Cumulative prospect theory and stock returns

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Abstract

Barberis et al. (2016) finds that the stocks with high (low) prospect theory (Tversky-Kahneman) values earn lower (higher) returns. We find that the long-short TK portfolio returns are highly volatile yet predictable using the estimated probability weights from empirical pricing kernel. Allowing time-varying probability weights strengthens the support for cumulative prospect theory in explaining stock returns beyond existing return predictors and time-varying stock characteristics. We also develop a trading strategy that significantly improves the performance of TK portfolios.

Keywords: cumulative prospect theory, time-varying, probability weighting, TK anomaly

JEL Codes: G02, G12, G14

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1 Introduction

Investor's risk attitudes are fundamental in explaining stock returns. Most of available asset pricing models make the assumption in expected utility framework. The prospect theory due to (Kahneman and Tversky, 1979) and (Tversky and Kahneman, 1992) greatly expands the framework of expected utility to accommodate empirical findings in asset pricing. In a direct application of prospect cumulative prospect theory, Barberis et al. (2016) evaluate the stocks based on their past return distributions' prospect theory (Tversky-Kahneman) values and demonstrate that stocks with high TK values earn lower returns, an evidence of investors tilting towards these stocks. Among the various components in the cumulative prospect theory, Barberis et al. (2016) find that probability weighting, the component capturing investors' attitudes towards the tails of a return distribution, contributes most in predicting stock returns. In this paper, we investigate the time-series of returns of the stocks of different TK values, and find that the returns of the long-short TK portfolio vary greatly over time and are predictable using the estimated probability weights. Our evidence highlights the importance of incorporating time-varying demand generated from cumulative prospect theory in asset pricing.

If the aggregate demand for stocks of high TK values is time-varying, the variables correlated with the demand can predict excess returns in these stocks. One such variable is the estimated probability weighting function from empirical pricing kernel. The empirical pricing kernel captures the representative preference, and the implied aggregate attitudes toward tail events could vary through the aggregation of heterogeneous preferences and expectations. We estimate the probability weighting function from the S&P 500 index options and the S&P 500 index returns, and find considerable variations in the estimates, with periods when investors overweight or underweight the tails. The overweight of tails is the common assumption when applying the prospect theory to stock returns. But Polkovnichenko and Zhao (2013) show that the implied probability weight from the S&P 500 index options suggests that during some periods investors may not overweight the tails or even underweight the tails when the investors are relatively complacent believing tail events improbable. Variations in probability weights could well generate time-varying demand for stocks with high TK values. Similar evidence exists for the demand for active portfolio management. Polkovnichenko et al. (2019) find that the implied probability weighting function covaries with investor demand for mutual funds that can outperform in the tails of return distribution.

Our first set of results demonstrates the variations and predictability of stock returns with different TK values. Within the framework of the prospect theory, we construct the long-short TK portfolio following Barberis et al. (2016) from 1996 to 2020 and compute monthly returns for value-weighted and equal-weighted portfolios. Table 1 reports the average returns for the whole sample and sub-samples when investors over-weigh or under-weigh tail events. We find that the performance of the long-short TK portfolio varies significantly between two sub-samples and the positive returns of long-short TK portfolio are much larger in the overweight subsample, whereas the average return of value-weighted portfolio is small and insignificant in the whole sample.

[Insert Table 1 here]

Furthermore, we find strong predictability of the long-short TK portfolio returns using probability weights implied from empirical pricing kernel. Table 2 shows the predictive power of the dummy variable, an indicator for overweight periods, for the long-short TK portfolio returns over 3-, 6-, and 12-month horizon. The predictive power is statistical and economically significant, and persistent over various forecasting horizon.

[Insert Table 2 here]

We next investigate how variations in investor's attitude toward tail events could affect demand and expected returns on stocks with different TK values. Conceptually, investor risk attitudes based on the Expected Utility theory and the prospect theory can jointly affect the long-short TK portfolio returns. Stock characteristics in the portfolios based on the TK values can affect returns through channels distinct from the probability weighting component. We next illustrate the effect of variations in probability weights in the equilibrium following the modelling framework of Barberis et al. (2021). We calculate the model predicted CAPM alpha spread for the long-short TK portfolio and demonstrate that a large drop in the spreads from positive to negative values when investors change risk attitudes from over-weighing to under-weighing tail events.

We conduct additional empirical analysis of stock returns to draw the contrast between overweight periods and underweight periods. Besides the long-short TK portfolio, we examine the decile TK portfolio performance, and find opposite patterns in the relationship between portfolio alphas and TK values. We further investigate whether the variations in the long-short TK portfolio returns are the result of exposure to common risk factors, or time-varying stock characteristics. Additionally, we conduct Fama-MacBeth analysis to control for known predictors of stock returns, including capital gains overhang (CGO) and other lottery-related stock characteristics. Our findings suggest that time-varying probability weights provide a robust and distinct channel for explaining the variations in long-short TK portfolio returns.

As an application of our finding, we propose a trading strategy that substantially outperforms the original long-short TK portfolio. For the value-weighted returns, the Sharpe ratio is raised from 0.196 for the original portfolio to 0.492 with our strategy. Our findings are robust to the alternative estimation methods for the probability weights.

This paper is related to several strands of literature. First, this paper follows closely the literature on applying the prospect theory to explain stock returns and stock anomalies, such as Barberis and Huang (2008), Barberis et al. (2016), and Barberis et al. (2021). While the probability weights are constant in these studies, we allow the probability weights to be time-varying to reflect the fluctuations in the risk attitudes of the representative agent towards tail events and study the implications. Our findings not only strengthen the empirical support for the prospect theory, but also highlight the importance of studying the variations in the predictions from the prospect theory.

Several studies investigate the time-series variations in asset returns related to the prospect theory. Baele et al. (2018) apply the cumulative prospect theory to simultaneously explain low returns on both out-of-the-money put and out-of-the-money call options, and combine probability weighting and time-varying equity return volatility to match the time-series pattern of the variance premium. In contrast to their finding that time-varying probability weights are not important in explaining the dynamics of variance premium, we find that time-varying probability weights are important to explain the dynamics of stock returns. Our paper is also different from Liu et al. (2020) in that they focus on the variations around earnings announcements. An et al. (2020) find that several lottery-related anomalies are state-dependent and stronger among stocks where investors have lost money. Barberis et al. (2021) also have the CGO variable in their model to capture prior trading gains or losses. We find that the explanatory power of the time-varying probability weights is little affected by the CGO variable.

This paper is also related to the literature on estimating probability weighting functions, such as Tversky and Kahneman (1992), Gonzalez and Wu (1999), and Prelec (1998) using laboratory experiments, and Polkovnichenko and Zhao (2013) using prices from stock indices and index options.

The rest of the paper is organized as follows. In Section 2, we review the conceptual framework of this paper and show the implication of the model. Section 3 describes the data and variables and how we estimate the parameter. In Section 4, we do empirical tests on our hypothesis. We do further analysis in Section 5 and conclude in Section 6.

2 Conceptual Framework

Our aim is to study how time-varying probability weighting preferences affect the implication of the cumulative prospect theory in asset pricing. In this section, we review the construction of portfolio based on the prospect theory (Barberis et al., 2016) and Barberis et al. (2021) model that incorporates prospect theory in equilibrium structure. We also illustrate how the model implies time-varying probability weighting preference.

2.1 Portfolio construction based on prospect theory

Probability weighting is one of the two important parts in the prospect theory. The first version of the prospect theory is proposed by Kahneman and Tversky (1979) to serve as an alternative model of the expected utility in explaining people's decision under risk. To incorporate cumulative functional (rank dependent) and extend the model to risky prospects with more than two outcomes, Tversky and Kahneman (1992) develop an advanced version, the cumulative prospect theory, which is the version we use in this paper[1].

[insert Figure 1 here]

To calculate the value of any given gamble under the cumulative prospect theory, as well as the expected utility, investors have to consider two parts, the utility of outcomes and their probabilities. Under the expected utility, the utility function is typically carrying for the final wealth, differentiable everywhere, and concave everywhere, and the probabilities are objective. On the contrary, under the cumulative theory, the utility function (also called value function) is carrying the gains and losses, kinked at the origin (so that the losses are more sensitive to individuals), concave only for gains but convex for losses, and the probabilities are transformed under some functions. Figure 1 shows how the value function and the probability function would look like. In our concern (see Section 3.2 for details), where probability weighting function is time-varying, it is possible that the function curve has inverse S-shape ($\delta < 1$) if investors overweight tails, linear-shape ($\delta = 1$) if investors don't transform the probabilities, and S-shape ($\delta > 1$) if investors underweight tails.

Applying to the stock returns, Barberis et al. (2016) suggest to calculate the prospect theory value (TK value) for each stock using past five years monthly return in excess of

[1]See Barberis et al. (2016) section 1 for a simple but more detailed description.

the market. First, they sort the sixty excess returns in increasing order, which starts with the most negative returns through to the most positive returns. Second, they give each return equal weight in the distribution, so each return has a physical probability of 1/60. To illustrate this, suppose that m of these returns are negative, while n=60-m of these returns are positive. Label the most negative return as r_{-m}, the second most negative one as r_{-m+1}, and so on, through to r_n, the most positive return, where r is a monthly return in excess of the market. The stock's historical return distribution is then

$$(r_{-m}, \frac{1}{60}; r_{-m+1}, \frac{1}{60}; ...; r_{-1}, \frac{1}{60}; r_1, \frac{1}{60}; ...; r_{n-1}, \frac{1}{60}; r_n, \frac{1}{60}). \quad (1)$$

Third, calculate the TK value for each stock-month using value function and probability weighting function

$$\begin{aligned} TK \equiv & \sum_{i=-m}^{-1} v(r_i) \left[w^- \left(\tfrac{i+m+1}{60}\right) - w^- \left(\tfrac{i+m}{60}\right)\right] \\ & + \sum_{i=1}^{n} v(r_i) \left[w^+ \left(\tfrac{n-i+1}{60}\right) - w^+ \left(\tfrac{n-i}{60}\right)\right], \end{aligned} \quad (2)$$

where $v(.)$ is the value function with the form

$$v(x) = \begin{cases} x^\alpha, & x \geq 0 \\ -\lambda(-x)^\alpha, & x < 0 \end{cases} \quad (3)$$

and w^+ and w^- are the probability weighting function with the form

$$w^+(P) = \frac{P^\phi}{(P^\phi + (1-P)^\phi)^{1/\phi}}, \quad w^-(P) = \frac{P^\delta}{(P^\delta + (1-P)^\delta)^{1/\delta}}. \quad (4)$$

They use the parameters estimated by Tversky and Kahneman (1992), namely,

$$\alpha = 0.88, \quad \lambda = 2.25$$
$$\phi = 0.69, \quad \delta = 0.61.$$

Fourth, at the start of each month, they sort stocks into deciles based on TK value and construct value-weighted and equal-weighted portfolios within each decile for the following month. A long-short strategy portfolio in this paper based on their method is to long the low-TK decile portfolio and short the high-TK decile portfolio.

