Reverse Mortgages, Housing and Consumption: An Equilibrium Approach*

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PRELIMINARY VERSION

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Abstract

Reverse mortgages (RMs) offer eligible senior homeowners liquidity of their home equity without them moving out and repayments before loan termination. By incorporating RMs into a quantitative equilibrium life-cycle model, we assess their impacts on household decisions, the mortgage and housing market. We show that retired RM borrowers experience enhanced significant consumption smoothing. Additionally, the presence of RMs in the mortgage market enhances the perceived value of houses to households, making homeownership a more financially attractive option and stimulating demand for housing. This also leads to increased overall household welfare in our model, highlighting the positive impact of RMs.

JEL Classification: G21, E21, J14

Key Words: Reverse Mortgage, Mortgage, Housing, Retirement, Welfare

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1 Introduction

The global challenge of managing later-life financial security exists widely: The problem arises from accelerating life expectancy and socioeconomic inequality in later lives, coupled with insufficient public resources to fund social security. The long-term trend of the growing ageing population has posed a two-fold problem: On the one hand, considerable families' wealth has accumulated in the form of housing estates (or home equity), an illiquid asset, which has experienced significant price growth over the past decades (1960-2006) during the housing market boom in the US.[1] On the other hand, retirees need to make intelligent decisions on their pension plans, and their under-saving for retirement results in insufficient retirement cash flow. These factors make ordinary families "equity-rich and cash poor"(see Caplin (2002)). Reverse mortgage loans (RMs) present a potential solution for older homeowners to access their home equity and fund their retirement consumption.[2] Granular loan-level data such as CoreLogic and Black Knights, on open liens including mortgages (first liens,) and RMs (second liens), show that RMs have become an increasingly important retirement solution, with a growing presence in the financial and housing markets. As a result, their impacts on the financial and housing markets are non-negligible, especially in the long-term. While the RMs market's rapid growth improves the financial well-being of older homeowners,[3], the complexities of RMs hinder our understanding of their interaction with the broader mortgage and housing markets, as well as their impact on inter-generational wealth transfers and overall societal welfare improvements. Thus, key questions remain unanswered: How do RMs affect the mortgage and housing market, and what are the implications for societal welfare gains?

Our paper addresses these questions with a quantitative equilibrium model featuring heterogeneous households and competitive lenders, with endogenous house prices and rates of conventional

[1] Housing estates are in the order of magnitude of 10 times liquid assets such as cash and equivalents (financial securities (equities, and bonds), according to Chen et al. (2020). Similarly, Nakajima and Telyukova (2020) states that excluding housing wealth, retirees' net worth would be 28% - 44% lower, depending on age groups. Iacoviello (2011) also estimates that about one-half of the total household net worth with the U.S. data.

[2] In the US, RMs are commonly FHA-insured Home Equity Conversion Mortgages (HECMs), available to individuals aged 62 and older, primarily used to finance retirement consumption. Home equity financing solutions encompass various instruments, including non-age-restricted home equity loans (HELs), Home Equity Lines of Credit (HELOCs), and second mortgages that utilize housing equity as collateral. Similar retirement products, albeit under different names, are offered in other developed countries and regions, such as the UK, Canada, Australia, and the EU, as well as emerging economies such as Brazil, South Africa, China, Thailand and India.

[3] See Nakajima and Telyukova (2017), Nakajima and Telyukova (2020), and Chen et al. (2020).

mortgages and RMs. Besides, we introduce cyclical interest rates and allow the economy to transit between booms, recessions, and crises. Lenders set spreads for mortgages and reverse mortgages to break even in equilibrium. In this economy, households' decisions influence the equilibrium house prices and spreads of mortgage and reverse mortgage, which feed back to household decisions. Our model considers multiple sources of household risk, including un-insurable labour income risk, health risk, longevity risk, interest rate risk, and uncertainty in house prices and the macroeconomic environment. Based on households' expectation of equilibrium house prices, mortgage and reverse mortgage rates, households make decisions on consumption, saving, homeowner-ship (purchasing v.s. renting), and (re)financing through conventional mortgages and RMs, facing property maintenance costs, ownership preferences and bequest motives.

At a high level, our general equilibrium model has three unique features. First, to the best of our knowledge, our model is one of the first few to clear the housing and the credit market simultaneously with both conventional mortgages and RMs. In comparison, Nakajima and Telyukova (2020) and Nakajima and Telyukova (2017) model home equity in retirement with RM and deterministic house prices, and Cocco and Lopes (2020) model the demand of RMs and incorporate a random walk process of house prices. Moreover, most papers on RMs focus on the demand side of the equation and analyze the reasons for the low demand despite obvious benefits for eligible senior households.[4] These papers follow a partial equilibrium life-cycle model, which focuses on the household balance sheet, without consideration of the interactions with the financial market (mortgage market) and the real economy (housing market).[5]

Second, we allow households to load RMs after retirement to repay conventional mortgages subject to specific loan-to-value ratio (LTV) constraints, which is also feasible in the real market. This feature depicts the possibility of an alternative 'refinancing' for mortgage borrowers. Unlike refinancing conventional mortgages, RMs exempt borrowers from monthly repayments and thus substantially mitigate the liquidity issues faced by senior households. Moreover, we calibrate our model with proprietary datasets from CoreLogic and Black Knights, which offer detailed and

[4]The reasons for the low RM take-up rates include bequest motives, precautionary savings, uncertainty in longevity and medical expenses, etc.

[5]See Cocco and Lopes (2020)'s model on the impact of maintenance costs on RM demand, and Nakajima and Telyukova (2017)'s model on the impact of bequest motives and initial set-up costs on RM demand.

comprehensive information on anonymous households, ensuring granularity and completeness.[6]

The third main feature is that our calibrated model generates homeownership rate, households' net wealth, and Loan-To-Value (LTV) ratio close to the Survey of Consumer Finances (SCF). In addition, our model result matches Payment-to-Income (PTI) ratio with the empirical evidence very well, with the introduction of RM. However, Guren et al. (2021)'s model generates a PTI ratio growing exponentially during retirement, which deviates significantly from the empirical evidence. Moreover, the refinancing rate of conventional mortgages in our model remains relatively stable and moderate upon retirement. These findings differ from the study by Guren et al. (2021), which indicates a low refinancing rate during the working period followed by a sudden spike after retirement. The difference may results from the introduction of RMs into our model, such that households can use RMs as a substitution for refinancing with the additional benefit of no monthly repayments and an embedded put option of housing price; thus the refinancing rate becomes moderate after retirement.

Our main findings suggest that RMs play a significant role in effectively smoothing household consumption over the household life cycle. The consumption growth volatility of households with RMs is reduced by 17.04% (working) and 8.36% (retiring) because RMs allow senior households to extract home equity and address the reduction in income that typically occurs after retirement. In addition, RMs provide liquidation for senior households, which, in their earlier years, enables them to save slightly less or extend their mortgage borrowing period. This enhanced flexibility contributes to the smoothing of their consumption not only during retirement but also throughout their working period.

One possible limitation of our model is that our model generates a relatively high level of RM take-up rate of 10% among eligible homeowners aged 62 to 65, with a decreasing trend as they age. This take-up rate is higher compared to the rates reported in the existing literature on RMs. It is important to note that our model assumes perfect financial and housing market conditions while overlooking the initial RM set-up costs and product costs. When taking into account such costs, the take-up rate shall drop significantly and be closer to the rates in the existing literature,

[6]For example, CoreLogic and Black Knights both provide households' demographic, socioeconomic characteristics, household consumption and investment behaviours, alongside their housing condition, house prices, conventional (first lien) and reverse (second or junior lien) mortgage (if any) information.

such as Cocco and Lopes (2020) and Nakajima and Telyukova (2017). Another reason for the high take-up rate in our model is that under our equilibrium setting, households in our panel assume perfect access to housing and credit markets without friction and information asymmetry, such that they can make rational decisions on savings, consumption, mortgages, and RMs, as well as housing. This is in line with the statements that the current RM market is far from saturated; hence the take-up rate in the housing and credit market equilibrium in our model shall be higher than the rates reported in the data and other papers that specifically target the take-up rates.

To evaluate the effect of RMs on both the housing market and households' welfare, we evaluate an alternative economy without RMs in the credit market, with all other parameters held constant as in our economy with RMs. We find that given the same housing supply, house prices increase significantly (from 0.66% to 1.90%) in the economy with RMs. Meanwhile, homeowners attain substantial welfare gains measured by the equivalent consumption variation as 1.29% (age <62), 4.77% (age 62 - 75) and 7.47% (age >75). Renters, on the other hand, enjoy marginal welfare improvement. Moreover, households in the economy with RMs experience enhanced consumption smoothing. Therefore, these findings suggest that the introduction of RMs as a financial product for trading liquidations enhances the efficiency of the housing market and benefits the households.

Our paper builds on a large body of literature that studies senior households' consumption and investment behaviours and household balance sheet dynamics such as Cocco et al. (2005) and Gourinchas and Parker (2002). These papers emphasize the importance of household balance sheet dynamics through the life cycle, and the transition from the young, working and saving phase into the retirement and consumption phase. Most of these papers overlook the role of home equity and its interaction with other factors, such as the bequest motive and medical expense risk, in shaping homeownership decisions of households. In contrast, our paper takes into account households' decisions regarding homeownership and the utilization of home equity, including the consideration of reverse mortgages. Therefore, our paper provides a more comprehensive understanding of the dynamics and implications of homeownership decisions, as well as the potential benefits and challenges associated with RMs.

Further, our paper contributes to another strand of papers on household decisions on mortgage default, which emphasizes the role of house prices and home equity extraction, such as the earlier empirical papers, Campbell (1983), and later Deng et al. (2000) states borrowers do not default when home equity becomes negative immediately, but wait until the default is irreversible and their option to default is deep in the money. Chen et al. (2020) follows Campbell and Cocco (2003)'s partial equilibrium approach and integrates mortgage refinancing and home-equity-based borrowing (HELOC). During the recession, when facing negative income shock, liquidity-driven households refinance, even with higher borrowing costs particularly more pronounced. This is in contrast with traditional models that predict refinancing activities are mainly driven by lowered interest rates. Our paper contributes to this strand of literature that in our household panel, households make decisions to buy or rent houses, borrow mortgages and RMs, under standard life-cycle utility maximizing conditions, and housing and credit market equilibria.

Our paper is also related to the literature on reverse mortgages. For example, Cocco and Lopes (2020) derives the household utility from remaining in their residential home, dampened by large product costs and maintenance costs as a standard requirement of RM, leading to a low borrowing rate.[7] In addition, they find RMs with insurance against forced home sales (due to large medical expenses) induced improving demand for RMs. The estimates of the group of households who may potentially benefit from RM varies significantly due to different assumptions.[8] Nakajima and Telyukova (2017) find that senior households face complex trade-offs considering various factors such as bequest motives, the desire to *age in place* termed by Cocco and Lopes (2020) (strong to stay in current residence for retirement long-term), retirement cash shortfall, longevity and health uncertainty, as well as interest rate and inflation rate uncertainty. Nakajima and Telyukova (2017)'s key model results include ex ante and ex post welfare benefit of RM borrowers.[9] Our model results are in comparison to Nakajima and Telyukova (2017)'s on welfare

[7] Campbell and Cocco (2003) estimate only of 2-3% senior homeowners among eligible elderly homeowners borrow RM, as reported by the US Census Bureau. On the other hand, Nakajima and Telyukova (2017) estimate this ratio to be 1.9% in 2013, down from 2.1% in 2011.

