Lending Competition and Funding Collaboration

Abstract

We study competition and collaboration between a bank and a shadow bank that lend in the same market plagued by adverse selection. The bank has cheaper funding, whereas the shadow bank is endowed with a better screening technology. Our innovation is to allow the bank to lend to the shadow bank, i.e., to finance its competitors. This interbank arrangement lowers shadow bank's funding cost and reduces the bank's incentive to compete. We show two lenders collaborate when the average quality of the borrower pool is low but compete when the quality gets high. While the shadow bank always benefits from interbank financing, the bank receives more profits only when the average quality is high, at the expense of higher interest rates faced by the borrowers.

Keywords: shadow bank, lending competition, interbank funding, winner's curse, financial inclusion.

1 Introduction

The rise of shadow banks has posed serious challenges to traditional banks. As documented by Buchak et al. (2018), their market share in the residential mortgage market has nearly doubled from 2007-2015. Recent evidence shows that shadow banks finance themselves primarily with short-term credit lines from banks (Jiang et al., 2020). In fact, often times, the shadow banks are funded by the *very* banks with which they compete in the *same* lending market (Jiang, 2019). Why do banks finance (and collaborate with) their competitors?[1]

One answer is that by financing competitors, banks get to share the shadow banks' profits from direct lending. Indeed, Buchak et al. (2018) show that a large fraction of shadow banks are also fintech firms with superior information technology (such as relying on artificial intelligence, machine learning, and big data) to identify qualified borrowers who could not obtain loans from traditional banks.[2] Meanwhile, due to deposit insurance and other implicit forms of government guarantee (such as bailouts), banks have access to arguably cheaper funding than shadow banks. Naturally, there is room for the these two types of lenders to collaborate in that banks can provide interbank funding to shadow banks. When do we expect to observe competition and/or collaboration between these two types of lenders? How does the interbank funding affect the borrowers' payoff and lenders' profits, and ultimately, the overall efficiency?

This paper develops a model to address these questions. Our theory builds upon the comparative advantage of the two lenders: banks have cheaper funding, whereas shadow banks have better screening technology. They directly compete to lend to the same set of borrowers who suffer from adverse selection. Moreover, shadow banks could also obtain funding from banks, reducing the effective funding costs. Our theory highlights two important channels that arise from interbank funding. On the one hand, the reduced funding cost increases the shadow bank's competitiveness and thereby intensifies direct lending competition. On the other hand, interbank funding offers the bank an alternative avenue to earn profits: by lending to the shadow bank, the bank could also indirectly enjoy the surplus generated by the shadow bank's superior information technology. Therefore, the bank has lower incentives to compete, but, instead, would like to collaborate with its competitor. We show that the magnitudes of these two channels depend on the degree of adverse

[1] The example is not only restricted to mortgages. In personal loans, Avant, one of the largest online lenders, offered credit products to consumers throughout the United States through its partnership with WebBank, an FDIC-insured, state-chartered industrial bank. Another fintech company, GreenSky, is financed by federally-insured, federal and state-chartered financial institutions such as Regions Financial Corp. and SunTrust Banks

[2] For example, Berg et al. (2020) show that digital footprints can be informative in predicting consumer default in additional to traditional credit scores. Frost et al. (2019) show that machine learning and data from e-commerce platform are better at predicting losses. Also see Gambacorta et al. (2020), Agarwal et al. (2020), Di Maggio et al. (2022).

selection, approximated by the average quality of the borrower pool. As a result, one should expect to observe different levels of collaboration and competition in different markets: collaboration (competition) prevails when the average quality is low (high). Moreover, we show that while interbank funding always increases the shadow bank's profits, it might reduce the borrowers' payoff as well as the bank's profits.

Let us be more specific. We model a continuum of borrowers with projects that are either of high or low quality, and only a high-quality project has a positive net present value (NPV). Each borrower seeks credit from either a bank or a shadow bank. We assume the funding cost of the bank falls below that of the shadow bank. Meanwhile, the shadow bank has an information advantage. Specifically, we assume that the bank cannot differentiate the borrowers' types and therefore only lend blindly. By contrast, the shadow bank is endowed with a screening technology that generates a private signal on each borrower. Given this, the bank suffers from the winner's curse when it directly competes with the shadow bank for borrowers: whenever it wins the competition, chances are that it will lend to a low-quality borrower and suffer losses. This setup is reminiscent of the problem of common-value auctions under asymmetric information (Milgrom and Weber, 1982) and the applications to bank lending (Broecker, 1990; Hauswald and Marquez, 2003). Our setup allows lenders (or equivalently, bidders) to have both different information technology and funding costs.

A main departure of our model from the existing literature is that, we allow the bank to finance the shadow bank. Specifically, the shadow bank can borrow a fraction of its funding from the bank to reduce its effective funding cost. This faction can be motivated in various ways, such as the development of the interbank relationship, the search friction, or agency frictions that require the shadow bank to have enough skin in the game. The presence of interbank funding allows the two lenders to collaborate in addition to directly competing for lenders.

Our first set of results concerns the relative degree of collaboration and competition between the two lenders. We show that collaboration dominates when the average quality of the borrower pool is very low. Intuitively, the bank wouldn't lend blindly because, most likely, the borrower is a low-quality one. In this case, direct lending generates losses in expectation. For this set of borrowers, the shadow bank's screening technology is particularly useful in that it helps identify high-quality ones, just as finding a needle in the haystack. By offering interbank funding and charging a spread, the bank could also earn some profits generated from the screening technology. As a result, one should only observe the shadow bank lending to high-quality borrowers with funding partly from the bank – collaboration. When the average quality of the borrower pool gets very high, screening becomes less useful compared to strategy of blind lending. In other words, the information advantage of the shadow bank is mitigated and could be dominated by the bank's funding advantage. As a result, one should only observe the bank lending to all borrowers. For this set of borrowers, even though the shadow bank does not lend in equilibrium, its presence poses important threats to the bank, forcing the latter lender to charge a lower loan rate – competition. Finally, when the average quality of the borrower pool is neither too high nor too low, the shadow bank's information advantage and the bank's funding cost advantage are comparable. As a result, competition and collaboration coexist, and one could observe lending by both lenders.

The results on competition and collaboration imply that shadow banks are more likely to enter into markets whose average quality is neither too high nor too low. Indeed, we show that the shadow bank's profits are non-monotonic in the average quality of the borrower pool. At low average quality, there are very few high-type borrowers in the pool. Consequently, both the lending volume and profits are low for the shadow bank. By contrast, when the average quality becomes very high, the competition from the bank is intensified, and bank's funding advantage dominates the shadow bank's informatoin advantage. Therefore, the shadow bank's profit margin gets substantially squeezed. If either screening or entry entails a cost to the shadow bank, our model predicts the following lending patterns. When the average quality is very low, neither lender is active, and borrowers are credit rationed, as in Stiglitz and Weiss (1981). When the average quality gets a bit higher, the shadow bank charges a high interest rate to high-quality borrowers, and the bank offers interbank funding to the shadow bank. When the average quality further improves, both lenders compete to lend, and high-quality borrowers can always receive financing. Because the bank lends blindly, low-quality borrowers might also receive financing. Finally, when the average quality gets to the highest region, only the bank lends to all borrowers.

In the first-best benchmark, all high-quality borrowers should obtain funding from the bank, whereas all low-type borrowers should not be financed. The equilibrium in our model therefore features two types of inefficiency. The funding inefficiency arises whenever the shadow bank lends using its own funding, which is more costly; the lending inefficiency arises whenever the bank lends blindly because it often ends up offering credit to low-quality, negative NPV projects. In the collaboration region, there is only funding inefficiency, whereas, in the competition region, there is only lending inefficiency. Interestingly, the degree to which the two sources of inefficiency prevail in equilibrium is also non-monotonic in the average quality of the borrower's pool. Funding inefficiency is closely related to the lending volume of the shadow bank, and consequently, it peaks when the average quality is neither too high nor too low. By contrast, lending inefficiency depends on the total lending volume of the bank and the fraction of low-type projects present in the market. These two variables move in opposite directions: as the average quality of the pool goes up, the volume of bank lending increases, but the share of the low-type projects decreases. In general, lending inefficiency also reaches its maximum when the average quality is neither too high nor too low.

Our second set of results highlights the role of the interbank funding market. The introduction of the interbank market always reduces the shadow bank's effective funding cost, increasing its profits. Somewhat surprisingly, the presence of the interbank market could reduce the payoff of borrowers and the bank. In general, the interbank market introduces two forces. On the one hand, it reduces the shadow bank's funding cost and allows it to be more competitive against the bank. This channel intensifies competition between the two lenders, reducing the bank's profits and benefiting the borrowers through lower loan rates. On the other hand, it allows the bank to make profits not only from direct lending to the borrower but also from offering interbank funding to the shadow bank. Ceteris paribus, the bank competes less aggressively in the direct lending market. This channel mitigates competition between the two lenders, which could increase the bank's profits but harm the borrowers. The two channels interact differently in markets with different average quality. When the quality is low, competition is low. The introduction of the interbank market allows the bank to earn some profits from the set of borrowers it would not lend anyway. This increases the bank's profits but reduces the borrower's payoff. By contrast, information asymmetry is less severe and competition is high when the average quality is high. The introduction of the interbank market reduces the shadow bank's effective funding cost, which gets passed through to borrowers as lower loan rates. Therefore, the interbank funding erodes the bank's profits but increases borrowers' payoff.

Our modeling framework follows from Broecker (1990) and Hauswald and Marquez (2003), which comes from the literature on common-value auctions with asymmetric information (Milgrom and Weber, 1982). This literature has established that there is no pure-strategy equilibrium in the bidding game, and the equilibrium must be one with mixed strategies. We depart in two aspects. First, we allow for bidders with both heterogeneous costs and information sets. As a result, we show that when the funding advantage of the less-informed bidder becomes sufficiently high, the equilibrium is one with pure strategies. Second, we allow the two bidders to collaborate and share the surplus. By doing so, we can weigh the relative magnitudes of competition and collaboration and show how both depend on the degree of adverse selection.

Our paper contributes to a growing literature that studies the effect of fintech on lending. Parlour et al. (2020) highlight the information spillovers from payment processing to lending while He et al. (forthcoming) analyze the consequences of proposed open banking regulation that allows customers to share information across lenders. Both papers highlight the potential downsides of consumer data portability. We do not allow for information sharing but instead focus on the effects of interbank funding and the welfare implications. Similarly, our analysis presents a cautionary tale of how the presence of interbank lending can hurt borrowers. Huang (2022) analyzes competition between a traditional bank and fintech, who rely on different kinds of lending technologies (collateral for bank and information for fintech). In contrast, in our model, both a shadow bank and a bank lend based on information but differ in the quality of information acquisition technology and funding costs. Moreover, we allow the lenders to collaborate via the interbank lending market - a channel absent in Huang (2022). Interbank lending is the central focus of Jiang (2019). In her model, there is no adverse selection, and banks and shadow banks offer differentiated products. By casting the model in the context of information asymmetry, our paper offers new predictions on the degree of collaboration and competition in different markets and the entry patterns by shadow banks. Moreover, we highlight how interbank funding affects lending competition through the two channels highlighted earlier.

2 The Model

We introduce a model with two dates $t = 0, 1$ and three sets of players. All players are risk-neutral, have limited liabilities, and do not discount the future. One bank and one shadow bank compete to lend to borrowers that are of two types. Meanwhile, the bank may also lend to the shadow bank in the interbank funding market. Figure 1 lays out the building blocks of the model.

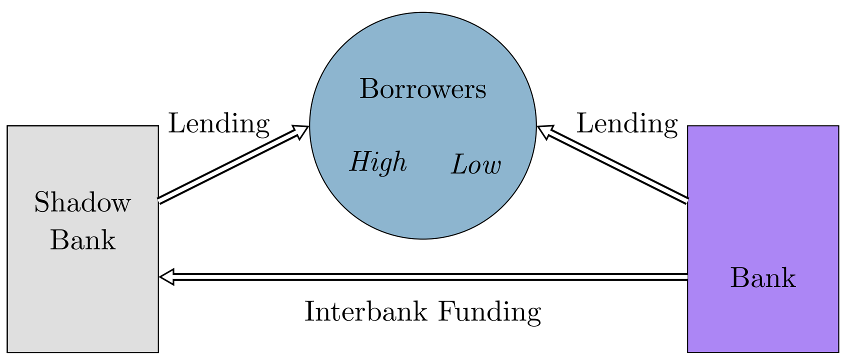

Figure 1: Model Overview

2.1 Borrowers and Projects

We model a continuum $i \in [0, 1]$ of penniless borrowers that are of two types: high and low. Borrower's type is private information and only known by the borrower herself. Let μ be the fraction of high-type borrowers. Each borrower is infinitesimal and has access to a fixed-scale investment technology that requires \$1 at $t = 0$. Once the investment is made, the project generates R with probability p_{θ_i} and 0 with probability $1 - p_{\theta_i}$, where $\theta_i \in \{h, l\}$ stands for the borrower's type. For the rest of the paper, we assume without loss of generality that $p_h = 1$ and $p_l = p \in (0, 1)$. In our model, borrowers shall be interpreted as either small businesses or consumers who seek personal loans.

