Optimising Currency Factors[⋆,⋆⋆]

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Abstract

We propose a novel framework of dynamic optimization schemes for currency market factors including carry, momentum, and value. We examine the performance of 24,336 portfolio optimization approaches and find that the optimized currency factors significantly outperform the naive factors after correcting for data snooping bias. We find that the superior performance of the optimal factor portfolios is derived from the standard deviation of the abnormal return distribution, driven primarily by emerging market currencies. An out-of-sample procedure that aggregates all our outperforming optimization approaches validates the economic significance of our optimized currency factor portfolio.

Keywords: Currency factor; Foreign exchange; Multiple hypothesis testing; Portfolio optimization.

JEL: C11, F31, G11

[⋆]The original version: June, 2021. This version: October 2023.

[⋆⋆]We thank seminar participants at the Doctoral Consortium of 2021 FMA European Conference.

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1. Introduction

Modern portfolio theory, as introduced by Markowitz (1952), has served as a guiding framework for the construction of diversified investment portfolios using expected returns and covariance matrices. However, scholars have found that the use of portfolio optimisation approaches often leads to poor out-of-sample performance (Black and Litterman, 1992, Kan and Zhou, 2007), mainly due to estimation errors (Merton, 1980, Jobson and Korkie, 1981). Empirical studies such as DeMiguel et al. (2009) and Hsu et al. (2018) further revealed that optimised portfolios often fail to outperform the naive diversification approach in equity markets. However, this dynamic is not observed in currency markets, where mean-variance optimisation has been shown to deliver consistently significant out-of-sample profits (Baz et al., 2001, Della Corte et al., 2009, Ackermann et al., 2017, Daniel et al., 2017, Maurer et al., 2023).

Studies of currency market predictability have predominately focused on the cross-sectional sorting of currency characteristics, the most famous of which are carry (Lustig et al., 2011), momentum (Menkhoff et al., 2012*a*,*b*) and value (Asness et al., 2013) factors. However, more recently, Chernov et al. (2023) advocate that currency market studies should shift away from equity-based factor models and find that an unconditional mean-variance efficient portfolio can effectively price currency strategies. To the best of our knowledge, none of the existing studies extensively examine the combined effect of currency factors and optimisation.

In this paper, we introduce a novel approach for optimising currency factors to fill the literature gap mentioned above. For each currency factor, we perform 24,336 combinations of optimisation approaches and find that the best-optimised currency factor portfolios outperform their naive diversification factor portfolios by at least 7% a year. Through rigorous data snooping bias tests, we show that the outperformance of our optimised factor portfolios arises from the inherent merit of the optimisation process rather than luck. Moreover, we propose an out-of-sample (OOS) approach by selecting and aggregating all outperforming optimisation methods. Our OOS portfolios also show significant outperformance compared to the original naive factor portfolios.

The currency market is the world's largest financial market, with an average daily trading volume of $1.5 trillion. The pivotal work of Ackermann et al. (2017) demonstrated that mean-variance portfolio optimisation distinctly outperforms naive weighting in the currency market, shedding light on possible studies about optimising currency factors. The currency market features a small cross-section, which offers a unique advantage by mitigating estimation errors in expected return and covariance estimators. This naturally addresses the concerns raised by DeMiguel et al. (2009) and Barroso and Saxena (2021), who claimed that estimation errors can significantly erode the outperformance of optimised portfolios. However, the currency market has its own source of uncertainty due to the dynamic nature of its sample size, e.g., the joining of many emerging currencies and the introduction of the Euro causing a decline in currency numbers. These features motivate us to dissect how global economic events influence portfolio optimisation dynamics, which will help demystify the intricate relationship between currency factors and portfolio optimisation.

Both our in-sample and out-of-sample investigations of currency factors affirm the superiority of the optimised portfolio over the simplistic 1/N construction. We construct optimised portfolios for three of the most extensively documented currency anomalies, namely carry, momentum and value. Our universe of optimisation approaches cover most of the well-known portfolio construction methods from the literature, including DeMiguel et al. (2009), Ardia et al. (2017), Ardia et al. (2017) and Hsu et al. (2018), leading to a universe of 24,336 optimised factor portfolios. Our paper contributes to the literature on currency market anomalies and portfolio optimisation. To the best of our knowledge, we are the first to combine factor trading strategies with optimal portfolio constructions and explore whether the optimal factor portfolios outperform naive factor portfolios. Moreover, our finding holds in asymmetric currency portfolios with unequal weights for long and short legs, such as the time series momentum of Moskowitz et al. (2012) and return signal momentum of Papailias et al. (2021).

To examine the source of the outperformance of the optimal factor strategies, we divide our sample into developed and emerging currencies according to the International Monetary Fund (IMF) classification. We find that the superiority of optimised factor portfolios vanishes when the sample mainly consists of developed currencies. As most emerging currencies have been available since 1997 and the Euro was introduced in 1999, the proportion of developed currencies in our sample declined from 91% to 45% of the available currencies. We find that none of the optimised factor portfolios reported superior profitability before most emerging currencies became available and the superiority only appeared since the sample mostly consisted of emerging currencies. Our findings are in line with the global trade networking studies, e.g. Richmond (2019) and Babiak and Baruník (2021), who find that the currencies from developed economies (close to the centre of the trade network) show lower risk premia than emerging currencies. Therefore, we conclude that the superiority of the optimised factor strategies is sourced from emerging currencies.

The cross-sectional standard deviation of CAPM α plays an important role in the performance of optimal portfolios. DeMiguel et al. (2009) used simulated returns series assuming exactly zero CAPM α and found that the optimal portfolios consistently failed to outperform the naive portfolio. This assumption is unrealised, although the α in the real world is very close to zero (Jarrow, 2010). Platanakis et al. (2021) run similar tests but allow for a zero-mean distribution of CAPM α and a standard deviation of 30 basis points. Their results show that the optimal portfolios significantly outperform the 1/N rule.

Our empirical finding is consistent with Platanakis et al. (2021) in which the outperformance of optimised factor portfolios is due to the growth in the cross-sectional standard deviation of CAPM α. According to our results of the F-test, the cross-sectional standard deviation of emerging currencies' CAPM α significantly exceeds that of the developed sample at the level 1%, due to the growing count in emerging currencies and the declining count in developed currencies between 1997 and 1999. We use a simple linear model to show that the performance of optimised factor portfolios is positively and significantly related to the CAPM α standard deviation.

Given the large universe of our candidate optimisation approaches, we further investigate whether the outperformance of optimised factor portfolios is subject to data-snooping bias by using the multiple hypothesis testing (MHT) approaches.[1] As Leamer (1983) defined, data-snooping bias arises when some significant outperforming models are chosen by luck as opposed to the inherent merit of the trading rules. We conducted the Superior Predictive

[1]Data snooping test has been extensively used in the literature when comparing a large number of candidate models, see, e.g., factor zoo problem (Harvey, 2017), stock return predictability (Lo and MacKinlay, 1990), technical trading rules (Sullivan et al., 1999), and mutual fund (Barras et al., 2010).

Ability (SPA) test controlling the false discovery proportion (FDP-SPA) proposed by Hsu et al. (2014) and the SPA that controls the family error rate (FWER-SPA) of Romano and Wolf (2005). We found at least 24 significant outperforming optimised factor portfolios for a given factor.

Finally, based on MHT approaches, we construct an optimal OOS portfolio by aggregating all the optimisation approaches that exhibit statistically significant outperformance. Results of the OOS test further suggest the superiority of the optimal currency factors. Our aggregated optimised factors effectively mitigate potential look-ahead bias, ensuring their applicability as a promising real-world trading strategy for practitioners.

The remainder of this paper is organised as follows. In Section 2, we present our method of building the optimised factor portfolios. We then describe our dataset, currency return calculations, and how to build the factor trading signals in Section 3. Next, Section 4 evaluates the performance of the optimised factor portfolios and compares their performance with naive diversification of factors. We explore the causes of superiority in optimal factor portfolio performance in Section 5. In Section 6, we present the results of the robustness tests that include the OOS performance of the optimal factor portfolios and its applicability to asymmetric currency factors. Finally, we conclude our findings in Section 7. All supplementary results are provided in Appendix A to Appendix E.

2. The optimised factor portfolio constructions

We start by proposing our procedure for optimising the currency market factor portfolios. A cross-sectional market factor trading strategy has symmetric portfolio construction. Assuming N assets are available at time t, we can consider all assets as a universe (U), shown as:

$$U = \{a_1, a_2, \cdots, a_N\} \tag{1}$$

where a_N denotes the N-th asset and the assets are sorted in descending order based on asset characteristics, e.g., lagged returns and carry, or ascending order, e.g., value. Here, we assume N as an even integer to simplify the following procedure. Other assumptions and procedures won't be affected if N is an odd integer. When a factor portfolio covers all the available assets, half of the available assets are allocated to the long portfolio, and the other half are to the short portfolio, where both the long and short portfolios have $\frac{N}{2}$ assets. The long/short portfolios can be expressed as two subsets of U as follows:

$$\begin{aligned} A &= \{a_1, a_2, \cdots, a_{N/2}\}, \\ B &= \{a_{N/2+1}, a_{N/2+2}, \cdots, a_N\}, \end{aligned} \tag{2}$$

where A contains all assets taking long positions, and B includes all assets in the short portfolio. In a factor portfolio, a given asset can not be allocated into two different portfolios simultaneously so that we can derive a crucial property,

$$A \cap B = \emptyset. \tag{3}$$

For our assumption in Equation 2, we additionally have $A \cup B = U$.

According to Equation 3, the long portfolio is independent of the short portfolio, indicating that the long and short portfolios can be optimised separately. Then, the weights of the long/short portfolio (w_{long}/w_{short}) can be expressed as two independent vectors,

$$\begin{aligned} w_{long} &= [w_1, w_2, \cdots, w_{N/2}], \\ w_{short} &= [w_{N/2+1}, w_{N/2+2}, \cdots, w_N], \end{aligned} \tag{4}$$

where w_N denotes the weight of a given asset. According to Goyal and Jegadeesh (2017), the asset weights in the long (short) portfolio should add up to one (minus one), so we hold $\sum_{i=1}^{N/2} w_i = 1$ and $\sum_{i=N/2+1}^{k} w_i = -1$. In an original factor portfolio, the weight of each asset in the long (short) portfolio is denoted by $\frac{1}{N/2}$ $(-\frac{1}{N/2})$.

Embracing the inherent characteristics of cross-sectional portfolios, wherein the long and short portfolios exhibit independence, we introduce a groundbreaking methodology that distinctively applies portfolio optimisation procedures to these two segments. Moreover, the distinctive attributes inherent in the long and short portfolios. For instance, Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) find that the momentum losers rebound much faster than the winners after the market panic. Fan et al. (2022) further find that the short portfolios gather higher volatility assets than the long portfolios. An optimised portfolio construction that applies the same scheme to both the long and short portfolios cannot estimate the asset weights efficiently based on such different return-risk patterns. Therefore, we argue that investors should adopt two portfolio constructions for long and short portfolios.

Our research target is to implement optimised portfolio constructions to determine w_{long} and w_{short}. After the asset weights are measured, we can compute the returns of the optimised factor portfolios as the returns of the optimised long portfolio minus that of the optimised short portfolio. To achieve our target, we propose the procedure of optimising the factor portfolio as follows:

- 1) At the time t, determine the long/short trading position of each asset based on the cross-sectional characteristics of assets at time $t-1$, e.g., currency's lagged returns, carry, and value.
- 2) Adopt various covariance estimators, expected return estimators, and optimisation approaches to estimate w_{long} and w_{short}. The asset weights in the long (short) portfolio add up to one (minus one).
- 3) Calculate the return of the optimised long portfolio and that of the optimised short portfolio, and the long-minus-short return as the optimal factor portfolio return.
- 4) Repeat the above steps at time $t+1$.

Our procedure is also suitable for factor portfolios with unequal long and short proportions of the available assets. Regardless of how many assets are allocated to the long/short portfolios, it is reasonable to adopt our optimisation procedure, which determines the weights of assets in long and short portfolios separately. Therefore, our optimised procedure can be applied to factors that have asymmetric portfolios, e.g., the time-series momentum (TSMOM) of Moskowitz et al. (2012) and the return-signal momentum (RSMOM) of Papailias et al. (2021). A vital nature of an asymmetric factor portfolio is that the number of assets in the long portfolio can be different from the number of assets in the short portfolio, which results in a non-zero net investment (Goyal and Jegadeesh, 2017). Therefore, Equation 2 changes to:

$$A = \{a_1, a_2, \cdots, a_M\},$$
$$B = \{a_{M+1}, a_{M+2}, \cdots, a_N\}, \tag{5}$$

where M refers to the number of assets taking long trading positions. According to the nature of the asymmetric portfolios, we have $0 \leq M \leq N$. In cases of $M = 0$ and $M = N$, all assets are taking short and long positions, respectively. Switching from Equation 2 to Equation 5, our optimisation procedure can also be applied to the asymmetric factors as Equation 3 still holds. Differently, since M (N) is dynamic across various time horizons, we need to consider circumstances when the number of assets in the long or short portfolios is extremely low so that optimisation methods cannot be employed. To implement our procedure, we only adopt the optimisation in the long (short) portfolio if $M \geq 3$ ($M - N \geq 3$) and use equally weighted portfolios otherwise. Hence, the procedure for optimising asymmetric factor portfolios is shown as follows:

- 1) At the time t, determine the trading position of each asset based on the characteristics of assets at time $t-1$.
- 2) If $3 \leq M \leq N-3$, move to step 3. If $M < 3$ ($M > N-3$), apply equal weight to the long (short) portfolios and adopt various covariance estimators, expected return estimators, and optimisation approaches to estimate w_{short} (w_{long}). The asset weights in the long (short) portfolio add up to one (minus one). Then, move to step 4.
- 3) Adopt various covariance estimators, expected return estimators, and optimisation approaches to estimate w_{long} and w_{short}. The asset weights in the long (short) portfolio add up to one (minus one).
- 4) Calculate the return of the optimised long portfolio and that of the optimised short portfolio, and the long-minus-short return based on the proportion of longing assets and shorting assets in step one as the optimal factor portfolio return.
- 5) Repeat the above steps at time $t+1$.

The distinct feature for optimising the asymmetric factors is detecting whether the number of assets in the long/short portfolios is large enough to be optimised. Notably, in this paper, we mainly focus on the symmetric portfolio as it is widely used as the standard factor portfolio construction and demonstrates the application in the asymmetric factors in Section 6.2.

Table 1 lists all estimators and optimising approaches we used in step 2. We implement a comprehensive comparison by applying 12 covariance estimators, four expected return estimators, and seven optimising approaches. After combining various estimators/approaches, we achieve 156 basic optimal portfolios for a given currency factor.[2] These optimal portfolio constructions are extensively investigated in previous studies, e.g., DeMiguel et al. (2009), Ardia and Boudt (2015), Ardia et al. (2017), and Hsu et al. (2018). Appendix B presents the details of listed estimators and optimising approaches. It is worth noting that we take the negative values of assets' returns in the short portfolio in estimating w_{short} as the lower the expected return, the better it is for the assets in the short portfolio. In addition, the weight of a single currency is constrained to a range between 0.01 and 0.5 to allow for diversification, and the estimation window is 60 months.[3] Lastly, we allow the long and short portfolios to have different optimised portfolio constructions, so our optimised factor pool for a given market anomaly contains $156 \times 156 = 24,336$ optimised portfolios in total.

3. Data and the cross-sectional trading signals

In this section, we present our data sample and describe how we generated the factor trading signals. First, Sub-section 3.1 presents the data source and the calculation of monthly excess return. The methods used to determine different factor trading positions are illustrated in Sub-section 3.2. Lastly, Sub-section 3.3 reports the measurement of transaction costs in the FX market.

3.1. Foreign exchange rates

We employ an extensively used dataset proposed by Menkhoff et al. (2012*a*) and Menkhoff et al. (2012*b*), which consists of the following 48 currencies, including Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Croatia, Cyprus, Czech Republic, Denmark, Egypt, Euro

[2] For the details of combinations, please see Appendix B.

