Can we afford defined benefit pensions in low interest rate environments?

Woon K. Wong

Cardiff Business School
Aberconway Building, Colum Drive, Cardiff CF10 3EU
Email: wongwk3@cf.ac.uk

Abstract

This paper shows how time diversification limits the cost of guarantying DB pensions to no more than the required cash contributions to pay for outgo, which can be significantly cheaper than the risk-free bond approach. The discrepancy can be explained by the incompleteness of financial markets for pensions and hence the breach of Ross's (1976) arbitrage-free conditions to value DB benefits using interest rates. Under low interest rate environments, this article shows that it is optimal to hold more equity as the increased risk can be mitigated by cash contributions, a higher funding ratio and long investment horizon.

JEL: G1, G22, G23, G3

Keywords: pension, defined benefit, low interest rate, cost of guarantee, prudence

First draft: 31 October 2018
This version: 17 December 2022

Can we afford defined benefit pensions in low interest rate environments?

Abstract

This paper shows how time diversification limits the cost of guarantying DB pensions to no more than the required cash contributions to pay for outgo, which can be significantly cheaper than the risk-free bond approach. The discrepancy can be explained by the incompleteness of financial markets for pensions and hence the breach of Ross's (1976) arbitrage-free conditions to value DB benefits using interest rates. Under low interest rate environments, this article shows that it is optimal to hold more equity as the increased risk can be mitigated by cash contributions, a higher funding ratio and long investment horizon.

JEL: G1, G22, G23, G3

Keywords: pension, defined benefit, low interest rate, cost of guarantee, prudence

1. Introduction

The intergenerational risk sharing of defined benefit (DB) schemes is welfare enhancing; see for example Gordon and Varian (1988), Shiller (1999), Gollier (2008), Weil (2008), Cui *et al*. (2011) and Chen *et al*. (2016). Despite the finding, the provision of DB pensions has declined in recent decades.[1] One reason for the decline is rising cost due to falling interest rates. While it may be reasonable to assume that the increased cost is attributable to the amount of bonds held by DB schemes, many academics and practitioners argue that because defined benefits are guaranteed, the liabilities should be priced as if they were funded by a 100% risk-free bond portfolio.[2] According to this approach, the UK interest rate as of 31/3/2020 would imply a contribution or saving rate of 48% of payroll for a typical DB scheme (henceforth the "Scheme") considered in this paper.

However, a recent statement by the UK Pensions Regulator (TPR) suggests that holding riskless bonds may not be the only means of guarantying DB pensions:

> Truly open schemes with a strong flow of new entrants may always be immature… might invest in more illiquid and volatile assets in the expectation of a higher return. They might anticipate that higher return in their discount rate and consequently set

[1] See for example Schrager (2009), Maer and Thurley (2009), Cobb (2015) and Breedon and Larcher (2021).

[2] Bodie (1990, 2006), Gold and Hudson (2003) and Wilcox (2006) provide the economic rationales. Joliffe (2005) and Ralfe (2005) put the bond-based approach into practice in a commercial firm.

> lower technical provisions [or liability]. The rationale being that there is sufficient time for the scheme to ride out that volatility.[3]

Following the above principle, this paper shows how time diversification limits the cost of guarantying defined benefits to no more than the saving rate required for the cash contributions necessary to pay for outgo; for the Scheme, the maximum required saving rate is 34.7% of payroll. Note that this is neither a Pay-As-You-Go nor a Ponzi approach, as the Scheme can be ensured to be fully funded (asset larger than liability) if it invests sufficiently in risky assets.

The discrepancy is unsurprising since competitive financial markets are incomplete for long-term products such as pensions (Diamond, 1977; Gordon and Varian, 1988; Shiller, 1999) and thus the arbitrage free pricing conditions of Ross (1976) are not met to use interest rates to price DB pensions. Governments enable risk sharing between different generations via a Pay-As-You-Go pension plan, or other fiscal policy and public debt management.[4] As a private arrangement, DB schemes can exploit their intergenerational risk sharing ability to invest in risky assets to earn higher returns; see for example Dyck and Pomorski (2016). Financial institutions make arbitrage profit by borrowing at short-term interest rates and making long-term loans at higher interest rates. Likewise, when interest rates are low, an optimally invested DB fund arbitrages to benefit its members.

Both analytical and simulation analyses have been carried out to help understand the role of cash contributions in time diversification. The analyses complement the findings of Gollier (2002) in that cash contributions are shown to lessen liquidity constraints, making risks (including those of extreme downturns) diversifiable through time. A high funding ratio also serves the same purpose, as it makes liquidity constraints less binding by providing a buffer against losses; see Epstein (1983) and Gollier (2002). Consequently, DB schemes with low cash outflow (due to cash contributions) and a high funding ratio can effectively mitigate the tail risks due to volatile assets and enjoy a higher return. At current low interest rates, a full equity investment strategy is found to be optimal for the studied Scheme.

While the arbitrage free pricing approach adds an interesting perspective to the widely debated issue of how to set the discount rate for pension liabilities, there are policy implications that mean the risks of volatile assets can be effectively mitigated by open DB schemes.[5] TPR encourages DB schemes to set Long-Term Objectives (LTO) in order to achieve low dependency on the employer, which can be very expensive if a low-risk investment strategy is adopted.[6] On the other hand, legislation in the UK requires DB schemes to be prudently funded, which implies that the discount rate must be set below the expected return of the scheme's risky assets. This paper shows that with prudent funding,

[3] David Fairs, 2020, DB funding code: busting a few myths, TPR, 8 December 2020, Accessed 31 January 2022, https://blog.thepensionsregulator.gov.uk/2020/12/08/db-funding-code-busting-a-few-myths/.

[4] See Bohn (2009) on fiscal policy, Weiss (1979) on monetary policy, and Ball and Mankiw (2007), Krueger and Kubler (2006) on Pay-As-You-Go pension plan.

[5] See Breedon and Larcher (2021) for a recent summary on how to set a discount rate for pension liabilities.

[6] See p11 of https://www.thepensionsregulator.gov.uk/-/media/thepensionsregulator/files/import/pdf/quick-guide-db-funding-consultation.ashx

an optimal investment strategy enables the DB Scheme to raise its funding ratio and lower its cash outflow, thereby making LTO on low dependency affordable in a low interest rate environment.

The above findings can be used to shed light on recent disputes about the valuation of the Universities Superannuation Scheme (USS), the largest privately funded DB scheme in the UK. Essentially, the nature of the dispute may be understood as whether USS should be regarded as a closed or a truly open scheme for valuation purposes. Treating USS as a closed scheme, both Miles and Sefton (2021) and this paper find that although it is likely to pay pensions in full, the tail risk may be high enough to justify a deficit. However, if USS is regarded as a truly open scheme, for reasons explained above, the risk of paying pensions in full drops to a level that is likely lower than low dependency or self-sufficiency would imply. Treating USS as a truly open scheme is consistent with the message expressed by the Joint Expert Panel that USS "*can afford to take a very long-term view*".[7] By overlooking the role of cash contributions in the mitigation of risks, less relevant factors are considered, thereby producing the past and current deficits of USS. In particular, the large deficits (as much as £17.9bn) and high contribution rates (up to 67.9% of payroll) in the 2020 valuation may serve as an example of "*combinations of extreme values that rarely or never occur together in practice*," and go against the guidance on producing quality analysis for the UK government.[8] This paper provides quantified evidence that supports the recent joint letter by the Universities of Cambridge, Oxford and Imperial College which expresses concerns over USS's proposal to increase de-risking of its portfolio.[9]

Finally, Boubaker *et al.* (2018) and Lu *et al.* (2019) find that pension funds take more investment risks to search for yield in a low interest rate environment. This paper finds that low interest rates cause Defined Contribution (DC) schemes to save more as well as making riskier investments. Despite these efforts, lower interest rates reduce members' social welfare by 12%. If affordability is considered, the welfare loss increases to 22%. These findings illustrate the undesirable effects studied by Hacker (2008) and Kalleberg (2009, 2011) when individuals are forced to shoulder market uncertainties that were once born by institutions. On the other hand, by virtue of intergenerational risk sharing, the Scheme suffers considerably less welfare loss. This suggests that policies should enable DB schemes to remain open so that the welfare-enhancing feature of intergenerational risk sharing can continue to benefit pensioners in a low interest rate environment.

The rest of the paper is organised as follows. Section 2 introduces a model for the Scheme and provides further information on USS. Section 3 discusses prudence and its implications. Section 4 obtains the cost of guarantee and investigates the various factors that determine

---

[7] The Joint Expert Panel was formed in response to the largest ever industrial action in the UK HE sector caused by the 2017 valuation of USS. Its report is available at https://www.ussemployers.org.uk/tags/jep-2018-report

[8] See point 8.30 in the Aqua Book: guidance on producing quality analysis for government. Available at https://www.gov.uk/government/publications/the-aqua-book-guidance-on-producing-quality-analysis-for-government

[9] The increased de-risking is to be achieved by increasing the weight of inflation-linked bonds in portfolio. The joint letter is available at https://www.imperial.ac.uk/human-resources/pay-and-pensions/pensions/uss/changes/what-the-college-is-doing/joint-letter/

the risk to accrued benefits. Section 5 compares the impact of low interest rates on the Scheme and DC schemes, and provides a simulation study to show how the former can achieve low dependency as LTO at an affordable saving rate. Finally, Section 6 concludes. Note that unless otherwise stated, all discussions, analyses and reported figures are in real terms.