2.2 Lenders, Screening, and Interbank Funding

One bank and one shadow bank compete to lend to borrowers. To raise \$1, the bank needs to promise a gross interest payment r_B to its financiers, whereas the shadow bank needs to promise r_S. One can interpret r_B as the return to a riskless storage technology that is available to the financiers. We assume $r_S > r_B \geq 1$ to reflect the idea that the bank has a funding advantage, which could come from government subsidy, deposit market power, liquidity insurance or a better network in attracting deposits. The difference $r_S - r_B$ is an iceberg cost needed for the shadow bank to raise financing. Whereas the bank has a funding advantage, the shadow bank – sometimes interpreted as a fintech firm – has a better screening technology. Specifically, we assume the bank can not screen any borrower, whereas the shadow bank has a costless screening technology. In particular, the technology generates a private signal on each borrower that is either good g or bad b (we assume that the signals are *i.i.d.* across borrowers). Specifically,

$$\Pr(b|h) = e_1, \qquad \Pr(g|l) = e_2, \tag{1}$$

where e_1 and e_2 are the probability of a type-I and type-II error, respectively. Let q_g (q_b) the total measure of borrowers who receive a good (bad) signal. Following the law of large numbers

$$q_g = \mu(1 - e_1) + (1 - \mu)e_2, \qquad q_b = \mu e_1 + (1 - \mu)(1 - e_2). \tag{2}$$

Conditional on a good/bad signal, the shadow bank's posterior of a borrower being a high type is

$$\mu_g = \frac{\mu\,(1 - e_1)}{q_g}, \qquad \mu_b = \frac{\mu e_1}{q_b}. \tag{3}$$

We assume $e_1 + e_2 < 1$ so that $\mu_b < \mu < \mu_g$. For the baseline analysis, we assume $e_1 = e_2 = 0$, so that screening generates a perfect signal. In this case, $q_g = \mu$, $q_b = 1 - \mu$, $\mu_g = 1$ and $\mu_b = 0$. To simplify notation, we define

$$P(\tilde{\mu}) = \tilde{\mu} + (1 - \tilde{\mu})p, \ \forall \tilde{\mu} \tag{4}$$

as the conditional probability of producing R if the average quality is $\tilde{\mu}$. We introduce the following assumption throughout the paper.

Assumption 1. *The funding costs satisfy*

$$r_S < P(\mu_g) \cdot R, \ r_B > P(\mu_b) \cdot R. \tag{5}$$

In words, it is profitable for the shadow bank to lend using its own funding to a borrower conditional on a good signal. Yet, it is unprofitable for a bank to lend to a borrower conditional on a bad signal. Note under $r_S > r_B$, Assumption 1 implies that for both lenders, a borrower's project has a positive NPV under a good signal but has a negative NPV under a bad signal.

Even though the shadow bank has a funding disadvantage, it can borrow from the bank to (partially) offset the disadvantage. Specifically, the shadow bank can borrow a fraction $\lambda \leq 1$ of its funding from the bank, where $1 - \lambda$ could be interpreted as the shadow bank's skin in the game. The interest rate of this interbank lending is determined via Nash Bargaining. Let α and $1 - \alpha$ be the bargaining power of the shadow bank and the bank. Clearly, the gross interest rate of the interbank lending satisfies $\alpha r_B + (1 - \alpha) r_S$.

2.3 Equilibrium

The timing goes as follows.

- $t = 0$
  - The shadow bank screens and obtains a signal on each borrower.
  - The bank and the shadow bank simultaneously make interest rate offers to each borrower. The borrower decides which offer to accept.
  - The interbank market opens, and the shadow bank can borrow a maximum fraction of λ of its funding from the bank.[3]
- $t = 1$
  - The project's outcome is realized, the borrower repays the loan, and the shadow bank repays the interbank loan.

There are no active decisions to be made at $t = 1$. At $t = 0$, the result in the funding market is straightforward. Let ℓ_S and ℓ_B be the equilibrium amount of lending made by the shadow bank

[3] Results of the paper stay largely unchanged if the interbank market opens before the lending competition. The equilibrium identified later still survives in such an alternative timing arrangement, but there might be some other self-fulfilling equilibria. For example, the shadow bank could choose not to establish an interbank funding arrangement because it anticipates to lose in the direct lending competition, and due to its high effective funding cost, it will indeed lose in the competition.

and the bank. Due to risk neutrality, the shadow bank always chooses to borrow $\lambda \ell_S$ from the bank and fund the remaining $(1-\lambda)\ell_S$ from its own financiers. Let us define

$$r_E = \lambda \alpha r_B + (1 - \lambda\alpha) r_S \tag{6}$$

as the effective funding cost of the shadow bank. For each loan made by the shadow bank, the bank makes expected profits

$$\Pi_B = \lambda(1-\alpha)(r_S - r_B) \tag{7}$$

through interbank lending. Sometimes, it is useful to write $r_E = \Pi_B + \lambda r_B + (1-\lambda) r_S$ so that it becomes clear part of the shadow bank's effective funding costs arises from the bank's profits from interbank lending.

Let R_B and R_S be the gross interest rate offer made to the borrower by the bank and the shadow bank, respectively. Clearly, $R_B \in [0, R] \cup \{+\infty\}$, $R_S \in [0, R] \cup \{+\infty\}$, and both offers can be stochastic[4]. When $R_S \to +\infty$ ($R_B \to +\infty$), we say the shadow bank (bank) does not make an offer. As a result, it is convenient to define the cumulative distribution functions (CDFs) $F_B(\cdot)$ and $F_S(\cdot)$ to be the strategies of the bank and shadow bank. The borrower's decision is straightforward: she should accept the offer with a lower interest rate. For simplicity, we assume whenever there is a tie, the borrower opts to accept the offer from the bank. This assumption can be motivated by the other non-pecuniary services offered by the bank and it is made without loss of generality. In the appendix, we show that the expected profits and payoff to all parties are identical if ties are randomly broken between the bank and the shadow bank.

Let $\tilde{V}_B(i)$ and $\tilde{V}_S(i)$ be bank's and shadow bank's the expected payoff from lending to an individual borrower $i \in [0, 1]$. We have

$$\tilde{V}_S(i) = \mathbb{1}_{R_B(i) > R_S(i)} \cdot \Big[(p + (1-p)\mathbb{1}_{\theta_i = h}) R_S - r_E\Big] \tag{8}$$

$$\tilde{V}_B(i) = \mathbb{1}_{R_B(i) \leq R_S(i)} \cdot \Big[(p + (1-p)\mathbb{1}_{\theta_i = h}) R_B - r_B\Big]. \tag{9}$$

Let us define V_B and V_S as the bank's and shadow bank's profits from lending to borrowers. Aggregating across all borrowers, we have

$$V_J = \int_0^1 \tilde{V}_J(i) di, \text{ and} \qquad \ell_J = \int_0^1 \mathbb{1}_{R_J(i) > R_{J'}(i)} di \qquad \text{for } J \neq J' \in \{S, B\}.$$

[4]When the bank (or shadow bank) offer is stochastic, each borrower i receives an *i.i.d.* realization $R_B(i)$ (or $R_S(i)$) from the offer distribution. This result allows us to use the exact law of large numbers in the cross-section of borrowers and obtain bank profits, borrower surplus and welfare that are deterministic.

We look for a Bayesian Nash Equilibrium, where the shadow bank's interest rate offer $R_S \sim F_S(\cdot)$ maximizes V_S and the bank's interest rate offer $R_B \sim F_B(\cdot)$ maximizes $V_B + \ell_S \Pi_B$. In particular, (8) shows that while making the interest offer, the shadow bank takes into account that it has an effective rate of r_E instead of r_S. Meanwhile, the bank also takes into account that for the borrowers it loses to the shadow bank, it still profits from lending to the shadow bank in the interbank market. Therefore, it aims to maximize $V_B + \ell_S \Pi_B$.

3 Solution

This section solves the model under assumption that screening generates a perfect signal to the shadow bank, i.e., $e_1 = e_2 = 0$. The results under general type-I and type-II errors are presented in Section 4.1.

3.1 Lending Competition

This subsection solves the problem that the bank and the shadow bank compete for one representative borrower. The law of large numbers allows us to apply the results to the entire set of borrowers later on. This problem has been studied in the previous literature such as Hauswald and Marquez (2003) and Rajan (1992). Our problem differs in two aspects. First, the bank and the shadow bank have different funding costs. Second, even in the case that the bank loses the borrower to the shadow, the bank can still make profits from the interbank funding market.

Let us start by putting a lower bound on the bank's payoff and therefore its interest-rate bid in the lending competition. When the bank does not make an offer (equivalently $R_B \to +\infty$), the shadow bank lends to the borrower with probability μ, that is, when the realized signal from screening is good.Subsequently, the bank's expected profit from the interbank lending is $\mu \Pi_B$. Meanwhile, by offering an interest rate R_B and winning the lending competition, the bank's expected profits are at most $P(\mu)R_B - r_B$, where $P(\mu)$ is the probability that the representative borrower will repay the loan. Note that these profits would be further reduced after taking into account the winner's curse effect. Whenever a bank makes a potentially winning bid in the lending competition, it must be that

$$P(\mu)R_B - r_B \geq \mu \Pi_B \Rightarrow R_B \geq \underline{R}_B := \frac{r_B + \mu \Pi_B}{P(\mu)} \tag{10}$$

Depending on the parameters, $\underline{R}_B$ may be greater or less than R. For the rest of this paper, we refer to $\underline{R}_B$ as the bank's *curse-free bid*, because it is the lowest bid by the bank even without the winner's curse effect.

Similarly, there is a lower bound on the shadow bank's bid in the lending competition. Following (1), we know that a shadow bank may only bid after it has received a good signal about the borrower. An offer R_S generates expected profits $R_S - r_E$. Meanwhile, by not making an offer (or equivalently, $R_S \to +\infty$), the shadow bank receives zero. Therefore, R_S must satisfy

$$R_S - r_E \geq 0 \Rightarrow R_S \geq \underline{R}_S := \frac{r_E}{P(\mu_g)} = r_E. \tag{11}$$

Clearly, $\underline{R}_S < R$ follows from Assumption 1 and $r_E < r_S$. For the rest of this paper, we refer to $\underline{R}_S$ as the shadow bank's *break-even bid*.

A comparison between $\underline{R}_B$ and $\underline{R}_S$ highlights the relative advantages of the bank and the shadow bank. Comparing the denominators, $P(\mu) < P(\mu_g) = 1$ reflects the shadow bank's information advantage and a higher $P(\mu_g)$ further increases this advantage. In terms of the numerators, $r_B < r_E$ captures the bank's funding advantage, and a lower r_B further increases the advantage. Finally, the term $\mu \Pi_B$ captures the bank's potential interbank profits, which are the profits if the shadow bank manages to lend to all borrowers with a good signal (so $\ell_S = \mu$).

The equilibrium outcome depends on the comparison between $\underline{R}_B$ and $\underline{R}_S$. Let us elaborate.

Case 1 $\underline{R}_B \leq \underline{R}_S$: dominating funding cost advantage

Knowing that the shadow bank's bid always exceeds $\underline{R}_S$, the bank would never make any bid $R_B < \underline{R}_S$. Interestingly, the bank would never make an interest-rate offer strictly above $\underline{R}_S$, either. Intuitively, any bid above $\underline{R}_S$ could suffer from the winner's curse. Below we illustrate this result in a few steps.

Lemma 1. *If $\underline{R}_B \leq \underline{R}_S$, the bank never offers an interest rate $\hat{R}_B \in (\underline{R}_S, R]$ with a positive probability mass.*

The intuition behind Lemma 1 is clear. Whenever the bank bids some $\hat{R}_B > \underline{R}_S$, the shadow bank with a good signal always has incentives to undercut the bank by bidding slightly below $\hat{R}_B$, which increases its profits by a large amount. Our next result shows that the bank cannot bid over an interval with a finite probability density, either.

Lemma 2. *If $\underline{R}_B \leq \underline{R}_S$, the bank never offers an interest rate $\hat{R}_B \in (R_1, R_2)$ with a positive probability density.*

The proof of the Lemma 2 relies on the following contradiction. If the bank offers a random interest rate in (R_1, R_2) with a finite probability density, the shadow bank must also make a random offer in (R_1, R_2) with a finite probability density; otherwise the bank would increase its bids without affecting the probability of winning. Next, it must be that $R_2 = R$; otherwise the bank would increase its bids from just below R_2 to R and increase its profits. However, high bids close to R almost never win, hence, the shadow bank cannot be making positive profits. Consequently, none of the shadow bank's bids in (R_1, R) can ever win. That is impossible when the bank bids in (R_1, R) with positive probability density.

Given Lemma 1 and 2, the bank adopts a pure strategy by offering an interest rate $\underline{R}_S$ and always wins over the borrower. The shadow bank, even though it never wins the lending competition, must also offer an interest rate on $[\underline{R}_S, R]$. This offer deters the bank from deviating and charging an interest rate higher than $\underline{R}_S$, and the distribution of this offer is not uniquely determined.

Case 2 $\underline{R}_S < \underline{R}_B < R$: comparable funding and information advantage

Knowing that the bank's bid always exceeds $\underline{R}_B$, the shadow bank would never make any bid $R_S < \underline{R}_B$. In this case, it is unavoidable for the bank to suffer from the winner's curse. In contrast to the previous case, both lenders must adopt mixed strategies in equilibrium.

Lemma 3. *If $\underline{R}_S < \underline{R}_B < R$, the bank can not offer an interest rate with a positive probability mass.*

The intuitions behind Lemma 3 are identical to those behind Lemma 1. Meanwhile, it cannot be that the bank never bids: if so, the shadow bank with a good signal always offers an interest rate R, which then induces the bank to bid R. Therefore, in equilibrium, the bank must adopt a mixed strategy. The only way such mixing by the bank can be incentive compatible is that the shadow bank is randomizing its bid on the same interval as well. Our next result shows that the mixed strategies must have a continuous CDF on the interval $[\underline{R}_B, R]$.