[3] When an asset can be selected by the factor portfolio but without sufficient data for the expected return and covariance estimators, we allocate $\frac{1}{N/2}$ to such an asset and optimise the remaining selected assets.

Table 1: List of estimators and portfolio constructions

Models	Abbreviation	Source
Covariance estimators		
Sample-based covariance matrix	BasicCov	-
Exponential weighted moving average	EWMACM	-
Bayes-Stein shrinkage	BS	Jorion (1986)
Ledoit-Wolf shrinkage (factor model target)	LW(factor)	Ledoit and Wolf (2003)
Ledoit-Wolf shrinkage (constant target)	LW(constant)	Ledoit and Wolf (2003)
Ledoit-Wolf shrinkage (pre-set correlation)	LW(PC)	Ledoit and Wolf (2003)
Ledoit-Wolf shrinkage (one-parameter)	LW(one)	Ledoit and Wolf (2003)
Ledoit-Wolf shrinkage (diagonal method)	LW(dia)	Ledoit and Wolf (2003)
Ledoit-Wolf shrinkage (large method)	LW(large)	Ledoit and Wolf (2003)
Oracle Approximating Shrinkage	OAS	Chen et al. (2010)
Rao-Blackwell Ledoit-Wolf	RB	Chen et al. (2010)
Adaptive Thresholding	ADA	Cai and Liu (2011)
Expected return estimators		
Sample based expected return	BasicR	-
Exponential weighted moving average returns	EWMARE	-
Bayes-Stein shrinkage	BSR	Jorion (1986)
Implied return	IR	Martellini (2008)
Optimising approaches		
1/N	Naïve	-
Mean-variance	MV	Markowitz (1952)
Global minimum variance	GMV	Markowitz (1952)
Maximum diversified	MD	Choueifaty and Coignard (2008)
Equal risk contribution	ERC	Maillard et al. (2010)
Risk efficient	RE	Amenc et al. (2011)
Maximum decorrelation	MAD	Christoffersen et al. (2012)
Volatility timing	VT	Kirby and Ostdiek (2012)

This table lists all the optimising asset-allocation models we consider. The second column of the table presents the abbreviation used to refer to each of the optimising models in the table and the third column lists their references.

area, Finland, France, Germany, Greece, Hong Kong, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Japan, Kuwait, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Russia, Saudi Arabia, Singapore, Slovakia, Slovenia, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Ukraine, and the United Kingdom. The sample period starts in November 1983 and ends in November 2020.[4] The spot and 1-month forward exchange rates of each currency are obtained from Barclays Bank International (BBI) and WM Reuters (WMR) via Datastream.

We first measure the monthly gross return by buying one unit of foreign currency. Following previous literature since Fama (1984), we work on the logarithms of spot rate and compute the monthly gross return of currency k in month t as $r_t^k = s_t^k - s_{t-1}^k$, where s_t^k is the end-of-month spot exchange rate (U.S dollar per unit of foreign currency k) in log. When r_t^k is negative, the currency k depreciates against the U.S. dollar.

Next, we measure the monthly excess returns as,

$$rx_t^k = (i_{t-1}^k - i_{t-1}) - r_t^k, \tag{6}$$

where i_{t-1}^k is the foreign interest rate in region k in month $t-1$; i_{t-1} is the U.S interest rate. The $i_{t-1}^k - i_{t-1}$ refers to the interest differential between economy k and the U.S. We follow Taylor (1989), Akram et al. (2008), Menkhoff et al. (2012*a*) and Menkhoff et al. (2012*b*) to estimate the interest rate differentials using the forward discounts assuming the covered interest parity holds. Thus, we have the short-term interest differential $i_{t-1}^k - i_{t-1} = f_{t-1}^k - s_{t-1}^k$, where f_{t-1}^k is the one-month forward exchange rate between currency k and U.S dollar in month $t-1$ (foreign exchange per unit of U.S dollar) in log. Then, we compute the monthly excess returns of currency k in month t as:

$$rx_t^k = (i_{t-1}^k - i_{t-1}) - r_t^k \approx f_{t-1}^k - s_t^k, \tag{7}$$

where all notations remain as in Equation 6.

[4]The summary statistics are reported in Appendix A.

3.2. Currency anomalies

This section demonstrates how we determine the trading positions of various currency anomalies. A factor portfolio is usually constructed by sorting on a currency characteristic and calculating the return difference between high and low portfolios (Fama and French, 1992). We implement three extensively studied currency market anomalies in this work, including the carry factor of Lustig et al. (2011) and Menkhoff et al. (2012*a*), the currency momentum factor of Menkhoff et al. (2012*b*), and the currency value factor of Moskowitz et al. (2012) and Menkhoff et al. (2017).

Carry in the FX market refers to the disparity between a spot rate and a one-month forward rate, as defined by Lustig et al. (2011). Consequently, a carry strategy involves taking a long position on currencies with higher interest rates and a short position on currencies with lower interest rates. This suggests that higher interest-rate currencies tend to exhibit strength, while lower-interest-rate currencies tend to display weakness. Consistent with the approach of Menkhoff et al. (2012*a*), we approximately estimate the currency carry as the forward discount, $f_t - s_t$, when covered interest parity holds closely at the frequency employed in our study (Akram et al., 2008). A carry factor portfolio is to buy (sell) the currencies with high (low) carries.

Momentum refers to the currency market's cross-sectional momentum trading strategy (Mom). Momentum is a pervasive and persistent financial market anomaly that has been extensively investigated in academia and industry. In a momentum strategy, as is defined by Jegadeesh and Titman (1993), abnormal profits are generated by buying the securities that are past winners and selling those that are past losers. More recently, Menkhoff et al. (2012*b*) and Asness et al. (2013) uncover the momentum profits in the currency market. In line with Menkhoff et al. (2012*b*), we adopt a momentum strategy with a three-month formation period as the proxy of the currency momentum anomaly. A momentum factor portfolio is to long (short) the currencies with high (low) cumulative returns over the formation period.

Value is another important currency anomaly introduced by Asness et al. (2013) and Menkhoff et al. (2017). The value premium is sourced from the real exchange rate (RER) as follows:

$$RER_t = s_t \times \frac{c_t}{c_{f,t}}, \tag{8}$$

where $c_{f,t}$ and c_t refer to the foreign and domestic inflation rate in month t, respectively. $\frac{c_t}{c_{f,t}}$ denotes the Purchasing Power Parity (PPP). In line with Asness et al. (2013), we estimate the currency value as the log difference in the real exchange rates over the past 60-month (5 years) as:

$$V_t = log(\frac{spot_t * c_t}{c_{f,t}}) - log(\frac{spot_{t-60} * c_{t-60}}{c_{f,t-60}}), \tag{9}$$

where $spot_{t-60}$ is the average of spot rates between 5.5 and 4.5 years ago. Currencies with low RER against the U.S. dollar have higher returns as the value strategy is long the low-value currencies and short the high-value currencies. To build the portfolio for each of the above-mentioned currency anomalies, we construct high-minus-low portfolios (or a low-minus-high portfolio for value) by dividing the whole sample into two equal portfolios. This measure ensures our empirical results are consistent with our assumptions in Section 2.

3.3. Transaction costs

The previous data description holds the zero-transaction cost assumption. In practice, transaction cost has significant effects on measuring investment revenues in the FX market. Timmermann and Granger (2004) documented that the excess returns generated by exchange rate spreads may be easily eliminated against the transaction costs. Burnside et al. (2007) further indicated that this phenomenon is more likely to appear in emerging markets.

Menkhoff et al. (2012*b*) estimated the transaction costs through the full quoted bid-ask spread, but Neely and Weller (2013) proposed that the full quoted bid-ask spreads tend to be larger than the effective spreads rates that are actually traded. Thus, we follow Neely and Weller (2013) and compute the one-way approximate transaction cost as one-third of the one-month forward bid-ask spread. Our transaction costs are, therefore, much lower than those used by Menkhoff et al. (2012*b*).

This transaction cost measurement is extensively used in FX studies, e.g., Hsu et al. (2016) and Filippou et al. (2018). The transaction-adjusted portfolio return is:

$$R_t^p = ln\{1 + \sum_{k=1}^{N}[exp(rx_t^k) - 1]w_t^k - c_t^k \times \sum_{k=1}^{N}|w_{t+1}^k - w_t^k|\}, \tag{10}$$

where N refers to the number of currencies in the portfolio, c_t^k represents the transaction cost of currency k in month t, w_t^k is the weight of currency k before the rebalancing at the end of month t, and w_{t+1}^k is the weights in the coming month, respectively. w_t^k is positive (negative) if the asset k is assigned to the long (short) portfolio. We obtain the monthly end bid and ask for quotes of a one-month forward exchange rate from 1983 via Datastream. In the next section, we present the performance of the naive and optimised factors.

4. Characterising optimised factor returns in the FX market

In this section, we report our major empirical results regarding the performance of the naive (equally weighted) currency factor strategies, the performance of optimised currency factor strategies, and their differences. Since more than 20,000 optimised portfolios are built for each currency anomaly, the comparison between the naive and optimised factors becomes a multiple comparison procedure. To mitigate the data mining issue, we apply multiple hypothesis testing approaches to assess whether the differences between the naive and optimised factors are truly significant.

4.1. Performance Comparison: Naive vs Optimised Portfolios

First, we assess the profitability of the naive and optimised currency factors. As our optimised factor portfolios are available from the 61st month of the full sample period due to the use of a 60-month estimation window, we also measure the performance of the naive factor portfolios starting from the 61st month to ensure the two portfolios are measured over the same sample period.

Table 2 presents an overview of the performance of the naive and optimised factors portfolios, incorporating transaction costs. The performance metrics we use include annualised average return (Average), standard deviation (Volatility), t-value, Sharpe ratio (Sharpe), maximum/minimum monthly return (Max/Min) and maximum drawdown (Drawdown). In Panel A, we present the summary statistics for the naive factor portfolios with equally weighted schemes in both the long and short legs. The momentum factor exhibits the strongest performance among the three factors, with the highest average annual return of 4.86% (t =4.290) and a Sharpe ratio of 0.949. Following closely, the carry factor displays the second strongest profitability with an average return of 3.80% (t =3.428) and a Sharpe ratio of 1.061. Notably, both the carry and momentum factors report statistically significant profits at the 1% level. Conversely, the value factor generates an insignificant and negative annual return at -0.31% (t =-0.325).

In Panel B and C, we provide a performance summary of the optimised factor portfolios, focusing on their portfolio returns (Panel B) and Sharpe ratio rankings (Panel C) as selection criteria, respectively. Panel B reveals that the optimised factors based on mean return consistently yield at least 7% annual return higher than their naive counterparts. The most profitable factor, carry, stands out with impressive growth in annual return from 3.80% to 11.94%. The optimised value portfolio leads to a return of 7.60% per annum (t = 2.710), resulting in a significant improvement to its naive version. While the optimised factors exhibit doubled volatility compared to the naive factors, the increased profits sufficiently compensate for the heightened volatility, leading to higher Sharpe ratios. The most profitable value factor, in particular, exhibits the most significant growth in the Sharpe ratio, increasing from -0.070 to 0.686, highlighting its superior risk-adjusted performance. The higher values of Max and lower values of Min monthly returns further validate the heightened volatility of the optimised factor returns compared to the naive factor returns. Nevertheless, the substantial rise in profitability makes the optimised factor portfolios a compelling investment choice for consideration.

In Panel C, we identify the best risk-adjusted optimised factor portfolios based on the portfolio's Sharpe ratio. Notably, the best profitable portfolio is the same as the best risk-adjusted portfolio for the momentum factor. Despite the fact that the optimised factor portfolios in Panel C exhibit lower average returns and volatilities compared to those in Panel B, the optimised factors still significantly outperform the naive factors in terms of excess returns and risk-adjusted profits. For all three factors, the Sharpe ratios witness a notable increase from 0.660, 0.949, and -0.070 in Panel A to 1.134, 1.134, and 0.688 in Panel C, respectively. This signifies better risk-adjusted performance for optimised factor portfolios. The improvement in Sharpe ratios further supports the effectiveness of the optimisation approaches in generating attractive risk-adjusted profits for currency factors.

Table 3 further presents the portfolio constructions of the best profitable/ risk-adjusted optimal factor portfolios shown in Table 2. The estimators and portfolio constructions are

Table 2: Performance metrics of the naive and optimised factors

Factor	Average	Volatility	T-value	Sharpe	Max	Min	Drawdown
Panel A: Naïve portfolio							
Carry	3.80%	0.058	3.428	0.660	6.51%	-5.76%	0.096
Mom	4.86%	0.051	4.290	0.949	4.55%	-5.66%	0.149
Value	-0.31%	0.045	-0.325	-0.070	3.98%	-3.74%	0.366
Panel B: Best optimised portfolio based on mean returns							
Carry	11.94%	0.118	3.942	1.012	13.21%	-14.47%	0.147
Mom	11.95%	0.105	4.439	1.134	13.01%	-17.89%	0.221
Value	7.60%	0.111	2.710	0.686	10.21%	-15.87%	0.344
Panel C: Best optimised portfolio based on Sharpe ratios							
Carry	11.20%	0.099	4.336	1.134	9.57%	-17.02%	0.133
Mom	11.95%	0.105	4.439	1.134	13.01%	-17.89%	0.221
Value	7.48%	0.109	2.680	0.688	10.12%	-15.79%	0.394

This table summarises the performance of the naive and optimised factor portfolios with transaction costs over the entire sample period from November 1989 to November 2020. *Average*, *Volatility*, and *Sharpe* denotes the annualised average returns, standard deviation, and Sharpe ratio of portfolio returns; *Max* and *Min* denote the maximised and minimised monthly returns; *Drawdown* denotes the maximum drawdown. *T-value* is measured by the heteroskedasticity- and autocorrelation-consistent (HAC) standard errors described by Newey and West (1987).

divided into two panels based on the optimal factor portfolio's profitability and risk-adjusted performance. In long portfolios, regardless of the performance metrics considered, there is consistency in the dominance of the mean-variance (MV) optimisation approach. However, for short portfolios, the optimisation approach varies across factors. In Panel A, the MV approach remains successful in the carry short portfolio, while the Maximum Diversified (MD) portfolio excels in Panel B. For the momentum and value short portfolios in both Panel A and B, the Maximum Decorrelation (MAD) and Volatility Timing (VT) portfolios demonstrate the best performance.

Next, we examine the statistical significance of the superior performance exhibited by the best profitable and risk-adjusted optimised factors. Specifically, we assess the return differences between the optimised factor portfolios and the naive portfolios and report the results in Table 4. Overall, the six optimised factor portfolios deliver profits of at least 7.09% higher annually than the benchmark. All these return differences exhibit statistical significance at the 1% level, as evidenced by the t-values. These findings conclusively demonstrate the significant outperformance of the optimised factor portfolios over the equally weighted benchmarks.

In addition to our investigation into the best-performing optimal factor portfolios, we

Table 3: Best performing optimal factor portfolio construction

Factors	Carry		Mom		Value	
	Long	Short	Long	Short	Long	Short
Panel A: Best profitable portfolios						
Opt	MV	MV	MV	MAD	MV	VT
ER	IR	BasicR	BSR	-	IR	-
Cov	LW(dia)	EWMA	LW(dia)	EWMA	BS	EWMA
N	-	-	-	-	-	4
Panel B: Best risk-adjusted portfolios						
Opt	MV	MD	MV	MAD	MV	VT
ER	BasicR	-	IR	-	BSR	-
Cov	RBLW	LW(con)	LW(dia)	EWMA	BS	EWMA
N	-	-	-	-	-	2

This table reports the portfolio construction and estimators of the best-performing optimal factor portfolios in terms of the portfolio profits and risk-adjusted performance in Panel A and B, respectively. The sample period ranges from November 1989 to November 2020. *Opt, ER, Cov,* and *N* denote the optimisation approach, expected return estimator, covariance estimator, and the tuning parameters, respectively. The details of each abbreviation refer to Appendix B.

