The Price of the Voluntary Disclosure

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Abstract

This study explores the complex interplay between firms' disclosure strategies and investors(VC)' comprehension within a contract theory framework. It sheds light on the strategic execution of an optimal direct disclosure contract, underlining its consistent implementability regardless of the informativeness of the disclosure. Furthermore, the inclusion of an additional transfer, contingent on the investor's anticipated value to the firm, can sustain an optimal contract even with partial or independent disclosure. The article makes a significant academic contribution by weaving investor comprehension into disclosure strategy discourse and applying the contract theory framework in a novel context.

JEL: D82, G11, G20, M40

Keywords: Information asymmetry; Voluntary disclosure; Direct mechanism; Optimal implementation

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1 Introduction

Voluntary disclosure of R&D information serves as a double-edged sword for firms, enhancing the capacity to secure essential investments for its sustained growth, while exposing the firm to competitors, potentially undermining its competitive advantage by revealing innovative technologies (Darrough and Stoughton (1990), Bhattacharya and Ritter (1983) and Mohammadi and Khashabi (2021)). The transition in the United States patent system from the "first to invent" system under the American Inventors Protection Act (AIPA) to the "first to file" system under the America Invents Act (AIA) in 2012 intensified concerns, particularly for startup technology firms, regarding the preservation of their competitive advantage. Historically, startups benefited from the flexibility afforded by the "first-to-invent" paradigm, allowing them to disclose innovations without immediate patent apprehensions. However, the shift to the "first-to-file" framework mandates a meticulous strategic realignment, a challenging task for financially constrained startup companies.

In response to this dilemma, this paper suggests a strategic solution through the theoretical development of an optimal and feasible disclosure contract, issued by the startup and accessible to potential investors, usually venture capitalists (VCs), upon payment of a fee. These contracts, rather than public disclosure, facilitate the direct communication of sensitive intellectual property information to specific potential investors, thereby efficiently mitigating information asymmetry between the firm and the investor without the inherent risk of information leakage.

From the VCs' perspective, assessing the value of acquiring the disclosure contract is crucial. VCs' holding extreme pessimistic or optimistic views about the startup's future performance may view obtaining additional information through disclosure contract as redundant expenditure, as positive or negative new information does not significantly sway their investment decisions. However, for VCs who is uncertain about their beliefs regarding the startup's future performance, this contract may be viewed as beneficial, enabling them to acquire more information before making a final decision.

The startup, who hold the superior information (Lu et al. (2023)), can formulate its disclosure strategy. They can either adopt a uniform approach, disseminating identical details to all potential investors, or choose a tailored method, customizing information for specific individuals based on their distinct needs. To precisely align the disseminated information with the needs of potential investors, the startup can solicit investors about their initial beliefs regarding the startup's potential payoff. Subsequently, the startup can design a tailored disclosure strategy for a specific investor to minimizes the disparity between her belief and the startup's own assessment.

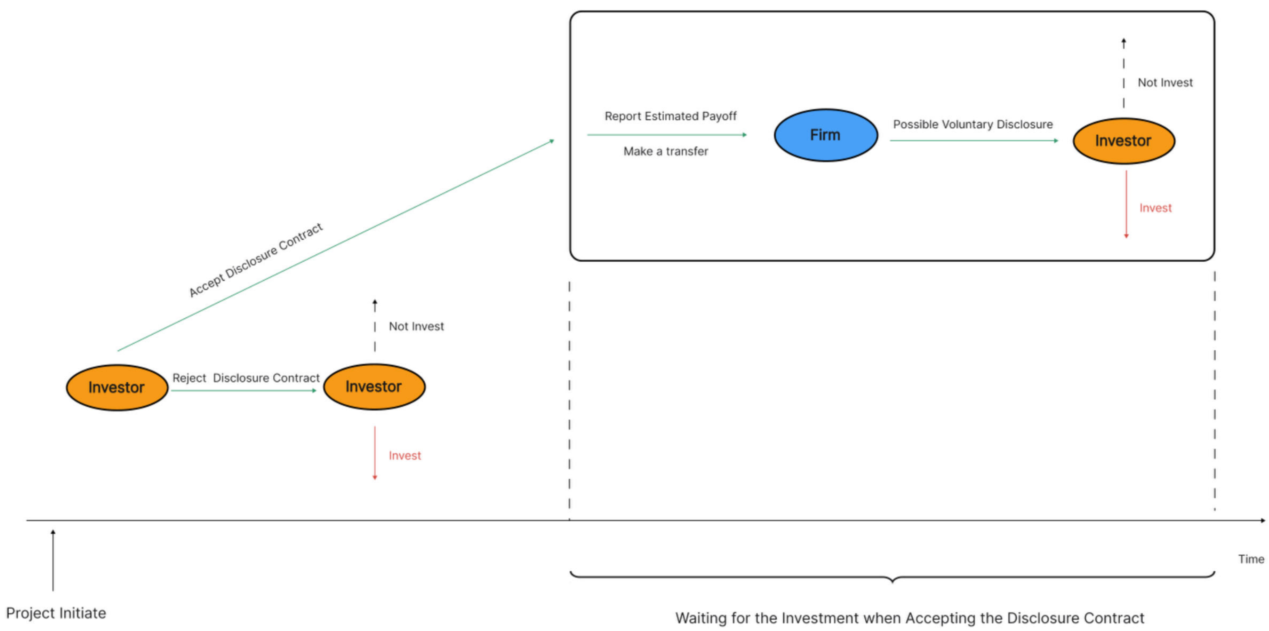

Figure 1 – Structure of a voluntary disclosure scheme

Under our framework, we examine a scenario involving two risk-neutral entities: a startup acting as the principal and a VC acting as agent. The startup oversees itself as a potential investment project characterized by an uncertain ex-post payoff. Initially, the VC forms her own projections regarding the potential payoff of the startup. Then, VC receives the disclosure contract issued by the startup. If the VC declines this contract, her investment decision relies solely on the initial belief. However, upon payment of a fee for the contract, she can communicates her belief regarding the estimated investment payoff to the firm. As a result, the startup can generate a signal reflecting the discrepancy between the VC's estimation and the startup's internal understanding of its future potential payoff. Ultimately, the VC can refines her belief in response to the additional signal received and makes her final investment decision based on the updated belief. This unique interplay between the startup's disclosure strategy and the VC's response underscores the nuanced dynamics of information asymmetry within the principal-agent framework, thereby serving as the foundation of our study. Figure 1 summarizes the overall structure of our model.

The disclosed signal is determined by the startup, which can be either perfect or imperfect. Empirical evidence from Tan and Yeo (2023) emphasizes the substantial impact of disclosure tone on investor perceptions, illuminating the intricate challenges encountered by firms. A theoretical framework established by Lichtig and Weksler (2023) investigates the effectiveness of information transmission in voluntary disclosure games, providing essential context for this intricate landscape. Thus, in this paper we define that the signal's quality is measured by how closely it reflects investors' estimated error: the greater the signal's informativeness, the better the quality. These contracts play a crucial role in adjusting investors' expected returns based on the quality of the disclosed signal, thereby aligning the two parties' interests, even in the presence of asymmetric information about the project's payoff.

Startup can tailors the disclosed signal with distinct objectives in mind. First, the aim of the startup can be persuade all potential investors. Second, startup can selectively convinces VCs who overestimate the startup's value than itself only, presenting an appealing investment prospect and potentially enhancing the chance of securing funding. Third, the startup could strategically disclose confidential information to persuade VCs who undervalue its potential, with the goal of eliciting investment interest.

Our primary finding is that by assuming the risk-neutral investors, the optimal disclosure contract offered by the startup entails a binary strategy regarding the choice of disclosure and the corresponding interpretability. The startup may discloses with full interpretability for certain types of investors, denoted by their estimated payoff, and may refrain from disclosing or disclose uninterpretable signals for other types of investor. This fresh perspective sheds light on prevalent binary disclosure practices, aligning with empirical observations in corporate settings (Allee et al. (2018), Liang (2023), and Wang (2023)). Such practices are particularly prominent in the finance and investment industries (Blankespoor et al. (2020) and Kanodia and Sapra (2016)).

Furthermore, within our model, we observe a consistent pattern where disclosure remains stable in response to accurate estimates. However, substantial errors in estimation are penalized through non-disclosure. This is consistent with Jiang and Yang (2017), which remarks that the predominant application of the binary disclosure mechanism revolves around mitigating potential losses. This distinctive characteristic of our research contributes significantly to the theoretical discourse, providing valuable insights into the complex dynamics governing how startups strategically attract investors through their disclosure strategies.

Our unique approach to model "voluntary disclosure"—where the startup may opt to disclose a signal that only the VC can interpret—is innovative and perhaps unconventional, but accurately mirrors real-world situations. Naturally, startups lack knowledge of the VC's initial value estimate or how the disclosed signal will influence the VC's investment interest, as this information is private to the VC. In this study, we aim to tackle a fundamental question: What is the startup's optimal disclosure contract if the investor's action (whether or not to invest) is contractible but the effect of the startup's disclosed signal on the investor's valuation is not? This unique setup offers a novel framework for exploring how startups can strategically utilize their informational advantage to influence investors' investment choices.

Expanding this further, our voluntary disclosure model is inspired by the prevailing understanding that startups, firms in general, assist investors by refining their ideas and recognizing their potential in strategic and investment contexts. Instead of delving into specific investor issues, firms often focus on presenting general information correctly to help investors' decision-making (e.g., what sorts of trade-offs to contemplate, common misconceptions). These serve as valuable signals to refine investors' knowledge about their investments (Bushee et al. (2011), Clor-Proell et al. (2023), Eilifsen et al. (2021), and Tan and Yeo (2023)). However, the exact impact of these signals on investors' valuation remains uncertain to the firm. In this study, we aim to explore this form of information transmission, where only investors' observable actions are contractible.

Our research align closely with existing models of voluntary disclosure, particularly in the context of the firm-investor relationship. Previous studies have observed firms issuing contingent disclosures, signals that vary based on project success or failure. These practices are explained through economic theories such as firm's risk-sharing preference, liquidity constraints, and moral hazard issues (Verrecchia (2001)). Contingent disclosure contracts are also considered optimal in cases of asymmetric information between firms and investors. For instance, Dye (1985) proposed a model where firms possess private information about their capabilities, while investors are better informed about potential of the firm's project. According to Verrecchia (2001), the firm acquires superior information regarding the project's potential after issuing a voluntary disclosure. In both models, contingent disclosure contracts arise in equilibrium.