2. A model and USS

2.1 A model of the Scheme

As in Cui *et al.* (2011) and Chen *et al.* (2016), the Scheme has 60 overlapping generations, each with 1 member who is either working age between 30 and 66 or retired age between 67 and 89. They are n = 37 workers and m = 23 pensioners in the Scheme. Each year, the oldest member who has just turned 90 dies and a new member who has just turned 30 joins the Scheme. In year t, the working generation earns £1 and saves £s_t for which retirement pension £b_t plus a one-off lumpsum £xb_t are accrued.

Let A_t and r_t denote the Scheme's assets and returns respectively in year t. Then, the asset evolves according to

$$A_{t+1} = (A_t + ns_t - B_t)(1 + r_{t+1}), \tag{1}$$

where ns_t is the cash contribution and the outgo is

$$B_t = (m + x)nb \tag{2}$$

if the accrual or replacement rate is constant. For variable accrual rates,

$$B_t = (x + 1)B_{t,0} + \sum_{a=1}^{m-1} B_{t,a}, \tag{3}$$

where for $a \in [0, m - 1]$,

$$B_{t,a} = \sum_{i=1}^{n} b_{t-a-i} \tag{4}$$

is the accrued benefit a retired member age $67 + a$ would receive in a year. Note that any change in b_t affects only future benefits but not the benefits that have already been accrued.

Unlike Cui *et al.* and Chen *et al.*, $\mu = E(r_t)$ instead of risk-free interest rate is used as the discount rate. In the case of a constant accrual rate and non-zero μ, the present value of the accrued benefit due to a member's n years of savings is given by

$$L = nb\big(x + \mu^{-1}(1 + \mu)(1 - (1 + \mu)^{-m})\big). \tag{5}$$

Given a constant saving rate and static returns, the asset saved by the member is

$$A = s\mu^{-1}(1+\mu)((1+\mu)^n - 1). \quad (6)$$

μ is the break-even discount rate if the above two equations equate by adjusting s. Since every member works and retires for the same number of years, μ is also the break-even discount rate of the Scheme. The liability of the Scheme is provided in the Appendix. Legislation in the UK requires DB pensions to be funded with prudence. This means that the asset, if risky, must exceed the liability.

Assumption of asset returns

The asset returns, r_t, are assumed to be independent and identically distributed (IID) as normal. A normal tail implies a higher downside risk than lognormal, which is assumed by Cui *et al.* (2011), Chen *et al.* (2016) and Miles and Sefton (2021). Price reversal is acknowledged by practitioners and is widely documented in the literature; see for example De Bondt and Thaler (1985), Lakonishok *et al*. (1994), and Balvers *et al*. (2000). The IID normal assumption means that the model would have larger long-term tail risk than the evidence of price reversal would imply.

The Scheme invests in two assets, namely equity and gilt. The analyses in this paper are based on the estimates provided by USS as of 31 March 2020, as shown in Table 1, where e, y and μ are respectively the expected return of equity, gilt and USS's Reference Portfolio.[10] The volatility is denoted by σ whereas ρ refers to the correlation between equity and gilt.

Table 1. Expected returns and volatilities of assets held by USS as of March 2020

e	y	μ	σ_e	σ_y	ρ
0.0439	−.0114	0.0303	0.15	0.05	0.15

The table provides the expected return and volatility of equity and gilt held by USS as of 31 March 2020. e, y and μ are the expected return of equity, gilt, and USS's Reference Portfolio respectively. σ denotes the standard deviation and ρ is the correlation between equity and gilt.

The expected equity return of 4.39% in Table 1 is lower than the 6% used in Cui *et al.* (2011) and 5% in Chen *et al.* (2016). While both papers use a risk-free interest rate of 2% with zero standard deviation, a lower gilt yield of −1.14% is used here, reflecting the current negative interest rate environment. Although gilt is default free, this study follows USS to assume a non-zero σ_y, as gilt returns vary from year to year by virtue of marking to market.

The assets held by USS are approximated by 60% equity and 40% gilt, which according to Table 1, yields an expected return of 2.18%. Due to the rebalancing premium and leverage, USS's Reference Portfolio yields a higher expected return of 3.03%. The former expected return is used in Section 3 and 5 where there is a focus on optimal asset allocation. The latter expected return is used in Section 4 to study risks of underfunding and insolvency.

[10] The Reference Portfolio provides a benchmark investment strategy that is consistent with USS's risk appetite under the 2018 valuation. It invests roughly 60% equity and 40% gilt; see Universities Superannuation Scheme (2020).

2.2 USS

USS is the largest privately funded DB scheme in the UK. It is recognised by the Pension Protection Fund as a multi-employer scheme with a joint liability based on the 'last-man standing' principle, which means members' accrued benefits will be honoured by the last remaining employer. Being a pension fund for the higher education sector, several of USS's employers enjoy the highest credit rating. Therefore, USS fits well with what the Pensions Regulator (TPR) in the UK regards as a truly open scheme with a strong flow of new entrants. Currently, USS is immature with cash contributions projected to exceed outgo for almost the next two decades.

Prior to the 2017 valuation, the contribution or saving rate of USS was 26% of payroll and benefits were accrued at a replacement rate of 1/75 for salary below the threshold of £55,000. For salary above the threshold, 20% of the contribution was saved in a DC fund and the remaining 6% was added to the DB fund as subsidy with no benefit accrued. The accrued benefits increased with CPI inflation in full between 0% and 5%, half between 5% and 10%, and zero beyond 10%. The accrued benefits would not decline if there were deflation. A one-off lumpsum worth 3 times the annual pension would be paid at the start of retirement. These translate for the Scheme approximately as s = 0.26, b = 1/75 and x = 3. For simplicity, no threshold and full inflation-linked pension are assumed in the study. When the saving rate is 26%, equating Eqn. 5 to Eqn. 6 gives the break-even discount rate of the Scheme as 0.996%.

Self-sufficiency, covenant and the 2020 valuation

Although not required by legislation, self-sufficiency is introduced to DB schemes in the UK so that reliance on the sponsoring employer can be kept to a minimal level.[11] For USS, self-sufficiency means a low-risk investment strategy of holding mainly gilts and gilts-like securities to achieve a low, 5% chance of requiring further contributions from employers to meet all accrued benefits.[12] Self-sufficiency was incorporated into the 2014, 2017 and 2018 valuations of USS by de-risking its Reference Portfolio (replacing equity with gilt) so that the assets were more like those for self-sufficiency. Consequently, interest rates drove the past valuations of USS. This is evidenced from Wong (2018) which found that the index-linked gilt yield and a constant explained up to 99% of the variations of USS's liabilities between 2011 and 2017. Although the 'de-risking towards self-sufficiency' methodology was replaced by a dual-discount rate approach in the 2020 valuation, interest rates continue to play an important role in that a significant part of the discount rate is determined by the self-

[11] According to the submission of the Association of Pension Lawyers to TPR on the DB funding code of practice, legislation does not stipulate the requirement of self-sufficiency for DB schemes in the UK; see https://henrytapper.com/2020/09/15/association-of-pension-lawyers-dubious-about-tprs-db-funding-code/.

[12] See p.42 of Universities Superannuation Scheme (2017).

sufficiency discount rate. As a result, £14.9bn has been added to the liability of USS because of falls in gilt yields between 2018 and 2020.[13]

Another related factor that also plays a key role in USS's valuation is covenant, which is the legal obligation and ability of an employer to support a DB scheme during, for example, an extreme downturn. In its consultation for the 2020 valuation USS states *"[w]ithout a covenant, we would effectively have to pursue a 'self-sufficiency' investment strategy, involving a much higher funding target and commensurately higher contributions."*[14] To put it another way, a covenant would *not* be required if USS were self-sufficient. This will be used as a principle in Section 4 to show that the covenant should play a lesser role in valuation. As the covenant could change from strong to tending-to-strong in the 2020 valuation, Panel A in Table 2 shows how a weaker covenant lowers the discount rate but raises the liability, deficit, and contribution rates. Note the highest contribution rate is 67.9% of payroll.

Panel B reports the difference between the discount rate and the expected return of the Reference Portfolio (3.03%). In the 2014 valuation, a prudence of just over 1% per annum is used to lower the expected return of the scheme asset to obtain the initial year of discount rate.[15] De-risking is applied to further lower discount rates over a 20-year period. The difference in Panel B can be understood in the same way; it comprises of prudence and the effect of the dual-discount rate valuation methodology. Note that both the de-risking and dual-discount rate approach are for valuation purposes; they do not change the investment strategy and the expected return of the asset. The next three sections of this paper show that the large mark down of expected return required to arrive at the discount rate is unnecessary and reduces the members' welfare.