Table 4: Superiority of the best-performing optimised factors

Factor	Average	Volatility	T-value	Sharpe	Max	Min	Drawdown
Panel A: Best profitable portfolio minus naive factor portfolio							
Carry	8.14%	0.100	3.060	0.814	11.59%	-12.83%	0.253
Mom	7.09%	0.086	3.336	0.824	9.45%	-15.62%	0.335
Value	7.91%	0.099	3.233	0.797	9.68%	-15.75%	0.224
Panel B: Best risk-adjusted portfolio minus naive factor portfolio							
Carry	7.40%	0.080	3.223	0.929	10.69%	-15.37%	0.484
Mom	7.09%	0.086	3.336	0.824	9.45%	-15.62%	0.335
Value	7.79%	0.096	3.219	0.814	9.68%	-15.68%	0.244

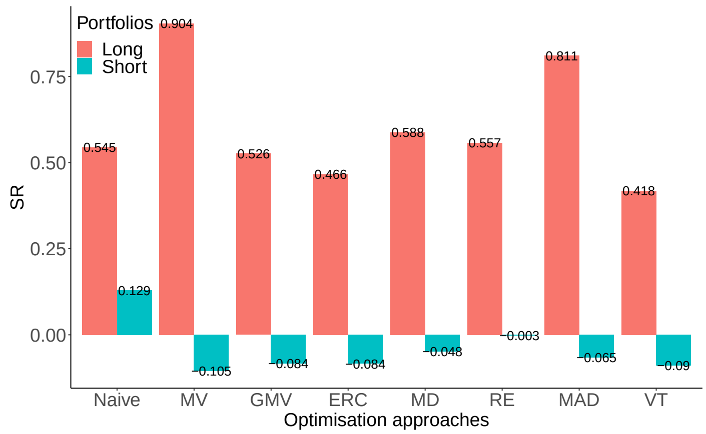
This table summarises the superiority of the best-performing optimised factor portfolios over naive factor portfolios with transaction costs. *Average*, *Volatility*, and *Sharpe* denotes the annualised average returns, standard deviation, and Sharpe ratio of portfolio returns; *Max* and *Min* denote the maximised and minimised monthly returns; *Drawdown* denotes the maximum drawdown. *T-value* is measured by the heteroskedasticity- and autocorrelation-consistent (HAC) standard errors described by Newey and West (1987).

categorise all optimised portfolios into seven distinct groups based on their optimisation approaches, namely MV, GMV, MD, ERC, RE, MAD, and VT, as illustrated in Table 1. By calculating the average annualised Sharpe ratios for each category, we aim to provide a comprehensive overview of the risk-adjusted performance associated with each optimisation approach. Furthermore, we delve into a detailed analysis of the long and short portfolios individually, recognising that these two segments may employ distinct weighting schemes, offering a more nuanced perspective on their performance characteristics.

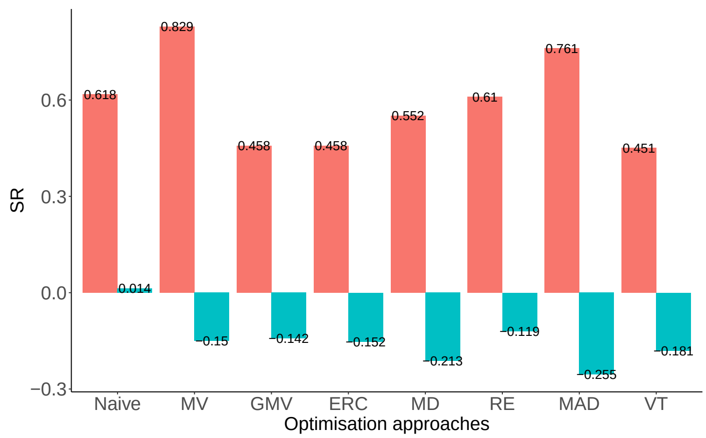
Figure 1 visually presents the average portfolio Sharpe ratios for each factor. Our observations in Table 3 are further affirmed as the mean-variance (MV) approach consistently emerges as the preferred dynamic weighting method for the long portfolio, yet it's noteworthy that the choice of the most effective approach fluctuates across different factors. We argue that such a pattern is linked to the unique utility function, which addresses the balance between expected return and volatility, serving as a foundation for the mean-variance (MV) approach. In contrast, other optimisation methods focus on risk diversification. As we can discern from Figure 1, the profits of factor portfolios primarily stem from the long positions, where considering the trade-off between return and risk appears more favourable than risk-focused approaches. Conversely, according to Barroso and Santa-Clara (2015), Daniel et al. (2017), and Fan et al. (2018), short portfolios exhibit higher volatility compared to long ones, so optimisation approaches emphasising risk management can outperform the MV approach in the short portfolio.

This remarkable statistical significance underscores the superiority of the optimised factor portfolios, bolstering the relevance and importance of our novel method. The economic implications of such pronounced outperformance are noteworthy, as they indicate that the optimisation approach, especially the MV approach, can generate substantial incremental profits compared to conventional naive strategies. Consequently, our finding provides valuable insights into the field of investment, presenting a compelling methodology to enhance risk-adjusted returns and guide investors in their portfolio allocation decisions.

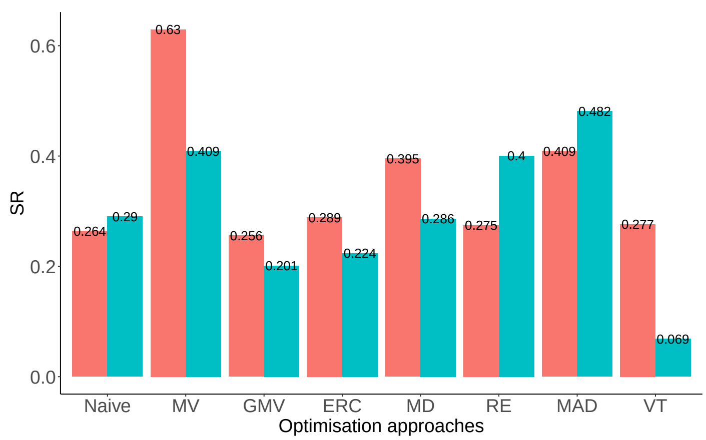
Figure 1: Aggregated portfolio Sharpe ratios based on various optimisation approaches



Carry



Momentum



Value

These plots illustrate the Sharpe ratios associated with various optimisation approaches for factors. To compute these Sharpe ratios, we aggregate the portfolio performance of a particular optimisation approach by taking the average Sharpe ratios of all optimised portfolios that employ this approach. For each of the carry, momentum and value strategies, these aggregated Sharpe ratios are plotted with respect to both long and short portfolios separately.

4.2. Data snooping bias

As mentioned above, we implement 24,336 optimal portfolios for each currency factor and focus on the best-performing ones. When testing such a large number of models, there are some instances when an optimisation approach significantly outperforms the benchmark by chance rather than the inherent merit of its portfolio construction. These potential issues are known as data snooping bias, data dredging, or p-hacking (Leamer, 1983). To manage such a bias, we implement MHT approaches, i.e., data snooping bias tests, that incorporate the bootstrap procedure of Politis and Romano (1994) in order to determine the adjusted critical values of significance based on bootstrapped distributions.

Recent literature, e.g., Chordia et al. (2020) and Harvey et al. (2020), assessed the test power of various approaches in the stock market. They find that the stepwise bootstrap reality check (RC) controlling false discovery proportion (FDP-RC) of Romano et al. (2007) is the most desirable approach for market participators. However, Hsu et al. (2014) argued that the RC has weaker test power than the stepwise superiority predictive ability (SPA) controlling false discovery proportion (FDP-SPA). This approach is extensively used in assessing the funding managers' skills (Kearney et al., 2014) and the profits of the trading rules in the FX market (Hsu et al., 2016). In line with the existing literature, we apply FDP-SPA to assess the outperformance of optimised factor portfolios. In addition to controlling the false discovery proportion (FDP), we apply the SPA controlling the familywise error rate (FWER) as additional evidence to ensure that our conclusion is not affected by the control target. The procedures of the two approaches are detailed in Appendix C.

To ensure that the outperformance of the optimised factor portfolios is not caused by luck, we apply data-snooping bias tests selected by portfolios' profits. The null and alternative hypotheses are:

$$\begin{aligned} H_0: &\quad R_k \leq R_n; \\ H_1: &\quad R_k > R_n, \end{aligned} \tag{11}$$

where R_k is the mean of the k-th optimise factor portfolio excess returns, and R_n refers to the average excess return of the naive factor portfolio. We can further convert the null and alternative to new forms which measure the relative performance as:

$$\begin{aligned} H_0 &: \quad R_k - R_n \leq 0; \\ H_1 &: \quad R_k - R_n > 0. \end{aligned} \tag{12}$$

The null and alternative are constant across different factor trading strategies, where the model sample size remains 24,336 and the tests are repeated three times for carry, momentum, and value factors. Table 5 presents the results of the data snooping bias tests based on portfolios' Sharpe ratio. For each of the three currency factors, *No.reject* reports the number of significant outperforming optimal factor portfolios uncovered by the FDP controls (columns two) and FWER controls (columns five) at the 10% level. *k.stopped* denotes the number of false rejections allowed and CV represents the adjusted critical values for each test.

Table 5: Data snooping bias tests

Factor	FDP			FWER		
	No.reject	*k_stopped*	*CV*	*No.reject*	*k_stopped*	*CV*
Panel A: Long-short portfolio (totaled 24,336)						
Carry	8,141	408	2.699	4,388	1	3.826
Mom	48	3	3.858	24	1	3.882
Value	4,072	204	3.538	2,408	1	4.073
Panel B: Long portfolio (totaled 156)						
Carry	48	3	2.512	48	1	2.839
Mom	12	1	3.156	12	1	3.156
Value	48	3	2.566	48	1	2.822
Panel C: Short portfolio (totaled 156)						
Carry	0	1	2.538	0	1	2.538
MOM	1	1	2.150	1	1	2.150
Value	0	1	3.137	0	1	3.137

This table summarises the number of portfolios rejecting the null as shown in Equation 11. The data-snooping test results based on long-short, long, and short portfolios are presented in Panels A, B, and C, respectively. *No.reject* counts the number of rejections for a specific factor strategy. *k.stopped* denotes the number of false rejections allowed. *CV* represents the adjusted critical values for each test. For more details, please see Appendix C. Columns two to four report the results based on the SPA test controlling the FDP of Hsu et al. (2010). Columns five to seven report the results based on the SPA test controlling the FWER of Hsu et al. (2014).

In our multiple comparison procedure, the critical values for significance at the 10% level are substantially increased, now standing at least at 2.699 for the long-short portfolio compared to the original value of 1.655. Moreover, the critical values of the FDP control approaches are lower than the critical values of the FWER control approaches in general. This aligns with the more lenient nature of the FDP controls, as they allow for a higher tolerance of false discoveries compared to FWER controls (Romano and Wolf, 2005, Hsu et al., 2014). This difference can be validated by the variation in k.stopped, which reflects how many false discoveries (false rejections) are allowed by each test. The FDP controls generally allow more than one false discovery, whereas the FWER controls allow only one false rejection.

Despite the increased critical values, we observe that each of the three factors still yields a considerable number of rejections in Panel A of Table 5, demonstrating significant outperformance. Specifically, for the carry portfolios, we find that 8,141 (FDP) and 4,388 (FWER) out of 24,336 portfolio optimisation approaches exhibit significantly better profitability than their naive counterparts. For the value factor, the MHT approaches with FDP control identify 4,027 optimised factor portfolios that outperform the naive factor portfolio, and the MHT approaches with FWER control uncover 2,408 outperforming optimised factor portfolios. Finally, for the momentum factor, the number of rejections is only 48 and 24 based on FDP and FWER control, respectively.

Our observation of fewer instances of significance within the momentum portfolios is not unexpected. According to Filippou et al. (2018), currency momentum encompasses the influence of global policy risks that are particularly tied to the US dollar in the sorting process. These global policy risks materialise in the form of exchange rate fluctuations for various currencies in relation to the US dollar and are reflected in the covariance matrix. When optimising a currency momentum portfolio, the extent to which portfolio performance can be enhanced through optimisation is relatively limited compared to the improvements achieved when optimising other factor portfolios. Nevertheless, although the number of rejections in the momentum factor is much less than those observed in the carry and value factors, these outperformed optimised momentum portfolios still provide evidence that our optimisation procedure can improve the factor portfolio performance as illustrated previously in Table 4.

Tables 5 Panels B and C further present results of data snooping analysis for the long and short portfolios, respectively. Interestingly, it's evident that a significant number of optimised long portfolios - at least 12 out of 156 - outperform the naive long portfolio. In contrast, for the short leg, only the momentum exhibits one outperformed optimised portfolio, whereas carry and value do not. These findings reinforce that the superior profits of the optimal factor portfolios are indeed sourced from the optimised long positions, aligning well with the trends highlighted in Figure 1.

Of particular note is the observation that the MV approach consistently stands out, generating the most significant outperformance in the long portfolios. We propose the hypothesis that MV portfolio construction plays a pivotal role in driving the substantial outperformance of the optimal factor portfolios. To put this hypothesis to the test and eliminate alternative explanations, we present a comprehensive summary of rejections based on the FDP-SPA test for each optimisation approach in Table 6.[5] A clear pattern emerges - it is only the MV approach that consistently produces significant outperformance, as well as rejecting the null hypothesis in Equation 11. The data in Panels A and C solidify this, demonstrating that all 48 MV long portfolios significantly outperform the naive carry and value long portfolios. In Panel B, we see that 12 out of 48 MV long portfolios exhibit significant outperformance against the naive momentum long portfolios. Additionally, a single MAD short portfolio significantly outperforms the naive one. In conclusion, these results substantiate our hypothesis: MV portfolio construction is the central driving force behind the notable outperformance observed in the optimal factor portfolios.

Moreover, the change in control targets does not eliminate the number of rejections, implying that the significant outperformance of optimised factor portfolios holds regardless of the statistical control employed. These findings underscore the robustness of our results and contribute valuable insights to the understanding of optimised factor performance in financial markets.

Overall, the performance of the optimised factor portfolios is superior to that of the naive factor portfolios both in terms of profitability and Sharpe ratio. Our data snooping bias tests further justify that, besides the best-performing optimised portfolios, there is a large number of alternative optimisation approaches can also significantly beat the naive

[5] The FDP-SPA and FWER-SPA tests exhibit the same reactions with various critical values, so we only report the FDP-SPA results.

Table 6: Data snooping bias tests for each optimisation approach

	MV (48)	GMV (12)	ERC (12)	MD (12)	RE (12)	MAD (12)	VT (48)
Panel A: Carry							
Long	48	0	0	0	0	0	0
Short	0	0	0	0	0	0	0
Panel B: Mom							
Long	12	0	0	0	0	0	0
Short	0	0	0	0	0	1	0
Panel C: Value							
Long	48	0	0	0	0	0	0
Short	0	0	0	0	0	0	0

This table summarises the number of rejections for each optimisation approach in long and short portfolios. The total number of optimisation approaches is 156. The number of portfolios is presented in each column title, e.g., MV (48) denotes 48 mean-variance portfolios. The data-snooping test results for the carry, momentum, and value factor portfolios are presented in Panels A, B, and C, respectively. FDP-SPA refers to the SPA test controlling the FDP of Hsu et al. (2010). The details of the seven optimisation approaches are presented in Appendix B.3.

factors. Therefore, the superiority of the optimised factor portfolio is caused by the inherent merit of adopting our optimisation procedure instead of luck.

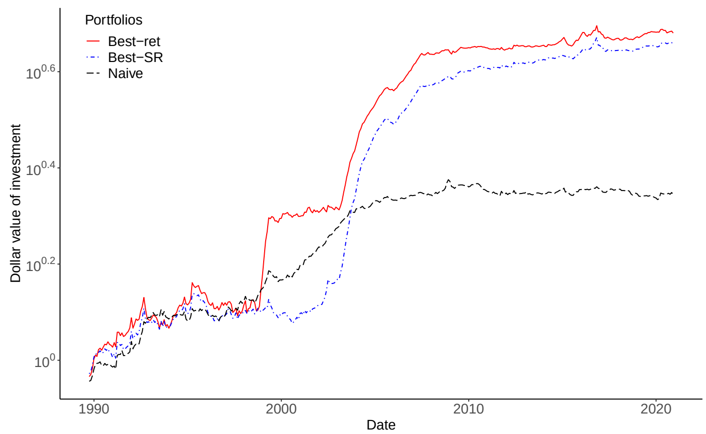
5. Optimal v.s. naive: the source of differences

We next investigate the source of the optimised factors' superiority. First, we plot the cumulative performance of the best-performing optimised factor portfolios for each factor (the naive factor is used as the benchmark) to identify when the optimised factors show superiority and link such differences to economic events to explore the economic drivers of outperformance. Then, we analyse the outperformance by linking it to the asset pricing model so that we can achieve the drivers from the econometric aspect.