Recent research has provided intriguing insights into voluntary disclosure practices. Bertomeu et al. (2021) show that strategic management of information production adds value in scenarios where managers can claim ignorance and withhold unfavorable news. Their model illustrates that imprecise reporting may be more advantageous than reporting bad news in a voluntary disclosure environment, and may enhance the quality of public signals in view of strategic disclosure effects. Kumar et al. (2017) find that informed managers strategically choose disclosure and share repurchases based on firm value, suggesting that full disclosure may not always occur when repurchases are used to extract information rents, and that disclosure decreases as stock trading liquidity increases. Finally, Beyer and Guttman (2012) uncover interdependencies between managers' disclosure and investment decisions, revealing unexpected scenarios such as dual non-disclosure intervals and bypassing of profitable opportunities.

In contrast to to existing models, our study delves into the interaction between startup' disclosure strategies and investors' investment decisions, emphasizing an additional signal related to investors' estimation errors about the startup. We suggest that investor types can be characterized by their estimated payoffs of startup, recognizing the presence of inherent estimation error. Additionally, we allow the startup's disclosed signal about this estimation error to vary, ranging from perfectly informative to entirely uninformative. An entirely uninformative signal is completely independent of the investor's estimated error. Moreover, we discuss the interpretability of the disclosure, considering the possibility of comprehension asymmetry where investors may not fully understand the information even when presented, thus introducing to potential disparities in understanding.

First, our work contributes to voluntary disclosure literature by investigating how startups may provide investors with more insights into their potential development without gaining superior information. As previously highlighted, our most intriguing finding is that the firm's information level becomes irrelevant when disclosing information to investors, since identical expected payoff can be realized through the corresponding optimal disclosure contract in both instances. Our model builds upon existing literature but with two key distinctions: first, our model incorporates a menu of contingent disclosures; and second, it links payoffs to observable investor actions, such as their investment decisions.

In addition, our model mirrors real-world contract features. First, extensive literature indicates that firms commonly operate within contingent-disclosure contracts, which are influenced by investors' reported valuations, such as their optimism or pessimism about the firm's future. The terms of these contracts often differ based on individual investors' expectations. Thus, our model enables optimal contract to be a menu of contingent disclosures, allowing investors to select based on their initial estimations of the startup's potential payoff before initiating communication with the firm. Second, fees tied to investors' actions are commonly seen in the real-world, such as those in financial and investment advice. For example, in initial public offerings, financial advisors receive a "success fee" upon successful listing, acting as disclosures contingent on investors' actions, since investors make ultimate investment decisions. In stock market, a financial advisor might receive less commission fees if the investors choose more affordable stock after considering the advice. Our model suggests this outcome can be the result of an optimal disclosure contract: the firm, who hold superior information, should reveal all available information, regardless of the effect on investors' estimates.

Furthermore, our model stands out due to its unique feature: the voluntary disclosure mechanism. Even though our explanation of the optimal disclosure strategy aligns with the the principal–agent framework, where the value of investors' alternative options depends on the their types. Unlike traditional frameworks, our approach allows the firms to voluntarily disclose a signal, aligning the investor's beliefs about the firm's future payoff closer to reality. The firm's disclosure decision depends on the investor's estimated payoff and can directly influences the investor's investment decision.

Last, despite extensive exploration of principal–agent models in this domain (e.g., Glover and Xue (2022), Wagenhofer (2011), and Frederickson and Waller (2005)), previous studies have not adequately addressed the voluntary disclosure context on which we focus, where the principal has more project-specific information and is able to influence the agent's decision by strategically revealing signals about the investor's estimation error. We endeavor to fill this gap in the literature. We show that firms may tailor their disclosure strategies to investors' estimates to influence their decisions, and that firms can obtain higher expected payoffs, even if the disclosed signals are imperfectly correlated with investors' estimation errors. Our research makes a significant contribution to the extant literature by elucidating the nuanced strategy of voluntary disclosures in a principal–agent model.

2 Model

2.1 The setting

Firm f and investor i are both risk-neutral. The firm proposes a project, and the investor chooses whether or not to invest in it. The cost of the investment, represented as c, belongs to a set of positive real numbers, R^+, and is common knowledge to both firm and investor. The firm's project generates an ex-post payoff P, which is not known by both agents. The investor can only make an estimate of P, denoted as p. Specifically, P equals $p+u+z$, where p is the investor's estimate of the payoff, u is the estimated error term, and z represents an exogenous shock to the payoff.[1]

The estimated payoff, p, is private information held by the investor and is not observable by the firm. As a result, the most effective form of communication that the firm can provide to the investor involves voluntarily disclosing a perfect or imperfect signal, θ, regarding the estimated error, u.[2] The firm incurs a cost, D, for this voluntary disclosure, and θ may be a function of u. However, for the investor, the firm's voluntary disclosure is not always informative owing to the varying interpretability of the signal. This means that

[1] Importantly, we do not assume that the final payoff is a linear combination of $p+z$ and u. Rather, we define u as an estimation error by the investor, such that $u = P - p - z$.

[2] The signal θ is also independent of p because p is unobservable from the firm's perspective. In the extreme case where p is observable by the firm, the signal is perfect and fully informative, implying that $\theta = u$.

the investor may not glean any additional information from the disclosure and may still behave as if no disclosure has been made. Drawing on studies by Lang and Stice-Lawrence (2015) and Acharya and Ryan (2016), which correlate disclosure quality with economic outcomes and financial stability, we assume that the interpretability of the disclosed signal is negatively correlated with the exogenous shock: the poorer the economic conditions, the more challenging for the investor to interpret the disclosed signal. A notable aspect is that if the firm has a thorough understanding of how the investor estimates the project's payoff (implying that $\theta = u$), our model reveals that the firm's optimal disclosure choice leads to a payoff that does not depend on the informativeness of θ. A similar result is obtained if the firm has no knowledge about the investor and consequently discloses purely random signals. The findings from these two extreme scenarios suggest that the firm is able to design an optimal disclosure scheme that induces the same payoff as a perfectly informative signal where $\theta = u$.

Let F_i represent the cumulative distribution function (CDF) of the investor's estimated payoff p on the unit interval, $[0, 1]$.[3] The corresponding probability density function (PDF), f_i, is log-concave and twice differentiable. Log-concavity is a standard assumption in contractual settings with incomplete information, as indicated by the relation $\frac{d^2 ln f_i(p)}{dp^2} < 0$ (see Riordan (1996), Povel and Raith (2004) and Povel and Singh (2010)). In the following analysis, we denote $\frac{k-F_i(p)}{f_i(p)}$ for $k \in [0, 1]$ as the "reservation payoff". Given that $\frac{d^2 ln f_i(p)}{dp^2} < 0$, the investor's belief generates a reservation payoff that is weakly decreasing, i.e., $\frac{d\frac{k-F_i(p)}{f_i(p)}}{dp} \leq 0$ for $k \in [0, 1]$.[4]

The investor's estimation error u follows a distribution denoted by Q_i with the domain $(-\infty, \infty)$. This assumption means that the project's ex-post payoff may be negative, corresponding with a bad real-world investment caused by the investor's misunderstanding of the

[3] In general, the domain of the investor's estimated payoff can be an arbitrarily bounded interval. However, all our results still hold after normalizing the corresponding bounded interval as the unit interval $[0, 1]$.

[4] Most commonly used probability density functions adhere to the log-concavity principle (Riordan (1996)).

firm's project. Since u is an estimation error, it is not correlated with p, and the corresponding mean based on Q_i is $E_Q[u] = 0$. Meanwhile, the exogenous shock captures the financial crisis, trade wars, COVID-19, and other events that may impact on the payoff from the firm's project but are not included in our framework. Thus, we assume that z is independent of u and p, and $E_G[z] = -\bar{z} < 0$, where the belief about the exogenous shock, G, is common knowledge to both firm and investor.[5] The negative mean reflects that exogenous shocks often tend to harm the economy.

2.2 Contract formulation

Following Dye and Hughes (2018), the firm's voluntary disclosure, represented by θ, may assist in refining investors' beliefs about the ex-post payoff, despite the presence of estimation error u. Our goal is to employ a contract framework to model the firm's optimal disclosure scheme concerning the estimation error. To ensure the effectiveness of our model, we introduce the following assumption.

Assumption 1. *(a) The firm does not have moral hazard issues, meaning it will not change its ex-ante disclosure choice. The investor's estimation error u cannot be affected by the firm after the contract has been made.*
(b) The investor's investment cost and the mean of the exogenous shock satisfy $1 - c - \bar{z} > 0$.

