An Accelerated Static Hedging Method for the Analytic Valuation of American Options

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Current version: December, 2022

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Abstract

This article proposes an accelerated static hedge portfolio (SHP) method for evaluating American options based on stochastic volatility and double jump processes. Our proposed model is a generalization of the static hedge portfolio approach of Derman, Ergener, and Kani to evaluate American options by utilizing the Richardson extrapolation. Numerical results demonstrate that the numerical efficiency of our accelerated static hedge portfolio approach is comparable to the least-squares Monte Carlo simulation method. Numerical results reveal that our proposed method is efficient and accuracy in evaluating American options with stochastic volatility and double jump processes.

JEL Classification: G13

Keywords：American option; Stochastic volatility; Jumps; Static hedging

I. Introduction

Pricing American options is an important problem in the financial research. In the past three decades, many studies have provided analytical approximation formulae and numerical solution methods to evaluate American options. For example, recent numerical approaches include the Monte Carlo simulation methods of Broadie and Glasserman (1997) and Longstaff and Schwartz (2001) and the Gaussian quadrature integration scheme of Sullivan (2000). Most analytical approximation formulae can price American options efficiently under Black-Scholes framework. However, the assumption of a lognormal diffusion process is not consistent with empirical characteristics of the underlying asset. Since some closed-form solutions for European option prices based on the general diffusion processes have been derived in recent years, analytical approximation formulae for pricing American option should be developed with greater generality.

Analytical approximations are well documented in the option pricing literature. One stream of the American option pricing literature is to utilize the quadratic approximation scheme for pricing American option values, see e.g., Barone-Adesi and Whaley (1987), Ju and Zhong (1999), Chang, Kang, Kim, and Kim (2007), and Guo, Hung, and So (2009). While the above mentioned methods can price American options efficiently under the Black-Scholes model, it should be to extend most of them to other stochastic processes, e.g. the stochastic volatility models with double jump of Duffie, Pan, and Singleton (2000), Guo, Hung, and So (2009), and Chang, Guo, and Hung (2016).

In contrast, the static hedge portfolio (SHP) method proposed in this article is not only efficient but also applicable for more general processes beyond the Black-Scholes model. Moreover, the hedge problem can be solved at the same time when the static hedge portfolio is found. Static hedge is the approach, developed by Bowie and Carr (1994), Derman, Ergener, and Kani (1995), Carr, Ellis, and Gupta (1998), for pricing and hedging options. This approach is to create a static portfolio of standard European options whose values match the payoff of the option been hedged at expiration and along the boundary. In comparison to dynamic hedging, static hedging is considerably cheaper than dynamic hedging when the transaction cost is large. Dynamic hedging may have substantial hedging errors due to discrete trading, see for example Primbs and Yamada (2006). In addition to the previous findings, Chung and Shih[1] (2009) show that the static hedging approach may also serve as a good pricing method for American options. The specific advantage of applying static hedge approach to pricing American options is that unlike the numerical methods, the computation of the American option price is as easy as the valuations of European options because there is no need to solve the static hedge portfolio again and the value of the static hedge portfolio is the summation of the European option prices in the portfolio.

Most of the earlier studies listed above are based on the assumption that the underlying asset follows a lognormal diffusion process. Nevertheless, the above mentioned methods have not developed in conjunction with the rapid growth of option pricing models in the stochastic volatility framework. Many empirical studies demonstrate that an option pricing model allowing for stochastic volatility and double jumps significantly reduces pricing errors, see e.g., Bakshi, Cao, and Chen (1997), and Broadie, Chernov, and Johannes (2007). Stochastic volatility and double jumps have been proposed as other factors that illustrate the biases in option pricing. These empirical studies show the importance of extending the static hedge method for

[1] Ruas, Dias, and Nunes (2013) evaluate American-style options through the static hedge approach proposed by Chung and Shih (2009) under the jump to default extended CEV (JDCEV) model of Carr and Linetsky (2006).

pricing American options based on these processes.