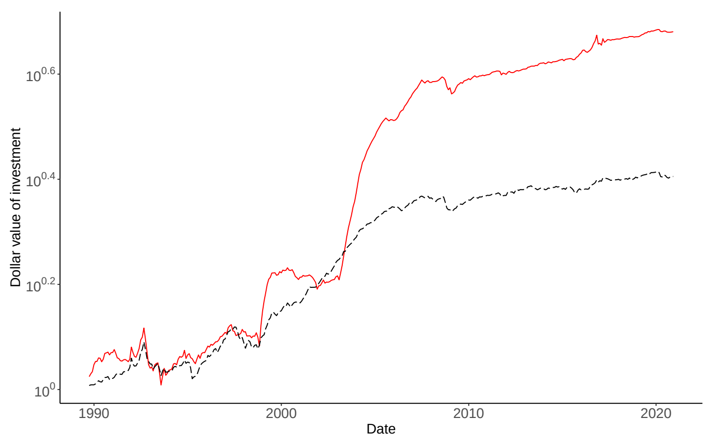
5.1. Cumulative performance

Figure 2 plots the cumulative performance of the factor portfolios over the entire investment horizon after adjusting for the transaction costs. For the momentum factor, we only report one cumulative performance of the optimised factor, as the best-profitable optimised portfolio is the same as the best risk-adjusted optimised portfolio. We highlight that the optimised factor portfolios (red and blue lines) generate better cumulative performance than the original factor portfolios, which aligns with our findings in Table 2.

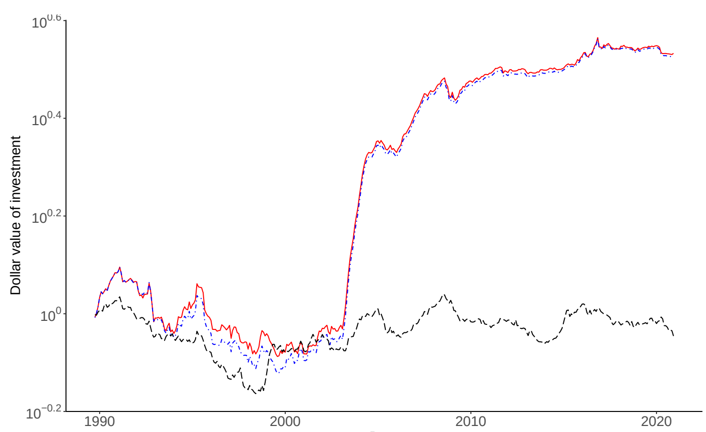
Figure 2: Cumulative performance of the currency factors portfolios



Carry



Momentum



Value

These plots exhibit the cumulative performance of the best-performing optimal and the naive factor portfolios throughout the sample period. The dollar value of investment (y-axis) is logarithmically scaled, given the massive difference between the two weighting schemes. Best-ret, Best-SR, and Naive refer to the optimised factor with the highest portfolio returns, the optimised factor with the highest Sharpe ratio, and the original equal-weighted factor, respectively. For the carry and momentum factors, we focus on the best-performing optimised factors based on the profit and the Sharpe ratio. For the momentum factor, we only plot the cumulative performance of the Best-ret to avoid duplicated lines.

For carry and momentum factors, the dollar values of investment of the optimised portfolio are similar to the naive portfolio before 1997. However, the performance boosts after a large number of emerging currencies became available in 1997. For instance, the optimised carry portfolios even underperformed the corresponding naive portfolios in the early 1990s. The dollar values of investment in optimised carry portfolios increased and exceeded that of naive carry portfolios after 1997. For the Value factor, the boost appears around 2003. As the available currencies in our data universe vary over time, we hypothesise that the performance of the optimised factor portfolios is correlated to the composites of the available currency sample.

To validate this, we divide the sample into two sub-samples based on currencies' economic features, namely developed and emerging samples, according to the definition of advanced economies supplied by the International Monetary Fund (IMF). 29 out of 48 currencies are in the developed sample (developed currencies), including those of Australia, Austria, Belgium, Canada, Cyprus, Czech, Denmark, Finland, France, Germany, Greece, Hong Kong, Iceland, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Slovak, Slovenia, Spain, Sweden, Swiss, Taiwan and UK. The remaining 19 currencies are defined as emerging currencies.

We can link the dynamic outperformance of the optimised factors to the economic events. Two remarkable changes in our data universe occurred in the late 1990s. First, most emerging currencies in the sample crashed in 1997. Second, due to the introduction of the Euro, ten European currencies, mainly from the developed countries, were omitted in 1999. These two changes reduce the proportion of the developed currencies in the sample. Figure 3 plots the number of available currencies (solid red line) and developed currencies (dashed blue line) over the entire sample period. In January 1997, ten emerging currencies started to be available. Following the introduction of the Euro, nine developed currencies were omitted in January 1999: those of Austria, Belgium, Germany, Finland, France, Ireland, Italy, Portugal, and Spain. Before these two changes, the sample mainly consisted of developed currencies. For instance, between 1983 and 1996, 20 out of 22 currencies were from the developed economies. By contrast, only 13 out of 24 currencies are from the developed sample after the introduction of the Euro. Since the Value factor needs at least a 5.5-year formation period to generate the first signal, such changes can also explain the boost of the optimised value factor cumulative performance around 2003.

Since the cumulative performance of the optimised factor portfolio began to launch when the data universe changed around 1998, we argue that the superiority of optimised factor portfolios arises with the reduction in the proportion of developed currencies or the increase in the proportion of emerging currencies. At the beginning of our sample period, the available sample mainly consisted of developed currencies, and the optimised factors failed to outperform the naive ones. Once the majority of the sample is comprised of emerging currencies, the superiority of the optimised factor portfolio appears.

Our findings receive support in the existing literature. Firstly, Menkhoff and Taylor (2007) and Hsu et al. (2016) have documented a diminishing trend in profits generated by holding developed currencies over time. Secondly, developed economies, positioned at the heart of global networks, typically exhibit lower interest rates and currency risk premia as highlighted by Richmond (2019). In contrast, emerging currencies, often characterised as volatility takers, tend to have higher currency risk premia as indicated by Babiak and Baruník (2021). This contextual background helps elucidate our observed patterns and underscores the relevance of our findings within the broader landscape of currency market dynamics. In the next sub-section, we explore the econometric causes of such a relationship between the performance of the optimised factors and the construction of the available currency sample.

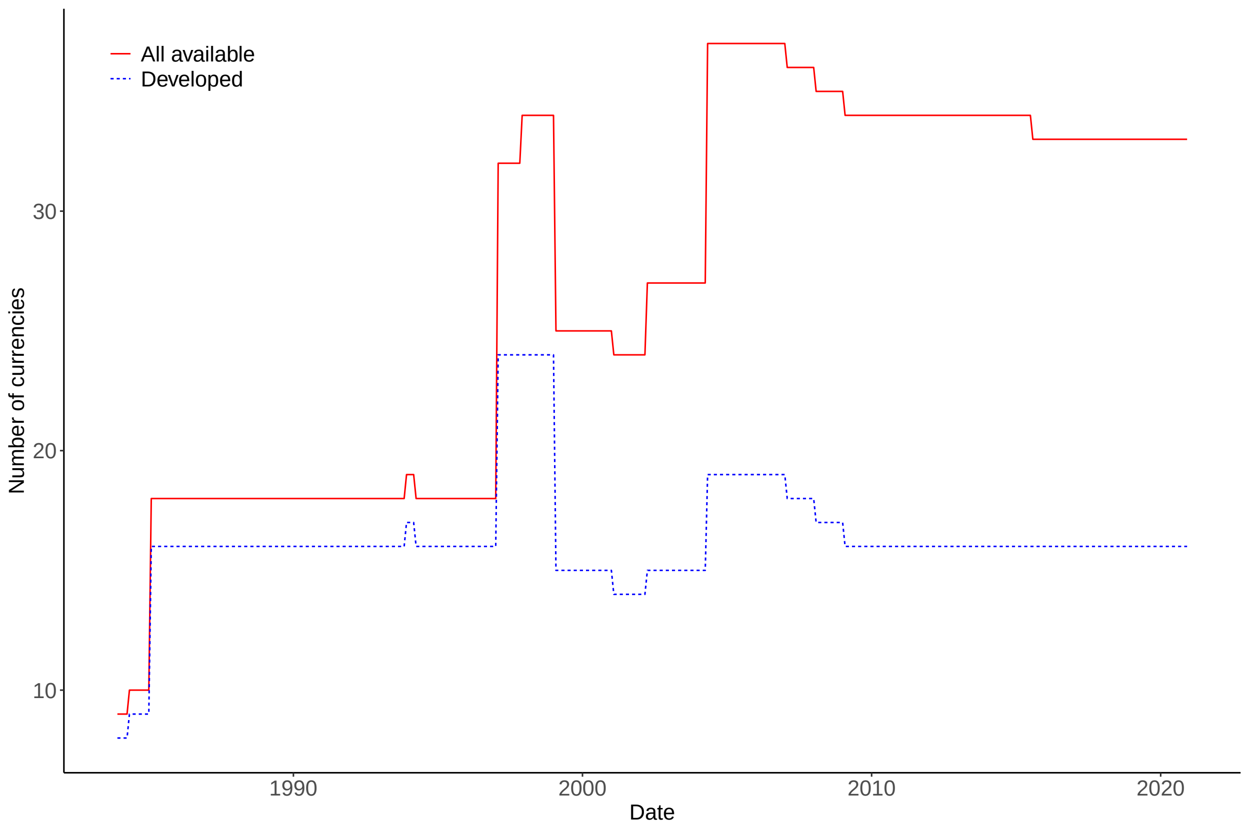
5.2. The distribution of abnormal profits matters

To further uncover the econometric insights of the superiority of the optimised factor portfolios, we consider links between the performance of the optimal portfolios and the distribution of abnormal profits based on the capital asset pricing model (CAPM), namely, the CAPM α. We extend the rationales of DeMiguel et al. (2009) and Platanakis et al. (2021) to the FX market and estimate the CAPM α for each currency, as:

$$r_{k,t} = \alpha + \beta r_{mkt,t} + \epsilon_t, \tag{13}$$

where $r_{k,t}$ is the return of currency k in month t; α represents the CAPM α; β is the factor loading; r_{mkt} refers to the excess returns of a market portfolio (we employ the log

Figure 3: Number of available currencies



This plot exhibits the number of currencies available for momentum investors. The blue dashed line counts the number of currencies from the developed economies across the sample period. The solid red line counts the total number of currencies available (developed and emerging market currencies).

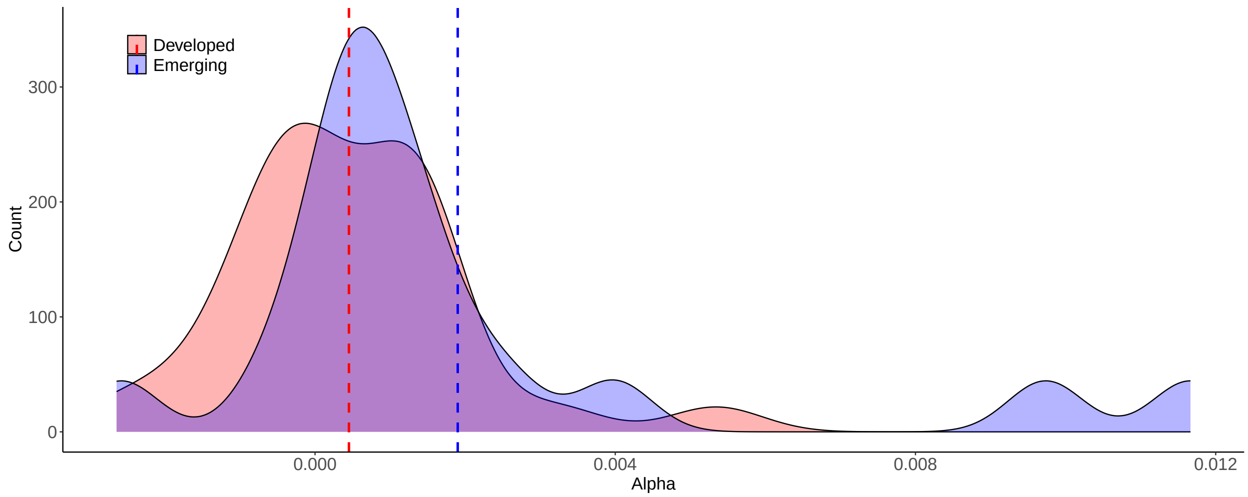
excess returns of the U.S. Dollar Index (DXY) here), and ϵ_t is the noise term distributed as $\epsilon_t \sim N(0, \sigma_\epsilon)$.

We first find that the standard deviation of CAPM α in the emerging sub-sample is significantly higher than that in the developed sub-sample. Figure 4 plots the distribution of CAPM α for both the developed and emerging samples. Previous literature, e.g., DeMiguel et al. (2009), showed that the optimal portfolios fail to significantly outperform the 1/N rule based on simulated return series with zero CAPM α, namely the zero-alpha assumption. Nevertheless, as argued by Jarrow (2010), the zero-alpha assumption is so static that "in the complexity of actual markets, their violation, although anathema to economists, is not logically unreasonable." Platanakis et al. (2021) further revealed that the performance of optimal portfolios is subject to the standard deviation of CAPM α across available assets. They abandon the zero-alpha assumption and re-assume that the CAPM α of various assets fits a normal distribution with a mean of zero and a standard deviation of 30 basis points, namely, the zero-mean assumption. In the context of this new assumption, the optimal portfolios significantly outperform the naive ones. We find that the zero-alpha assumption of DeMiguel et al. (2009) is too stringent to reflect the distribution of CAPM α in practice. The means of α in both the developed (dashed red line) and emerging (dashed blue line) samples exceed zero, and the mean in the emerging sample is slightly higher than in the developed sample. Due to the visible fat left tails, the standard deviation of CAPM α in the emerging sample is higher than in the developed sample. The results of the F-test indicate that the variance in either sample is significantly different from zero at the 1% level. Both distributions are in line with the zero-mean assumption of Platanakis et al. (2021) that CAPM α is not exactly equal to zero.[6]

Next, we explore how the increase in the standard deviation of CAPM α affects the performance of the optimised factor portfolios. We follow Platanakis et al. (2021) and conjecture that the improved performance of optimised factor portfolios after most of the emerging currencies became available derives from an increase in the standard deviation of CAPM α.

[6] We find that the CAPM α of five developed currencies and seven emerging currencies are significantly different from zero at the 10% level. We report the CAPM α of each currency in Appendix E.

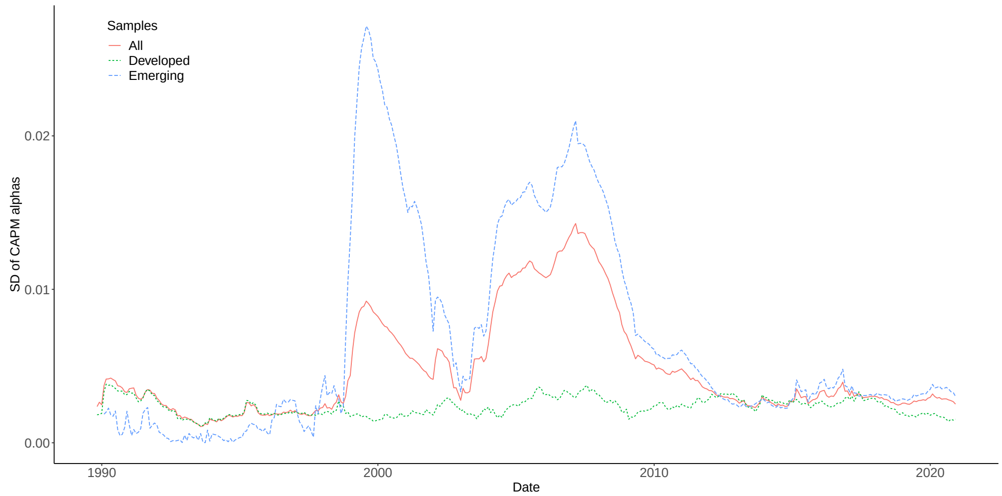
Figure 4: Distribution of alphas for both developed and emerging samples



This plot illustrates the destiny-fitted areas of distributions of CAPM alphas. The dashed lines locate the mean of each distribution. The red area presents the distributions based on developed sub-samples. The blue area denotes the distributions based on emerging sub-samples.