Lemma 4. *If $\underline{R}_S < \underline{R}_B < R$, both the bank and the shadow bank adopt mixed strategies with a probability density on $[\underline{R}_B, R]$. The shadow bank must offer R with a positive probability mass.*

Given the structure of the equilibrium, the CDFs of the interest rate offers can be uniquely determined by the opposite player's indifference condition. The shadow bank is indifferent between bidding $\underline{R}_B$ and winning almost for sure and bidding $\tilde{R} \in [\underline{R}_B, R]$ and winning with probability $1 - F_B(\tilde{R})$:

$$\underline{R}_B - r_E = \left(1 - F_B(\tilde{R})\right)(\tilde{R} - r_E). \tag{12}$$

Thus, the bank's distribution satisfies

$$F_B(\tilde{R}) = 1 - \frac{\underline{R}_B - \underline{R}_S}{\tilde{R} - \underline{R}_S} = \frac{\tilde{R} - \underline{R}_B}{\tilde{R} - \underline{R}_S}, \; \tilde{R} \in [\underline{R}_B, R]. \tag{13}$$

With a mass probability $1 - F_B(R) > 0$, the bank does not bid.

The bank is indifferent between not bidding at all and collecting profits $\mu\Pi_B$ from the interbank market and bidding any $\tilde{R} \in [\underline{R}_B, R]$. As a result

$$\mu\Pi_B = (1-\mu)\left(p\tilde{R} - r_B\right) + \mu\left[F_S(\tilde{R})\Pi_B + \left(1 - F_S(\tilde{R})\right)\left(\tilde{R} - r_B\right)\right], \tag{14}$$

which implies the offer by the shadow bank must satisfy the following distribution:

$$F_S(\tilde{R}) = \frac{\mu\Pi_B - \left(P(\mu)\tilde{R} - r_B\right)}{\mu\left[\Pi_B - \left(\tilde{R} - r_B\right)\right]}, \ \tilde{R} \in [\underline{R}_B, R). \tag{15}$$

With a mass probability $1 - F_S(R) > 0$, the shadow bank bids R.[5]

Case 3 $\underline{R}_S < R < \underline{R}_B$: dominating information advantage

In this remaining case, the bank strictly prefers to lose the bidding game even with the highest feasible interest rate R. As a result, the bank does not participate in the lending competition and the shadow bank always offers R to a good-signal borrower and nothing to a bad-signal one.

Note that the comparison among $\underline{R}_S$, $\underline{R}_B$, and R depends on μ. Given that $\underline{R}_B$ decreases with μ,[6] let us define

$$\bar{\mu} \equiv \frac{(1-\lambda\alpha p)r_B - p(1-\lambda\alpha)r_S}{\lambda(1-\alpha p)r_B + [(1-p) - \lambda(1-\alpha p)]r_S}$$
$$\underline{\mu} \equiv \frac{r_B - pR}{(1-p)R - \lambda(1-\alpha)(r_S - r_B)}.$$

Under Assumption 1, it is clear that $0 < \underline{\mu} < \bar{\mu} \leq 1$. Simple derivations show that $\underline{R}_B \leq \underline{R}_S$ for $\mu > \bar{\mu}$, $\underline{R}_S < \underline{R}_B < R$ for $\mu \in (\underline{\mu}, \bar{\mu})$, and $\underline{R}_B \leq R$ for $\mu \leq \underline{\mu}$. Given this result, we summarize the preceding discussion below.

Proposition 1 (Equilibrium Lending Competition)**.** *The lending bidding game has an essentially unique equilibrium.*

[5]Both numerator and denominator in (15) are negative since $\mu\Pi_B - (P(\mu)\tilde{R} - r_B) < 0$, $\forall\tilde{R} > \underline{R}_B$ and is a stronger condition than $\Pi_B - (\tilde{R} - r_B) < 0$.

[6]We can show $\frac{d\underline{R}_B}{d\mu}$ is proportional to

$$p(1-\alpha)\lambda(r_S - r_B) - (1-p)r_B < p(R - r_B) - (1-p)r_B = pR - r_B < 0.$$

1. *Collaboration: for* $\mu \in [0, \underline{\mu}]$*, the bank never bids and the shadow bank lends to a good-signal borrower at a rate* R *and does not lend to a bad-signal borrower.*

2. *Collaboration/Competition: for* $\mu \in (\underline{\mu}, \bar{\mu})$*, the bank randomizes between the bids in* $[\underline{R}_B, R]$ *with CDF* F_B *characterized by (13). With probability* $1 - F_B(R)$ *the bank does not bid at all. The shadow bank's bid distribution on* $[\underline{R}_B, R)$ *follows* F_S *characterized by (15). With a mass probability* $1 - F_S(R)$*, the shadow bank bids* R*.*

3. *Competition: for* $\mu \in [0, \bar{\mu}]$*, the bank always bids* $\underline{R}_S$ *and lends to all borrowers with probability* 1*.*

Proposition 1 shows how the equilibrium competition and collaboration depend on the average quality of the borrower pool. When the average quality μ is sufficiently low ($\mu < \underline{\mu}$), the bank does not participate in lending competition. Three factors affect this decision: (a) low quality of the pool implies that lending blindly to an average borrower is not very profitable to begin with, (b) the winner's curse effect implies that the pool of borrowers attracted to the bank's offer is even worse than the average, further reducing potential profits from lending, and finally (c) the option to lend to the borrowers indirectly through the interbank market crowds out the incentives to participate in the lending competition. Absent competition from the bank, the shadow bank charges monopolistic rate R and provides funding after observing a good signal. Lending in this region is inefficient due to the fact that the shadow needs to finance a fraction $1 - \lambda$ of the loan using its own costly funding.

When the average quality of the pool μ is sufficiently high ($\mu > \bar{\mu}$), information advantage of the shadow bank diminishes and becomes dominated by the funding cost advantage of the bank. As a result, the bank always outbids the shadow bank in equilibrium and provides lending. Despite the fact that shadow bank does not provide lending in this region, its presence does shape the equilibrium outcome by capping the rate that the bank could charge the borrower, preventing the bank from acting like a monopolist. Lending in this region is inefficient due to the blind uninformed of credit: the lack of screening by the bank results in financing negative NPV projects.

Finally, in the intermediate region, $\underline{\mu} < \mu < \bar{\mu}$, the information advantage of the shadow bank and the funding cost advantage of the bank are comparable. As a result, both institutions lend in equilibrium. Because both parties actively bid and can win with some positive probability, the equilibrium lending has two sources of inefficiency. Whenever the shadow bank offers a winning bid, lending is excessively costly; whenever the bank offer a winning bid, funds can be wasted on the negative NPV projects.

Proposition 1 implies that the competition between the bank and the shadow bank should only be observed by an econometrician in borrower pools whose average quality is neither too high nor too low. The bank retreats from borrower pools with low average quality, whereas shadow bank retreats from those with high average quality.

Next, we show how the equilibrium cutoffs $\underline{\mu}$ and $\bar{\mu}$ depend on the primitive model parameters.

Corollary 1 (Effect of the Funding Cost)**.** *Equilibrium cutoff $\bar{\mu}$ is decreasing in the funding cost of the shadow bank r_S and increasing in funding cost of the bank r_B. Equilibrium cutoff $\underline{\mu}$ is increasing in both the funding cost of the shadow bank r_S and the funding cost of the bank r_B.*

An increase in r_S (or decrease in r_B) magnifies the funding cost advantage that the bank has over the shadow bank. This makes the shadow bank less competitive relative to the bank and increases the size of region where the bank can outbid the shadow bank, i.e., decreases $\bar{\mu}$. Meanwhile, an increase in the bank's funding cost r_B makes low-quality pools further less attractive to the bank. As a result, the region $[0, \underline{\mu}]$ in which the shadow bank can charge the monopolistic rate expands. Shadow bank's funding cost r_S affects the cut-off $\underline{\mu}$ through the bank's choice between direct lending and interbank lending. Higher r_S increases interbank lending profits and reduces the bank's incentives to compete for direct lending. In response, the bank optimally cedes a larger region $[0, \underline{\mu}]$ to the shadow bank and only focuses on the interbank market.

3.2 Welfare Implications

Having characterized the equilibrium in Proposition 1, we now turn to its welfare implications. It is straightforward to see that in the first-best benchmark, all the high-type borrowers should obtain funding from the bank, whereas all the low-type borrowers should not be financed. Therefore, the equilibrium identified in Proposition 1 features two sources of inefficiency. First, whenever $\lambda < 1$, lending by the shadow bank requires to use the more costly sources of financing. Below we refer to it as *funding inefficiency*. Second, lending by the bank often ends up funding negative NPV projects. Below we refer to it as *lending inefficiency*. The degree to which the two sources of inefficiency are prevalent in the equilibrium depends on the average quality of the borrower's pool.

When the average quality of the pool is very low $\mu < \underline{\mu}$, the shadow bank is effectively a monopolist and lends at an interest rate R after receiving a good signal. Therefore, a mass q_g of all borrowers are able to receive funding, and all borrowers receive a zero payoff. The shadow bank makes expected profits

$$V_S = \mu \cdot (R - r_E),$$

and the bank earns interbank profits $\mu\Pi_B$. Therefore, the resulting welfare is

$$W = \mu\Pi_B + \mu \cdot (R - r_E) = \mu \cdot (R - (1-\lambda)r_S - \lambda r_B). \tag{16}$$

Equation (16) shows that in this region, the welfare loss is driven by the funding inefficiency, i.e., the fact that the shadow bank must finance a fraction $1-\lambda$ of its loans using its own funding, which is more costly.

When the average quality of the pool is very high $\mu > \bar{\mu}$, the bank always wins by bidding $\underline{R}_S$. In this case, the shadow bank makes zero profit, whereas a borrower of type $\theta_i \in \{h, l\}$ receives a payoff $p_{\theta_i}(R - \underline{R}_S)$. The bank receives no profits from interbank lending but earns $P(\mu)\underline{R}_S - r_B$ from directly lending to the borrowers. The resulting welfare is

$$W = P(\mu)R - r_B. \tag{17}$$

Equation (17) shows that in this region, the welfare loss is driven by the lending inefficiency, i.e., by the fact that low-type borrowers also receive funding.

In the intermediate region $\underline{\mu} < \mu < \bar{\mu}$, both bank and shadow bank actively bid and win with positive probabilities. A borrower with a good signal can always receive financing and therefore receives a payoff $-\int_{\underline{R}_B}^{R}(R-\tilde{R})d\left((1-F_B(\tilde{R}))\cdot(1-F_S(\tilde{R}))\right)$.[7] By contrast, a borrower with a bad signal can only be financed by the bank, so that the expected payoff is $\int_{\underline{R}_B}^{R}(R-\tilde{R})dF_B(\tilde{R})$. Given that the shadow bank places a random bid following a good signal, any bid $\tilde{R}_S \in [\underline{R}_B, R]$ must generate the same profits. When the shadow bank bids $\tilde{R}_S = \underline{R}_B$, it always wins over the borrower and receives expected profits

$$V_S = \mu\left(\underline{R}_B - r_E\right) \tag{18}$$

The bank places a random bid in $[\underline{R}_B, R] \cup \{\infty\}$. As a result, its total profits from direct lending and interbank market are the same regardless of whether the bank is bidding or not. Not bidding would generate the total profits equal to

$$V_B + V_i = \mu \cdot \Pi_B. \tag{19}$$

Summing up, we get the total welfare

$$W = \mu\left[R - (\lambda r_B + (1-\lambda)r_S)\int_{\underline{R}_B}^{R}\left(1 - F_B(\tilde{R}_S)\right)dF_S(\tilde{R}_S) - r_B\int_{\underline{R}_B}^{R}F_B(\tilde{R}_S)dF_S(\tilde{R}_S)\right] + (1-\mu)F_B(R)(pR - r_B). \tag{20}$$

The next proposition describes how equilibrium objects, bank profits, interbank profits, bor-

[7]Since CDF for $\Pr(\min\{R_S, R_B\} \le \tilde{R}) = 1 - (1 - F_S(\tilde{R}))(1 - F_B(\tilde{R}))$.

rower and total welfare, vary with the average quality of the borrower pool μ.

Proposition 2. *The bank's total profit strictly increases in μ. By contrast, the shadow bank's profit is non-monotonic in μ: it increases for $\mu < \underline{\mu}$ and could be non-monotonic on $\mu \in [\underline{\mu}, \bar{\mu}]$. The payoff of both high- and low-type borrowers increases in μ.*

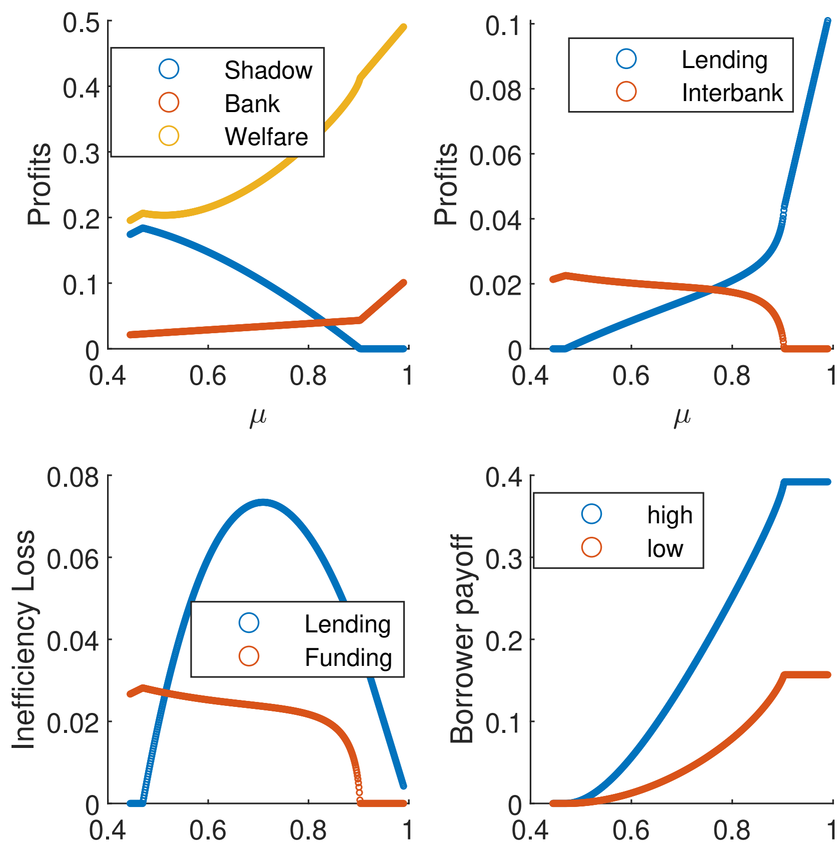

Figure 2: Profits, Welfare, and Lending under Different μ

This figure describes the equilibrium profits and welfare when the average quality of the borrower pool μ varies. The left panel plots the profits and welfare, and the right panel decomposes the bank profits into direct lending and interbank lending. The parameters in this figure are as follows: $R = 2.0$, $p = 0.1$, $r_S = 1.5$, $r_B = 1.0$, $\lambda = 0.8$, $\alpha = 0.2$, $e_1 = 0$, $e_2 = 0$.