First, we analyze the value of the firm's voluntary disclosure from the perspective of a social planner. The premise is that reducing the noise associated with the estimation error has the most value for the investor when the estimated payoff, p, is closely aligned with the sum of the disclosure cost, c, and the average exogenous impacts, $\bar{z}$. In an ideal scenario, the social planner observes a perfect disclosure signal, $\theta = u$, but the exogenous shock, z, remains unobserved. Therefore, the planner will enforce investment in the firm's project

[5] Similar to the setting of Krähmer and Strausz (2011) and Krähmer and Strausz (2015), normalizing the mean of all kinds of error terms to a constant will not reduce the model's generality.

when $p + u \geq c + \bar{z}$. As a result, the ex-ante expected social surplus prior to realization of the estimation error, u, and the exogenous shock, z, equals:

$$\int_Z \int_{u \geq c+\bar{z}-p}^{\infty} (p + u + z - c) dQ_i(u) dG(z) = \int_{u \geq c+\bar{z}-p}^{\infty} (p + u - c) dQ_i(u) - \bar{z}, \tag{1}$$

where Z is the domain of the exogenous shock z, and the equality is due to $E_G[z] = -\bar{z}$. Without a social planner, neither the estimation error nor the exogenous shock can be observed. Hence, the investor will invest when $p \geq c + \bar{z}$ and withhold investment when $p < c + \bar{z}$. Thus, the social value of the voluntary disclosure when $p < c + \bar{z}$ is equal to:

$$DV_1 = \int_{u \geq c+\bar{z}-p}^{\infty} (p + u - c) dQ_i(u) - \bar{z} - 0, \tag{2}$$

where $-\bar{z}$ still corresponds with the total payoff when $p < c + \bar{z}$. In particular, $p < c + \bar{z}$ means that there is no voluntary disclosure, since in this case the investor does not invest owing to the negative total payoff. However, the investor does choose to invest when $p \geq c + \bar{z}$ without the disclosure, and the corresponding expected social surplus is $\int_Z \int_U (p + u + z - c) dQ_i(u) dG(z)$. Thus, when the investor invests with $p \geq c + \bar{z}$, the social value of the voluntary disclosure will become:

$$\begin{aligned} DV_2 &= \int_{u \geq c+\bar{z}-p}^{\infty} (p + u - c) dQ_i(u) - \bar{z} - \int_Z \int_U (p + u + z - c) dQ_i(u) dG_i(z) \\ &= \int_{u \geq c+\bar{z}-p}^{\infty} (p + u - c) dQ_i(u) - (p - c), \end{aligned} \tag{3}$$

where U denotes the bounded domain of the estimation error u, and the second equality is because $E_Q[u] = 0$. Therefore, the social value of the voluntary disclosure, conditional on the investor's estimated payoff p, the investment cost c, and the mean of the exogenous impact $\bar{z}$, can be written as:

$$DV(p,c,\bar{z}) = \int_{u \geq c+\bar{z}-p}^{\infty} (p+u-c)dQ_i(u) - \bar{z} - (p-c-\bar{z})I_{p-c-\bar{z}\geq 0}, \tag{4}$$

where $I_{[\cdot]}$ denotes an indicator function equal to one when the condition in the bracket is true. Consider the monotonicity of $DV(p,c,\bar{z})$ in the investor's estimated payoff p. With fixed $\bar{z}$ and c, $DV(p,c)$ strictly decreases in p when $p \geq c+\bar{z}$ and increases in p when $p < c+\bar{z}$. Intuitively, the firm has more knowledge about the project's payoff than the investor. Therefore, we assume that the social value of a voluntary disclosure is always higher for the investor than for the firm, regardless of the investor's estimated payoff. Alternatively, it would be a socially optimal choice for the planner always to elicit voluntary disclosure relating to the estimation error, leading to the following assumption:[6]

Assumption 2. *For $\forall c, \bar{z}$, the social value of the voluntary disclosure is always higher than the disclosure cost:*

$$DV(0,c,\bar{z}) > D, DV(1,c,\bar{z}) > D. \tag{5}$$

Nevertheless, the firm does not always voluntarily disclose, even though it is definitely a Pareto improvement. We discuss this result in our extension.

Next, we aim to solve the voluntary disclosure scheme that maximizes the investor's payoff using a contract theory framework. In accordance with standard settings in contract theory, there is an interim stage during which investors generate their own estimated payoff p. In the following analysis, we define this as the "disclosure contract," which is based on the disclosed signal θ rather than the unobserved estimation error u. The disclosure choice is also a term of the contract and is fully committed. The investor must provide incentives to the firm to disclose a certain signal θ. We represent this incentive scheme with a monetary transfer determined by the investor's investment choice, a very general setting that encompasses

[6]However, in the decentralized case, always disclosing may not be an optimal choice for the firm.

various real-world incentive schemes. The detailed structure of the contract is presented in the solution to the model.[7]

One intuitive real-world scenario corresponding with our model is a merger and acquisition (M&A). Suppose the M&A is the project proposed by the firm. The investor is considering acquiring the firm but needs more information about its value, which can be obtained from the firm's disclosure. The investor's estimate of the target firm's value equals p, and the cost of the acquisition is c. Moreover, p is the investor's private information, whereas c and $\bar{z}$ are commonly known. The firm, knowing its own value better than the investor, can disclose a signal reflecting the investor's valuation error at a fixed cost of D. However, this disclosure may range from being perfectly informative about the investor's estimation error to being a purely random signal.

The investor's decision to acquire is observable and can be contractually stipulated. Given the information asymmetry regarding the firm's valuation, the target firm presents a disclosure contract to the investor, delineating its disclosure choices under the terms of the acquisition. The optimal disclosure contract is designed to maximize the firm's payoff during the acquisition process.

The firm's information advantage over the investor regarding the project's payoff endows the firm with monopolistic power in designing the disclosure contract. This setup corresponds with real-world scenarios such as the previously mentioned acquisition, in which the target firm has a more comprehensive understanding of its own worth than the investor, and has multiple potential investors interested in the acquisition. However, a more generalized specification allows investor and firm to negotiate over the total surplus of the disclosure during the contracting process, with each party having non-zero bargaining power. Returning to the acquisition example, this means that when multiple investors are competing for the

[7] First, we solve the model when the signal is fully informative about the estimation error u. We then extend the model to a partially informative case and argue that the payoff in the fully informative case can be achieved with an appropriate disclosure contract.

acquisition, the target firm wields full bargaining power and offers the disclosure contract. Conversely, if a particularly influential investor dominates the acquisition market and is the only party interested in acquiring the target firm, the investor may take the initiative to offer the disclosure contract, wielding full bargaining power.

The simplest case of our model is when the investor has full bargaining power. This means the investor will simply offer a contract for the firm to disclose a signal about the estimation error at the firm's disclosure cost of D. Thus, in the following analysis, we focus on the scenario in which the firm has full bargaining power in the design of the disclosure contract. This means that the firm tells the investor what type of voluntary disclosure it will make, given the investor's corresponding investment choice. In general, a firm-dominated disclosure contract can be considered a sub-game of a larger game following negotiation about the surplus voluntary disclosure (Eső and Szentes (2007)).

Suppose the firm is responsible for designing the disclosure contract. Given the social value of the voluntary disclosure, as defined in Equation (4), the distribution of the investor's estimated payoff p, and beliefs about the exogenous shock z, we can calculate the price of the firm's signal of the estimated error, irrespective of the signal's informativeness. This price is what the firm can charge the investor for its voluntary disclosure. Moreover, the optimal disclosure contract considered here explores the extent to which the firm can maximize its payoff by implementing a disclosure scheme conditional on a contractible investment choice by the investor. Specifically, we demonstrate that continuously charging the investor according to $D(p, \bar{z})$ is not a dominant contract design strategy for the firm.

Consider the contract structure when the signal is fully informative, i.e., $\theta = u$. The firm generates the signal $\theta = u$ at the cost of D through voluntary disclosure. However, it cannot guarantee that the investor will fully comprehend and interpret the signal accurately. If the firm makes a voluntary disclosure and the investor interprets it successfully, the investor will also fully observe the estimation error, $u = \theta$. Conversely, if the interpretation fails, the investor will gain no further information for the valuation. In summary, we define voluntary disclosures that are interpretable for investors as *informative disclosure*, and if they are not interpretable, we term them *uninformative disclosure*, which encompass both no disclosure and non-interpretable disclosures.

The firm offers the disclosure contract to the investor at an interim stage in which the disclosure signal has not yet been observed, whereas the investor knows that p, c, and $\bar{z}$ are common knowledge to both agents. Since our setting satisfies Sugaya and Wolitzky (2021) regularity condition, the revelation principle applies, and a direct mechanism can be implemented. We define this as the direct disclosure mechanism, characterized by the following five functions:

Disclosure Mechanism 1. *(1) t(p):[0,1]→ R denotes the expected transfer value that the firm asks the investor to pay for the voluntary disclosure.*
(2) $\beta(p)$:[0,1]→ [0,1] denotes the probability of voluntary disclosure, meaning that the firm investigates the estimation error and sends signal $\theta = u$ to the investor.
(3) $\iota(p,u)$:[0,1]× U→ [0,1] stands for the probability that the investor will choose to invest in the firm's project if the firm discloses $\theta = u$.
(4) $\bar{\iota}(p)$:[0,1]→[0,1] equals the probability that the investor will invest in the firm's project even without an informative disclosure by the firm.
(5) $\alpha(p) : [0,1] \to [0,\bar{z}]$ is the function governing the probability of interpretability and validity of voluntary disclosure information from the investor's perspective.

In the fully informative case, the firm makes a take-it-or-leave-it offer characterized by Disclosure Mechanism 1 $\{t, \beta, \iota, \bar{\iota}, \alpha\}$ to the investor. If the investor rejects it, then the corresponding payoff for the investor is $\max\{p - c - \bar{z}, 0\}$.

Accepting the offer implies that the investor will truthfully reveal the estimated payoff p and pay the transfer, defined by $t(p)$, to motivate the firm's voluntary disclosure. The probability that the firm will make a voluntary disclosure based on evaluation of the final payoff and will send signal $\theta = u$ to the investor depends on the investor's estimated payoff and is denoted by $\beta(p)$. The probability that the investor will correctly interpret the voluntary disclosure and obtain an informative disclosure is $\alpha(p)/\bar{z}$, which is inversely proportional to the average exogenous impacts outlined in our setting. Conversely, there remains a probability of $1 - \alpha(p)/\bar{z}$ that the investor will garner no additional information from the voluntary disclosure, resulting in an uninformative disclosure. On the other hand, the investor will choose to invest in the firm with a probability of $\iota(p, u)$ with voluntary disclosure and $\bar{\iota}(p)$ without disclosure. The incentive rationality condition is satisfied when the investor chooses to accept the offer irrespective of the estimated payoff, while the incentive compatibility condition holds true when the disclosure mechanism ensures the investor's truthful reporting.