Our static hedge portfolios of American options utilize based on standard European options with multiple strikes and multiple maturities, which provided by Chung and Shih (2009). The reason for using standard options with multiple strikes and multiple maturities is because the early exercise boundary of the American option is time variant. Unlike the static hedge of exotic options where the boundary is usually known, the static hedge of the American option involves a free boundary problem. We solve this problem by using two well-known conditions on the early exercise boundary. It is worth noting that value-matching and smooth-pasting conditions are widely applied in American option pricing literature. For example, Barone-Adesi and Whaley (1987) approximate the early exercise premium of an American option with a quadratic function and they applied these two conditions to solve the early exercise premium and the critical stock price. At the maturity date, if the American option is not exercised earlier, its boundary condition is exactly the same as the corresponding European option. Therefore our static hedge portfolio starts with one unit of the corresponding European option.

Huang, Subrahmanyam, and Yu (1996) utilize an accelerated recursive method that employs a three-point Richardson extrapolation scheme[2] to price American options. Chang, Chung, and Stapleton (2007) demonstrate that the modified Geske-Johnson formula is a better approximation of the value of American-style option, especially for nonstandard American options whose exercise boundary is discontinuous. The reason is that the modified Geske–Johnson formula avoids the non-uniform convergence problem. Therefore, we employ the Richardson

[2] Geske and Johnson (1984) provide an analytical formula for American options with the concept of compound options and they make use of a three-point Richardson extrapolation method to evaluate American options. By utilizing different extrapolation, the Geske and Johnson method has been expanded in a series of articles by Bunch and Johnson (1992), Chung (2002), and Chang and Hung (2007).

extrapolation technique to evaluate American options, which can improve the computational accuracy for the static hedge method proposed by Chung and Shih (2009).

The goal of this study is to provide an accelerated static hedge method for evaluating American options in a general framework that allows for stochastic volatility, return jumps, and volatility jumps. Several stochastic volatility models with double jump have been widely introduced in the literature. Stochastic volatility models with double jumps proposed by Duffie, Pan, and Singleton (2000), Guo, Hung, and So (2009), and Chang, Guo, and Hung (2016) are used as examples to highlight the generalization of the static hedge portfolio method, followed by a comparison with the least-squares Monte Carlo simulation method proposed by Longstaff and Schwartz (2001). The numerical results of our comparison show that the accelerated static hedge method is accurate and efficient in pricing American options based on these diffusion processes. Our proposed model is also applied to other stochastic volatility models with jumps.

The remainder of this article is organized as follows: Section II briefly describes the stochastic volatility model with correlated double jumps and the static hedging method for other jump models. Section III shows the accelerated static hedging method for pricing American options. Section IV compares the accelerated static hedging method with the least-squares Monte Carlo simulation approach and the quadratic approximation method. Conclusions are presented in Section V.

II. Analytical American Option Approximations for the Stochastic Volatility Model with Double Jumps

Many previous studies have provided analytical solutions for pricing American options, indicating that the American option price can be evaluated as the corresponding European option plus a recursive integration term that represents the early exercise premium, see e.g., Bowie and Carr (1994), Derman, Ergener, and Kani (1995), Carr, Ellis, and Gupta (1998). The primary expression of the stochastic volatility model with double jumps follows the framework of Duffie, Pan, and Singleton (2000) and Chang, Guo, and Hung (2016). We show our approach on call options. Put options will be treated in a similar way except with different boundary conditions.

Under the risk-neutral measure, the underlying asset price, $S(t)$, is assumed to follow a geometric jump diffusion with the instantaneous conditional variance, $V(t)$, following a mean-reverting square root jump process:

$$\frac{dS(t)}{S(t)} = (r-q)dt + \sqrt{V(t)}dZ_S(t) + (e^{x(t)}-1)d\hat{q}_S(t) - E^Q\left[(e^{x(t)}-1)d\hat{q}_S(t)\right] \tag{1}$$

$$dV(t) = (\overline{V} - \kappa_V V(t))dt + \sigma_V \sqrt{V(t)}dZ_V(t) + v(t)d\hat{q}_v(t), \quad t \geq 0 \tag{2}$$

where the risk-free rate, r, and the continuous dividend yield rate, q, are assumed constant. $x(t)$ represents a percentage jump in the stock price and follows a normal distribution, $N(\mu_0 + \mu_{x,v}v, \sigma_{x,v}^2)$, where $v(t)$ is a level jump in the volatility and follows an exponential distribution, $Exponential(\theta_v)$. $\hat{q}_S(t)$ and $\hat{q}_V(t)$ are two correlated Poisson counters with intensity $\lambda_{x,v}$. $Z_S(t)$ and $Z_V(t)$ are standard Brownian motions with $\text{cov}(dZ_S(t), dZ_V(t)) = \rho dt$. In order to retain the Martingale property, the compensator, $E^Q\left[(e^{x(t)}-1)d\tilde{q}_S(t)\right] = \lambda_{x,v}\left(\exp[\mu_0 + 0.5\sigma_{x,v}^2]\big/\left(1-\theta_v\mu_{x,v}\right)-1\right)dt$, is subtracted from the stock price process, such that the drift of the stock return rate is equal to $r-q$.