2.2 Model and implication

Barberis et al. (2021) present a model that incorporates all of the elements of prospect theory, and this model can help explain a majority of the common anomalies. In this subsection, we will first briefly describe their model and then show its implication of time-varying probability weighting preferences.

Consider an economy with three dates, t = -1, 0, and 1, and investors make decision at date 0. There is a risk-free asset with gross per-period return R_f and N risky assets with gross per-period return $\tilde{R}_i$ for risky asset i. Let $\tilde{R} = (\tilde{R}_1, ..., \tilde{R}_N)^0$ be the return vector and has cumulative distribution function $P(\tilde{R})$. The vector of expected returns on the risky assets is $\bar{R} = (\bar{R}_1, ..., \bar{R}_N)$ and the covariance matrix of returns is Σ.

Assume investors in the economy are identical in their preferences, wealth at time -1 ($W_{\ 1}$), and wealth at time 0 (W_0). Let the fraction of time 0 wealth that an investor allocates to risky asset i is Θ_i, and the allocation vector is $\Theta = (\Theta_1, ..., \Theta_N)^0$. Thus, wealth at time 1 is

$$\tilde{W}_1 = W_0((1 - 1^0\Theta)R_f + \Theta^0\tilde{R}). \tag{5}$$

At date 0, each investor solves the following objective function to determine the allocation:

$$\begin{aligned} &\max_{\Theta_1,...,\Theta_N} E(\tilde{W}_1) - \tfrac{\gamma}{2} Var(\tilde{W}_1) + b \textstyle\sum_{i=1}^N V(\tilde{G}_i) \\ &= \max_{\Theta_1,...,\Theta_N} W_0((1 - 1^0\Theta)R_f + \Theta^0\bar{R}) - \tfrac{\gamma}{2} W_0^2 \Theta^0 \Sigma \Theta + b \textstyle\sum_{i=1}^N V(\tilde{G}_i), \end{aligned} \tag{6}$$

where

$$\tilde{G}_i = W_0\Theta_i(\tilde{R}_i - R_f) + W_{\ 1}\Theta_{i,\ 1}g_i. \tag{7}$$

In equation (6), the first two terms are the traditional mean-variance preferences and the third term captures both prospect theory and narrow framing. γ measures the aversion to portfolio risk and b decides the weight of prospect theory in decision. The third term is the sum of $V(\tilde{G}_i)$, which corresponds to asset i, respectively. Specifically, G_i is the potential gain or loss on asset i, which is the sum of potential future gain or loss to asset i ($\Theta_i(\tilde{R}_i - R_f)$) and the experienced gain or loss in investor's holdings of asset i prior to time 0 ($W_{\ 1}\Theta_{i,\ 1}g_i$). $V(\tilde{G}_i)$ is the cumulative prospect theory value of this gain or loss, incorporating loss aversion, diminishing sensitivity, and probability weighting.

For $\Theta_i > 0$, $V(\tilde{G}_i)$ can be written as

$$\begin{aligned} &-\lambda W_0^{\alpha} \int_{1}^{R_f \quad \Theta_{i,\ 1} g_i/\Theta_i} (\Theta_i(R_f - R_i) - \Theta_{i,\ 1} g_i)^{\alpha} dw(P(R_i)) \\ &\quad -W_0^{\alpha} \int_{R_f \quad \Theta_{i,\ 1} g_i/\Theta_i}^{1} (\Theta_i(R_i - R_f) + \Theta_{i,\ 1} g_i)^{\alpha} dw(1 - P(R_i)). \end{aligned} \tag{8}$$

Let $\Theta_{M,R} = \sum_{i=1}^{N} \Theta_{M,i}$, where $\Theta_{M,i}$ is the market value of asset i divided by the total market value of all traded assets. Define $\theta_i = \Theta_i/\Theta_{M,R}$, $\theta_{M,i} = \Theta_{M,i}/\Theta_{M,R}$, and $\theta_{i,\ 1} = \Theta_{i,\ 1}/\Theta_{M,R}$. With a few assumptions, Barberis et al. (2021) show a bounded rationality with heterogeneous holdings equilibrium structure that can help to quantitatively predict the cross-section of average returns when investors evaluate risk according to prospect theory[2]:

$$\begin{aligned} &\theta_i \left(\mu_i + \frac{\nu\zeta_i}{\nu\ \ 2} - R_f\right) - \frac{\hat{\gamma}}{2}(\theta_i^2\sigma_i^2 + 2\theta_i(\beta_i\sigma_M^2 - \theta_{M,i}\sigma_i^2)) \\ &\quad -\lambda\hat{b} \int_{1}^{R_f \quad \theta_{i,\ 1} g_i/\theta_i} (\theta_i(R_f - R_i) - \theta_{i,\ 1} g_i)^{\alpha} dw(P(R_i)) \\ &\quad -\hat{b} \int_{R_f \quad \theta_{i,\ 1} g_i/\theta_i}^{1} (\theta_i(R_i - R_f) + \theta_{i,\ 1} g_i)^{\alpha} dw(1 - P(R_i)), \end{aligned} \tag{9}$$

where μ_i, ζ_i, and ν are location parameter, asymmetry parameter, and degree of freedom scalar, respectively, assuming one-dimensional GH skewed t distribution for return distribution. $\hat{\gamma} = \gamma W_0 \Theta_{M,R}$ and $\hat{b} = bW_0^{\alpha\ \ 1}\Theta_{M,R}^{\alpha\ 1}$. Following the process in their paper, if we assume stocks in each anomaly decile are identical, we can then calculate the expected return of all stocks in each decile by inputting average beta, average standard deviation, average skew-

[2] See Barberis et al. (2021) section 2 for more details.

ness, average capital gain overhang, and market capitalization of the decile. Note that the probability weighting parameters ϕ (for right tail) and δ (for left tail) mentioned in Section 2.1 are both set to be equal to 0.65 in the model, and the value function parameters (α, λ) are set to be equal to (0.7, 1.5) to reflect the recent findings in the literature. We follow these settings in our model for implication but change the probability weighting parameter for our purpose. Since the probability weighting parameters for left tail and right tail are set to be equal, we use δ to denote this parameter in this paper.

We sort all stocks in deciles based on TK value calculated in Section 2.1 and calculate the inputs[3] To emphasize the role of prospect theory in picking stocks, we set the capital gain overhang input in all deciles as the negative of average market return. It serves as a common reference point for investors when they treat the expected returns. On the contrary, if we use the capital gain overhang calculated in each decile as the paper uses, the difference between low-TK decile and high-TK decile in our sample period will be fifty percent, attributing almost all the prediction power of TK to capital gain overhang[4]. We also try different combinations of the parameter settings for $\hat{\gamma}$ and $\hat{b}$ to see whether the weights of risk aversion and prospect theory would significantly affect the implication of the model. With moderate weights combination, we should see similar implication from the model.

[insert Figure 2 here]

To see whether different value of δ will affect the prediction of CAPM adjusted alpha spread, we vary it from 0.55 to 2.50 in the model. Figure 2 shows that the alpha spread, in general, decreases with δ, and it jumps down around $\delta = 1$. When $\delta \leq 1$, the long-short portfolio based on TK should have positive alpha; when $\delta > 1$, the long-short portfolio should have zero or negative alpha. This result is in line with the theory in Barberis et al.

[3]We follow Barberis et al. (2021) strictly to calculate beta and market capitalization, but use past 5 years monthly returns to calculate standard deviation and skewness rather than use cross-sectional returns as in the paper. We argue that time-series volatility and skewness are better measures for a typical stock in the anomaly decile from the sense of picking stocks. However, we also admit that backward-looking method has its own disadvantages. See Section 3 for more discussion about the choice in their paper.

[4]The low-TK decile has average capital gain overhang of thirty-four percent, which is too high for a typical stock return to be treated as right tail.

(2016): when investors overweight tails, the high-TK stocks are overvalued by individual stocks so that, on average, they earn lower return than the low-TK stocks in subsequent months; but when investors underweight tails, such anomaly disappears. Along with the empirical findings that investors sometimes underweight tails, it helps to explain why TK anomaly, on average, has lower average return in recent years.

3 Data and estimation of probability weights

3.1 Data

We have two groups of data. One group is for empirical tests, including portfolio construction, model inputs calculation, time-series tests, and Fama-MacBeth tests. It consists of firm-level characteristics and market-wide factors. The other group is for time series of probability weighting parameter estimation. It consists of option data and market returns. We obtain stock return and accounting data from CRSP and Compustat. Market return is the value-weighted return including distribution (vwretd) from CRSP. S&P 500 index return is the return on S&P composite index (sprtrn) from CRSP. Our universe contains all common stocks that are publicly traded on NYSE, AMEX, and NASDAQ from January 1996 to December 2020[5]. To be consistent with Barberis et al. (2016), we also requires that the stocks have at least five years of monthly return data. We obtain monthly risk-free rate, Fama-French 3-factors (MktRf, SMB, HML), and Carhart momentum factor (UMD) from French Data Library[6]. The Pastro and Stambaugh (2003)'s liquidity factor is downloaded from WRDS. Cyclically adjusted price earnings ratio (CAPE) is obtained from Online Data Robert Shiller[7]. Variance risk premium (VRP) is from Hao Zhou's personal homepage[8].

The main variable to construct portfolio, as well as to do Fama-MacBeth tests, is TK. It

[5]We choose such a relatively short sample periods since the option data used to estimate δ starts from January 1996.

[6]Website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[7]Website: http://www.econ.yale.edu/ shiller/data.htm.

[8]Website: https://sites.google.com/site/haozhouspersonalhomepage.

is the prospect theory value defined in equation (2). We use past five years monthly return to calculate it following Barberis et al. (2016).

For model inputs, Beta, standard deviation, and skewness are average of firm-level characteristics within each TK decile across the whole sample periods. Firm-level Beta is CAPM beta computed using daily returns over the following year, following Barberis et al. (2021). Firm-level standard deviation and skewness are calculated using past five years monthly returns and are annualized assuming returns are independent. The spreads of these inputs are the difference between low-TK decile and high-TK decile (i.e., decile 1 minus decile 10). Capital gain overhang is the average of whole sample periods previous year market return and are the same for all deciles. Market capitalization is the average of whole sample periods previous month ratio of the decile market capitalization over the total market capitalization.