Figure 2 plots the profits and welfare when the average quality of the borrower pool varies. The top-left panel describes the profits of the bank and the shadow bank, as well as the total welfare. Given the parameter values (available in the caption), the two thresholds are $\underline{\mu} = 0.5405$ and $\bar{\mu} = 0.8956$. There are a few interesting observations. First, the shadow bank's profits are non-monotonic in μ: it increases for $\mu < \underline{\mu}$; for $\mu \in [\underline{\mu}, \bar{\mu}]$, it could also be non-monotonic. Intuitively, this result holds because for $\mu < \underline{\mu}$, there is no competition and, as a result, the shadow bank earns a monopoly rent. An increase in μ results in the shadow bank lending to and profiting from larger pool of high-type borrowers. By contrast, for $\mu \in [\underline{\mu}, \bar{\mu}]$, there are both competition and collaboration. A higher μ in general leads to more high-type borrowers, which increases the shadow bank's potential profits. Meanwhile, a higher μ intensifies the bank's competition, which reduces the shadow bank's per-borrower profits $\underline{R}_B - r_E$. We have shown that $\underline{R}_B$ decreases with μ. Intuitively, when μ goes up, the bank is more likely to be repaid when it lends blindly. Therefore, the curse-free bid $\underline{R}_B$ at which it is indifferent between lending blindly and interbank lending goes down. Combining both effects, the overall effect $(\mu(\underline{R}_B - r_E))$ can be non-monotonic. Finally, when μ rises above $\bar{\mu}$, the shadow bank completely retreats from lending and therefore makes zero profits.

By contrast, the bank's total profits always increase in μ, and the slope becomes even higher for $\mu > \bar{\mu}$. Intuitively, whenever $\mu < \bar{\mu}$, the bank always earns the same payoff as if it loses the competition and simply profits from interbank lending, which increases with μ at a slope Π_B. However, interbank lending profits of the bank are limited by the total borrowing capacity of the shadow bank λ and the shadow bank's bargaining power α. In other words, the bank is willing to share the profits with the shadow bank due to the high level of adverse selection. For $\mu \geq \bar{\mu}$, the bank's profit increases with μ at a slope $(1-p)r_E$. The interbank profit Π_B that accrues to the bank is a source of the shadow bank's effective funding cost, that is $r_E > r_B + \Pi_B$. For $p = 0$ this intuition shows that the slope of the total bank profit is increasing with μ. For $p > 0$ we use the upper bound imposed on p by Assumption 1 to show the result Pasha: Made edits The top-right panel of Figure 2 decomposes the bank's total profits into those from direct lending and those from interbank lending. Whereas the profits from direct lending increase in μ, the profits from interbank lending are, again, non-monotonic in μ, because when μ goes up, there are both more high types as well as more competition.

Finally, the yellow line of the top-left panel plots the total welfare against μ. Interestingly, it is also non-monotonic $\mu \in [\underline{\mu}, \bar{\mu}]$. The bottom-left panel decomposes the welfare losses into lending and funding inefficiency, respectively. The red line measures funding inefficiency, defined as the equilibrium amount of shadow bank lending times $(1-\lambda)\cdot(r_S - r_B)$, which monotonically decreases. Therefore, a higher μ reduces shadow bank lending and thereby reduces funding inefficiency. The blue line measures lending inefficiency, defined as the probability of low types being financed times $r_B - pR$. Clearly, lending inefficiency is non-monotonic in μ. On one hand, a higher μ means that the fraction of low-type borrowers gets lower. On the other hand, a higher μ increases lending competition by banks, which results in a higher probability that low types get financed. Therefore, the overall efficiency loss can be non-monotonic in μ. Finally, as illustrated in the bottom-right panel, the borrower's payoff increases in μ and the high-type borrower receives a higher payoff than a low-type borrower.

Remark 1. *Note that the welfare function has only included the payoff to borrowers, the bank and the shadow bank, but not the creditors to the bank/shadow bank. One interpretation is $r_B - 1$ and $r_S - 1$ are the iceberg costs of the bank and the shadow bank in taking one unit of deposit, and their creditors only receive 1. If, instead, we interpret r_B and r_S as the rates required by the creditors, then the funding costs are merely transfers from the institutions to their creditors. In this case, the welfare loss only comes from the bank funding bad projects, which depends on μ. When μ gets higher, the bank is more likely to submit a bid that is weakly less than R to finance the bad project, which increases the welfare loss. Meanwhile, when μ gets higher, there are fewer bad projects to begin with. The overall effects can be non-monotonic, as shown by the blue line of the right panel of Figure 2.*

3.3 Costly Screening and Entry

Our baseline model has taken as given the lending market structure, which is populated by a bank and a shadow bank. Moreover, we have assumed that screening entails no cost to the shadow bank, i.e., the shadow bank is endowed with a costless screening technology. We relax both assumptions in this subsection.

Suppose instead, screening entails a physical cost. Without screening, the shadow bank is dominated by the bank: it has neither funding nor informational advantage. In this case, it retreats from lending regardless of μ and receives a zero payoff. To screen, the shadow bank needs to have profits sufficiently high. As shown by Proposition 3 and illustrated by Figure 3, the shadow bank's profits always decline in μ without the interbank market. With the interbank market, however, the shadow bank's profits first increase then decrease in μ.

The analysis when entry is costly is isomorphic to a screening cost. Without entry, the shadow bank earns zero profits. With entry, the net profits depend on the profits from lending and the entry cost. Therefore, our model with either screening cost or entry cost yields two predictions. First, the presence of interbank market encourages entry by a shadow bank. Second, without the interbank market, shadow bank mainly enters into market with a high fraction of low-type borrowers. With the interbank market, the shadow bank enters into market of which the fraction of low-type borrowers is neither too high nor too low.

Remark 2. *We have assumed that screening by the shadow bank is observable by the bank. Results are similar if instead, screening becomes unobservable. To see this, note that if the shadow bank* *never screens or always screens in equilibrium, then observing screening or not does not affect the biding decision made by the bank. Therefore, results can only be affected if the shadow bank uses a mixed strategy, which implies it is indifferent between screening and not. Note that this case is a knife-edge one: it requires that the profits screening are exactly identical to the screening cost. Moreover, the results are qualitatively unchanged. Essentially, the bank knows that with some probability, it bids against an uninformed shadow bank and therefore bids a bit more aggressively.*

In this benchmark model, the shadow bank's profits are non-monotonic: it peaks when the average quality of the borrower pool is neither too high nor too low. The reason is, when the average quality gets too low, there are not many high-type borrowers to begin with, so the shadow bank's profits are low. By contrast, when the average quality gets too high, the competition from the bank gets intensified, and the shadow bank's profits are low again. Therefore, if either screening or entry entails a physical cost to the shadow bank, our model predicts that the shadow bank will be active in markets where the average quality is neither too high nor too low. In general, when the average quality is very low, neither lenders is active. When the average quality gets higher, the shadow bank active lends, with financing from the bank. When the average quality gets further improved, both lenders are actively competing and also collaborating. Finally, when the average quality gets to the highest region, only the bank lends.

3.4 The Effect of Interbank Market

In this subsection, we explore the effect of the interbank lending between the bank and the shadow bank. Surprisingly, the introduction of interbank market can often times make either the borrowers or the bank worse-off.

We begin with the effect on the thresholds $\{\underline{\mu}, \bar{\mu}\}$.

Corollary 2 (Effect of the Interbank Market)**.** *Equilibrium cutoffs $\underline{\mu}$ and $\bar{\mu}$ are increasing in the fraction of funds λ provided via the interbank market. When $\lambda = 0$, $\underline{\mu} = \mu_{\min}$ and the collaboration region disappears. When $\lambda = 1$, $\bar{\mu} = 1$ and the competition region disappears.*

A decrease in the fraction of funds λ provided via the interbank market has two effects. First, it makes interbank lending less attractive to the bank. Lower potential profits from interbank lending incentivize the bank to bid more aggressively due to the increased importance lending directly to the borrowers. As a result, the monopoly region of the shadow bank $[0, \underline{\mu}]$ becomes smaller. Second, an increase in the shadow bank's skin in the game $1 - \lambda$ (decrease in λ) increases the shadow bank's effective cost of funding r_I, making it less competitive against the bank. This force increases the region in which the bank can outbid the shadow bank $[\bar{\mu}, 1]$.

Below, we first specialize to the two corner cases $\lambda \in \{0, 1\}$, which corresponds to the situation without and with the interbank market, respectively. Note that when $\lambda = 1$, $\bar{\mu} = 1$, so that the equilibrium only features two regions $[\mu_{\min}, \underline{\mu}]$ and $[\underline{\mu}, 1]$. In other words, under $\lambda = 1$, the bank no longer possesses funding advantage, so that the last region of competition vanishes. When $\lambda = 0$, it is easily shown that $\underline{\mu} = \mu_{\min}$, so that the equilibrium also only features two regions: $[\underline{\mu}, \bar{\mu}]$ and $[\bar{\mu}, 1]$. In other words, without the interbank market, the collaboration region completely vanishes.

Proposition 3 (The Effect of Interbank Market)**.**

1. *There exists a μ_θ^* such that a type-θ borrower is better off with the interbank market if and only if $\mu \geq \mu_\theta^*$.*
2. *The shadow bank always receives more profits with the interbank market.*
3. *There exists a μ_B^* such that the bank receive more profits with the interbank market if and only if $\mu < \mu_B^*$.*

In general, the presence of interbank market introduces two forces. On one hand, it reduces the shadow bank's funding cost so that it can better compete with the bank. In other words, the interbank market allows the shadow bank to fund the borrower with a lower cost, which, due to competition, can be passed onto the high-type borrowers. Moreover, this channel forces the bank to offer a lower bid to compete with the shadow bank, which also benefits indirectly the low-type borrowers. This channel by itself leads to intensified competition. On the other hand, the interbank market allows the bank to make profits not from directly lending to the borrower. In other words, when the bank receives profits from interbank lending, it therefore has lower incentives to compete with the shadow bank in the direct lending market. This channel by itself leads to mitigated competition. Proposition 3 shows how these two channels interact differently in markets characterized by different levels of average quality. When μ is low, there is not much competition, and the interbank market allows the bank to profit additionally at a cost of the borrower's payoff. By contrast, when μ is very high, competition is high, and the interbank market further intensifies competition, which benefits the borrower but reduces the bank's profits.

The results in Proposition are illustrated by Figure 3. The top-left panel compares a high-type borrower's payoff with and without the interbank market. Results are similar for a low-type borrower. Clearly, the interbank market benefits borrowers in a low μ pool more than borrowers in a high μ pool.

The top-right panel compares the shadow bank's profits. It is not surprising that the shadow bank is always better off with the interbank, due to its lower funding cost. Intuitively, both channels work in the favor of the shadow bank: the lower funding cost allows it to compete more aggressively, and the interbank lending reduces the competition from the bank. Therefore, the shadow bank shall be better-off for any μ. Turning to the bottom-left panel. Interestingly, the bank receives more profits without the interbank market when μ is high, i.e., when the adverse-selection problem is only mild. Intuitively, the presence of interbank introduces mostly competition, which reduces its profits. For borrowers in a low μ pool, it was not profitable for the borrower to extend funding to begin with. In this case, the presence of the interbank market allows the bank to indirectly benefit from the shadow bank's superior screening technology, thereby increasing its profits. Finally, the bottom-right panel compares the total welfare, and the result is intuitive: the interbank market eliminates the funding inefficiency and therefore can only increase the welfare.

Our results imply that borrowers benefit from the interbank market when the average quality of the borrower pool is high but suffer when the average quality is low. By contrast, the bank has incentives to lend to the shadow bank when the average quality of the borrower pool is low but not when the average quality is high. In other words, if the bank could commit not lending to the shadow bank in the interbank market, it would have strict incentives to do so.

We conclude this subsection by showing the comparative statics with respect to λ. This exercise corresponds to an improvement in the interbank lending technology such as a reduction in the search friction or a better interbank monitoring technology. As shown in Figure 4, the profits of the bank and the shadow bank, the payoff of both types of borrowers, and the total welfare all show some non-monotonicities in λ. The non-monotonicities, in turn, are driven by the interaction of the two forces that change with λ. Higher λ lowers the funding cost of the shadow bank and increases the potential interbank profits at the same time.

Figure 4 starts with $\lambda = 0$ in the competition region, i.e. $\mu > \bar{\mu}$. A marginal increase in λ in this cases has only one effect - it decreases shadow bank funding cost. The second force is absent since the interbank profits are 0. Lower r_E implies increased competition for the borrowers, which leads to a reduction in bank's profits and an increase in borrowers' payoffs. Only the equilibrium bank bid is affected by λ in this region and, as a result, welfare, shadow bank profits and inefficiencies remain constant.

