitable upwards deviation in the price—unless firms already charge the monopoly price. Given the assumptions of constant price elasticities and constant marginal costs, the monopoly price is independent of consumer types. In equilibrium, consumer beliefs about firm pricing must be correct, which is only true if all firms charge the monopoly price in the final period and consumers expect them to do so. Thus, all consumers face the same price in the final period. Giving away information in the first period therefore does not harm consumers. Consumers again only consider product prices when deciding on their initial search and operate a cut-off rule having obtained a price offer. This leads to equilibrium monopoly pricing also in the first period.

Given the search frictions and assumptions on utility and marginal costs, firms obtain full market power, but do not price discriminate. Thus, high valuation consumers do not have any incentives to mimic the low valuation consumers' actions in equilibrium. The advantage is model tractability: the framework includes notions of imperfect competition and information, and is flexible enough to be built into a standard monetary framework. The disadvantage is that the model is silent on strategic consumer behaviour.[20]

[20] See Bonatti and Cisternas (2020) for a recent example of price discrimination based on past purchase behaviour.

3.5 Equilibrium advertising strategies

Having characterised equilibrium prices allows us to characterise equilibrium advertising strategies. Since all firms charge the monopoly price, profits on a consumer of type θ are given by $\Pi(\theta) = \kappa\theta$. When devising an advertising strategy, firms aim to maximise the value of their customer base: profits are increasing in the average type of their customers. The firms' relative levels of information thus determine the breakdown of profits.

Consider first a firm that has not introduced a payment technology, $\gamma^{i,int} = 0$. This firm neither knows their customers' identity, nor their type θ, and thus randomly advertises among the general population of consumers. Consider next a firm that has obtained information on their customer base and is able to distinguish consumers of types θ_L and θ_H. The firm advertises to consumers of known high valuation, and uses the remaining advertising capacity to advertise to consumers for which they did not obtain any information.[21]

3.6 Payment technology introduction and acceptance strategies

Having solved for equilibrium prices, advertising strategies and thus the value of information, this subsection characterises the firms' payment technology introduction and acceptance strategies in equilibrium.

Proposition 1 (No interoperability). *Firms do not accept their competitor's payment technology:*

$$\gamma^{i,acc} \;=\; 0, \quad i \in \{f, g\} \tag{15}$$

The proof is provided in Appendix C.3. Intuitively, more information is always better, as it helps to improve the customer base at the expense of the competitor firm. Therefore, firms have no incentive to generate information for other firms. I strongly expect this result to hold up in more

[21] To see that this strategy is optimal for a given firm i, consider first their randomising competitor $-i$. The probability of double-advertising is equal for all consumers, and thus the informed firm aims to advertise to as many high type consumers as possible. If the competing firm is also informed, consider the choice between advertising to a consumer of known low type, and a consumer of unknown type. Begin with a consumer of unknown type to which firm i advertises. Since the competitor $-i$ is informed, they know their type exactly. If $\theta = \theta_L$, the competitor does not advertise and firm i sells to this consumer with probability one. If $\theta = \theta_H$, the competitor $-i$ advertises with probability one, and firm i wins this customer with probability $\frac{1}{2}$. Fixing firm $-i$'s advertising strategy for a previous customer of firm i, switching to a consumer of unknown type reduces the probability of being matched with the known low type by $\frac{1}{2}$. At the same time, it increases the probability of being matched with an unknown type by $\frac{1}{2}$. The proof is completed by realising that the unknown type has a higher expected valuation $\mathbb{E}[\theta] > \theta_L$.

general settings. A firm should not accept a competitor payment technology in any environment in which the usage of this technology generates a direct benefit to the competitor.

The model therefore predicts that digital giants including Amazon, Apple and Google should not accept Diem as a payment technology upon its introduction. This prediction is consistent with observations from the real world. In China, the market for digital payments is dominated by Alipay and WeChat Pay. The social media platform WeChat resembles Facebook, and their payment technology cannot be used to purchase goods with Alibaba, the owner of Alipay. In other parts of the world, Amazon does not accept payments with Google Pay and PayPal, which until recently belonged to eBay.

The equilibrium payment technology introduction decisions are characterised in full in Appendix A. In summary, either both firms introduce a payment technology if the fixed cost of doing so is sufficiently low, or neither firm does. Industry profits are fixed at $\Pi = \kappa\mathbb{E}[\theta]$ by Lemma 1; it follows that any information gains are mere redistributions from one firm to another.

4 General equilibrium: Monetary framework

This section develops a general equilibrium framework which perfectly nests the partial equilibrium model of the previous section. The payment technology is specified as money which consumers need to hold in order to transact. The government supplies public currency. Firms choose whether to introduce a private currency, and whether to accept the public and competitor private currencies. Crucially, currencies compete. Consumers hold the currency in which they face the lowest total cost of purchasing consumption goods. In the general equilibrium framework, this total cost consists of two objects: the price charged by firms and the opportunity cost of holding money. Money does not pay interest, implying that consumers have to forego income. Thus, the opportunity cost of a particular currency is given by the nominal interest rate of bonds in this denomination. Importantly, the central bank conducts monetary policy by setting the nominal interest rate—and hence affects the opportunity cost of holding public currency. At the same time, firms issuing private currency maximise the sum of product profits and seignorage revenues. They jointly choose their product price and implement a private monetary policy that achieves this goal. Since currencies compete, the existence of a private currency may have stark consequences for the central bank which finds itself unable to implement their desired interest rates. In fact, this section shows that optimal private monetary policy forces the central bank to set interest rates to zero, leading to deflation.

4.1 Environment

4.1.1 Households

The model features overlapping generations (OLG) of consumers who live for three periods and discount the future at rate β. They consume a credit good X and a money good C, and supply labour N_t. At each point in time, three cohorts born in three different periods co-exist. Their age is denoted by $A \in \{y, m, o\}$. Period utility for consumer j is given by

$$U_{A,j} \;=\; U(X) \;+\; \theta_{A,j}^{1-\alpha} \left(C\right)^{\alpha} \;-\; N \tag{16}$$

The market for the money good corresponds to the market of Section 3. Consumers derive utility from money good consumption for the first two periods of their life during which they value consumption heterogeneously according to their type θ_j. They do not value consumption of the money good in the final period of their life:

$$\left(\theta_{y,j},\, \theta_{m,j},\, \theta_{o,j}\right) \;=\; \left(\theta_j,\, \theta_j,\, 0\right) \tag{17}$$

The two firms $i \in \{f, g\}$ introduced in Section 3 supply the money good and charge a price p_t^i. Consumers again search sequentially for price offers within a given period. To simplify without loss of generality, I assume that the first search is free, and the second search is infinitely costly. In the second period of their lives, consumers receive an advertising message sent by firms, as before. Consumers also decide which payment technology, now modelled as money, to use at a given firm. This decision is explained in detail in subsection 4.1.3.

The OLG structure is useful to replicate the setting of the previous section: it cuts off the purchase history and thus limits the degree of learning to the first period of consumer lives; it also allows the game among firms and consumers to be solved backwards from the final period in which consumers derive utility from money good consumption. Yet the model requires an infinite horizon for money to achieve its equilibrium value (see subsection 4.1.3 for details).

Turning to the credit good, I assume that the Inada conditions hold for the utility function, $U(X)$. This implies the existence of a consumption level X^* satisfying $U'(X^*) = 1$.[22] The market for the credit good is a useful modelling device. First, the credit good serves as the numeraire. Consumers supply labour and are compensated with a real wage w_t which allows them to purchase exactly as many units of the credit good. Money good producers charge real prices p_t in units of the credit good. Second, the market for the credit good pins down the real wage w_t, described in detail in the following subsection. Third, the separability and quasi-linearity of the utility function ensure that credit good consumption is pinned down in equilibrium. Intuitively, consumers can always supply an additional unit of labour at constant marginal disutility in order to purchase w_t more units of the credit good. This pins down the real interest rate. Effectively, the credit side of the economy is super-neutral with respect to monetary policy which allows me to focus on its direct effect on the levels of money good consumption—and thus on the role of money as medium of exchange.

Consider consumer j who has visited firm i and learnt their price p_t^i. This consumer forms a portfolio consisting of money and bonds in three currencies. Let the public currency be denoted by $M^{\$}$. Firm f issues private currency $M^{≋}$, firm g issues currency $M^{\mathbb{G}}$. Going forward, I refer to the public currency as the *Dollar* and to firm $f's$ private currency as *Diem*. Omitting all (A, j)-subscripts to indicate individual decision and state variables of consumer j aged A, let their total money holdings in Dollar values be denoted by $\boldsymbol{e}_t \boldsymbol{M}_t$:

$$\boldsymbol{e}_t \boldsymbol{M}_t \ = \ M_t^{\$} \ + \ e_t^{≋} M_t^{≋} \ + \ e_t^{\mathbb{G}} M_t^{\mathbb{G}} \tag{18}$$

where the exchange rates $(e_t^{≋}, e_t^{\mathbb{G}})$ denote the price of private currencies in terms of the Dollar. The price level P_t is the price of the Dollar in terms of the credit good; dividing nominal money balances in Dollar values by P_t thus yields real money balances in units of the numeraire. Consumers face non-negativity constraints on their real money balances for each currency. Turning to bonds, the

[22] The consumer's period utility corresponds to a buyer's period utility in Lagos and Wright (2005). In their paper, sellers in the decentralised market pay a utility cost for every unit produced. Instead, I assume that firms produce according to a production function outlined below, using labour which is supplied by the household. These assumptions on the utility function buy tractability.

Dollar value of total bond holdings, $\boldsymbol{e}_t\boldsymbol{Q}_t\boldsymbol{B}_t$, is given by:

$$\boldsymbol{e}_t\boldsymbol{Q}_t\boldsymbol{B}_t \;=\; Q_t^{\$}B_t^{\$} \;+\; e_t^{\approxeq}Q_t^{\approxeq}B_t^{\approxeq} \;+\; e_t^{\mathbb{G}}Q_t^{\mathbb{G}}B_t^{\mathbb{G}} \tag{19}$$

where Q_t denotes the prices of bonds issued at time-t, to mature in the following period. Bond prices are inversely related to the interest rate prevailing in the respective currencies:

$$Q_t^z = \frac{1}{1+i_t^z}\,, \qquad z \in \{\$, \approxeq, \mathbb{G}\} \tag{20}$$

In sum, consumer j's budget constraint, all in terms of the credit good, is then given by:

$$X_t \;+\; p_t^i C_t \;+\; \frac{\boldsymbol{e}_t\boldsymbol{M}_t}{P_t} \;+\; \frac{\boldsymbol{e}_t\boldsymbol{Q}_t\boldsymbol{B}_t}{P_t} \;\leq\; w_t N_t \;+\; \frac{\boldsymbol{e}_t\boldsymbol{M}_{t-1}}{P_t} \;+\; \frac{\boldsymbol{e}_t\boldsymbol{B}_{t-1}}{P_t} \;+\; T_t \tag{21}$$

where T_t denotes the total real lump-sum transfer from firms and government to consumer j. I assume that firms and governments transfer all proceeds to the young in equal proportions.[23]

4.1.2 Firms

There are two sectors, the credit good and the money good sector. All firms maximise lifetime profits.[24] The firms in the money good sector are the ones introduced in Section 3: two firms $i \in \{f, g\}$ compete for consumer demand, choose a price p^i, and send an advertising message $a^{i,j}$ to middle-aged consumers. They decide whether to introduce and accept payment technologies, outlined in detail in the subsection 4.1.3.

In the general equilibrium framework, each sector produces given production functions that are linear in the only input factor labour:

$$Y_t^X \;=\; N_t^X \qquad\qquad Y_t^C \;=\; N_t^C \tag{22}$$

I assume perfect competition in the credit good market. Given the linearity of the production

[23]If households receive large transfers when they are old, they may want to supply negative amounts of labour in order to achieve the optimal level of credit good consumption. As long as X^* is sufficiently high, this assumption ensures that the labour supply is always positive, without affecting any other equilibrium outcomes.

[24]In equilibrium, consumer discount factors are equalised. Firms use this representative discount factor.

function, the real wage and thus real marginal costs for all firms in this economy are given by

$$w_t = mc_t = 1 \qquad \text{for all } \ t \tag{23}$$

4.1.3 Payment technologies: Money

The payment technology of Section 3 is now modelled as money: consumers need to hold currency in order to facilitate transactions of the money consumption good. In particular, they face a cash-in-advance (CIA) constraint. The timing assumption is that of Lucas and Stokey (1987): the "cash market" opens after the "credit market". Search takes place when the credit market is open, and consumers choose their cash balances in order to purchase goods from the firms they have visited. Consumption finally takes place at the end of the period.[25]

Firm f's currency introduction and acceptance decision is given by

$$\Gamma^f \ = \ \left(\gamma^{\approx}, \gamma^{f,\$}, \gamma^{f,\mathbb{G}}\right) \tag{24}$$

Conditional on all private currencies being introduced, the CIA constraint faced by consumer j at firm f is therefore given by

$$p_t^f C_t^f \ \leq \ \gamma^{f,\$} \frac{M_t^{\$}}{P_t} \ + \ \frac{e_t^{\approx} M_t^{\approx}}{P_t} \ + \ \gamma^{f,\mathbb{G}} \ \frac{e_t^{\mathbb{G}} M_t^{\mathbb{G}}}{P_t} \tag{25}$$

with a corresponding currency decision and resulting CIA constraint for firm g. One unit of real money allows for one unit of real money good expenditure. For a given currency, this is only true if the firm also accepts this currency. As an example, if $\gamma^{f,\$} = 0$, then Dollar holdings do not enable consumption purchases with firm f. The CIA constraint above imposes that firms always accept their own private currency as means of payment.

Assumption. *Consumers perfectly anticipate which currencies each firm accepts.*

This assumption ensures that firms do not forgo any revenues by not accepting a particular currency. Since PCDC are newly being introduced, this assumption initially boils down to the public currency being widely used as an alternative form of payment. Casual empiricism from the world of digital giants, as discussed in the previous section, further suggests that the network effects of not accepting

[25] An alternative timing specification is presented in Appendix E.1. The results are unchanged.

PCDC are not reducing revenues to the extent that firms want to accept competitor technologies. Perhaps more controversially, this assumption also implies that firms are more willing to form digital currency areas by not accepting the public currency. I certainly plan to revisit the question of such network effects, particularly the effects of not accepting PCDC on smaller firms, in future work.

Given this setup, the OLG structure helps address a feature of the cash-in-advance model: consumers need to hold real money balances proportional to their consumption expenditure but there is no direct exchange of money and goods. Consumers enter the following period with exactly the same amount of nominal money even if they have consumed. Including a third period in which money holdings unravel is useful since money has no value to consumers when they have died, and they would otherwise strategically reduce their money good consumption when middle-aged. Furthermore, the model requires an infinite horizon for money holdings of old consumers to be valued: younger generations need to demand money in order to purchase consumption goods.

4.2 Money balances and demand schedules

Given the problem's set-up, this section provides a summary and intuitive discussion of the consumer optimality conditions. A formal, step-by-step solution to the consumer problem, including the full set of optimality conditions, is presented in Appendix D. The equilibrium of this economy is defined in Appendix D.1.

Importantly, the monetary framework nests the equilibrium mechanism of Section 3. As in the partial equilibrium model, firms have market power due to the search frictions and charge monopoly prices which are independent of consumer types—all consumers face the same prices in all time periods. Hence, the consumers' decisions when young do not affect future payoffs. It follows that consumers maximise present utility in each time period when determining money balances and interacting with producers of the money good.