For time-series tests, past n to m month market excess return is the difference between past n to m month market cumulative return and risk-free cumulative return. Standardized physical moments are calculated using past 5 years monthly S&P 500 index return. Standardized risk-neutral moments are calculated using S&P 500 index option with 30-days maturity (see Section 3.2 for data source). GFC is a dummy variable control for 2008 financial crisis, which equlas 1 if the month falls in October 2008 to March 2009 and 0 otherwise.

We include several common control variables in Fama-MacBeth tests. Beta is calculated from monthly returns over the previous five years, following Fama and French (1992). Size is the log market capitalization at the end of the previous month. Bm is the log of book value of equity scaled by market value of equity, computed following Fama and French (1992) and Fama and French (2008). When the book value of equity is missing from Compustat, we use data from Davis et al. (2000); observations with negative book value are removed. Mom is the cumulative return from the start of month t-12 to the end of month t-2. Cgo is the capital overhang in month t-1, following Barberis et al. (2021). It is computed as $(P_i - R_i)/R_i$, where P_i is the stock's current price and R_i is investors' average purchase price. It is slightly different from Grinblatt and Han (2005), but is a more precise match for the capital gain variable in the cumulative prospect theory model. Rev is the return in month t-1. Illiq is Amihud (2002)'s measure of illiquidity, scaled by 10^5. Lt rev is the cumulative return from the start of month t-60 to the end of month t-13. Ivol is idiosyncratic return volatility, as in Ang et al. (2006). Max and Min are the maximum and the negative of the minimum daily returns in month t-1, as in Bali et al. (2011). Skew is the skewness of monthly returns over the previous five years. Eiskew is expected idiosyncratic skewness, as in Boyer et al. (2010). Coskew is coskewness, computed as in Harvey and Siddique (2000) using five years of monthly returns.TK, Mom, Rev, Ivol, Max, and Min are scaled up by 100. The sample period runs from March 1996 to December 2020[9]. All variables are winsorized at 1 and 99 percent in each month, and TK is standardized after the winsorization.

3.2 Estimation of probability weighting function

We obtain the S&P 500 index option (symbol SPX) data from OptionMetrics and the S&P 500 index return from CRSP. We extract the needed information for option from the implied volatility surface.

The estimation of probability weighting parameter, δ, includes two parts: the risk-neutral distribution and the physical distribution. Through stochastic discount factor, Polkovnichenko and Zhao (2013) show that the probability weighting function has the following relationship with the risk-neutral distribution and the physical distribution. Let $P(.)$ and $p(.)$ be the physical distribution function and its density, $Q(.)$ and $q(.)$ be the risk-neutral distribution function and its density, and $G(.)$ be the probability weighting function. For a specific P_0 with corresponding gross return R_0 such that $P(R_0) = P_0$, we have

$$G(P_0) = c \int_0^{R_0} \frac{q(R)}{u^{'}(R)} dR, \tag{10}$$

[9]It starts from March 1996 since the benchmark three-month moving average δ starts from March 1996

where $u^{0}(R)$ is the marginal utility and $c = (\int_{0}^{1}(q(R)/u^{0}(R))dR)\ \ ^{1}$ is the normalizing constant. Apply equation (10) to the prospect theory described in Section 2.1, where $v(.)$ is the value function[10], w^{+} and $w\ \ $ are the probability weighting function for positive returns and negative returns starting from the two tails, respectively, and $r_i = R_i - 1$ is the net return. For a specific P_i with corresponding to positive net return r_i such that $P(r_i) = P_i$ and a specific $P_{\ \ j}$ with corresponding to negative net return $r_{\ \ j}$ such that $P(r_{\ \ j}) = P_{\ \ j}$[11], we have

$$\begin{aligned} w^{+}(P_i) &= k \sum_{l=i}^{n} \frac{q(R_l)}{v^{0}(r_l)} \\ w\ \ (P_{\ \ j}) &= k \sum_{l=\ \ m}^{\ \ j} \frac{q(R_l)}{v^{0}(r_l)} \end{aligned} \tag{11}$$

where $k = (\sum_{l=\ \ m}^{n} q(R_l)/v^{0}(r_l))\ \ ^{1}$ is the normalizing constant.

For risk-neutral distribution, we follow the procedure of Polkovnichenko and Zhao (2013)[12]. To match the monthly return, we use the S&P 500 index option with 30 days to expiration[13]. First, we estimate the risk-neutral moments based on Bakshi and Madan (2000) and Bakshi et al. (2003). Define the $(T-t)$ period log return as $R_t(T) = ln(F_T(T)/F_t(T))$. Then, we can compute the risk-neutral moments, $\mu_{R,n} = E_t^{Q}[R_t^{n}(T)]$, from the out-of-money call and put prices. Second, we estimate the risk-neutral density from the moments through Gram-Charlier series expansion (GCSE), which is a semi-parameter method. We calculate both A-type GCSE following Jarrow and Rudd (1982) and C-type GCSE following Rompolis and Tzavalis (2008), and then pick the best estimate based on the information criteria, such as the Akaike information criterion (AIC) or Schwarz criterion (SC), as the guidance for a nonparametric method, the constrained local polynomial method. Finally, we use the constrained local polynomial method to estimate the risk-neutral density. It is estimated by taking derivatives of call price function with respect to strike based on the idea that

[10]Note that we use a different pair of parameter value for the value function in benchmark following Barberis et al. (2021), namely, $\alpha = 0.7$ and $\lambda = 1.5$, to reflect the findings in recent literature (see Walasek et al. (2018) and Chapman et al. (2018)). We also use the values in Tversky and Kahneman (1992) for robustness test.

[11]The return distribution follows equation (1)

[12]See Section 3 and Appendix B in their paper for process and technique details

[13]Note that the monthly return from CRSP is simple return while the estimation in the following procedure uses log return, so one may need to unify them when using the distribution.

the risk-neutral density is the scaled second derivative of the call option price with respect to the call strike price (Breeden and Litzenberger, 1978).The procedure follows Aït-Sahalia and Duarte (2003) but when implement it, we estimate the call price function using the GCSE method rather than simulating data from the Black and Scholes formula with implied volatility modeled as a parametric function of moneyness.

For physical distribution, in order to be consistent with prospect theory, we use past five years monthly returns with equal probability, which is the same distribution as equation (1). One may argue to use forward-looking approaches instead, such as GARCH model, but such methods may not be appropriate under the circumstance described by Barberis et al. (2016), where the model is for individual investors rather than professional economists. We also try GARCH(1,1) for physical distribution, the trend of moving average δ is close to the benchmark while the level of it varies. Since we only need to separate the high and low δ periods, the results based on alternative estimation methods are very similar.

We then use the pairs of transformed probability $w^{+}(P_i)$ $(w \ \ (P_{\ j}))$ and physical probability P_i $(P_{\ j})$ in each month to fit equation (4). Assuming $\phi = \delta$ as in Barberis et al. (2021) so that we can use all the sixty pairs[14], we use nonlinear least-squares[15] to estimate the parameter

$$\min_{\delta} \sum_{i= \ m}^{n} \left[w(P_i) - \frac{P_i^{\delta}}{(P_i^{\delta} + (1-P_i)^{\delta})^{1/\delta}} \right]^2 . \tag{12}$$

Since the estimated δ is volatile while the true preferences may not change as sharply as the estimation, we use 3-month moving average.[16]

The trend of the estimated δ is shown in the bottom panel of Figure 3. There are three periods that δ is generally greater than 1, November 2004 to March 2006, March 2012 to July 2015, and June 2016 to January 2018, which indicate that the "representative" investor

[14] In robustness, we release this assumption and estimate the parameters for the two tails separately.

[15] Use nl command in Stata with initial level 0.65. The initial level, 0.65, is picked so that the estimation is as close to experimental level as possible. One may try other initial level that possibly lead to different estimation, but need to restrict specific level of R-square to obtain reasonable estimation

[16] It is the average of δ in past two months and the current month. Since the data is available at the beginning of each month, our estimated δ is also available at the beginning of each month. In robustness, we also try 1- (no moving average), 6-, and 12-month moving average.

may not overweight tails during these three periods. We can also see that δ drops quickly from March 2020, which is consistent with the narrative that investors reassess the tail probabilities during the COVID-19 crisis.

In comparison to the estimated δ, we plot the time series of the cumulative gross return of value-weighted long-short TK portfolio in the top panel of Figure 3. The cumulative gross return reaches over 400% in the earlier years, and then gradually decreases in the later years. The falling returns appear not caused by 2008 financial crisis, as we can see a recovery after 2008. The cumulative return keeps falling till 2020. We observe that the time periods with $\delta < 1$ are often accompanied with rising returns, in support of the prediction from the cumulative prospect theory.

[insert Figure 3 here]

4 Empirical analysis

In this section, we study how time-varying probability weight δ affect stock returns predicted by the prospect theory TK values. As explained in Section 2, investors overweight tails when $\delta < 1$ and underweight tails when $\delta > 1$. Our main hypothesis is that the predictive power of TK values on stock returns is stronger during the overweight periods.

4.1 Decile portfolio performance

In this section, we test our hypothesis using decile sorts by the stocks' TK values. Following Barberis et al. (2016), we sort stocks into deciles based on TK at the start of each month from March 1996 to December 2020. We then compute both value-weighted and equal-weighted average return of each TK-decile portfolio over the next month, giving us a time-series of monthly returns for each TK decile. Using δ estimated in Section 3.2, we divide the time series of monthly returns into two subsample periods: the overweight periods (the months with $\delta < 1$) and the underweight periods (the months with $\delta > 1$).[17] We calculate the average return of each decile over the whole sample, the overweight period, and the underweight period. In Table 3, we report the average return of each decile in excess of the risk-free rate; the four-factor alpha for each decile following Carhart (1997) (the return adjusted by the Fama and French (1993) three factors and a momentum factor); and the five-factor alpha for each decile (the return adjusted by the Fama and French (1993) three factors, the momentum factor, and the Pastro and Stambaugh (2003) liquidity factor). We also report the difference between the returns of the lowest TK decile and the returns of the highest TK decile in the right most column. The long-short portfolio is a zero investment portfolio that longs the stocks in lowest TK decile and shorts the stocks in highest TK decile (low-minus-high portfolio).