The price space of American options can be separated into two regions: the exercise region, $\mathrm{D} \equiv \left\{ \mathrm{B}_t^* \le \mathrm{S}_t < \infty, t \in [0,\infty) \right\}$, and the continuous region $L \equiv \left\{ 0 < \mathrm{S}_t < \mathrm{B}_t^*, t \in [0,\infty) \right\}$. B_t^* [3] denotes the optimal exercise boundary above which the option should be exercised immediately. Within the continuous region, the partial integro-differential equation for a contingent claim price, C, on the underlying asset is given by:

$$
\begin{aligned}
0 = \frac{1}{2} C_{SS} S^2 \mathrm{V} + \frac{1}{2} C_{VV} \sigma_V^2 V + C_{SV} \rho \sigma_V S V \\
+ C_S ((r-q) - \lambda^{x,v} E^Q \left[e^x - 1 \right]) S + C_V (\overline{V} - \kappa_V V) - C_T - rC \\
+ \lambda^{x,v} \int_0^\infty \int_{-\infty}^\infty \left[C(Se^x, V+v) - C(S,V) \right] \Phi(x,v) dx dv .
\end{aligned}
\tag{3}
$$

with boundary conditions:

$$
\begin{aligned}
& C(S,V,T;K) = E^Q \left[e^{-rT} \max \left\{ S(T) - K, 0 \right\} \right] \\
& \lim_{S \downarrow 0} C(S,V,t;K) = 0 \\
& \lim_{S \downarrow \mathrm{B}_t^*} C(S,V,t;K) = \mathrm{B}_t^* - K \\
& \lim_{S \downarrow \mathrm{B}_t^*} \frac{\partial C(t)}{\partial \mathrm{S(t)}} = 1
\end{aligned}
\tag{4}
$$

The present value of the European call option can be computed by:[4]

$$
\begin{aligned}
C^E(S,V,T;K) &= E^Q \left[e^{-rT} \max \left\{ S(T) - K, 0 \right\} \right] \\
&= \frac{1}{2} \psi(T;-i) - \frac{1}{\pi} \int_0^\infty \frac{\mathrm{Im} \left[\psi(T;-i-\upsilon) e^{i\upsilon \log[K]} \right]}{\upsilon} d\upsilon \\
&\quad - K \left(\frac{1}{2} \psi(T;0) - \frac{1}{\pi} \int_0^\infty \frac{\mathrm{Im} \left[\psi(T;-\upsilon) e^{i\upsilon \log[K]} \right]}{\upsilon} d\upsilon \right),
\end{aligned}
\tag{5}
$$

where $\psi(T;\varphi)$ is the characteristic function of the state density. The characteristic

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[3] In this case, B_t^* is not only a free boundary, but also a random process itself.

[4] Based on the closed-form solution for the European call option, the formula for the present value of the European put option can be obtained by the put-to-call parity.

function is given by

$$\psi(T;\varphi) = e^{[M(T;\varphi)+N(T;\varphi)Y]}\left(S\right)^{i\varphi}, \tag{6}$$

where

$$\begin{aligned} M(T;\varphi) \equiv & \left(i\varphi(r-q)-r\right)T \\ & -\frac{\overline{V}}{\sigma_V^2}\left[\left(\varepsilon+i\varphi\sigma_V\rho-\kappa_V\right)T+2\log\left[1-\frac{\left(\varepsilon+i\varphi\sigma_V\rho-\kappa_V\right)\left(1-e^{-\varepsilon T}\right)}{2\varepsilon}\right]\right] \\ & -i\varphi\lambda^{x,v}\left(\frac{e^{\mu_0+\frac{1}{2}\sigma_{x,v}^2}}{1-\theta_v\mu_{x,v}}-1\right)T-\lambda^{x,v}T \\ & +\frac{\lambda^{x,v}\left(2\varepsilon-b\right)e^{\left(i\varphi\mu_0-\frac{1}{2}\varphi^2\sigma_{x,v}^2\right)T}}{\tilde{p}} \\ & +\frac{2\lambda^{x,v}\theta_v i\varphi\left(i\varphi-1\right)e^{i\varphi\mu_0-\frac{1}{2}\varphi^2\sigma_{x,v}^2}}{\tilde{p}\tilde{q}}\log\left(\frac{\tilde{p}+\tilde{q}e^{-\varepsilon T}}{\tilde{p}+\tilde{q}}\right), \end{aligned} \tag{7}$$