First, the optimality conditions for credit good consumption establish the equilibrium relationship between nominal and real interest rates as well as the inflation rate. Assuming separability and quasi-linearity of the utility function, combined with constant wages $w_t = 1$, renders the credit side of the economy super-neutral with respect to monetary policy. It follows that credit good consumption is equal for all time periods t and consumer ages A:

$$X_{A,t} = X^* \tag{26}$$

Intuitively, given the unit real wage, consumers can always purchase one more unit of the credit good by supplying an additional unit of labour. The real interest rate of the economy is then pinned down by the time rate of preference: $1 + r_t = \beta^{-1}$. For all currencies $z \in \{\$, \approx, \mathbb{G}\}$, define the inflation rate π^z_{t+1} as the change in their price relative to the credit good over time. The first order conditions for bonds for all consumers, regardless of their type and asset holdings, simplify to the Fisher equation (here expressed in bond prices):

$$Q^z_t = \beta(1 + \pi^z_{t+1})^{-1} \tag{27}$$

The price of bonds needs to compensate for the fact that consumers discount the future and that nominal bonds change real value over time, captured by the inflation rate.

Second, consumers only hold money in order to enable consumption purchases. To illustrate, let $\tau^z_t = (1 - Q^z_t)$ denote the difference in prices of money and bonds. Money is dominated by bonds in terms of returns whenever $Q^z_t < 1$. Thus, τ^z_t exactly captures the cost of holding money. Intuitively, whenever $\tau^z_t > 0$ for all currencies z, consumers do not hold currencies that are not accepted by firm i; if multiple currencies are accepted, they hold the currency with the lowest cost. Furthermore, consumers do not hold real money balances in excess of their real money good expenditure. Let τ^i_t denote the lowest opportunity cost among all currencies accepted by firm i. Appendix D.2 formally shows that consumer j's CIA constraint is binding whenever holding money to purchase consumption goods from firm i is costly, $\tau^i_t > 0$:

$$m_{A,j,t} = p^i_t \, C_{j,t}(\theta_{A,j}, p^i_t, \tau^i_t) \tag{28}$$

where $m_{A,j,t}$ denotes the equilibrium real money portfolio of the consumer at time-t. If $\tau^i_t = 0$, there is no opportunity cost of holding money and the CIA constraint is slack.[26]

Third, the demand schedule for the money good that maximises present money good utility is

[26]The solution also implies a zero lower bound on nominal interest rates for all currencies $z \in \{\$, \approx, \mathbb{G}\}$: $Q^z_t \leq 1 \Leftrightarrow i^z_t \geq 0$. For negative interest rates, markets for bonds and money do not clear: consumers want to borrow infinite amounts at negative rates to purchase money which pays zero interest.

given by

$$C(\theta_{A,j}, p_t^i, \tau_t^i) \;=\; \theta_{A,j} \left[\frac{\alpha}{p_t^i (1+\tau_t^i)} \right]^{\frac{1}{1-\alpha}} \tag{29}$$

Compared to Section 3, the demand schedule is now a function of the *seignorage-adjusted price*: firms charge a price p^i which is scaled up by the opportunity cost of having to hold a currency that is accepted in exchange. If $\tau_t^i = 0$, bonds and money have the same return. There is no opportunity cost of money and consumers pay the real price only once. If $\tau_t^i > 0$, consumers pay the full price once to firms, and another (τ_t^i)-times to the issuer of currency. As τ_t^i approaches one—or equivalently, the corresponding bond price approaches zero and inflation in this currency approaches infinity—consumers have to hold an asset that fully loses its real value in the process. Thus, they pay the full price a second time.

Finally, consumers visit the firm which they expect to charge the lowest seignorage-adjusted price. This is true for any age. When consumers are middle-aged, they receive an advertising message from firms. For equal seignorage-adjusted prices, they visit the more heavily advertising firm. Otherwise they randomly choose a firm. When consumers are young, they do not receive an advertising message and randomise immediately for equal seignorage-adjusted prices. Due to the assumption of infinite search costs, consumers only ever visit one firm to obtain a price offer in a given period. As before, let the consumer's initial search strategy be denoted by $\psi^j \in \{f, g\}$.

4.3 Producers issuing PCDC remove the opportunity cost of money

Given search frictions and resulting market power, firms set their prices in each period as if they were monopolists. In contrast to Section 3, the model now contains two types of (money good) producers: those that do not issue private currency and need to accept the public currency; and those that have introduced a PCDC. The consumer demand schedule of Equation (29) reveals that consumers consider two factors: the real price p_t^i in terms of the numeraire, and the opportunity cost of money captured by τ_t^i. For a firm without private currency, the firm sets the price and the central bank implements a particular seignorage tax rate. A producer that issues private currency chooses both: it charges a price and controls monetary policy for the currency used in the transaction. This subsection first characterises monopoly pricing for firms that transact in the public currency. It then jointly characterises product pricing and private monetary policy for firms that transact in their private currency.

Consider firm i that transacts in the public currency and does not obtain seignorage revenues on a given transaction with consumer j. The firm's corresponding profits are given by

$$\Pi_t^i \;=\; \left(p_t^i - 1\right) C\left(\theta_{A,j}, p_t^i, \tau_t^{\$}\right) \tag{30}$$

Firms charge the same price as in the partial equilibrium Diamond search game, but profits are distorted by the opportunity cost of holding Dollars. Taking public monetary policy as given, firms maximise profits. Importantly, firms optimally do not internalise the Dollar opportunity cost, and profits are directly reduced by the seignorage tax:

$$\tilde{p} \;=\; arg\,\max_{p_t^i}\; \Pi_t^i \qquad \Rightarrow \qquad \Pi^i\big(\theta_{A,j}, \tau_t^{\$}\big) \;=\; \kappa\theta_{A,j}\big(1+\tau_t^{\$}\big)^{\frac{1}{\alpha-1}} \tag{31}$$

Next consider firm f which issues Diem. The opportunity cost of money is exactly the seignorage income of the issuer of money.[27] Diem is modelled as a stablecoin: it is backed with Diem-denominated bonds that have been issued by the household.[28,29] For every unit of money issued at price one, firm f purchases a unit of bonds at price $Q_t^{\approxeq}$. The firm's total seignorage revenues, $s_t^{\approxeq}$, are then given by $s_t^{\approxeq} = \tau_t^{\approxeq} m_t^{\approxeq}$. Positive seignorage revenues require $\tau_t^{\approxeq} > 0$, implying a binding CIA constraint. Equation (29) stresses how seignorage acts as a tax on consumption: the opportunity cost of money directly scales up the firms' prices.[30] The expression for seignorage revenues also stresses the link between the seignorage tax rate, bond prices and inflation. As $\tau_t^{\approxeq}$ increases, $Q_t^{\approxeq}$ falls; by Equation (27), this corresponds to an increase in the inflation rate $\pi_{t+1}^{\approxeq}$.

Given Proposition 1, I postulate that firm g does not accept Diem in order not to generate information for their competitor. I verify this postulate having jointly characterised optimal product pricing and private monetary policy for firm f, and having discussed the consequences for public

[27] A full derivation of seignorage income from the firm's flow budget constraint is provided in Appendix D.5.

[28] Section 6.2 discusses an economy in which only Dollar-denominated bonds exist.

[29] Households are perfectly happy supplying bonds in exchange for money as long as the real bond return does not exceed $1 + r_t = \beta^{-1}$. Since $w_t = 1$, every unit of interest payments will have to be made up by supplying one unit of labour in the future, but the disutility from supplying labour is discounted by the rate of time preference β.

[30] Although not explicitly modelled here, the government's seignorage income can be obtained in perfect analogy. If the government issues bond to borrow from households, then the seignorage income is the interest expenditure saved when issuing money, a zero-interest liability.

monetary policy. It follows that—whenever seignorage revenues are positive and the CIA constraint binds—Diem holdings are exactly equal to purchases from firm f using Diem:

$$s_t^{≋} \;=\; \tau_t^{≋} p_t^f \int_{\{\psi^j = f\}} C\left(\theta_{A,j}, p_t^f, \tau_t^{≋}\right) dj \tag{32}$$

The profit-maximisation problem of firm f is then given by

$$\max_{p_t^f, \tau_t^{≋}} \;\; \left(p_t^f(1+\tau_t^{≋}) - 1\right) \int_{\{\psi^j = f\}} \theta_{A,j} \left[\frac{\alpha}{p_t^f(1+\tau_t^{≋})}\right]^{\frac{1}{1-\alpha}} dj \tag{33}$$

The following proposition jointly characterises private monetary policy and product pricing:

Proposition 2 (**Profit-maximising private monetary policy).** *A producer of the consumption good, who also supplies the money used in the transaction and controls the associated inflation rate, chooses a seignorage-adjusted price satisfying*

$$p_t^f(1+\tau_t^{≋}) \;=\; \tilde{p} \tag{34}$$

While continuing to charge a seignorage-adjusted price corresponding to the monopoly price, the firm optimally removes the opportunity cost of money. It does so by providing product discounts, pursuing a private monetary policy of $\tau^{≋} = 0$, or implementing a combination of the two.

Firm f charges a real price p_t^f. At the same time, consumers need to hold money to purchase consumption goods and are subject to the associated opportunity cost. This is captured by $(1+\tau_t^{≋})$. Consumers pay the real price once directly to firms, and another $(\tau_t^{≋})$-times indirectly to the issuer of currency as seignorage tax. Importantly, if a transaction takes place in private currency, the tax rate is also set by firm f. As the supplier of Diem and therefore recipient of seignorage revenues, firm f maximises the total sum of producer profits and seignorage tax income. In doing so, they perfectly internalise any equilibrium effects. Thus, the firm optimally implements a private currency variant of the Friedman rule, removing the tax income altogether. Firms obtain a degree of freedom: they can implement any private monetary policy and then set prices accordingly. Firm f charges exactly that seignorage-adjusted price—consisting of a real product price and the opportunity cost of money—which maximises total profits. The breakdown between product profits and seignorage revenues is irrelevant to firm f.

The result of Proposition 3 appears surprising in the context of proposed regulation. Regulators worry that issuers of Diem inflate away the value of their currency in order to increase profits, hurting consumers in the process. The model suggests that the equilibrium effect goes in the exact opposite direction: firms remove the opportunity cost of holding money, which in the case of $\tau_t^{\approx} = 0$ is associated with deflation (Equation 27).

Corollary 1 (Currency design equivalence). *If firms were allowed to pay interest on private currency in the model, Proposition 2 would extend to interest payments. Firms are indifferent between implementing $\tau_t^{\approx} = 0$, or fully compensating for inflation using either price discounts or interest payments on currency.*

Given the private currencies' digital nature, it is technologically feasible to pay interest on money holdings. Corollary 1 helps explain why the Libra White Paper v2.0 (April 2020) proposes to design Diem as non-interest-bearing currency: consumers can be incentivised to hold Diem through different measures other than interest payments. Considering the regulators' worries about the consequences arising from the introduction of PCDC, this result is again surprising. One suggested policy to mitigate the consequences is to prevent issuers of PCDC from paying interest. The model suggests that such a policy does not have any bite. It prevents issuers of PCDC from adding a feature to their currency which they do not need to add. It also cannot avoid the consequences for monetary policy outlined below.

Crucially, the results obtained in this section rely on the fact that the issuer of PCDC obtains seignorage revenues corresponding exactly to its product sales. Consumers only hold Diem in order to transact with firm f. This section should therefore be considered a benchmark. Section 5 discusses currency consortia more generally in which seignorage revenues may not be distributed according to sales shares. Section 6 introduces policies that affect seignorage revenues, i.e. through macroprudential policies which limit how firms can invest the proceeds from issuing money.

4.4 The presence of a PCDC pushes nominal interest rates to zero

Having characterised the seignorage-adjusted prices that firms charge in different currencies, I am now ready to derive the consequences for private and public monetary policy. Suppose only firm f has introduced their private currency Diem. Since I fix the firms' introduction decisions, let me call this scenario *partial equilibrium.* The partial equilibrium is interesting to discuss for various reasons. First, understanding partial equilibrium payoffs is required to fully characterise the best response of a competing firm upon the introduction of a private currency. Second, if seignorage does not provide an advantage over the competitor in equilibrium (which I show below), then counter-innovations in payment technologies need not take the form of currencies. Lastly, there may be first mover advantages outside of the model: examples include network effects, or the regulator's lack of appetite for another digital currency run by some digital conglomerate in the future.

Corollary 2 (Search and choice of currency). *Suppose* $\tau_t^{\$} > 0$. *Consumers only visit firms which have introduced their own private currencies. Whenever these firms accept the Dollar and their private currency, consumers prefer to transact using the private currency.*

Proposition 3 (Monetary policy consequences for one-sided introduction). *Suppose the government supplies a positive amount of money,* $M_t^{\$,S} > 0$. *Then money markets only clear if*

$$\tau_t^{\$} = 0 \qquad \Leftrightarrow \qquad i_t^{\$} = 0 \tag{35}$$

Given the pricing formula for bonds, this policy is associated with deflation: $\pi_{t+1}^{\$} = \beta - 1 < 0.$

Consumers rationally form beliefs about firms' prices. Given optimal product pricing combined with private monetary policy, and unless $\tau_t^{\$} = 0$, purchasing at firm f using Diem is less costly than a) using the Dollar at firm f, and b) purchasing at firm g. Suppose $\tau_t^{\$} > 0$. No consumer visits firm g's store, and thus aggregate consumption of the money good provided by firm g is zero. Since no consumer uses the Dollar at firm f, the non-negativity constraint binds, and $M_t^{\$} = 0$. Therefore money markets cannot clear if the government supplies a positive Dollar supply, $M_t^{\$,S} > 0$, unless $\tau_t^{\$} = 0$. Seignorage revenues accruing to the currency-supplying firm lower their seignorage-adjusted prices; these prices are only matched by the competing firm in the absence of any government seignorage revenues. That is, the central bank's interest rate needs to satisfy $i_t^{\$} = 0$.

4.5 Model equivalence for Friedman rule

Upon the introduction of a PCDC, the obtained consumption levels are those that would be achieved in a pure Dollar economy in which the central bank follows the Friedman rule. Here, in equilibrium, this Friedman rule is privately enforced. The PCDC disciplines the public currency, and the central is forced to remove the opportunity cost of holding money by setting nominal interest rates to zero.

Corollary 3. *By Proposition 3, the government is forced not to raise seignorage taxes. Once all seignorage taxes are removed, the competitor firm's problem in the monetary framework mirrors the problem in the partial equilibrium model of Section 3.*

Given the monetary framework's equivalence to the partial equilibrium model once the opportunity cost of money has been removed, the firms' currency acceptance and advertising decisions are unchanged relative to Section 3. Firms never want to generate information for their competitors, and hence do not accept other firms' currencies in analogy to Proposition 1. This verifies the postulate that firms do not accept competitor currencies:

$$\gamma^{g,\approx} \; = \; 0 \tag{36}$$

4.6 Welfare and efficiency

Forcing the government to implement the Friedman rule is welfare-improving. Money serves the vital role of facilitating transactions, and levying a tax on money balances reduces consumption. However, the allocations achieved following the Friedman rule are not efficient due to the search frictions, giving rise to monopoly pricing. Firms still sell their products at prices exceeding marginal costs.

4.7 Both firms introduce a private currency if information rents are large

The equilibrium private currency introduction decisions are discussed in detail in Appendix B. In short, the model features a first mover advantage. Both firms reap the equilibrium seignorage rents equally from a one-sided introduction of PCDC. Firm f forces the government to implement the Friedman rule and remove their seignorage tax, which by Equation (31) leads to an increase in firm g's profits. However, once information on consumers is being generated, firm f exploits it to improve their customer base at the cost of firm g. The first mover therefore trades off the sum of seignorage and information gains from introducing a private currency against a fixed cost of doing so. The second mover then only trades off the information gains against the fixed cost. Only if these information gains are large, both firms benefit equally. If the information gains are small relative to the fixed cost, only one firm introduces a PCDC and gains an information advantage on the competitor firm.