[insert Table 3 here]

Barberis et al. (2016) show that the average return of the long-short TK portfolio is positive, and more strongly for the equal-weighted portfolio returns. As shown in Table 1, during our sample period, we also find significant positive returns for the equal-weighted portfolios, but small and insignificant for the value-weighted returns. However, during the overweight periods, the value-weighted average return doubles in magnitude and becomes significant (at 10% level for excess return and 5% level for four-factor and five-factor adjusted return). On the other hand, the value-weighted average return during the underweight periods is significantly negative. The differences in average returns between the two periods are about four times of the average returns during the whole sample return for both equal-weighted and value-weighted, and highly significant.

[insert Figure 4 here]

Figure 4 plots the results of four-factor alphas in Table 3. The alphas of the ten decile portfolios, according to Barberis et al. (2016), should decline monotonously moving from

[17] We do not have months with $\delta = 1$ in our sample. In Section 5.1 and Section 4.3, we further divide the sample into three periods using 0.9 and 1.1 as the cutoff values for δ.

low TK portfolio to the high TK portfolio. This trend does not show up during sample period for value-weighted portfolios. However, the monotonic pattern shows up during the overweight periods, and the inverse pattern shows up during the underweight periods, which explains the non-monotonic pattern in whole sample. The equal-weighted portfolios exhibit the monotonic pattern in the whole sample, but we still find flat pattern during the underweight periods, and more pronounced monotonous decline during the overweight periods.

4.2 Variations in the long-short TK portfolio returns

We now investigate the time-series variations in the long-short TK portfolio returns. Specifically, we examine whether the variations are the result of exposure to common risk factors, or time-varying stock characteristics.

Table 2 reports the predictive power of the time-series of probability weights for the long-short TK portfolio returns over 3-, 6-, and 12-month horizon. In the predictive regression, we include a constant and the overweight dummy for the months with $\delta < 1$ (Over-Under). The portfolio returns are over the period of 3, 6, and 12 months after the portfolio construction. All the coefficients for overweight dummy are significant and the magnitudes are proportional to the horizon, a sign of persistent predictive power over long horizon.

[insert Table 4 here]

The time-series variations in long-short TK portflio returns could also be explained by exposures to risk factors for cross-sectional stock returns. In Table 4, we gradually include Carhart (1997)'s four factors (MktRf, SMB, HML, and UMD), GFC, and the interaction terms between the factors and probability weights. The dummy variable indicating overweight periods captures the differences between overweight and underweight periods since we include a constant in the regression. The estimates for the overweight dummy variable remain robust in magnitudes and significance for both value-weighted and equal-weighted long-short portfolio. Interestingly, the momentum factor can explain about one-fifth of the variations since the high TK stocks tend to be momentum stocks. The effects of conditional factor exposures during overweigtht and underweight periods, captured by the interaction terms, are insignificant.

[insert Table 5 here]

Another source of time-series variations in long-short TK portfolio returns could be the time-varying stock characteristics in the long and short portfolios. As illustrated in the model in Section 2.2, the differences between the low-TK decile returns and the high-TK decile returns are determined by the differences of firms' characteristics between the two deciles. In Table 5, we include the time-series of these differences in market beta, return volatility and skewness, as well as variance risk premium (Baele et al. (2018)), into the regression. The coefficient estimate of overweight dummy variable remains significant when we include these additional control variables, and the time-series of the differences in return volatilities actually reduce long-short portfolio returns. This result shows that time-varying probability weights, not time-varying stock characteristics, drive the results.

4.3 Fama-MacBeth tests

We now examine the effect of time-varying probability weights using Fama-MacBeth methodology to control for known predictors of stock returns in Table 6. In the first step, we regress excess returns on stocks' TK values and a long list of characteristics. In the second step, we regress the TK coefficients from the first step on three specifications: a constant for whole sample period (Whole); a constant and an overweight dummy indicating $\delta < 1$ (Over-Under by 1); a constant, an overweight dummy indicating $\delta < 0.9$ and a middle dummy indicating $0.9 \leq \delta \leq 1.1$ (Over-Under by 0.9 and 1.1). We report the estimate of the constant for the whole sample as benchmark and the estimates on overweight dummy variables to test the hypothesis. A lower cutoff value for δ is closer to the parameter value used in the theoretical model, but results in fewer observations. We discuss this further in Section 5.1.

[insert Table 6 here]

As in Barberis et al. (2016), the results of whole sample indicate that TK has significant predictive power after controlling for the known predictors in our sample periods. In terms of stock characteristics, past returns (Rev) decrease the economic magnitude of the coefficient on TK; lottery-like features (IVOL, Maxret, and Minret) increase the economic magnitude; the reference point in prospect theory value function (CGO) also increases the economic magnitude. In summary, TK remains its predictive power in economic magnitude and statistical significance even after the inclusion of known stock return predictors.

To focus on the time-varying probability weights, we examine the coefficients on the overweight dummy variable for both cutoff values for δ. During the overweight periods, the economic magnitudes of the TK coefficients are larger than those in the whole sample. Consistent with our model, a lower cutoff value generates higher magnitude of the TK coefficients. Although the economic magnitude of the TK coefficient changes with the included control variables, the estimate of the overweight dummy remains significant. This finding indicates that the time-series variations in TK portfolio returns cannot be explained away by the characteristics that are also time-varying.

One such stock characteristics is CGO as shown in An et al. (2020) that several lottery-related anomalies are state-dependent and stronger among stocks where investors have lost money. Barberis et al. (2021) also have CGO in their model to capture prior trading gains or losses. Therefore, it is possible that the variations in the TK coefficients can be explained by stock's CGO. After including CGO, we observe the time-series variations in TK coefficients are little affected.

Additionally, many characteristics for lottery-like stocks could be time-varying, such as IVOL, Maxret, and Eiskew. Were these anomalies also stronger during the overweight periods, the estimates for the overweight dummy variables would be less significant. However, we observe that the estimates are still significant controlling for these characteristics, and the magnitudes become even larger when we use $\delta < 0.9$ as the cutoff.

5 Further analysis

5.1 Trading strategy

We know from Section 4.1 that the TK prediction power is different between overweight periods and underweight periods, making the average return in whole sample periods decreases. It helps explain why we see a low total return in value-weighted long-short TK portfolio in a twenty-five years investment horizon shown in Figure 3. In this section, we propose a new trading strategy to overcome the problem.

The basic idea is to flip the original long and short positions when investors underweight tails. However, simply flipping the positions using $\delta = 1$ as the trigger is not a good choice. First, there are time periods that δ jumps up and down frequently around 1, such as 2004 to 2007. It is hard to implement the trading strategy following δ with such frequent change of position. Second, the estimation of δ usually include noise such that the true δ deviates from what we estimated. It is especially problematic when the estimated δ is close to 1. Third, it is possible that investors actually do not act when probability weighting preferences are slight (i.e., δ is close to 1). Then, flipping the positions is just adding noise to the strategy.

With the arguments above in mind, we propose to use 0.9 and 1.1 as triggers. We begin with original long-short TK portfolio in March 1996, which has $\delta \leq 1$. Later, if $\delta > 1.1$ in a month, then we flip the positions in that month and the following months: long high-TK decile stocks and short low-TK decile stocks. After this flip, if $\delta < 0.9$ later, then we flip the long and short position back as the original portfolio: long low-TK decile stocks and short high-TK decile stocks. Do the same flip strategy if triggers appear again.

[insert Table 7 here]

We report the characteristics of the original long-short TK portfolio and the strategical long-short TK portfolio in Table 7. While the standard deviations and skewness are similar between original portfolio and strategical portfolio, the average return increases significantly in strategical portfolio, leading to a much higher Sharpe ratio. Such increase is even greater if we adjust the return by four-factor model or five-factor model.

[insert Figure 5 here]

Figure 5 shows the results of the trading strategy, on both value-weighted (left panels) and equal-weighted basis (right panels). While the original value-weighted long-short portfolio cumulative return generally goes down in the past ten years, the strategical value-weighted long-short portfolio cumulative return goes up. The gross return of the strategical portfolio would increase to 1600% in the past twenty-five years. There is a large drop for both original and strategical portfolio in 2008, which is coincident with the financial crisis. However, if we use four-factor model to adjust related factor risks, the large drop disappears, and the strategical portfolio would lead to a greater cumulative return. The equal-weighted strategical portfolio's cumulative return almost overlaps with the equal-weighted original portfolio in the first fifteen years but diverges gradually in the last ten years. This is consistent with the results of estimated δ shown in Figure 3, in which almost no δ is greater than 1.1 in the first fifteen years. It is also striking that, in the past ten years, the strategical portfolio cumulative return quadruples while the original portfolio cumulative return has no growth.

5.2 Robustness of probability weights

Since our outcomes rely heavily on the estimated δ, we provide several robustness checks for the time-series results. In panel A of Table 8, we check whether δ using different estimating method would influence the effect of differentiating overweight periods and underweight periods on long-short TK portfolio returns. In panel B, we check whether the number of months used to smoothing preferences would affect the effect.

[insert Table 8 here]

First, literature after Tversky and Kahneman (1992) that focuses on probability weighting function usually uses two-parameter model, rather than single-parameter model, to separately control for the curvature and the elevation of the function[18]. We can interpret the curvature as reflecting how much is the investor discriminating between probabilities (the magnitude of overweight or underweight) and the elevation as how attractive is the chance domain of prospect to the investor (the crossover point between the curve and the 45 degree line). Here, we use Gonzalez and Wu (1999)'s linear in log odds probability weighting function to substitute the one in Tversky and Kahneman (1992) to estimate the parameter for curvature, which is used to differentiate overweight periods and underweight periods. The differences in average return for both value-weighted and equal-weighted portfolio are close to the ones in benchmark (even a little bit better in the sense of the magnitude and significance of difference) while the number of months in overweight decreases. This result indicate that changing model would not influence our conclusion on differentiating overweight periods and underweight periods, and there are indeed some months that investors have slight probability weighting preferences (δ is close to 1).

Second, while probability weighting function subjects to the restriction of parameter value, value function also has the same problem. In Section 3.2, we use $\alpha = 0.7$ and $\lambda = 1.5$ in benchmark, and now we use the original value in Tversky and Kahneman (1992), namely, $\alpha = 0.88$ and $\lambda = 2.25$. Again, the results are similar, and it only changes the belonging of months that δ is close to 1.

Third, we release the assumption that the probability weighting parameters are equal for both tails. We find that they have similar trend while the parameter of left tail is generally smaller than the parameter of right tail. To have meaningful number of months in each group, we use only right tail to differentiate overweight periods and underweight periods. The difference in average return still holds its economic magnitude and statistical significance.

[18]See, for example, Prelec (1998), Gonzalez and Wu (1999), , Zhang and Maloney (2012), etc.