$$N(T;\varphi) \equiv \frac{i\varphi(i\varphi-1)\left(1-e^{-\varepsilon T}\right)}{2\varepsilon-\left(\varepsilon+i\varphi\sigma_V\rho-\kappa_V\right)\left(1-e^{-\varepsilon T}\right)}$$

$$\varepsilon \equiv \sqrt{\left(i\varphi\sigma_V\rho-\kappa_V\right)^2-i\varphi(i\varphi-1)\sigma_V^2}$$

$$b \equiv \varepsilon+i\varphi\sigma_V\rho-\kappa_V$$

$$\tilde{p} \equiv 2\varepsilon(1-\theta_v i\varphi\mu_{x,v})-\tilde{q},$$

$$\tilde{q} \equiv i\varphi(i\varphi-1)\theta_v+b(1-\theta_v i\varphi\mu_{x,v}).$$

The early exercise boundary has to be determined at the same time when the static hedge portfolio is established. We solve the free boundary problem by the value-matching and smooth-pasting conditions. At the maturity date T, if the American option is not exercised earlier, its terminal condition is exactly the same as the corresponding European option. Therefore, our static hedge portfolio starts with one unit of the corresponding European option. Suppose that the static hedge portfolio matches the boundary conditions of the American option before maturity at n evenly-spaced time points, i.e. $\mathrm{t}_0 = 0, \mathrm{t}_1, \ldots, \mathrm{t}_{\text{n-1}} = \mathrm{T} - \Delta\mathrm{t}$, where $\Delta\mathrm{t} = \frac{T}{N}$. To match the unknown exercise boundary B_i at time t_i (i = 0, 1, . . . , n-1), we add w_i units of a standard European option, maturing at time t_{i+1} and with a strike price equaling B_i, into the static hedge portfolio. We then solve w_i and B_i using the value-matching and smooth-pasting conditions. Similar to the lattice models for American options, we work backward to determine the number of the standard European options and their strike prices for the above n-point critical exercise price B_{n-1}, the value-matching and smooth-pasting conditions imply that

$$\mathrm{B}_{n-1} - K = C^E(\mathrm{B}_{n-1}, V, T - \mathrm{t}_{n-1}; K) + w_{n-1} C^E(\mathrm{B}_{n-1}, V, T - \mathrm{t}_{n-1}; \mathrm{B}_{n-1}) \tag{8}$$

$$1 = C_S^E(\mathrm{B}_{n-1}, V, T - \mathrm{t}_{n-1}; K) + w_{n-1} C_S^E(\mathrm{B}_{n-1}, V, T - \mathrm{t}_{n-1}; \mathrm{B}_{n-1}) \tag{9}$$

where C^E and C_S^E are the formulae of the price and delta of European options, respectively.

Based on Eq.(9), we can obtain that

$$w_{n-1} = \frac{1 - C_S^E(\mathrm{B}_{n-1}, V, T - \mathrm{t}_{n-1}; K)}{C_S^E(\mathrm{B}_{n-1}, V, T - \mathrm{t}_{n-1}; \mathrm{B}_{n-1})} \tag{10}$$