5 Industrial organisation of currency consortia

In the benchmark of Section 4, firms internalised the opportunity cost of holding money due to seignorage revenues. Monetary policy was forced to follow suit and implement a zero interest rate policy. Inspired by the institutional set-up of Diem as a currency consortium consisting of multiple firms and initiated by Facebook, I now consider seignorage dividend structures more generally, and discuss their effect on equilibrium inflation outcomes.

5.1 Environment

In this section, consumption utility is derived from *two* money goods:

$$U_{A,j,t} = U(X_t) + \left(C_t^f\right)^\alpha + (\chi\theta_{A,j})^{1-\alpha}\left(C_t\right)^\alpha - N_t \tag{37}$$

For simplicity, and to isolate the mechanism, I assume that firm f supplies the first money good, C^f, as a monopolist. The market for the second money good, C, mirrors the money good market of the previous section, and so do the consumers' optimality conditions. Two firms (f^*, g) produce the second money good, and consumers have to sequentially search for price offers from these firms. The parameter χ determines the relative size of the second market. Let θ^* denote the firm f^*'s average customer base. Information helps improve this customer base. The parameter thus captures the information gains in the second money good market relative to the information gains in the first money good market.

Consider the *partial equilibrium* scenario in which firms (f, f^*) have formed a currency consortium that issues Diem, the only private money in the economy. I assume that firm f is the *consortium leader*, deciding on Diem monetary policy and thus on the corresponding seignorage tax rate.[31] I refer to firm f^* as the *consortium member*.

[31] One interpretation is that the leading firm determines the initial private currency set-up, including private monetary policy, and the second firm joins the currency consortium afterwards taking this set-up as given. The Diem

As by Proposition 1 and in Section 4, competitor firm g does not accept Diem given the consortium's information rents. The consortium only obtains seignorage revenues that correspond to the Diem transactions with consortium firms. The parameter η characterises the seignorage dividend structure: the consortium leader receives a share η of the total Diem seignorage revenues. The leader's total profits are then given by:

$$\Pi_t^f \;=\; \big[p^f(1+\eta\,\tau^{\approxeq}) - mc\big]\; C^f\big(p^f(1+\tau^{\approxeq})\big) \;+\; \eta\,\tau^{\approxeq}\,p^{f^*}\,C\big(p^{f^*}(1+\tau^{\approxeq}),\chi\theta^*\big) \tag{38}$$

The consortium leader maximises profits with respect to its price p^f and the Diem seignorage tax rate $\tau^{\approxeq}$, subject to the consortium member's optimal pricing strategy and an upper bound on $\tau^{\approxeq}$:

$$\tau^{\approxeq} \;\leq\; \bar{\tau}\big(\tau^{\$},\eta\big) \tag{39}$$

The consortium member only sells positive amounts of the consumption good if they charge a weakly lower seignorage-adjusted price than their competitor. If the Diem seignorage tax rate is too high, the consortium member prefers not to be part of the currency consortium and does not generate any seignorage revenues. It follows that the Diem seignorage tax rate is bounded from above.[32] To complete the set-up, I define a notion of private currency market power:

Definition (Ownership concentration). *Let the transaction share of the consortium leader relative to the consortium member at a Diem seignorage tax rate of $\tau^{\approxeq} = 0$ be denoted by*

$$\varphi^* \;=\; \frac{1}{1+\chi\theta^*} \tag{40}$$

Ownership of seignorage dividend claims is concentrated whenever the consortium leader's dividend share exceeds their transaction share absent of a Diem seignorage tax:

$$\eta \;>\; \varphi^* \tag{41}$$

Ownership is not concentrated whenever $\eta \;=\; \varphi^*$.

consortium's proposal to issue many public currency-denominated stablecoins before all of the proposed 100 members have joined is one such scenario.

[32] Appendix F.1 provides the corresponding profit function for firm f^* and derives equilibrium pricing strategies for both firms, yielding the leader's profits as a function purely of the Diem seignorage tax rate and parameters (η, χ, θ^*), as well as the upper bound on $\tau^{\approxeq}$.

5.2 Inflationary pressures for large consortia and if ownership is concentrated

Given this set-up, I am now ready to characterise sufficient conditions such that the consortium leader levies seignorage taxes on its consortium member firms.

Proposition 4 (**Diem inflationary pressures**). *The consortium leader implements a strictly positive seignorage tax, inducing inflationary pressures, if at least one of two sufficient conditions is met:*

1. *Ownership is concentrated.*

2. *The Diem transaction share of the consortium leader is sufficiently small:* $\varphi^* < \underline{\varphi}$

Proof: see Appendix F.2. Intuitively, the consortium leader trades off maximising its own product profits and corresponding seignorage revenues against collecting seignorage income generated from the consortium member's transactions. The former is maximised as $\tau_t^{\approx} = 0$ but this sets the latter to zero. As ownership becomes concentrated, the trade-off tips in the favour of the latter: the consortium leader gains ability to internalise a positive seignorage tax while receiving a larger portion of the tax on the consortium member's transactions. Similarly, as φ^* decreases, the tax base available to the consortium leader grows relative to its own product profits, and so does the temptation to levy a seignorage tax. One interpretation of Proposition 4 is that inflationary pressures arise as the private currency becomes more commonly used in the economy. Another interpretation is that information becomes inflationary:

Corollary 4 (**Inflationary information**). *When information collection leads to a relative increase in the consortium member's transactions relative to the consortium leader, increasing the seignorage tax base, then information becomes inflationary.*

The optimal Diem seignorage tax rate— if the consortium leader were unconstrained by central bank policy—is the solution to a non-linear problem which can only be characterised analytically for a subset of parameter combinations. Hence Figure 1 plots the numerical solution as a function of dividend and transaction shares.[33] It also shows the upper bound on $\tau^{\approx}$, here illustrated for a

[33]The Figure includes two plots for two values of the curvature parameter $\alpha \in \{0.75, 0.9\}$. The higher the curvature parameter, the higher the elasticity of substitution between the credit and money goods. Consumer are more willing to switch to credit good consumption, and hence the optimal tax rate, keeping other parameters fixed, is decreasing in α.

desired central bank policy of $\tau_t^{\$} = 0.06$. Whenever the unconstrained Diem seignorage tax rate lies above the upper bound, then public monetary policy disciplines private monetary policy. Central bank enjoys full policy autonomy.

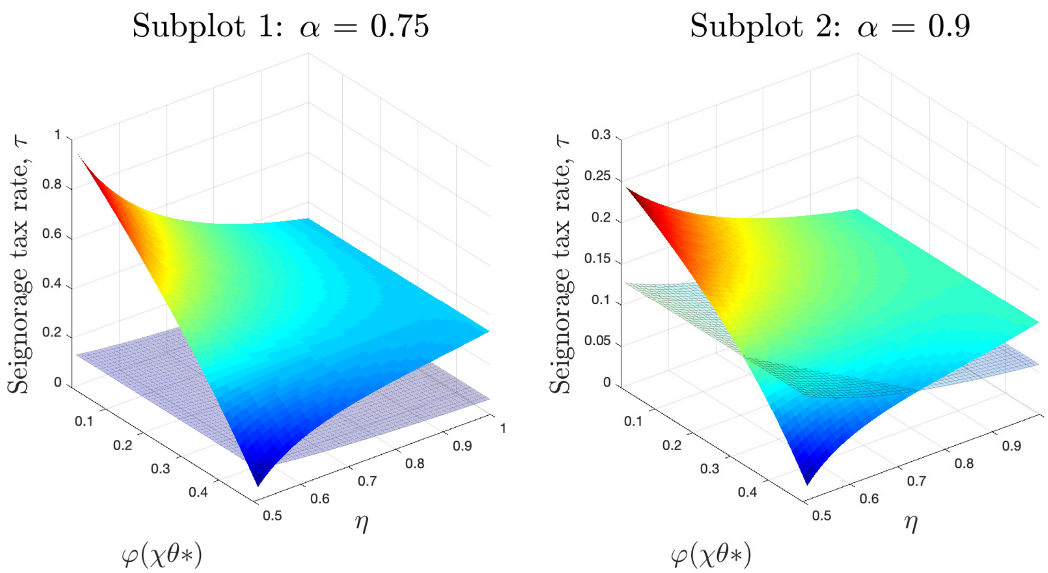

Figure 1: Unconstrained profit-maximising Diem seignorage tax rate and its upper bound

5.3 Comparative statics

Figures 2 and 3 show comparative statics of the unconstrained Diem seignorage tax rate for the combined market size and relative information gain parameters $\chi\theta^*$ as well as the dividend share parameter η. At an equal dividend and transaction share of $\eta = \varphi^* = \frac{1}{2}$, the profit-maximising seignorage tax rate is zero. At this point, decreasing the leader's transaction share or increasing its ownership share leads to an increase in $\tau^{\approx}$. Figure 2 demonstrates that the effect of decreasing φ^*—either increasing the size of the consortium member market χ or increasing their market share θ^* due to information—remains positive over the entire parameter space. New consortium members and larger information gains are always (weakly) inflationary.

The effect of increasing the leader's seignorage dividend share η is ambiguous. At a seignorage tax rate of zero, the effect of increasing ownership concentration on seignorage tax rates is positive and large. The consortium leader obtains a larger share of the total seignorage income, gains in ability to internalise the tax' equilibrium effect, and thus prefers to raise taxes. Away from the

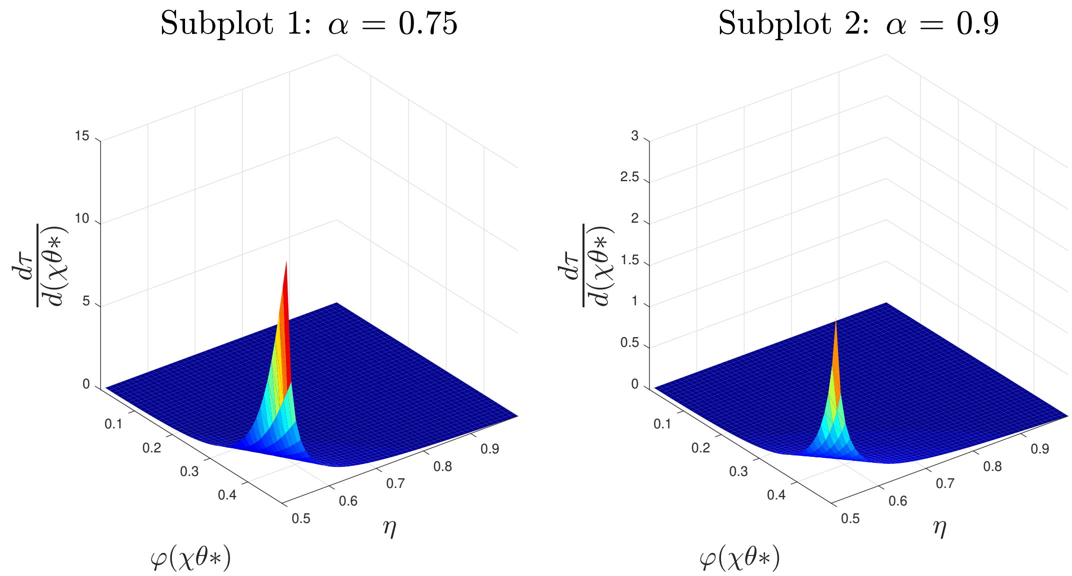

Figure 2: Comparative statics: increasing the consortium member's transaction share

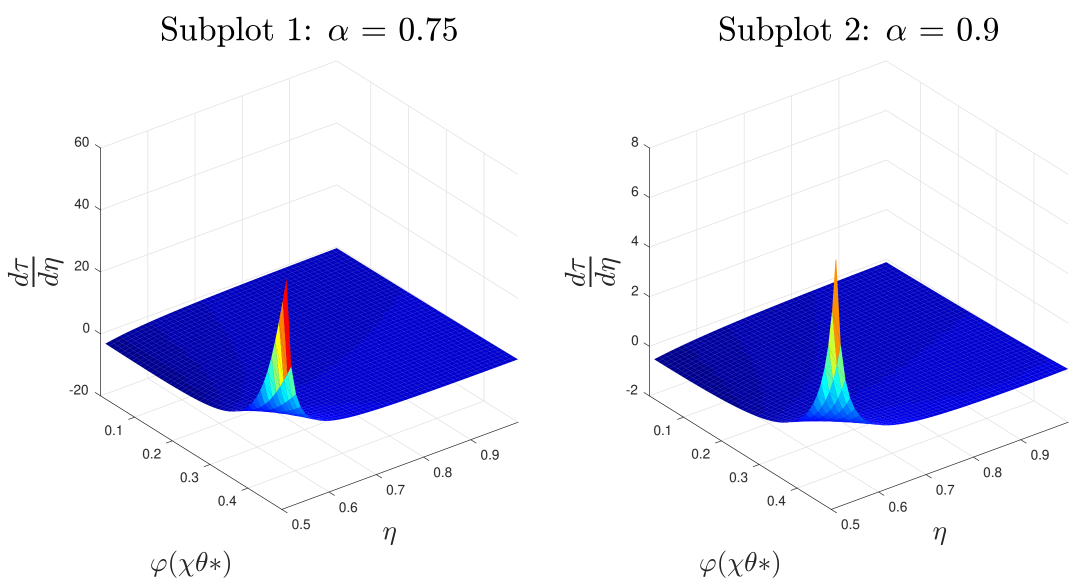

Figure 3: Comparative statics: increasing the consortium leader's dividend share

Friedman rule, the effect is negative, especially if the consortium member's transaction share is large. The reasons is that the consortium leader—if unconstrained by the central bank—charges a high seignorage tax rate. Reducing the consortium member's seignorage dividend share implies that they are less able to internalise the effect of the seignorage tax. Hence, the consortium leader optimally would like to charge a lower seignorage tax rate.

5.4 Discussion of the proposed set-up of Diem

The fact that the consortium leader may benefit from private currency inflation has implications for the optimal design of private currencies. In particular, the model offers an explanation why the Diem consortium are planning on issuing many stablecoins denominated in public currencies, effectively adopting the public currency as unit of account. Through the lens of the model, this corresponds to a scenario in which the public currency disciplines the PCDC, and the consortium leader maximises their profits by implementing the government's preferred inflation policy. In the same vein, the model helps explain why the Diem association plan on designing their currency as non-interest-bearing.

Information may well be the reason why Diem are willing to adopt public monetary policy. The Diem consortium is lead by Facebook, a firm in the business of collecting information on consumers in order to tailor advertising. If information helps increase its consortium members' market shares, then this incentivises the consortium leader to implement a seignorage tax, which in turn reinstates central bank autonomy.

In sum, this section demonstrates that private currency market power creates inflationary pressures. The general equilibrium concerns remain: in partial equilibrium, firm f optimally implements strictly positive seignorage tax rates. The counter-innovation incentives for firm g therefore increase relative to Section 4.

6 Extensions

This section presents a variety of further extensions to the benchmark outlined in Section 4. First, paying interest on public currency allows the central bank to escape the zero interest rate environment. Second, regulators fear that issuers of PCDC may be tempted to inflate away the value of their currency if it is backed with assets denominated in another currency. I show that firms cannot increase profits through such capital gains. Lastly, subsection 6.3 evaluates a frequent argument for the success of public currencies: the government can force firms to pay sales taxes in the public currency. Similarly, as seen in China for Alipay and WeChat Pay, the regulator could impose macroprudential policies which force firms to invest proceeds from issuing private currency in central bank reserves.

6.1 Interest-bearing CBDC

In Section 4, the central bank was forced to set a zero nominal interest rate. Here I show that central bank digital currency, paying an interest rate $i_t^{M^\$}$, allows central banks to internalise the opportunity cost of holding public money without giving up central bank autonomy.