Last, we test whether changing the number of months in smoothing process would affect our conclusion. We find that even no smoothing (1-month) can result in similar findings. Although the difference in average return of value-weighted portfolio is not significant, it still has a large magnitude; the difference in average return of equal-weighted portfolio is significant. When we increase the number of months in moving average to six and twelve, the results are close to the one in benchmark. It indicates that to have an efficient differentiation on preferences, we need some smoothing process, but how long to smooth does not matter much. It also implies that our findings mainly come from the trend of time-varying preferences rather than the zigzags of the time series estimation curve.

5.3 Sources of time variation in probability weighting

We have shown that time-varying probability weighting preferences affect the prediction of TK by differentiating overweight periods and underweight periods. In this section, we explain the sources of probability weighting time variation, and study whether it can be backed out by related factors.

[insert Table 9 here]

We regress the dummy variable of overweight, which equals 1 if $\delta \leq 1$ and 0 otherwise, on related moments and variables. In the first three columns in Table 9, we tests the effect of first moments, namely, the past market excess returns. It shows that past 1 month market excess return and past 2 to 12 month market excess return have significantly negative relationship with overweight while 2 to 60 month market excess return does not have significant relationship, implying that low near past returns may boost investors' overweight preferences on tails of the distribution. Column (4) to (6) study the effect of higher moments of physical and risk-neutral distribution. When regressing together, it shows that physical variance, risk-neutral skewness, and risk-neutral kurtosis have significantly negative sign while physical kurtosis is significantly positive at marginal level. The negative sign of physical variance means that investors tend to not overweight when the market is volatile, illustrating that investors avoid lotteries when risk is high. The positive sign of physical kurtosis implies that investors prefer to overweight tails when small events indeed happens more frequently. It seems counter-intuitive that risk-neutral skewness and kurtosis have negative relationship with overweight preferences since we expect investors to overweight tails when the risk premium on the third and fourth moments are high. One potential explanation is that the out-of-money options, which mainly reflect the third and fourth risk-neutral moments, are illiquid when implied volatility is low. As a result, the risk premium measured by risk-neutral skewness and kurtosis includes not only the risk premium we want but also the liquidity premium we don't want. It is hard to say which premium increases when they are mixed. In column (7), we includes all the moments. The coefficients of past market excess returns are not significant anymore, implying the effect of the first moments are covered by higher moments. In column (8), we also include previous month CAPE and GFC. The previous month CAPE does not significantly explain the time variation of overweight preferences, showing that overweight preferences are not determined by whether stocks are relatively expensive or cheap. GFC is negatively correlated with overweight preferences, meaning that after control other moments, financial crisis does not necessarily stimulate investors to overweight tails. Also note that all the columns have adjusted R-square lower than 0.5. It means that we cannot simply back out the overweight dummy by linearly combining the moments. The overweight time variation does include additional information out of the moments.

6 Conclusion

We propose that the probability weighting preferences in prospect theory are empirically time-varying and the asset pricing implication based on prospect theory is different between overweight periods and underweight periods. We use option data to estimate the time-varying preferences based on the relationship among risk-neutral distribution, physical distribution, and probability weighting function and suggest to smooth the estimated parameter. We find that in our shorter but newer sample, the predictive power of prospect theory value generally holds but weakens. After differentiating overweight periods and underweight periods, the predictive power resumes in overweight periods but shows no predictive power or reverse direction of predictive power in underweight periods.

Our results support the effect of prospect theory in asset pricing. Moreover, our findings show the importance of differentiating time periods based on probability weighting preferences in later research on prospect theory.

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Table 1
TK long-short portfolio performance

	Whole	Over	Under	Over-Under
	Panel A. Value-weighted			
Mean	0.56	1.44*	-1.00*	2.44**
	(0.98)	(1.74)	(-1.77)	(2.44)
# of months	298	190	108	
	Panel B. Equal-weighted			
Mean	1.18***	2.05***	-0.35	2.40***
	(2.67)	(3.18)	(-0.86)	(3.15)
# of months	298	190	108	

The table reports average monthly returns differentiated by δ, on both an equal-weighted (EW) and value-weighted (VW) basis, of long-short portfolios of TK. TK portfolio is formed following Barberis et al. (2016). All months belong to whole sample group (Whole). A month is classified into overweight group (Over) if $\delta \leq 1$ in that month. A month is classified into underweight group (Under) if $\delta > 1$ in that month. Over-Under is the difference of the mean between Over and Under. t-statistics, in parentheses, are based on the heteroskedasticity-consistent standard errors of White (1980).

Table 2
Long horizon prediction

	Panel A. Value-weighted			Panel B. Equal-weighted		
	(1)	(2)	(3)	(4)	(5)	(6)
	3-month	6-month	12-month	3-month	6-month	12-month
Over-Under	0.075***	0.121**	0.226**	0.083***	0.159***	0.321***
	(2.74)	(2.34)	(2.22)	(3.15)	(2.86)	(2.84)
Constant	-0.032*	-0.047	-0.082	-0.016	-0.023	-0.038
	(-1.88)	(-1.55)	(-1.60)	(-1.26)	(-0.90)	(-0.80)
Observations	296	293	287	296	293	287
Adjusted R-square	0.036	0.045	0.080	0.058	0.071	0.110

The table reports the results of long horizon prediction using overweight dummy (Over-Under). The dummy variable equals 1 if $\delta \leq 1$ and 0 otherwise. t-statistics, in parentheses, are Newey-West adjusted with n+3 lags where n equals the number of months used to calculate the long-horizon return.

Table 3
Decile portfolio analysis

			P1 Low TK	P2	P3	P4	P5	P6	P7	P8	P9	P10 High TK	TK Low-high portfolio
Excess return	VW	Whole	1.253	0.817	0.671	0.963	0.863	0.808	0.952	0.725	0.755	0.697	0.556
			(1.84)	(1.58)	(1.67)	(2.62)	(2.71)	(2.84)	(3.58)	(2.89)	(3.17)	(2.45)	(0.98)
		Over	2.004	1.199	0.899	1.260	1.023	0.859	1.009	0.703	0.781	0.564	1.440
			(2.01)	(1.60)	(1.53)	(2.33)	(2.21)	(2.07)	(2.60)	(1.94)	(2.30)	(1.39)	(1.74)
		Under	-0.069	0.145	0.270	0.440	0.582	0.718	0.851	0.762	0.710	0.930	-1.00
			(-0.11)	(0.27)	(0.66)	(1.25)	(1.79)	(2.50)	(3.16)	(2.83)	(2.58)	(2.86)	(-1.77)
	EW	Whole	1.880	1.194	1.008	1.046	0.961	1.089	0.975	0.859	0.862	0.702	1.178
			(3.03)	(2.66)	(2.64)	(2.97)	(3.04)	(3.74)	(3.63)	(3.32)	(3.40)	(2.09)	(2.67)
		Over	2.624	1.486	1.168	1.189	1.017	1.179	0.991	0.827	0.836	0.576	2.047
			(2.89)	(2.28)	(2.11)	(2.34)	(2.23)	(2.81)	(2.59)	(2.25)	(2.32)	(1.18)	(3.18)
		Under	0.571	0.680	0.726	0.794	0.864	0.930	0.947	0.914	0.907	0.923	-0.351
			(0.94)	(1.47)	(1.79)	(2.09)	(2.51)	(2.90)	(3.06)	(3.03)	(3.02)	(2.67)	(-0.86)
Four-factor alpha	VW	Whole	0.413	-0.059	-0.047	0.317	0.208	0.171	0.371	0.108	0.160	-0.082	0.496
			(1.04)	(-0.24)	(-0.29)	(2.10)	(1.80)	(1.75)	(4.34)	(1.40)	(1.80)	(-1.00)	(1.22)
		Over	0.998	0.205	0.105	0.561	0.347	0.214	0.459	0.128	0.258	-0.141	1.139
			(1.80)	(0.60)	(0.48)	(2.73)	(2.17)	(1.54)	(3.87)	(1.22)	(2.20)	(-1.34)	(2.03)
		Under	-0.753	-0.414	-0.372	-0.123	-0.138	0.168	0.185	-0.008	-0.090	0.016	-0.769
			(-2.05)	(-1.54)	(-2.22)	(-0.69)	(-0.96)	(1.79)	(1.97)	(-0.08)	(-0.81)	(0.12)	(-1.82)
	EW	Whole	1.146	0.471	0.275	0.344	0.270	0.436	0.342	0.215	0.191	-0.121	1.267
			(3.08)	(2.74)	(2.38)	(3.72)	(3.60)	(6.62)	(5.52)	(3.25)	(2.78)	(-1.11)	(3.62)
		Over	1.671	0.619	0.322	0.393	0.257	0.470	0.327	0.166	0.159	-0.310	1.981
			(3.29)	(2.54)	(2.01)	(3.05)	(2.47)	(5.09)	(3.85)	(1.80)	(1.69)	(-2.07)	(4.09)
		Under	0.026	0.116	0.133	0.171	0.201	0.330	0.299	0.255	0.208	0.156	-0.130
			(0.06)	(0.74)	(1.10)	(1.87)	(2.28)	(5.06)	(4.46)	(3.39)	(2.28)	(1.37)	(-0.34)
Five-factor alpha	VW	Whole	0.397	-0.110	-0.082	0.289	0.224	0.169	0.360	0.107	0.146	-0.101	0.498
			(0.99)	(-0.42)	(-0.52)	(1.99)	(1.88)	(1.76)	(4.15)	(1.37)	(1.67)	(-1.22)	(1.22)
		Over	0.974	0.111	0.063	0.525	0.361	0.214	0.441	0.128	0.227	-0.180	1.154
			(1.73)	(0.30)	(0.29)	(2.64)	(2.17)	(1.57)	(3.61)	(1.19)	(1.94)	(-1.65)	(2.02)
		Under	-0.742	-0.421	-0.423	-0.145	-0.123	0.161	0.180	-0.016	-0.091	0.014	-0.756
			(-2.01)	(-1.56)	(-2.68)	(-0.85)	(-0.81)	(1.69)	(1.87)	(-0.16)	(-0.81)	(0.10)	(-1.79)
	EW	Whole	1.097	0.424	0.241	0.326	0.254	0.419	0.336	0.201	0.176	-0.121	1.218
			(3.09)	(2.61)	(2.24)	(3.66)	(3.49)	(6.61)	(5.38)	(2.98)	(2.59)	(-1.13)	(3.62)
		Over	1.567	0.526	0.253	0.356	0.219	0.433	0.306	0.127	0.121	-0.321	1.888
			(3.23)	(2.30)	(1.71)	(2.86)	(2.17)	(4.87)	(3.55)	(1.34)	(1.30)	(-2.20)	(4.07)
		Under	0.042	0.110	0.129	0.169	0.202	0.328	0.300	0.263	0.217	0.169	-0.127
			(0.11)	(0.69)	(1.07)	(1.78)	(2.29)	(5.10)	(4.55)	(3.61)	(2.49)	(1.45)	(-0.33)

The table reports average monthly excess returns and monthly alphas differentiated by δ, on both an equal-weighted (EW) and value-weighted (VW) basis, of portfolios of stocks sorted on TK. All months belong to whole sample group (Whole). A month is classified into overweight group (Over) if δ 1 in that month. A month is classified into underweight group (Under) if $\delta > 1$ in that month. Each month, all stocks are sorted into deciles based on TK. For each of the decile portfolios, P1 (low TK) through P10 (high TK), we report the average excess return, four-factor alpha following Carhart (1997), and five-factor alpha (Carhart four-factor model augmented by Pastro and Stambaugh (2003) liquidity factor). The sample runs from March 1996 to December 2020. t-statistics, in parentheses, are based on the heteroskedasticity-consistent standard errors of White (1980).