For $\lambda > 0.25$ the equilibrium outcome in Figure 4 is competition with collaboration. Higher λ now reduces competition through the interbank market channel. Lower competition translates into lower borrower's payoff, as shown in the center panel of Figure 4. Both bank and shadow bank benefit from less competitive bidding, as show in the left panel of Figure 4. The total welfare is affected by the behavior of funding inefficiencies plotted in the right panel of Figure 4. Lending inefficiency declines since the bank increasingly retreats from direct lending in favor of interbank lending. Funding inefficiency, however, exhibits non-monotone behavior. Higher λ reduces per dollar funding inefficiency, since the shadow bank can increasingly rely on interbank

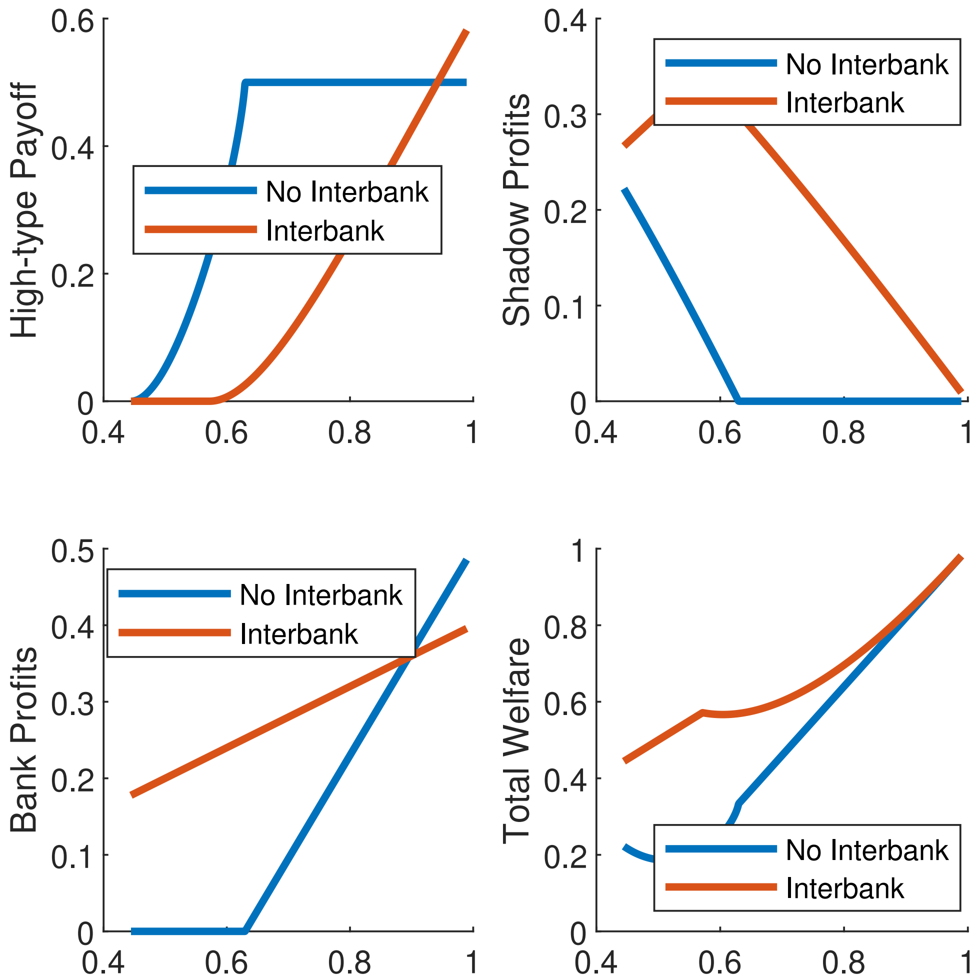

Figure 3: Profits and Welfare with and without the interbank market

This figure describes the equilibrium profits and welfare with and without the interbank market, when the average quality of the borrower pool μ varies. The left panel plots the profits and welfare, and the right panel decomposes the bank profits into direct lending and interbank lending. The parameters in this figure are as follows: $R = 2.0$, $p = 0.1$, $r_S = 1.5$, $r_B = 1.0$, $\alpha = 0.2$, $e_1 = 0$, $e_2 = 0$. $\lambda = 1$ and $\lambda = 0$ respectively stand for with and without the interbank market.

loans. However, higher λ also increases the total volume of the shadow bank lending and amplifies funding inefficiency as a result.

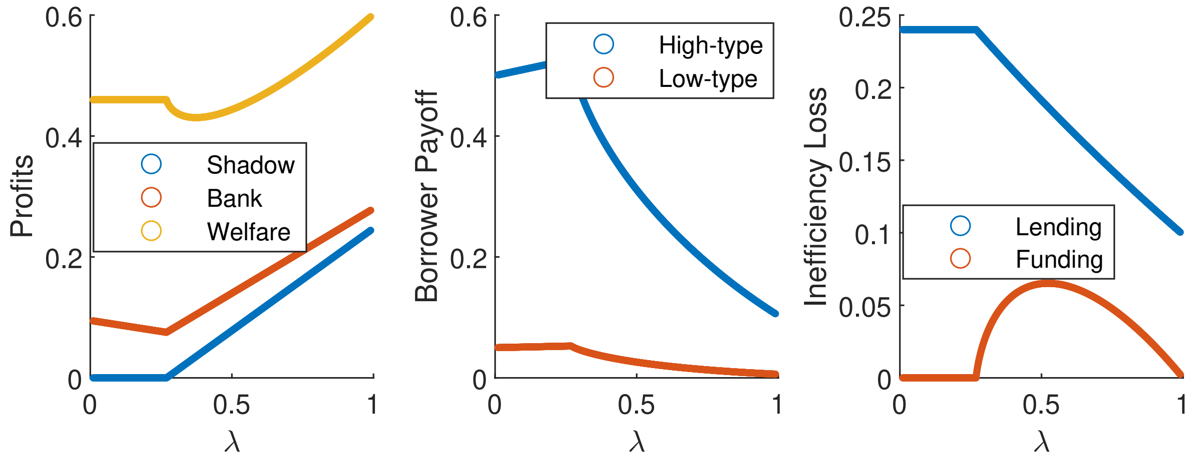

Figure 4: Comparative Statics w.r.t. λ

3.5 Interbank Financing from A Non-Competing Bank

In the benchmark model, we have assumed that the shadow bank always borrows from the same bank with whom it competes in direct lending. This subsection introduces the possibility that the shadow bank could obtain financing from a non-competing bank, i.e., a bank that does not lend in the same market. One can think of this bank as some lender with special expertise in a different geographical location. We are going to show that the profits of both the shadow bank and the incumbent bank get lower. Therefore, if the shadow bank had a choice, it would choose to borrow from the direct competing bank. However, this will come as a cost: borrowers are strictly worse-off compared to the situation that the shadow bank obtains financing from the non-competing bank.

Let us first consider the model in which the shadow bank obtains a fraction λ of the interbank financing from a third, non-competing bank. Borrowing from the third-party bank allows the shadow bank to reduce its funding cost from r_S to r_E without affecting the competing bank's incentives via interbank profits. Note that in such a model, the per-loan interbank profits are $\Pi_B = 0$. The bank's curse-free bid becomes $\underline{R}_B = \frac{r_B}{P(\mu)}$, whereas the shadow bank's break-even bid $\underline{R}_S$ stays unchanged. The two critical thresholds in the average quality are

$$\underline{\mu}_{NC} = \underline{\mu}_0 = \frac{r_B/R - p}{1-p} < \underline{\mu}_1 \qquad \bar{\mu}_{NC} = \frac{r_B/r_E - p}{1-p} \in (\bar{\mu}_0, \bar{\mu}_1),$$

where the subscripts NC stand for non-competing. Our next result compares the payoffs of the different market participants in the benchmark model to the one with third party financing.

Proposition 4 (Interbank Financing from a Non-Competing Bank)**.**

Compared to the benchmark model with and without the interbank financing:

1. *Borrowers are weakly worse off if the shadow bank obtains financing from a non-competing bank.*
2. *The shadow bank receives weakly more profits if it obtains financing from a non-competing bank.*
3. *The bank also receives weakly more profits if the shadow bank obtains financing from a non-competing bank.*

Proposition **??** implies that the shadow bank always prefers to borrow from the same bank with whom it competes to some other banks who do not directly participate in lending competition. Non-competing bank financing isolates the effect of bank funding cost from the lending competition. Relative to the case without interbank market, non-competing bank financing only reduces the effective funding cost of the shadow bank but does not mitigate the bank's incentives for competition. This change increases shadow bank profits and forces the bank to bid more aggressively; hence, the bank profits decrease. The borrowers benefit from the increased competition and between the bank and the shadow bank.

4 Robustness and Extensions

4.1 The Effect of Signal Error

In this subsection, we examine how the two types of errors, e_1 and e_2 affect the equilibrium outcome and welfare. Assumption 1 imposes an upper bound and a lower bound on μ. Specifically,

$$\mu \in \left[\max\left\{\frac{r_B/R-p}{1-p}, \frac{(r_S/R-p)e_2}{(1-e_1-e_2p)-(1-e_1-e_2)r_S/R}\right\}, \frac{(1-e_2)(r_B/R-p)}{(e_1-(1-e_2)p)+(1-e_1-e_2)r_B/R}\right].$$

The two lower bounds come from $r_S < P(\mu_g) \cdot R$ and $r_B < P(\mu) \cdot R$, whereas the upper bound comes from $r_B > P(\mu_b) \cdot R$.

We first show that the equilibrium characterized in Proposition 2 is robust to the introduction of the type I and type II errors. While the equilibrium retains its general structure, type I and type II errors affect profits of the bank and shadow bank in a way that we describe below.

Type I error

Let us start with type-I error $e_1 > 0$ and $e_2 = 0$, so that a shadow bank might receive a bad signal when facing a high-type borrower. In this case, $q_g = \mu(1-e_1)$, $q_b = \mu e_1 + (1-\mu)$, $\mu_g = 1$, and $\mu_b = \frac{\mu e_1}{q_b}$. The equilibrium is still characterized by two thresholds $\{\bar{\mu}, \underline{\mu}\}$, and we have the following results.

Corollary 3. *With type-I error, the equilibrium consists of three regions: collaboration for $\mu < \underline{\mu}$, collaboration / competition for $\mu \in (\underline{\mu}, \bar{\mu})$, and competition $\mu > \bar{\mu}$ similar to Proposition 1. Both thresholds $\bar{\mu}$ and $\underline{\mu}$ decrease with e_1. Moreover, equilibrium shadow bank profits are strictly and bank profits are weakly decreasing in e_1 in every region.*

Intuitively, a higher e_1 reduces the likelihood of the good signals, and as a result it also reduces the equilibrium amount of lending by the shadow bank. Lower shadow bank lending volume implies a lower demand for interbank loans and, hence, lower interbank profits for the bank. Consequently, the bank has more incentives to compete with the shadow bank as opposed to offer interbank funding. Therefore, the region that the bank dominates in lending shall increase, leading to a decrease in $\bar{\mu}$. A similar reason leads to a decrease in $\underline{\mu}$.

The negative effect of e_1 on the bank profits stems only from the interbank market. It is absent, whenever the interbank markets are closed, i.e., $\lambda = 0$. The negative effect of e_1 on the shadow bank profits stems from both the reduced lending due to a lower likelihood of observing a good signal, and stronger bank competition.

Type II error

Now, we turn to the case of type-II error $e_1 = 0$ and $e_2 > 0$, so that a shadow bank might receive a good signal when facing a low-type borrower. In this case, $q_g = \mu + (1-\mu)e_2$, $q_b = (1-\mu)(1-e_2)$, $\mu_g = \frac{\mu}{q_g}$, and $\mu_b = 0$. The equilibrium is again characterized by two thresholds $\{\bar{\mu}, \underline{\mu}\}$, and we have the following results.

Corollary 4. *With type-II error, the equilibrium consists of three regions: collaboration for $\mu < \underline{\mu}$, collaboration / competition for $\mu \in (\underline{\mu}, \bar{\mu})$, and competition $\mu > \bar{\mu}$ similar to Proposition 1. The upper threshold $\bar{\mu}$ is decreasing in e_2 and the lower threshold $\underline{\mu}$ is increasing with e_2.*

Moreover, equilibrium shadow bank profits are decreasing and bank profits are increasing in e_2 in every region.

Intuitively, an increase in the type-II error has two effects. First, it reduces average quality of the pool conditional on good signal μ_g. As a result, shadow bank's informational advantage is mitigated and it bids less aggressively (direct effect). Second, higher e_2 increases the likelihood of observing the good signal q_g. Keeping the bidding strategies fixed this would translate into an increase in the volume of shadow bank lending and consequently, interbank profits, reducing the bank's incentives to compete (indirect effect).

The threshold $\underline{\mu}$ is driven by the bank's incentives. Higher e_2 translates into an increase in the volume of the interbank lending through q_g and strengthens the bank's incentives to withdraw from direct lending in favor of interbank lending. As a result, $\underline{\mu}$ increases in e_2. The other threshold $\bar{\mu}$ is pinned down by the competitive incentives of the bank and shadow bank respectively. For μ close to $\bar{\mu}$ the volume of interbank lending is close to zero, hence, and increase in q_g does little to deter the bank from competing. Lower information advantage of the shadow bank due to lower μ_g, however, decreases the competitiveness of the shadow bank. Hence, the region where the bank dominates expands with e_2 and the upper threshold $\bar{\mu}$ decreases with e_2.

Total bank profits are affected in the same direction by both forces. The bank benefits from a less competitive shadow bank and from higher potential interbank market, hence, its profits are increasing in e_2. For the shadow bank the two forces are working in the opposite directions. However, the direct channel dominates and the shadow bank profits are decreasing in e_2. To see the intuition consider the two corner cases $\mu = \bar{\mu}$ and $\mu = \underline{\mu}$. As discussed earlier, at $\mu = \bar{\mu}$ interbank profits are zero and the indirect interbank channel is absent. Hence the shadow bank profits are decreasing in e_2 due to the direct effect. At $\mu = \underline{\mu}$ the interbank profits are high and indirect effect should increase the probability of the shadow bank winning the bidding game. However, the shadow bank wins it with probability 1 to begin with, hence the indirect effect is muted and the profits are decreasing due to the direct effect again.