Table 2. Key figures in the 2020 valuation of USS

	Covenant 1 (TTS)	**Covenant 2**	**Covenant 3**	**Covenant 4 (Strong)**
Panel A				
Discount rate	CPI+0.0%	CPI+0.2%	CPI+0.4%	CPI+0.5%
Liability	£84.4bn	£81.4bn	£78.8bn	£76.3bn
Deficit	£17.9bn	£14.9bn	£12.3bn	£9.8bn
Contribution rate, 8 years $DR^{0\%}$	67.9%	59.7%	52.2%	45.4%
Contribution rate, 8 years $DR^{0.5\%}$	63.0%	54.9%	47.5%	40.8%
Contribution rate, 10 years $DR^{0\%}$	60.3%	53.5%	47.2%	41.5%
Panel B				
$\mu -$ discount rate	3.0%	2.8%	2.6%	2.5%
Panel C				
Future service cost, USS	37.6%	34.5%	31.8%	29.4%
Future service cost, m = 23	34.7%	32.7%	30.9%	30.0%

[13] See p.78 of Universities Superannuation Scheme (2020).
[14] See p.18 of Universities Superannuation Scheme (2020).
[15] See p.11 of Universities Superannuation Scheme (2014).

Future service cost, m = 24	36.0%	34.0%	32.0%	31.1%
Future service cost, m = 25	37.3%	35.2%	33.1%	32.2%

The figures in the table are sourced from Universities Superannuation Scheme (2020). In Panel A, the liabilities are obtained using the stated discount rates. The deficits are obtained as assets (£66.5bn) minus liabilities. The reported contribution rates include deficit recovery contributions (denoted as DR), of which the superscript indicates outperformance over the discount rate in row 1. The difference in Panel B is obtained by subtracting the discount rate in Panel A from the expected return of the Reference Portfolio (3.03%) in Table 1. The difference reflects prudence and the effect of de-risking for valuation purposes. In Panel C, the first row is FSC of USS whereas the next three rows are the Scheme's FSC obtained by setting μ in Eqn. 5 and 6 as the discount rate in Panel A. m is the number of pensioner members.

The liabilities in Panel A are highly sensitive to the discount rate; a rise of 0.5% in discount rate from Covenant 1 to Covenant 4 lowers the liability by £8.1bn. If no valuation-methodological lowering of discount were applied, USS would be in a comfortable surplus at the 2014 level of prudence.

Finally, for benefits to be accrued in the future, the saving rate s in Eqn. 6 is also referred to as the Future Service Cost (FSC). Despite the simplicity of the DB Scheme, Panel C shows that its FSC is reasonably close to that of USS. m = 23 is used in this paper's analysis as it gives the closest FSC at 0.5% discount rate, which is lower than the expected return of the Reference Portfolio. Although the analysis conducted on the Scheme cannot be relied on to provide accurate actuarial figures, useful qualitative inferences can be made.

3. Prudence

This section considers the return-distribution measure of prudence which facilitates the analyses in Section 4 and 5. A lower interest rate is found to encourage holding more equity. While this lowers the required level of asset, a larger tail risk is entailed. The implications of prudence for FSC and long-term trends of funding ratios (asset over liability) are also investigated.

3.1 Cost of prudence

To express prudence in terms of return distribution, consider the annualised rate of return after holding asset for T years

$$R_T = \left(\prod_{t=1}^{T}(1+r_t)\right)^{1/T} - 1. \qquad (7)$$

A prudent discount rate at confidence level p in year T, denoted as $\varrho_T(p)$, may be defined as the (1−p)-th percentile of distribution of R_T. The liability at 100p% prudence, denoted as $L(p)$, is the sum of present values of outgo O_t (accrued benefits payable in year $t = 1,..,T$),

$$L(p) = \sum_{t=1}^{T} \frac{O_t}{\left(1 + \varrho_t(p)\right)^t}. \qquad (8)$$

A prudent discount rate $\varrho_T(p)$ requires $p > 0.5$ and is less than the expected return discount rate. Assuming that asset returns are normally distributed, Table 3 gives the liability of the Scheme for various equity weights w when discount rates based on expected return, 67% and 95% levels of prudence are used. Consistent with modern portfolio theory, the liability falls with expected return but rises with volatility and prudence.

The 2017 valuation of USS was conducted at 67% prudence.[16] At this level of prudence and a 2% interest rate, a half-equity asset achieves a minimal liability of 274.2. When the interest rate falls to −1.14%, the liability rises to 351. Despite the higher volatility, the liability lowers to 286 if the asset is fully equity. However, at 95% prudence, an all-equity asset has more tail risk thereby giving rise to a higher liability. If the weight of equity can be reduced by half, the liability reduces to a minimum of 520.5. Finally, the last two columns indicate the required funding (asset to liability) ratios for prudence at 67% and 95% levels.

Table 3. The return-distribution prudence

		$y = 2\%$				$y = -1.14\%$					
$\boldsymbol{w}$	$\boldsymbol{\sigma}$(%)	$\boldsymbol{\mu}$(%)	$\boldsymbol{L(\mu)}$	$\boldsymbol{L(67)}$	$\boldsymbol{L(95)}$	$\boldsymbol{\mu}$(%)	$\boldsymbol{L(\mu)}$	$\boldsymbol{L(67)}$	$\boldsymbol{L(95)}$	$\boldsymbol{L(67)/L(\mu)}$	$\boldsymbol{L(95)/L(\mu)}$
0.0	5.00	2.00	279	303	378	-1.14	469	536	699	1.14	1.49
0.1	4.95	2.24	270	291	362	-0.59	423	478	617	1.13	1.46
0.2	5.35	2.48	261	283	358*	-0.03	384	434	568	1.13	1.48
0.3	6.10	2.72	253	278	363	0.52	350	400	540	1.14	1.54
0.4	7.10	2.96	245	275	376	1.07	320	373	526	1.17	1.64
0.5	8.25	3.20	238	274.2*	394	1.63	295	351	520.5*	1.19	1.77
0.6	9.51	3.43	231	274	417	2.18	272	333	521	1.22	1.92
0.7	10.8	3.67	224	276	446	2.73	253	318	527	1.26	2.08
0.8	12.2	3.91	218	278	480	3.28	235	305	537	1.30	2.29
0.9	13.6	4.15	212	281	521	3.84	220	295	551	1.34	2.50
1.0	15.0	4.39	206	286	570	4.39	206	286*	570	1.39	2.77

w is weight of equity in portfolio, giving rise to values of expected portfolio return μ and standard deviation σ based on statistics provided in Table 1. $L(\mu)$ is liability of the Scheme obtained by using μ as discount rate. $L(100p)$ refers to liability at $100p$% prudence. The liabilities are obtained by simulation using normal returns with mean μ and volatility σ based on 10,000 simulations.

Table 4 provides for USS its asset A, reported liability L^*, single equivalent discount rate SEDR, break-even discount rate BEDR, expected return of asset μ, difference between μ and

[16] See p.20 of Universities Superannuation Scheme (2020).

SEDR, liability $L(\mu)$, and funding ratios $A/L(\mu)$ and $L^*/L(\mu)$. The liabilities L^* and $L(\mu)$ are obtained by discounting the projected outgo of USS using SEDR and μ respectively. As explained in Section 2, $\mu -$ SEDR contains both prudence and lowering of expected return for valuation purposes. It is highest in 2020 when the Covid-19 pandemic began.[17]

Table 4. A short time series of some key statistics of USS

Year	A	L^*	SEDR	BEDR	μ	μ–SEDR	$L(\mu)$	$A/L(\mu)$	$L^*/L(\mu)$
2017	60.0	65.1	3.01%	3.43%	5.00%	1.99%	45.5	1.319	1.431
2018	63.6	67.3	2.94%	3.23%	5.26%	2.32%	44.8	1.420	1.502
2020	66.5	79.4	2.40%	3.31%	5.13%	2.73%	48.8	1.363	1.627
2021	80.6	95.8	1.97%	2.79%	4.53%	2.56%	58.3	1.383	1.643

The reported figures are USS's asset A, reported liability L^*, single equivalent discount rate SEDR, break-even discount rate BEDR, expected return of asset μ, difference between μ and SEDR, liability $L(\mu)$ obtained by using μ as discount rate, and funding ratios $A/L(\mu)$ and $L^*/L(\mu)$.

$\mu >$ BEDR means that USS has more than enough assets to pay for its liabilities if investment returns are static. Allowing for risky returns, a comparison of $A/L(\mu)$ with $L(67)/L(\mu)$ in Table 3 suggests surpluses for USS at a moderate, 67% level of prudence. As stated in the preceding section, prudence in terms of discount rate in the 2014 valuation is just over 1% per annum. Therefore, assuming there is no de-risking, a similar conclusion of surplus can be reached since $\mu -$ BEDR is considerably greater than 1%. Finally, note that the Dutch pension regulation requires the funding ratio to be 1.3 for solvency purposes; see Franzen (2010). All the funding ratios $A/L(\mu)$ exceed 1.3, indicating that USS has sufficient assets to pay for liabilities.

However, the concern for tail risk causes USS to 'de-risk' for valuation purposes, which is equivalent to using a gilts-based method to set its liabilities. Therefore, recent falls in interest rates have caused its reported liability L^* and the required funding ratio $L^*/L(\mu)$ to increase significantly, giving rise to large deficits. Notwithstanding the above discussion, the next section shows that a low-risk self-sufficiency portfolio is not the only way to manage tail risks. Intergenerational risk sharing enabled by working members' cash contributions is found to mitigate extreme downturns effectively for a risky investment strategy.