Proposition 5. *The central bank can escape the zero lower bound by issuing interest-bearing digital currency. The interest rate on digital currency must match the interest rate paid on bonds:*

Appendix G.1 presents two variants of the model which formally prove Proposition 5. For a two-sided introduction of private currency, the Dollar loses its role as medium of exchange. However, the central bank can convince consumers to hold Dollars even though it does not facilitate transactions. As long as the interest paid on bonds does not exceed the interest received on money, consumers are happy to scale up their balance sheets by purchasing government currency. For a one-sided introduction, consumers only hold Dollars if accepting firms do not charge strictly higher seignorage-adjusted prices. Since issuers of private currency internalise the opportunity cost of holding money, the central bank of Section 4 was forced to follow suit and set $\tau_t^\$ = 0$ ($i_t^\$ = 0$). Equivalently, the central bank can compensate consumers by paying sufficient interest on money.

6.2 Capital gains due to higher private currency inflation

Previously I assumed that the assets held by firms, household-issued bonds, were of the same denomination as their liabilities, private currency. In this subsection, I assume that bonds may only be issued in Dollars. This could give rise to a motive for higher inflation on the private currency: firms can effectively force consumers to hold the private currency by not accepting the Dollar; then generating a high inflation rate on private currency can lead to an appreciation of the Dollar-denominated assets relative to the liabilities denominated in the private currency. The firm obtains capital gains. This section establishes that issuers of PCDC cannot increase their profits relative to the benchmark using capital gains.

Proposition 6. *Suppose the firm only holds Dollar-denominated assets. Then, in equilibrium and relative to the benchmark of Proposition 2, the firm cannot increase profits through capital gains resulting from higher inflation on its own currency.*

The formal proof is provided in Appendix G.2. Intuitively, total profits are broken down into three components: product profits, direct seignorage revenues, and indirect capital gains. However, the profit function mirrors the one in Equation (33). Thus, the result on the jointly optimal private monetary policy and product pricing of Proposition 2 still holds.

$$\begin{aligned} \Pi_t^{f,total} &= \underbrace{\Pi_t^f\left(p_t^f, \tau_t^{≋}\right)}_{\text{product profits}} + \underbrace{\tau_t^{\$} m_t^{≋}}_{\text{seignorage}} + \underbrace{\left(\tau_t^{≋} - \tau_t^{\$}\right) m_t^{≋}}_{\text{capital gains}} \\ &= \Pi_t^f\left(p_t^f, \tau_t^{≋}\right) + \tau_t^{≋} m_t^{≋} \end{aligned} \tag{42}$$

To deepen the intuition, note that seignorage revenues are governed by the Dollar seignorage tax rate $\tau_t^{\$}$. Capital gains however are governed by the difference in the Dollar and Diem seignorage tax rates: for every unit of real Diem balances the firm holds real Dollar bonds, and their relative value increases proportionally in the Diem inflation rate and therefore its seignorage tax rate. On the consumer's side, money holdings are determined by the opportunity cost of holding Diem. The breakdown between direct seignorage revenues and indirect capital gains, captured by the Dollar seignorage tax rate, is irrelevant for consumers. Thus, profits are only affected by the Diem seignorage tax rate and the firm implements a private monetary policy as characterised in Proposition 2.

The general equilibrium consequences for monetary policy are unchanged. Firm f perfectly internalises the opportunity cost of holding Diem. Firm g is priced out of the market unless $\tau_t^{\$} = 0$, forcing the central bank to follow suit.

6.3 Forcing firms to hold public currency

A popular argument for the success of government fiat currency is the fact that governments can demand taxes to be paid in public currency. In this section firms need to pay a fraction λ of their sales in tax, payable in Dollars:

$$\lambda p_t^f C_t^f \leq \frac{M_t^{f,\$}}{P_t} \tag{43}$$

where $M_t^{f,\$}$ denotes the Dollar holdings of firm f. It is immediately clear that such a policy leads to positive demand for Dollars whenever firm f sells any goods to consumers, even if they only accept Diem. This section establishes intuitively that sales taxes payable in Dollars lead to high levels of private currency inflation which, surprisingly, are welfare-improving. A formal derivation of the result is presented in Appendix G.3.

Consider a binding CIA constraint for which real expenditure on consumption goods with firm f correspond to the economy's real Diem balances: $p_t^f C_t^{m,f} = m_t^{\approx}$. Firms prefer to hold interest-bearing bonds over money, and thus the constraint in Equation (43) binds:

$$\lambda m_t^{\approx} = \frac{M_t^{f,\$}}{P_t} \qquad (1-\lambda) m_t^{\approx} = \frac{B_t^f}{P_t} \tag{44}$$

where B_t^f denotes firms f's Dollar bond holdings.[34] Total firm profits are then given by:

$$\Pi_t^{f,total} = \underbrace{\Pi_t^f\left(p_t^f, \tau_t^{\approx}, \lambda\right)}_{\text{product profits}} + \underbrace{(1-\lambda)\tau_t^{\$} m_t^{\approx}}_{\text{seignorage}} + \underbrace{\left(\tau_t^{\approx} - \tau_t^{\$}\right) m_t^{\approx}}_{\text{capital gains}} \tag{45}$$

Forcing firms to hold a part of their asset portfolio in public currency acts as a tax on direct seignorage revenues, breaking the one-for-one relationship with capital gains of the previous subsection. The firm collects interest only on its bond holdings, but the capital gains accrue for both money and bond holdings. The balance of internalising the opportunity cost of money either through discounts or through deflation tips in favour of high inflation and corresponding product discounts, in order to achieve capital gains.

Proposition 7. *Suppose the government forces the firm to pay sales taxes in Dollars. Then both firm profits and consumer welfare are increasing in Diem inflation, and are maximised as*

$$\tau^{\approx} = 1 \qquad \Leftrightarrow \qquad \pi^{\approx} \to \infty \tag{46}$$

It follows that private monetary policy is characterised by infinite inflation, associated with higher consumption and thus higher consumer welfare. Consumers have to hold an asset that fully loses its real value in order to purchase a consumption good; however, this good is sold at a low price, possibly below marginal costs, to compensate for the inflation losses. Effectively, capital gains allow the firm to circumvent both the sales tax and the enforced opportunity cost of holding Dollars. Both consumers and firm benefit.

[34] Without loss of generality, I maintain the assumption that all household bonds are issued in Dollars.

General equilibrium. The option to circumvent the sales tax through capital gains combined with product discounts has stark consequences for the competitor firm g, who is fully priced out of the market (see Appendix G.3). Thus, the incentive to counter-innovate and also introduce a digital private currency is extremely high. Since the government forces firms to hold Dollars to pay taxes, the central bank does not need to set nominal interest rates to zero. While public money may lose its role as medium of exchange between firms and consumers, it finds a new role as medium of exchange between firms and government.

Macroprudential policy. Regulating agencies in Europe and the US are in the process of drawing up a rule book for companies issuing stablecoins such as Diem, including restrictions on asset investments. In the model, the equilibrium logic of this subsection is unchanged when firms are forced to hold government currency as a macroprudential measure. Issuers of private currency aim to circumvent the policy by inflating away the value of their currency, achieving capital gains; consumers are compensated with discounts. Profits and welfare are increasing in private currency inflation.[35]

Macroprudential policy for a widely used PCDC. Alipay and WeChat Pay are subject to very tight macroprudential regulation imposed by the People's Bank of China (PBoC). In particular, they need to hold 100% of their proceeds as reserves with the PBoC.[36] The model helps explain why the value of private currency—the tokens held in digital wallets—is pegged to the value of the public currency, the Renminbi (RMB). Both Alipay and WeChat Pay are widely used with third-party firms. These firms do not give product discounts as they do not receive any seignorage revenues or capital gains. The issuers of PCDC therefore trade off the increase in capital gains against the seignorage and information rents obtained when other firms transact using their technologies. If the latter outweigh the former, then firms prefer to also implement the public interest rate set by the PBoC and peg the value of their tokens to the value of the RMB. If capital gains are sufficiently large, then these firms would prefer to inflate away the value of their currency. In that case, no transactions with other third-party firms would take place using their technologies.

[35] See Appendix G.4 for a formal derivation of the result.

[36] Source: General Office of the People's Bank of China (2018), issue no. 114.

7 Conclusion

This paper is the first work to formally analyse the relationship between information and seignorage for private digital currencies issued by firms. The model highlights two important interactions between information and seignorage. First, seignorage revenues accrue to the issuer of private currency, but information limits the degree to which firms can collect such seignorage proceeds. The resulting private and public inflation outcomes depend on whether large firms issue currencies on their own, or whether many smaller firms form currency consortia which are dominated by one firm. The central bank loses its policy autonomy in the former case but regains it in the latter. In the second case, information becomes inflationary when it increases the seignorage tax base which the consortium leader can exploit using higher inflation rates. The model helps explain the Diem consortium's plans to issue public currency-denominated stablecoins.

In my analysis, information affects the size of the seignorage tax base, leading to different private and public monetary policy outcomes depending on the institutional set-up of the issuer of private money. Looking ahead, I consider exploring the formation of currency consortia as well as the interaction between information and seignorage in the presence of price discrimination and network effects important and fruitful avenues of future research.

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Appendices

A Partial Equilibrium payment technology introduction decisions

Lemma 2 (**Symmetric payment technology introduction decision).** *Consider a fixed cost $k > 0$ to introduce the payment technology. Let the value of information in the partial equilibrium model be denoted by Δ. Then both firms introduce the payment technology if $\Delta \geq k$; otherwise no firm does so.*

The proof is provided in Appendix C.4. By Lemma 1, industry profits are fixed at $\Pi = \kappa\mathbb{E}[\theta]$. Whenever firms have equivalent sets of information—either because no firms have generated transaction data, or both have generated equal amounts—the expected payoffs for the two firms equal. Firms evenly divide the market in half. It follows that any information gains are mere redistributions from one firm to another. When firm i introduces the technology, their profits increase at the expense of their competitor. When the competitor also introduces their own technology, both firms again obtain information of the same value: firm i loses exactly those profits initially gained from firm $-i$.

Lemma 2 states that there is no first mover advantage in introducing payment technologies. The result resonates with the observation that Alipay and WeChat Pay share the market for digital payments in China. With the imminent introduction of Diem, all of the digital giants Apple, eBay, Google and Facebook will have introduced payment technologies. However, this result is less likely to hold up in more general environments than the one presented here. Having to handle an increasing number of payment technologies, or currencies in Section 4, could become increasingly costly for consumers. General adoption also depends on network effects: consumers only want to use a payment technology (hold a particular currency) if they expect others to accept it. The number of equilibrium payment technologies may therefore well be limited once these considerations are included in the framework. In a similar vein, Section 4 presents a first mover advantage arising due to the second benefit of introducing a PCDC: seignorage revenues.

B General Equilibrium PCDC introduction decisions

Using the results of subsection 4.4, I now allow firms to counter-innovate by also introducing a private currency, thus calling this scenario *general equilibrium*. I highlight a first mover advantage in the monetary framework: while information rents reduce the competing firm's profit, seignorage revenues accrue to both firms in equilibrium. The second mover therefore may not find it profitable to also introduce a PCDC.

The first mover gains can be neatly decomposed into two components: information rents in a zero interest rate environment, and seignorage revenues. Prior to the introduction of a private currency, firms split the market equally. By Equation (31), the opportunity cost of holding Dollars acts as a tax on consumption, lowering profits for all firms in every time period. Proposition 3 showed that this opportunity cost is fully removed for all currencies upon the introduction of one private currency. Both firms benefit equally and achieve the seignorage gains, denoted by $\Delta^{\$}\left(\left\{\tau^{\$}_{t+s}\right\}_{s\geq 0}\right)$.

Turning to information, let Δ^I denote the lifetime information rents for an economy with zero interest rates.[37] The first mover trades off the total gains from introducing a private currency against a fixed cost of doing so. The second mover then trades off only the information gains Δ^I against the fixed cost.

Proposition 8 (**First mover advantage).** *If the fixed cost of introducing the currency is smaller than the lifetime information gain, $k \leq \Delta^I$, then both firms introduce private digital currencies. Neither accept the Dollar nor their competitor's currency. The Dollar loses its role as medium of exchange, and thus money market clearing for a positive supply $M_t^{\$,S} > 0$ again requires $\tau_t^{\$} = 0$.*

For an intermediate cost k, with $\Delta^I < k \leq \Delta^I + \Delta^{\$}\left(\left\{\tau^{\$}_{t+s}\right\}_{s\geq 0}\right)$, only one firm introduces a private currency. The competitor firm does not accept it. The first mover achieves both information and seignorage rents. While information rents impose a negative externality on the competitor firm, forcing the central bank to implement the Friedman rule imposes a positive externality.

Whether the public currency loses its role as medium of exchange depends on the size of the economy's information rents. For sufficiently large information rents, all firms form digital currency

[37] Let Δ denote the period information gain for an economy with zero interest rates. It corresponds to the one-time information gain in Section 3. Firms discount future profits using the household's real discount factor, given by β in equilibrium. Since the firm benefits from information for the first time in the period after introducing the currency, the total lifetime information gain is given by $\Delta^I = \frac{\beta}{1-\beta}\,\Delta$.

areas as introduced by Brunnermeier et al. (2019): although transactions take place within one economy, they are settled using different currencies in different marketplaces. Information generated by purchases is valuable, and thus firms aim to maximise their own information set while minimising that of the competitor. This is achieved if each firm only accepts their private currency. It follows that firms prefer not to transact in the Dollar even for a central bank monetary policy of zero interest rates. However, given the privately-enforced Friedman rule result, payment technologies introduced by second movers need not take the form of private currencies. One currency already disciplines the government, and firms can rely on other technologies to generate transaction data.

Proposition 8 also shows that it is more profitable to introduce a PCDC in countries with high inflation. Incentives are higher if the central bank is ill-disciplined and monetary policy effectively taxes transactions between firms and consumers. Even in the absence of any PCDC, public currency interest rates are capped. As $\tau_t^{\$}$ increases—or equivalently as $\pi_{t+1}^{\$}$ increases—profits decrease. Unless the cost of introducing a PCDC approaches infinity, there exists a threshold level of public currency ill-discipline at which one firm introduces a PCDC, disciplining the government forever. This finding resonates with the observed dollarisation in economies which experienced high inflation rates.

Finally, the model's first mover advantage also resonates with frequently made arguments for the clear dominance of one currency in the majority of economies.[38] In all likelihood, handling multiple currencies is mentally costly, and the marginal cost of holding another currency is increasing. General adoption also depends on network effects: consumers only want to hold a particular currency if they expect others to accept it. Interestingly, the People's Bank of China is planning on issuing CBDC very shortly. One doubt about its future success is whether the CBDC can compete with Alipay and WeChat Pay which have successfully established themselves economy-wide. The example of China shows that two digital payment technologies can co-exist, but the number of currencies that can circulate in an economy may well be limited. Such considerations strengthen this paper's result. Together, they help explain why Facebook is pushing ahead with their currency project, although they have been unable to convince the desired amount of 100 firms to join their consortium so far.

[38] One argument for the success of public currencies is that the government can force agents to pay taxes in the public currency. I evaluate the equilibrium effects of such a policy in Section 6.3.