In relation to the standard principal–agent framework, we can regard the firm as the principal and the investor as the agent, since we assume that the firm initiates the contract offer. However, there are two differences from the textbook setting. First, for a price, the firm can obtain an additional signal reflecting the final payoff, $\theta = u$ in the fully informative case. Second, the investor's outside option, $\max\{p - c - \bar{z}, 0\}$, is not an exogenous constant but depends on the investor's estimated payoff.[8]

We define $\Xi(p)$ as the expected probability that an investor with estimated payoff p will invest in the firm's project:

$$\Xi(p) = \beta(p)\alpha(p)/\bar{z} \int_U \iota(p, u) dQ_i(u) + [\beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p))]\bar{\iota}(p). \tag{6}$$

Suppose the investor lies about the estimated payoff by reporting p' instead of p. Then the

[8] In the textbook principal–agent model, we usually assume that the agent's reservation utility is fixed and does not vary with the type of agent.

firm's payoff is given by:

$$r(p,p') = \underbrace{\beta(p')\alpha(p')/\bar{z}\int_U (p+u-c-\bar{z})\iota(p',u)dQ_i(u)}_{\text{Firm's expected payoff when the investor interprets the disclosure correctly}} \\ + \underbrace{[\beta(p')(1-\alpha(p')/\bar{z}) + (1-\beta(p'))]\bar{\iota}(p')(p-c-\bar{z})}_{\text{Firm's expected payoff without extra information}} - \underbrace{t(p')}_{\text{Transfer}}. \tag{7}$$

The incentive compatibility condition for truth-telling requires $r(p,p') \leq r(p,p)$ for any p and p'. For simplicity, we assume that the investor will opt for honesty when indifferent between deviation and truthful reporting. The individual rationality condition mandates that it is profitable for the investor to contract with the firm, i.e., $r(p,p) \geq \max\{p - r - \bar{z}, 0\}$ for all p. For convenience, we define $R(p) = r(p,p)$ as the investor's expected payoff when telling the truth. Following Dikolli and Vaysman (2006), under the direct disclosure mechanism defined above, the monetary transfer $t(p)$ corresponds one-to-one with $R(p)$ due to the revelation principle. Therefore, the disclosure contract offered by the firm can be characterized by a set of functions $\beta, \iota, \bar{\iota}, \alpha, R$. Moreover, the incentive compatibility condition that enforces this disclosure contract satisfies the following result:

Lemma 1. [9] *The incentive-compatible direct disclosure mechanism satisfies the following condition:*

$$R(p_1) - R(p_2) = \int_{p_2}^{p_1} \Xi(u)du, \tag{8}$$

where $\Xi(u)$ is as defined in Equation (6) and p_1 and p_2 are two arbitrary estimated payoffs in the domain of the investor's estimation.

The contract functions β, ι, $\bar{\iota}$, and α characterize the social surplus created by the interaction between firm and investor. Since, according to Equation (8), the investor's payoff is upper bounded by a constant, the firm's payoff is equal to the social surplus minus the

[9] This is a standard result in the contract theory literature. The difference here is that our probability of investment $\Xi(p)$ is defined specifically to our setting.

investor's payoff:

$$V(p) = \beta(p)\alpha(p)/\bar{z} \int_U (p + u - c - \bar{z})\iota(p, u)dQ_i(u) \\ + [\beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p))]\bar{\iota}(p)(p - c - \bar{z}) - R(p). \tag{9}$$

Since our model satisfies Mirrlees (1971) regularity condition, we can use the result of Lemma 1 to characterize the firm's disclosure contract in a standard mechanism framework. With the investor's incentive compatibility condition characterized by Equation (8) and the individual rational condition defined by $R(p) = r(p, p) \geq \max\{p - r - \bar{z}, 0\}$, the firm chooses a set of "behavior functions" $\{\beta, \iota, \bar{\iota}, \alpha\}$ to determine the expected probability that the investment will happen $\Xi(p)$, and the investor's "boundary payoffs" $\{R(0), R(1)\}$ to optimize its ex-ante expected payoff denoted by $\bar{V} = \int_0^1 V(p) f_i(p)dp$.

Consider the geometric feature of the investor's truth-telling payoff $R(p)$. Since $\Xi(p)$ denotes the expected probability that the investor will invest, $\Xi(p) \in [0, 1]$, then, according to Equation (8), the first-order derivative of $R(p)$ is also in $[0, 1]$. However, the right-hand side of the firm's individual rational condition, $\max\{p - c - \bar{z}, 0\}$, is not differentiable when the investor's estimated payoff is $p = c + \bar{z}$. Its first-order derivative is 0 when $p < c + \bar{z}$ and 1 when $p > c + \bar{z}$.

These geometric features suggest that the optimal disclosure contract offered by the firm should induce a bounded individual rationality condition for investors with $p = 1$ or $p = 0$, or possibly both. Technically, the investor's individual rationality condition might be bounded for any open interval, encompassing either $p = 0$ or $p = 1$. Consequently, in the next sections, in discussing the optimal disclosure contract, we consider different boundary conditions for the investor's individual rationality condition.

3 The Solution to the Optimal Disclosure Contract

3.1 The left-bounded individual rationality

Assuming that the investor's individual rational condition is bounded when the estimated payoff is $p = 0$, whereas $p = 1$ is not a boundary point, we define the interval $[0, \underline{p}]$ as the bounding interval, such that $p - c - \bar{z} \leq 0$ for $p \in [0, \underline{p}]$. Lemma 1 then implies that the investor's ex-ante expected truth-telling payoff is equal to the following expression:

$$\int_0^1 R(p) f_i(p) dp = R(0) + \int_0^1 (1 - F_i(p)) \Xi(p) dp. \tag{10}$$

Then the firm's corresponding ex-ante expected payoff is equal to:

$$\begin{aligned} \bar{V} = \int_0^1 \{ &\beta(p)\alpha(p)/\bar{z}[\int_U (p + u - c - \bar{z} - \frac{1 - F_i(p)}{f_i(p)})\iota(p, u) dQ_i(u) - D] \\ &+ [\beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p))](p - c - \bar{z} - \frac{1 - F_i(p)}{f_i(p)})\bar{\iota}(p)\} f_i(p) dp - R(0), \end{aligned} \tag{11}$$

where $\frac{1-F_i(p)}{f_i(p)}$ denotes the reservation payoff of the investor's estimated payoff, conditional on the left-bounded individual rationality. We plug in the expression for the expected probability of investment $\Xi(p)$ defined by Equation (6), and the firm's payoff function $V(p)$ defined by Equation (9) into the previous equation, $\bar{V} = \int_0^1 V(p) f_i(p) dp$.

Let $\bar{V} + R(0)$ be the ex-ante expected social surplus of the voluntary disclosure scheme. Suppose the realization of the investor's estimated payoff is p. The project's net surplus is then equal to $(p + u - c - \bar{z} - \frac{1-F_i(p)}{f_i(p)})\iota(p, u) - D$ if the investor interprets the voluntary disclosure correctly, and $(p - c - \bar{z} - \frac{1-F_i(p)}{f_i(p)})\bar{\iota}(p)$ if the voluntary disclosure does not occur or if disclosed signal is not interpretable to the investor.

The firm's objective is to maximize $\bar{V}$ when offering the disclosure contract. Specifically, it selects a set of "behavior functions" $\{\beta, \iota, \bar{\iota}, \alpha\}$ and the boundary value of the investor's truth-telling payoff $R(0)$. Without loss of generality, we normalize the investor's truth-telling payoff when the estimated payoff p equals 0, i.e., $R(0) = 0$. We then address the optimal disclosure contract using a two-step approach. Initially, we assume an arbitrary form for β and α. As the integrand in Equation (11) is continuous, based on our assumption about the distribution of the investor's estimated payoff F_i, we can achieve pointwise maximization by setting $\iota(p, u) = 1$ when the project's net value with disclosure is positive, i.e., $p + u - c - \bar{z} - \frac{1-F_i(p)}{f_i(p)} \geq 0$, and $\iota(p, u) = 0$ when it is negative, i.e., $p + u - c - \bar{z} - \frac{1-F_i(p)}{f_i(p)} < 0$. The intuition is that the investor's behavior with estimated payoff p can be characterized by a twopoint distribution: the investment will go ahead if the realized value of the firm's project surpasses the corresponding reservation payoff $\frac{1-F_i(p)}{f_i(p)}$ with or without the addition of information from the firm's disclosure, and otherwise the investment will be withheld. Consequently, it is optimal to set $\beta(p)\alpha(p)/\bar{z} = 1$ and $\beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p)) = 0$ when the term in the bracket multiplied by $\beta(p)\alpha(p)/\bar{z}$ is larger than the term multiplied by $\beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p))$, and vice versa. Let $\Sigma^1(p) = \frac{1-F_i(p)}{f_i(p)}$ be the reservation payoff. We then have the following system of equations to characterize the optimal β and α under different conditions:

$$\int_U max\{p+u-c-\bar{z}-\Sigma^1(p), 0\}dQ_i(u) - D \geq max\{p-c-\bar{z}-\Sigma^1(p), 0\} \rightarrow \begin{cases} \beta(p)\alpha(p)/\bar{z} = 1 \\ \beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p)) = 0 \end{cases} \tag{12}$$

$$\int_U max\{p+u-c-\bar{z}-\Sigma^1(p), 0\}dQ_i(u) - D < max\{p-c-\bar{z}-\Sigma^1(p), 0\} \rightarrow \begin{cases} \beta(p)\alpha(p)/\bar{z} = 0 \\ \beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p)) = 1 \end{cases} \tag{13}$$

Suppose the investor's estimated payoff is p. Since $\beta \in [0, 1]$ and $\alpha \in [0, \bar{z}]$, if $\int_U \max\{p + u - c - \bar{z} - \Sigma^1(p), 0\}dQ_i(u) - D \geq \max\{p - c - \bar{z} - \Sigma^1(p), 0\}$, it results in $\alpha(p) = 1, \beta(p) = \bar{z}$. Conversely, if $\int_U \max\{p + u - c - \bar{z} - \Sigma^1(p), 0\}dQ_i(u) - D < \max\{p - c - \bar{z} - \Sigma^1(p), 0\}$, it implies either $\alpha(p) \in [0, 1], \beta(p) = 0$ or $\alpha(p) = 0, \beta(p) \in [0, 1]$.