Substituting Eq. (10) into Eq. (8) leads to a nonlinear equation of B_{n-1} which can be solved numerically based on the Newton-Raphson method. Then we have w_{n-1} by substituting B_{n-1} into Eq. (10). Using similar procedures, we work backward to determine the number of units of the standard European option, w_i, and its strike price, B_i, at time t_i, i = n-2, n-3, . . . , 0. Through solving all w_i and B_i (i = 0, 1, . . . , n-1), the value of the n-point static hedge portfolio C_n at time 0 is obtained as follows:

$$C_n = C^E(S_0,V,T;K) + w_{n-1}C^E(S_0,V,T;B_{n-1}) + w_{n-2}C^E(S_0,V,t_{n-1};B_{n-2}) + ...$$

$$+ \quad w_0 C^E(S_0,V,t_1;B_0) \tag{11}$$

where the value-matching and smooth-pasting conditions are given in the following:

$$B_0 - K = C^E(S_0,V,T;K) + w_{n-1}C^E(S_0,V,T;B_{n-1}) + w_{n-2}C^E(S_0,V,t_{n-1};B_{n-2}) + ...$$

$$+ \quad w_0 C^E(S_0,V,t_1;B_0) \tag{12}$$

$$1 = C_S^E(S_0,V,T;K) + w_{n-1}C_S^E(S_0,V,T;B_{n-1}) + w_{n-2}C_S^E(S_0,V,t_{n-1};B_{n-2}) + ...$$

$$+ \quad w_0 C_S^E(S_0,V,t_1;B_0) \tag{13}$$

Our pricing model can be easily applied to other famous models, such as those developed by Duffie, Pan, and Singleton (2000), Bakshi, Cao, and Chen (1997), and Bates (1996). An example considered for an illustration is the model in which return-jumps and volatility-jumps are non-simultaneous and independent. Let λ^x and λ^v, respectively, denote the arrival rates of the return-jump and the volatility-jump. The distribution of the return-jump amplitude is assumed to be:

$$x(t) \sim N\left(\log(1+\mu_x) - \frac{1}{2}\sigma_x^2, \sigma_x^2\right), \tag{14}$$

and v follows an exponential distribution, $Exponential(\theta_v)$. Then, the characteristic function of the state density must satisfy:

$$0 = \frac{1}{2}C_{SS}S^2\mathrm{V} + \frac{1}{2}C_{VV}\sigma_V^2 V + C_{SV}\rho\sigma_V SV$$

$$+ C_S((r-q) - \lambda^x E^Q\left[e^x - 1\right])S + C_V(\overline{V} - \kappa_V V) - C_T - rC$$

$$+ \lambda^x \int_{-\infty}^{\infty}\left[C(Se^x) - C(S)\right]\Phi(x)dx + \lambda^v \int_0^{\infty}\left[C(V+v) - C(V)\right]\Phi(v)dv\,. \tag{15}$$

where $\Phi(x)$ and $\Phi(v)$ denote the density functions of x and v, respectively. The characteristic function of Eq. (6) still satisfies Eq. (15). This model is called the stochastic volatility model with independent double jumps. The characteristic function of this model is provided in Appendix A.

III. Accelerated Static Hedging Method

Once the critical exercise boundary can be obtained numerically by working backward using Eq. (12) and (13) for call options, the price of the American option under the stochastic volatility model with double jumps can be calculated by Eq. (11). In order to be efficient so as to rapidly evaluate American options without approximating the whole early exercise boundary, Geske and Johnson (1984) first use the three-point Richardson extrapolation to evaluate American options. Huang, Subrahmanyam, and Yu (1996) also provide a simple method that utilizes a three-point Richardson extrapolation to accelerate the recursive integration method. The Richardson extrapolation scheme gains efficiency without sacrificing much accuracy.

Let C_n^{GJ} be the value of a Bermudan call option considered in Geske and Johnson (1984) which can only be exercised at one of the n-exercisable time points: Δt, $2\Delta t$,..., T=nΔt. It is worth comparing our n-point static hedge portfolio to the Bermudan call option with n-exercisable time points. First of all, the values $C_1^{GJ}, C_2^{GJ}, C_3^{GJ}, C_4^{GJ}, \ldots$, define a sequence with the limit equaling the American call value. Our static hedge portfolio value, C_n, is simply the summation of European call prices. Based on the geometric-spaced exercise points employed in the modified Geske-Johnson method, define C_1^{GJ} to be the European option value permitting exercise only at period *T*, where *T* is the maturity of the option, C_2^{GJ} , the