C Appendix to Section 3

C.1 Consumer strategies in full formality

Given the demand schedule $c(p, \theta_j)$, let $V_j(p)$ denote the value function of a consumer of type θ_j:

$$V_j(p) \;=\; u_j\big(c(p, \theta_j)\big) \tag{IC.1}$$

Consumers shop at the firm where they have obtained the lowest price offer. Since time-1 is the final period, decisions have no consequences for any future payoffs. Thus, consumers first visit the firm which they expect to charge lower prices. Let $\mu^j(p)$ denote the beliefs of consumer j over the firms' prices, and $\psi^j(\mu^j(p))$ their initial search strategy. If they expect both firms to charge the same price, the relative advertising intensity determines the first search; otherwise consumers randomise:

$$\psi^j(a^j, \mu^j(p)) \;=\; \begin{cases} i & \text{if } \;\mathbb{E}\Big[p^i|\mu^j\Big] \;<\; \mathbb{E}\Big[p^{-i} \mid \mu^j\Big] \\ i & \text{if } \;\mathbb{E}\Big[p^i|\mu^j\Big] \;=\; \mathbb{E}\Big[p^{-i} \mid \mu^j\Big], \text{ and } a^{i,j} \;>\; a^{-i,j} \\ \begin{cases} i \quad \text{with prob. } \frac{1}{2} \\ -i \quad \text{with prob. } \frac{1}{2} \end{cases} & \text{if } \;\mathbb{E}\Big[p^i \mid \mu^j\Big] \;=\; \mathbb{E}\Big[p^{-i} \mid \mu^j\Big], \text{ and } a^{i,j} \;=\; a^{-i,j} \end{cases} \tag{IC.2}$$

Let $\omega^j(p^i, \mu^j(p^{-i}))$ denote the consumer's continuation search decision, having learnt about firm i's price. Consumers decide not to search whenever they believe that obtaining another price offer is not profitable. Optimally, they follow a cut-off rule, trading off paying the additional search cost S and achieving higher utility due to a lower price offer:

$$\omega^j(p^i, \mu^j(p^{-i})) \;=\; \begin{cases} 0, \;\text{ if } \;V_j(p^i) \;\geq\; \mathbb{E}\big[V_j(p^{-i}) \mid \mu^j(p^{-i})\big] \;-\; S \\ 1, \;\text{ o/w} \end{cases} \tag{IC.3}$$

C.2 Proof of Lemma 1

First, I show that charging the monopoly price is indeed the unique equilibrium outcome in the final period (again omitting time subscripts for readability). Since the firm's profit function has a unique maximum at $\tilde{p}$ for all consumer types θ, firms have no incentive to increase the price. Given consumer beliefs $\mu(p) = \tilde{p}$, the consumers' initial search strategy is either decided through advertising, or via a coin toss. Thus firms also have no incentive to reduce the price to attract more consumers. Consumers, having obtained one monopoly price quote, have no incentive to search any further, given their beliefs to obtain the exact same price quote a second time. Therefore no agents have any incentive to deviate unilaterally.

Now consider an equilibrium candidate for which firm i charges a price $p^i < \tilde{p}$, and the competitor $-i$ charges a price weakly larger than firm i: $\mu(p^i) = p^i \leq p^{-i} = \mu(p^{-i})$. According to the consumers' initial search strategies, and given consumer beliefs, all consumers visit firm i first if the above inequality is strict. By the law of large numbers, half of the measure of consumers visit firm i if the inequality holds with equality. Consider next the consumers' cut-off rule $\omega\big(p^i, \mu(p^{-i})\big)$. Let $\bar{p}$ denote the price that makes the consumer exactly indifferent between obtaining another price quote or purchasing from firm i, satisfying:

$$V^j(\bar{p}) \;=\; \mathbb{E}[V^j(p^{-i}) \mid \mu(p^{-i})] \;-\; S \tag{IC.4}$$

Note that $V^j(p^i)$ is continuous in p^i. Further note that the profit function $\Pi(p^i, \theta)$, which captures profits conditional on selling, is also continuous in p^i. It then follows that there exists an $\varepsilon > 0$ for every $p^i < \tilde{p}$ such that charging a price $\tilde{p} = p^i + \varepsilon \leq min\{\bar{p}, \tilde{p}\}$ yields strictly higher profits: $\Pi(\tilde{p}, \theta) > \Pi(p^i, \theta)$. Firm i thus unilaterally deviates, and there can be no equilibrium for which $p^i \leq p^{-i} \leq \tilde{p}$, with at least one inequality strict.

It follows that consumers of both types anticipate to face the same price $\tilde{p}$ at both firms in the final period. Therefore their time-0 strategies cannot affect future utility, removing all strategic considerations other than searching for the lower price. The conditional demand schedules and the continuation search strategies at time-0 mirror those of time-1. The initial search strategy is given by

$$\psi_0^j\big(\mu^j(p_0)\big) = \begin{cases} \begin{cases} i & \text{with prob. } \frac{1}{2} \\ -i & \text{with prob. } \frac{1}{2} \end{cases} & \text{if } \mathbb{E}\Big[p_0^i \mid \mu_0^j\Big] \;=\; \mathbb{E}\Big[p_0^{-i} \mid \mu_0^j\Big] \\ i & \text{if } \mathbb{E}\Big[p_0^i \mid \mu_0^j\Big] \;<\; \mathbb{E}\Big[p_0^{-i} \mid \mu_0^j\Big] \end{cases} \tag{IC.5}$$

Following the same steps as above, I find that both firms charge the monopoly price at time-0, and thus in all time periods.

C.3 Proof of Proposition 1

In the initial period, given that consumers randomise between firms in equilibrium, firm f is visited by exactly half of the measure of consumers, with the average type of their customers equalling the population average type. Assume that all consumers use firm f's technology at firm f (without loss of generality). With a payment technology generating information, firm f thus learns that $\frac{q}{2}$ consumers have high valuations, while $\frac{1-q}{2}$ consumers have low valuations. If $\xi > \frac{q}{2}$, some advertising capacity remains after firm f has advertised all of its previous high valuation customers. Suppose that firm g accepts their competitor's technology. Let the share of purchases with firm g conducted using firm f's technology be denoted by $\rho > 0$. Since $\xi > \frac{q}{2}$, it must be that $\xi > \frac{(1+\rho)q}{2}$ for some $\rho > 0$. Information accrues to the owner of the technology used to transact. Not accepting the competitor technology, $\gamma^{g,acc} = 0$, corresponds to a share of $\rho = 0$. Effectively, I compare payoffs for firm g for these two values of ρ.

Consider first a firm g that does not operate a payment technology. This firm cannot match consumer ID and purchase, and therefore does not know who their previous customers are. Thus, it randomises among the general population of consumers when advertising. If $\rho = 0$, firm f optimally advertises to a fraction $\frac{q}{2}$ of all consumers, and optimally does not advertise to a fraction of $\frac{1-q}{2}$. The remaining capacity is used for a total measure of $\xi - \frac{q}{2}$ of consumers that were firm g's customers in the previous period. Thus firm f has no information on their types: some low valuation consumers receive firm f advertising. Now suppose $\rho > 0$. Firm f learns the valuation for an additional $\frac{\rho}{2}$ measure of consumers, and therefore identifies an additional $\frac{\rho q}{2}$ measure of high valuation types and directs advertising towards them. Given the randomisation strategy of firm g, the probability of double advertising to a previous customer of g's with a high valuation increases. The probability of double advertising to a low type decreases in exactly the same proportion as it increases for high types. Overall, the expected match quality for firm g decreases as ρ becomes strictly positive, and so do profits.

The above reasoning is demonstrated algebraically below for the case in which $\xi > \frac{(1+\rho)q}{2}$. Firm g randomises, advertising to each consumer with a probability of ξ. Thus, the matching probability conditional on firm f's advertising decision is given by

$$P\big[\psi^j = f | a^{f,j} = 1\big] = (1-\xi) + \frac{\xi}{2} \qquad P\big[\psi^j = f | a^{f,j} = 0\big] = \frac{1-\xi}{2} \tag{IC.6}$$

Consumers with known type generate profits of $\kappa\theta_H$ and $\kappa\theta_L$, respectively; consumers with unknown type generate $\kappa\mathbb{E}[\theta]$ in expectation. Firm f identified a measure $\frac{(1+\rho)q}{2}$ of consumer to be the high type, and a measure of $\frac{(1+\rho)(1-q)}{2}$ of consumers to be the low type. Since $\xi > \frac{(1+\rho)q}{2}$, firm f advertises to all high types, and uses the remaining capacity for consumers that have not been f's customers in the previous period. Expected profits of firm f are thus given by:

$$\begin{aligned}\Pi^f \;=&\; \frac{(1+\rho)q}{2}\left[(1-\xi) + \frac{\xi}{2}\right]\kappa\theta_H \;+\; \frac{(1+\rho)(1-q)}{2}\left[\frac{1-\xi}{2}\right]\kappa\theta_L \\ &+\; \left[\xi - \frac{(1+\rho)q}{2}\right]\left[(1-\xi) \;+\; \frac{\xi}{2}\right]\kappa\mathbb{E}[\theta] \;+\; \delta\left[\frac{1-\xi}{2}\right]\kappa\mathbb{E}[\theta] \\ &\text{where} \quad \delta = 1 - \frac{(1+\rho)(1-q)}{2} - \xi\end{aligned} \tag{IC.7}$$

$$\frac{\partial \Pi^f}{\partial \rho} \;=\; \frac{\kappa}{4} q(1-q)\big[\theta_H - \theta_L\big] > 0 \tag{IC.8}$$

The derivative of the profit function with respect to ρ yields that firm f's profits are increasing in ρ. Given that equilibrium industry profits are fixed at $\Pi = \kappa\mathbb{E}[\theta]$, it must be that firm g's profits are decreasing in ρ. It follows that $\gamma^{g,acc} = 0$.

If $\xi < \frac{(1+\rho)q}{2}$, the relevant comparison of payoffs is for the case where firm g does not accept firm f's payment technology ($\rho = 0$); and the case where firm f identifies more high valuation types than they can advertise to, corresponding to a maximum value of ρ given by $\bar{\rho}$, satisfying $\xi = \frac{(1+\bar{\rho})q}{2}$.

The algebraic proof for a firm that has also introduced a payment technology is not presented here. Noting that the information loss is even stronger if firm g is collecting information themselves, the claim immediately follows. Above I showed that more competitor information and a constant information for firm g negatively affected firm g's profits. With a technology, not only does firm g generate information for competitor firm f when accepting their technology, but they also directly reduce the information they themselves collect. The losses in profits are even stronger for a firm that has introduced a payment technology and thus $\gamma^{g,acc} = 0$.

C.4 Proof of Lemma 2

The first mover gains in profits from introducing the payment technology, denoted by Δ^{1st}, are given by

$$\Delta^{1st} = \Pi^i\big[(1,0),(0,0)\big] - \Pi^i\big[0,0\big] \tag{IC.9}$$

Since industry profits are fixed at $\Pi = \kappa\mathbb{E}[\theta]$ in any scenario, and the firms' payoffs always equal for fully symmetric strategies and equivalent levels of information, it must be that[39]

$$\begin{aligned}\Delta^{2nd} &= \Pi^i\big[(1,0),(1,0)\big] - \Pi^i\big[(0,0),(1,0)\big] \\ &= \Pi^i\big[(1,0),(1,0)\big] - \Big[\Pi^i\big[0,0\big] - \Delta^{1st}\big]\Big] \\ &= \Delta^{1st}\end{aligned} \tag{IC.10}$$

First and second mover gains, denoted by Δ^{2nd}, are equal. If one firm finds it profitable to introduce the payment technology, so does the other: $\Delta = \Delta^{1st} = \Delta^{2nd} \geq k$. Any potential advantages that the payment technology delivers are only redistributions of equilibrium profits from one firm to another. When firm i introduces the technology, their profits increase at the expense of their competitor. When the competitor also introduces their own technology, the game is symmetric again: firms have information of equal value, profits are shared equally, and firm i loses exactly those profits initially gained from firm $-i$.

[39]Information is not symmetric because firms will always obtain information on different individual consumers. However it is equivalent if both firms have collected information on half of the population, where each half is of the same average type $\mathbb{E}[\theta]$.

D Appendix to Section 4

D.1 Equilibrium definition

The competitive equilibrium of this economy is given by

1. Set of firm strategies that solve the profit maximisation problems, given beliefs μ_t^i

$$\left(\Gamma^i, p_t^i, a_t^i, \tau_t^z\right)_{i\in\{f,g\},\, z\in\{\approx,\mathbb{G}\},\, t\geq 0}$$

2. Set of consumer strategies that solve the utility maximisation problem, given beliefs μ_t^j

$$\left(\psi_{j,y,t}, \psi_{j,m,t}, C_{j,A,t}, X_{j,A,t}, M^z_{j,A,t}, B^z_{j,A,t}, N_{j,A,t}\right)_{j\in[0,1],\, A\in\{y,m,o\},\, z\in\{\$,\approx,\mathbb{G}\},\, t\geq 0}$$

3. Set of prices $\left(w_t, P_t, \tau_t^{\$}, e_t^z\right)_{z\in\{\approx,\mathbb{G}\},\, t\geq 0}$

such that the markets for labour, the credit good, the money good, bonds and money clear. Beliefs are formed rationally, and updated according to Bayes' Law.

D.2 The middle-aged consumer's maximisation problem

Consider the second period of consumer j's life, born at time $t-1$, having visited firm f's shop and learnt their price p_t^f. The consumer maximises remaining lifetime utility subject to budget constraints when middle-aged and old (Equation 21) and the CIA constraint when middle-aged (Equation 25). As in the main body of text, drop all j-subscripts for readability. Consumer j's total money holdings in Dollar values is denoted by $e_t M_t$:

$$\boldsymbol{e_t M_{m,t}} \;=\; M^{\$}_{m,t} \;+\; e_t^{\approx} M^{\approx}_{m,t} \;+\; e_t^{\mathbb{G}} M^{\mathbb{G}}_{m,t} \tag{ID.1}$$

where the exchange rates $(e_t^{\approx}, e_t^{\mathbb{G}})$ denote the price of private currencies in terms of the Dollar. Formally, the non-negativity constraints on their real money holdings reads:

$$\frac{M^{\$}_{m,t}}{P_t},\ \frac{e_t^{\approx} M^{\approx}_{m,t}}{P_t},\ \frac{e_t^{\mathbb{G}} M^{\mathbb{G}}_{m,t}}{P_t} \;\geq\; 0 \tag{ID.2}$$

The Dollar value of total bond holdings, $e_t Q_t B_t$, is given by:

$$\boldsymbol{e_t Q_t B_{m,t}} \;=\; Q_t^{\$} B_{m,t}^{\$} \;+\; e_t^{\approxeq} Q_t^{\approxeq} B_{m,t}^{\approxeq} \;+\; e_t^{\mathbb{G}} Q_t^{\mathbb{G}} B_{m,t}^{\mathbb{G}} \tag{ID.3}$$

where Q_t denotes the prices of bonds issued at time-t, to mature in the following period. Bond prices are inversely related to the interest rate prevailing in the respective currencies:

$$Q_t^z = \frac{1}{1+i_t^z} \qquad z \in \{\$, \approxeq, \mathbb{G}\} \tag{ID.4}$$

Lastly, summarise money holdings accepted in exchange by firm f's as

$$\frac{\boldsymbol{\gamma^f e_t M_{m,t}}}{P_t} \;=\; \gamma^{f,\$} \frac{M_{m,t}^{\$}}{P_t} \;+\; \frac{e_t^{\approxeq} M_{m,t}^{\approxeq}}{P_t} \;+\; \gamma^{f,\mathbb{G}} \frac{e_t^{\mathbb{G}} M_{m,t}^{\mathbb{G}}}{P_t} \tag{ID.5}$$

The above imposes that firm f accepts their own currency Diem. The utility maximisation problem is then given by

$$\max_{\substack{\{X_{m,t}, X_{o,t+1}, C_{m,t}, N_{m,t},\\ N_{o,t+1}, B_{m,t}, M_{m,t}\}}} \; U(X_{m,t}) \;+\; \theta_j^{1-\alpha}(C_{m,t})^{\alpha} \;-\; N_{m,t} \;+\; \beta\Big[U(X_{o,t+1}) - N_{o,t+1}\Big]$$

$$\begin{aligned}
\text{s.t.} \quad X_{m,t} + p_t^f C_{m,t} + \frac{\boldsymbol{e_t(M_{m,t} + Q_t B_{m,t})}}{P_t} \;&\leq\; N_{m,t} + \frac{\boldsymbol{e_t(M_{y,t-1} + B_{y,t-1})}}{P_t} \\
X_{o,t+1} \;&\leq\; N_{o,t+1} + \frac{\boldsymbol{e_{t+1}(M_{m,t} + B_{m,t})}}{P_{t+1}} \\
p_t^f C_{m,t} \;&\leq\; \frac{\boldsymbol{\gamma^f e_t M_{m,t}}}{P_t} \\
\frac{M_{m,t}^{\$}}{P_t}, \; \frac{e_t^{\approxeq} M_{m,t}^{\approxeq}}{P_t}, \; \frac{e_t^{\mathbb{G}} M_{m,t}^{\mathbb{G}}}{P_t} \;&\geq\; 0
\end{aligned} \tag{ID.6}$$