[8] Using different assumptions, Rasmussen et al. (1995) uses public microdata and estimate potential uses of RMs, assuming retired households with home equity exceeding $30,000 and without mortgage loans potentially benefit from RMs, while Merrill et al. (1994) assumes potential RM borrowers with (1) home equity ranges from $100,000 - $200,000 and (2) with annual income less than $30,000, (3) with strong intention of *aging in place* (4) and owns home outright, Merrill et al. (1994) estimate of 9% of eligible homeowners could benefit from RMs, while Rasmussen et al. (1995) argue this ratio is about 80%.

[9] Their key model results include ex ante welfare benefit of RM for retirees aged 65, which is equivalent to a lump-sum transfer of $252 per retired homeowner (0.84% of median annual after-tax income); the ex post welfare gains for actual RM borrowers, equivalent to a lump-sum of $1,770 per borrower (5.1% of median annual income) Also, the lowest income group (with a take up rate of 2.2%) with low wealth, poor health use RM to support

gains such that RMs affect households in our panel with a welfare gain for RM borrowers. Finally, both Cocco and Lopes (2020) and Nakajima and Telyukova (2017)'s model generate low demand for RM that matches well with empirical data[10]. However, they assume exogenous house prices, mortgage rates, and reverse mortgage (RM) rates, while our model takes a different approach by endogenizing both the housing and credit markets. This allows us to capture the feedback effects and feedback loops that exist between these markets, leading to a more accurate assessment of the impacts and outcomes of RMs on household decisions and welfare.

Lastly, our paper is also part of the literature on the interaction between mortgage and housing market. Several papers on conventional mortgages provide an earlier general equilibrium framework with dynamic factors such as household characteristics and financial variables such as interest rate, inflation, and house price. For example, Campbell and Cocco (2015) solve for households' optimal decision on mortgage choice (FRM and ARM) and default decisions, incorporating households' labour income, house price, interest rate and inflation rate risk. Under the zero-profit lender function framework, they solves for equilibrium mortgage premia through a micro-founded model. In addition, Kung (2014) develop an equilibrium model of housing and mortgagemarkets where houseprices, mortgage interestrates, and leverage ratios are all determined endogenously. Our paper introduces another mortgage product into the market. By considering the interaction between multiple mortgage products, such as conventional mortgages and reverse mortgages, our model captures the dynamics and complexities of the mortgage market more comprehensively.

The rest of the paper is organized as follows. Section 2 highlights the importance of RM from the data. Section 3 introduces the model's setup. Section 4 shows the calibration, and Section 5 presents the quantitative analysis. Section 6 concludes.

2 Empirical Motivation

RMs have emerged as a popular financial instrument for senior households to finance their costs and consumption during retirement. Since the first RM was issued in the 1960s in US and UK,

consumption and medical expenses, although the general eligible population has a take-up rate of less than 1%

[10]Nakajima and Telyukova (2017)'s model generates low demand of RM (0.89%), which matches with empirical evidence (0.84%) from 1997-2013, and they find the key factors that dampen demand of RM are bequest motives and up-front costs, and analyzed the quantitative impacts of these factors on RM demand

these products have gained popularity in developed economies globally, including the US, UK, Australia, Canada, Hong Kong, South Africa over the past two decades due to their unique feature of allowing homeowners to extract the value of their homes without moving out. Many emerging markets, such as Brazil, Chile, South Africa, and Thailand, have also adopted RMs. In this paper, we focus on the US markets. Using proprietary dataset through Corelogic and Black knights, we have access to a complete and granular dataset on household mortgage open liens (up to four liens) information, including first lien which is the conventional mortgage, and reverse mortgage are identified as "Mortgage Subordinate Type" loans.

The advantage of our dataset is that ours covers the actual mortgage and RM borrowers' property information such as their historical and market value, housing type and condition, as well as all mortgage and reverse mortgages (initial balance, interest rate, terms, etc.) secured on the properties in the entire country during our research period, which covers almost all of the properties secured on such mortgages. Other datasets used in previous literature such as Panel Study Income Dynamics (PSID) and Health Retirement Study (HRS) are surveys only covering a subset of the population. These types of surveys use estimation and various weighting methodologies which can sometimes make the model results biased because RM is a relatively small market compared to conventional mortgages and other more developed financial products. This makes our contribution distinct and essential.

As depicted in Figure 1, calculated using the number of RMs in any given year throughout the research period, divdided by eligible senior households in the US (statistics estimated from Census), we first notice that the RM take-up rate in the US has increased almost ten-fold, from approximately 0.3% in 1997 to 2.8% in 2021. Over the course of its development, RM has experienced several major milestones. The first FHA-insured HECM was issued in 1989, followed by the first major news of HECM become permanent product by Housing and Urban Department (HUD) Appropriations Act in 1994. During the 2000s' housing market boom, demand for RM surged such that homeowners take advantage of the insurance of housing value embedded in the RM contract. At the same time, more lenders join the RM game as HUD increase origination fees and engage American Association of Retired Persons (AARP) to improve counselling services to boost the RM market. More rules were relaxed such as refinancing existing HECM became possible. RMs became ever more important for the elderly to boost their retirement incomes, and proprietary RMs specifically designed for those who are not eligible for HECM schemes emerged in the market.

A second observation on Figure 1 is the obvious drop (from 1.4% to 1%) from 2008 to 2011. Since the GFC in 2008, some lenders left the market, and proprietary RM disappeared for a while. This results in RM plan issuance in the US reduced following the GFC (see Cocco and Lopes (2020)). There is a similar effect in the UK market, which is evidenced in Figure 2 panel (a) from 2007 - 2011, during which the market contract by almost 2/3. This is due to the correction of the US and international housing market following the previous artificial housing boom from 1990s to 2006. RMs were hit harder because the lending criteria is mainly based on LTV which heavily relied on housing market performance. The US RM market gradually recovered in 2011 and has increased significantly over the past decade. Moreover,the newly approved legislation in 2015 of RM being used as down payment for a property of the eligible senior homeowners' offspring gives another boost of this market.

A third observation on Figure 1 is the un-interrupted growth in the US RM market following the Coronavirus crisis (COVID-19). This is in contrast to the market contraction following GFC. The UK RM market initially suffered a minor contraction (2019-2020) but quickly recovered from 2020 onwards. These can be seen in the panels of Figure 2 panel (a). Post-COVID-19 crisis is a regime of the high interest rate as the central banks increase base rate multiple times in short period of time, in order to cool off the inflation build up due to the pro-longed extra-low interest rate regime following the GFC. The empirical experience of international RM market growth following the COVID-19 crisis is in contraction to some prediction that usually hightened interest rates cool down the housing market and mortgage market, due to higher (re)financing costs. However, the surge of RM in the post-COVID-19 crisis regime supports the conclusion by Hurst and Stafford (2004) such that enabling households to converting home equity into liquid assets re-adjusts households' decision to re-finance, even at the costs of higher borrowing costs. This is one of the reasons that RM becomes increasingly important in household finance and credit literature such that RMs provide a direct way to borrow against home value, as an alternative to mortgage refinancing. For example, Chen et al. (2020) overshoot the average refinancing rate (11%

by the model versus 7% in the empirical data) and the size of cash-outs conditional on refinancing (by nearly a factor of three). Such overestimation can be explained by the existence of RMs in the credit market which boost the household's refinancing activities. Another direct factor that boost UK RM market surge in the post-COVID-19 regime is the combination of negative labour income shock, inflated living costs, and energy crisis in Europe exacerbated by the war between Russia and Ukraine.

In summary, the two crises periods (GFC in 2008 and COVID-19) have caused opposite RM market movements due to varying dominant factors of central bank base rates, housing market performances and labour income instability. These dynamics on the household balance sheet and decision choices make the integration of consideration of LTV and LTI constraints and economic states essential and highlight our contribution to this end.

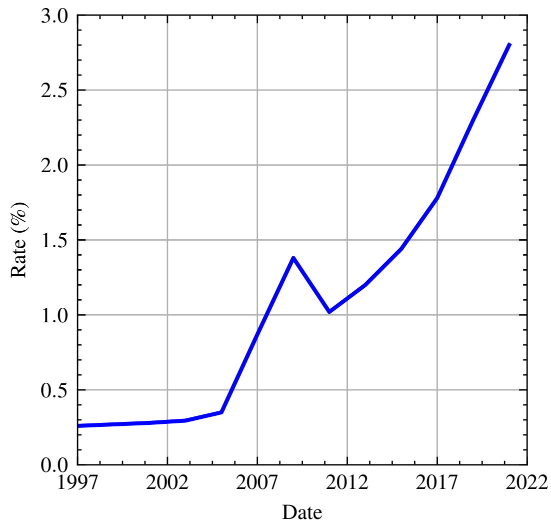

Figure 1: Take-up Rate of RM in the US

At the same time, the recent boom (in the past decade) in the international RM markets is captured in Figure 2 (all panels). The consistent growth pattern has already been observed in several developed countries such as the UK, Australia and Singapore (See details in Appendix), with similar socio-economic, demographic shifting trend, pension system, as well as financial market advancement and consumer financial literacy level. As a high-level snapshot, Figure 2 panel (a) and (c) shows the number of new RM plans (including all types such as lump sums, draw downs, withdrawals and home reversion plans) in the UK and Hong Kong, respectively, during the 2005 - recent period. In comparison, panel (b) shows the active number of RM plans in any given year in Australia for the same period, allowing expired plans to drop out from the database. In addition, according to Securities et al. (2018), from 2013 to 2017, there are more than 17,000 RM plans issued by lenders, hence the market has the potential to reach up to 58,000 active plans in 2017, presented by the dash line from 2013-2017. All panels show significant growth in our studies during the past two decades. This signals strong and growing demand and supply of the RM products in these countries, although with some fluctuations (caused by crises), which shows potential for this product to be popular in other countries with growing ageing population, housing market and homeownership rate, as well as the (lack of) social security system. This motivates our interest in quantitatively assess the impacts of RM on household decisions, the housing market, and conventional mortgage rates. Moreover, while the take-up rate is small in the US, the size of the RM market that it implies is nontrivial.

Finally, Figure 3 shows the growing trends of the conventional mortgage rates, reverse mortgage rates, and the housing price index (HPI) in the US since 2005 to recently. It reveals that, as anticipated, reverse mortgage rates generally exceed conventional mortgage rates to compensate the riskiness of such instruments. Although fluctuating, there has been notable declines during the past two decades, mainly due to increase of lenders' appetite and improved regulation and consumer knowledge on the product. In some periods, reverse mortgage rates fall below conventional mortgage rates during housing and credit market frictions, e.g. the period following the GFC in 2008, during which HPI drops significantly, coupled with extra low interest rates. The inherent reason of the lowered RM rate is driven by both the low central bank base rate and lack of appetite of both consumers and lenders. RM rates recovered as the HPI gradualy picked up in 2010, and stayed above the conventional mortgage rates. Another period of RM rates sharp dropping below the conventional mortgage rates is the period immediately following the Coronavirus crisis (COVID-19) period in late 2019, but it quickly picked up and followed the pattern of ARM since then. This is a short-term negative shock on the RM rates possibly due to the unstable market activities due to COVID-19 crisis.