Table 4
Contemporary exposure tests

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Value-weighted						
Over-Under	0.024**	0.025**	0.022***	0.018***	0.017**	0.018***
	(2.38)	(2.57)	(2.92)	(2.75)	(2.58)	(2.70)
MktRf		0.839***	0.704***	0.323**	0.340***	0.305**
		(4.74)	(5.15)	(2.58)	(2.65)	(2.31)
SMB			1.042***	1.124***	1.124***	1.094***
			(5.72)	(8.45)	(8.40)	(4.79)
HML			1.048***	0.645***	0.667***	0.782***
			(5.21)	(3.92)	(4.00)	(3.22)
UMD				-0.975***	-0.967***	-0.725***
				(-9.73)	(-9.24)	(-5.15)
GFC					0.022	0.021
					(0.67)	(0.64)
MktRf*Over						0.034
						(0.17)
SMB*Over						0.028
						(0.10)
HML*Over						-0.125
						(-0.40)
UMD*Over						-0.268
						(-1.48)
Constant	-0.010	-0.016**	-0.015***	-0.006	-0.007	-0.008*
	(-1.49)	(-2.54)	(-3.59)	(-1.49)	(-1.53)	(-1.76)
Observations	298	298	298	298	298	298
Adjusted R-square	0.011	0.163	0.345	0.562	0.561	0.557
Panel B. Equal-weighted						
Over-Under	0.024***	0.024***	0.023***	0.019***	0.018***	0.019***
	(2.77)	(2.94)	(2.97)	(2.99)	(2.82)	(2.97)
MktRf		0.516***	0.452***	0.145	0.179	0.158
		(3.77)	(3.93)	(1.28)	(1.54)	(1.31)
SMB			0.515***	0.581***	0.582***	0.647***
			(2.62)	(4.42)	(4.44)	(3.14)
HML			0.554***	0.230	0.273	-0.010
			(3.17)	(1.36)	(1.64)	(-0.04)
UMD				-0.785***	-0.768***	-0.506***
				(-7.14)	(-7.28)	(-2.79)
GFC					0.044	0.047
					(1.35)	(1.44)
MktRf*Over						0.021
						(0.12)
SMB*Over						-0.050
						(-0.19)
HML*Over						0.361
						(1.11)
UMD*Over						-0.293
						(-1.39)
Constant	-0.004	-0.007	-0.007*	0.000	-0.000	-0.001
	(-0.74)	(-1.61)	(-1.68)	(0.04)	(-0.05)	(-0.33)
Observations	298	298	298	298	298	298
Adjusted R-square	0.020	0.114	0.190	0.423	0.427	0.429

The table reports the results of contemporary exposure tests. We includes overweight dummy (Over-Under), Fama and French (1993) three factors (MktRf SMB HML), Carhart (1997) momentum factor (UMD), 2008 financial crisis dummy (GFC), and interaction term between factors and overweight dummy. Over-Under (same as Over) equals 1 if δ 1 and 0 otherwise. GFC equals 1 if the month falls in October 2008 to March 2009 and 0 otherwise. t-statistics, in parentheses, are Newey-West adjusted with four lags.

Table 5
Time-varying weights or characteristics tests

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Value-weighted						
Over-Under	0.024**	0.025**	0.021**	0.024**	0.028**	0.022*
	(2.38)	(2.34)	(2.03)	(2.35)	(2.43)	(1.86)
Beta spread		0.006				0.009
		(0.41)				(0.55)
Std Dev spread			-0.051			-0.124**
			(-1.58)			(-1.98)
Skewness spread				-0.032		0.088
				(-0.77)		(0.96)
VRP					-0.000	-0.000
					(-0.93)	(-1.00)
Constant	-0.010	-0.012	-0.003	-0.017	-0.006	0.028
	(-1.49)	(-1.58)	(-0.35)	(-1.35)	(-0.76)	(0.86)
Observations	298	298	298	298	298	298
Adjusted R-square	0.011	0.008	0.015	0.009	0.027	0.034
Panel B. Equal-weighted						
Over-Under	0.024***	0.025***	0.022**	0.024***	0.026***	0.023**
	(2.77)	(2.65)	(2.37)	(2.73)	(2.84)	(2.16)
Beta spread		0.008				0.011
		(0.56)				(0.61)
Std Dev spread			-0.038			-0.101*
			(-1.26)			(-1.82)
Skewness spread				-0.020		0.073
				(-0.55)		(0.85)
VRP					-0.000	-0.000
					(-0.93)	(-0.96)
Constant	-0.004	-0.006	0.002	-0.008	-0.000	0.027
	(-0.74)	(-0.95)	(0.35)	(-0.80)	(-0.07)	(0.88)
Observations	298	298	298	298	298	298
Adjusted R-square	0.020	0.019	0.023	0.017	0.036	0.045

The table reports the results of competition between time-varying weights and time-varying characteristics. Over-Under is the overweight dummy, which equals 1 if $\delta \leq 1$ and 0 otherwise. Beta, Std Dev, and Skewness are the average of firm-level characteristics within the deciles sorted on TK following Barberis et al. (2021) as inputs of their model. Firm-level Beta is calculated using daily returns over the following year. Firm-level Std Dev and Skewness are calculated using monthly returns over the past five years. Spread is the difference between low-TK decile and high-TK decile. VRP is the previous month variance risk premium from https://sites.google.com/site/haozhoushomepage. t-statistics, in parentheses, are Newey-West adjusted with four lags.

Table 6
Fama-MacBeth regression analysis

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Whole	-0.279*	-0.257**	-0.302***	-0.229**	-0.238**	-0.284**	-0.290**	-0.285*	-0.248**	-0.289**
	(-1.78)	(-2.03)	(-2.58)	(-2.07)	(-2.15)	(-2.28)	(-2.33)	(-1.81)	(-2.09)	(-2.33)
Over-Under by 1	-0.619**	-0.550***	-0.577***	-0.468***	-0.454***	-0.391**	-0.388**	-0.442*	-0.371**	-0.394**
	(-2.42)	(-2.75)	(-3.18)	(-2.69)	(-2.60)	(-2.02)	(-2.00)	(-1.79)	(-1.96)	(-2.05)
Over-Under by 0.9 and 1.1	-0.758**	-0.680***	-0.715***	-0.586***	-0.576***	-0.680***	-0.680***	-0.716**	-0.687***	-0.685***
	(-2.38)	(-2.78)	(-3.21)	(-2.74)	(-2.68)	(-2.72)	(-2.71)	(-2.31)	(-2.81)	(-2.76)
Variables in first step										
TK	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Beta	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
Size	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
Bm	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
Mom	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
Cgo	N	N	Y	Y	Y	Y	Y	Y	Y	Y
Rev	N	N	N	Y	Y	Y	Y	Y	Y	Y
Illiq	N	N	N	N	Y	Y	Y	Y	Y	Y
Lt rev	N	N	N	N	N	Y	Y	Y	Y	Y
Ivol	N	N	N	N	N	Y	Y	Y	Y	Y
Max	N	N	N	N	N	N	Y	Y	Y	Y
Min	N	N	N	N	N	N	Y	Y	Y	Y
Skew	N	N	N	N	N	N	N	Y	N	N
Eiskew	N	N	N	N	N	N	N	N	Y	N
Coskew	N	N	N	N	N	N	N	N	N	Y

The table reports the results of Fama-MacBeth regressions. The dependent variable is percentage return. In the first step, we regress excess return on firm characteristics. In the second step, we regress the coefficients from the first step on 1 without constant for whole sample period (Whole) or on overweight dummy (and middle dummy for Over-Under by 0.9 and 1.1) with constant for the difference between overweight periods and underweight periods (Over-Under). For Over-Under by 1, overweight dummy equals 1 if $\delta \quad 1$ and 0 otherwise. For Over-Under by 0.9 and 1.1, overweight dummy equals 1 if $\delta < 0.9$ and 0 otherwise and middle dummy equals 1 if $0.9 \quad \delta \quad 1.1$ and 0 otherwise. TK is the prospect theory value of a stock's historical return distribution, following Barberis et al. (2016). Beta is calculated from monthly returns over the previous five years, following Fama and French (1992). Size is the log market capitalization at the end of the previous month. Bm is the book value of equity scaled by market value of equity, computed following Fama and French (1992) and Fama and French (2008); when the book value of equity is missing from Compustat, we use data from Davis et al. (2000); observations with negative book value are removed. Mom is the cumulative return from the start of month t-12 to the end of month t-2. Cgo is the capital overhang in month t-1, following Barberis et al. (2021). It is computed as $(P_i \quad R_i)/R_i$, where P_i is the stock's current price and R_i is investors' average purchase price. It is slightly different from Grinblatt and Han (2005), but is a more precise match for the capital gain variable in the cumulative prospect theory model. Rev is the return in month t-1. Illiq is Amihud (2002)'s measure of illiquidity, scaled by 10^5. Lt rev is the cumulative return from the start of month t-60 to the end of month t-13. Ivol is idiosyncratic return volatility, as in Ang et al. (2006). Max and Min are the maximum and the negative of the minimum daily returns in month t-1, as in Bali et al. (2011). Skew is the skewness of monthly returns over the previous five years. Eiskew is expected idiosyncratic skewness, as in Boyer et al. (2010). Coskew is coskewness, computed as in Harvey and Siddique (2000) using five years of monthly returns. TK, Mom, Rev, Ivol, Max, and Min are scaled up by 100. The sample period runs from March 1996 to December 2020. All variables are winsorized at 1 and 99 percent in each month, and TK is standardized after the winsorization. t-statistics, in parentheses, are Newey-West adjusted with four lags.