The optimal choices of β and α suggest that the firm will make the disclosure and maximize its interpretability when the project's ex-ante expected net value is higher in the scenario with the disclosure than without it. Conversely, if the ex-ante expected net value is higher without an informative disclosure, the firm will either choose not to disclose or will disclose an uninterpretable signal. Most significantly, the firm's voluntary disclosure choice is determined by a threshold of the estimated payoff reported by the investor, denoted as $\bar{p}$. The value of $\bar{p}$ is obtained by equating the expected net value of the project with and without the disclosure. If the investor's reported estimated payoff surpasses $\bar{p}$, the firm opts for disclosure, otherwise it does not.[10]

The next question is: when will the individual rational condition be bounded only in an interval starting from $p = 0$ but slack in the neighborhood of $p = 1$? To simplify the exposition, from now on we call an investor with estimated payoff p as a type p investor. The two-step method above does not consider the incentive compatibility and individual rational condition when maximizing the firm's ex-ante expected payoff. Therefore the $\{\beta, \iota, \bar{\iota}, \alpha\}$ solved by this method demonstrates that a type p investor's expected probability of investing when the voluntary disclosure is informative is equal to:

$$\Xi(p) = 1 - Q_i(c + \bar{z} + \Sigma^1(p) - p) \tag{14}$$

and if neither the disclosure is informative nor the firm chooses to disclose. Hence $\Xi(p)$ increases weakly with the investor's estimated payoff p. Suppose the incentive compatibility condition is true according to Lemma 1. The individual rational condition will then be true for all $p \in [0, 1]$:

$$\begin{aligned} R(p) = \int_0^p \Xi(\mu) d\mu &\geq \max\{p - c - \bar{z}, 0\}, \forall p \in [0, 1] \\ \rightarrow \int_{\bar{p}}^1 1 - Q_i(c + \bar{z} + \Sigma^1(p) - p) dp &\geq 1 - c - \bar{z}. \end{aligned} \tag{15}$$

Note that Equation (15) depends on the threshold of disclosure $\bar{p}$ defined above, and is a

[10] This implies that $\beta(p)\alpha(p) = \bar{z}$ and $\beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p)) = 0$ if $p \geq \bar{p}$.

sufficient and necessary condition for the individual rationality of investors with all potential estimated payoff. A useful remark is that Equation (15) is the individual rational condition for an investor with an estimated payoff of $p = 1$. The necessity of Equation (15) results from the following argument. Since we normalize the investor's truth-telling payoff with $p = 0$ as $R(p) = 0$ and the first-order derivative of $R(p)$ is in $[0, 1]$ based on Lemma 1, the individual rational condition is always true if $p \leq c + \bar{z}$. Suppose the individual rationality is violated at some point when $p \in [c + \bar{z}, 1]$ and Equation (15) still holds. However, now the first order derivative of $\max\{p - c - \bar{z}, 0\}$ is one, which is weakly greater than the first order derivative $R^{'}(p) = \Xi(p) \in [0, 1]$. Then simple algebra implies that $\int_{\bar{p}}^{1} 1 - Q_i(c + \bar{z} + \Sigma(p) - p)dp < 1 - c - \bar{z}$, which is a contradiction. Therefore, we only need the inequality in Equation (15) to guarantee the individual rationality of all types of investor when the investor's individual rational condition is only bounded in the neighborhood of 0.

3.2 The right-bounded individual rationality

Suppose the investor's individual rational condition is bounded in the neighborhood of $p = 1$ but not around $p = 0$. Then, following the same procedure as for left-bounded individual rationality, we have the following equality based on Lemma 1:

$$\int_0^1 R(p) f_i(p)dp = R(1) - \int_0^1 \Xi(p)Q_i(p)dp. \tag{16}$$

Again, we plug in the expressions $\Xi(p)$ and $V(p)$ based on Equation (6) and Equation (9) to obtain the firm's ex-ante expected payoff in this scenario:

$$\begin{aligned}\bar{V} = \int_0^1 \{&\beta(p)\alpha(p)/\bar{z}[\int_U (p + u - c - \bar{z} + \frac{F_i(p)}{f_i(p)})\iota(p, u)dQ_i(u) - D] \\ &+ [\beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p))](p - c - \bar{z} + \frac{F_i(p)}{f_i(p)})\bar{\iota}(p)\} f_i(p)dp - R(1).\end{aligned} \tag{17}$$

Unlike $\bar{V}$ in the case of left-bounded individual rationality, we carry out the integration from the upper bound of the estimated payoff, $p = 1$, since now the individual rational condition is bounded on the right with a constant $R(1)$.[11]

The right-bounded individual rational condition implies that $R(1) = 1 - r - \bar{z}$ for the investor, while the firm still aims to maximize $\bar{V}$. Following the same procedure in the left-bonded scenario, $\bar{V}$ is point-wise maximized on the domain of the estimated payoff by setting $\iota(p,u) = 1$ when the net value of the firm's project with disclosure is non-negative, i.e., $p + u - c - \bar{z} + \frac{F_i(p)}{f_i(p)} \geq 0$, and 0 otherwise. Similarly, the firm chooses $\bar{\iota}(p) = 0$ if the corresponding net value of the firm's project without disclosure is non-negative, meaning $p - c - \bar{z} + \frac{F_i(p)}{f_i(p)} \geq 0$ and 0 otherwise. The intuition is that the investor will invest in the firm's project only with an estimation that the value of the firm's project is higher than the "reservation payoff" $\Sigma^0(p) = \frac{F_i(p)}{f_i(p)}$.

We still follow the two-step method, as stated before. After determining ι and $\bar{\iota}$ by point-wise maximization of $\bar{V}$, we solve the optimal choice of β and α by comparing the ex-ante total surplus with disclosure to the case without disclosure:

$$\int_U max\{p+u-c-\bar{z}+\Sigma^0(p),0\}dQ_i(u)-D \geq max\{p-c-\bar{z}+\Sigma^0(p),0\} \rightarrow \begin{cases} \beta(p)\alpha(p)/\bar{z} = 1 \\ \beta(p)(1-\alpha(p)/\bar{z}) + (1-\beta(p)) = 0 \end{cases} \tag{18}$$

$$\int_U max\{p+u-c-\bar{z}+\Sigma^0(p),0\}dQ_i(u)-D < max\{p-c-\bar{z}+\Sigma^0(p),0\} \rightarrow \begin{cases} \beta(p)\alpha(p)/\bar{z} = 0 \\ \beta(p)(1-\alpha(p)/\bar{z}) + (1-\beta(p)) = 1 \end{cases} \tag{19}$$

Consider a type p investor. Given that $\beta \in [0,1]$ and $\alpha \in [0,\bar{z}]$, if $\int_U \max\{p + u - c - \bar{z} + \Sigma^0(p), 0\}dQ_i(u) - D \geq \max\{p - c - \bar{z} + \Sigma^0(p), 0\}$, then $\alpha(p) = 1$ and $\beta(p) = \bar{z}$. Conversely, when $\int_U \max\{p + u - c - \bar{z} + \Sigma^0(p), 0\}dQ_i(u) - D < \max\{p - c - \bar{z} + \Sigma^0(p), 0\}$, it indicates either $\alpha(p) \in [0,1]$ and $\beta(p) = 0$, or $\alpha(p) = 0$ and $\beta(p) \in [0,1]$.

[11]If $R(0)$ and $R(1)$ are both known constants, then Equation (17) is equivalent to Equation (11), as will be shown in the third scenario.

Analogous to $\bar{p}$ in the case of left-bounded individual rationality, a voluntary disclosure threshold, denoted as $\tilde{p}$, can also be determined. If the investor's reported estimated payoff p is less than or equal to $\tilde{p}$, the firm makes the disclosure and maximizes its interpretability. Conversely, when the reported estimated payoff is greater than $\tilde{p}$, the firm either abstains from disclosing or discloses an uninterpretable signal.

Regarding the condition of guaranteeing that the individual rational condition holds for investors with all possible estimated payoffs, we first derive the probability of investment according to the optimal functions $\{\beta, \iota, \bar{\iota}, \alpha\}$ for an arbitrary $p \in [0, 1]$, which is equal to $\Xi(p) = 1 - Q_i(c + \bar{z} + \Sigma^0(p) - p)$ with voluntary disclosure, and 0 otherwise. Then we claim that the following inequality is a sufficient and necessary condition for the individual rationality of investors with all possible estimated payoffs:

$$\tilde{p} - c - \bar{z} \geq \int_0^{\tilde{p}} [1 - Q_i(c + \bar{z} - \Sigma^0(p) - p)]dp. \tag{20}$$

Observe that Equation (20) represents the investor's individual rational condition when the reported payoff $p = 0$. The incentive compatibility of this disclosure contract is underpinned by Lemma 1 and the fact that $\Xi(p)$ weakly increases with p. The sufficiency of Equation (20) can be proven by contradiction. Assume that the investor's individual rational condition is violated for some p in the range $[0, c + \bar{z}]$. However, for p in the range $[c + \bar{z}, 1]$, the individual rational condition holds true since $R(1) = 1 - \bar{z} - c$ and $\Xi(p) = R^{'}(p)$ falls within the range $[0, 1]$, as per Lemma 1. If the individual rational condition fails for any point in the range $[0, c + \bar{z}]$, it would also be false when $p = 0$. This contradiction of the condition implied by Equation (20) (i.e., individual rationality is bounded at $p = 0$), establishes that Equation (20) is sufficient to sustain the optimal disclosure contract denoted by $\{\beta, \iota, \bar{\iota}, \alpha\}$ when the individual rational condition is left-bounded.

3.3 The two-side-bounded individual rationality

Let the investor's individual rational condition be bounded on both sides of the domain of the estimated payoff, which are two intervals, $[0, p_{k1}]$ and $[p_{k2}, 1]$, with $p_{k1} < p_{k2}$, and $k \in (0, 1)$ being the index of the disclosure contract in this case.