The first order conditions (FOCs) are then given by

$$X_{m,t}: \quad U'(X_{m,t}) \;=\; \lambda_{m,t} \tag{ID.7}$$

$$X_{o,t+1}: \quad U'(X_{o,t+1}) \;=\; \lambda_{m,t+1} \tag{ID.8}$$

$$C_{m,t}: \quad \alpha\theta_j^{1-\alpha}(C_{m,t})^{\alpha-1} \;=\; p_t^f(\lambda_{m,t}+\nu_{m,t}) \tag{ID.9}$$

$$N_{m,t}: \quad 1 \;=\; \lambda_{m,t} \tag{ID.10}$$

$$N_{o,t+1}: \quad 1 \;=\; \lambda_{o,t+1} \tag{ID.11}$$

$$B^z_{m,t}: \quad Q^z_t \;=\; \beta\,\frac{\lambda_{o,t+1}}{\lambda_{m,t}}\,\frac{e^z_{t+1}}{e^z_t}\,\frac{P_t}{P_{t+1}} \tag{ID.12}$$

$$M^z_t: \quad 1 \;=\; \beta\frac{\lambda_{o,t+1}}{\lambda_{m,t}}\frac{e^z_{t+1}}{e^z_t}\frac{P_t}{P_{t+1}} \;+\; \gamma^{f,z}\nu_{m,t} \;+\; \rho^z_{m,t} \tag{ID.13}$$

for all currencies $z \in \{\$, \approx, \mathbb{G}\}$. The Lagrange and Kuhn-Tucker multipliers of the budget and CIA constraints are denoted by $\lambda_{A,t}$ and $\nu_{m,t}$. The Kuhn-Tucker conditions for the CIA and the non-negativity constraints are given by

$$\nu_{m,t}\left(\frac{\boldsymbol{\gamma^f e_t M_{m,t}}}{P_t} - p_t^f C_{m,t}\right) = 0 \qquad \text{and} \qquad \nu_{m,t} \geq 0 \tag{ID.14}$$

$$\rho^z_{m,t}\,\frac{e^z_t M^z_{m,t}}{P_t} = 0 \qquad \text{and} \qquad \rho^z_{m,t} \geq 0 \qquad \forall\, z \tag{ID.15}$$

Combining FOCs for consumption of the credit good and labour supply immediately yields that $X_{m,t} = X_{o,t+1} = X^*$. Because consumption of the credit good is equal across time, the real interest rate of the economy is pinned down by the discount factor β. Defining inflation as $1+\pi^z_{t+1} = \frac{P^z_{t+1}}{P^z_t}$, the FOCs for bonds in all currencies simplify to

$$Q^z_t = \beta(1+\pi^z_{t+1})^{-1} \tag{ID.16}$$

Using this expression for bond prices, money FOCs become

$$\gamma^{i,z}\nu_{m,t} = 1 - Q^z_t - \rho^z_{m,t} \tag{ID.17}$$

Consumers only hold currencies accepted by firms, and only hold the one with the lower inflation rate (higher bond price) if multiple currencies are accepted. Consider first the case where firm f only accepts one currency: $\gamma^f = (\gamma^{f,\$}, \gamma^{f,\approx}, \gamma^{f,\mathbb{G}}) = (0,1,0)$. The LHS of the above is zero, requiring $(\rho_t^{\$}, \rho_t^{\mathbb{G}}) > 0$ whenever $(Q_t^{\$}, Q_t^{\mathbb{G}}) < 1$; it follows that $M_{m,t}^{\$} = M_{m,t}^{\mathbb{G}} = 0$. Consider next a firm that accepts multiple currencies, i.e. the Dollar and Diem. Combining the two FOCs for $M^{\$}$ and $M^{\approx}$ shows that whenever $Q_t^{\$} < Q_t^{\approx}$, it must be that $\rho_{m,t}^{\$} > \rho_{m,t}^{\approx}$; since $\rho_{m,t}^{\approx} \geq 0$, this requires $\rho_{m,t}^{\$} > 0$, yielding $M_{m,t}^{\$} = M_{m,t}^{\mathbb{G}} = 0$.

The FOC for money also implies a zero lower bound on the nominal interest rate. All of $\gamma^{f,z}$, $\nu_{m,t}$ and $\rho_{m,t}^z$ are non-negative, and hence Q_t^z can take a maximum value of one (i_t^z can take a minimum value of zero).

Denote the lowest seignorage tax rate with firm f by τ_t^f. The FOC for this money then reads

$$\nu_{m,t} = \tau_t^f$$

which implies that $\nu_{m,t} > 0$ whenever $\tau_t^f > 0$. Then the CIA constraint holds with equality:

$$m_{m,t} = p_t^f C_{m,t}(p_t^f, \tau_t^f)$$

where $m_{m,t}$ denote real money balances of currencies held by the consumer. Turning to the FOC for money good consumption, the demand schedule is given by

$$C_{m,t}(p_t^f, \tau_t^f) = \theta_j \left[\frac{\alpha}{p_t^f (1 + \tau_t^f)} \right]^{\frac{1}{1-\alpha}} \tag{ID.18}$$

Combining all of the equilibrium conditions—see Appendix D.4 for a full derivation—I can write down the middle-aged household's value function at time-t as

$$V_{m,t}^j(\boldsymbol{M_{y,t-1}}, \boldsymbol{B_{y,t-1}}) = \bar{V}_m \; + \; \frac{\boldsymbol{e_t(M_{y,t-1} + B_{y,t-1})}}{P_t} \; + \; \theta_j \tilde{\kappa} \Big[p_t^f (1 + \tau_t^f) \Big]^{\frac{\alpha}{\alpha-1}} \tag{ID.19}$$

where $\tilde{\kappa} > 0$ is a constant, and function of α only. Utility derived from the money good consumption at time-t is only affected by the price that consumers face at the firm they visit, and by the inflation rate on the money that they need to hold. Clearly, consumers visit the firm which they expect to charge the lowest seignorage-adjusted real price. For equal seignorage-adjusted prices, they visit the more heavily advertised firm:

$$\psi^j_{m,t}\big(a^j_t, \mu^j_t(p_t), \tau_t\big) = \begin{cases} i & \text{if} \quad \mathbb{E}\Big[p^i_t(1+\tau^i_t) \mid \mu^j_t\Big] \;=\; \mathbb{E}\Big[p^{-i}_t(1+\tau^{-i}_t) \mid \mu^j_t\Big], \text{ and } a^{i,j}_t \;>\; a^{-i,j}_t \\ i & \text{if} \quad \mathbb{E}\Big[p^i_t(1+\tau^i_t) \mid \mu^j_t\Big] \;<\; \mathbb{E}\Big[p^{-i}_t(1+\tau^{-i}_t) \mid \mu^j_t\Big] \end{cases} \tag{ID.20}$$

If both seignorage-adjusted price and advertising intensity equal, consumers randomly choose a firm.

D.3 The young consumer's maximisation problem

The middle-aged consumer's value function is independent of any consumer's decisions taken when young, apart from asset holdings. Consider a consumer born at time t. Given the equilibrium pricing by a producer issuing private currency, utility is affected by the currency introduction decisions Γ. In the absence of private currencies, Dollar inflation also affects utility:

$$V^j_{m,t+1} = \bar{V}_m\big(\Gamma, \tau^{\$}_{t+1}, \theta_j\big) + \frac{\boldsymbol{e_{t+1}(M_{y,t} + B_{y,t})}}{P_{t+1}} \tag{ID.21}$$

Having visited firm i's shop and learnt their price p^i_t, the young consumer's utility maximisation problem is given by

$$\begin{aligned} \max_{\{X_{y,t}, C_{y,t}, N_{y,t}, \boldsymbol{B_{y,t}}, \boldsymbol{M_{y,t}}\}} \quad & U(X_{y,t}) \;+\; \theta_j^{1-\alpha}(C_{y,t})^{\alpha} \;-\; N_{y,t} \;+\; \beta\, V^j_{m,t+1}(\boldsymbol{M_{y,t}}, \boldsymbol{B_{y,t}}) \\ \text{s.t.} \quad X_{y,t} + p^i_t C_{y,t} + & \frac{\boldsymbol{e_t(M_{y,t} + Q_t B_{y,t})}}{P_t} \le N_{y,t} \;+\; T_{y,t} \\ & p^i_t C_{y,t} \le \frac{\boldsymbol{\gamma^i e_t M_{y,t}}}{P_t} \\ & \frac{M^{\$}_{y,t}}{P_t}, \; \frac{e^{\approx}_t M^{\approx}_{y,t}}{P_t}, \; \frac{e^{\mathbb{G}}_t M^{\mathbb{G}}_{y,t}}{P_t} \;\ge\; 0 \end{aligned} \tag{ID.22}$$

The resulting equilibrium conditions below, together with the budget and CIA constraint holding with equality, mirror those for the middle-aged consumer:

$$X_{y,t} = X^* \tag{ID.23}$$

$$C_{y,t}(\theta_j, p_t^i, \tau_t^i) = \theta_j \left[\frac{\alpha}{p_t^i(1+\tau_t^i)} \right]^{\frac{1}{1-\alpha}} \tag{ID.24}$$

Combining all of the above, fully derived in Appendix D.4, the value function of consumer j is given by

$$V_{y,t}^j = \bar{V}_y \;+\; T_{y,t} \;+\; \theta_j \tilde{\kappa} \Big[p_t^i (1+\tau_t^i) \Big]^{\frac{\alpha}{\alpha-1}} \;+\; \beta\, \bar{V}_m \big(\Gamma, \tau_{t+1}^{\$}, \theta_j \big) \tag{ID.25}$$

Since firms do not advertise in the consumer's initial period, consumers simply visit firms seeking to minimize the seignorage-adjusted cost of purchasing the money good, and randomise if indifferent:

$$\psi_{y,t}^j \big(\mu^j(p_t), \tau_t \big) = \; i \qquad \text{if} \quad \mathbb{E}\Big[p_t^i (1+\tau_t^i) \mid \mu_t^j \Big] \;<\; \mathbb{E}\Big[p_t^{-i} (1+\tau_t^{-i}) \mid \mu_t^j \Big] \tag{ID.26}$$

D.4 Deriving the consumer's value functions

The value function of the middle aged consumer j at time-t is given by

$$V_{m,t}^j = U(X^*) \;+\; \theta_j^{1-\alpha} (C_{m,t}^i)^\alpha \;-\; N_{m,t} \;+\; \beta \big[U(X^*) - N_{o,t+1} \big] \tag{ID.27}$$

where

$$N_{m,t} = X^* + p_t^i C_{m,t}^i + \frac{\boldsymbol{e_t (M_{m,t} + Q_t B_{m,t})}}{P_t} - \frac{\boldsymbol{e_t (M_{y,t-1} + B_{y,t-1})}}{P_t} \tag{ID.28}$$

$$N_{o,t+1} = X^* - \frac{\boldsymbol{e_{t+1} (M_{m,t} + B_{m,t})}}{P_{t+1}} \tag{ID.29}$$

$$C_{m,t}^i = \theta_j \left[\frac{\alpha}{p_t^i (1+\tau_t^i)} \right]^{\frac{1}{1-\alpha}} \tag{ID.30}$$

Plugging in and rearranging, the expression becomes

$$V^j_{m,t} = \bar{V}_m + \theta_j \left[\frac{\alpha}{p^i_t(1+\tau^i_t)}\right]^{\frac{\alpha}{1-\alpha}} - p^i_t\theta_j \left[\frac{\alpha}{p^i_t(1+\tau^i_t)}\right]^{\frac{1}{1-\alpha}} \tag{ID.31}$$

$$- \frac{\boldsymbol{e_t(M_{m,t} + Q_t B_{m,t})}}{P_t} + \frac{\boldsymbol{e_t(M_{y,t-1} + B_{y,t-1})}}{P_t} + \beta\frac{\boldsymbol{e_{t+1}(M_{m,t} + B_{m,t})}}{P_{t+1}} \tag{ID.32}$$

By the nominal stochastic discount factors, I know that

$$\frac{e^z_t Q^z_t}{P_t} = \beta\frac{e^z_{t+1}}{P_{t+1}} \tag{ID.33}$$

for all $z \in \{\$, \approx, \mathbb{G}\}$. Therefore $\boldsymbol{B_{m,t}}$ cancels out, and the expression for $\boldsymbol{M_{m,t}}$ simplifies substantially:

$$V^j_{m,t} = \bar{V}_m + \theta_j \left[\frac{\alpha}{p^i_t(1+\tau^i_t)}\right]^{\frac{\alpha}{1-\alpha}} - p^i_t\theta_j \left[\frac{\alpha}{p^i_t(1+\tau^i_t)}\right]^{\frac{1}{1-\alpha}} \tag{ID.34}$$

$$- \frac{\boldsymbol{(1-Q_t)e_t M_{m,t}}}{P_t} + \frac{\boldsymbol{e_t(M_{y,t-1} + B_{y,t-1})}}{P_t} \tag{ID.35}$$

Optimally consumers only bring currencies accepted by the firm, choose the one subject to the lower inflation rate than other currencies accepted. The cash-in-advance constraint is holding with equality. It follows that

$$p^i_t C_{m,t} = \frac{\boldsymbol{e_t M_{m,t}}}{P_t} \tag{ID.36}$$

and the expression for the value function becomes

$$V^j_{m,t} = \bar{V}_m + \theta_j \left[\frac{\alpha}{p^i_t(1+\tau^i_t)}\right]^{\frac{\alpha}{1-\alpha}} - p^i_t(1+\tau^i_t)\theta_j \left[\frac{\alpha}{p^i_t(1+\tau^i_t)}\right]^{\frac{1}{1-\alpha}} + \frac{\boldsymbol{e_t(M_{y,t-1} + B_{y,t-1})}}{P_t} \tag{ID.37}$$

Combining the two middle terms yields the expression as in Appendix D.2, with $\tilde{\kappa} = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha) > 0$.

The value function of the young consumer j at time-t is found in analogy to the middle age value function above. Starting point is the middle-age value function:

$$V^j_{m,t+1} = \bar{V}_m\big(\Gamma, \tau^{\$}_{t+1}, \theta_j\big) + \frac{\boldsymbol{e_{t+1}(M_{y,t} + B_{y,t})}}{P_{t+1}} \tag{ID.38}$$

Given the time-$(t+1)$ equilibrium outcomes for consumption of the credit and money goods, time-t decisions only affect future utility through asset holdings: for every unit of real assets in the next period, consumers need to supply one unit less labour.

$$V^j_{y,t} = U(X^*) + \theta_j^{1-\alpha} C_{y,t} - N_{y,t} + \beta\left[\bar{V}_m\big(\Gamma, \tau^{\$}_{t+1}, \theta_j\big) + \frac{\boldsymbol{e_{t+1}(M_{y,t} + B_{y,t})}}{P_{t+1}}\right] \tag{ID.39}$$

where

$$N_{y,t} = X^* + p^i_t C_{y,t} + \frac{\boldsymbol{e_t(M_{y,t} + Q_t B_{y,t})}}{P_t} - T_{y,t} \tag{ID.40}$$

Plugging in the expression for $N_{y,t}$ and $C_{y,t}$, and again making use of the expression for the nominal stochastic discount factor and equilibrium money holdings gives

$$V^j_{y,t} = \bar{V}_y + \theta_j\left[\frac{\alpha}{p^i_t(1+\tau^i_t)}\right]^{\frac{\alpha}{1-\alpha}} - p^i_t(1+\tau^i_t)\theta_j\left[\frac{\alpha}{p^i_t(1+\tau^i_t)}\right]^{\frac{1}{1-\alpha}} + T_{y,t} + \beta\bar{V}_m\big(\Gamma, \tau^{\$}_{t+1}, \theta_j\big) \tag{ID.41}$$

As for the middle age value function, combining the middle terms yields the expression as in Appendix D.3.