3.2 Future service cost (FSC)

A high level of prudence was used in the 2020 valuation which took place at the trough of the financial market during the pandemic. The subsequent rise in the scheme's assets prompted a call for a 2021 actuarial valuation.[18] This was rejected by USS because the

[17] The reported liability £79.4bn in 2020 is based on Report and Accounts of USS for the year ended 31 March 2020; see https://www.uss.co.uk/about-us/report-and-accounts.

[18] The statistics for 2021 in Table 4 are based on the so-called monitoring valuation that uses the assumptions of the 2020 valuation. An actuarial valuation will review and revise these assumptions.

improved funding position was offset by a high FSC mainly due to prudence. Below shows an economic rationale for setting FSC with little or no prudence.

Consider a saving rate s_o such that the next year's asset is expected to remain unchanged, i.e., $E(A_{t+1}) = E(A_t) = A_o$. Assuming a constant accrual rate, we have from Eqn. 1 and 2

$$A_o\mu = \big((m+3)nb - ns_o\big)(1+\mu). \tag{9}$$

A_o and s_o are called steady state asset and saving rate respectively by Chen *et al.* (2016). Suppose a surplus S is added to the asset. To maintain expected asset at $A_o + S$, the saving rate s needs to satisfy

$$(A_o + S)\mu = \big((m+3)nb - ns\big)(1+\mu). \tag{10}$$

Equating the above two equations gives

$$s = s_o - S\mu/(n(1+\mu)), \tag{11}$$

which states when there is a surplus on expectational basis, the saving rate needs to be lowered by $S\mu/(n(1+\mu))$ to maintain the same level of surplus. This result is supported by the simulation study in Section 5. In short, while prudence is reasonable for past accrued benefits, it may not be required for setting FSC. This is a view shared by some actuaries in the UK.

The above analysis also has the following implication. If $s = s_o$ in the right-hand side of Eqn. 10, then the surplus will be expected to increase from S to $S + S\mu$, which in turn means a higher funding ratio in the next period. Therefore, while market volatility implies there will be ups and downs, the funding ratio of a prudently funded DB scheme will likely be on a rising trajectory. Table 4 shows this is indeed the case for USS. Although the funding ratio $A/L(\mu)$ falls in 2020 due to the pandemic, it remains higher than that in 2017. Despite the large rise in $L(\mu)$ in 2021, a larger increase in assets raises the funding ratio further. To sum up, if the saving rate remains at a reasonable level, say $s \geq s_o$, surplus due to prudence will place the funding ratio on a rising path. This could eventually lead to a funding level that is high enough to make the scheme self-sufficient. The next two sections show this indeed could be the case.

4 Cost of guarantee

The preceding section studies prudence based solely on return distribution; an approach criticised by Miles (2018) for underestimating the multiyear tail risk of USS. However, the return distribution approach and its critique are predicated on an incomplete set of factors to determine whether an open DB scheme would be able to meet its liabilities. This section shows that cash contributions and funding ratios also play a central role in setting prudence. Based on these additional factors, by virtue of time diversification, a maximum required saving rate exists to guarantee payment of accrued benefits thereby eliminating any risk of extreme downturn. The implications for pricing and management of DB pensions are discussed.

4.1 Maximum required saving rate

Without loss of generality, a constant accrual rate is assumed here for ease of exposition. If the saving rate is

$$s = (m + x)b, \tag{12}$$

then $ns - (m + x)nb = 0$, i.e., cash contribution pays for outgo. Let t denote now. From Eqn. 1, the asset in T years from now is

$$A_{t+T} = A_t \prod_{\tau=1}^{T} (1 + r_{t+\tau}). \tag{13}$$

Let z be a standard normal random variate. The risk of underfunding (asset less than liability) T years from now can be expressed as

$$P(A_{t+T} < L) \approx P\left(\sum r_{t+\tau} < ln(L/A_t)\right) = P\left(z < \frac{ln(L/A_t) - T\mu}{\sqrt{T}\sigma}\right), \tag{14}$$

which tends to zero for large T if $\mu > 0$. To sum up the above, Eqn. 12 guarantees payment of accrued benefits when they fall due whereas Eqn. 14 ensures sufficient funding by virtue of time diversification.

Although Gollier (2002) has provided a theoretical foundation for time diversification, some clarification is helpful here because of the controversy surrounding the topic in the literature. Consider the per year average rate of independent risky returns which has a smaller standard deviation if the holding horizon is longer. As Samuelson (1963, 1989a, b) and Bodie *et al.* (2014) rightly argue, what usually matters for investors is total return which has a larger standard deviation thereby offsetting the benefits of the average return's smaller standard deviation. However, whether time diversification is beneficial depends on the objective function; see Thorley (1995), Strong and Taylor (2001) and Boyle and Guthrie (2005). In Eqn. 14, it is used to secure sufficient funding and hence payment of accrued benefits. Note that although the risk $\sqrt{T}\sigma$ increases with time horizon T, the return $T\mu$ increases at a rate faster by a factor of $\sqrt{T}$.

Time diversification under liquidity constraints

The result presented in Eqn. 14 is a working of time diversification in the absence of liquidity constraints, i.e., no asset is sold to pay for outgo. According to Gollier (2002), liquidity constraint reduces a DB scheme's *implicit time horizon* or ability to diversify risk through time. Conversely, cash contributions lessen liquidity constraints and increase the implicit time horizon. Also, as Gollier points out, a higher level of asset makes liquidity constraints less binding by providing more buffer against losses; see also Epstein (1983). Their role in mitigating risk can be seen in the equation for the one-period-ahead underfunding risk

$$P(A_{t+1} < L) = P\left(z < \frac{L/A_t - 1 + d_t}{\sigma(1 - d_t)} - \frac{\mu}{\sigma}\right) \approx P\left(z < \frac{(L/A_t - 1) + d_t - \mu}{\sigma}\right), \quad (15)$$

where d_t is cash outflow divided by asset as shown below

$$d_t = \big((m + x)nb_t - ns_t\big)/A_t. \quad (16)$$

Note that d_t is small because DB liabilities span over a long time horizon. The above shows that in addition to the effects of expected return and volatility as analysed in Section 3, a lower cash outflow (due to cash contributions) and a higher level of asset or funding ratio decrease the next-period underfunding risk.

The insolvency risk, denoted as $P(A_t < 0)$, is the probability of failure to pay for pensions when they fall due. For a multiyear horizon, Table 5 reports the risks of underfunding and insolvency for the DB Scheme in T years from now at various funding ratios and saving rates for an investment strategy that yields 3.03% return and 10% per annum standard deviation. As stated earlier, as of March 2020, the Reference Portfolio of USS has a similar return distribution and funding ratio, 1.363. The saving rates 26% and 30.7% at 2017 and 2020 valuation respectively are also considered in the simulation study.

The first two rows of Table 5 report the case when the Scheme is closed so that the saving rate is zero and liquidity constraint is the largest. As it takes 60 years to pay all benefits earned to date, the reported probabilities represent the insolvency risks when the Scheme is closed. Liquidity constraints render the risks higher than those implied by the distribution-based prudence discussed in Section 3. For example, according to the distribution measure, a unit funding ratio suggests a 50% insolvency risk. However, the actual insolvency risk is 62.4%. Consistent with the analysis above, the Scheme's insolvency risk reduces to 36.7% if the funding ratio is raised to 1.363. Based on the 2020 projected outgo of USS, Miles and Sefton (2021) obtains a slightly higher insolvency risk, at 40%. The difference can be explained by a higher portfolio volatility assumed by Miles and Sefton.

The expected return of 3.03% implies a steady state saving rate at 14.3%. For non-stochastic returns, at unit funding ratio, the asset with zero saving rate is just enough to pay for outgo whereas the asset with 14.3% saving rate is constant from year to year. Note that saving rates higher than 14.3% would raise the funding ratio which would be an additional factor to affect the insolvency risk. Therefore, the reduction in insolvency risk from 62.4% to 49.9% provides a measure of the impact of a reduced liquidity constraint when the saving rate increases from nil to 14.3%. As liquidity constraints are further lessened by a higher funding ratio (f = 1.363), the insolvency risk further declines to 20.7%.

Table 5. Risks of underfunding and insolvency

			60	120	240	800
s=0%	**f=1.363**	$\boldsymbol{P(A_T < 0)}$	0.367			
Closed scheme	**f=1.000**	$\boldsymbol{P(A_T < 0)}$	0.624			

s=14.3%	f=1.363	$P(A_T < L)$	0.405	0.462	0.482	0.484
		$P(A_T < 0)$	0.207	0.418	0.478	0.484
	f=1.000	$P(A_T < L)$	0.698	0.736	0.749	0.750
		$P(A_T < 0)$	0.499	0.706	0.746	0.750
s=26.0%	f=1.363	$P(A_T < L)$	0.082	0.057	0.046	0.045
		$P(A_T < 0)$	0.002	0.027	0.044	0.045
	f=1.000	$P(A_T < L)$	0.202	0.152	0.133	0.132
		$P(A_T < 0)$	0.018	0.089	0.128	0.132
s=30.7%	f=1.363	$P(A_T < L)$	0.028	0.011	0.003	0.002
		$P(A_T < 0)$	0.000	0.001	0.002	0.002
	f=1.000	$P(A_T < L)$	0.076	0.024	0.009	0.008
		$P(A_T < 0)$	0.000	0.003	0.008	0.008

The reported underfunding and insolvency risks of the DB Scheme are obtained using 10,000 simulated scenarios of normally distributed returns with mean 3.03% and standard deviation 10%. The Reference Portfolio of USS as of March 2020 has approximately 60% equity and similar return distribution. s and f are saving rate and initial funding ratio respectively.