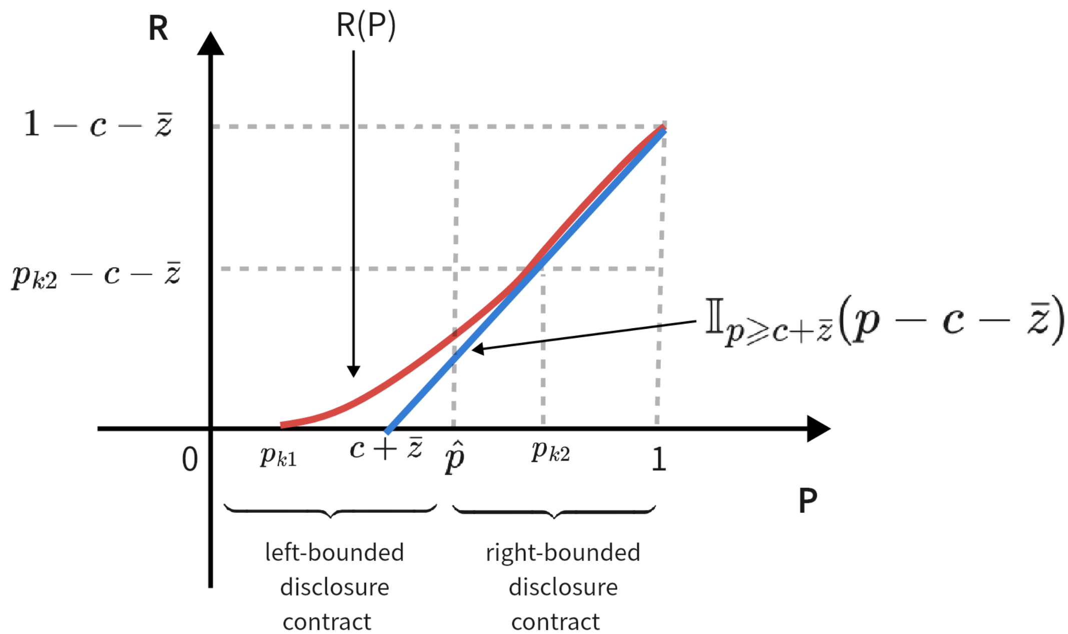

Figure 2 – Disclosure contract with two-side-bounded individual rationality

Let $p_{k1} < \hat{p} < p_{k2}$. Figure 2 demonstrates that the investor's truth-telling payoff resembles the feature for left-bounded individual rationality on the interval $[p_{k1}, \hat{p})$ and the feature for right-bounded individual rationality on $[\hat{p}, 1]$. In Lemma 2, we show that there is an optimal $\hat{p}$, such that the disclosure contract will be split into two parts: one for investors reporting $p < \hat{p}$ and the other one for investors with $p \geq \hat{p}$. Alternatively, the optimal direct disclosure mechanism is that the firm makes two types of take-it-or-leave-it offer to the investor, depending on whether or not the reported estimated payoff is higher than $\hat{p}$.

Consider the optimal disclosure contracts in the two intervals. The conditional distribution function of the estimated payoff on the interval $[0, \hat{p}]$ is given by $\underline{F}_i(p) = \frac{F_i(p)}{F_i(\hat{p})}$, the corresponding probability density function is denoted as $\underline{f}_i(p)$ for $p \in [0, \hat{p}]$. In particular, we have the following relationship between $\underline{F}_i$ and F_i:

$$\frac{1 - \underline{F}_i(p)}{\underline{f}_i(p)} = \frac{F_i(\hat{p}) - F_i(p)}{f_i(p)}. \tag{21}$$

Adopting the solution for left-bounded individual rationality, we initially maximize the firm's ex-ante expected payoff on a point-wise basis. We set $\iota(p, u) = 1$ if the expression $p + u - c - \bar{z} - \frac{1 - \underline{F}i(p)}{\underline{f}i(p)}$ holds true, and zero otherwise. In addition, we establish $\bar{\iota}(p) = 1$ for conditions where $p - c - \bar{z} - \frac{1 - \underline{F}i(p)}{\underline{f}i(p)} \geq 0$, and zero otherwise. Thus, an investor with an estimated payoff p in the range $[0, \hat{p}]$ will opt to invest in the firm's project if the project's value exceeds a "reservation payoff", represented as $\underline{\Sigma}(p) = \frac{F_i(\hat{p}) - F_i(p)}{f_i(p)}$, irrespective of any additional information from the firm's disclosure. Furthermore, the firm's disclosure probability and the probability of the investor correctly interpreting the firm's disclosure are determined by a system of equations. These hinge on whether the project's ex-ante expected net value is higher with disclosure than without. This is similar to Equation 13, except that $\Sigma(p)$ is replaced with $\underline{\Sigma}(p)$.

$$\int_U max\{p+u-c-\bar{z}-\underline{\Sigma}(p), 0\} dQ_i(u) - D \geq max\{p-c-\bar{z}-\underline{\Sigma}(p), 0\} \rightarrow \begin{cases} \beta(p)\alpha(p)/\bar{z} = 1 \\ \beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p)) = 0 \end{cases} \tag{22}$$

$$\int_U max\{p+u-c-\bar{z}-\underline{\Sigma}(p), 0\} dQ_i(u) - D < max\{p-c-\bar{z}-\underline{\Sigma}(p), 0\} \rightarrow \begin{cases} \beta(p)\alpha(p)/\bar{z} = 0 \\ \beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p)) = 1 \end{cases} \tag{23}$$

By equating the project's ex-ante expected net value with and without the firm's voluntary disclosure, we can also derive a threshold p_0 for the firm's disclosure. This threshold is such that the firm will opt for a fully interpretable disclosure when the investor reports $p \in [p_{k1}, \hat{p}]$. Conversely, for $p \in [0, \hat{p})$, the firm will either not disclose or choose to disclose an uninterpretable signal. It is important to note that p_0 is dependent on $\hat{p}$.

For $p \in [\hat{p}, p_{k2}]$, we first define the conditional distribution function and the corresponding probability density function for $p \in [\hat{p}, p_{k2}]$ as $\bar{F}_i(p) = \frac{F_i(p) - F_i(\hat{p})}{1 - F_i(\hat{p})}$ and $\bar{f}_i(p)$. The following identity characterizes the relationship between $\bar{F}_i(p)$ and $F_i(p)$

$$\frac{\bar{F}_i(p)}{\bar{f}_i(p)} = \frac{F_i(p) - F_i(\hat{p})}{f_i(p)}. \tag{24}$$

Then we adopt the procedure for right-bounded individual rationality to solve the optimal disclosure contract when $p \in [\hat{p}, 1]$. However, now the firm's reservation payoff is equal to $\bar{\Sigma}(p) = -\frac{\bar{F}_i(p)}{\bar{f}_i(p)}$. The firm's ex-ante expected payoff is pointwise maximized by setting $\iota(p, u) = 1$ and $\bar{\iota}(p) = 1$ when $p + u - r - \bar{z} - \bar{\Sigma}(p) \geq 0$ and $p - r - \bar{z} - \bar{\Sigma}(p) \geq 0$, and zero otherwise. Moreover, the disclosure probability function β and the likelihood function of the disclosure's interpretability α still follow the system characterized by Equation (22) and Equation (23) since the reservation payoff $\bar{\Sigma}$ on $[\hat{p}, 1]$ is equal to the reservation payoff $\underline{\Sigma}$ on $[0, \hat{p}]$.

On integrating the results from intervals $[0, \hat{p})$ and $[\hat{p}, 1]$, it is clear that the firm will chooses to disclose the information if the investor's reported estimated payoff falls within the range $p \in [p_{k1}, p_{k2}]$. The investor, on the other hand, will decide to invest in the firm's project only if its net surplus is non-negative, regardless of whether or not the voluntary disclosure is informative.

With an estimated payoff of p, the investor will correctly interpret the firm's voluntary disclosure and decide to invest if the following condition is met: $p + u - c - \bar{z} - \underline{\Sigma}(p) - D \geq 0$. Conversely, an investor faced with an uninformative disclosure or no disclosure at all will choose to invest when this condition is fulfilled: $p - c - \bar{z} - \underline{\Sigma}(p) \geq 0$.

The cutoff point $\hat{p}$ is determined such that the investor's ex-ante expected payoff is equivalent under both optimal disclosure contracts for the domains $p \in [0, \hat{p})$ and $p \in [\hat{p}, 1]$, as solved earlier. The parameters p_1 and p_2 are chosen to ensure that the ex-ante expected social surplus when there is an informative disclosure exceeds that when there is not, for any arbitrary value of $p \in [p_{k1}, p_{k2}]$. Consequently, the following inequality holds true:

$$\int_U max\{p + u - c - \bar{z} - \underline{\Sigma}(p), 0\} dQ_i(u) - D \geq max\{p - c - \bar{z} - \underline{\Sigma}(p), 0\}. \tag{25}$$

We summarize our analysis of the three scenarios with different bounding conditions for the investor's individual rationality in the following proposition, which is the main result of this paper.

Proposition 1. *Let the distribution of the investor's estimated payoff F_i be common knowledge. In the fully informative case, such that the firm's disclosure signal is equal to the investor's estimation error, $\theta = u$, and the disclosure cost D and the belief about the exogenous shock are both common knowledge, the optimal direct disclosure mechanism can be characterized by a set of functions, $\{\beta, \iota, \bar{\iota}, \alpha\}$, which satisfies:*

(a) For any coefficient $k \in [0, 1]$, the functions ι and $\bar{\iota}$ satisfy $\iota(p, u) = I_{p+u-c-\bar{z}-\Sigma^k(p) \geq 0}$, $\bar{\iota}(p) = I_{p-c-\bar{z}-\Sigma^k(p) \geq 0}$ with $\Sigma^k(p) = \frac{k - F_i(p)}{f_i(p)}$. The I_x is an indicator function equal to one when x is true.

(b) There exists a pair of coefficients $0 \leq p_{k1} < p_{k2} \leq 1$ such that the $\beta(p)$ and $\alpha(p)$ are determined by a system of equations conditional on the ex-ante total surplus with and without informative disclosure

$$\int_U max\{p+u-c-\bar{z}-\Sigma^k(p), 0\} dQ_i(u) - D \geq max\{p-c-\bar{z}-\Sigma^k(p), 0\} \rightarrow \begin{cases} \beta(p)\alpha(p)/\bar{z} = 1 \\ \beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p)) = 0 \end{cases} \tag{26}$$

$$\int_U \max\{p+u-c-\bar{z}-\Sigma^k(p), 0\} dQ_i(u) - D < \max\{p-c-\bar{z}-\Sigma^k(p), 0\} \rightarrow \begin{cases} \beta(p)\alpha(p)/\bar{z} = 0 \\ \beta(p)(1 - \alpha(p)/\bar{z}) + (1 - \beta(p)) = 1 \end{cases} \tag{27}$$

(c) The investor's truth-telling payoff R satisfies the incentive compatibility characterized by Equation (8)

(d) The expected probability of investment $\Xi(p)$ for an investor with an estimated payoff p is defined by Equation (6).