D.5 *Deriving the expression for seignorage revenues*

This section of the appendix formally derives the expression for seignorage revenues. Consider firm f that is conducting a transaction in its private currency. The firm generates profits from selling the money good, Π^f_t, transfers resources to the young households, issues money and holds assets in the form of bonds. The flow budget constraint at time-t is given by

$$\Pi^f_t + \frac{e^{\approx}_t\big(M^{S,\approx}_t - M^{S,\approx}_{t-1}\big)}{P_t} = T^f_t + \frac{\boldsymbol{e_t\big(Q_t B^f_t - B^f_{t-1}\big)}}{P_t} \tag{ID.42}$$

where $M_t^{S,\approxeq}$ denotes the Diem supply.[40] Diem is modelled as stablecoin: it is backed with Diem-denominated bonds that have been issued by the household.[41,42] For every unit of money issued, the firm purchases a unit of Diem-denominated bonds: $M_t^{S,\approxeq} = B_t^{f,\approxeq}$. Jumping ahead to impose the market clearing condition, $M_t^{S,\approxeq} = M_t^{\approxeq}$, and defining $m_t^{\approxeq} = \frac{e_t^{\approxeq} M_t^{\approxeq}}{P_t}$, the firm's flow budget constraint simplifies to

$$\Pi_t^{m,f} + (1 - Q_t^{\approxeq}) m_t^{\approxeq} = T_t^f$$

where $(1 - Q_t^{\approxeq}) m_t^{\approxeq}$ are the firm's seignorage revenues.

E Alternative specifications of the CIA model

E.1 Alternative timing assumption

For simplicity, and without loss of generality, assume that the firm is a monopolist producer of the money good and also the sole supplier of money in this economy. Ignore the OLG structure and any consumer heterogeneity: a representative consumers purchases consumption goods from a monopolist and holds their money Diem. In the full model as developed in the main text, firms issuing PCDC charge seignorage-adjusted prices as if they were monopolists. Here I demonstrate that this monopoly price is unchanged in the alternative CIA specification.

Suppose the money market opens before the credit market. Money then enables transactions with a one-period delay:

$$p_t C_t \;\leq\; \frac{M_{t-1}^{\approxeq}}{P_t^{\approxeq}} \tag{IE.1}$$

where $P_t^{\approxeq}$ is the price of Diem in terms of the numeraire. The consumer's utility maximisation

[40] Such a budget constraint is relevant if firms also provide currency. Otherwise they simply transfer profits to the household.

[41] An alternative backing of the currency is discussed in Appendix E.2. The results are unchanged.

[42] I discuss an economy in which only Dollar-denominated bonds exist in Section 6.2.

problem becomes

$$\mathcal{L}_0 \;=\; \max_{\{X_t, C_t, N_t, B_t^{\approxeq}, M_t^{\approxeq}\}_{t\geq 0}} \sum_{t=0}^{\infty} \Bigg\{ U(X_t) \;+\; (C_t)^{\alpha} \;-\; N_t$$
$$+\; \lambda_t \Bigg(N_t \;+\; \frac{B_{t-1}^{\approxeq}}{P_t^{\approxeq}} + \frac{M_{t-1}^{\approxeq}}{P_t^{\approxeq}} \;+\; T_t^f \;-\; X_t \;-\; p_t C_t \;-\; \frac{M_t^{\approxeq}}{P_t^{\approxeq}} \;-\; \frac{Q_t^{\approxeq} B_t^{\approxeq}}{P_t^{\approxeq}} \Bigg)$$
$$+\; \nu_t \Bigg(\frac{M_{t-1}^{\approxeq}}{P_t^{\approxeq}} \;-\; p_t C_t \Bigg) \Bigg\} \tag{IE.2}$$

where λ and ν denote the Lagrange and Kuhn-Tucker multipliers corresponding to the budget and cash-in-advance constraints.[43] The first order conditions are unchanged relative to the full model, with the exception of (the now only type of) money $M_t^{\approxeq}$:

$$\lambda_t \frac{1}{P_t^{\approxeq}} \;=\; \beta \frac{1}{P_{t+1}^{\approxeq}} \big[\lambda_{t+1} \;+\; \nu_{t+1} \big] \tag{IE.3}$$

Rearrange, and use $\lambda_t = 1$ and $Q_t^{\approxeq} = \beta(1 + \pi_{t+1}^{\approxeq})^{-1}$, to find that

$$\nu_{t+1} \;=\; \frac{1}{Q_t^{\approxeq}} \;-\; 1 \tag{IE.4}$$

The bond price is inversely related to the nominal interest rate: $Q_t^{\approxeq} \;=\; \frac{1}{1+i_t^{\approxeq}}$. The first order condition for consumption of the money good is given by

$$u'(C_t) \;=\; p_t(\lambda_t + \nu_t) \;=\; p_t(1 + i_{t-1}) \qquad \Rightarrow \qquad C_t \;=\; \left[\frac{\alpha}{p_t(1 + i_{t-1})} \right]^{\frac{1}{1-\alpha}} \tag{IE.5}$$

Comparing the demand schedule to the one obtained in the main body of the paper, the opportunity cost of having to hold Diem is now captured by the interest rate of the previous period. Turning to the firm's problem, write the flow budget constraint as

$$\Pi_t \;+\; \frac{M_t^{\approxeq} \;-\; M_{t-1}^{\approxeq}}{P_t^{\approxeq}} \;=\; T_t^f \;+\; \frac{B_t^{f,\approxeq} \;-\; (1 + i_{t-1}^{\approxeq}) B_{t-1}^{f,\approxeq}}{P_t^{\approxeq}} \tag{IE.6}$$

[43] For simplicity, I omit a money non-negativity constraint which never binds in equilibrium.

The currency is backed according to $M_t^{\approxeq} = B_t^{f,\approxeq}$. Define $m_t^{\approxeq} = \frac{M_{t-1}^{\approxeq}}{P_t^{\approxeq}}$ to find

$$\Pi_t \; + \; i_{t-1} m_t^{\approxeq} \; = \; T_t^f \tag{IE.7}$$

Consider a binding cash-in-advance constraint: $m_t^{\approxeq} = p_t C(p_t, i_{t-1})$. Plugging this expression into the flow budget constraint shows that total profits are given by

$$\Pi_t \;=\; \left[p_t(1+i_{t-1}^{\approxeq}) - 1\right] C(p_t, i_{t-1}^{\approxeq}) \;=\; \left[p_t(1+i_{t-1}^{\approxeq}) - 1\right] \left[\frac{\alpha}{p_t(1+i_{t-1}^{\approxeq})}\right]^{\frac{1}{1-\alpha}} \tag{IE.8}$$

Firms choose an optimal seignorage-price tuple (p_t, i_{t-1}) satisfying

$$p_t(1+i_{t-1}) = \tilde{p} \tag{IE.9}$$

This variant of the model delivers a solution that is equivalent to the monopoly solution in the main model: firms perfectly internalise the opportunity cost of holding money. Seignorage discounts here correspond to the previous period's interest rate. Importantly, firms can internalise the effect of non-zero interest rates when setting their price, thus manipulating consumer demand. If firms would be unable to do so, they can always implement a zero interest rate policy which, combined with real monopoly prices for the consumption good, also satisfies the above.

E.2 Backing the currency according to $M_t = Q_t B_t$

As in Appendix E.1, consider the model of a representative consumer and a firm that is the only supplier of the money good and currency. Begin with the flow budget constraint, but assume that the stablecoin is backed according to $M_t^{\approxeq} = Q_t^{\approxeq} B_t^{f,\approxeq}$. Effectively, firms spend all of the funds raised by issuing money in financial markets, and receive interest gains from bond purchases in the following period. Plugging in and rearranging, the flow budget constraint becomes

$$\Pi_t \; + \; \left[\frac{1}{Q_{t-1}^{\approxeq}} - 1\right] \frac{M_{t-1}^{\approxeq}}{P_t^{\approxeq}} \; = \; T_t^f \tag{IE.10}$$

Rearranging and using $m^{\approxeq}_{t-1} = \frac{M^{\approxeq}_{t-1}}{P^{\approxeq}_{t-1}}$, the expression is given by

$$\Pi_t \; + \; \left[\frac{1 - Q^{\approxeq}_{t-1}}{Q^{\approxeq}_{t-1}}\right] \frac{m^{\approxeq}_{t-1}}{1 + \pi^{\approxeq}_t} \; = \; T^f_t \tag{IE.11}$$

The firm's problem becomes dynamic, but only in the sense that some revenues accrue with a one-period delay (see also Section 6.2 and Appendix G.2). Firms discount future profits at rate β, the appropriate discount factor implied by the consumer maximisation problem.[44] Since $Q^{\approxeq}_t = \beta(1 + \pi^{\approxeq}_{t+1})^{-1}$ and $\tau^{\approxeq}_t = 1 - Q^{\approxeq}_t$, the profit maximisation problem can be written as

$$\underset{p_t, i^{\approxeq}_t}{max} \;\; (p_t - 1)\, C_t(p_t, \tau^{\approxeq}_t) \; + \; \tau^{\approxeq}_t m^{\approxeq}_t \tag{IE.12}$$

Plugging in the demand curve for C_t and the CIA constraint holding with equality, the problem becomes

$$\underset{p_t, \tau^{\approxeq}_t}{max} \;\; (p_t(1 + \tau^{\approxeq}_t) - 1) \left[\frac{\alpha}{p_t(1 + \tau^{\approxeq}_t)}\right]^{\frac{1}{1-\alpha}} \tag{IE.13}$$

Clearly, the solution perfectly matches the solution of the main specification.

F Appendix to Section 5

F.1 Derivation of optimal pricing strategies and of the upper bound on the Diem seignorage tax rate

Consider firm f^*'s profit function:

$$\Pi^{f^*}_t \; = \; (p^{f^*}_t - 1)\, C^{f^*}_t \; + \; (1 - \eta)\, \tau^{\approxeq}_t \Big[p^f_t C^f_t \; + \; p^{f^*}_t C^{f^*}_t\Big] \tag{IF.1}$$

[44] Firms are owned by the household and thus use their discount factor.

The demand schedules, conditional on purchase using Diem, match the demand schedules of the previous sections:

$$C_t^f(p_t^f, \tau_t^{\approx}) \;=\; \left[\frac{\alpha}{p_t^f(1+\tau_t^{\approx})}\right]^{\frac{1}{1-\alpha}} \tag{IF.2}$$

$$C_t^{f^*}(\theta_{A,j}, p_t^{f^*}, \tau_t^{\approx}) \;=\; \chi\theta_{A,j}\left[\frac{\alpha}{p_t^{f^*}(1+\tau_t^{\approx})}\right]^{\frac{1}{1-\alpha}} \tag{IF.3}$$

Note that there is no heterogeneity among consumers in the first money good market. The firms' first order conditions then reveal that firms charge the following prices in Diem:

$$p_t^f(\eta, \tau_t^{\approx}) = \frac{\tilde{p}}{1+\eta\tau_t^{\approx}} \qquad p_t^{f^*}(\eta, \tau_t^{\approx}) = \frac{\tilde{p}}{1+(1-\eta)\tau_t^{\approx}} \tag{IF.4}$$

The equilibrium profit function of the consortium-leading firm f is then given by:

$$\Pi_t^f\left(\tau_t^{\approx}, \eta, \chi\theta^*\right) = \kappa\left[\frac{1+\eta\tau_t^{\approx}}{1+\tau_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \;+\; \eta\,\tau_t^{\approx}\,\chi\,\theta^*\frac{\kappa}{1-\alpha}\left[\frac{\left[1+(1-\eta)\tau_t^{\approx}\right]^{\alpha}}{1+\tau_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \tag{IF.5}$$

where the first term captures firm f's product profits and seignorage revenues due to its own transactions; the second term captures the seignorage revenues generated by firm f^* which accrue to firm f. Firm f^*'s customer base is denoted by θ^*.

To derive the lower bound on the Diem seignorage tax rate, begin by noting that firm f^* only wants to remain part of the consortium if they pay weakly lower seignorage-adjusted prices than their competitor—otherwise they do not sell any goods and prefer to only accept the Dollar.

$$p_t^{f^*}(1+\tau_t^{\approx}) \;\leq\; \tilde{p}(1+\tau_t^{\$}) \tag{IF.6}$$

where the above expression imposed that firm g charges a real product price of $\tilde{p}$ as by Equation (31). Given firm f^*'s pricing strategy as above, rearrange to find

$$\tau_t^{\approx} \;\leq\; \frac{\tau_t^{\$}}{1-(1+\tau_t^{\$})(1-\eta)} \tag{IF.7}$$

whenever this expression's numerator is strictly greater than zero. If the numerator is weakly less than zero, Diem inflation is unconstrained by central bank policy.

F.2 Proof of Proposition 4

Given the firms' optimal pricing strategies, the consortium leader's profits are given by

$$\Pi^{f,total} = \frac{1-\alpha}{\alpha}\left(\frac{1+\eta\tau^{\approx}}{1+\tau^{\approx}}\right)^{\frac{1}{1-\alpha}} + \frac{\eta\tau^{\approx}\chi\theta^{*}}{\alpha}\left(\frac{[1+(1-\eta)\tau^{\approx}]^{\alpha}}{1+\tau^{\approx}}\right)^{\frac{1}{1-\alpha}} \quad \text{(IF.8)}$$

Taking the first order condition with respect to $\tau^{\approx}$, and evaluating it at zero reveals that $\tau^{\approx} = 0$ is a critical point whenever the consortium leader's dividend share corresponds to their transaction share ($\eta = \varphi^{*}$). Evaluating the second order conditions at zero and imposing $\eta = \varphi^{*}$ reveals that there exists a consortium leader transaction share $\underline{\varphi}$ such that

- For all $\varphi^{*} > \underline{\varphi}$, $\tau^{\approx} = 0$ is a maximum.
- For $\varphi^{*} = \underline{\varphi}$, $\tau^{\approx} = 0$ is a saddle point.
- For all $\varphi^{*} < \underline{\varphi}$, $\tau^{\approx} = 0$ is a minimum.

Since implementing a seignorage tax of zero for a sufficiently small transaction share minimises profits, it must be that there is a positive seignorage tax rate $\tau^{\approx} > 0$ associated with higher profits. It follows that the consortium leader implements a positive seignorage tax if $\varphi^{*} < \underline{\varphi}$ and their dividend share corresponds to their transaction share.

Next, I show that the optimal seignorage tax rate for concentrated ownership is strictly positive. Take the derivative of profits with respect to the Diem seignorage tax rate and evaluate it at zero. This derivative is strictly greater than zero whenever ownership is concentrated:

$$\eta > \varphi^{*} \quad \Rightarrow \quad \left.\frac{d\Pi^{f,total}}{d\tau^{\approx}}\right|_{\tau^{\approx}=0} > 0 \quad \text{(IF.9)}$$

This implies that there is a $\tau^{\approx} > 0$ in the neighbourhood of zero associated with larger profits. Hence it must be that the maximum of the profit function over the permissible domain of $\tau^{\approx}$ is strictly positive.