Bermudan-style option value permitting exercise only at period $\frac{T}{2}$ and T, and C_4^{GJ}, the Bermudan-style option value permitting exercise at period $\frac{T}{4}$, $\frac{2T}{4}$, $\frac{3T}{4}$ and T only. Again, we can apply the Repeated-Richardson extrapolation technique to derive one 3-point modified Geske–Johnson formula as follows:

$$C^A = \frac{8}{3}\mathrm{C}_4^{\mathrm{GJ}} - 2\mathrm{C}_2^{\mathrm{GJ}} + \frac{1}{3}\mathrm{C}_1^{\mathrm{GJ}} \tag{14}$$

where

$$C_1^{GJ} = C^E(S_0, V, T; K) + \omega_0 C^E(S_0, V, T; B_0)$$

$$C_2^{GJ} = C^E(S_0, V, T; K) + \omega_1 C^E(S_0, V, T; B_1) + \omega_2 C^E(S_0, V, \frac{T}{2}; B_0) \tag{15}$$

$$C_4^{GJ} = C^E(S_0, V, T; K) + \omega_3 C^E(S_0, V, T; B_3) + \omega_2 C^E(S_0, V, \frac{3T}{4}; B_2) + \omega_1 C^E(S_0, V, \frac{T}{2}; B_1) + \omega_0 C^E(S_0, V, \frac{T}{4}; B_0)$$

IV. Numerical Results and Comparisons

In order to confirm the accuracy of our proposed method, we compare the estimators of the least-square Monte Carlo simulation (LSMC) of Longstaff and Schwartz (2001) with those calculated by our proposed analytical formula. Longstaff and Schwartz (2001) provide a least-square Monte Carlo approach to evaluate American options. Our numerical results provide a comparison of our proposed model, the LSMC method, and the quadratic approximation scheme proposed by Guo, Hung, and So (2009) for the stochastic volatility models with jumps. The parameters reported in Bakshi and Cao (2003) and Guo, Hung, and So (2009) are used to compute the numerical results. The least-squares Monte Carlo simulation is based on 1,000,000 (500,000 plus 500,000 antithetic) paths for the stock price, using 24 exercise points per year.

The accelerated static hedging method, the LSMC method, and the quadratic approximation provided by Guo, Hung, and So (2009) are represented by Static Hedging American, LSMC American, and BAW American, respectively. The standard errors of the simulation estimates (s.e.) are given in parentheses. Static Hedging Diff and BAW Diff are defined as the difference between the LSMC estimate (i.e. numerical solution) and our estimate, and the difference between the LSMC estimate and a quadratic approximation estimator. The sum of the squared percentage relative errors (RMS) is defined as $RMS = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(C_i - \tilde{C}_i)^2 / C_i^2}$, where C_i is the option price obtained from the least-squares Monte Carlo method, and $\tilde{C}_i$ is the American option price computed by the analytical approximation.

[Table 1 is here]

The parameters used are adopted from Table 2 of Guo, Hung, and So (2009) to compute values of American options. As shown, the values of the American options from our proposed method are close to those obtained from the LSMC method. The differences between the LSMC estimates and ours are generally small. The Static Hedging Diff is typically smaller than the BAW Diff in all cases. The RMS calculated from our proposed method is generally smaller than the RMS computed from the quadratic approximation provided by Guo, Hung, and So (2009), which demonstrates that our proposed method can give accurate price estimates for pricing American options. The results shown in Table 1 demonstrate that our proposed method can provide accurate estimations for evaluating American options. In view of the computation time, our proposed method is more numerically efficient than the LSMC Based on the results shown in Table 1, these findings demonstrate that our proposed method can still give accurate price estimations for pricing American options under the stochastic volatility model with correlated double jumps.

[Table 2 is here]

Table 2 gives a comparison of the LSMC method, our proposed model, and the quadratic approximation for the stochastic volatility model with independent double jumps using parameters adopted from Table 3 of Guo, Hung, and So (2009). The differences between the LSMC estimates and our estimates are generally small. The RMS computed from our proposed method is similar to those obtained from the quadratic approximation provided by Guo, Hung, and So (2009), which demonstrates that our proposed method can give accurate price estimates for pricing American options. The parameters listed in Bakshi and Cao (2003) and Guo, Hung, and So (2009) are used to calculate the numerical results. In general, the RMS calculated from our proposed method is small. Based on the results presented in Table 2, our proposed method can provide accurate price estimations for evaluating American options under the stochastic volatility model with independent double jumps.