In recent years, there has been a remarkable increase in the HPI accompanied by significant decreases in both mortgage and reverse mortgage rates. This observation emphasizes the dynamic

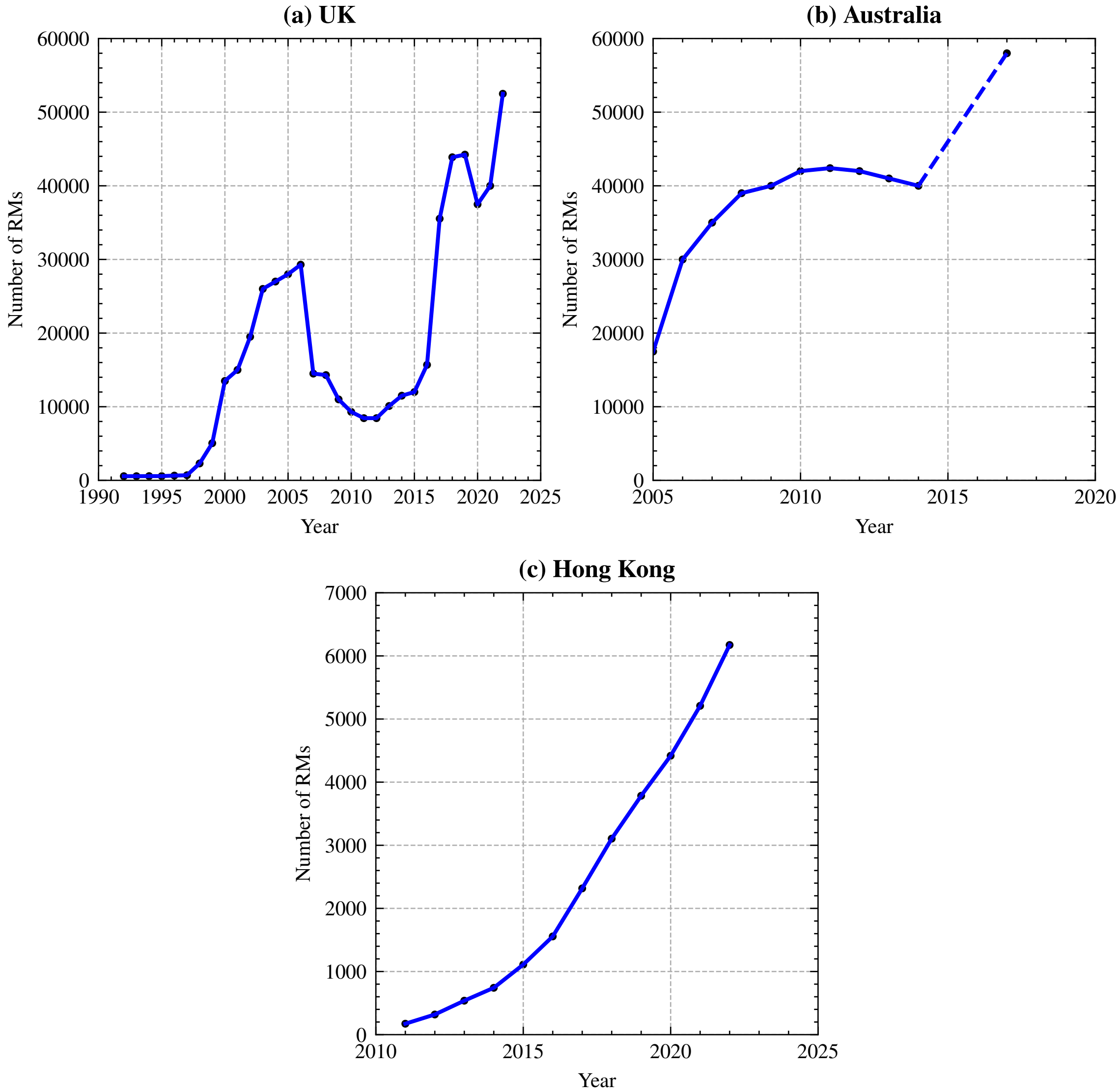

Figure 2: Number of RMs in Different Markets

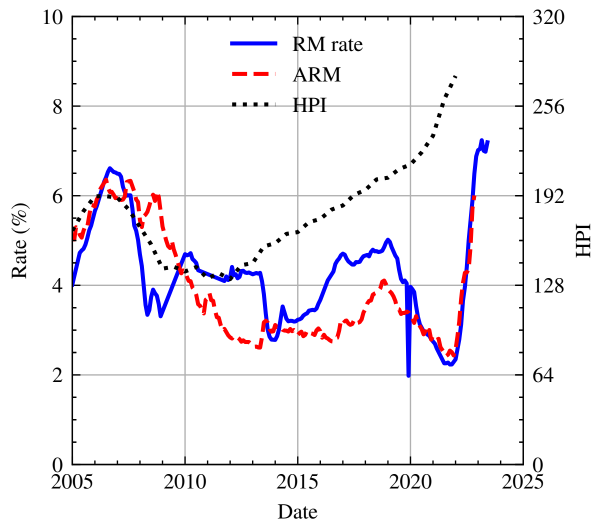

Figure 3: Mortgage Rates, Reverse Mortgage Rates, and Housing Price Index in the US

nature of these variables and calls for integrated framework when considering reverse mortgages. Such a framework should consider the interplay between the mortgage market and the housing market, recognizing their simultaneous influence on reverse mortgage dynamics.

3 Model

This section presents an annual equilibrium model considering the decisions of households and financial intermediaries. Households have a finite living span divided into two phases: the working period and the retirement period. Over the lifecycle, households earn labor income (pension in retirement) and allocate wealth to consumption, financial investment, and housing. The financial market only provides a riskless bond. In addition, households can finance for housing by loaning a mortgage in the working period and accessing home equity using a reverse mortgage with monthly payments after retiring. Financial intermediaries decide mortgage and reverse mortgage spreads, while the riskless interest rate is exogenous and depends on the economic state.

3.1 Economy

We use a discrete Markov process $\{\Theta_t\}$ to describe the macroeconomic situation including five states: (1) Crisis With Tight Credit; (2) Recession With Tight Credit; (3) Expansion with loose credit; (4) recession with tight credit; (5) Expansion with tight credit. We further assume that Θ_t determines the real interest rates (including riskless rate R_t^f, mortgage rate R_t^m and reverse mortgage rate R_t^{rm}) and real aggregate labor income $Y_t^{(agg)}$.

The house price P_t is determined by the equilibrium of the housing market each period. The equilibrium also depends on the households' distribution Π_t. In summary, we denote the aggregate state as a vector $\Sigma_t = (\Theta_t, P_t, \Omega_t)'$ and the house price function as $P(\Omega_t)$.

3.2 Households

Following the convention in life-cycle models, we assume households start working at 20 ($t_0 = 0$), retire at 65 ($K = 45$), and can live up to 100 ($T = 80$).

3.2.1 Labor Income

Before retirement, households' labor income is exposed to both aggregate shocks from the economic state and idiosyncratic shocks. Specifically, the labor income Y_{it} is determined by the following equation:

$$\ln Y_{it} = \ln Y_{it}^p + \epsilon_{it}, \quad t \leq R, \tag{1}$$

where the transient shock ϵ_{it} follows $N(0, \sigma_\epsilon^2)$ and the permanent labor income Y_{it}^p is given by:

$$\ln Y_{it}^p = \ln Y_t^{(agg)}(\theta_t) + u_{it}, \tag{2}$$

where $Y_t^{(agg)}$ is a function of macroeconomic situation θ_t and u_{it} is normally distributed as $N(0, \sigma_u^2(\theta))$.

After retirement, households still face aggregate and keep the same idiosyncratic income they had at age R, reduced by ρ log points:

$$Y_{it} = \lambda Y^{p}_{iR}, \quad t > R, \tag{3}$$

There is a progressive tax system for households given by:

$$\tau(Y_{it}) = \tau_y Y_{it}. \tag{4}$$

3.2.2 Housing and Rental Market

Households can decide the housing size H_{it} and to be a homeowner (denoted as $o = 1$) or a renter(denoted as $o = 0$), of which the cost and utility are different. Each period, homeowners must pay a maintenance cost $C_{ma}(H_{it}|P_t)$, while renters face a renting cost C_r:

$$C_{ma}(H_{it}|P_t) = c_{ma} H_{it} P_t, \tag{5}$$

$$C_r = q. \tag{6}$$

The homeowners get a utility benefit as u_o. We also assume that households may be shocked and forced to move with a probability p_m with moving cost $C_{ms}(H_{it}|P_t)$:

$$C_{ms}(H_{it}|P_t) = k_{ms} Y^{p}_{it} + c_{ms} H_{it} P_t. \tag{7}$$

In our paper, we do not differentiate the owner-occupied house price and the rental property price and only consider the equilibrium of owner-occupied housing stock.

3.2.3 Mortgage

In our modeling framework, working households have access to conventional residential mortgages when purchasing houses, and can decide the fraction of down payments constrained by Loan-to-value ratio (LTV) ϕ^m determined by the macroeconomic state Θ. Specifically, denote M_{it} to be the mortgage balance for household i at the start of period t, and for newly issued mortgages, the initial balance M_{it} satisfies:

$$M_{it} \leq \phi^m(\theta_t) P_t H_{it}. \tag{8}$$

Required mortgage payments depend on the type of mortgage. For simplicity, we only model one type of the main residential mortgage, which is the long-term fixed-rate mortgage (FRM). Households with an FRM bear the same mortgage rate determined by the aggregate shock Θ_t at origination. Here we assume that mortgage rates do not change among individuals in the same economy.

According to Guren et al. (2021), with the timing assumption that households pay their interest between periods t and $t+1$ in advance at time t, the minimum payment on a mortgage for an agent who does not move or refinance with age a_{it} is given by:

$$M_{it-1} - M_{it}(1 - R_t^m) \geq M_{it-1} \frac{R_{it}^m (1 + \left(\frac{R_{it}^m}{1 - R_{it}^m}\right)^{T - a_{it} + 1}}{\left(\frac{R_{it}^m}{1 - R_{it}^m}\right)^{T - a_{it} + 1} - 1}, \tag{9}$$

where T is the term of the mortgage.

3.2.4 Reverse Mortgage

During their whole life, homeowners can liquidate the home value for consumption by selling. In our model, we also allow the retiree to access the home value without selling it through RMs. In the real market, lenders provide several types of payments for borrowers to choose from. For simplicity, we only introduce a lump-sum RM, which is a single large payout at closing and accumulates with variable interest rates until termination. Denote the RM balance as M_{it}^{rm}, and the initial balance is constrained by a function of age a_{it} and the macroeconomic state Θ_t:

$$M_{it}^{rm} \leq \phi^{rm}(\Theta_t, a_{it}) H_{it} P_t. \tag{10}$$

The balance M_{it}^{rm} includes the initial loan fee rate (arrangement and valuation fees) c_0^{rm} and thus the household will receive $(1 - c_0^{rm}) M_{it}^{rm}$ at closing of the origination.