When the saving rate is 26% or higher, as explained in Section 3, the underfunding risk falls with T because the surplus (due to prudence) raises the funding ratio over time. Because the Scheme is now prudently funded, the insolvency risks drop to a level more secured than is required for self-sufficiency or low dependency. Note that at the 14.3% saving rate, since there is no surplus, liquidity constraints cause the underfunding risk to rise with T. In the long run, both underfunding and insolvency risks converge to the same probability, as the Scheme will either run out of money or be massively overfunded with outgo fully paid.

Finally, as the time horizon increases and the saving rate approaches the maximum required saving rate, both underfunding and insolvency risks not only decrease, the difference between them also diminishes. Moreover, the role of funding ratio declines, which can be explained by the disappearance of $ln(L/A_t)$ in the further simplification of Eqn. 14 to

$$P(A_{t+T} < L) \approx P\left(z < \frac{-\sqrt{T}\mu}{\sigma}\right). \quad (17)$$

In short, these observations are consistent with the existence of a maximum required saving rate and the role of cash contributions in mitigating risks in the DB Scheme.

4.2 Implications

The implications of the above findings for pricing and management of DB pensions are discussed below.

Interest rate does not price DB benefits

Bodie (1990, 2006), Gold and Hudson (2003), Wilcox (2006) among others propose to use a 100% risk-free bond portfolio to value DB pensions. Using the real gilt yield of −1.14% in Table 1, this approach would require 48% of payroll to pay for the accrued benefits of the DB Scheme. According to Eqn. 12, this is higher than the maximum required saving rate of 34.7%. The discrepancy can be explained by the fact that competitive financial markets are incomplete for long-term products such as pensions and thus Ross's (1976)'s arbitrage-free conditions are not sufficiently met to use interest rates to price open DB schemes. Financial institutions make arbitrage profit by borrowing at short-term interest rates and making long-term loans at higher interest rates. Likewise, when interest rates are low, DB schemes can exploit intergenerational risk sharing to benefit pensioners by investing in risky assets.

The above does not mean that interest rate is inappropriate to price, for example, the buy-out of accrued benefits with an insurance company for purposes of scheme closure. This is illustrated by the simulation results in Table 5 which show high insolvency risks for a closed scheme.

Tail risk of equity

Section 3 on prudence shows that the tail risk of equity can be large. Instead of a low-risk self-sufficiency portfolio, the above study in this section shows that cash contributions from working members can help remove the tail risk of equity faced by pensioner members. Indeed, the maximum required saving rate puts a cap on the cost of eliminating the risks of extreme downturns if a risky investment strategy is adopted.

Self-sufficiency and covenant

As noted in Section 2, self-sufficiency is not required by legislation. Notwithstanding this, the simulation study shows that at 26% and 30.7% saving rates, the risks to the accrued benefits of the Scheme are lower than 5% which is expected of self-sufficiency. Since the cost of tail risk is limited to the maximum required saving rate, de-risking a truly open DB scheme like USS towards a low-risk self-sufficiency is unnecessarily expensive at low interest rates and goes against TPR's statutory objective to minimise any adverse impact on the growth of an employer.[19]

For USS, a revision from strong to tending-to-strong (TTS) covenant reduces the additional employers' 10% of payroll contributions from 30 years to 20 years and increases the 2020 deficit by £8.1bn. However, no further funding from the employer is needed in the simulation study to make the accrued benefits more secured than that of a self-sufficient scheme. Hence for open schemes, rather than in financial terms, the nature of employers' business and the likelihood of employers to simply continue such activities play a more important role in determining the strength of a covenant. This is especially relevant for USS

[19] The statutory objective is in relation to the exercise of TPR's functions under Part 3 of the Pensions Act 2004 only. See https://www.thepensionsregulator.gov.uk/en/document-library/codes-of-practice/code-3-funding-defined-benefits-

whose sponsoring employers comprise essentially the UK Higher Education Sector. Even if we allow for the possibility that the cash contributions of USS were to decline in future, the role of a covenant is unlikely to be as large as that in the 2020 valuation.

Estimate of maximum required saving rate for USS

The relevance of the above discussion for USS may be further appreciated if an estimate of its maximum required saving rate can be obtained. When $\mu = 0$, the liability in Eqn. 5 reduces to $nb(m + x)$ and the asset in Eqn. 6 is simply ns. Equating them gives rise to the FSC which happens to be the same as the maximum required saving rate in Eqn. 12. Therefore, the 37.6% of payroll FSC at CPI+0% discount rate in Table 2 serves as an estimate for the maximum required saving rate of USS.

Table 5 shows that the 26% saving rate, which is 8.7% below the maximum required saving rate, makes the Scheme as secured as a closed self-sufficient scheme. USS could have the same level of security for its accrued benefits at a 30.7% saving rate, which is 6.9% lower than its estimated maximum required saving rate.

Deficit recovery

As long as the Scheme invests sufficiently in risky assets so that $\mu > 0$, Eqn. 17 shows that the security of accrued benefits is independent of funding ratio. This implies that the deficit recovery of a truly open scheme could be 'more generous' in terms of outperformance over the discount rate and number of years for recovery. For example, a unit funding ratio would be indicative of a deficit. Table 5 shows that at the 26% saving rate the insolvency risk when the funding ratio is 1 amounts to only 13.2% for the Scheme; at the 30.7% saving rate the insolvency risk reduces to 0.8% which is lower than the level represented by self-sufficiency. In the 2020 valuation of USS, despite the large $\mu -$ discount rate that ranges from 2.5% to 3%, only 0% and 0.5% outperformance over the discount rate are considered, thereby giving rise to contribution rates that exceed the maximum required saving rate; see Table 2.

Omission of cash contributions in risk mitigation

What are the reasons for the large difference between the results of this paper and USS? To answer this question, first note that if the Scheme were to be closed, the insolvency risk would be similar to that found by Miles and Sefton (2021), who agree with the deficits of USS. However, Miles and Sefton are analysing the projected outgo of USS as if from a closed scheme. USS is clearly not a closed scheme. Furthermore, despite recommendations made by stakeholders, USS does not properly consider the role of cash contributions in mitigating risks; see First Actuarial (2017) and Universities UK (2020). As a result of omitting a relevant factor (cash contributions), other less relevant factors (a strong versus a tending-to-strong covenant; de-risking towards self-sufficiency for valuation purposes) come into play.

Consequently, the 2020 valuation produces large deficits (as much as £17.9bn) and high contribution rates (up to 67.9% of payroll) that are, even allowing for fluctuations in cash contributions in the real world, more like the "*combinations of extreme values that rarely or never occur together in practice*" than what is truly needed for USS.[20] As stated above, the high contribution rates are the result of not recognizing that the tail risks of extreme downturns can be fully diversified by intergenerational risk sharing enabled by cash contributions in a truly open scheme.

5. Long-term objectives under a low interest rates environment

This section applies the findings presented in the last two sections to show, under the current low interest rate environment, how (tail) risks from a high-equity investment strategy can be mitigated and the various possible affordable Long-Term Objectives (LTO) on funding and investment strategies realised. For comparative purposes, the impact of low interest rates on defined contribution schemes is first investigated.

5.1 Defined contribution scheme

When at work, a member in a DC scheme saves £s in a portfolio of equity and bond, leaving £$c_t = £(1-s)$ for consumption. The savings give rise to asset A_t, which is used to provide a pension drawdown £a_t during retirement. Thus, the consumption is

$$c_t = \begin{cases} 1-s & 1 \le t < T_R \text{(at work)} \\ a_t & T_R \le t < T_D \text{(retirement)}, \end{cases} \tag{18}$$

and the asset evolves according to

$$A_{t+1} = \begin{cases} (A_t + s)(1 + r_{t+1}) & 1 \le t < T_R \\ (A_t - a_t)(1 + r_{t+1}) & T_R \le t < T_D. \end{cases} \tag{19}$$

In the above two equations, t = 1, T_R = 37 and T_D = 60 correspond to age 30, 67 (first year of retirement) and 90 (death) respectively. The expected return on asset is given by $E(r_t) = (1-w_t)y + w_t\mu_e$, where $w_t \in [\omega, 1]$ is the weight of equity in the asset. Like Feldstein and Ranguelova (1998) and Chen *et al.* (2016), a_t is the value of an annuity that costs A_t, has term $T_D - t$ and a discount rate equal to the geometric mean of $E(r_t)$ during the period $[t, T_D)$.[21]

Members feature a constant relative risk aversion (CRRA) utility given below:

[20] The quoted scenario is what the guidance on producing quality analysis for government tries to warn against; see footnote 6.
[21] Feldstein and Ranguelove (1998) and Chen *at al.* (2016) assume a constant expected return.

$$U = E_0 \left\{ \sum_{t=1}^{T_D - 1} e^{-\delta t} \frac{c_t^{1-\gamma}}{1-\gamma} \right\}, \tag{20}$$

where the risk aversion parameter γ = 5, and the discount factor δ = 4%. ω_0 of savings are invested in equity from the beginning until just before year T_a, then decline linearly to a constant weight ω_1 in year T_b. Optimisation over the consumption and portfolio choice (i.e., $\omega_0, \omega_1, T_a, T_b$ and s) is carried out so that the maximum expected utility over the lifetime of the member can be obtained.