Table 7
Original long-short portfolio and strategic long-short portfolio performance

Portfolio	Mean	Std	Sharpe	Skewness	four-factor alpha	five-factor alpha
Original Value-weighted	0.067	0.340	0.196	0.518	0.059	0.060
	(0.98)				(1.22)	(1.22)
Strategical Value-weighted	0.166	0.337	0.492	0.478	0.174	0.174
	(2.45)				(3.04)	(3.01)
Original Equal-weighted	0.141	0.264	0.536	0.545	0.152	0.146
	(2.67)				(3.62)	(3.62)
Strategical Equal-weighted	0.194	0.261	0.746	0.533	0.211	0.203
	(3.72)				(4.64)	(4.63)

The table reports the portfolio performance on both an equal-weighted (EW) and value-weighted (VW) basis. For original portfolio, long low-TK decile stocks in each month for all months and short high-TK decile stocks in each month for all months. For strategical portfolio, begin with the original portfolio, which has $\delta \leq 1$ in March 1996, and use 0.9 and 1.1 as triggers. If $\delta > 1.1$ in a month, then long high-TK decile stocks and short low-TK decile stocks in the following months (flip the original long and short position). If $\delta < 0.9$ later in a month, flip the long and short position back as the original portfolio. Do the same flip strategy if triggers appear again. t-statistics, in parentheses, are based on the heteroskedasticity-consistent standard errors of White (1980).

Table 8
Robustness

	Value-weighted				Equal-weighted			
	Whole	Over	Under	Over - Under	Whole	Over	Under	Over - Under
Panel A. Different estimating method								
GW two-parameter model								
Mean	0.56	1.83*	-0.75	2.58**	1.18***	2.58***	-0.26	2.85***
	(0.98)	(1.85)	(-1.43)	(2.30)	(2.67)	(3.37)	(-0.67)	(3.31)
# of months	298	151	147		298	151	147	
TK with alternative value function parameters								
Mean	0.56	1.78*	-0.70	2.48**	1.18***	2.54***	-0.22	2.76***
	(0.98)	(1.78)	(-1.37)	(2.21)	(2.67)	(3.28)	(-0.59)	(3.21)
# of months	298	151	147		298	151	147	
Right tail only								
Mean	0.56	1.06	-1.11	2.17**	1.18***	1.72***	-0.64	2.36***
	(0.98)	(1.50)	(-1.50)	(2.13)	(2.67)	(3.17)	(-1.15)	(3.05)
# of months	298	229	69		298	229	69	
Panel B. Different moving average								
1-month								
Mean	0.54	1.08	-0.39	1.47	1.21***	1.88***	0.04	1.84**
	(0.96)	(1.30)	(-0.73)	(1.48)	(2.75)	(2.92)	(0.09)	(2.39)
# of months	300	190	110		300	190	110	
6-month								
Mean	0.51	1.46*	-1.17**	2.64***	1.13**	2.16***	-0.67	2.83***
	(0.89)	(1.75)	(-2.15)	(2.64)	(2.54)	(3.34)	(-1.62)	(3.69)
# of months	295	188	107		295	188	107	
12-month								
Mean	0.62	1.34	-0.82	2.15**	1.21***	2.19***	-0.76*	2.94***
	(1.07)	(1.63)	(-1.39)	(2.14)	(2.67)	(3.45)	(-1.80)	(3.87)
# of months	289	193	96		289	193	96	

The table reports robustness checks of average monthly returns differentiated by δ, on both an equal-weighted (EW) and value-weighted (VW) basis, of long-short portfolios of TK. In panel A, we estimate δ by different methods. GW two-parameter model follows Gonzalez and Wu (1999). TK with alternative value function follows the same method as benchmark but changes the value function parameters to (0.88, 2.25) as Tversky and Kahneman (1992). Right tail only follows the same method as benchmark but estimates parameters separately on the two tails, and uses the δ estimated from right tail. In panel B, we follows the same method as benchmark but changes the moving average to 1-, 6-, and 12-month respectively. t-statistics, in parentheses, are based on the heteroskedasticity-consistent standard errors of White (1980).

Table 9
Sources of probability weighting preferences time variation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Past 1 month market excess return	-1.005**	-0.927**	-0.837**				0.238	-0.283
	(-2.03)	(-2.48)	(-2.25)				(0.58)	(-0.69)
Past 2-12 month market excess return		-0.879***	-1.091***				-0.308	-0.329
		(-2.98)	(-3.18)				(-0.98)	(-1.16)
Past 2-60 month market excess return			0.168				0.046	-0.154
			(1.28)				(0.42)	(-1.14)
Standardized physical variance				-0.062		-0.113**	-0.102*	-0.140***
				(-0.71)		(-2.39)	(-1.72)	(-2.97)
Standardized physical skewness				-0.054		0.033	0.038	0.040
				(-0.67)		(0.73)	(0.74)	(0.79)
Standardized physical kurtosis				0.117		0.088*	0.090*	0.133**
				(1.29)		(1.88)	(1.86)	(2.35)
Standardized risk-neutral variance					-0.034	-0.069	-0.081	0.057
					(-0.46)	(-0.85)	(-1.20)	(0.72)
Standardized risk-neutral skewness					-0.194***	-0.155**	-0.155**	-0.097
					(-2.67)	(-2.01)	(-2.22)	(-1.56)
Standardized risk-neutral kurtosis					-0.452***	-0.467***	-0.458***	-0.335***
					(-4.74)	(-5.10)	(-5.43)	(-4.54)
CAPE								0.015
								(1.61)
GFC								-1.036***
								(-2.70)
Constant	0.645***	0.713***	0.638***	0.638***	0.638***	0.638***	0.635***	0.358*
	(8.53)	(12.62)	(8.20)	(9.45)	(11.65)	(13.81)	(8.43)	(1.71)
Observations	298	298	298	298	298	298	298	298
Adjusted R-square	0.006	0.091	0.124	0.119	0.344	0.427	0.430	0.487

The table reports the results of time variation analysis of probability weighting preferences. The independent variable is a dummy variable equals 1 if $\delta \leq 1$ and 0 otherwise. Past 1 month market excess return is the market excess return in previous month. Past 2-12 month market excess return is the difference between past 2 to 12 month market cumulative return and risk-free cumulative return. Past 2-60 month market excess return is the difference between past 2 to 60 month market cumulative return and risk-free cumulative return. Standardized physical moments are calculated using past five years monthly S&P 500 index return. Standardized risk-neutral moments are calculated using S&P 500 index option with 30-days maturity. CAPE is previous month cyclically adjusted price earnings ratio from http://www.econ.yale.edu/ shiller/data.htm. GFC is a dummy variable control for 2008 financial crisis, which equals 1 if the month falls in October 2008 to March 2009 and 0 otherwise. t-statistics, in parentheses, are Newey-West adjusted with twelve lags.

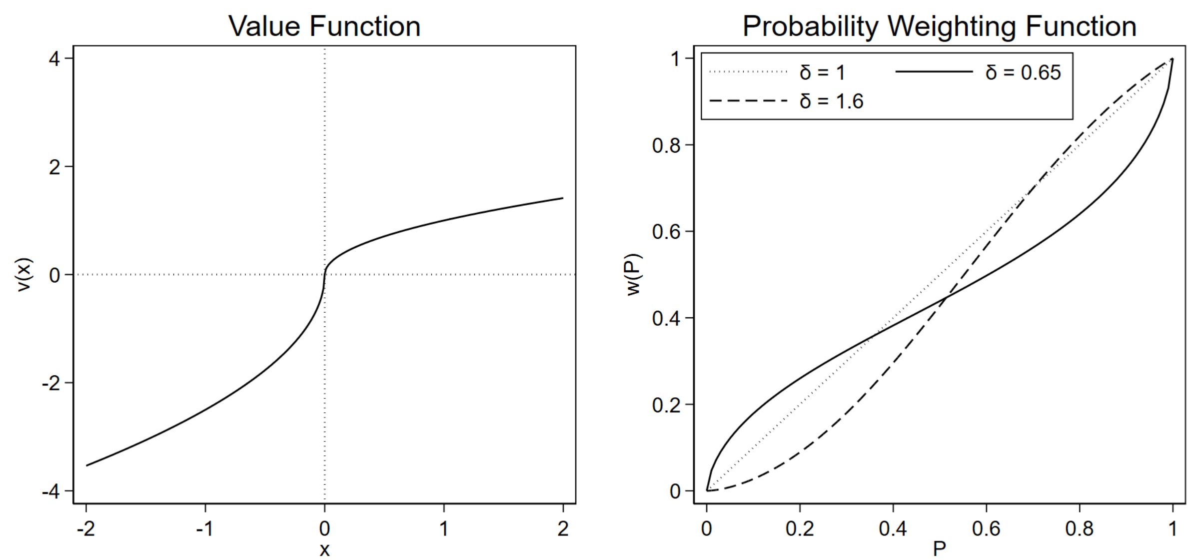

Figure 1
The prospect theory value function and probability weighting function
The left panel plots the value function proposed by Tversky and Kahneman (1992) as part of their cumulative prospect theory, namely, $v(x) = x^{\alpha}$ for $x \geq 0$ and $v(x) = -\lambda(-x)^{\alpha}$ for $x < 0$, for $\alpha = 0.5$ and $\lambda = 2.5$. The right panel plots the probability weighting function they propose, namely, $w(P) = P^{\delta}/(P^{\delta} + (1-P)^{\delta})^{1/\delta}$, for three different values of δ. The dotted line corresponds to $\delta = 1$, the solid line to $\delta = 0.65$, and the dash line to $\delta = 1.6$.