Interbank Market and Imperfect Signal

In absence of the interbank market, the Type I and Type II errors affect the equilibrium only through a direct channel. Namely, Type I error leads the shadow bank to miss out on some of the high quality borrowers and Type II error effectively reduces the quality of the pool that the shadow bank competes for. Importantly, the direct effect of Type I does not change the shadow bank incentives to compete, conditional on receiving a good signal. Because of that it also does not impact the incentives of the bank to compete. Formally, the bidding cdfs F_B and F_S do not depend on e_1, and the equilibrium outcome is not affected by the Type I error. This can be clearly seen in the left panel of Figure 5: total bank profits with perfect signal and $e_1 > 0$ coincide.

If the shadow bank were to choose among the class of information acquisition technologies with a fixed precision but different profile of errors, i.e., $e_1 + e_2 = e$, it would prefer to minimize the Type II error as can be seen in the right panel of Figure 5. This preference is driven by comparing the loss from investing in a negative NPV project to the unrealized gain form not investing in a negative NPV project.

The presence of interbank market introduces indirect effect. Type I (Type II) error decreases (increases) the potential size of the interbank market and increases (decreases) the bank's incentive

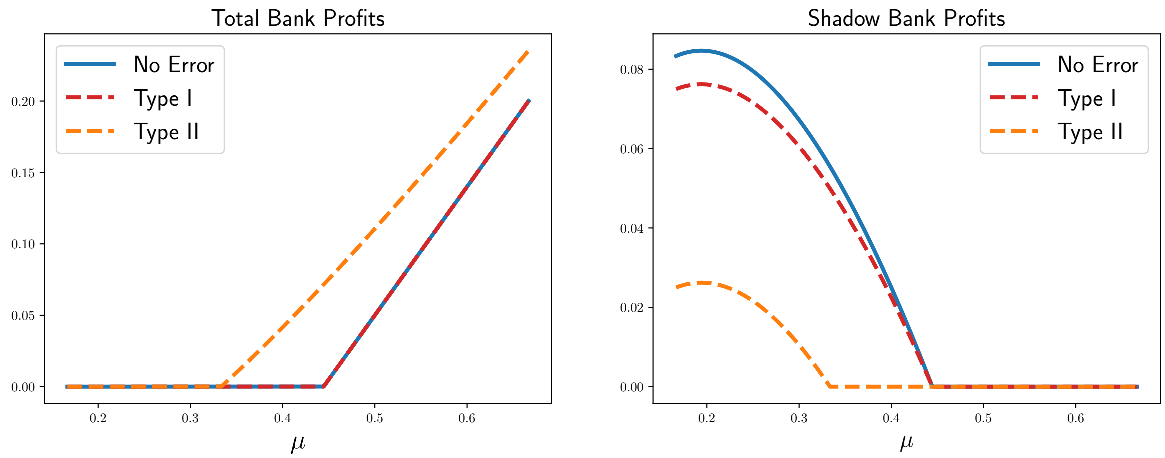

Figure 5: Total bank and shadow bank profits with imperfect signals in the absence of the interbank market ($\lambda = 0$).

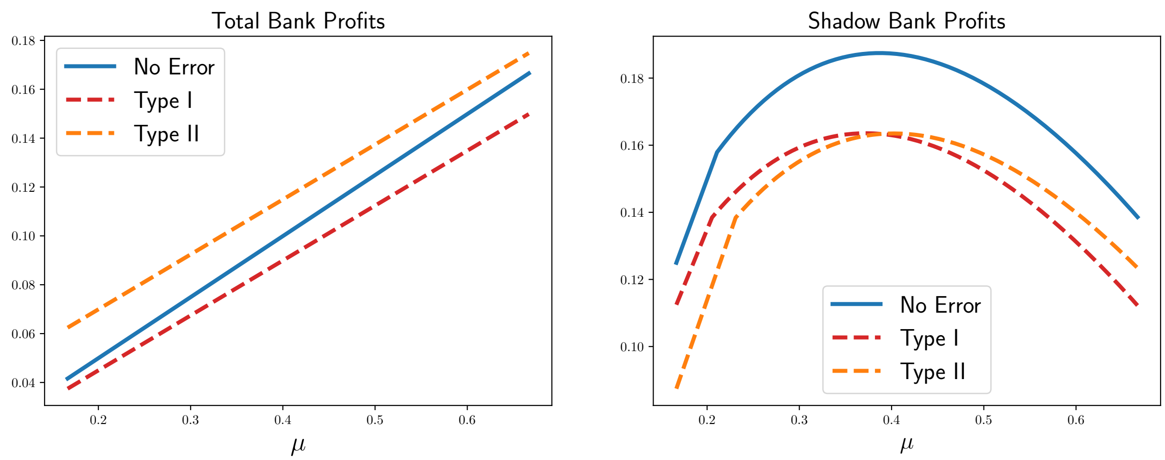

Figure 6: Total bank and shadow bank profits with imperfect signals with interbank market ($\lambda = 1$).

to compete against the shadow bank. Indirect effect further increases total bank's profits in case of Type II error and decreases profits in case of Type I error as can be seen in the left panel of Figure 6.

Indirect effect acts in the opposite direction on the shadow bank's profits. At $\mu = \underline{\mu}$ the bank bids with probability 0, and, as a result the indirect effect is muted. The right panel of Figure 6 shows that at $\mu = \underline{\mu}$ the ranking between Type I and Type II errors for the shadow bank coincides with the ranking in the absence of the interbank market on the right panel of Figure 5. However, as the average quality of the pool μ goes up the probability of the bank competing with the shadow bank increases and the indirect effect becomes more pronounced. For higher μ the ranking of the Type I and Type II order flips, since the shadow bank benefits from less aggressive bidding by the bank under Type II errors.

4.2 Non-Pecuniary Benefits

We interpret borrowers as either consumers or small-business owners who seek credit. In practice, these borrowers could value non-pecuniary benefits associated with different lenders. For example, it is well-known that lending relationships with banks are sticky, due to the other services offered and bundled. Moreover, the rise of fintech lending has been partially attributed to the convenience and speed offered by these lenders. We explore these issues in this subsection. Specifically, we differentiate between two cases, depending whether the bank or the shadow bank has the benefits.

Let us first consider the case that the bank has convenience benefits in that the shadow bank can only outbid the bank if its bid satisfies $R_S < R_B - \Delta$, where $\Delta > 0$ captures the benefits associated with bank lending, such as its extensive branch network or payment services. Once again, we define $\underline{R}_B$ and $\underline{R}_S$ as in (10) and (11). If Δ gets very high such that $\underline{R}_S + \Delta > R$, the shadow bank could never win the borrower over, and its presence does not pose a threat to the bank. If on the other hand $\underline{R}_S + \Delta \leq R$, The equilibrium is qualitatively similar to the one in Section 3, which is characterized by three regions. In the first region $\underline{R}_B > R$ (low average quality), the bank never lends, and the shadow bank acts as a monopolist lender who offers R to high-type borrowers. In the second region $\underline{R}_S + \Delta < \underline{R}_B < R$ (intermediate average quality), both lenders could win the borrowers. In equilibrium, the bank both offers R and retreats from lending with positive probability masses. As a result, whereas shadow bank introduces competition and the resulting winner's curse effect prevents the bank from always bidding, the convenience benefit Δ enhances the bank's monopoly power in that it still some times offer R on the equilibrium path. Finally, in the last region $\underline{R}_B < \underline{R}_S + \Delta$, the bank always outbids the shadow bank and offers an interest rate $\underline{R}_S + \Delta$. We supplement details in the appendix. The broader takeaway from this exercise is that, the convenience benefit essentially offers monopoly power to the bank, which stifles lending competition.

Next, we turn to the case that the shadow bank offers more convenience benefits in that the bank can only outbid the shadow bank if its bid satisfies $R_B < R_S - \Delta$, where Δ captures the speed and simplified procedures when borrowers apply for credit from the shadow bank. The equilibrium turns out very similar to the one described by Proposition 1, with the only exception that $\bar{\mu}$ is defined as the threshold such that $R_B = \underline{R}_S - \Delta$ holds. Intuitively, the convenience benefit reduces the shadow bank's effective funding cost by Δ, which further increases its competitiveness.

Remark 3. *In the case that the shadow bank offers more convenience benefits, if we take the limit of $\Delta \downarrow 0$, the result corresponds to the one in the benchmark model in which the tie favors the shadow bank. Therefore, our earlier results in section 3 are robust to the way that ties are settled.*

4.3 Competition within banks and shadow banks

In the benchmark model, we have taken as given the intermediation sector being populated by one bank and one shadow bank. In other words, the environment that we study is one with a pre-existing bank with some local monopoly power in lending, faced with a new shadow bank entering the market, consistent with Philippon (2016). In this subsection, we study how competition within banks and within shadow banks may alter the results.

Competition within shadow banks

Let us first consider the results with one bank and two shadow banks. For simplicity, we assume the two shadow banks observe the same signals from one borrower. Once informed, the two shadow banks engage in essentially Bertrand competition, which drives their expected profits to zero. That is, the shadow banks will offer interest rates $\underline{R}_S = \frac{r_E}{P(\mu_g)}$ upon observing a good signal but retreat from lending upon observing a bad signal. The bank, as before, would like to offer an interest rate that is at least $\underline{R}_B$.

The equilibrium turns out straightforward. As long as $\underline{R}_B \leq \underline{R}_S$, the bank lends at an interest rate $\underline{R}_S$ and keeps earning positive rents due to its lower funding cost. If $\underline{R}_B > \underline{R}_S$, the two shadow banks lend and offer an interest rate $\underline{R}_S$ while the bank retreats from direct lending to the interbank market. From here, we can see that the two shadow banks pose an effective threat to bank in the direct lending market only when $\underline{R}_B > \underline{R}_S$, just as the benchmark model. Moreover, compared to the one shadow bank case, total bank profits remain unchanged. However, competition between shadow banks reduces the their profits when shadow banks' informational advantage dominates their funding disadvantage, i.e., when $\underline{R}_B < \underline{R}_S$, which translates into the welfare gains of high-type borrowers.

The result that shadow banks earn zero profits is a bit special in that it depends on the assumption that the two shadow banks observe identical signals from a borrower. If alternatively the signals are correlated but not identical, they will earn some positive amount of profits. The higher the correlation, the lower the profits are, and therefore the higher gains that high-type borrowers will receive. The broader message here is that, competition between shadow banks reduces the rents associated with the shadow bank's information advantage but not the ones from the bank's funding advantage.

Competition within banks

Next, we consider the model that the lending market was originally populated with two banks competing with each other and one shadow bank entering. Given so, the shadow bank naturally has the bargaining power in the interbank funding market, so that $r_E = \lambda r_B + (1-\lambda) r_S$ and $\Pi_B = 0$. In other words, the two banks compete in offering funding to the shadow bank and end up earning zero profits from interbank lending.

The rest of the model can be solved as the one in the benchmark under the assumption $\alpha = 1$. In equilibrium, the bank never makes any profit, whereas the shadow bank earns some positive amount of profits as long as $\mu < \bar{\mu}$. Compared to the benchmark model with a single bank, competition between the two banks eliminates the bank's rents associated with funding advantage. However, competition between banks increases the profits associated with the shadow bank's informational advantage by reducing the interbank borrowing rate.

5 Conclusion

Motivated by the rise of shadow banks in the financial industry, we develop a theory that features competition and collaboration between shadow banks and the traditional banks. Our theory highlights the role of interbank lending, which allows shadow banks to borrow from banks. We show the two lenders collaborate when the average quality of the borrower pool is low but compete when the quality is high. The interbank funding increases the shadow banks' competitiveness in direct lending market and reduces the banks' incentives to compete. As a result, borrowers can only benefit if the average quality of the borrower pool is already sufficiently high. Otherwise, they can be worse-off. By contrast, the bank earns more profits if and only if the average quality of the borrower pool is sufficiently low. The presence of interbank lending shift shadow bank preferences towards technologies that make more type-II errors relative to type-I errors. Screening technology with type-II errors, increases the total volume of shadow bank and consequently interbank lending. Higher interbank profits discourage bank from competition to the benefit of the shadow bank.

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A Appendix

A.1 Proofs

Proof of Lemma 1

Proof. We prove by contradiction. There are two cases: $\hat{R}_B \in (\underline{R}_S, R)$ and $\hat{R}_B = R$.

Suppose the bank bids some $\hat{R}_B \in (\underline{R}_S, R)$ with a probability mass Δ_B, then for any $\varepsilon > 0$, the shadow bank must bid with a strictly positive probability on $(\hat{R}_B, \hat{R}_B + \varepsilon]$. If instead there exists an ε^* such that $F_B(\hat{R}_B + \varepsilon^*) - F_B(\hat{R}_B) = 0$, then the bank earns strictly higher profits by bidding $\hat{R}_B + \frac{\varepsilon^*}{2}$ compared to bidding $\hat{R}_B$, a contradiction. By bidding $\hat{R}_B + \varepsilon$, a shadow bank with a good signal receives profits $\Big(1 - F_B(\hat{R}_B + \varepsilon)\Big)(\hat{R}_B + \varepsilon) - r_E$. By bidding $\hat{R}_B - \varepsilon$, a shadow bank with a good signal receives profits $\Big(1 - F_B(\hat{R}_B - \varepsilon)\Big)(\hat{R}_B - \varepsilon) - r_E$. The difference between the two $\Big(1 - F_B(\hat{R}_B - \varepsilon)\Big)(\hat{R}_B - \varepsilon) - \Big(1 - F_B(\hat{R}_B + \varepsilon)\Big)(\hat{R}_B + \varepsilon) \to \Delta_B \hat{R}_B > 0$ as $\varepsilon \downarrow 0$, a deviation.