Table 6. Optimal defined contribution scheme

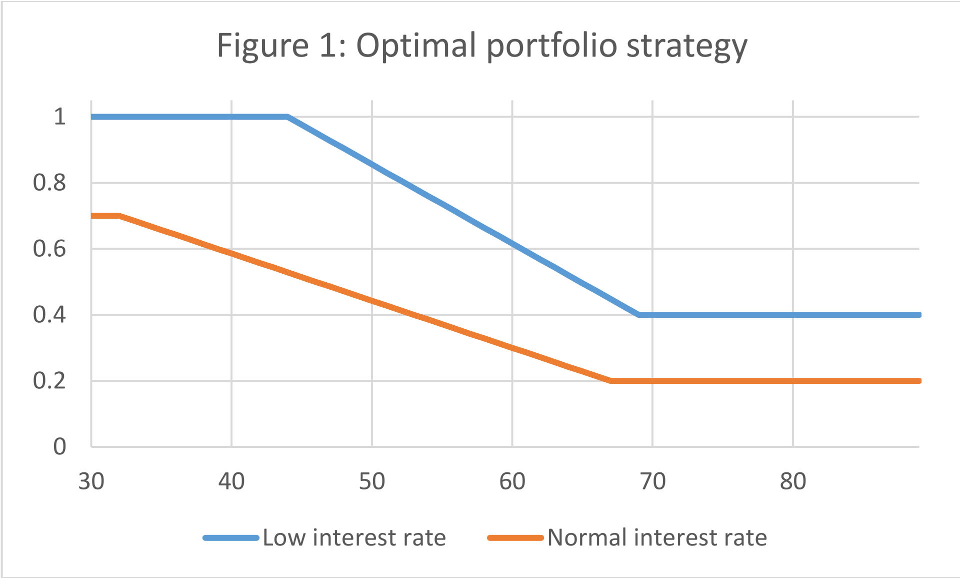
		s	ω_0	ω_1	T_a	T_b	CEC
Low interest rates:	Optimal s	0.36	1.0	0.4	44	69	0.2322
y = −1.14%, μ_e = 4.39%	2017 rate	0.26	1.0	0.4	44	69	0.2040
Normal interest rates:	Optimal s	0.29	0.7	0.2	32	67	0.2641
y = 2%, μ_e = 4.39%	2017 rate	0.26	0.7	0.2	32	67	0.2607

10,000 simulations were carried out the obtain the reported figures. s, ω_0, ω_1 and CEC are saving rate, weights of equity and certainty equivalent consumption. Before year T_a, the portfolio invests ω_0 of savings in equity. From year T_b onwards, the weight of equity remains constant at ω_1.

To understand the impact of low interest rates and restrictions due to members' affordability, the normal level of interest rate (y = 2%) used in Chen *et al.* (2016) and the 26% of payroll saving rate prior to USS's 2017 contribution rate are considered. For simplicity, optimisation is done by grid search over w_t (in steps of 10% increment), T_a and T_b. The optimal saving, portfolio strategy and welfare as measured by the certainty equivalent consumption (*CEC*) are reported in Table 6.[22]

For optimal strategies, the fall in interest rates lowers members' welfare by 12% from 0.2641 to 0.2322. Since a saving rate of 36% is likely unaffordable, restricting the saving rate to 26% increases the loss in welfare to 21.7%. The low interest rate also raises the optimal saving rate from 29% to 36% and encourages more risk taking as depicted in Figure 1.

[22] The certainty equivalent consumption is calculated as $CEC = \left(U(1-\gamma)\right)^{1/(1-\gamma)}$.



Grid search with 10,000 simulations was used to obtain the optimal portfolio strategy specified by T_a, T_b, ω_0 and ω_1. Before year T_a, the portfolio invests ω_0 of savings in equity. From year T_b onwards, the weight of equity remains constant at ω_1.

5.2 Defined benefit scheme

In the Scheme that offers defined benefits, a member's consumption is given by

$$c_t = \begin{cases} 1 - s_t & 1 \leq t < T_R \text{ (work)} \\ (x+1)B_{T_R,0} & t = T_R \\ B_{T_R,0} & T_R < t < T_D \text{ (retirement),} \end{cases} \tag{21}$$

where $x = 3$ and $B_{T_R,0}$ is given by Eqn. 4. The asset evolves according to

$$A_{t+1} = (A_t + ns_t - B_t)(1 + r_{t+1}), \tag{22}$$

where the outgo B_t is given in Eqn. 3. The investment risk to the scheme is mitigated by first obtaining the required adjustment rate below.

$$\xi_t = \alpha\left(A_t - f_p L_t\right)/n, \tag{23}$$

where f_p is a pre-determined funding ratio required for prudence, α is the deficit recovery rate and n is the number of working members. If the deficit (surplus) represented by a negative (positive) ξ_t is within $\pm\theta$, then only the saving rate will be adjusted according to the first line of Eqn. 24. For large ξ_t, adjustment to the saving rate is capped at $\pm\theta$ and the remaining adjustment within $\pm\vartheta$ is applied to the accrual rate, as given in the last two lines of the same equation. In practice, if any adjustment is required, s_t will more likely be considered first; changes to b_t are less frequent. The adjustment order described in Eqn. 24 reflects this arrangement. Any remaining adjustments that exceed $\pm\vartheta$ will impact on the asset. For a prudently funded DB scheme, the impact on assets will likely be positive, thereby giving rise to a rising funding ratio over time. This study follows follow Cui *et al.* (2011) in calling s_p and b_p as 'target' saving and accrual rates respectively.

$$\begin{array}{llll} \text{If } -\theta \le \xi_t \le \theta, & s_t = s_p - \xi_t, & b_t = b_p & \\ \text{If } \quad \xi_t \le -\theta, & s_t = s_p + \theta, & b_t = b_p - \min(\theta - \xi_t, \vartheta)/(m+3) & \quad (24) \\ \text{If } \quad \xi_t \ge \theta, & s_t = s_p - \theta, & b_t = b_p + \min(\xi_t - \theta, \vartheta)/(m+3) & \end{array}$$

Due to prudence and other regulatory concerns, this study does not follow Cui *et al.* (2011) and Chen *et al.* (2016) in determining an optimal DB scheme over the various parameters in the above two equations. Rather, the focus here is to study whether a risky investment strategy enables a DB scheme to meet TPR's LTO on funding and investment strategies under a low interest rate environment. Like self-sufficiency, LTO allow DB schemes to reach a low dependency on the sponsoring employers in the long-term.

Unless stated otherwise, the parameter values are s_p = 26%, b_p = 1/75, f_p = 1.363, α = 0.05, θ = 9% and ϑ = 5%, and asset begins at f_0 = 1.363 times the initial liability. Asset returns on portfolios of bond and equity are simulated using the return distributions given in Table 2. These parameter values are relevant to members of USS which has a similar benefit level, funding ratio and return distribution as of March 2020. Based on 10,000 simulated scenarios, Table 7 reports for various weights of equity w, expected return μ, liability L_0, average saving rate s, accrual rate b_T, cash outflows d_0 and d_T, funding ratios f_T and $f_{T(0.1)}$, and certainty equivalent consumption CEC. The subscripts (0.1), 0 and 1 of the above variables denote the tenth percentile, the beginning, and the end of lifetime respectively.

Others being equal, a member's welfare is lower (higher) if the asset finishes higher (lower) at the end of his lifetime. In Panel A, where accrual rate is fixed, the target saving rate s_* is adjusted so that the average asset at end of lifetime, A_T, equals the initial asset. Consistent with the analysis on prudence in Section 3, as the weight of equity increases, the higher returns lower the average saving rate and raise welfare as measured by the CEC. The gain in welfare, however, is accompanied by large cash outflow and a low tenth percentile funding ratio.[23] This renders the strategies in Panel A not suitable as LTO to achieve low dependency.