To validate our inference that the performance of the optimised factors portfolios is positively related to the standard deviation of CAPM α, we first estimate the time-varying standard deviation of CAPM α over the entire sample period. In line with our estimation window in Section 2, for each currency, we implement the 60-month rolling window to estimate the monthly CAPM α. Figure 5 plots the time-dynamic standard deviations of CAPM α for samples consisting of all (solid red line), developed (green dotted line), and emerging (blue dashed line) currencies. Before 1997, the blue line was even lower than the red and green lines, indicating a lower standard deviation of α for emerging currencies. Then, there was a sudden spike in standard derivation in the emerging sample around 1997, and the blue line moved above the green and red lines afterwards. The standard deviation of CAPM α in the emerging sample persistently exceeds that of the developed sample during 2000-2010. During the 1997 Asian financial crisis, five important emerging currencies, namely Indonesian, Malaysian, Philippine, South Korean and Thai, extremely depreciated against the US dollar during this panic period and pushed up the standard deviation in the emerging sample. While the standard deviation across the full sample was consistently below 0.0043 before most emerging currencies became available, the full sample standard deviation reached 0.014 after the joining of these emerging currencies.

Figure 5: Time-varying standard deviations of CAPM α distribution



This plot displays the standard deviations of CAPM α from November 1989 to November 2020 based on the three samples. The red solid line represents the time-dynamic standard deviation for the entire sample. The blue and green lines refer to the time-dynamic standard deviation for the developed and emerging samples, respectively.

Next, to quantify the relationship between the standard deviation of CAPM α and the outperformance of the optimised factor portfolios, we regress the superiorities of the optimised factor portfolios on the lagged time-varying standard deviation of full-sample CAPM α, given as:

$$r_{t+1}^{diff} = \alpha + \beta * SD_{t,p} + \epsilon, \tag{14}$$

where r_{t+1}^{diff} represents the differences between the returns of the best-performing optimised factor portfolio and those of the naive factor in month $t+1$, and $SD_{t,p}$ refers to the standard deviation of CAPM α over the 60-month estimation window based on sample p in month t. We take a month lag between the CAPM SD and r_{t+1}^{diff} as the asset weights in month $t+1$ are determined by the information in month t.

Table 7 reports the beta coefficients and corresponding t-statistics for each optimised factor in terms of Equation 14. In Panel A, the correlation coefficients are positive and significantly different from zero at the 1% level according to the t-statistics, so the superiorities of the optimised factor portfolios in month $t+1$ are positively subject to the standard deviation of currencies' CAPM α in month t. The positive relationship is consistent with our previous findings in Figure 5 that the increase in cumulative performance of the optimised

Table 7: Relationship between the outperformance of the optimised portfolios and the standard deviation of CAPM α

Dependents		Carry	Mom	Value
Panel A: full sample				
Best profitable	β	2.754***	1.853***	1.634***
	t	(4.927)	(3.842)	(2.798)
Best risk-adjusted	β	2.146***	1.853***	1.702***
	t	(3.736)	(3.842)	(2.953)
Panel B: developed sample				
Best profitable	β	1.965	1.623	3.986
	t	(0.55)	(0.632)	(1.353)
Best risk-adjusted	β	2.882	1.623	4.167
	t	(0.989)	(0.632)	(1.415)
Panel C: emerging sample				
Best profitable	β	1.187***	0.599**	0.346
	t	(3.088)	(2.163)	(0.927)
Best risk-adjusted	β	0.515	0.599**	0.396
	t	(1.583)	(2.163)	(1.089)

This table reports the regression results of Equation 14. For each factor, we report the correlation coefficients, β, and the corresponding t-values, t. The t-value is measured by the heteroskedasticity- and autocorrelation-consistent (HAC) standard errors described by Newey and West (1987). '*', '**', '***' represent that the t-values of α are statistically significant at the 10%, 5% and 1% levels.

portfolios appears at the same time as the growth in standard deviation.

We further explore the relationship between the SD of CAPM α and the outperformance of the optimised factors in the developed and emerging sub-samples. Panel B reports the beta coefficients and corresponding t-statistics of Equation 14 based on the developed sub-sample. Regardless of factors and performance metrics, the beta coefficients are positive but statistically insignificant at the 10% level. Panel C shows that, in the emerging sub-sample, the beta coefficients are only positive and significant in the best profitable carry factor and the two momentum factors. These results explain our abovementioned finding that the outperformance of the optimised factors is invisible when the sample mainly consists of developed currencies.

Evidence from both the plot in Figure 5 and regression results in Table 7 suggests that the outperformance of the optimised factor portfolios is positively related to the standard deviation of CAPM α. We conclude that the superior profitability of the optimised factors mainly stems from the increased number of emerging currencies. When the sample mainly consists of developed currencies, the optimised factors fail to show outperformance due to the low standard deviation of CAPM α. The increased proportion of emerging currencies raises the standard deviation of CAPM α, resulting in the improved performance of the optimised factors, which is consistent with the finding of Platanakis et al. (2021).

6. Robustness check

In this section, we implement two robustness checks to validate our main findings of the optimised factor portfolios. First, we extend our analysis to the out-of-sample (OOS) period by constructing an aggregate optimised factor portfolio that encompasses all portfolios displaying statistically significant outperformance according to the MHT tests. Secondly, we apply our innovative optimisation approach to the asymmetric factor portfolios, examining the generalisability of our method.

6.1. Out-of-sample conduction and the aggregate optimised factors

The preceding analysis has been exclusively centred on the in-sample period, a practice that inherently carries the risk of look-ahead bias. To avoid such bias and provide feasible insights for practitioners, we introduce an OOS approach in forming aggregate optimised factors based on data snooping bias tests.

The core assumption behind our OOS procedure aligns with market participants' expectations that portfolio construction approaches demonstrating superiority during the in-sample period can sustain similar levels of profitability in the subsequent OOS period. At the outset of each month t, we identify the significantly outperforming optimised portfolios at the 5% significance level based on performance over the past 60 months, from $t-60$ to $t-1$. Subsequently, we construct the OOS aggregate optimised factor portfolios by equally allocating investments among these selected portfolios for the forthcoming month t. The aggregate portfolio return in month t is:

$$R_{oos,t} = \frac{\sum(I_{t-1} * R_{opt,t})}{N_{I=1}}, \tag{15}$$

where $R_{\mathrm{opt},t}$ denotes the optimised factor portfolio return in month t, I_{t-1} is set to one if a given optimised factor portfolio significantly outperforms the naive factor portfolio from $t-60$ to $t-1$, and zero otherwise. $N_{I=1}$ counts the number of significantly outperforming optimised factor portfolios. Essentially, the aggregate optimised factor portfolio return in month t is the simple average of returns in month t across all significantly outperforming optimised factor portfolios from $t - 60$ to $t - 1$.

It is worth noting that we use a rolling window of only 60 months in the OOS portfolio selection process in order to align that of the covariance estimation window. This might cause the bootstrapped MHT results, such as FWER-SPA and FDP-SPA, as employed in Section 4.2, to be biased. To facilitate this concern, we adopt two alternative methods, 1) Bonferroni corrections of Bonferroni (1936) controlling for Familywise Error Rate (FWER), and 2) the method proposed by Benjamini and Hochberg (1995) (BH) heading for the False Discovery Rate (FDR). Additional details regarding these methods can be found in Appendix D.

As suggested in the results from Figure 1, the outperformance of the optimised factors remains statistically insignificant prior to 1998, so it is meaningless to form the aggregate factors before most emerging currencies are available.

As indicated by the results in Figure 3, the majority of emerging currencies become available after 1997. To ensure ample data for the 60-month in-sample selection window, we initiate the construction of aggregate out-of-sample (OOS) factors in January 2002. Table 8 provides an overview of the performance of the aggregate OOS factors, spanning from January 1998 to November 2020. The aggregate optimised carry and momentum factors consistently yield significantly positive profits according to both MHT approaches, with t-values exceeding 3.964. Conversely, the aggregated optimised value factor fails to generate statistically significant profits due to the negative returns generated by the original value factor. Regardless of the selection method, the aggregate optimised factors consistently outperform the naive factors at the 10% significance level, with all t-values of differences (Diff.t) exceeding 1.919.

Overall, our OOS analysis provides a crucial solution to the influence of look-ahead bias, bolstering the credibility and robustness of our findings. By demonstrating that the optimised currency factors consistently show their outperformance in the OOS test, we underscore their economic significance in the foreign exchange market and their applicability for market participants.

Table 8: Performance metrics of the OOS aggregate optimised factors

	Carry		Mom		Value	
	Bonferroni	BH	Bonferroni	BH	Bonferroni	BH
Average	4.58%	4.32%	5.61%	5.64%	0.59%	0.80%
T-value	4.349	3.964	6.002	6.010	0.749	0.960
Volatility	0.064	0.066	0.057	0.057	0.048	0.051
Sharpe	0.713	0.650	0.985	0.986	0.123	0.157
Diff_t	4.404	2.839	2.012	1.919	2.940	2.824

This table summarises the performance of the OOS aggregate optimised factors with transaction costs. *Average*, *Volatility*, and *Sharpe* denotes the annualised average returns, standard deviation, and Sharpe ratio of the aggregate optimised portfolio returns; Diff_t refers to the t-statistics of the differences between the aggregate optimised factors and naive ones.

6.2. Asymmetric currency anomalies

All the three factors we have studied above are symmetric portfolios based on cross-sectional sorting of currency returns, and therefore, have the same weight for long and short legs. In this section, we further validate our optimisation procedure by applying it to those currency factor portfolios that are asymmetric. We implement the time-series momentum (TSMOM) of Moskowitz et al. (2012), and the return signal momentum (RSMOM) of Papailias et al. (2021). As discussed by Goyal and Jegadeesh (2017), the TSMOM and RSMOM are sorted based on time series returns and are not net-zero investment strategies in which the number of assets in the long and short portfolios is not equal.

First, we build the TSMOM portfolio following Moskowitz et al. (2012). The TSMOM trading signals are determined by currencies period returns over the past j month formation period, as follows:

$$Sign_t^i = \begin{cases} 1, & \text{if } R_{t-j,t-1}^i > 0; \\ -1, & \text{otherwise,} \end{cases} \tag{16}$$

where $R_{t-j,t-1}^i$ is the period return of asset i over j month formation period. Next, for the RSMOM, the trading positions are determined by the return sign P over the formation period. For a particular asset i, the return sign of j months formation period is:

$$P_{t-j,t-1}^i = \frac{1}{J} \sum_{i=t-j}^{t-1} v_k, \tag{17}$$

where v_i is the return sign in month k, it is one if the return in that month is positive or 0 otherwise. Then we buy asset i with $P^i_{t-j,t-1}$ greater than a threshold p and sell if others. We set the threshold $p = 0.4$, which is the optimal threshold estimated by Papailias et al. (2021). Hence, the trading position of an RSMOM strategy is:

$$Sign^i_t = \begin{cases} 1, & \text{if } P^i_{t-j,t-1} > 0.4; \\ -1, & \text{otherwise.} \end{cases} \tag{18}$$

As there is no standard formation set for the TSMOM and RSMOM, we employ the formation period, j, as 1, 3, 6, 9, and 12 months, which is in line with the set of Moskowitz et al. (2012) and Papailias et al. (2021). To simplify the strategy name, TSMOM-j refers to the TSMOM with j months formation period, and the same manner is applied to represent RSMOM strategies. When $j = 1$, the RSMOM signals are identical to the TSMOM signals, so we report the results based on $j = \{3, 6, 9, 12\}$ for RSMOM.

We first apply data snooping bias tests to ascertain the remarkable performance of optimised TSMOM and RSMOM portfolios.[7] Reflecting the methodologies outlined in Section 4.2, we control for both FDP and FWER to ensure the integrity of our findings against various control targets. Table 9 reports the outcomes of the data snooping tests for the asymmetric currency factors. First, for TSMOM, we find that, even with the elevation of the critical value to 4.024, a significant number of optimised factors, no less than 1,152, outperform the naive factor. In the case of TSMOM-1, 7,532 and 5,332 optimised portfolios emerge as significant outperformers based on FDP-SPA and FWER-SPA, respectively. Similar patterns are shown in the results of RSMOM. The surge in the adjusted critical value to 3.314 does not deter the emergence of at least 824 optimised factors, each displaying notable outperformance at least at the 10% significance level. Importantly, these outcomes are consistently observed across various formation periods, further solidifying the credibility of our optimisation procedure for asymmetric currency factors. Overall, the results of data snooping tests suggest our optimisation approach can effectively uncover a substantial number of outperforming optimised factors against naive factors.

[7] Other supplementary results are shown in Appendix F.

Table 9: Data snooping bias test for asymmetric factors

J	TSMOM					RSMOM			
	1	3	6	9	12	3	6	9	12
Panel A: Controlling for FDP									
No.reject	7,532	5,592	3,888	4,548	5,196	7,536	5,936	3,328	3,028
k.stopped	377	280	195	228	260	377	297	167	152
CV	2.708	2.937	3.348	3.143	3.188	2.474	2.643	2.770	2.987
Panel B: Controlling for FWER									
No.reject	5,332	2,468	1,352	1,152	2,728	7,360	4,348	824	1,028
k.stopped	1	1	1	1	1	1	1	1	1
CV	3.368	3.627	3.921	4.024	3.710	3.219	3.402	3.314	3.394

This table summarises the number of portfolios rejecting the null as shown in Equation 11. No.reject counts the number of rejections for a specific factor strategy. k.stopped denotes the number of false rejections allowed. CV represents the adjusted critical t-values for each test. For more details, see Appendix C.

7. Conclusion

This study explores optimised portfolio construction for three well-studied FX market anomalies. Through an exhaustive evaluation encompassing over 24,000 optimised factor portfolios, we find compelling evidence to support the superiority of optimised currency factors over simplistic naive diversification. In particular, this superiority is rooted in the dynamic variations of currency CAPM α, a phenomenon propelled by the presence of emerging currencies. This increase in CAPM standard deviation α distinctly improves the profitability of optimised factor portfolios, providing a novel lens through which to comprehend their robust performance.

Importantly, when the currency landscape predominantly features developed currencies, none of the optimised factors exhibit statistically significant superior performance. This nuanced association aligns with prior research, such as Platanakis et al. (2021), who find that optimal portfolio performance is intricately linked to the standard deviation of CAPM α. By subjecting our findings to rigorous data snooping bias tests, following Romano and Wolf (2005) and Hsu et al. (2014), we confirm that the supremacy of optimised factors transcends mere chance, firmly anchoring itself in the inherent virtues of our refined portfolio construction methodologies. Furthermore, it elucidates a pivotal connection between optimal portfolio performance and the standard deviation of CAPM α, a revelation poised to invigorate future investigations. These findings collectively call for an exploration into the applicability of optimal asset allocation models for various financial anomalies across diverse asset classes, thus enriching our understanding of portfolio optimisation in broader financial contexts.

This study makes notable contributions to the literature on market efficiency and portfolio optimisation. Central to its innovation is the fusion of market factor trading positions with optimal portfolio construction, offering a fresh perspective on how the information in currencies' co-movement can improve market factor portfolios. We believe that this paper paves the way for future innovative research on the dynamics of factor portfolio construction in other asset classes such as commodity and bond. Furthermore, our attempt to employ data snooping tests to develop OOS optimal currency factor portfolios sheds light on a potential application of data snooping bias tests in currency portfolio selection.