The reverse mortgage balance accumulates with the reverse mortgage rate R_t^{rm} and the annual loan fee rate c_a^{rm}. Households are allowed to prepay at each period and the balance after the settlement satisfies:

$$M_{it}^{rm} \leq M_{it-1}(1 + R_t^{rm} + c_a^{rm}). \tag{11}$$

The RM will terminate whenever the borrower moves or dies and will get paid at most of the home value. The potential loss is insured by the insurance company. Thus, the mortgage rate R_t^{rm} includes both RM spread and insurance premium.

3.2.5 Health Risk and Medical Cost

Older households face medical expense conditional on health states. We assume that the health state I_{it}^{health} of household i in period t is a Markov process independent of the economic state and idiosyncratic states. The health state I_{it}^{health} can be either good ($I_{it}^{health} = 0$) or bad ($I_{it}^{health} = 1$). We further assume households start to have health risk after retiring and the probability of transitioning from a good to a bad health state is 0.2 in each period. Households are assumed to be in good health prior to retirement and keep the bad health state whenever they enter it. Household i pays a medical expense $c_{med}(I_{it}^{health}, a_{it})Y_{it}$ with a rate c_{med} determined by the health state I_{it}^{health} and the age a_{it}.

3.3 Households Optimization Problem

At the start of each period t before making decisions, we can define household i's idiosyncratic state S_{it} including age a_{it}, idiosyncratic labor income Y_{it}^p, saving W_{it}, ownership o_{it}, mortgage balance M_{it}^m, mortgage accumulated rate R_{it}^m, mortgage default flag d_{it}, RM balance M_{it}^{rm}, and moving indicator m_{it} as:

$$S_{it} = (a_{it}, W_{it}, o_{it}, M_{it-1}, R_{it-1}^m, M_{it-1}^{rm}, m_{it})'. \tag{12}$$

And household i makes decisions A_{it} of saving, housing, loans' states:

$$A_{it} = (I_{it}, s_{it+1}, M^{m}_{it+1}, M^{rm}_{it+1})', \tag{13}$$

where I_{it} takes -1 if defaulting, 0 if renting, 1 if remaining the previous housing state, 2 if moving, 3 if buying a new house as a renter, 4 if refinancing and 5 if initializing reverse mortgage. Given S_{it} and A_{it}, household i faces total cost $\phi_c(S_{it}, A_{it})$ from housing, moving (shocked or voluntary moving) and financial service (refinance):

$$\begin{aligned} \phi_c(S_{it}, A_{it}) =& o_{it+1} c_{ma} H_{it} P_t + (1 - o_{it+1}) q + m_{it}(k_{ms} + c_{ms} P_t) \\ &+ (I_{it} = 4)(k_{re} + c_{re} M^{m}_{it+1}). \end{aligned} \tag{14}$$

And the consumption C_{it} determined by S_{it} and A_{it} is constrained to be positive:

$$\begin{aligned} C_{it}(S_{it}, A_{it}|\Sigma_t) =& [Y_{it} - \tau(Y_{it})] - \left[\frac{W_{it+1}}{1 + R_{it}} - W_{it}\right] - \phi_c(S_{it}, A_{it}) + (o_{it+1} - o_{it}) P_t \\ &+ (I_{it} \geq 0) \left[(1 - R^{m}_{it+1}) M_{it+1} - M_{it}\right] + \left[M^{rm}_{it+1} - (1 + R^{rm}_{t}) M^{rm}_{it}\right]. \end{aligned} \tag{15}$$

One can understand equation (15) as different channels to finance consumption, cost, and interest payments: (1) after-tax labor income; (2) saving; (3) house selling; (4) net cash flow from mortgages. The total wealth Q_{it} of household i can be computed by:

$$Q_{it}(S_{it}|\Sigma_t) = W_{it} - M^{m}_{it} + o_{it} \max(P_t - M^{rm}_{it}, 0). \tag{16}$$

The households' value function is defined as follows:

$$V(S_{it}|\Sigma_{it}) = \max_{I_{it}} \left\{ V_{I_{it}}(S_{it}|\Sigma_{it}) \right\}. \tag{17}$$

We first denote p_s is the survival probability to be alive at period $s+1$, $\{C_{it}\}$ is the consumption level, H_{it} is the housing size, Q_{it} is the amount of bequeathed wealth, β_i is the discount factor, γ_i is the coefficient of relative risk aversion, u_0 describes the utility from homeownership, ξ is the bequest motive shifter.

Households lacking current housing and devoid of credit record defaults are faced with a significant choice: either to acquire a house through a new mortgage or to continue renting. Renters with credit record defaults are precluded from pursuing homeownership. In the event that homeowners experience a moving shock, they are confronted with the decision of either defaulting on their existing mortgage and transitioning to renting or paying off the mortgage balance. By opting for the latter, homeowners gain the flexibility to freely choose between renting or obtaining financing for a new home purchase. Conversely, if homeowners do not encounter a moving shock, they must evaluate the alternatives of defaulting, refinancing, or fulfilling the minimum mortgage balance.

Every period, households that remain to be renters have the value function

$$V_0(S_{it}|\Sigma_{it}) = \max_{w_{it+1}} \Bigg\{ \frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + \alpha_{ia} H_{it} + u_o o_{it+1} \tag{18}$$

$$+ \beta_i \left[p_s(a_{it+1}) \mathbb{E}_t[V(S_{it+1}|\Sigma_{it+1})] + (1 - p_s(a_{it+1})) \psi^{bq} \frac{(Q_{it} + \xi)^{1-\gamma_i}}{1-\gamma_i} \right] \Bigg\} \tag{19}$$

$$s.t. \quad o_{it+1} = M_{it+1}^{m} = M_{it+1}^{rm} = 0, \tag{20}$$

$$d_{it+1} = 0, \tag{21}$$

$$C_{it}(S_{it}, A_{it}|\Sigma_t) > 0. \tag{22}$$

Alternatively, households that decide to purchase houses and take on the mortgage have the value function

$$V_4(S_{it}|\Sigma_{it}) = \max_{w_{it+1}} \Bigg\{ \frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + \alpha_{ia} H_{it} + u_o o_{it+1} \tag{23}$$

$$+ \beta_i \left[p_s(a_{it+1}) \mathbb{E}_t[V(S_{it+1}|\Sigma_{it+1})] + (1 - p_s(a_{it+1})) \psi^{bq} \frac{(Q_{it} + \xi)^{1-\gamma_i}}{1-\gamma_i} \right] \Bigg\} \tag{24}$$

$$s.t. \quad o_{it+1} = 1, \tag{25}$$

$$M_{it}^{m} \leq \phi^{m}(\theta_t) P_t H_{it}, \tag{26}$$

$$R_{it}^{m} = R^{m}(\theta_t), \tag{27}$$

$$M_{it}^{rm} = 0, \tag{28}$$

$$C_{it}(S_{it}, A_{it}|\Sigma_t) > 0. \tag{29}$$

Subsequent to the initial purchase, households that select to retain their homes without engaging in refinancing and without encountering any moving shocks have the value function

$$V_1(S_{it}|\Sigma_{it}) = \max_{w_{it+1}} \Bigg\{ \frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + \alpha_{ia} H_{it} + u_o o_{it+1} \tag{30}$$

$$+ \beta_i \left[p_s(a_{it+1}) \mathbb{E}_t[V(S_{it+1}|\Sigma_{it+1})] + (1 - p_s(a_{it+1})) \frac{(Q_{it} + \xi)^{1-\gamma_i}}{1-\gamma_i} \right] \Bigg\} \tag{31}$$

$$s.t. \quad o_{it+1} = o_{it}, \tag{32}$$

$$M_{it-1}^m - M_{it}^m (1 - R_t^m) \geq M_{it-1}^m \frac{R_{it}^m (1 + \left(\frac{R_{it}^m}{1 - R_{it}^m} \right)^{T - a_{it} + 1}}{\left(\frac{R_{it}^m}{1 - R_{it}^m} \right)^{T - a_{it} + 1} - 1}, \tag{33}$$

$$M_{it}^{rm} \leq M_{it-1}^{rm} (1 + R_t^{rm}), \tag{34}$$

$$d_{it+1} = 0, \tag{35}$$

$$C_{it}(S_{it}, A_{it}|\Sigma_t) > 0. \tag{36}$$

Households that experience default may lose their homes, but their savings remain intact. These households, when faced with default, make decisions regarding consumption and savings while considering their value function

$$V_{-1}(S_{it}|\Sigma_{it}) = \max_{w_{it+1}} \Bigg\{ \frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + \alpha_{ia} H_{it} + u_o o_{it+1} - u_d \tag{37}$$

$$+ \beta_i \left[p_s(a_{it+1}) \mathbb{E}_t[V(S_{it+1}|\Sigma_{it+1})] + (1 - p_s(a_{it+1})) \psi^{bq} \frac{(Q_{it} + \xi)^{1-\gamma_i}}{1-\gamma_i} \right] \Bigg\} \tag{38}$$

$$s.t. \quad o_{it+1} = M_{it+1}^m = M_{it+1}^{rm} = 0, \tag{39}$$

$$d_{it+1} = 1, \tag{40}$$

$$C_{it}(S_{it}, A_{it}|\Sigma_t) > 0. \tag{41}$$

Households that refinance make the same choices, but pay the fixed and variable costs of refinancing (which can be rolled into their new mortgage) and face the LTV constraint rather than the constraint. They have the value function

$$V_3(S_{it}|\Sigma_{it}) = \max_{w_{it+1}} \Bigg\{ \frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + \alpha_{ia} H_{it} + u_o o_{it+1} \tag{42}$$

$$+ \beta_i \left[p_s(a_{it+1}) \mathbb{E}_t [V(S_{it+1}|\Sigma_{it+1})] + (1 - p_s(a_{it+1})) \psi^{bq} \frac{(Q_{it} + \xi)^{1-\gamma_i}}{1-\gamma_i} \right] \Bigg\} \tag{43}$$

$$s.t. \quad o_{it+1} = 1, \tag{44}$$

$$M_{it}^m \leq \phi^m(\theta_t) P_t H_{it}, \tag{45}$$

$$R_{it}^m = R^m(\theta_t), \tag{46}$$

$$M_{it}^{rm} = 0, \tag{47}$$

$$C_{it}(S_{it}, A_{it}|\Sigma_t) > 0. \tag{48}$$

Households that take up RMs solves

$$V_5(S_{it}|\Sigma_{it}) = \max_{w_{it+1}} \Bigg\{ \frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + \alpha_{ia} H_{it} + u_o o_{it+1} \tag{49}$$

$$+ \beta_i \left[p_s(a_{it+1}) \mathbb{E}_t [V(S_{it+1}|\Sigma_{it+1})] + (1 - p_s(a_{it+1})) \psi^{bq} \frac{(Q_{it} + \xi)^{1-\gamma_i}}{1-\gamma_i} \right] \Bigg\} \tag{50}$$

$$s.t. \quad o_{it+1} = 1, \tag{51}$$

$$M_{it}^m = 0, \tag{52}$$

$$M_{it}^{rm} \leq \phi^{rm}(\theta_t, a_{it}) H_{it} P_t, \tag{53}$$

$$C_{it}(S_{it}, A_{it}|\Sigma_t) > 0. \tag{54}$$

In our model, households borrowing on a RM can flexibly decrease their loan balance at any time. However, it is worth noting that in reality, reverse mortgage lines of credit often prohibit partial repayment until the loan is fully settled. Furthermore, in reality, households have the capacity to accumulate financial assets while simultaneously borrowing against an RM, which is not accounted for in our model. Consequently, we interpret the flexible adjustment of the RM loan balance in our model as an approximation of this absent supplementary saving avenue.