Table 7. Long-Term Objectives of the DB Scheme

	w	μ(%)	L_0	s	$1/b_T$	d_0	d_T	f_T	$f_{T(0.1)}$	CEC
Panel A: Average end-period asset equal initial asset										
y=-1.14%	0.0	-1.14	469	0.547	75	-1.154	-1.293	1.362	1.117	0.1810
f_0=1.363	0.2	-0.03	384	0.353	75	-0.042	-0.187	1.364	1.083	0.2702
A_T=fL_0	0.4	1.07	320	0.223	75	1.082	0.800	1.361	0.960	0.3050
θ=35%	0.6	2.18	272	0.137	75	2.254	1.682	1.360	0.767	0.3215
ϑ=0	0.8	3.28	235	0.081	75	3.514	2.564	1.366	0.496	0.3304
	1.0	4.39	206	0.055	75	4.888	4.805	1.359	0.131	0.3356
Panel B: Rising asset level, low interest rate										
y=-1.14%	0.0	-1.14	469	0.334	75	0.502	-0.039	0.648	0.379	0.2886
f_0=1.363	0.2	-0.03	384	0.302	75	0.613	0.172	1.070	0.677	0.2952

[23] Note that the expected return of 40% equity strategy is roughly the same as that of the Scheme studied. Because of 'surplus' due to prudence, as explained in Section 3, the average saving rate of 40% equity strategy is lower than the steady state saving rate of 26%.

s_p=26%	0.4	1.07	320	0.252	75	0.735	0.615	1.648	0.914	0.3023
θ=9%	0.6	2.18	272	0.226	75	0.864	0.735	2.954	1.064	0.3060
ϑ=0	0.8	3.28	235	0.215	75	1.000	0.688	5.704	1.172	0.3077
	1.0	4.39	206	0.211	75	1.141	0.618	11.121	1.257	0.3085
Panel C: Rising asset level, normal interest rate										
y=2%	0.0	2.00	279	0.206	75	0.843	0.981	2.369	1.427	0.3107
f_0=1.363	0.2	2.48	261	0.196	75	0.900	0.908	3.099	1.618	0.3126
s_p=26%	0.4	2.96	245	0.196	75	0.959	0.797	4.258	1.679	0.3122
θ=9%	0.6	3.43	231	0.200	75	1.019	0.715	5.901	1.619	0.3111
ϑ=0	0.8	3.91	218	0.205	75	1.079	0.659	8.135	1.464	0.3098
	1.0	4.39	206	0.211	75	1.141	0.618	11.121	1.257	0.3085
Panel D: Rising asset level and improving accrual rate										
y=-1.14%	0.0	-1.14	469	0.331	86.7	0.502	-0.639	0.789	0.504	0.2806
f_0=1.363	0.2	-0.03	384	0.299	80.1	0.613	-0.074	1.139	0.829	0.2922
s_p=26%	0.4	1.07	320	0.253	73.2	0.735	0.598	1.558	1.012	0.3033
θ=9%	0.6	2.18	272	0.227	69.7	0.864	0.814	2.608	1.112	0.3102
ϑ=5%	0.8	3.28	235	0.216	68.2	1.000	0.784	4.932	1.188	0.3138
	1.0	4.39	206	0.212	67.6	1.141	0.701	9.587	1.260	0.3155
Panel E: Asset begins at unit funding ratio										
y=-1.14%	0.0	-1.14	469	0.335	75	0.684	-0.055	0.469	0.277	0.2889
f_0=1.0	0.2	-0.03	384	0.307	75	0.836	0.182	0.750	0.486	0.2949
s_p=26%	0.4	1.07	320	0.264	75	1.001	0.758	1.069	0.639	0.3013
θ=9%	0.6	2.18	272	0.236	75	1.178	1.007	1.800	0.720	0.3051
ϑ=0	0.8	3.28	235	0.223	75	1.363	0.997	3.438	0.773	0.3069
	1.0	4.39	206	0.218	75	1.555	0.924	6.771	0.801	0.3077

The bond and equity return distributions in Table 2 are used to generate the asset returns. 10,000 scenarios of asset return were simulated to obtain the reported statistics. The subscripts (0.1), 0 and T denote the tenth percentile, the beginning, and the end of lifetime respectively. w, μ, L, s, b, d, f and CEC are weight of equity, expected portfolio return, liability, average saving rate, average accrual rate, average cash outflow, average funding ratio and certainty equivalent consumption respectively. A_T in Panel A is the average of asset at end of lifetime. y is the interest rate whereas θ and ϑ are bounds on saving rate and accrual rate respectively.

The parameters in Panel C are the same as Panel B except for the higher interest rate. As expected, a higher interest rate improves welfare and lowers risk. Possibly due to a lower risk premium of 2.39%, 20% equity becomes the optimal investment strategy. A caveat here is that the analysis assumes more funds are available for a low-risk strategy as the asset begins at the same multiple of initial liability.

Panel D allows assets to accumulate and the accrual rate to increase. This results in higher CECs than Panel B. While higher accrual rates imply higher maximum required saving rates, the funding ratio remains high whereas cash outflow is low. Because the benefit level is allowed to rise, the all-bond and 20%-equity strategies produce a higher welfare than those in Panel A. Compared with the optimal DC scheme, the all-equity strategy improves members' welfare by 19%.

Assets begin at unit funding ratio in Panel E. Compared with Panel B, the difference due to a lower funding ratio is surprisingly small: less than half a percent drop in welfare at the expense of a small rise in contributions. For weight of equity more than 40%, liquidity constraints in terms of cash outflow and funding ratios at end of lifetime lessen, albeit not to the same extent as in Panel B.

To sum up the above, a full-equity investment strategy is optimal in the current low interest environment. In the case of Panel B, C and E, $s_p + \theta$ = 35% ensures the contribution rate is aways large enough to guarantee payment for benefits accrued at rate b_p = 1/75. For Panel D, tail risks could be mitigated by a high funding ratio and low cash outflow. Also, DB schemes offer significant welfare improvements over DC schemes, which suffer from volatile consumption. These findings are consistent with Gollier (2008) who shows that the intergenerational risk sharing of DB schemes enhances welfare by generating a higher return as well as smoothing volatile consumption. Finally, Panel B and D illustrate the various possible ways to achieve both affordability and LTO on funding and investment strategies.

6. Conclusion

The widely held view that guaranteed pensions should be priced using interest rates renders DB schemes exorbitantly expensive under a low interest rate environment. However, it is shown that interest rate fails to meet the no arbitrage pricing conditions necessary to price an open DB scheme.

At negative interest rates, DC schemes require their members to save more and invest in riskier assets. Despite these efforts, the reduction in welfare is significant. On the other hand, by virtue of time diversification, the impact of low interest rates on DB schemes is minimal if a risky investment strategy is adopted. This paper finds that it is optimal for DB schemes to pursue an equity investment strategy under a low interest rate environment, as the risks (including those of extreme downturns) can be effectively diversified across different generations via cash contributions.

This paper also finds that future service costs need not be priced prudently if, for example, affordability is an issue.

The funding ratio of a prudently funded DB scheme is on a rising trajectory. A simulation study was carried out to show how prudence can be divided between adding further prudence, lowering the saving rate and raising the benefit level for future generations. By means of a higher funding ratio and lower cash outflow to minimise the liquidity constraints, Long-Term Objectives to achieve low dependency can be affordable without compromising the security of accrued benefits. An affordable DB pension resolves several issues faced by the current pension industry, such as strained industrial relations in the Higher Education Sector and the high opt-out rate among younger members.

The valuations of USS overlook the role of cash contributions in mitigating risk. This results in less relevant factors such as

i. a strong versus a tending-to-strong covenant
ii. de-risking to self-sufficiency for valuation purposes
iii. significant holding of self-sufficiency securities in dual-discount rate approach

being given undue importance. As a result, the welfare enhancing feature of intergeneration risk sharing cannot be realised. Even allowing for a possible decline in cash contributions in the real world, the exorbitantly high contribution rates in the 2020 valuation are more likely an outcome of combinations of extreme scenarios that rarely or never occur together in practice. This goes against the guidelines on producing quality analysis for the UK government.

Reference

Ball, L. and Mankiw, N.G., 2007. Intergenerational risk sharing in the spirit of Arrow, Debreu, and Rawls, with applications to social security design. *Journal of Political Economy*, *115*(4), pp.523-547.

Balvers, R., Wu, Y. and Gilliland, E., 2000. Mean reversion across national stock markets and parametric contrarian investment strategies. *The Journal of Finance*, *55*(2), pp.745-772.

Bodie, Zvi. "The ABO, the PBO and pension investment policy." *Financial Analysts Journal* 46, no. 5 (1990): 27-34.

Bodie, Z., 2006. On asset-liability matching and federal deposit and pension insurance. Review-Federal Reserve Bank of Saint Louis, 88(4), p.323.

Bodie, Z., Kane, A. and Marcus, A., 2014. *EBOOK: Investments-Global edition*. McGraw Hill.

Bohn, H., 2009. Intergenerational risk sharing and fiscal policy. *Journal of monetary Economics*, *56*(6), pp.805-816.

Boubaker, S., Gounopoulos, D., Nguyen, D.K. and Paltalidis, N., 2018. Assessing the effects of unconventional monetary policy and low interest rates on pension fund risk incentives. *Journal of Banking & Finance*, 92, pp.340-357.

Boyle, G.W. and Guthrie, G.A., 2005. Human capital and popular investment advice. *Review of Finance*, *9*(2), pp.139-164.

Breedon, F. and Larcher, L., 2021. Discounting and the market valuation of defined benefit pensions. *European Financial Management*.

Chen, D.H., Beetsma, R.M., Ponds, E.H. and Romp, W.E., 2016. Intergenerational risk-sharing through funded pensions and public debt. *Journal of Pension Economics & Finance*, *15*(2), pp.127-159.