G Appendix to Section 6

G.1 Formal model demonstrating Proposition 5: CBDC

Consider the following variant of the model, demonstrating how the central bank can escape the zero interest rate environment if the public currency has lost its role as medium of exchange. A representative consumer derives period utility according to

$$U(X_t) + \big(C_t\big)^{\alpha} - N_t \tag{IG.1}$$

The model features two currencies: the Diem issued by firm f, and the Dollar issued by the central bank. The Dollar here is a digital currency that pays interest at rate $i_t^{M^{\$}}$. The budget constraint is thus given by

$$X_t + p_t C_t + \frac{Q_t^{\$} B_t^{\$}}{P_t} + \frac{e_t^{\text{≋}} Q_t^{\text{≋}} B_t^{\text{≋}}}{P_t} + \frac{M_t^{\$}}{P_t} + \frac{e_t^{\text{≋}} M_t^{\text{≋}}}{P_t} \leq w_t N_t + \frac{B_{t-1}^{\$}}{P_t} + \frac{e_t^{\text{≋}} B_{t-1}^{\text{≋}}}{P_t} + \frac{(1 + i_{t-1}^{M^{\$}}) M_{t-1}^{\$}}{P_t} + \frac{e_t^{\text{≋}} M_{t-1}^{\text{≋}}}{P_t} \tag{IG.2}$$

Firms produce according to the linear production functions as in the main body of the text. The credit market is perfectly competitive, and the real wage is again pinned down as $w_t = 1$. Firm f supplies the money good as a monopolist at price p_t and does not accept the Dollar in exchange. The consumer thus faces a CIA constraint as below:

$$p_t C_t \leq \frac{e_t^{\text{≋}} M_t^{\text{≋}}}{P_t} \tag{IG.3}$$

Real money balances in both currencies need to be weakly positive at all points in time:

$$\frac{e_t^{\text{≋}} M_t^{\text{≋}}}{P_t}, \frac{M_t^{\$}}{P_t} \geq 0 \tag{IG.4}$$

The resulting optimality conditions are largely unchanged. The first order condition for Dollar bonds simplifies to

$$Q_t^{\$} = \beta(1 + \pi_{t+1}^{\$})^{-1} \tag{IG.5}$$

The simplified first order condition for Dollar balances is given by

$$1 \;=\; \beta \frac{1 + i_t^{M^\$}}{1 + \pi_{t+1}^\$} \;+\; \rho_t^\$ \tag{IG.6}$$

where $\rho_t^\$$ is the multiplier of the non-negativity constraint on real Dollar balances. Combining the two equations yields

$$1 \;=\; Q_t^\$ \, (1 + i_t^{M^\$}) \;+\; \rho_t^\$ \tag{IG.7}$$

The Kuhn-Tucker conditions for Dollar balances imply that

$$\frac{M_t^\$}{P_t} \, \rho_t^\$ \;=\; 0 \tag{IG.8}$$

And hence positive real money balances, for which $\rho_t^\$ = 0$, require that

$$Q_t^\$ \, (1 + i_t^{M^\$}) = 1 \qquad \Rightarrow \qquad i_t = i_t^{M^\$} \tag{IG.9}$$

For a one-sided introduction, the main body of text established that the issuer of PCDC charges a seignorage-adjusted price satisfying

$$p^f (1 + \tau_t^{\approx}) \;=\; \tilde{p} \tag{IG.10}$$

While still achieving monopoly rents, the firm fully removes the opportunity cost of holding money. A firm that transacts in the public currency does not internalise the opportunity cost of holding Dollars and charges a real price $p^g = \tilde{p}$. Thus, no consumer visits firm g if the opportunity cost of holding Dollars is positive. In Section 4, the central bank did not have the option to pay interest on its money and therefore was forced to set the nominal interest rate on bonds to zero. Consider now the problem for the consumer as in subsections 4.1.1 and 4.2 but with two currencies in circulation, the Dollar and Diem. The Dollar pays interest, and the period budget constraint is given by Equation (IG.2). Firm g only accepts the Dollar, and so $(\gamma^{g,\$}, \gamma^{g,\approx}) = (1, 0)$. Having visited their shop, the first order condition for Dollar holdings for a middle-aged consumer becomes

$$1 \;=\; \beta\frac{(1+i_t^{M\$})P_t}{P_{t+1}} \;+\; \nu_{m,t} \;+\; \rho_{m,t}^{\$} \tag{IG.11}$$

where ν and ρ denote the Kuhn-Tucker multipliers of the CIA and non-negativity constraints. Having visited firm g, the consumer can only purchase goods from this firm and thus holds positive real Dollar balances. Combining with the first order condition for Dollar-denominated bonds, the above becomes

$$1 - Q_t^{\$}(1+i_t^{M\$}) \;=\; \nu_{m,t} \tag{IG.12}$$

First of all, $Q_t^{\$}(1+i_t^{M\$}) = 1$ already implies that $\nu_{m,t} = 0$. The CIA constraint does not bind: there is no opportunity cost of holding Dollars that pay sufficient interest and the constraint forcing money holdings is slack. Combining Equation (IG.12) with the first order conditions outlined in Appendix D.2, the consumer's money good demand becomes

$$C_{m,t}(p_t^g, Q_t^{\$}, i_t^{M\$}) = \theta_j\left[\frac{\alpha}{p_t^g\Big(2 - Q_t^{\$}(1+i_t^{M\$})\Big)}\right]^{\frac{1}{1-\alpha}} \tag{IG.13}$$

The seignorage-adjusted price face by consumer j at firm g is thus given by $p_t^g\Big(2 - Q_t^{\$}(1+i_t^{M\$})\Big)$. Consumers visit firm g's store, leading to positive demand for Dollar goods in equilibrium, if

$$p_t^g\Big(2 - Q_t^{\$}(1+i_t^{M\$})\Big) \;=\; \tilde{p}\Big(2 - Q_t^{\$}(1+i_t^{M\$})\Big) \;\le\; p^f(1+\tau_t^{\approx}) \;=\; \tilde{p} \tag{IG.14}$$

which requires $Q_t^{\$}(1+i_t^{M\$}) = 1$, or equivalently, $i_t^{\$} = i_t^{M\$}$.

G.2 Derivations for subsection 6.2: Capital gains

Consider the flow budget constraint of a firm that issues Diem currency, $M_t^{\approx}$, and holds household bonds denominated in the Dollar:

$$\Pi_t^f \;+\; \frac{e_t^{\approx}(M_t^{\approx,S} \;-\; M_{t-1}^{\approx,S})}{P_t} \;=\; T_t^f \;+\; \frac{Q_t^{\$}B_t^f \;-\; B_{t-1}^f}{P_t} \tag{IG.15}$$

where Π_t^f denote firm f's product profits. As before, there is full backing of the currency: $e_t^{\approx} M_t^{\approx,S} = B_t^f$. Plugging in the market clearing condition for Diem, $M_t^{\approx,S} = M_t^{\approx}$, the Diem exchange rate, $e_t^{\approx} = \frac{P_t}{P_t^{\approx}}$, and the definition of real Diem balances, $m_t^{\approx} = \frac{e_t^{\approx} M_t^{\approx}}{P_t}$, the flow budget constraint becomes:

$$\Pi_t^f \; + \; \tau_t^{\$} m_t^{\approx} \; + \; \left[1 - \frac{e_t^{\approx}}{e_{t-1}^{\approx}}\right] \frac{e_{t-1}^{\approx} M_{t-1}^{\approx}}{P_t} \; = \; T_t^f \tag{IG.16}$$

By the definitions of the exchange rate, the inflation rates in both currencies and real Diem balances, the above becomes

$$\Pi_t^f \; + \; \tau_t^{\$} m_t^{\approx} \; + \; \left[1 - \frac{1 + \pi_t^{\$}}{1 + \pi_t^{\approx}}\right] \frac{m_{t-1}^{\approx}}{1 + \pi_t^{\$}} \; = \; T_t^f \tag{IG.17}$$

From consumers first order conditions, I obtain $Q_t^{\$} = \beta(1 + \pi_t^{\$})^{-1}$. Since this economy does not feature Diem-denominated bonds, define $Q_t^{\approx} = \beta(1 + \pi_{t+1}^{\approx})^{-1}$. Effectively, if a Diem bond would exist, its price would account for the time rate of preferences and the change in the value of Diem relative to the numeraire. Using the definition of seignorage tax rates τ, I obtain

$$\Pi_t^f \; + \; \tau_t^{\$} m_t^{\approx} \; + \; \frac{\tau_{t-1}^{\approx} - \tau_{t-1}^{\$}}{\beta} m_{t-1}^{\approx} \; = \; T_t^f \tag{IG.18}$$

where the $\frac{\tau_{t-1}^{\approx} - \tau_{t-1}^{\$}}{\beta}$ captures the capital gains due to inflation rate differentials on the two currencies. Note how higher inflation on Diem, or equivalently $\tau_{t-1}^{\$} < \tau_{t-1}^{\approx}$, yields capital gains in the following period. The firm's profit maximisation problem is now dynamic: a choice of $(Q_t^{\approx}, p_t^f)$ affects both profits today and tomorrow. Since there is no meaningful economic connection between time periods other than the fact that some profits only accrue tomorrow, I the firm's total profit maximisation problem becomes:

$$\begin{aligned}
\max_{p_t^f, \tau_t^{\approx}} \Pi_t^{f,total} \; &= \; \max_{p_t^f, \tau_t^{\approx}} \Pi_t^f \; + \; \tau_t^{\$} m_t^{\approx} \; + \; \beta \frac{\tau_t^{\approx} - \tau_t^{\$}}{\beta} m_t^{\approx} \\
&= \; \max_{p_t^f, \tau_t^{\approx}} \Pi_t^f \; + \; \tau_t^{\approx} m_t^{\approx} \\
&= \; \max_{p_t^f, \tau_t^{\approx}} \left(p_t^f (1 + \tau_t^{\approx}) - 1\right) \left[\frac{\alpha \theta^{1-\alpha}}{p_t^f (1 + \tau_t^{\approx})}\right]^{\frac{1}{1-\alpha}}
\end{aligned} \tag{IG.19}$$

where tomorrow's profits are discounted at rate β. The problem and also its solution are exactly as before.

G.3 Derivations for subsection 6.3: Sales taxes payable in public currency

Consider firm f's flow budget constraint:

$$\Pi_t^f \ + \ \frac{e_t^{\approx}(M_t^{S,\approx} - M_{t-1}^{S,\approx})}{P_t} \ = \ T_t^f \ + \ \frac{Q_t^{\$} B_t^f - B_{t-1}^f}{P_t} \ + \ \frac{M_t^{f,\$} - M_{t-1}^{f,\$}}{P_t} \tag{IG.20}$$

Diem is backed using dollar-denominated assets: $e_t M_t^{S,\approx} = B_t^f + M_t^{f,\$}$. Since the firm achieves seignorage returns when holding bonds, the cash-tax constraint always binds. Thus, the firm holds government currency and bonds according to

$$\lambda \frac{e_t^{\approx} M_t^{S,\approx}}{P_t} \ = \ \frac{M_t^{f,\$}}{P_t} \qquad (1-\lambda)\frac{e_t^{\approx} M_t^{S,\approx}}{P_t} \ = \ \frac{B_t^f}{P_t} \tag{IG.21}$$

The firm's profit function is derived in analogy to Appendix G.2. Plugging in the firm's government currency and bond holdings, using definitions as above and rearranging, the flow budget constraint becomes:

$$\Pi_t^f \ + \ (1-\lambda)\tau_t^{\$} m_t^{\approx} \ + \ \left[1 - \frac{e_t^{\approx}}{e_{t-1}^{\approx}}\right] \frac{e_{t-1}^{\approx} M_{t-1}^{\approx}}{P_t} \ = \ T_t^f \tag{IG.22}$$

Direct seignorage revenues now only accrue on a fraction $(1-\lambda)$ of Diem balances, since a fraction λ need to be held in non-interest-bearing public currency. Following the same steps as before, the firm's flow budget constraint becomes

$$\Pi_t^f \ + \ (1-\lambda)\tau_t^{\$} m_t^{\approx} \ + \ \frac{\tau_{t-1}^{\approx} - \tau_{t-1}^{\$}}{\beta} \, m_{t-1}^{\approx} \ = \ T_t^f \tag{IG.23}$$

Again, the firm's profits are only dynamic in the sense that some profits accrue with a one-period delay. The profit function is thus given by

$$\Pi_t^{f,total} \ = \ \left[p_t^f(1-\lambda) - 1\right] C(\theta_{A,j}, p_t^f, \tau_t^{\approx}) \ + \ \left[(1-\lambda)\tau_t^{\$} \ + \ \left(\tau_t^{\approx} - \tau_t^{\$}\right)\right] p_t^f C(\theta_{A,j}, p_t^f, \tau_t^{\approx}) \tag{IG.24}$$

which yields the expression as in the main body of text. Firm f optimally charges a real price given by

$$p_t^f = \frac{\alpha^{-1}}{\left[(1-\lambda) + \tau_t^{\approx} - \lambda \tau_t^{\$}\right]} \tag{IG.25}$$

Plugging this expression into the profit function and the consumer's money good consumption function, reveals both are increasing in $\tau_t^{\approx}$:

$$\Pi^f \;=\; \kappa \theta_{A,j} \left[1 \;-\; \lambda \, \frac{1+\tau_t^{\$}}{1+\tau_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \qquad C_t \;=\; \theta_{A,j} \left[1 \;-\; \lambda \, \frac{1+\tau_t^{\$}}{1+\tau_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \tag{IG.26}$$

where κ is constant as in the main body of text. It follows that Diem monetary policy is characterised by the corner solution $\tau_t^{\approx} = 1$, corresponding to infinite inflation.

The competitor is fully priced out of the market. Given the tax, firm g charges a seignorage-adjusted price satisfying

$$p^g(1+\tau_t^{\$}) \;=\; \frac{1+\tau_t^{\$}}{\alpha(1-\lambda)} \tag{IG.27}$$

As $\tau_t^{\approx} \to 1$, firm f charges a seignorage-adjusted price of

$$p_t^f(1+\tau_t^{\approx}) \;\to\; \frac{2}{\alpha[2-\lambda(1+\tau_t^{\$})]} \tag{IG.28}$$

Comparing the two expressions reveals that firm g can only compete if two conditions are met: a) in the absence of a tax ($\lambda = 0$); and b) if $\tau_t^{\$} = 0$. Effectively, there cannot be an opportunity cost to hold Dollars, and there cannot be capital gains discounts that firm f provides but firm g does not.

G.4 Derivations for subsection 6.3: Macroprudential policies

Diem firm f faces a macroprudential constraint:

$$\lambda e_t^{\approx} M_t^{S,\approx} \;\leq\; M_t^{f,\$} \tag{IG.29}$$

The firm again prefers to invest in bonds rather than currency, and the macroprudential constraint holds with equality (see Equation IG.21). Manipulating the flow budget constraint (Equation IG.20) following exactly the same steps as before, the profit maximisation problem is given by

$$\underset{p_t^{\approx}, \tau_t^{\approx}}{max} \;\; [p_t^f - 1] C(p_t^f, \tau_t^{\approx}) + (1-\lambda)\tau_t^{\$}\, p_t^f C(p_t^f, \tau_t^{\approx}) + \left(\tau_t^{\approx} - \tau_t^{\$}\right) p_t^f C(p_t^f, \tau_t^{\approx}) \tag{IG.30}$$

Relative to the case of sales taxes payable in Dollar, profits are untouched. However, direct seignorage revenues are again reduced by a factor of λ which introduces a capital gains motive, as in the previous appendix subsection. Given optimal product discounts, profits and consumption are increasing in Diem inflation:

$$\Pi_t^f \;=\; \kappa\theta \left[1 - \lambda\, \frac{\tau_t^{\$}}{1+\tau_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \qquad C_t \;=\; \theta \left[1 - \lambda\, \frac{\tau_t^{\$}}{1+\tau_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \tag{IG.31}$$

Capital gains and corresponding product discounts allow firms to circumvent the enforced opportunity cost of holding public currency.