Suppose the bank bids $\hat{R}_B = R$ with a probability mass Δ_B. Then bidding $R - \varepsilon$ strictly dominates R for the shadow bank. Therefore, the shadow bank never bids R in equilibrium. By bidding R, the bank only lends to a borrower with a bad signal and makes a loss, a contradiction. □

Proof of Lemma 2

Proof. We prove by contradiction. Let (R_1, R_2) be the interval where the right boundary R_2 is closest to R.[8] This implies the bank is indifferent between any bid $\hat{R}_B \in (R_1, R_2)$. Then the shadow bank must also bid on the same interval (R_1, R_2) without any gaps $(\hat{R}_1, \hat{R}_2)$. Otherwise, bidding $\hat{R}_2$ strictly dominates $\hat{R}_1$ for the bank.

Next, it must be that $R_2 = R$. If not, bidding R is dominated by bidding R_2 from the perspective of the shadow bank.

Finally, we show that $R_2 = R$ leads to a contradiction. Depending on whether the shadow bank bids R with a probability mass or not, there are two cases. If the shadow bank does not bid R with a probability mass, then by bidding R the bank almost surely lends to a borrower with a bad signal, which results in a loss - a contradiction. If the shadow bank bids R with a probability mass, by bidding R, it can only win if the bank does not bid at all with a positive probability (since the ties are broken in favor of the bank, and bank bid $R_B \leq R$ would win against shadow bank's $R_S = R$). The fact that not bidding is part of the bank's equilibrium strategy is a contradiction because not bidding and getting zero expected profits is strictly dominated by bidding $\underline{R}_S$ and receiving positive expected profits. Hence, by bidding R the shadow bank never wins and receives zero profits. Since the shadow bank is indifferent between any bids it makes it must be that shadow bank makes zero profits for any bid $R_S \in (R_1, R)$. This is possible only if the shadow bank never wins for any of those bids which is impossible when the bank is also bidding in (R_1, R) without mass points. □

[8] Since we have ruled out mass probabilities, the same result holds if the interval is half-open or open.

Proof of Lemma 3

Proof. The proof should be same as Lemma 1. □

Proof of Lemma 4

Proof. We follow similar steps as the proof in Lemma 2 to show that there are no holes, and the right boundary of the interval must be R. Therefore, when both institutions randomize their bids, the interval is $[R_1, R]$ for $R_1 \geq \underline{R}_B$ since the bank must make non-negative profits. When both the bank and the shadow bank randomize their bids in $[R_1, R_2]$, it must be that $R_2 = R$. Otherwise, offering R_2 is strictly dominated by bidding R and the shadow bank would prefer to bid R instead of mixing in $[R_1, R_2]$. Randomizing by both parties in $[R_1, R]$ also implies that the shadow bank must have a positive mass of bids at R. Absent such a mass, when the bank's bid gets close to R, almost surely it can only win when the shadow bank receives a bad signal, which results in a loss.

To pin down R_1 we exploit the indifference condition of the shadow bank. On the one hand, the shadow bank can bid R_1 and win the bidding game almost for sure, generating expected profits $P(\mu_g)R_1 - r_E > 0$. On the other hand, the shadow bank can bid R and win whenever the bank bids below R, generating expected profits $(1 - F_B(R)) \cdot (P(\mu_g)R - r_E)$. Indifference between the two options implies that $F_B(R) < 1$, i.e., that the bank does not bid at all with a positive probability. This can only happen when the bank is indifferent between winning and losing for every bid it submits. As a result, $R_1 = \underline{R}_B$. □

Proof of Corollary 1

Proof. It is easily derived that

$$\frac{d\bar{\mu}}{dr_S} = \frac{-(1-p)(1-\lambda)r_B}{\{\lambda(1-\alpha p)r_B + [(1-p) - \lambda(1-\alpha p)]r_S\}^2} < 0$$

$$\frac{d\bar{\mu}}{dr_B} = \frac{(1-p)(1-\lambda)r_S}{\{\lambda(1-\alpha p)r_B + [(1-p) - \lambda(1-\alpha p)]r_S\}^2} > 0$$

$$\frac{d\underline{\mu}}{dr_B} = \frac{(1-p)R - \lambda(1-\alpha)(r_S - pR)}{[(1-p)R - \lambda(1-\alpha)(r_S - r_B)]^2}.$$

To show the last term is positive, it suffices to show that

$$(1-p)R > (r_S - pR) \Longleftrightarrow R > r_S.$$

Finally, $\frac{d\underline{\mu}}{dr_S} > 0$ is obvious. □

Proof of Corollary 1

Proof. We can show

$$\frac{d\bar{\mu}}{d\lambda} = \frac{(r_S - r_B)[(1-p\alpha)r_B - p(1-\alpha)r_S]}{\{\lambda(1-\alpha p)r_B + [(1-p) - \lambda(1-\alpha p)]r_S\}^2}$$
$$\frac{d\bar{\mu}}{d\alpha} = \frac{p(1-\lambda)\lambda * (r_B - r_S)^2}{\{\lambda(1-\alpha p)r_B + [(1-p) - \lambda(1-\alpha p)]r_S\}^2} > 0.$$

Meanwhile, we know

$$(1-p\alpha)r_B - p(1-\alpha)r_S > \left(\frac{(1-p\alpha)p(1-\lambda\alpha)}{1-\lambda\alpha p} - p(1-\alpha)\right) r_S$$
$$= \frac{(1-p)p\alpha(1-\lambda)}{1-\lambda\alpha p} r_S > 0,$$

so that $\frac{d\bar{\mu}}{d\lambda} > 0$. The inequality follows from $\bar{\mu} > 0$. Finally, $\frac{d\underline{\mu}}{d\lambda} > 0$ and $\frac{d\underline{\mu}}{d\alpha} < 0$ are fairly obvious. □

Proof of Proposition 2

Proof. Start with total bank profits:

$$V_B + V_i = \begin{cases} \mu\Pi_B & \text{for } \mu \leq \bar{\mu} \\ P(\mu)r_E - r_B & \text{for } \mu \geq \bar{\mu} \end{cases} \tag{21}$$

where we used that for $\mu < \underline{\mu}$ we have $V_B = 0$ and $V_i = \mu\Pi_B$. For $\mu \in (\underline{\mu}, \bar{\mu})$ the bank is indifferent between losing and winning the bidding competition, hence the total profits are equal to profits when the bank always loses, i.e. $\mu\Pi_B$. Finally, for $\mu > \bar{\mu}$ the bank always bids $\underline{R}_S = r_E$ and wins. Clearly, $V_B(\mu) + V_i(\mu)$ is continuous and increasing.

Next, turn to shadow bank profits:

$$V_S = \begin{cases} \mu(R - r_E) & \text{for } \mu \leq \underline{\mu} \\ \mu(\underline{R}_B - r_E) & \text{for } \mu \in (\underline{\mu}, \bar{\mu}) \\ 0 & \text{for } \mu \geq \bar{\mu} \end{cases} \tag{22}$$

Clearly, V_S is positive and increasing for $0 < \mu < \underline{\mu}$. And it equal to 0 for $\mu > \bar{\mu}$. So, overall it is non-monotone.

In $\mu \in (\underline{\mu}, \bar{\mu})$ the shadow bank's profits are

$$\begin{aligned} V_S &= \mu(\underline{R}_B - r_E) \\ &= \mu\left(\frac{r_B + \mu\Pi_B}{p + \mu(1-p)} - r_E\right) \\ &= \mu\frac{r_B + \mu\Pi_B - (p + \mu(1-p)r_E}{p + \mu(1-p)} \\ &= \mu\frac{r_B - pr_E - \mu((1-p)r_E - \Pi_B)}{p + \mu(1-p)} \\ &\sim a \cdot \mu \cdot \frac{b - \mu}{\mu + c} \end{aligned}$$

At $\mu = \bar{\mu}$ the shadow bank profit should be decreasing in μ, since $V_S(\bar{\mu}) = 0$ and $V_S(\mu) > 0$ for $\mu < \bar{\mu}$. Given the shape of V_S as a function of μ (linear minus a $1/\mu$ term) the shadow bank profits can either be (a) decreasing everywhere in $(\underline{\mu}, \bar{\mu})$ or (b) be hump-shaped, i.e., increasing in $(\underline{\mu}, \mu^*)$ and decreasing in $(\mu^*, \bar{\mu})$.

Next we consider the borrower surplus. To prove that it is increasing in μ we will rely on first order dominance of the cdfs F_B and F_S.

$$F_B(x) = \frac{x - \underline{R}_B}{x - \underline{R}_S} = \frac{x - \underline{R}_B(\mu)}{x - r_E}$$

Since $\underline{R}_B(\mu)$ is decreasing in μ, we have $F_B(x)$ is increasing in μ. Hence F_B at μ' dominates F_B at $\mu < \mu'$ in the FOSD sense.

Similarly,

$$\begin{aligned} F_S(x) &= \frac{P(\mu)x - r_B - \mu\Pi_B}{\mu(x - r_B - \Pi_B)} \\ 1 - F_S(x) &= \frac{1-\mu}{\mu} \cdot \frac{r_B - px}{x - r_B - \Pi_B}. \end{aligned}$$

Since $(1-\mu)/\mu = 1/\mu - 1$ is decreasing in μ, $F_S(x)$ is increasing in μ. Hence F_S at μ' dominates F_S at $\mu < \mu'$ in the FOSD sense.

Since both cdfs increase in the FOSD with μ, both types of borrowers prefer lower rates to higher, their surplus is increasing in μ.

□

Proof of Proposition 3.

Proof. First, establish the cutoffs of different lending regions. Define $\underline{\mu}_\lambda$ as a solution to

$$\underline{R}_B(\mu, \lambda) = R \tag{23}$$

and $\bar{\mu}_\lambda$ as a solution to

$$\underline{R}_B(\mu,\lambda) = \underline{R}_S(\mu,\lambda). \tag{24}$$

Then

$$\underline{\mu}_0 = \frac{r_B/R - p}{1-p} \qquad \bar{\mu}_0 = \frac{r_B/r_S - p}{1-p} \qquad \underline{\mu}_1 = \frac{r_B/R - p}{1 - p - (1-\alpha)(r_S - r_B)/R} \qquad \bar{\mu}_1 = 1 \tag{25}$$

1. Start with borrowers.

   Two cases are possible. If at $\bar{\mu}_0 < \underline{\mu}_1$ then only the shadow bank participates and bids R in which case the both types of borrowers get a zero payoff. Clearly, they are better at $\mu = \bar{\mu}_0$ without the interbank market.

   If $\bar{\mu}_0 > \underline{\mu}_1$ then both parties bid with CDFs F_B and F_S at $\mu = \bar{\mu}_0$ with $\lambda = 1$ and the high type's payoff is

$$V_H = R - \mathbf{E}[\min(\tilde{R}_B, \tilde{R}_S)]. \tag{26}$$

   First, let's simplify the expected minimal bid:

$$\begin{aligned}
&\mathbf{E}[\min(\tilde{R}_S, \tilde{R}_S)] = \int_{\underline{R}_B}^{R} x d[1 - (1 - F_S(x))(1 - F_B(x))] \\
&= -\int_{\underline{R}_B}^{R} x d[(1 - F_S(x))(1 - F_B(x))] \\
&= R(F_S(R) - F_S(R-))(1 - F_B(R-)) - \int_{\underline{R}_B}^{R-} x d[(1 - F_S(x))(1 - F_B(x))] \\
&= R(F_S(R) - F_S(R-))(1 - F_B(R-)) - x[(1 - F_S(x))(1 - F_B(x))]\Big|_{\underline{R}_B}^{R-} \\
&+ \int_{\underline{R}_B}^{R-} [(1 - F_S(x))(1 - F_B(x))]dx \\
&= \underbrace{R(F_S(R) - F_S(R-))(1 - F_B(R-)) - R[(1 - F_S(R-))(1 - F_B(R-))]}_{=0} + \underline{R}_B \\
&+ \int_{\underline{R}_B}^{R-} [(1 - F_S(x))(1 - F_B(x))]dx \\
&= \underline{R}_B + \int_{\underline{R}_B}^{R-} [(1 - F_S(x))(1 - F_B(x))]dx.
\end{aligned}$$

   The high-type borrower in a pool characterized by $\bar{\mu}_0$ is better off without the interbank market if

$$\begin{aligned}
r_S &< \underline{R}_B + \int_{\underline{R}_B}^{R-} [(1 - F_S(x))(1 - F_B(x))]dx \\
r_S - \underline{R}_B &< \int_{\underline{R}_B}^{R-} [(1 - F_S(x))(1 - F_B(x))]dx.
\end{aligned}$$

We evaluate the LHS at $\bar{\mu}_0$:

$$r_S - \underline{R}_B = r_S - \frac{r_B + \bar{\mu}_0 \Pi_B}{r_B/r_S} = -\frac{\bar{\mu}_0 \Pi_B}{r_B/r_S} < 0.$$

For the RHS, we know it is positive, so the inequality always holds at $\bar{\mu}_0$.

For the low-type borrower, the payoff with interbank market is

$$\begin{aligned} V_L(\lambda = 1) &= (1 - F_B(R)) \cdot p(R - \mathbf{E}[\tilde{R}_B \,|\tilde{R}_B < \infty]) \\ &< p(R - \mathbf{E}[\tilde{R}_B \,|\tilde{R}_B < \infty]) \\ &< p(R - \mathbf{E}[\min(\tilde{R}_S, \tilde{R}_B)]) \\ &< p(R - r_S) \\ &= V_L(\lambda = 0). \end{aligned}$$

We have established that when $V_\theta(\lambda = 1) < V_\theta(\lambda = 0)$ at $\mu = \bar{\mu}_0$. For $\mu > \bar{\mu}_0$ borrower's surplus $V_\theta(\lambda = 0)$ is constant, while $V_\theta(\lambda = 1)$ is strictly increasing.