[Table 3 is here]

Based on the specification of volatility jumps ($\lambda^{v}=0$), our proposed model reduces to the stochastic volatility model with return jumps that is empirically examined in Bakshi, Cao, and Chen (1997) and Bates (1996). The parameters used are adopted from Table 4 of Guo, Hung, and So (2009). As shown in Table 3, the differences between the LSMC estimates and our estimates are small. The American option prices acquired from our proposed method are close to those from the LSMC method. The RMS computed from our proposed method is smaller than the RMS calculated from the quadratic approximation provided by Guo, Hung, and So (2009). Based on the results shown in Table 3, our proposed method can give accurate price estimations for evaluating American options under the stochastic volatility model with return jumps.

[Table 4 is here]

Table 4 shows a comparison of the LSMC method, our proposed model, and the quadratic approximation for the stochastic volatility model with volatility jumps ($\lambda^{x} = 0$) using parameters adopted from Table 5 of Guo, Hung, and So (2009). The differences between the LSMC estimates and our estimates are small. The American option prices acquired from our proposed method are close to those from the LSMC method. In general, the RMS calculated from our proposed method is small. These findings demonstrate the accuracy of our proposed model for pricing American options.

V. Conclusions

Although most of the previous work has been done for pricing American options under the Black-Scholes framework, analytical pricing models of American options under stochastic volatility and double jump processes are relatively scarce. More importantly, incorporating stochastic volatility and double jump processes are provided to improve pricing errors in the earlier empirical studies. In this article, we described the application of a static hedging method to obtain efficient analytic formulae for pricing American options on processes permitting stochastic volatility and double jumps in order to illustrate its generality, and the general Static Hedging approximations are provided. This approximation scheme is a generalization of the static hedge portfolio approach of Derman, Ergener, and Kani's approach.

Comparisons with the least-squares Monte Carlo approach and the quadratic approximation method show that the general accelerated static hedge approximations scheme is accurate and efficient in pricing American options with stochastic volatility and double jumps. Our proposed model introduces a new direction for pricing American options. Given the generalization of the stochastic volatility model with double jumps, our proposed model can serve as a convenient tool to evaluate

American options. Furthermore, our proposed model can be extended to more complicated models. For example, it is possible to add the stochastic interest rate into our pricing model in the future.

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Appendix A

Stochastic Volatility Model with Independent Double Jumps

The characteristic function of the state density has the same functional form as in Eq. (7). However, the component function $M(T;\varphi)$ is somewhat different than Eq. (A1).

$$M(T;\varphi) \equiv \left(i\varphi(r-q)-r\right)T - i\varphi\lambda^{x}\mu_{x}T$$

$$+\lambda^{x}\left[\left(1+\mu_{x}\right)^{i\varphi} e^{\left[\frac{1}{2}i\varphi\left(i\varphi-1\right)\sigma_{x}^{2}\right]}-1\right]T$$

$$-\frac{\overline{V}}{\sigma_{V}^{2}}\left[\left(\varepsilon+i\varphi\sigma_{V}\rho-\kappa_{V}\right)T+2\log\left[1-\frac{\left(\varepsilon+i\varphi\sigma_{V}\rho-\kappa_{V}\right)\left(1-\exp\left[-\varepsilon T\right]\right)}{2\varepsilon}\right]\right]$$

$$-\lambda^{v}T+\frac{\lambda^{v}\left(2\varepsilon-b\right)}{2\varepsilon-\tilde{q}}T+\frac{2\lambda^{v}(\tilde{q}-b)}{(2\varepsilon-\tilde{q})\tilde{q}}\log\left[\frac{2\varepsilon-\tilde{q}(1-\exp\left[-\varepsilon T\right])}{2\varepsilon}\right] \qquad \text{(C1)}$$

$$N(T;\varphi) \equiv \frac{i\varphi(i\varphi-1)\left(1-e^{-\varepsilon T}\right)}{2\varepsilon-\left(\varepsilon+i\varphi\sigma_{V}\rho-\kappa_{V}\right)\left(1-e^{-\varepsilon T}\right)} \qquad \text{(C2)}$$

$$\varepsilon \equiv \sqrt{\left(i\varphi\sigma_{V}\rho-\kappa_{V}\right)^{2}-i\varphi(i\varphi-1)\sigma_{V}^{2}} \qquad \text{(C3)}$$