(e) The investor's truth-telling payoff satisfies $R(p_{k1}) = 0$ if $p_{k1} \geq 0$ and $R(p_{k2}) = 1 - c - \bar{z}$ if $p_{k2} \leq 0$.

The generality of Proposition 1 lies in the first scenario with left-bounded individual rationality corresponding with the case where $c > 1, k = 1$ and $p_{k2} = 1$, and the second scenario with right-bounded individual rationality is captured by the parameterization of $c < 0, k = 0$ and $p_{k1} = 0$. The two-side-bounded case is simply a combination of the two. More interestingly, if the firm discloses a signal at zero cost, we have $p_{k1} = 0$ and $p_{k2} = 1$. This implies that with a costless voluntary disclosure scheme, the firm will disclose, regardless of the estimated payoff reported by the investor. Furthermore, with an estimated payoff outside the interval $p \notin [p_{k1}, p_{k2}]$, the investor's individual rational condition is always bounded, meaning that $R(p) = \max\{p - c - \bar{z}, 0\}$ since no informative disclosure is received from the firm. Thus this type of investor behaves as if there is no disclosure contract. In particular, an investor with $p < p_{k1}$ will never invest in the firm's project, and an investor with $p > p_{k2}$ will invest anyway. This means that the net value of the project without informative disclosure is always negative $p - c - \bar{z} - \Sigma^k(p) < 0$ for $p \in [0, p_{k1})$ and always positive $p - c - \bar{z} - \Sigma^k(p) > 0$ for $p \in (p_{k2}, 1]$.

Hence with the investment cost c, the belief about the exogenous shock z, and the distribution of the investor's estimated payoff F_i being common knowledge, Proposition 1 demonstrates that if there is no informative disclosure from the investor's perspective, the investor will choose not to invest in the firm's project when the estimated payoff is lower than the sum of the investment cost and the expected value of the exogenous shock, i.e., $p < c + \bar{z}$ regardless of the firm's disclosure decision. This investment strategy is socially optimal when the firm's disclosure is not informative or there is no disclosure.

In the fully informative case, the firm can obtain a signal that perfectly reflects the investor's estimated error at a fixed cost of D. In other words, $\theta = u$. To illustrate the optimal disclosure mechanism in this scenario, we introduce Disclosure Mechanism 1, the optimality of which has been established in our previous analysis. Furthermore, the fully informative optimal disclosure mechanism provides a benchmark for determining the firm's maximum possible payoff, given that the voluntary disclosure is perfectly informative. The logic here is that even in a fully informative situation, the firm may still disclose a signal that is not fully correlated with u. However, in the optimal disclosure mechanism, the firm opts to disclose $\theta = u$. Since our model complies with the regularity condition, the weak axiom of revealed preference (WARP) indicates that $\theta = u$ yields the highest ex-ante expected firm payoff.

A natural question that arises is can the firm achieve the highest payoff, even if it cannot disclose a fully informative signal through the disclosure contract? The answer is affirmative, as it can be demonstrated that the lower and upper bounds are identical. Therefore, even if the firm's disclosed signal θ is independent of u, the firm can still implement the contract that induces the highest payoff. In other words, the firm's payoff under the optimal disclosure contract is independent of θ. The rationale behind this is that investors' information rents from the firm's voluntary disclosure are fully appropriated by the firm, and investors' only source of information rents comes from their own estimated payoffs. The only difference arises in the incentive compatibility condition, which will be more stringent when θ and u are independent than in the case where $\theta = u$. It is important to note that the assumption that both the exogenous effect and the investment cost are common knowledge is critical for this conclusion.

4 Imperfect Disclosure

The result in the fully informative case is predicated on the assumption that the firm can perfectly disclose the investor's estimated error, i.e., $\theta = u$. Now consider an arbitrary optimal disclosure contract denoted by k. Proposition 1 shows that an investor with an estimated payoff of $p \in [p_{k1}, p_{k2}]$ will invest in the firm's project as long as the project's net value is non-negative.[12] We can also implement this optimal disclosure contract by requiring type p investor to pay $\Sigma^k(p)$ to receive informative disclosure from the firm, should they decide to invest in the project. Assuming that this approach guarantees incentive compatibility, such that the type p investor will indeed pay $\Sigma^k(p)$ based on the corresponding contract indexed by k, the investment will take place if and only if the project's net value is non-negative. Therefore, the firm's ability to ascertain the investor's estimation error u and provide a perfect disclosure is not a prerequisite for identifying an incentive-compatible direct disclosure contract for implementation.

The imperfect signal can be modeled based on our setting with the perfect signal, as follows. An investor with an estimated payoff of $p \notin [p_{k1}, p_{k2}]$ does not receive any informative disclosure from the firm. Such investors' investment decisions are entirely dependent on their own belief and are unrelated to any additional information from the firm. Specifically, they choose to invest whenever they determine that the project's net value is non-negative, i.e., $p - c - \bar{z} \geq 0$. Consequently, the firm may simply choose not to offer any disclosure contract to this type of investor to implement the optimal direct disclosure contract as described in Proposition 1.

To obtain the same payoff as in the fully informative case, we make an assumption that the firm, given its imperfect disclosure, devises an extended disclosure contract, indexed by k. This ensures that the investor's payoff is consistent with the benchmark case outlined in Proposition 1. Specifically, this contract necessitates that the investor with $p \in [p_{k1}, p_{k2}]$

[12] This implies that $p + u - c - \bar{z} - \Sigma^k(p) \geq 0$ for an investor with an estimated payoff of p.

makes an additional transfer to the firm, dependent on p. This is denoted by $e(p)$. Consequently, the investor's truth-telling payoff equates to:

$$R(p) = \int_U \max\{p + u - c - \bar{z} - \Sigma^k(p), 0\} dQ_i(u) - e(u), \tag{28}$$

where $\Sigma^k(p) = \frac{k - F_i(p)}{f_i(p)}$ for a suitable $k \in [0, 1]$ denotes the "reservation payoff" under the extended disclosure contract. Therefore, this extended disclosure contract introduces an additional function, e. Naturally, compared with the fully informative case, we need an additional incentive compatibility condition to encourage truthful reporting by the investor. We summarize the features of this extended disclosure contract in the following proposition.

Proposition 2. *The direct disclosure mechanism presented in Proposition 1 can be implemented even if the firm's disclosed signal θ is not perfectly correlated with the investor's estimation error. The implementation is as follows.*

(a) Suppose an arbitrary correlation exists between the firm's disclosed signal θ and the investor's estimation error u. The voluntary disclosure contract under the fully informative case is indexed by $k \in [0, 1]$, and the corresponding disclosure interval is $[p_{k1}, p_{k2}]$. The investor reports an estimated payoff of p. When $p \notin [p_{k1}, p_{k2}]$, the investor will not receive an informative disclosure.

(b) While $p \in [p_{k1}, p_{k2}]$, the investor pays the extra transfer of $e(p)$ to the firm to obtain the estimation error u disclosed by the firm.

(c) An investor choosing to invest in the firm's project has to pay the "reservation payoff", denoted by $\Sigma^k(p) = \frac{k - F_i(p)}{f_i(p)}$.

Proposition 2 shows that the optimal allocation in a fully informative case can also be facilitated via a direct disclosure mechanism. This mechanism is employed even when the firm's disclosure is not fully informative. It is worth noting that common knowledge still prevails regarding the belief about the exogenous shock, the investment cost, and the distribution of the investor's estimated payoff. The firm's disclosure contract, indexed by k, includes a series of take-it-or-leave-it offers, which the investor selects based on the estimated payoff. The set of offers specifically consists of $(e(p), \Sigma^k(p))$ with $p \in [p_{k1}, p_{k2}]$. The investor then decides whether to accept an offer from the set or proceed without a disclosure contract. When an offer $(e(p), \Sigma^k(p))$ is chosen, the investor initially pays an extra transfer $e(p)$ to acquire the estimation error u. If the investor decides to invest after understanding u, the reservation payoff $\Sigma^k(p)$ will be paid.

A significant pattern of an offer $(e(p), \Sigma^k(p))$ is that the extra transfer increases with p, while the reservation payoff $\Sigma^k(p)$ decreases with p. This feature is crucial for the implementation of the disclosure contract. Given that investors with high estimated payoffs have confidence in the firm's project and are likely to invest and pay the reservation payoff, they have a stronger incentive than investors with lower estimated payoffs to increase the extra transfer, thereby reducing the reservation payoff.

Let us consider scenarios with extreme investment costs, specifically where $c > 1 - \bar{z}$ and $c < -\bar{z}$. In the situation where $c > 1 - \bar{z}$, the right-hand side of the investor's individual rational condition, $\max\{p - c - \bar{z}, 0\}$, is always zero. This scenario corresponds with left-bounded individual rationality, where the disclosure contract is indexed by $k = 1$, and the reservation payoff for an investor with estimated payoff p is $\Sigma^1(p)$. The intuition is that the extremely high investment cost reflects investors' pessimism about the firm's project, as they believe that even if the project generates the highest payoff, it will still result in a net loss for them. In other words, they will not invest without an additional informative disclosure from the firm. In such situations, investments only occur when investors learn that the estimation error u is sufficiently positive. As a result, an investor with estimated payoff p will need to pay a "fee" to the firm for the voluntary disclosure if the investment takes place, given that the reservation payoff is $\Sigma^1(p) = \frac{1-F_i(p)}{f_i(p)}$.

If $c < -\bar{z}$, it indicates an extremely optimistic investor where $p - c - \bar{z} > 0$ holds true for all estimated payoffs. This corresponds with right-bounded individual rationality. Under such circumstances, the reservation payoff for all estimated payoffs p, represented as $\underline{\Sigma}(p)$, is below zero. Hence, in scenarios where investors choose to invest, they end up making higher payments to the firm than in situations where they refrain from investing.

In the two extreme scenarios discussed above, investors either refrain from investing entirely, irrespective of the estimated payoff, or elect to invest, again independent of the payoff. The cost of voluntary disclosure signifies the extent to which the firm's informative disclosure may influence the investor's decision making. Nevertheless, if the investment cost is positioned between these two extremes, the firm cannot accurately ascertain, ex-ante, the impact of voluntary disclosure on an investor's decision without a truthful report from the investor.