Households that move choose their consumption, savings, mortgage balance if they purchased the houses, and RM balance if they took up the RMs. They have the value function

$$V_2(S_{it}|\Sigma_{it}) = \max_{w_{it+1}} \Bigg\{ \frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + \alpha_{ia} H_{it} + u_o o_{it+1} \tag{55}$$

$$+ \beta_i \left[p_s(a_{it+1}) \mathbb{E}_t [V(S_{it+1}|\Sigma_{it+1})] + (1 - p_s(a_{it+1})) \psi^{bq} \frac{(Q_{it} + \xi)^{1-\gamma_i}}{1-\gamma_i} \right] \Bigg\} \tag{56}$$

$$s.t. \quad M_{it}^m \leq \phi^m(\theta_t) P_t H_{it}, \tag{57}$$

$$M_{it}^{rm} = 0, \tag{58}$$

$$C_{it}(S_{it}, A_{it}|\Sigma_t) > 0. \tag{59}$$

3.4 Mortgage Lenders

We assume that mortgages are supplied by competitive lenders who discount payoffs using an SDF $m_{t,t+1}$, which is a function of today's aggregate state Θ_t and tomorrow's state Θ_{t+1}. We will calibrate the SDF from the data and use it as the pricing kernel in the equilibrium.

The net present value $\Pi(S_{jt}|\Sigma_t)$ of the expected payments from the mortgage originated to household j before termination is defined iteratively as:

$$\Pi^m(S_{jt}|\Sigma_t) = \begin{cases} p_s(a_{jt}) \big[(I_{it} = 1) \left(M_{t-1} - M_t(1 - R_t^m) + \mathbb{E}_t \left[m_{t,t+1} \Pi^m(S_{jt+1}|\Sigma_{t+1}) \right] \right) & \\ +(I_{it} = 0) \lambda P_t (I_{it} = 2, 3) M_{t-1} \big] + (1 - p_s(a_{jt})) \min(M_{t-1}, w_{jt} + P_t), & a_{jt} < T_m \\ p_s(T_m) \left((I_{it} = 0) \lambda P_t + (I_{it} \neq 0) M_{t-1} \right) + (1 - p_s(T_m)) \min(M_{t-1}, w_{jt} + P_t), & a_{jt} = T_m \end{cases} \tag{60}$$

$$\Pi^{rm}(S_{jt}|\Sigma_t) = p_s(a_{jt}) \mathbb{E}_t \left[m_{t,t+1} \Pi^{rm}(S_{jt+1}|\Sigma_{t+1}) \right] + (1 - p_s(a_{jt})) \min(M_{t-1}^{rm}, P_t) \tag{61}$$

where λ is the foreclosure sale recovery rate.

3.5 Equilibrium

A competitive equilibrium for this economy consists of a law of motion for the aggregate state Σ_t, a house price function $P(\Sigma_t)$, mortgage rates $R^m(\Theta_t)$. $R^{rm}(\Theta_t)$, and an optimal decision rule A_{it} for households, which satisfies:

- Given the fixed supply of homes, the housing market clears:

$$\int o_{it} d\Omega_t = \int o_{it+1} d\Omega_{t+1}. \tag{62}$$

- The mortgages' lenders reach (exogenous) returns under the mortgages rates:

$$\mathbb{E}_{\Theta_t=\Theta_i}\left[\mathbb{E}_{\Omega_t^{orig}(S_{jt})}\left[m_{t,t+1}\Pi^x(S_{jt+1}|\Sigma_{t+1}) - (1 - R^x(\Theta_i))M_{jt}^a\right]\right] = 0, x \in \{m, rm\}, \forall\Theta_i, \tag{63}$$

where Ω_t^{orig} is the distribution of newly originated mortgages at time t.

3.6 Solution Method

Solving the equilibrium requires households to forecast the law of motion for Σ_t, which, however, includes an infinite-dimensional object Ω_t. To address the problem, we follow the implementation of the Krusell and Smith (1998) algorithm in Kaplan, Mitman and Viotante(2019). We focus directly on the law of motion for home prices and assume that households use a simple AR(1) forecast rule that depends on the economic states:

$$\ln P_{t+1} = f_{(\Theta_t,\Theta_{t+1})}(\ln P_t). \tag{64}$$

We parameterize f as a linear spline:

$$f_{(\Theta_t,\Theta_{t+1})}(x) = b^{hp} + \sum_{i=1}^{n-1} k_i^{hp}(x - x_i), \tag{65}$$

where b^{hp}, k_i^{hp} depends on (Θ_t, Θ_{t+1}).

Denote the endogenous parameters of the equilibrium as $\Xi = (b^{hp}, k_i^{hp}, R^m, R^{rm})^T$. We further solve the equilibrium numerically. First Initialize Ξ as Ξ_0, and for $n = 1, 2, \cdots$

i) Solve the household's optimization problem under Ξ_{n-1}.

ii) Simulate 100,000 households for 19,000 periods with the home price decided by (62) each period.

iii) Estimate Ξ as Ξ_n by (63) and (65) based on the simulation results.

iv) Terminate if:

$$||\Xi_n - \Xi_{n-1}||_\infty < \epsilon. \tag{66}$$

4 Calibration

We calibrate our baseline model with reverse mortgages using several data sources. To estimate the model, we adopt a two-stage strategy, each with distinct purposes. Within our models, there are different sets of variables related to macroeconomic states, human mortality and survival functions which are exogenous variables, as well as initial mortgages and reverse mortgages interest rates. There are also variables inherently estimated from our model setting such as mortgage and reverse mortgages interest rates at equilibrium states, as well as household labour incomes, and their decisions on consumption, financial and housing conditions, etc.

In the first stage estimation, we calibrate all the stand-alone parameters that can be clearly identified without explicitly referring to our model setting. For example, the mortality and survival probabilities are exogenous variables which do not depend on our model. Hence, we adopt such statistics published by the National Center for Health Statistics to parameterize the conditional survival probabilities in our transition matrix. In the second stage, we estimate the remaining parameters to align with specific moments of the empirical data, ensuring a overall goodness-of-fit between the model and empirical observations. Table 1 summarizes the calibrated parameter values.

4.1 First-stage Estimation

The first-stage calibration process aims to simulate macroeconomics environments such as peaks, troughs, recoveries and recessions for our model. In this process, we first choose aggregate and idiosyncratic shocks mimic the dynamics of modern business cycles in the US. subsequently, we exogenously calibrate a set of parameters to widely accepted values found in the macroeconomic and housing literature.

Following Guren et al. (2021), we calibrate the Markov transition matrix between macroeconomic states such as crisis, recession, and expansion based on the frequency and duration of such events defined by NBER. Accordingly, crises happen every 75 years and that all other NBER recessions are regular, cyclical recessions. Moreover, the economy switches to expansion following a crisis or recession. Another assumption is that if the NBER peak of the previous expansion takes place in the first half of a given

Table 1: Model Parameters in Baseline Parameterization

The table shows parameters for the baseline calibration. Average income is normalized to one. There are five aggregate states, $\theta_t \in$ {Crisis With Tight Credit, Recession With Tight Credit, Recession With Loose Credit, Expansion With Tight Credit, Expansion With Loose Credit}, but we assume that income and monetary policy are the same in a recession with loose or tight credit and in an expansion with loose or tight credit. The tuples of interest rates reflect the interest rate in a crisis, recession, and expansion, respectively.

Parameters	**Description**	**Value**
T	Years in life	80
R	Retirement age	62
ρ	Log income decline in retirement	0.35
τ_0	Constant in tax function	0.8
τ_1	Curvature tax function	0.18
γ	CRRA	3.0
ξ	Bequest motive shifter	0.5
ψ^{bq}	Bequest motive multiplier	250
u_o	Utility from homeownership	10
β	Discount factor	0.98
Υ	Foreclosure sale recovery	0.654
ϕ^m(Loose)	Max LTV, loose credit	0.95
ϕ^m(tight)	Max LTV, tight credit	0.80
ϕ^{rm}	Max LTV, reverse mortgage	0.50
c_{ms}	Variable moving cost rate	3.0%
k_{ms}	Fixed moving cost	0.1
c_{re}	Variable refinance cost rate	1.0%
k_{re}	Fixed refinance cost	0.04
u_d	Default penalty	10
q	Rent cost	0.20
c_{ma}	Maintenance cost rate	0.025
p_{ms}(working)	Prob. of moving, working	1/9
p_{ms}(retiring)	Prob. of moving, retiring	0.02
p_{md}	Prob. of default flag removed	0.1
c_0^{rm}	Initial fee of reverse mortgage	0.04
c_a^{rm}	Annual fee of reverse mortgage	0.025
H_r	Homeownership rate	0.65
r	Short rate	$[0.26\%, 1.32\%, 3.26\%]$
$Y^{(agg)}$	Aggregate income	$[0.0976, 0.1426, 0.1776]$
c_{me} (Age $[65, 85)$)	Medical cost rate	0.15(good), 0.2(bad)
c_{me} (Age $[85, 100)$)		0.25(good), 0.35(bad)

year, that year is classified as the first year of the new recession, whereas if the peak happens during the second half of a year, the recession follows in the subsequent year. The ending date of a recession is defined as the next year after the start year of the expansion announced by the NBER because the unemployment rate is a lagging variable and does not immediately fall after NBER troughs. According to this definition, recessions took place during the periods 1991-1992, 2001-2002, 2008-2010, and 2020.

Another condition in our model needs calibration is the the labor income process, in which the replacement ratio during retirement is set to 0.68, and the deterministic component of the labor income process is set to be the same as that in Cocco et al. (2005). We use 0.1^2 for the transitory variance, which is similar to the value used in Gourinchas and Parker (2002). For permanent income shocks, we rely on the estimates in Guvenen et al. (2014) and Shen (2022), who estimate a quantitative labor income model using a large and confidential US dataset. We allow skewness to depend not only on the business cycle but also on expected growth rates. The moments of permanent income shocks can be easily calculated based on these estimates, and therefore, the parameters with respect to the mixture of normal distributions during expansions and recessions can be calibrated. We then estimate the remaining moments to match the first four moments during expansions and the first four moments during recessions. We adjust the income process to ensure that the average aggregate income equals 1, as the normalization convention. The tax system is calibrated following the method proposed in Heathcote et al. (2017).