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Appendix A. Summary statistics

This section details the data measure and summary statistics. Our sample is in line with the sample of Menkhoff et al. (2012*a*) ranging from November 1983 to November 2020. Table A.1 exhibits a summary statistic of our foreign exchange universe. Most currencies yield positive annualised returns over the sample period, in which *Indonesia* reports the highest annualised return at 13.86% per annum. Only 11 out of 48 currencies produce negative returns, in which *Finnish* displays the lowest averaged return, at -7.06% per annum. For volatility, most currencies exhibit moderate risks, with the annualised standard deviation between 0.003 and 0.272 *Indonesian* shows the highest annualised standard deviation, at 0.272; *Saudi Arabia* reports the lowest value, at 0.003. Furthermore, *Slovak* illustrates the highest Sharpe ratio, at 1.150 per annum; *Finnish* shows the lowest value, at -1.078 per annum. Lastly, *Indonesian* shows the highest single-month excess return, 34.85%, and the lowest signal month return, at -59.81%.

Table A.1: Summary statistics

Regions	Statistics					Sample period	
	Mean	*SD*	*SR*	*min*	*max*	*Start.date*	*End.date*
Australian	2.41%	0.117	0.205	-15.11%	10.88%	Jan-85	Nov-20
Austrian	-6.08%	0.101	-0.605	-6.69%	5.66%	Jan-97	Dec-98
Belgian	3.82%	0.125	0.306	-10.34%	7.36%	Nov-83	Dec-98
Brazilian	4.51%	0.160	0.282	-15.64%	12.12%	Apr-04	Nov-20
Bulgarian	-0.36%	0.095	-0.038	-9.14%	10.29%	Apr-04	Nov-20
Canadian	0.64%	0.074	0.086	-11.66%	9.25%	Jan-85	Nov-20
Croatian	0.56%	0.099	0.057	-9.76%	7.82%	Apr-04	Nov-20
Cyprus	4.65%	0.074	0.630	-4.60%	4.42%	Apr-04	Dec-07
Czech	1.46%	0.125	0.117	-13.11%	10.10%	Jan-97	Nov-20
Danish	2.08%	0.105	0.198	-10.31%	10.12%	Jan-85	Nov-20
Egyptian	11.65%	0.152	0.766	-49.88%	16.59%	Apr-04	Nov-20
Euro	-0.58%	0.097	-0.060	-9.45%	9.89%	Jan-99	Nov-20
Finnish	-7.06%	0.102	-0.691	-6.72%	5.63%	Jan-97	Dec-98
French	4.04%	0.116	0.347	-9.99%	7.93%	Nov-83	Dec-98
German	2.14%	0.120	0.179	-10.45%	7.19%	Nov-83	Dec-98
Greek	-4.76%	0.111	-0.427	-10.88%	7.19%	Jan-97	Dec-00
Hong Kong	-0.25%	0.006	-0.402	-1.34%	0.79%	Nov-83	Nov-20
Hungarian	2.47%	0.138	0.178	-18.88%	11.10%	Nov-97	Nov-20
Icelandic	1.53%	0.149	0.102	-26.71%	20.41%	Apr-04	Nov-20
Indian	1.57%	0.073	0.217	-8.43%	8.16%	Nov-97	Nov-20
Indonesian	13.86%	0.272	0.509	-59.81%	34.85%	Jan-97	Nov-20
Irish	1.96%	0.081	0.243	-5.98%	5.33%	Nov-93	Dec-98
Israeli	1.92%	0.080	0.240	-7.61%	7.30%	Apr-04	Nov-20
Italian	4.13%	0.117	0.354	-14.11%	8.59%	Apr-84	Dec-98
Japanese	-0.16%	0.109	-0.014	-10.29%	16.14%	Nov-83	Nov-20
Kuwaiti	0.50%	0.022	0.225	-6.23%	2.15%	Jan-97	Nov-20
Malaysian	2.45%	0.158	0.155	-31.31%	12.57%	Jan-85	Nov-20
Mexican	3.01%	0.119	0.253	-20.38%	12.24%	Jan-97	Nov-20
Netherlands	2.28%	0.120	0.190	-10.48%	7.55%	Nov-83	Dec-98
New Zealand	4.86%	0.125	0.391	-14.74%	13.55%	Jan-85	Nov-20
Norwegian	1.89%	0.111	0.170	-11.71%	8.64%	Jan-85	Nov-20
Philippine	1.17%	0.076	0.154	-11.99%	9.28%	Jan-97	Nov-20
Polish	2.69%	0.139	0.194	-16.41%	10.45%	Mar-02	Nov-20
Portuguese	-5.26%	0.097	-0.543	-6.51%	5.51%	Jan-97	Dec-98
Russian	0.36%	0.149	0.024	-16.45%	11.89%	Apr-04	Nov-20
Saudi	0.12%	0.003	0.407	-1.02%	0.52%	Jan-97	Nov-20
Singapore	0.31%	0.055	0.056	-8.58%	6.08%	Jan-85	Nov-20
Slovak	13.37%	0.116	1.150	-9.69%	10.55%	Mar-02	Dec-08
Slovenian	2.45%	0.081	0.302	-4.54%	4.47%	Apr-04	Dec-06
South Africa	0.03%	0.157	0.002	-19.69%	13.92%	Nov-83	Nov-20
South Korean	1.70%	0.109	0.156	-13.72%	12.93%	Mar-02	Nov-20
Spanish	-4.89%	0.101	-0.483	-7.04%	5.70%	Jan-97	Dec-98
Swedish	1.20%	0.112	0.107	-14.49%	11.06%	Jan-85	Nov-20
Swiss	0.46%	0.114	0.040	-14.11%	12.65%	Nov-83	Nov-20
Taiwan	-1.20%	0.053	-0.226	-8.76%	5.72%	Jan-97	Nov-20
Thai	1.24%	0.099	0.125	-19.96%	16.33%	Jan-97	Nov-20
Ukraine	-3.43%	0.198	-0.174	-48.39%	12.77%	Apr-04	Jun-15
Uk	1.20%	0.103	0.117	-13.02%	13.74%	Nov-83	Nov-20

This table reports the summary statistics of monthly excess returns. Regions show the country/region name of the currencies. *Mean* denotes annualised excess return. *SD* is annualised standard deviation of excess returns. *SR* refers to the Sharpe ratio. *max* and *min* represent the highest and lowest single month excess return, respectively. *Start.date* and *End.date* are the start and end months of the sample period, respectively.

Appendix B. Estimator and optimising approaches

The conventional portfolio optimisation of Markowitz (1952) followed a three-step procedure, including expected return estimation, expected covariance matrix estimation, and asset allocation (optimisation). Following this rationale, we first document the estimators of expected returns and covariance matrix in Appendix B.1 and Appendix B.2, and then, summarise the optimising approaches in Appendix B.3. When N assets are available, we define R_t as the $N \times 1$ vector of excess returns in month t. Then, the expected returns vector is shown as μ_t, and Σ_t represents the $N \times N$ estimated out-of-sample covariance matrix, with their sample counterparts given by $\hat{\mu}_t$ and $\hat{\Sigma}_t$, respectively. M refers to the estimation window, and T represents the total number of observations. We employ the rolling estimation window with M as 60 for all estimators to ensure they account for the market dynamics.

Appendix B.1. Covariance estimators

We estimate the out-of-sample covariance matrix across selected currencies through 12 different estimators. These are classified into three categories: 1) two estimators, in which the in-sample covariance matrix exclusively determines the out-of-sample covariance matrix; 2) nine estimators that originate from the Shrinkage covariance estimation model (Stein, 1956, James and Stein, 1961); and 3) one sparse covariance estimator that estimates the out-of-sample covariance matrix by adjusting the in-sample one.

Sample based covariance matrix (BasicCov): According to Markowitz (1952), this is a basic estimation approach, in which the out-of-sample covariance matrix, Σ_t, is equal to the in-sample covariance matrix, $\hat{\Sigma}_t$, over the estimation window, as:

$$\hat{\Sigma}_t = \frac{1}{M-1} \sum_{s=t-M+1}^{t} (R_s - \hat{\mu})(R_s - \hat{\mu})'. \tag{B.1}$$

Exponential weighted moving average covariance matrix (EWMACM): a simple dynamic model in which recent returns have more weight than past returns in the estimation. The out-of-sample covariance matrix is measured as :

$$\Sigma_t = \lambda\hat{\Sigma}_t + (1-\lambda)\hat{\mu}_t{}'\hat{\mu}_t, \tag{B.2}$$

where $\hat{\mu}_t{}'$ is the transpose of return vector $\hat{\mu}_t$. Following Ardia and Boudt (2015), we determined λ at 0.94.

Bayes-Stein shrinkage estimator (BS): The Bayes-Stein estimator was designed to mitigate the estimation errors of out-of-sample Σ_t and $\hat{\mu}_t$, where the predictive distribution of returns are used (James and Stein, 1961, Stein, 1956), as:

$$\Sigma_t = \frac{1}{M-N-2} \sum_{s=t-M}^{t-1} (R_s - \hat{\mu}_t)(R_s - \hat{\mu}_t)', \qquad (b) \tag{B.3}$$

where all notions remain as above-mentioned.

Ledoit-Wolf shrinkage estimator (LW): This is a weighted average sample covariance matrix based on *'prior'* or *'shrinkage target'*, which is first built by Ledoit and Wolf (2003) and further developed by Ledoit and Wolf (2004*a*,*b*). In this study, the *'shrinkage target'* is determined by six different approaches: 1) factor approach, LW(factor), where the *'prior'* is given by a single-factor model of Sharpe (1964), and the factor is equal to the cross-sectional average of all the variables; 2) constant approach, LW(con), where the *'prior'* is determined by a constant correlation matrix; 3) pre-set correlation approach, LW(PC), where the *'prior'* is given by the constant correlation covariance matrix of Ledoit and Wolf (2004*a*); 4) one-parameter method, LW(one), where the *'prior'* is given by the one-parameter matrix, and all variances are the same and all covariances are zero; 5) diagonal method, LW(dia), where the *'prior'* is given by a diagonal matrix; 6) large approach, LW(large), this covariance estimator uses a weighted average of the sample covariance matrix where the *'shrinkage target'* is given by a one-factor model (similar to the LW(factor)), but the weight, or *'shrinkage intensity'* is chosen to minimise quadratic loss measured by the Frobenius norm. This covariance estimator is guaranteed to be invertible and well-conditioned when the number of variables exceeds that of observations.

Oracle Approximating Shrinkage (OAS) and Rao-Blackwell Ledoit-Wolf Estimator (RBLW): These two approaches were first introduced by Chen et al. (2010). Both algorithms are designed to minimise the mean squared error of covariance estimation. The authors proposed to estimate the out-of-sample covariance matrix by iteratively approximating the shrinkage with:

$$\Sigma_t = \rho \hat{F}_t + (1-\rho)\hat{\Sigma}_t \tag{B.4}$$

where $\rho \in (0,1)$ is a control parameter, and $\hat{F}_t$ is a target matrix. It is proposed to use a structured estimate $\hat{F}_t = \mathrm{Tr}(\hat{\Sigma}_t/p) \cdot I_{p\times p}$ where $I_{p\times p}$ is an identity matrix of dimension p, and $Tr(\bullet)$ represents the trace of a matrix. In our study, dimension p is equal to the length of the estimation window, M. However, the control parameter ρ is estimated via various approaches. For the *Oracle Approximating Shrinkage Estimator*, the control parameter is estimated as:

$$\rho = min(\frac{(\frac{1-2}{p})Tr(\hat{\Sigma}^2) + Tr^2(\hat{\Sigma})}{(\frac{N+1-2}{p})[Tr(\hat{\Sigma}^2) - \frac{Tr^2(\hat{\Sigma})}{p}]}, 1). \tag{B.5}$$

For the *Rao-Blackwell Ledoit-Wolf Estimator*, we determine the control parameter as:

$$\rho = \frac{(\frac{N-2}{N})Tr(\hat{\Sigma}^2) + Tr^2(\hat{\Sigma})}{(N+2)[Tr(\hat{\Sigma}^2) - \frac{Tr^2(\hat{\Sigma})}{p}]}, \tag{B.6}$$

where all notations remain the same.

Adaptive Thresholding Estimation (ADA): The idea of this approach is to apply the thresholding technique on the correlation matrix so that it becomes adaptive to each variable (Cai and Liu, 2011). In line with the above-mentioned definitions, each element in the in-sample covariance matrix is defined as $\hat{\sigma_{ij}}$, and, therefore, $\hat{\Sigma} = (\hat{\sigma_{ij}})_{N\times N}$. This method then links the out-of-sample covariance matrix to the in-sample one as follows:

$$\Sigma = (\sigma_{ij})_{N\times N} \quad with \quad \sigma_{ij} = s_{\lambda_{ij}}(\hat{\sigma_{ij}}), \tag{B.7}$$

where $\lambda_{ij} = \lambda_{ij}(\delta) = \delta\sqrt{\frac{\theta_{ij} log(N)}{M}}$, $\delta > 0$ is a regularisation parameter that is chosen by the data during estimation window, and $s_\lambda(\bullet)$ is a general thresholding function. In this study, we follow Cai and Liu (2011) and define:

$$s_{\lambda_{ij}}(\hat{\sigma_{i,j}}) = \hat{\sigma_{i,j}} I\{\hat{\sigma_{ij}} \geq \lambda_{ij}\}, \tag{B.8}$$

where $I\{\bullet\}$ is an indicator function whose value is one if the condition is satisfied.

Appendix B.2. Expected return estimators

In this study, we employ four different expected return estimators as follows.

Sample-based expected return (BasicR): The arithmetic mean of the in-sample returns, μ_t, equals the out-of-sample expected return, $\hat{\mu}_t$.

Exponential weighted moving average returns (EWMARE): Like the EWMACM estimator, the EWMARE estimator allocates higher weights to more recent returns, and then estimates the out-of-sample expected return by taking the sum of those weighted returns as follows:

$$\hat{\mu}_t = \sum_{i=1}^{T}(R_i \times \frac{\lambda_i}{\sum_{i=1}^{T}\lambda_i}) \tag{B.9}$$

where we hold $\lambda = 0.94$. This approach is extensively used in both financial academia and industry.

Bayes-Stein (BSR): The Bayes-Stein estimator introduced by Stein (1956) and James and Stein (1961), as

$$\hat{\mu}_t^{bs} = (1 - \hat{\phi}_t)\hat{\mu}_t + \hat{\phi}_t\hat{\mu}_t^{min}, \tag{B.10}$$

$$\hat{\phi}_t = \frac{N+2}{(N+2) + M \times (\hat{\mu}_t - \mu_t^{min})'\Sigma_t^{-1}(\hat{\mu}_t - \mu_t^{min})}, \tag{B.11}$$

where $0 < \hat{\phi}_t < 1$, Σ_t and $\hat{\mu}_t = \hat{\mu}_t^{bs}$ are measured by Equation B.3 (a), and $\mu_t^{min} \equiv \hat{\mu}_t'\hat{w}_t^{min}$ is the mean of the sample global minimum variance portfolio excess returns. The estimation of μ_t^{min} is consistent with the estimator introduced by Jorion (1986), which is the mean of the minimum-variance portfolio.

Implied return (IR): this model was first introduced by Martellini (2008), who found that the expected return of stock positively links to the total volatility over the estimation window. This is in line with the findings of Ang et al. (2006) that the assets with high volatility over the formation period yield outperforming profits afterwards. Therefore, Martellini (2008) demonstrates that the volatility of returns during the estimation horizon can be an expected return estimator as:

$$\hat{\mu}_t = \sqrt{\frac{\sum_{i=1}^{M}(R_i - \bar{R})^2}{M-1}} \tag{B.12}$$

where $\bar{R}$ refer to the mean of excess returns over the estimation window. In this approach, the expected return is equivalent to the ex-ante standard deviation over the estimation window.