Cobb, J.A., 2015. Risky business: The decline of defined benefit pensions and firms' shifting of risk. *Organization Science*, *26*(5), pp.1332-1350.

Cui, J., De Jong, F. and Ponds, E., 2011. Intergenerational risk sharing within funded pension schemes. *Journal of Pension Economics & Finance*, *10*(1), pp.1-29.

De Bondt, W.F. and Thaler, R., 1985. Does the stock market overreact? *The Journal of finance*, 40(3), pp.793-805.

De Jong, F., 2008. Pension fund investments and the valuation of liabilities under conditional indexation. *Insurance: Mathematics and Economics*, *42*(1), pp.1-13.

Diamond, P.A., 1977. A framework for social security analysis. *Journal of public economics*, *8*(3), pp.275-298.

Dyck, A. and Pomorski, L., 2016. Investor scale and performance in private equity investments. *Review of Finance*, *20*(3), pp.1081-1106.

Epstein, L.G., 1983. Decreasing absolute risk aversion and utility indices derived from cake-eating problems. *Journal of Economic Theory*, *29*(2), pp.245-264.

Feldstein, M. and Ranguelova, E., 2001. Individual risk in an investment-based social security system. *American Economic Review*, *91*(4), pp.1116-1125.

First Actuarial, 2017. Progressing the valuation of the USS. https://www.ucu.org.uk/media/8705/Progressing-the-valuation-of-the-USS-First-Actuarial-Sep-17/pdf/firstactuarial_progressing-valuation-uss_sep17.pdf. Accessed: 30 October 2018.

Franzen, D., 2010. Managing investment risk in defined benefit pension funds. OECD Working Papers on Insurance and Private Pensions, No. 38, OECD Publishing.

Gold, J. and Hudson, N., 2003. Creating value in pension plans (or, gentlemen prefer bonds). *Journal of Applied Corporate Finance*, *15*(4), pp.51-57.

Gollier, C., 2002. Time diversification, liquidity constraints, and decreasing aversion to risk on wealth. *Journal of Monetary Economics*, *49*(7), pp.1439-1459.

Gollier, C., 2008. Intergenerational risk-sharing and risk-taking of a pension fund. *Journal of Public Economics*, *92*(5-6), pp.1463-1485.

Gordon, R.H. and Varian, H.R., 1988. Intergenerational risk sharing. *Journal of Public economics*, *37*(2), pp.185-202.

Hacker, J.S., 2019. *The great risk shift: The new economic insecurity and the decline of the American dream*. Oxford University Press, New York.

Joliffe, A., 2005, Ex-Boots executive challenges pension trustees, *Financial Times* February 12, 2.

Kalleberg, A.L., 2009. Precarious work, insecure workers: Employment relations in transition. *American sociological review*, *74*(1), pp.1-22.

Kalleberg, A.L., 2011. *Good jobs, bad jobs: The rise of polarized and precarious employment systems in the United States, 1970s-2000s*. Russell Sage Foundation, New York.

Krueger, D. and Kubler, F., 2002. Intergenerational risk-sharing via social security when financial markets are incomplete. *American Economic Review*, *92*(2), pp.407-410.

Lakonishok, J., Shleifer, A. and Vishny, R.W., 1994. Contrarian investment, extrapolation, and risk. *The journal of finance*, *49*(5), pp.1541-1578.

Lu, L., Pritsker, M., Zlate, A., Anadu, K. and Bohn, J., 2019. Reach for yield by US public pension funds. *FRB Boston Risk and Policy Analysis Unit Paper No. RPA*, pp.19-2.

Maer, L. and Thurley, D., 2009. Defined Benefit pension schemes. *House of Commons Library, 9th March*.

Miles, D., 2018. The state of USS: some observations. https://medium.com/ussbriefs/the-state-of-uss-some-observations-ffc4b1400d1a. Accessed: 25 July 2022.

Miles, D. and Sefton, J.A., 2021. *How much risk is the USS taking?* National Institute of Economic and Social Research.

Ralfe, J., 2005. U.K. firms taking risks with equities. *Financial Times*, September 19, 2.

Ross, S., 1976. The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory*, *13*, pp.341-360.

Samuelson, P., 1963. Risk and uncertainty: A fallacy of large numbers. *scientia*, *57*(98).

Samuelson, P.A., 1989a. The judgement of economic science on rational portfolio man. *Journal of portfolio Management*, *16*(1), p.4.

Samuelson, P.A., 1989b. A case at last for age-phased reduction in equity. *Proceedings of the national academy of sciences*, *86*(22), pp.9048-9051.

Shiller, R.J., 1999, June. Social security and institutions for intergenerational, intragenerational, and international risk-sharing. In *Carnegie-Rochester Conference Series on Public Policy* (Vol. 50, pp. 165-204). North-Holland.

Schrager, A., 2009. The decline of defined benefit plans and job tenure. *Journal of Pension Economics & Finance*, *8*(3), pp.259-290.

Strong, N. and Taylor, N., 2001. Time diversification: empirical tests. *Journal of Business Finance & Accounting*, *28*(3-4), pp.263-302.

Thorley, S.R., 1995. The time-diversification controversy. *Financial Analysts Journal* 51, 68-76.

Universities Superannuation Scheme, 2014. A consultation for the 2014 valuation.

Universities Superannuation Scheme, 2017. A consultation for the 2017 valuation.

Universities Superannuation Scheme, 2020. A consultation for the 2020 valuation.

Universities UK, 2020. UUK Representatives' Views on Progress of the USS Valuation Methodology Discussion Forum (VMDF). https://www.ussemployers.org.uk/briefing-resources/background-and-briefings/uss-valuation-methodology-discussion-forum-2020-valuation. Accessed: 26 July 2022.

Weil, P., 2008. Overlapping generations: The first jubilee. *Journal of Economic Perspectives*, *22*(4), pp.115-34.

Weiss, L., 1980. The effects of money supply on economic welfare in the steady state. *Econometrica: Journal of the Econometric Society*, pp.565-576.

Wilcox, D.W., 2006. Reforming the defined-benefit pension system. *Brookings Papers on Economic Activity*, *2006*(1), pp.235-304.

Wong, W.K., 2018. The Discount Rate Debate and Its Implications for Defined Benefit Pensions. *Cardiff Economics Working Papers* E2018/12.

Appendix

Let $L_R(i)$ denote the liability due to a retired member whose life expectancy is i years and $L_W(j)$ be the liability due to a working member who has contributed j years. Let $\kappa_{\mu,m} = \mu^{-1}(1-(1+\mu)^{-m})$ denote the annuity. The liability of the DB Scheme is

$$\sum_{i=1}^{m} L_R(i) + \sum_{l=1}^{n} L_W(i), \tag{25}$$

where

$$\sum_{i=1}^{m} L_R(m) = nb\left(x + (1+\mu^{-1})(m - \kappa_{\mu,m})\right), \tag{26}$$

and

$$\sum_{l=1}^{n} L_W(i) = B\left(n\kappa_{\mu,n} - \mu^{-1}\kappa_{\mu,n} + n\mu^{-1}(1+\mu)^{-n}\right). \tag{27}$$

Proof:

Using the annuity formula, we have for $1 \leq i \leq m$

$$L_R(i) = nb\left(x + \mu^{-1}(1+\mu)\left(1-(1+\mu)^{-i}\right)\right). \tag{28}$$

The liability of all retired members is

$$\sum_{i=1}^{m} L_R(i) = nb\{x + \mu^{-1}(1+\mu)[m - (1+\mu)^{-m} - \cdots - (1+\mu)^{-1}]\}. \tag{29}$$

The proof of Eqn. 26 is complete by noting that $(1+\mu)^{-m} + \cdots + (1+\mu)^{-1} = \kappa_{\mu,m}$.

Let $B = b\left(x + (1+\mu)\kappa_{\mu,m}\right)$. For working members, we have

$$L_W(n) = (1+\mu)^{-1}nB \tag{30}$$

$$L_W(n-1) = (1+\mu)^{-2}(n-1)B \tag{31}$$

$$\vdots$$

$$L_W(1) = (1+\mu)^{-n}B \tag{32}$$

Let $a = (1+\mu)^{-1}$. Then

$$\begin{aligned}\sum_{l=1}^{n} L_W(i) &= B(an + a^2(n-1) + a^3(n-2) + \cdots + a^n) \\ &= B\left(n\kappa_{\mu,n} - \frac{a^2}{1-a}\frac{1-na^{n-1}+(n-1)a^n}{1-a}\right).\end{aligned} \tag{33}$$

Substituting $a = (1+\mu)^{-1}$ into above, we obtain

$$\begin{aligned}\sum_{l=1}^{n} l(i) &= B\left(n\kappa_{\mu,n} - \frac{(1-n(1+\mu)^{1-n} + (n-1)(1+\mu)^{-n})}{\mu^2}\right) \\ &= B\left(n\kappa_{\mu,n} - \mu^{-1}\left(\kappa_{\mu,n} + n(1+\mu)^{-n}\frac{1-(1+\mu)}{\mu}\right)\right),\end{aligned} \tag{34}$$

which gives Eqn. 27.