Subsequently, we calibrate the lender's Stochastic Discount Factor (SDF) to match the interest rate and mortgage spread. Following Backus et al. (2011), we set the risk price for the crisis state to be 6.1 times that of non-crisis states. By doing so, we make sure that lenders charge a fair premium for insurance against crisis states.

$$E_t\left[m(\Theta_t, \Theta_{t+1}\right] = \frac{1}{1 + R_t^f + \kappa} \tag{67}$$

where R_t^f is the riskless rate in state Θ_t. Lenders impose a requirement that guarantees a certain return on investment for a certain payoff of one unit at date $t+1$. This return is determined by adding the cost of making mortgage loans, denoted as κ, to the risk-free rate R_t^f. In this case, we have set κ to 125 basis points (bps) to ensure that the average difference between the FRM rate and a 10-year bond is 1.65%.

To determine the risk-free rate, we calibrate it based on the historical real interest rate observed between 1985 and 2022. Specifically, during recessions, the risk-free rate is set at 2.32%, reflecting the lower economic activity and market conditions. During expansions, when the economy is performing well, the risk-free rate is set at 4.26%. In the case of a crisis, we assume a significantly lower risk-free rate of 1.26% to account for the heightened uncertainty and economic instability.

Furthermore, we calibrate all the reverse mortgage parameters and age-dependent collateral constraints based on the existing RM contracts. Table 2 shows the principal limit factor for reverse mortgages from HECM. For example, for the age group 60-65 and RM interest rate of 4.26%, homeowners can only borrow 45.4% of their home equity. Our primary source of data on mortgage performance is CoreLogic and Black Knights. The RM interest premiums are endogenized, depending on the aggregate states and conventional mortgage rates. Unlike conventional mortgages, the amounts of loans given depends on the ages of the borrowers which increase as they age. The intuition is that as the borrowers' ages increase, the anticipated interest costs decrease and the expected future house price appreciation diminishes, and hence lower overall risk for the lenders. In general, the collateral constraint related to conventional mortgages becomes more stringent as the borrowers age. In contrast, RMs provide a more relaxed collateral constraint that loosens with age, resulting in a more favorable borrowing environment at all stages of life.

Table 2: Principal Limit Factor for Reverse Mortgages

	Age						
RM rate	60-65	65-70	70-75	75-80	80-85	85-90	90-95
4.26%	0.454	0.474	0.507	0.533	0.572	0.624	0.681
4.32%	0.454	0.474	0.507	0.533	0.572	0.624	0.681
5.26%	0.396	0.417	0.452	0.479	0.522	0.58	0.644

4.2 Second-stage Estimation

In the second-stage estimation, we estimate the rest of the parameters related to households' consumption, financial and housing decision with the method of simulated moments (MSM). In particular, we find the set of parameters yielding the simulated life-cycle decision profiles that match the profiles from the empirical data best. The mechanics of our MSM approach are fairly standard. We compute life-cycle histories for a large number of artificial households. Each of these households is endowed with a specific value for the state vector, and a series of idiosyncratic shocks are assigned to them in a manner consistent with the stochastic processes explicated in Section 3.

To facilitate the computational process, we discretize the state space and employ value function iteration to numerically solve the model. This iterative procedure enables us to derive a set of decision rules that govern the choices made by the households. By combining these decision rules with the simulated endowments and shocks, we are able to simulate various aspects of household behavior, including wealth accumulation, labor income, housing decisions, refinancing, and the decision to take up RMs. Subsequently, we compute age profiles based on the artificial household histories, employing the same methodology used in the analysis of the actual data. We adjust until the difference between the data and simulated profiles is minimized as follows

$$(\hat{m} - m)' W (\hat{m} - m), \tag{68}$$

where $\hat{m}$ refers to the simulated moments, m refers to the targets, and W is the inverse of the covariance matrix of the empirical moments, which is estimated by bootstrapping the true data.

The empirical evidence we are trying to match our model parameters are from the American Housing Survey (AHS) and the Survey of Consumer Finances (SCF). We assume that households start working at age 20, retire at age 65, may live up to age 100, and have CRRA type of utility. The estimated discount factor β is 0.98, which is within the accepted range of estimates in models of this kind. The coefficient of relative risk aversion is 3, which is in the middle of the spectrum in the literature.[11] We choose the bequest motive parameters to match the ratio of total net worth at age 60 to net worth at age 45 in the SCF.

The processes of household moving and refinancing are associated with both the fixed and variable costs. Specifically, we have established the fixed cost of moving to be equivalent to 10% of the average household annual income, approximately amounting to \$5,000. Additionally, proportional costs, incurred by both buyers and sellers, are set at 3% of the house value to account for expenses such as closing costs and realtor fees. Refinancing entails a fixed cost equal to 4% of the average annual income, roughly equating to \$2,000, along with a variable cost equivalent to 1% of the remaining mortgage balance. This variable cost aligns with the average refinancing costs reported by the Federal Reserve.

For renters, a monthly rent payment is set at 20% of their income to maintain a rent-to-income ratio of 20%. On the other hand, homeowners are required to contribute a maintenance cost of 2.5% of the house value annually. To ensure a realistic moving frequency, we calibrate the moving shock parameter

[11] Yogo (2016) uses Epstein-Zin preferences to study the portfolio choice in retirement with health risk and derives risk aversion coefficient to be 5. Nakajima and Telyukova (2020) only focus on retirees aged 65 and above, and calibrate the risk aversion to be 2 to fit the age profiles in the Health and Retirement Study (HRS).

in a manner that results in homeowners moving on average every nine years, as observed in the AHS. Finally, the homeownership rate is adjusted to match the long-term average homeownership rate of 65% in the United States. To be consistent with the average homeownership in the AHS, we assign a utility benefit of 10 to owning a home.

Following a foreclosure event, we assume that the bank recovers 65% of the property's market value. Additionally, we consider two scenarios: loosening credit v.s. tightening credit regimes. Under loosening credit regime, the maximum LTV at origination is set at 95%, meaning borrowers can obtain a loan up to 95% of the property's value. In contrast, under tightening credit regime, the maximum LTV is reduced to 80%, limiting borrowers to a loan amounting up to 80% of the property's value. To replicate the observed foreclosure rate of 8% in the housing stock between 2006 and 2013, we set the default penalty to 10.

5 Quantitative Analysis

In this section, we first investigate whether the model is able to quantitatively match the homeownership, wealth accumulation (decumulation) and other life-cycle patterns, and compare the model with and without RMs. Then we study the take-up rate as well as welfare.

5.1 Life-cycle Profiles

We simulate the decisions of saving, consumption, housing, and RMs of 100,000 households over the life cycle and present the average profiles in Figure 4. For comparision, we also report the average profiles in the SCF from 2007 to 2019. The model matches the life-cycle patterns in the SCF data well.

In Panel A of Figure 4, we present the homeownership rate patterns over the life cycle. The model's simulations yield an average homeownership rate of 65%, which aligns with the data. However, we observe that the model tends to underestimate the homeownership rate for very young households and overestimate it for middle-aged households.

Panels B and C of Figure 4 show the LTV and PTI ratio by age. Our model effectively captures the LTV ratios for young homeowners; however, it tends to overpredict the LTV ratios for older homeowners and increases steadily during the retirement. This discrepancy can be primarily attributed to the inclusion of RMs in our analysis. Reverse mortgages allow older homeowners to borrow against their home equity, which increases their available funds without the need to sell the property. When older homeowners opt

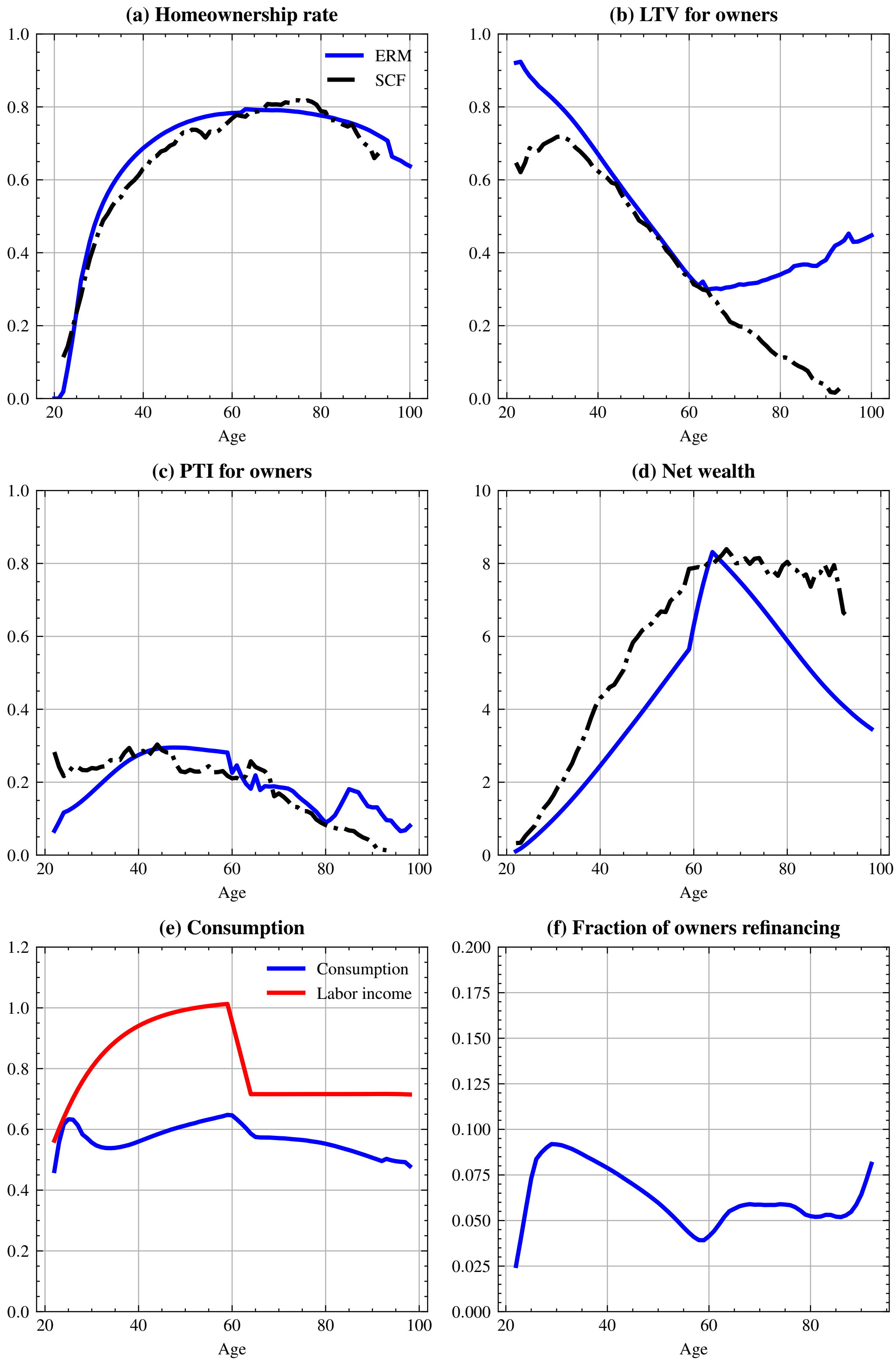

Figure 4: Life-cycle Profiles

for reverse mortgages, they receive funds based on the accumulated value of their homes. This influx of borrowed funds increases the overall value of available resources for these homeowners, resulting in a higher loan amount in the LTV ratio calculation. At the same time, since reverse mortgages do not require regular repayments like traditional mortgages, the property value remains relatively unchanged. As a result, the LTV ratio for older homeowners increases, indicating a higher proportion of borrowing relative to the value of their homes. Moreover, we observe a close alignment between the PTI ratios and the data for the elderly population, in contrast to the results presented in the study by Guren et al. (2021). This is because the fact that older homeowners have access to RMs, which serve as an additional source of funds for mortgage repayment. Consequently, their PTI ratios decrease as this supplementary income from RMs helps to alleviate the burden of mortgage repayments. This contrast in findings highlights the significance of considering the impact of reverse mortgages on the PTI ratios of older homeowners.