$$b \equiv \varepsilon + i\varphi\sigma_{V}\rho-\kappa_{V} \qquad \text{(C4)}$$

$$\tilde{q} \equiv b + i\varphi(i\varphi-1)\theta_{v}\,. \qquad \text{(C5)}$$

Tables

Table 1 Comparisons of American Call Options: Correlated Double Jumps

	S	Static Hedging	LSMC American	(s.e.)	BAW American	Static Hedging Diff	BAW Diff	European Option
$T = 0.25$	80	1.249	1.287	0.033	1.247	0.038	0.040	1.246
	90	3.459	3.517	0.053	3.454	0.058	0.063	3.451
	100	7.804	7.858	0.079	7.794	0.054	0.064	7.786
	110	14.311	14.351	0.069	14.286	0.040	0.065	14.268
	120	22.341	22.362	0.051	22.288	0.021	0.074	22.249
	RMS	0.0245%			0.0277%			

Note: $K = 100$, $r = 0.06$, $q = 0.06$, $\bar{V} = 0.49$, $V = 0.0968$, $\rho = -0.1$, $\lambda^{x,v} = 1.64$, $\mu_0 = -0.03$, $\mu_{x,v} = -7.87$, $\sigma_{x,v} = 0.22$, $\theta_v = 0.0036$, $\sigma_V = 0.61$, and $\kappa_V = 5.06$.

Table 2 Comparisons of American Call Options: Independent Double Jumps

	S	Static Hedging	LSMC American	(s.e.)	BAW American	Static Hedging Diff	BAW Diff	European Option
$T = 0.25$	80	1.046	1.049	0.009	1.049	0.003	0.000	1.045
	90	3.416	3.425	0.018	3.419	0.009	0.006	3.410
	100	7.870	7.877	0.019	7.863	0.007	0.014	7.843
	110	14.324	14.287	0.039	14.274	-0.037	0.013	14.232
	120	22.2977	22.171	0.027	22.145	-0.127	0.026	22.064
	RMS	0.0011%			0.0002%			

Note: $K = 100$, $r = 0.06$, $q = 0.06$, $\bar{V} = 0.49$, $V = 0.1623$, $\rho = -0.31$, $\lambda^x = 0.87$, $\mu_x = -0.014$, $\sigma_x = 0.04$, $\lambda^v = 2.43$, $\theta_v = 0.0036$, $\sigma_V = 0.54$, and $\kappa_V = 3.02$.

Table 3 **Comparisons of American Call Options: Stochastic Volatility with Jump in Return**

	S	Static Hedging	LSMC American	(s.e.)	BAW American	Static Hedging Diff	BAW Diff	European Option
$T=0.25$	80	1.868	1.893	0.038	1.866	0.025	0.027	1.862
	90	4.603	4.660	0.082	4.594	0.057	0.066	4.585
	100	9.206	9.206	0.109	9.200	0.000	0.006	9.181
	110	15.648	15.611	0.063	15.572	-0.037	0.039	15.537
	120	23.440	23.350	0.079	23.271	-0.090	0.079	23.207
	RMS	0.0069%			0.0084%			

Note: $K=100$, $r=0.06$, $q=0.06$, $\bar{V}=0.49$, $V=0.125$, $\rho=-0.16$, $\lambda^x=3.05$, $\mu_x=-0.03$, $\sigma_x=0.19$, $\lambda^v=0$, $\theta_v=0$, $\sigma_V=0.41$, and $\kappa_V=3.92$.

Table 4 **Comparisons of American Call Options: Stochastic Volatility with Jump in Volatility**

	S	Static Hedging	LSMC American	(s.e.)	BAW American	Static Hedging Diff	BAW Diff	European Option
$T = 0.25$	80	1.366	1.420	0.031	1.369	0.054	0.051	1.363
	90	3.948	3.983	0.053	3.945	0.035	0.038	3.932
	100	8.506	8.498	0.056	8.466	-0.008	0.032	8.439
	110	14.967	14.882	0.067	14.817	-0.085	0.065	14.766
	120	22.948	22.652	0.104	22.564	-0.296	0.088	22.472
	RMS	0.0344%			0.0286%			

Note: $K = 100$, $r = 0.06$, $q = 0.06$, $\bar{V} = 0.49$, $V = 0.189922$, $\rho = -0.26$, $\lambda^x = 0$, $\mu_x = 0$, $\sigma_x = 0$, $\lambda^v = 1.36$, $\theta_v = 0.0016$, $\sigma_V = 0.53$, and $\kappa_V = 2.58$.