When $\mu = 1$ the bank always bids r_E and always wins. With $\lambda = 1$ we have $r_E = r_B$ and with $\lambda = 0$ we have $r_E = r_S$, hence $V_\theta(\lambda = 1) > V_\theta(\lambda = 0)$. Hence, there exists $\mu^*_\theta > \bar{\mu}_0$ such that $V_\theta(\lambda = 1, \mu^*_\theta) = V_\theta(\lambda = 0, \mu^*_\theta)$. Borrowers are better off with interbank market for $\mu > \mu^*_\theta$ and better off without interbank market for $\mu < \mu^*_\theta$

For $\underline{\mu}_1 < \mu < \bar{\mu}_0$ we have non-trivial bidding by both bank and shadow bank regardless of λ. Hence the payoff of the low type player is determined via the expected minimal bid

$$\mathbf{E}[\min(\tilde{R}_S, \tilde{R}_S)] = \underline{R}_B + \int_{\underline{R}_B}^{R-} [(1 - F_S(x))(1 - F_B(x))]dx. \tag{27}$$

To see how the expected minimal bid varies with λ notice that we need to take only the derivative inside of the integral since the derivative w.r.t. $\underline{R}_B$ in the above expression cancels out.

Recall that

$$F_B(x) = \frac{x - \underline{R}_B}{x - r_E}.$$

Since $\underline{R}_B$ is increasing in λ and r_E is decreasing in λ, the cdf $F_B(x)$ is decreasing in λ. As a result, $1 - F_B(x)$ term is increasing in λ.

Similarly

$$\frac{\partial}{\partial \mu} F_S(x) \sim \frac{\partial}{\partial \Pi_B} F_S(x) \sim \frac{px - r_B}{()^2} < 0.$$

As a result, the term $1 - F_S(x)$ is increasing in μ.

Hence, the whole integral above is increasing in μ (it is a product of two non-negative increasing terms). Since the expected minimum bid is increasing in μ the payoff of the high type is decreasing in μ.

For the low type the welfare comparison comes from the fact that $F_B(x)$ id decreasing in λ. Hence $F_B(x)$ for $\lambda = 0$ dominates in the FOSD sense $F_B(x)$ for $\lambda = 1$. Since the low type prefers lower bids, it prefers the cdf F_B at $\lambda = 0$.

2. Next, consider the shadow bank. For $\lambda = 1$ the shadow bank profits are given by

$$V_S = \begin{cases} \mu(R - r_E) & \text{for } \mu \leq \underline{\mu}_1 \\ \mu(\underline{R}_B - r_E) & \text{for } \mu \geq \underline{\mu}_1 \end{cases} \tag{28}$$

with $r_E = \alpha r_B + (1-\alpha) r_S$ and $\underline{R}_B = \frac{r_B + \mu(1-\alpha)(r_S - r_B)}{P(\mu)}$. Note that $V_S = 0$ at $\mu = 0$ and at $\mu = 1$ and reaches it's maximum at $\mu = \underline{\mu}_1$.

For $\lambda = 0$ the shadow bank profits are given by

$$V_S = \begin{cases} \mu(R - r_S) & \text{for } \mu \leq \underline{\mu}_0 \\ \mu\left(\frac{r_B}{P(\mu)} - r_S\right) & \text{for } \mu \in (\underline{\mu}_0, \bar{\mu}_0) \\ 0 & \text{for } \mu \geq \bar{\mu}_0 \end{cases} \tag{29}$$

Note that $V_S = 0$ at $\mu = \bar{\mu}_0$ and reaches it's maximum at $\mu = \underline{\mu}_0$.

Comparison of V_S with and without the interbank market is obvious: for $\lambda = 1$ V_S starts with a higher slope, at $\mu = 0$, reaches its peak later (at $\underline{\mu}_1 > \underline{\mu}_0$) and stays positive for longer. Moreover, for $\mu \in [\underline{\mu}_1, \bar{\mu}_0]$, we have

$$V_S(\lambda = 1) - V_S(\lambda = 0) = \mu(\underline{R}_B - r_E) - \mu\left(\frac{r_B}{P(\mu)} - r_S\right).$$

To show this is positive, we need

$$\begin{aligned} \underline{R}_B - r_E &> \frac{r_B}{P(\mu)} - r_S \\ \frac{r_B + \mu(1-\alpha)(r_S - r_B)}{P(\mu)} - r_E &> \frac{r_B}{P(\mu)} - r_S \\ r_B + \mu(1-\alpha)(r_S - r_B) - P(\mu) r_E &> r_B - P(\mu) r_S \\ \mu(1-\alpha)(r_S - r_B) &> P(\mu)\left(r_E - r_S\right). \end{aligned}$$

The last inequality holds because the LHS is positive whereas the RHS is negative. Hence, it dominates V_S for $\lambda = 0$ everywhere.

3. Next, consider the bank. When $\lambda = 1$, then the total bank profits are

$$V_B + V_i = \mu \Pi_B = \mu(1-\alpha)(r_S - r_B). \tag{30}$$

For $\mu < \underline{\mu}_1$ this is correct since $V_B = 0$ and for $\mu > \underline{\mu}_1$ this is correct since the bank is indifferent between winning and losing the bidding game.

When $\lambda = 0$, then the total bank profits are

$$V_B + V_i = \begin{cases} 0 & \text{for } \mu \leq \bar{\mu}_0 \\ P(\mu) r_S - r_B & \text{for } \mu \geq \bar{\mu}_0 \end{cases} \tag{31}$$

For $\mu < \underline{\mu}_0$ this is correct since $V_B = V_i = 0$ and for $\mu \in (\underline{\mu}_0, \bar{\mu}_0)$ this is correct since the bank is indifferent between winning and losing the bidding game. Finally, for $\mu > \bar{\mu}_0$ the bank simply bids r_S and always wins.

To compare the total profits of the bank, we only need to check that at $\mu = 1$ the bank is better off without the interbank markets. This is true, since $r_S - r_B > (1 - \alpha)(r_S - r_B)$. Hence, there exists a $\mu_B^* \in (\bar{\mu}_0, 1)$ such that

$$\mu_B^*(1 - \alpha)(r_S - r_B) = P(\mu_B^*) r_S - r_B \tag{32}$$

and the bank is better off without interbank market for $\mu > \mu_B^*$ and is better off with interbank market for $\mu < \mu_B^*$.

4. Finally, consider overall welfare.

   Welfare is clearly higher with $\lambda = 1$ vs. $\lambda = 0$. With $\lambda = 1$ the high type projects are always funded at a cost r_B, hence the funding inefficiency does not exist, while it strictly positive for $\lambda = 0$.

   Similarly, lending inefficiency with $\lambda = 1$ is also smaller, since the probability that the bank bids $F_B(x)$ is decreasing in λ.

□

Proof of Proposition 4

Proof. Note that in the case of interbank financing under $\lambda = 1$, we have $\bar{\mu}_1$. Starting with the shadow bank. For $\mu < \underline{\mu}_0$ we have

$$V_S^0 = \mu(R - r_S) < V_S^{NC} = V_S^1 = \mu(R - r_E).$$

For $\mu \in (\underline{\mu}_0, \underline{\mu}_1)$ we have

$$V_S^0 = \mu \left(\frac{r_B}{P(\mu)} - r_S \right) < V_S^{NC} = \mu \left(\frac{r_B}{P(\mu)} - r_E \right) < V_S^1 = \mu(R - r_E).$$

For $\mu \in (\underline{\mu}_1, \bar{\mu}_0)$ we have

$$V_S^0 = \mu \left(\frac{r_B}{P(\mu)} - r_S \right) < V_S^{NC} = \mu \left(\frac{r_B}{P(\mu)} - r_E \right) < V_S^1 = \mu \left(\frac{r_B + \mu \Pi_B}{P(\mu)} - r_E \right).$$

For $\mu \in (\bar{\mu_0}, \bar{\mu}^{NC})$ we have

$$V_S^0 = 0 < V_S^{NC} = \mu \left(\frac{r_B}{P(\mu)} - r_E \right) < V_S^1 = \mu \left(\frac{r_B + \mu \Pi_B}{P(\mu)} - r_E \right).$$

For $\mu \in (\bar{\mu}^{NC}, \bar{\mu}_1)$ we have

$$V_S^0 = V_S^{NC} = 0 < V_S^1 = \mu \left(\frac{r_B + \mu \Pi_B}{P(\mu)} - r_E \right).$$

Next, consider the bank. With third party financing the total bank profits are given by

$$V_B^{NC} + V_i^{NC} = V_B^{NC} = \begin{cases} 0, & \text{if } \mu < \bar{\mu}^{NC} \\ P(\mu) r_E - r_B, & \text{if } \mu > \bar{\mu}^{NC} \end{cases}$$

Since $\bar{\mu}^0 < \bar{\mu}^{NC}$ and $r_E < r_S$ we have $V_B^{NC} \leq V_B^0$ for all μ. Moreover, since at $\mu = 1$ $V_B^{NC} = r_E - r_B = (1-\alpha)(r_S - r_B) = \Pi_B = V_B^1 + V_i^1$ we have $V_B^{NC} < V_B^1 + V_i^1$ for all $\mu < 1$.

Finally, consider the borrowers who face the bid distributions

$$F_S^0(x) = \frac{P(\mu)x - r_B}{\mu(x - r_B)} \qquad F_B^0(x) = \frac{x - r_B/P(\mu)}{x - r_S}$$
$$F_S^{NC}(x) = \frac{P(\mu)x - r_B}{\mu(x - r_B)} \qquad F_B^{NC}(x) = \frac{x - r_B/P(\mu)}{x - r_E}$$
$$F_S^1(x) = \frac{P(\mu)x - r_B - \mu\Pi_B}{\mu(x - r_B - \Pi_B)} \qquad F_B^1(x) = \frac{x - (r_B + \mu\Pi_B)/P(\mu)}{x - r_E}$$

Notice that moving from no interbank market to third party lending keeps F_S intact and increases F_B in the FOSD sense. Hence, the borrowers benefit from lower bids by the shadow bank. Moving from third party lending to the case with interbank lending reduces both F_S and F_B in the FOSD sense. Hence the borrowers suffer from higher bids by both the shadow bank and the bank.

□

Detailed Analysis of Subsection 4.2

Bank has the convenience benefits. The bank wins the bidding game whenever $R_B < R_S + \Delta$. The equilibrium has the following cases.

- Case 1a: $\underline{R}_B < \underline{R}_S + \Delta < R$. The bank can always undercut the shadow bank, hence in equilibrium it always wins with a bid $\underline{R}_S + \Delta$ and the shadow bank plays a mixed strategy to deter bank's deviations.
- Case 1b: $\underline{R}_B < R < \underline{R}_S + \Delta$. In this case the bank enjoys the monopoly and bids R while the shadow bank does not participate.
- Case 2: $\underline{R}_S + \Delta < \underline{R}_B < R$. In this case the shadow bank can always undercut the bank, and hence the bank earns zero profits in equilibrium. The bank bids in $(\underline{R}_B, R] \cup \{\infty\}$ and the shadow bank

bids in $(\underline{R}_B - \Delta, R - \Delta)$ with a mass point at R. Note that shadow bank does not want to bid in $(R - \Delta, R)$ since such bid only wins when the bank does not bid. Hence, bidding in $(R - \Delta, R)$ is dominated by bidding R.

The shadow bank's bidding strategy is pinned down similar to Section 3.1:

$$\mu \Pi_B = (1-\mu)\left(p\tilde{R} - r_B\right) + \mu\left[F_S(\tilde{R} - \Delta)\Pi_B + \left(1 - F_S(\tilde{R} - \Delta)\right)\left(\tilde{R} - r_B\right)\right],$$

For the bank we have for $\tilde{R} \in (\underline{R}_B - \Delta, R - \Delta)$

$$\underline{R}_B - \Delta - r_E = \left(1 - F_B(\tilde{R} + \Delta)\right)(\tilde{R} - r_E) \tag{33}$$

Shadow bank's bid $\tilde{R} = R - \Delta$ wins only when the bank bids R or ∞, hence

$$\underline{R}_B - \Delta - r_E = (1 - F_B(R-))(R - \Delta - r_E) \tag{34}$$

Shadow bank's bid $\tilde{R} = R$ wins only when the bank bids ∞, hence

$$\underline{R}_B - \Delta - r_E = (1 - F_B(R+))(R - r_E) \tag{35}$$

As a result, the bank needs to have a mass point bid at R and another mass point at $\{\infty\}$. This is always possible since $\underline{R}_B - \Delta < R - \Delta < R$ and hence, $F_B(R+) < 1$.

- Case 3: $R < \underline{R}_B$. In this case the bank never bids and since $\underline{R}_S < R$ the shadow bank is a monopolist with a bid R.

Shadow bank has the convenience benefits. The bank wins the bidding game whenever $R_B < R_S - \Delta$. The equilibrium has the following cases.

- Case 1: $\underline{R}_B < \underline{R}_S - \Delta$. In this case the bank can always undercut the shadow bank, hence in equilibrium it always wins with a bid $\underline{R}_S - \Delta$ and the shadow bank plays a mixed strategy to deter bank's deviations.
- Case 2: $\underline{R}_S - \Delta < \underline{R}_B < R$. In this case the shadow bank can always undercut the bank, and hence the bank earns zero profits in equilibrium. The bank bids in $(\underline{R}_B, R - \Delta) \cup \{\infty\}$ and the shadow bank bids in $(\underline{R}_B + \Delta, R]$ with a mass point at R. Note that bank does not want to bid anywhere in $(R - \Delta, R]$ since such bid would always lose against a shadow bank, and hence would attract only low types (which generates negative profits).
- Case 3: $R < \underline{R}_B$. In this case the bank never bids and since $\underline{R}_S < R$ the shadow bank is a monopolist with a bid R.