Figure 4, Panel D, shows the net wealth accumulation over the life cycle. We find that households are liquidity constrained during the first 15 years of their working lives. However, as time progresses, households start accumulating wealth at an accelerated pace. This accumulation of wealth serves as a crucial form of insurance, providing a cushion against adverse labor income shocks and uncertainties in the macroeconomic environment. During retirement, we observe a rapid decumulation of wealth. This decumulation reflects the utilization of accumulated assets to support living expenses and maintain the desired standard of living during the retirement phase. Overall, the dynamic patterns of wealth accumulation and decumulation throughout the life cycle emphasizes the significance of building a robust financial foundation and managing resources effectively to address income fluctuations and retirement needs.

Figure 4, Panel E, shows the profiles of consumption and income. Consumption closely tracks income over the life cycle. During the early working years, as households face liquidity constraints, their income levels are relatively lower. Consequently, their consumption levels also remain restrained. However, as households progress in their careers and accumulate wealth, their income increases, leading to a corresponding rise in consumption. As households continue to age, the impact of liquidity constraints on consumption diminishes. This is reflected in the consumption profile reaching a point where further increases cease, despite income still rising. This pattern suggests that as individuals advance in age, they become less constrained by liquidity concerns. During retirement, we observe a downward slope in the consumption path. This decline indicates a decrease in consumption as households rely on their accumulated wealth and retirement income sources to sustain their desired lifestyle. The reduced income flow during retirement necessitates a more prudent approach to consumption.

Figure 4, Panel F, shows the fraction of owners refinancing. Refinancing is relatively low across all ages. Compared with Guren et al. (2021), our model does not generate jumps at the retirement, as homeowners have additional options to smooth their consumption and manage their financial obligations. The availability of RMs serves as a supplementary avenue for homeowners to access their home equity and alleviate liquidity constraints, reducing the need for traditional refinancing. By considering the impact of reverse mortgages on homeowners' financial strategies, our model provides insights into a more nuanced understanding of refinancing behavior, highlighting the role of alternative mechanisms in managing liquidity.

5.2 Consumption Smoothing and Take-up Rate

Table 3, Panel A, reports the cross-sectional standard deviations of consumption growth for workers and retirees. We find that households utilizing reverse mortgages exhibit, on average, lower volatility in consumption growth throughout their life cycle. Interestingly, for workers, which have often been overlooked in previous literature on reverse mortgages, the availability of RMs as an additional option for accessing home equity leads to a more substantial reduction in volatility of consumption growth. Moreover, the presence of RMs serves to narrow the disparity in consumption growth volatility between working households and retirees. These findings underscore the significance of RMs in enhancing the consumption smoothing and bridging the gap between different household segments.

Table 3: Consumption Growth Volatility and Take-up Rates

Panel A: Consumption Growth Volatility

Age Groups	Households without RMs	Households with RMs	Difference
< 62	0.851	0.706	-17.04%
≥ 62	0.323	0.296	-8.36%

Panel B: Take-up Rate of RMs for Owners

Age groups	[62,65]	[66,70]	[71,75]	[76,80]	[81,85]	[86,90]	[91,95]	>95
Take-up Rate	10.71%	12.44%	10.24%	9.80%	9.64%	9.12%	8.62%	0.00%

Table 3, Panel B, shows the take-up rate of reverse mortgages across the age groups. We don't target the take-up rate of RMs by homeowners. Instead, we introduce a hypothetical assumption in our analysis, considering a scenario where no upfront costs are imposed on homeowners when they opt for RMs. Consequently, it is not surprising to observe a significantly higher take-up rate under this hypothetical condition. Consistent with previous studies such as Cocco et al. (2020) and Nakajima (2017), which have identified the high costs associated with RMs as a contributing factor to the low take-up rate. By considering the hypothetical elimination of upfront costs, our results provide further support to the notion that reducing the financial barriers linked to RMs could lead to an increased adoption rate among homeowners.

Additionally, we observe a gradual decline in the proportion of homeowners opting for RMs as they advance in age. Older homeowners are more hesitant to engage in RM agreements compared to their younger counterparts. Several factors could potentially contribute to this trend. First, homeowners approaching retirement age may have already made substantial progress in paying off their mortgage or accumulating home equity, thereby reducing the immediate need for RM products. Second, older homeowners may have concerns about the long-term financial implications and potential risks associated with RMs, leading them to opt for alternative strategies to access their home equity. The diminishing take-up rate of RMs among older homeowners highlights the significance of considering age-related factors when examining the adoption and utilization patterns of these financial instruments.

5.3 Mortgage, Reverse Mortgage, and House Prices

A significant aspect of our analysis, which distinguishes it from previous literature, lies in the equilibrium determination of conventional mortgage spreads, mortgage spreads, and housing prices. By considering the equilibrium determination of these factors, we gain a more comprehensive understanding of the interplay between mortgage rates, reverse mortgage rates, credit constraints, and housing market conditions. This approach allows us to capture the intricate relationships and feedback loops that exist within the housing and mortgage markets.

Table 4 shows the conventional mortgage rate, reverse mortgage rate and corresponding spread under different aggregate states. First, mortgage rates are influenced by the prevailing economic conditions and credit constraints. During the crisis, the mortgage rate reaches its lowest point, aligning with our expectations. This is in line with the policy objective of stimulating the economy and promoting borrowing by keeping interest rates low. In the recession phase with loose credit constraints, the mortgage rate increases to 4.28%. The more relaxed lending standards during this period lead to higher mortgage rates as lenders assume additional risk. Continuing into the expansion with loose credit constraints, the mortgage rate further increases. This increase is attributable to heightened demand for loans and increased economic activity, which exert upward pressure on interest rates. Conversely, when credit constraints tighten, the mortgage rate experiences a decline. This decline is more pronounced during the recession with tight credit constraints. Tight credit constraints signify lenders adopting a more cautious and stringent approach to loan approvals, resulting in lower mortgage rates as borrowing becomes more challenging.

Then we find that reverse mortgage rates consistently remain higher than conventional mortgage rates regardless of the aggregate states. Similar to the conventional mortgage, the reverse mortgage rate is lowest during the crisis. This aligns with the broader interest rate environment during crises, which aims to facilitate homeowners' access to their home equity at more favorable rates. Moreover, we find that the reverse mortgage rates only depend on the macroeconomic environment and are irrelevant to credit constraints. This distinction can be attributed to the unique characteristics of reverse mortgages. Unlike conventional mortgages, reverse mortgages typically do not involve stringent credit checks or income qualifications. Instead, eligibility is primarily determined based on factors such as the homeowner's age, home value, and other specific criteria related to reverse mortgages. Furthermore, reverse mortgages are structured as non-recourse loans. This means that the borrower's liability is limited to the value of the home. In situations where the loan balance exceeds the home's value upon sale, the lender absorbs the resulting loss instead of the borrower or their estate. This inherent feature of reverse mortgages further mitigates the impact of credit constraints on reverse mortgage rates. Considering these factors, it becomes evident that credit constraints play a minimal role in determining reverse mortgage rates. Instead, reverse mortgage rates primarily hinge on the prevailing macroeconomic environment and the specific attributes and regulations associated with reverse mortgage programs.

Moreover, the difference between conventional mortgage rates and reverse mortgage rates tends to be larger during periods of tight credit constraints. This is because conventional mortgages are more sensitive to credit conditions. When credit becomes scarce or restricted, lenders adjust their strategies to mitigate risk, often resulting in increased interest rates for conventional mortgages. In contrast, reverse mortgage rates are not directly influenced by credit constraints. As a result, the gap between conventional mortgage rates and reverse mortgage rates widens.

In addition to examining the dynamics of reverse mortgage rates, conventional mortgage rates, and interest rates, it is crucial to explore the implications of reverse mortgages on housing prices, as it sheds light on the broader impact of reverse mortgage programs on the overall housing market. We find that the inclusion of RMs in the mortgage market contributes to an overalll increase in housing prices across

Table 4: Mortgage and Reverse Mortgage Rates

	Mortgage		Reverse mortgage	
	Rate	Spread	Rate	Spread
Crisis	3.18%	1.92%	4.26%	3.00%
Recession and loose	4.28%	1.96%	4.32%	2.00%
Expansion and loose	5.12%	0.86%	5.26%	1.00%
Recession and tight	4.04%	1.72%	4.32%	2.00%
Expansion and tight	4.59%	0.33%	5.26%	1.00%

all aggregate states, which enhances the competitiveness of the housing market. This inclusion of RMs in the mortgage market has both direct and indirect effects on housing prices.

One direct effect is that reverse mortgages can provide homeowners with additional financial resources, allowing them to invest in home improvements or other expenditures that can increase the value of their property. This increased investment in housing puts upward pressure on housing prices.

Indirectly, RMs influence the supply and demand dynamics of the housing market. By enabling older homeowners to access their home equity without selling their homes, RMs reduce the supply of *available* housing for sale. With a limited supply of housing but consistent or increasing demand, the scarcity of available properties drives up prices.

Additionally, the presence of RMs in the mortgage market can influence the overall dynamics of housing transactions. As more homeowners opt for reverse mortgages, it can lead to a decrease in traditional home sales, limiting the turnover of properties in the market. Reduced turnover can contribute to a slower rate of housing entering the market, further exacerbating the scarcity of available homes and potentially pushing prices higher.

However, it is worth noting that there is notable heterogeneity among the aggregate states. Specifically, when credit conditions are tight, the housing price experiences a relatively smaller increase compared to other states. This aligns with the expectation that when credit conditions are tight, potential homebuyers may face greater difficulties in obtaining mortgage financing. This can lead to a decrease in overall demand for homes, including those financed through reverse mortgages. With lower demand, the upward pressure on housing prices is alleviated, resulting in a smaller increase in prices.

Meanwhile, lenders tend to tighten their lending standards and criteria during tight credit conditions. This means that borrowers, including those seeking reverse mortgages, may find it more challenging to qualify for loans. The reduced availability of credit can limit the number of potential buyers utilizing reverse mortgages, thereby mitigating the impact on house prices.