Appendix B.3. Optimising approaches

A cross-sectional portfolio is composed of a long portfolio and a short portfolio. The currency market anomalies hold in the optimal factor portfolio where the trading direction of each investment is still determined by the factor trading signals. Following DeMiguel et al. (2009) and Hsu et al. (2018), we implement seven portfolio optimising approaches, where the 1/N portfolio is used as the benchmark.

1/N portfolio (Naive): This is the most frequently used approach where all assets are equally weighted. This scheme is extensively applied as the benchmark by DeMiguel et al. (2009) and Hsu et al. (2018). The conventional currency momentum portfolio of Menkhoff et al. (2012*b*) also employs this basic portfolio construction. In line with this literature, we employ this portfolio construction as the benchmark weighting scheme.

Mean-variance portfolio (MV): The mean-variance portfolio was first introduced by Markowitz (1952). Under the extensively used approach to conditional mean-variance optimisation, the investors are assumed to determine the asset weights vector, w_t, to maximise their expected utility as:

$$max_{w_t} \quad w_t'\mu_t - \frac{\gamma}{2} w_t'\Sigma_t w_t, \tag{B.13}$$

where γ is the coefficient of the relative risk aversion. Here, in accordance with Ardia et al. (2017), we set γ as 0.89. The optimal w_t is not achievable unless we estimate μ_t and Σ_t. As abovementioned, we employ 12 covariance estimators and four expected return estimators, so there are 48 combinations in total.[8]

Global minimum variance portfolio (GMV): To implement the minimum-variance portfolio in practice, the investor chooses the portfolio that minimizes the variance of returns:

$$min_{w_t} \quad w_t'\Sigma_t w_t. \tag{B.14}$$

The portfolio weight, w_t is exclusively determined by the covariance matrix. In this study, we

[8]This portfolio family contains a number of well-known portfolio constructions. For instance, when both expected return and conditional covariance matrix are estimated through the Bayes-Stein approach, this is a Bayes-Stein shrinkage portfolio of Jorion (1986).

employed 12 covariance matrix estimators, so there are 12 different portfolios in this group.

Maximum diversified portfolio (MD): This optimisation approach was first introduced by Choueifaty and Coignard (2008). The authors defined the diversification ratio as $DR(w_t) = \frac{w_t'\sigma_t}{\sqrt{w_t'\Sigma_t w_t}}$, and the MDP is going to build the portfolio with maximum DR. The portfolio weight is computed by solving the following problem:

$$max_{w_t} \quad \frac{w_t'\sigma_t}{\sqrt{w_t'\Sigma_t w_t}}, \tag{B.15}$$

where σ_t is a vector of the sample standard deviation that is equilibrium to the square root of the dialogue of the in-sample covariance matrix $\hat{\Sigma}_t$, according to different covariance estimators, 12 portfolio constructions are obtained in this family.

Equal risk contribution portfolio (ERC): this portfolio construction was first documented by Maillard et al. (2010), who aimed to build a risk-balanced portfolio. Under this construction, all risky assets make the same contributions to the portfolio risks. In other words, under this construction, the percentage volatility risk contribution, $RC_i = \frac{w_{i,t}'[\Sigma_t w_t]}{w_t'\Sigma_t w_t}$, of any asset i in N available assets equals 1/N. The portfolio weight is measured by solving the optimisation problem:

$$min_{w_t} \quad \sum_{i=1}^{N} (RC_i - \frac{1}{N})^2 . \tag{B.16}$$

According to different covariance estimators, 12 portfolio constructions are obtained in this family.

Risk efficient portfolio (RE): This portfolio construction was first introduced by Amenc et al. (2011). The authors introduce construct a maximum Sharpe ratio portfolio under the assumption that the asset's expected return is a deterministic function of its semi-deviation and the cross-sectional distribution of semi-deviations. In particular, they sorted assets by their semi-deviation, formed decile portfolios and then computed the median semi-deviation of stocks in each decile portfolio (denoted by $\xi_j(j = 1, ..., 10)$). The so-called risk-efficient portfolio is achieved by solving:

$$max_{w_t} \quad \frac{w_t' J\xi}{\sqrt{w_t'\Sigma_t w_t}}, \tag{B.17}$$

where J is a $(N \times 10)$ matrix of zeros whose (i, j)th element is one if the semi-deviation of stock i belongs to decile j, $\xi = (\xi_1, ..., \xi_{10})$. We estimate the semi-deviations using the exponential weighted moving average approach proposed by Ardia et al. (2017), where the weighing parameter is 0.94. We implement 12 constructions with the different covariance estimators in this group.

Maximum decorrelation portfolio (MAD): This portfolio optimisation approach was proposed by Christoffersen et al. (2012). The maximum decorrelation portfolio is closely related to the minimum variance portfolio but attempts to reduce the number of input parameters. Instead of using the variance-covariance matrix Σ_t, the strategy assumes that individual asset volatilities are identical and solely uses the correlation matrix Ω as its main input. Hence, the optimal portfolio weights are given by the following optimisation problem:

$$min_{w_t} \quad w_t' \Omega w_t, \tag{B.18}$$

where Ω is the $N \times N$ correlation matrix. Therefore, in contrast to the MV and GMV portfolios that attempt to decrease risk by concentrating on low volatility assets, the MAD portfolio seeks to exploit risk reduction effects stemming from investing in assets with low correlations. While this approach avoids concentration in certain assets by ignoring differences in individual volatilities, the strategy can nevertheless result in high asset loading by focusing on assets with low correlations with other assets. With various correlation estimators, we conduct 12 portfolios in this family.

Volatility timing (VT): This optimisation was documented by Fleming et al. (2001), Fleming et al. (2003), and Kirby and Ostdiek (2012). These authors study a class of active diversification strategies characterized by a low turnover. They argue that these strategies outperform naive diversification, even in the presence of relatively high transaction costs. The volatility timing strategy uses the weights as follows:

$$w_{i,t} = \frac{(1/\sigma_{i,t}^2)^n}{\sum_{i=1}^{N}(1/\sigma_{i,t}^2)^n}, \tag{B.19}$$

where $n > 0$, and $\sigma_{i,t}$ is the estimated standard deviation of i-th asset in month t. The tuning parameter n reflects the aggressiveness of volatility timing. Here, similar to Kirby and Ostdiek (2012), we define the portfolio constructed by this equation as the VT(n) portfolio, where $n = 0.5, 1, 2, 4$.[9] In terms of various covariance estimators and the dynamic of n, we conduct 48 portfolio constructions in this group.

To sum up, we conduct 48+12+12+12+12+12+48=156 optimal portfolio constructions for the long or short portfolios. Further considering the different constructions applied between long and short legs, we achieve $156 \times 156 = 24,336$ optimised factor portfolios in total.

[9] Kirby and Ostdiek (2012) ranged N as integers from one to four. We adopt $n = 0.5$ which makes the portfolio equivalent to the inverse volatility weighted portfolio of De Carvalho et al. (2012).

Appendix C. Multiple hypothesis testing controlling the false discovery proportion

We define $f_{k,t}$ ($k = 1, 2, \ldots, m$ and $t = 1, 2, \ldots, n$) as series with n observations relative to the benchmark model for the k-th model over sample period. Then, m represents the total number of tested models, R is an observed variable that is the number of trading rules rejected by the SHT, V are falsely rejected hypotheses (Type I errors), T is the hypotheses should be rejected but are not (Type II errors), U and S represent the number of correct rejections when the null is true or not. Therefore, the V/R is the false discovery proportion (FDP).

White (2000) proposed the Bootstrap Reality check (RC) test to draw statistical performance in terms of an empirical distribution of the best model from a universe of alternative models. However, the test power of the RC test is adversely impacted by the poor performance models. To solve this issue, Hansen (2005) developed a more powerful method named the Superior Predictive Ability (SPA) test. Both MHT approaches are also known as data snooping bias tests. Both algorithms adjust the critical values based on bootstrapped distribution so that they can exclude the significant outperformance caused by chance.[10]

The null hypothesis of SPA test is that none of the involved m trading rules show significant outperformance, as:

$$H_0: \quad \max_{k=1,\ldots,m} \varphi_k \leq 0. \tag{C.1}$$

The alternative hypothesis is therefore: at least one of the m trading rules is significantly superior to the benchmark. When we measure the strategy performance according to the mean of excess returns, φ_k is equal to $\bar{f}_k = \frac{\sum_{t=1}^{n} f_{k,t}}{n}$. According to Equation C.1, the normalised maximum performance is:

$$RC_n = \quad \max_{k=1,\ldots,m} \sqrt{n}\bar{f}_k. \tag{C.2}$$

Then, the RC test employs the stationary bootstrap procedure of Politis and Romano (1994)

[10]The data-snooping bias is a well-discussed topic in financial academia, e.g., Lo and MacKinlay (1990) and Brock et al. (1992).

to measure the p-values of each hypothesis. We define $f_k^*(b)$ ($b = 1, 2, \ldots, B$) as the resampled series of model k after b times stationary bootstrap with a pre-specified block length Q, and $\bar{f}_k^*(b)$ is the performance of trading rule k at b-th bootstrap. The bootstrapped normalised maximum performance, $RC_n^\star$, is:

$$RC_n^\star = \max_{k=1,\ldots,m} \sqrt{n}(\bar{f}_k^*(b) - \bar{f}_k). \tag{C.3}$$

Then, the adjusted p-value is measured by comparing RC_n with the quantiles of the distribution of bootstrapped performance, $RC_n^\star$. We reject the null hypothesis when the p-value is below the pre-determined critical value, α. Notably, White (2000) apply the least favourable configuration (LFC), in which the expected return of k-th trading rule is equal to zero, $E(\bar{f}_k) = 0$. When a large number of poor performing trading rules ($E(\bar{f}_k) < 0$) are included by the model sample, the adjusted p-values will be very high, and few null hypotheses can be rejected.

To avoid LFC, Hansen (2005) introduced the Superior Predictive Ability (SPA) test based on the RC but minimises impacts of poorly performed trading rules. First, the SPA excludes the negatively performed model, so the normalised maximum performance of Equation C.2 is converted to:

$$SPA_n = \max(\max_{k=1,\ldots,m}(\bar{f}_k, 0)). \tag{C.4}$$

The SPA no longer requires the LFC by recentering the distribution of bootstrapped performance. For the k-th rule, we define $\bar{Z_{k,t}}^\star(b)$ as the performance measure of the recentered returns of the b-th bootstrapped series:

$$Z_{k,t}^\star(b) = f_{k,t}^*(b) - \bar{f}_k + \bar{f}_k I(\bar{f}_k \leq A_k), \tag{C.5}$$

where $A_k = -\hat{\sigma_k}\sqrt{2loglog(n)}$, and $\hat{\sigma_k}$ refers to the standard deviation of $\sqrt{n}\bar{f}_k$. Therefore, the bootstrapped normalised maximum performance of SPA, $SPA_n^\star$, is:

$$SPA_n^\star = \max(\max_{k=1,\ldots,m} \sqrt{n}(\bar{Z_{k,t}}^\star(b) - \bar{f}_k), 0). \tag{C.6}$$

Then, the adjusted p-values of SPA are measured through the same procedure of the RC test. Hansen (2005) also proposed that the performance used in RC test is not standardised which also reduces the test powers. In our study, we apply the standardised performance measures, t-statistics, for both RC and SPA tests to compare their test power in the same manner.

Romano et al. (2007) introduced the (FDP) control approaches to manage Type I errors. Such an approach is an extension of controlling the familywise error rate (FWER) representing the number of false rejections. The basic FWER control approach ensures that the probability of at least one false rejection is less than or equal to a given significance level α. However, those tests are static as the procedure only stop if all the null hypothesis are rejected. To build a more flexible data snooping test procedure, Romano et al. (2007) and Hsu et al. (2014) further developed generalisations of the Step-SPA tests known as Step-Step-SPA(K) tests to control the FWER at K level (FWER(K)), i.e. these tests stop when the number of rejection is less a specific value K. Thus, a Step-SPA(K) test allows at most K false rejections in the MHT procedure. K can be any positive integer, and a higher K refers to more tolerance of false rejections. The procedure for applying the Step-SPA(K)tests is as follows:

1. Reform $\bar{f}_k$ in descending order.

2. Rank $\widehat{c_{\alpha,spa}}$ for SPA test in descending order and collect its $(1-\alpha)$-th quantile as critical value $q_j(\alpha)$.

3. Reject the top performing model k if $\bar{f}_k$ is greater than the maximum between zero and critical value, $q_j(\alpha)$, for the SPA test. Stop the procedure if the number of rejected models is less than a given level K ($K \geq 2$); otherwise, move to the next step.

4. Create a sub-sample by removing the model k from the entire sample. Further, reject the model i if $\bar{f}_i$ is greater than the maximum between zero and the critical value for the SPA test based on the sub-sample. Stop the procedure if the number of rejected models is less than a given K; otherwise, move to the next step.

5. Repeat step **4** to reconstruct sub-samples and re-test until the procedure stops.

where $\bar{f}_k$ refers to the t-statistics of Sharpe ratios in our study, $\widehat{c_{\alpha,spa}} = (\bar{f}^*_k(b) - \bar{f}_k + \bar{f}_k I(\bar{f}_k \leq A_{n,k}))$.

However, previous literature, e.g., Romano et al. (2007) and Hsu et al. (2014), agreed that controlling the FWER(K) is still stringent when the number of false rejections is large but the K is fixed and small. When asymptotically controlling the FDP at a given level α, the probability function of FDP should follow limsup $P(FDP > \gamma) \leq \alpha$ where the number γ ranges from zero to one. According to this, Romano et al. (2007) and Hsu et al. (2014) developed an asymptotic controlling the FDP based on the Step-RC and Step-SPA tests, respectively. The FDP refers to the rate between the number of erroneously rejected null hypotheses and the total number of significant declarations as $P = V/R$. When there is no rejection ($R = 0$), the authors defined $P = 0$ as no false rejection occurs.

The FDP control approach is a direct extension of the abovementioned generalised FWER(K) control approach. For instance, assuming that the procedure is consistent in that every outperformed trading rule is rejected in the first step with probability approaching one and S=10, the FDP of MHT will be $V/(V + 10)$. Further image the $\gamma = 0.1$, $V/(V+10)$ is greater than γ as long as $V \geq 2$. As a consequence, the FDP control approach is asymptotically equivalent to the FWER(2) control. Generally, for any known S and γ, we conclude that the asymptotic FDP control is equivalent to FWER($\lceil \frac{\gamma * S}{1-\gamma} \rceil$). The MHT controlling the FDP, namely FDP-SPA, at the level α can be achieved by implementing the FWER(K) approach. In other words, the FDP approach controls the error proportions by applying various K in the generalised Step-SPA(K) tests across different data sets. Thus, the FDP-RC/SPA algorithm is constructed by repeating the Step-SPA(K). In the empirical tests, we set γ at 0.1. The procedure of the FDP-RC/SPA tests is as follows:

1. Start with $K = 1$ and a γ between zero and one.

2. With a given level of α, R denotes the number of the rejected hypothesis by the Step-SPA(K).

3. If $R < K/\gamma - 1$, stop and reject all hypotheses rejected by Step-SPA(K). If not, reset $K = K + 1$ and repeat Step 2.

The FDP-SPA test is designed to ensure that the realised FDP is below a specific threshold when the same hypothesis question is asked according to various datasets(Harvey et al., 2020). In other words, this control approach forces the tail of the FDP distributions to behave in a certain manner. Theoretically, this is a desirable method to control the Type I errors when the MHTs are applied to find the significance from m trading rule based on various data samples.

To implement the bootstrap procedure, we follow previous literature, namely, Hsu et al. (2014), Hsu et al. (2016), Chordia et al. (2020), and Harvey et al. (2020), and set $Q = 4$, $b = 500$, $\alpha = 0.1$ and $\gamma = 0.1$.

Appendix D. Bonfferoni corrections and the BH approach

The Bonferroni approach is a single-step procedure in which all tested statistics are compared to a single adjusted critical value (Bonferroni, 1936). If each hypothesis is tested at a given significance level α, and the probability of erroneous rejection is $\alpha^\star$, the expected number of false rejections $E(V)$ is equal to $m \times \alpha^\star$. Thus, this method rejects a specific hypothesis k if the p-value (p_k) is equal to or less than the adjusted critical value based on the total number of hypotheses, $\alpha^\star = \alpha/m$. This method is widely used due to its simplicity and inspires the FWER control methods described below.