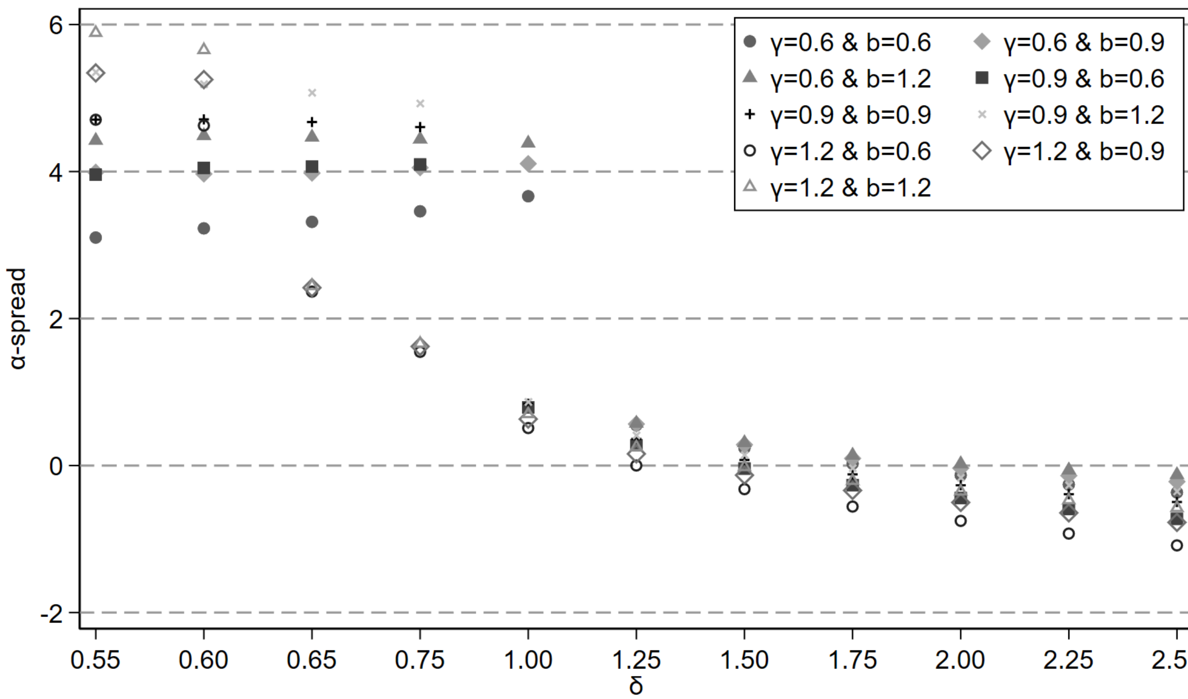

Figure 2

Model-predicted alpha spread with respect to δ

The figure plots the model-predicted alpha spread with respect to δ given different parameter settings. The model follows Barberis et al. (2021). δ, which is set to 0.55 to 2.50, respectively, represents the probability weighting. γ ($\hat{\gamma}$ in equation (9)), which is set to 0.6, 0.9, and 1.2, respectively, controls for the weight of risk aversion in the investor's objective function. b ($\hat{b}$ in equation (9)), which is set to 0.6, 0.9, and 1.2, respectively, controls for the weight of prospect theory in the investor's objective function.

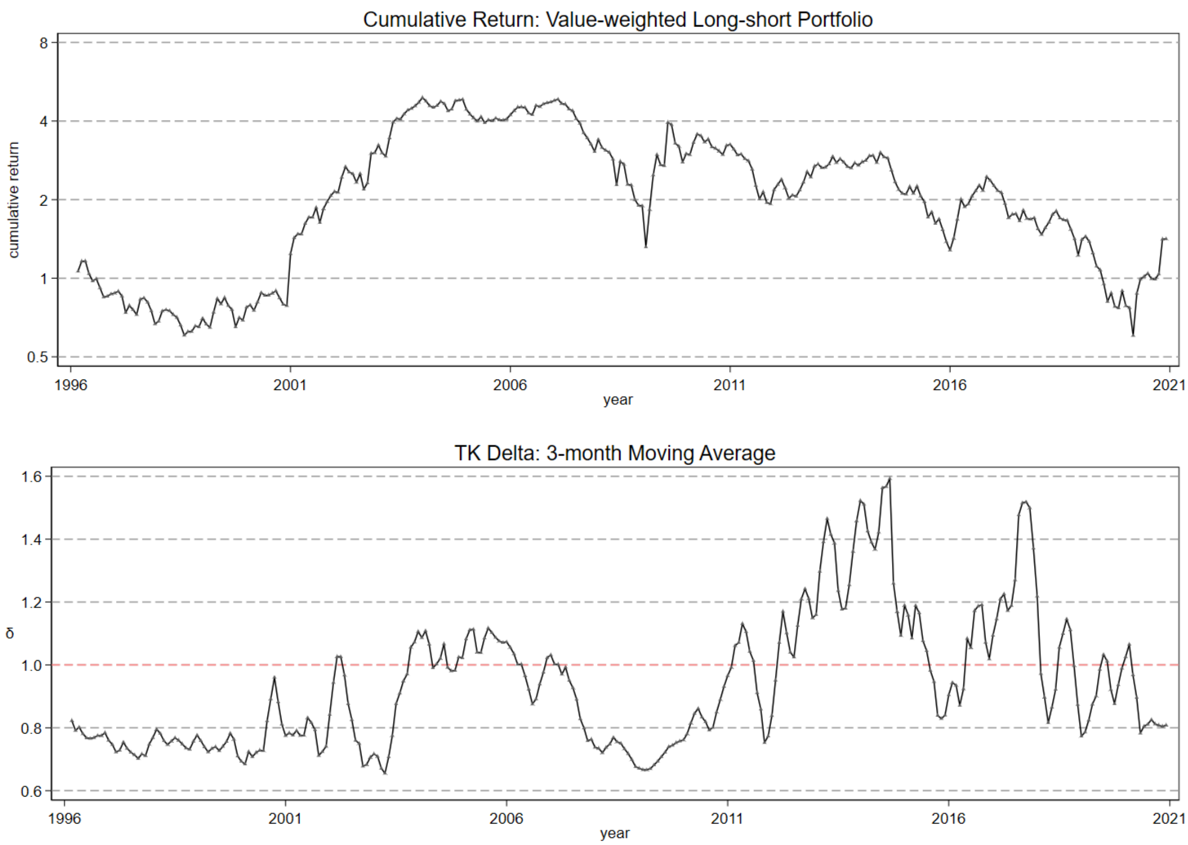

Figure 3
Performance of value-weighted long-short TK portfolio and empirical TK delta
The top panel plots the time series of the cumulative gross return of value-weighted long-short TK portfolio that starts from the beginning of March 1996. TK portfolio is formed following Barberis et al. (2016). The bottom panel plots the time series of parameter for measure of probability weighting (δ) estimated by the method described in Section 3.2. δ is estimated at the beginning of each month and the figure plots the moving average δ of the past two months and current month.

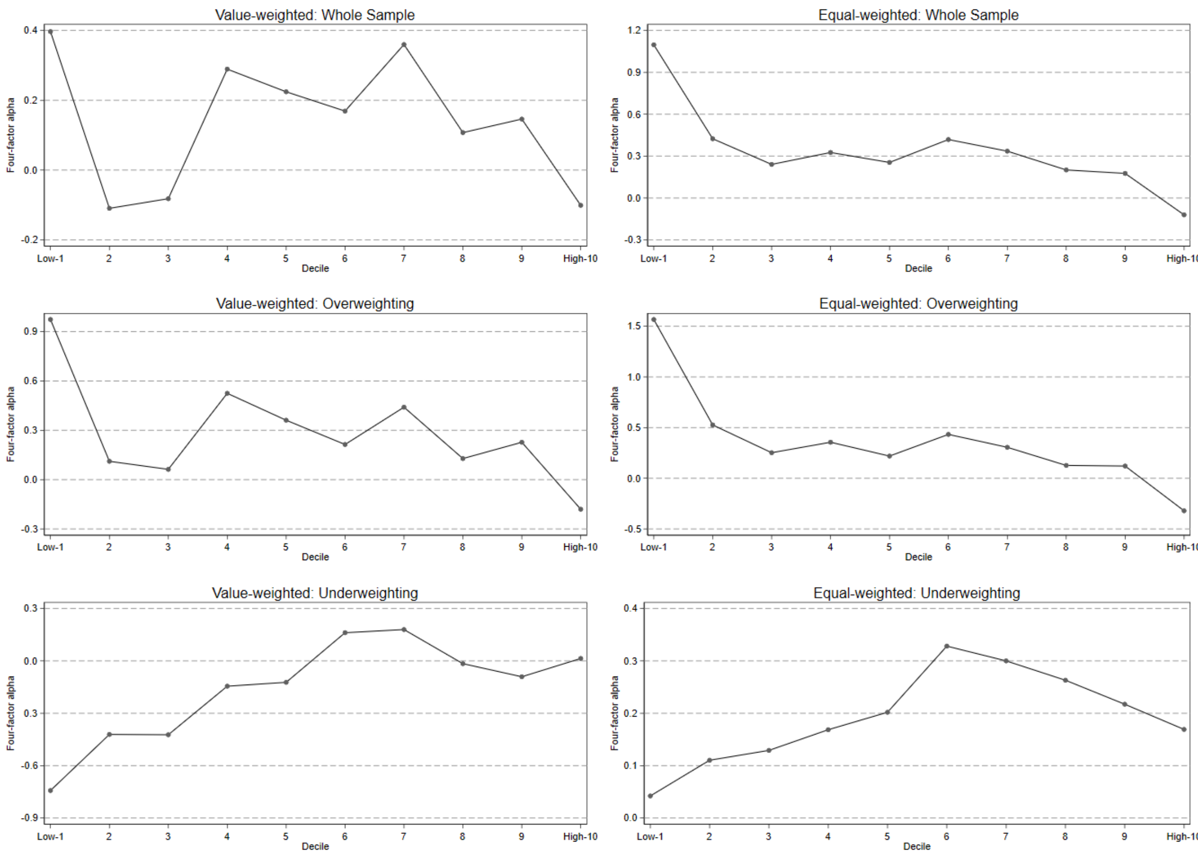

Figure 4
Performance of TK deciles
The figure plots the four-factor alpha, on both an value-weighted (left panels) and equal-weighted (right panels) basis, of TK decile portfolios using time series of average return differentiated by δ. The TK decile portfolios are formed following Barberis et al. (2016). The top panels are for whole sample periods; the middle panels are for overweight sample periods ($\delta \leq 1$); the bottom panels are for underweight sample periods ($\delta > 1$). The vertical axis is the monthly alpha, in percent; the horizontal axis marks the decile portfolio, from decile 1 (low TK) on the left to decile 10 (high TK) on the right.

Figure 5

Performance of trading strategy

The figure plots the time series of the cumulative gross return of value-weighted (left panels) and equal-weighted (right panels) long-short TK portfolios that starts from the beginning of March 1996. The top panels are for direct strategies; the bottom panels are for four-factor adjusted strategies, where four factors are MKT, SMB, HML, and UMD. In each panel, we present performance of both trading strategy proposed in the paper and original long-short strategy following Barberis et al. (2016). For the trading strategy portfolio, begin with the original portfolio, which has $\delta \leq 1$ in March 1996, and use 0.9 and 1.1 as triggers. If $\delta > 1.1$ in a month, then long high-TK decile stocks and short low-TK decile stocks in the following months (flip the original long and short position). If $\delta < 0.9$ later in a month, flip the long and short position back as the original portfolio. Do the same flip strategy if triggers appear again.