Table 5: House Prices

Econ state	Ignore RM	Recognize RM	Change	No. of periods
Crisis	3.70	3.77	1.90%	60
Recession and loose	4.71	4.80	1.91%	203
Expansion and loose	4.54	4.57	0.66%	1015
Recession and tight	4.43	4.44	0.25%	113
Expansion and tight	4.25	4.27	0.48%	608

5.4 Welfare Gains

When examining the presence of reverse mortgages, the assessment of welfare gain becomes important for comprehending the potential benefits and implications of these financial instruments. Table 6 reports the welfare gains associated with the availability of reverse mortgages. In our analysis here, we refer to the economy without reverse mortgages as Economy 0 and the economy with reverse mortgages as Economy 1. To quantify the household welfare gains, we calculate the equivalent consumption variation. This represents the amount by which households would need to increase their consumption in Economy 0 to achieve the same level of satisfaction as in Economy 1. Specifically, we denote the value functions under Economy 0 and Economy 1 as V^0 and V^1, respectively. The equivalent consumption variation $\Delta\left(S_{it}|\Sigma_t\right)$ is defined implicitly as

$$\begin{aligned} V^1\left(S_{it}|\Sigma_t\right) =& E_t\left\{\sum_{n=t}^{t+T-a_{it}} \beta^{n-t} \prod_{m=t+1}^{n} p_m(a_{im}) \left[\begin{array}{c} \frac{\left(C_n^0(1+\Delta(S_n|\Sigma_n))\right)^{1-\gamma_i}}{1-\gamma_i} + \alpha H_n^0 \\ + \left(I_{in}^0 > 0\right) u_o - \left(I_{in}^0 = -1\right) u_d \end{array}\right]\right. \\ &\left. + \beta^{n-t}\left(1 - \prod_{m=t+1}^{n} p_m(a_{im})\right) \psi \frac{\left(Q_{in}^0 + \xi\right)^{1-\gamma_i}}{1-\gamma_i}\right\} \\ =& V^0\left(S_{it}|\Sigma_t\right) + \left[\left(1 + \Delta\left(S_n|\Sigma_n\right)\right)^{1-\gamma_i} - 1\right] E_t\left\{\sum_{n=t}^{t+T-a_{it}} \beta^{n-t} \prod_{m=t+1}^{n} p_m(a_{im}) \left[\frac{\left(C_n^0\right)^{1-\gamma_i}}{1-\gamma_i}\right]\right\}, \end{aligned} \tag{69}$$

where C_n^0 and I_{in}^0 are the corresponding policies, and Q_{in}^0 is the total wealth under Economy 0. Consequently,

$$\Delta\left(S_{it}|\Sigma_t\right) = \left(1 + \frac{V^1\left(S_{it}|\Sigma_t\right) - V^0\left(S_{it}|\Sigma_t\right)}{E_t\left\{\sum_{n=t}^{t+T-a_{it}} \beta^{n-t} \prod_{m=t+1}^{n} p_m(a_{im}) \left[\frac{(C_n^0)^{1-\gamma_i}}{1-\gamma_i}\right]\right\}}\right)^{\frac{1}{1-\sigma}} - 1 \tag{70}$$

We report the welfare gains for four groups of households: (1) Young, working homeowners, (2) young, working renters, (3) Retired homeowners, (4) Retired renters. We find that the retired homeowners experience significant welfare gains. As homeowners age, welfare gains increase. On the other side, we also observe marginal improvement for the renters. While the rising housing prices may create additional challenges for renters, especially young households who often aspire to become homeowners, an economy equipped with reverse mortgages can provide some advantages for renters, albeit modestly. This positive spillover effect on the rental market might come from the fact that renters still have the potential to become homeowners in the future and utilize RMs as an additional option for accessing home equity. Understanding the potential benefits of RMs allows renters who have the intention to become homeowners to incorporate this option into their long-term financial planning and take advantage of the financial stability that RMs offer.

We can obtain additional insights into the differences between the two economies from Table 6. Here we show the cross-sectional standard deviations of consumption growth for workers and retirees under two economies. In the economy where RMs are accessible, homeowners, regardless of whether they choose to take up RMs or not, experience lower volatility in consumption growth compared to the economy where RMs are not available. This suggests that the presence of RMs has a stabilizing effect on consumption patterns for homeowners. On the other hand, it is indeed not surprising to find that the consumption smoothing effect is subtle for renters. The decrease in consumption volatility for both working and retired renters, amounting to 0.004 and 0.003 respectively, suggests a relatively modest impact. While renters may not have direct access to the consumption-smoothing benefits offered by reverse mortgages, they might become homeowners later. This contributes to a small but noticeable impact on consumption smoothing. Overall, an economy with reverse mortgages offers potential benefits and implications for various segments of the population, including homeowners and renters.

Table 6: Comparison between Two Economies

Panel A: Welfare Gains						
	Homeowners			Renters		
Age groups	<62	[62,75]	>75	<62	[62,75]	>75
Welfare Gains	1.29%	4.77%	7.47%	0.18%	0.31%	0.76%

Panel B: Consumption Growth Volatility						
	Homeowners			Renters		
Age groups	Ignore RM	Recognize RM	Change	Ignore RM	Recognize RM	Change
< 62	0.741	0.695	-6.21%	0.914	0.910	-0.44%
≥ 62	0.295	0.286	-3.05%	0.387	0.384	-0.78%

6 Conclusion

In this paper, we analyze the reverse mortgage using a quantitative model where households make decisions of saving, consumption, homeownership and reverse mortgages over the life cycle. By doing so, our model incorporates the mortgage market and housing market, enabling a comprehensive understanding of how the presence of RM influences both sectors.

Our analysis reveals that older households who opt for RMs experience enhanced consumption smoothing compared to those who do not participate in RM programs. Additionally, our model demonstrates that the inclusion of RMs has a significant impact on housing prices, particularly during the expansion with loose credit constraints. As a result, the housing market becomes more competitive, making homeownership more appealing. While these dynamics pose challenges for young renters, as they contend with rising house prices, it is important to consider their potential for future homeownership. In an economy with reverse mortgages, young renters have the opportunity to aspire to become homeowners and utilize RMs as an additional avenue for accessing home equity. Thus, even though they face some hurdles, young renters still enjoy marginal benefits in an economy that incorporates reverse mortgages. However, as expected, older homeowners experience very notable welfare gains. Overall, an economy that includes RMs exhibits superior total welfare benefits.

From a policy perspective, policymakers play a crucial role in promoting the understanding of RMs among households and minimizing the barriers that impede access to them, all while maintaining an effective risk control framework. Encouraging older households to consider RMs carries multiple advantages, benefiting not only the households themselves but also the broader society. Additionally, such encouragement helps alleviate the strain on public pension schemes, reducing the related pressures and challenges.

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Appendix: Reverse Mortgage Markets in Global Context

Compared to the case of US and other developed countries of similar socio-economic, demographic conditions and financial market development, **UK** has the most sophisticated and penetrated RM market in the world, defined by the product ranges and optional features such as repayment and withdrawal, number of lenders and consumers, as well as consumer awareness and product reputation and regulation. Since its inception in 1991, the RM market in the UK shows very healthy and significant growth trend, with several turning points. For example, the first downturn of the UK RM market is caused by the Great Financial Crisis (GFC) rooted by the subprime mortgage crisis in late 2007 in the US, causing a global housing market slump. The RM market gradually recovered since 2008 low point, and has remained healthy growth, with a threefold size in 2016, due to major new product introduction. Another major turning point for the UK RM market is the Coronavirus (COVID-19) crisis, which induced the prevailing extra-low interest rate regime during and after the crisis, boosting the housing market, causing a temporary contraction of the new lump sum plans in 2019, and growth of further advance contracts. Lump sum plans pick up the growth trend in 2020 during the living-cost crisis and stays as stable since then.

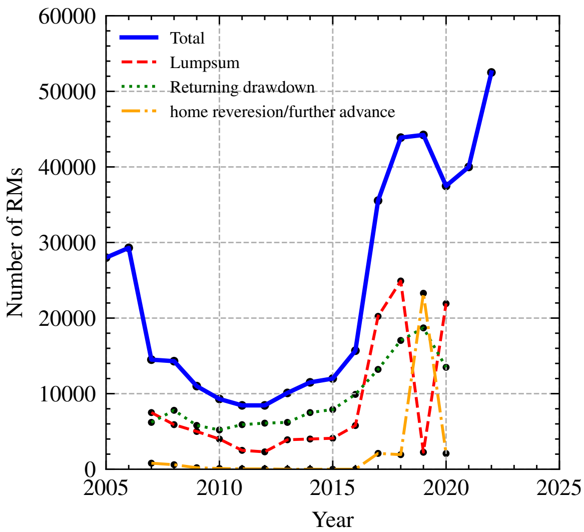

Figure 5: Number of RMs of Different Types in UK Market

Our estimates using the Equity Release Council (ERC) market report and Census data reveals that around 12% of eligible senior households have taken up RM in the UK, which is a relatively high and healthy number for this market and shows strong appetite by consumers and providers, thus a huge potential for this product to succeed in other countries in similar economic and social conditions.[12]

The case of **Hong Kong** is described in the Hong Kong Monetary Authority market reports. Although still small, the RM market has gained significant interest and growth since its inception in 2011. There are multiple factors contributing to this slow but upward growth trend: Firstly, the lack of social security (public pension plans) in Hong Kong forces senior households to consider alternative methods to increase retirement income. Secondly, the housing market boom in Hong Kong makes housing asset a significant part of family's wealth. However, the homeownership rate in Hong Kong is low compared to other developed countries such as US and UK, due to the un-affordability of housing for local citizens, stated by housing price to income ratio of 23.3.[13]

Australian RM regulatory body SEQUAL uses a different estimation methods from the US and UK. They calculate the RM participation rate using the population of suitable senior individuals instead of number of eligible households. This makes the estimated RM take-up rate lower and not comparable to those reported by the US and UK regulators or market participants. MortgageBusiness (2021) estimate about half of collective wealth in Australia is tied up in housing, and it is skewed towards the senior population, that is, around AUD$1 trillion. Out of this housing wealth, AUD$300 billion is being accessed by RM, which amounts to only about 1%-1.5% in 2021, although the market has grown by more than 5 times since its inception. Product ranges, consumer knowledge and brand reputation are not as strong as in the UK and US. The challenges of Australian RM market development is due to major lenders' unwillingness to offer such products, and hence only proprietary, niche lenders are willing to take the risk and offer such products. On the consumer side, consumer awareness, financial literacy and product education are not through.

[12]The recent UK RM take-up rate is estimated by the cumulative number of RM plans issued since its inception (1991-2022), e.g. according to ERC report, there are 592,000 RM plans issued during this period, divided by the number of eligible senior households who own their main house outright. To make this ratio comparable to the case of US and other countries, we use the number of households headed by someone 65+ by report Later Life in UK (2019), 6,500,000, with the homeowner ratios of 78%, and the fraction of those still paying mortgages 6%, this makes the RM eligible households to be 4,765,800. Hence the RM participation ratio is estimated to be 12% in 2022. However, if instead the denominator is the population of 65+, the number becomes significantly large, estimated to be 15,500,000 (p4 ERC report 2022), then the RM take-up rate becomes 3.8%.

[13]According to Secretariat (2021), in 2019, home ownership ratio in Hong Kong fell to the lowest point in the past two decades, as 49.8%, following a nearly four-fold growth in flat prices during that period. Although homeownership rate modestly grows to 51.2% during 2020, it is still far below the prevailing 60% level in developed countries. Ernst and Young (2022) states that the median house price to median income ratio of Hong Kong in a 2021 survey is 23.3, see (Cox (2023)), which compares house to income ratios over 92 major metropolitan areas. Hong Kong was the highest for consecutive 12 years.