However, the Bonferroni approach determines the critical values based exclusively on the total number of tests, and, hence, is too conservative to uncover all true significant rejections. The critical value of this method does not incorporate the cross-correlations across various trading rules that exist in most financial applications. According to this mechanism, a large m leads to a very small critical value for many financial applications, and only hypotheses that yield extremely small p-values can be rejected.

Differently, Benjamini and Hochberg (1995) (BH) introduce a more tolerant MHT approach. It is a step-up procedure that sorts all p-values in ascending order. This method first assesses the least significant hypothesis and then moves upwards to find the critical value. The critical value is equal to the p-value with order $j^\star$, which is computed as:

$$j^\star = \max\Big\{ j : p_j \leq \frac{j \times \delta}{m} \Big\}. \tag{D.1}$$

When the critical value is p_j, $(j-1)$ hypotheses are rejected. The expected number of false rejections is $m \times p_j$, and $(m \times p_j)/j$ refers to the FDR that is always less than δ as we start from the smallest p-value.

Appendix E. Regression results based on CAPM

Table E.1 summarises the coefficient correlations of CAPM for each currency over the entire sample period. Panel A contains the regression results of developed currencies defined in Section 5. Panel B lists the results of emerging currencies.

We find that the average alpha in the developed sample, -0.001, is higher than that in the emerging sample, -0.002. Five alphas in Panel A are statistically and significantly different from zero at the 10% level, and seven currencies are statistically and significantly different from zero at the 10% level in Panel B. The standard deviation of α in the emerging sample exceeds that in the developed sample, which supports our findings in Section 5. Moreover, the regression results further show that the σ_ϵ in the emerging sample, 0.029, is higher than that in the developed sample, 0.018. This is in line with the proposition of Platanakis et al. (2021) that the increased idiosyncratic volatility can benefit the performance of optimal portfolios.

Table E.1: CAPM regression

	α	$t(\alpha)$	β	$t(\beta)$	σ_ϵ
Panel A: Developed					
Australian	-0.001	-0.921	0.582	4.623	0.031
Austrian	0.000	-0.391	1.148	28.445	0.014
Belgian	-0.001	-1.256	1.128	25.997	0.015
Canadian	0.000	-0.015	0.307	5.645	0.019
Cyprus	-0.001	-1.636	0.915	10.836	0.009
Czech	-0.002	-1.573	1.237	23.766	0.020
Danish	-0.001	-1.433	1.116	40.064	0.013
Euro	0.000	1.004	1.136	32.254	0.010
Finnish	0.003*	1.916	1.127	21.333	0.009
French	-0.001	-0.730	1.091	29.526	0.013
German	0.001	0.713	1.145	29.701	0.013
Greek	-0.001	-0.486	0.946	14.303	0.021
Hong Kong	0.000*	1.787	0.007	1.829	0.002
Irish	-0.001	-0.916	1.041	31.122	0.016
Israeli	-0.002	-1.219	0.549	8.620	0.020
Italian	-0.002	-1.534	0.956	18.898	0.018
Icelandic	-0.002	-0.685	0.719	4.177	0.039
Japanese	0.001	0.812	0.707	9.247	0.028
Netherlands	0.000	0.213	1.134	28.055	0.014
New Zealand	-0.003**	-2.247	0.684	5.977	0.031
Norwegian	-0.001	-1.020	0.956	21.433	0.019
Portuguese	-0.002	-1.522	0.941	20.807	0.021
Singapore	0.000	0.230	0.380	10.672	0.013
Slovak	-0.005***	-2.806	1.165	15.044	0.017
Slovenian	-0.001	-1.071	0.914	8.995	0.009
Spanish	-0.002**	-1.962	0.926	20.618	0.021
Swedish	0.000	0.414	0.985	26.647	0.019
Swiss	0.001	0.785	1.129	26.066	0.019
Taiwan	0.001	1.148	0.309	8.914	0.014
Uk	0.000	-0.429	0.836	15.256	0.021
Average	-0.001	-0.494	0.874	18.296	0.018
SD	0.002	1.190	0.310	10.183	0.007
Panel B: Emerging					
Brazilian	-0.006**	-2.104	0.851	4.843	0.040
Bulgarian	0.000	-0.479	1.139	29.078	0.010
Croatian	-0.001*	-1.815	1.147	28.276	0.011
Egyptian	-0.009***	-4.832	0.106	1.245	0.046
Hungarian	-0.003*	-1.920	1.310	13.448	0.024
Indian	-0.001	-1.075	0.350	5.480	0.019
Indonesian	-0.013*	-1.763	0.900	5.320	0.080
Kuwaiti	0.000*	-1.792	0.174	7.996	0.005
Malaysian	-0.002	-0.273	0.349	3.398	0.046
Mexican	-0.003	-1.379	0.362	2.944	0.029
Philippine	-0.001	-0.460	0.224	4.583	0.023
Polish	-0.001	-0.544	1.317	14.173	0.025
Russian	-0.001	-0.299	0.867	6.266	0.037
Saudi	0.000*	-1.682	0.002	1.280	0.001
South Africa	0.000	0.024	0.745	8.174	0.040
South Korean	-0.001	-0.376	0.825	5.521	0.025
Thai	-0.001	-0.645	0.397	6.371	0.029
Ukraine	0.001	0.222	0.373	1.850	0.037
Average	-0.002	-1.177	0.635	8.347	0.029
SD	0.004	1.176	0.426	8.201	0.018

This table reports the regression results for each currency based on Equation 13. α and β represent the coefficients of intercept and slope, $t(\alpha)$ and $t(\beta)$ report the t-values measured by Heteroskedasticity- and autocorrelation-consistent (HAC) standard errors of Newey and West (1987). σ_ϵ refers to the standard deviation of noise term ϵ_t in Equation 13, which is also known as idiosyncratic volatility. '*', '**', '***' represent that the t-values of α are statistically significant at the 10%, 5% and 1% level.

Appendix F. Optimising the asymmetric currency anomalies

This section demonstrates the empirical results identifying the outperformance of the optimised factor portfolios for the asymmetric currency anomalies. Table F.1 presents the performance of the optimised and naive TSMOM and RSMOM factors. The table reports metrics, including annualised average return (Average), t-statistics (T-value), standard deviation (Volatility), and Sharpe ratio (Sharpe). In Panel A, we present the performance metrics for the naive TSMOM and RSMOM portfolios, wherein currencies are equally weighted. The TSMOM and RSMOM strategies generate significant profits at the 10% level, except for the RSMOM-9 strategy. Across various formation periods, the TSMOM-3 generates the highest average returns, 4.37% (t = 3.581). RSMOM-12 portfolio yields the second most profitable performance, with an average return of 3.96% (t=3.658) and a Sharpe ratio of 0.681, the highest Sharpe ratio across all portfolios.

Table F.1: Performance metrics of naive and optimised portfolios for asymmetric factors

	TSMOM					RSMOM			
	Formation periods					Formation periods			
	1	3	6	9	12	3	6	9	12
Panel A: Naïve portfolio									
Average	3.48%	4.37%	3.60%	3.83%	2.79%	2.99%	2.45%	1.99%	3.96%
Volatility	0.063	0.065	0.069	0.066	0.063	0.060	0.065	0.063	0.058
T-value	2.937	3.581	2.863	2.924	2.274	2.949	2.034	1.556	3.658
Sharpe	0.550	0.671	0.521	0.577	0.441	0.498	0.377	0.314	0.681
Panel B: Best profitable portfolio									
Average	11.67%	13.51%	12.65%	12.92%	11.58%	11.41%	9.94%	8.94%	11.56%
Volatility	0.101	0.106	0.115	0.104	0.104	0.100	0.116	0.109	0.104
T-value	4.550	5.168	4.374	4.761	4.264	4.681	3.642	3.326	4.469
Sharpe	1.152	1.271	1.101	1.244	1.118	1.139	0.858	0.818	1.109
Panel C: Best risk-adjusted portfolio									
Average	11.65%	13.51%	12.58%	12.59%	11.43%	11.30%	9.94%	8.94%	11.56%
Volatility	0.100	0.106	0.113	0.100	0.102	0.098	0.115	0.109	0.104
T-value	4.579	5.168	4.401	4.835	4.221	4.714	3.668	3.326	4.469
Sharpe	1.169	1.271	1.109	1.263	1.122	1.151	0.862	0.818	1.109

This table summarises the performance metrics of the TSMOM and RSMOM portfolios based on various formation periods. Panel A reports the performance of the original naive portfolios. Panel B and C report the performance of the best-performing optimised factors based on portfolio returns and Sharpe ratios, respectively. *Average* denotes the mean annualised factor returns. *Volatility* and *Sharpe* are the annualised standard deviation and Sharpe ratio. *T-value* is the t-statistic which is measured by the heteroskedasticity- and autocorrelation-consistent (HAC) standard errors described by Newey and West (1987).

In Panel B and C, we provide a performance summary of the best-performing optimised

TSMOM and RSMOM portfolios, focusing on their returns (best profitable portfolio) and Sharpe ratio rankings (best risk-adjusted portfolio), respectively. In each Panel, the optimised TSMOM and RSMOM portfolios show higher profits than their naive counterparts, and the annual returns are all statistically significant at the 1% level. When considering return as the performance metric, the most profitable TSMOM portfolio, TSMOM-3, results in impressive growth in profitability from 4.37% to 13.51%. Besides the profits, similar to our symmetric factor results, the optimised factors exhibit almost doubled volatility compared to the naive factors, but the increased profits sufficiently compensate for the increased volatility, leading to higher Sharpe ratios. The best risk-adjusted TSMOM-9 portfolio exhibits the most significant improvement in the Sharpe ratio from 0.577 to 1.263.

Table F.2: Outperformance of the optimised asymmetric factors

	TSMOM					RSMOM			
	Formation periods					Formation periods			
	1	3	6	9	12	3	6	9	12
Panel A: Best profitable portfolio									
Average	8.19%	9.14%	9.06%	9.09%	8.79%	8.41%	7.49%	6.95%	7.60%
Volatility	0.083	0.083	0.088	0.085	0.089	0.082	0.099	0.085	0.086
T-value	3.725	4.013	3.844	3.921	3.732	3.887	3.099	3.126	3.328
Sharpe	0.987	1.101	1.025	1.075	0.984	1.029	0.754	0.814	0.888
Panel B: Best risk-adjusted portfolio									
Average	8.17%	9.14%	8.99%	8.76%	8.64%	8.31%	7.49%	6.95%	7.60%
Volatility	0.082	0.083	0.087	0.083	0.088	0.081	0.099	0.085	0.086
T-value	3.716	4.013	3.850	3.924	3.672	3.881	3.129	3.126	3.328
Sharpe	0.993	1.101	1.035	1.050	0.981	1.029	0.760	0.814	0.888

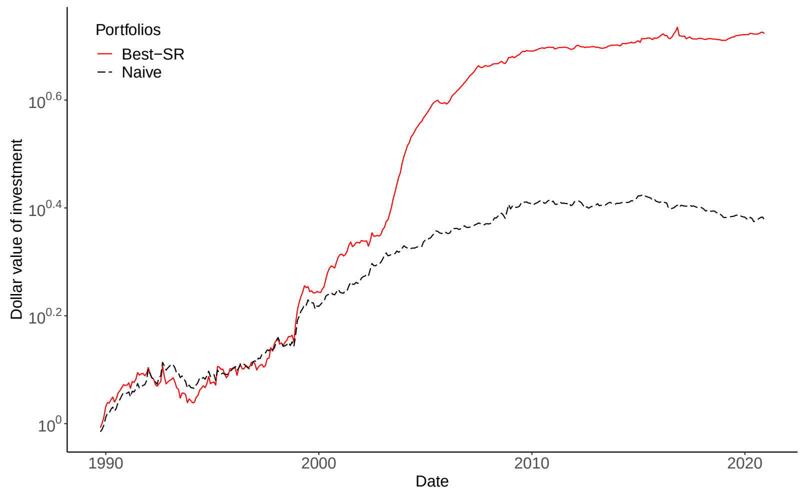
This table summarises the outperformance of the optimised TSMOM and RSMOM portfolios based on various formation periods. The original naive portfolios are the benchmarks. Panel A and B report the outperformance of the best-performing optimised factors based on portfolio profits and Sharpe ratios, respectively. *Average* denotes the mean annualised factor returns. *Volatility* and *Sharpe* are the annualised standard deviation and Sharpe ratio. *T-value* is the t-statistic which is measured by the heteroskedasticity- and autocorrelation-consistent (HAC) standard errors described by Newey and West (1987).

We further assess whether the outperformance of the optimised TSMOM and RSMOM are statistically significant. In line with the measurement in Section 4, we employ the naive TSMOM and RSMOM portfolios as the benchmarks and assess the differences between the optimised and benchmark portfolios. The results are presented in Table F.2, demonstrating the summary statistics of the outperformance. Impressively, the 18 optimised asymmetric factors deliver profits of at least 7.49% higher annually than the corresponding benchmarks. All these superiorities show statistical significance at the 1% level, according to the t-values.

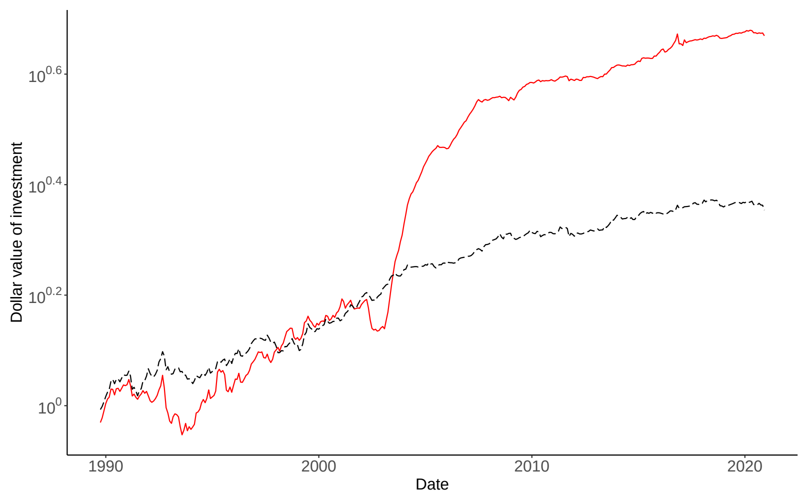
These results conclusively support our abovementioned finding in the symmetric factor portfolios.

The cumulative performance analysis provides additional evidence of the superior performance of optimised asymmetric factors. Figure F.1 presents the cumulative performance of the naive and best-performing optimised portfolio of TSMOM-3 and RSMOM, which reports the highest naive portfolio returns. The dollar investment values of the optimised portfolio exhibit comparable trends to the cumulative performance of the symmetric factors depicted in Figure 2. Upon closer examination, the optimised TSMOM-3 mirrors the performance of the naive TSMOM-3, particularly prior to the year 2000. Intriguingly, the cumulative investment trajectory of the optimised RSMOM-12 falls even below that of the naive RSMOM-12, particularly before 1995. In contrast, a dramatic rise in the performance of optimised factors is observed when considering the post-emergence of certain emerging currencies and the omission of numerous developed currencies. This reaffirms that the composition of available currencies significantly influences the optimised factor portfolio performance. Consisting with our conclusion in the main analysis, the superiority of optimised factor portfolios becomes especially apparent when the majority of the sample is composed of emerging currencies. This dynamic underscores the nuanced relationship between factor portfolio performance and the composition of currency markets.

Figure F.1: Cumulative performance of the TSMOM and RSMOM portfolios



TSMOM



RSMOM

These plots exhibit the cumulative performance of naive and the best-performing optimised TSMOM-3 and RSMOM-12 factors throughout the sample period. The dollar value of investment (y-axis) is logarithmically scaled, given the huge difference between the naive and optimised weighting schemes. Best-SR and Naive refer to the optimised factor with the highest portfolio Sharpe ratio and the original one, respectively. As the most profitable and best risk-adjusted strategies are the same for the selected asymmetric factors, we only plot the cumulative performance of the Best-SR to avoid duplicated lines.