5 Discussion

Integrating the findings from Proposition 1 and Proposition 2 naturally gives rise to a further question: can we always obtain the allocation of the disclosure contract from the fully informative case, regardless of how the firm's disclosed signal θ depends on the investor's estimation error u? In a more intuitive sense, is it invariably the case that the firm cannot secure any additional payoff by directly obtaining the estimation error u?

Under the same conditions as before, the investment cost c, the disclosure cost D, the belief about the exogenous shock z, and the distribution of the investor's estimated payoff F_i remain common knowledge. We then need to formulate a belief from the firm's perspective about the investor's estimation error Q_i, such that we can implement the direct disclosure contract in Proposition 1 in both the fully informative case and the independent case (where u and θ are independent).

Assuming that u and θ are independent, the firm can implement a direct disclosure contract indexed by k following a set of offers $(e(p), \Sigma^k(p))_{p\in[0,1]}$, akin to our discussion of imperfect disclosure. According to Proposition 2, the necessary and sufficient condition for implementing the disclosure contract in Proposition 1 is the following inequality:

$$p^{'} \geq p \Leftrightarrow 1 - Q_i(c + \bar{z} + \Sigma^k(p^{'}) - p) \geq 1 - Q_i(c + \bar{z} + \Sigma^k(p) - p), \forall p, p^{'} \in [p_{k1}, p_{k2}]. \quad (29)$$

Thus, our assumption that the reservation payoff $\Sigma^k(p)$ increases on the disclosure interval $[p_{k1}, p_{k2}]$ supports this sufficient and necessary condition.

If $u = \theta$, the expected probability of investment $\Xi(p)$ has to be a weakly increasing function of the investor's estimated payoff p to ensure the implementability of the disclosure contract in Proposition 1. For a disclosure contract indexed by k and corresponding disclosure interval $[p_{k1}, p_{k2}]$, $\Xi(p)$ defined by Equation (6) is continuous on $[0, 1]$ and weakly increases on $[0, p_{k1}]$ and $[p_{k2}, 1]$. Thus, the optimal disclosure contract is implementable if and only if Q_i satisfies:

$$p^{'} \geq p \Leftrightarrow 1 - Q_i(c + \bar{z} + \Sigma^k(p^{'}) - p^{'}) \geq 1 - Q_i(c + \bar{z} + \Sigma^k(p) - p). \quad (30)$$

Moreover, since we assume that the probability density function of the estimated payoff f_i is log-concave, $c + \bar{z} + \Sigma(p) - p = c + \bar{z} + \frac{k - F_i(p)}{f_i(p)} - p$ weakly increases in p on the interval $[p_{k1}, p_{k2}]$. Hence the inequality in Equation (30) holds.

Let us contemplate a disclosure contract designated by k. In the independent case, assume that the investor inaccurately reports a payoff estimate of $p^{'}$ when the true estimated payoff is p, and the firm is able to precisely obtain the estimation error. Under these circumstances, an investment will transpire only if $p^{'} + u - \bar{z} - c - \Sigma^k(p^{'}) \geq 0$. This suggests that investors' decisions are not contingent on their actual estimated payoffs. In the independent case, where the firm only receives an unrelated signal θ and has no access to u, it is infeasible for the firm to consistently maintain an incentive-compatible disclosure contract for the investor.

Consider the same scenario of misreporting in the fully informative case with voluntary disclosure. The investor's decision continues to hinge on the true estimated payoff, as the investment will proceed only if $p+u-c-\bar{z}-\Sigma^k(p) \geq 0$. Consequently, the same deviation in the independent case results in more benefits for investors than in the fully informative case, since they can incorporate their own estimated payoffs into the decision-making process in the former situation. Therefore, in answer to the question posited at the start of this section, the firm can derive more benefits by precisely observing the estimation error, as the condition for the investor's incentive compatibility is now more stringent than in the independent case.[13]

6 Conclusion

This paper contributes to the literature on voluntary disclosures in accounting by presenting a model that highlights the role of a firm's strategic disclosure of a signal regarding an investor's estimation error regarding the payoff from a proposed project. This marks a departure from previous models, which have not explicitly accounted for disclosure of such information. Our results shed light on the optimal disclosure contract that a firm can implement, where the firm's disclosure decision is contingent on the investor's estimated payoff. We also highlight how this disclosure signal may influence the investor's subsequent investment decisions.

A central finding of our study is that it is feasible to implement this optimal disclosure contract through a direct disclosure mechanism, even when the firm's signal is not perfectly correlated with the estimation error. This result serves to illuminate the complex interplay between firms' disclosure strategies and investors' investment decisions, presenting a more nuanced understanding of voluntary disclosures in an accounting context.

Theoretical advances notwithstanding, our research also has practical implications for

[13]This argument also explains that the condition in Equation (29) is stronger than that induced by Equation (26).

firms, investors, and regulators. For firms, understanding the strategic value of disclosing estimated error signals may inform their communication and engagement strategies with investors. Investors, on the other hand, will be better able to comprehend the nuances of firms' disclosures, and hence make more informed investment decisions. From a regulatory perspective, our results may guide policy decisions on firms' transparency and disclosure requirements.

In summary, we significantly advance understanding of voluntary disclosures in accounting, outlining a mechanism through which firms can strategically manage their disclosure strategies, given investors' estimated payoffs from projects. However, as with all theoretical work, the generalizability of our findings is limited to the assumptions and constraints of our model. Therefore, we encourage further research to extend our model and test the empirical validity of our theoretical predictions, thereby advancing discourse on voluntary disclosures in accounting.

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Appendix

Proof of Lemma 1: Lemma 1 requires that Equation (8) is sufficient and necessary for the investor's incentive compatibility. We first prove the necessity. Without loss of generality, we assume $p^{'} < p$. Then, according to the definition of $r(p, p^{'})$, we have the expression below

$$
\begin{aligned}
r(p,p^{'}) &= \beta(p^{'})\alpha(p^{'})/\bar{z}\int_U (p + p^{'} - p^{'} + u - c - \bar{z})\iota(p^{'}, u)dQ_i(u) \\
&+ [\beta(p^{'})(1 - \alpha(p^{'})/\bar{z}) + (1 - \beta(p^{'}))]\bar{\iota}(p^{'})(p + p^{'} - p^{'} - c - \bar{z}) - t(p^{'}) \\
&= \beta(p^{'})\alpha(p^{'})/\bar{z}\int_U (p^{'} + u - c - \bar{z})\iota(p^{'}, u)dQ_i(u) \\
&+ [\beta(p^{'})(1 - \alpha(p^{'})/\bar{z}) + (1 - \beta(p^{'}))]\bar{\iota}(p^{'})(p^{'} - c - \bar{z}) - t(p^{'}) \\
&+ (p - p^{'})\{\beta(p^{'})\alpha(p^{'})/\bar{z}\int_U \iota(p^{'}, u)dQ_i(u) + [\beta(p^{'})(1 - \alpha(p^{'})/\bar{z}) + (1 - \beta(p^{'}))]\bar{\iota}(p^{'})\} \\
&= R(p^{'}) + (p - p^{'})\Xi(p^{'}),
\end{aligned}
\tag{31}
$$

where the second equality is due to the definition of the investor's truth-telling payoff $R(p)$ and the third equality is based on the definition of the investor's expected probability of investment $\Xi(p)$. Then the incentive compatibility condition that $r(p, p^{'}) \leq R(p)$ implies that:

$$R(p) \geq R(p^{'}) + (p - p^{'})\Xi(p^{'}). \tag{32}$$

Correspondingly, $r(p, p^{'})$ can be rewritten as the following expression based on the same procedure:

$$r(p^{'}, p) = R(p) + (p^{'} - p)\Xi(p). \tag{33}$$

Again the investor's incentive compatibility condition implies that $r(p^{'}, p) \leq R(p^{'})$. Then we have:

$$\Xi(p^{'}) \leq \frac{R(p) - R(p^{'})}{p - p^{'}} \leq \Xi(p), \tag{34}$$

where the inequality is true because $p - p^{'} > 0$. Thus the expected probability $\Xi(p)$ weakly increases in the investor's estimated value p. Since $R(p)$ is continuous and differentiable, the intermediate theorem implies that the expected probability of investment is equal to:

$$\Xi(p) = \frac{dR(p)}{dp}. \tag{35}$$

Moreover, $\Xi(p)$ is bounded for all $p \in [0, 1]$ since $\Xi(p)$ is a probability. The continuity of $R(p)$ implies that $\Xi(p)$ is integrable. Therefore, we back up the Equation (8) by integrating Equation (35) from p_2 to p_1 stated in Lemma 1:

$$R(p_1) - R(p_2) = \int_{p_2}^{p_1} \Xi(u) du. \tag{36}$$

Thus the necessity is proved.

Consider the sufficiency. Equation (8) holds and $\Xi(p)$ is weakly increasing, and we wish to show that the corresponding disclosure contract is incentive-compatible. Without loss of generality, let p and $p^{'}$ be two arbitrary values estimated by the investor on the interval $[0, 1]$ such that $p^{'} < p$. Then the expected payoff of a type p investor reporting $p^{'}$ is equal to:

$$\begin{aligned} r(p, p^{'}) = \beta(p^{'})\alpha(p^{'}/\bar{z}) \int_U (p + u - c - \bar{z})\iota(p^{'}, u) dQ_i(u) \\ + [\beta(p^{'})(1 - \alpha(p^{'})/\bar{z}) + (1 - \beta(p^{'}))]\bar{\iota}(p^{'})(p - c - \bar{z}) - t(p^{'}). \end{aligned} \tag{37}$$

Using the definition of $R(p)$, $r(p, p^{'})$ can be expressed as:

$$r(p, p^{'}) = R(p^{'}) + (p - p^{'})\Xi(p^{'}). \tag{38}$$

Moreover, Equation (8) implies that:

$$R(p) = R(p^{'}) + \int_{p^{'}}^{p} \Xi(u)du \geq R(p^{'}) + (p - p^{'})\Xi(p^{'}), \tag{39}$$

where the inequality is because $\Xi(p)$ weakly increases in p. Thus we have:

$$R(p) \geq r(p, p^{'}). \tag{40}$$

Hence it is incentive-compatible for type p investors to report their type honestly under the disclosure contract. Following a similar procedure, we can show the incentive compatibility for type $p^{'}$ investors such that:

$$R(p^{'}) \geq r(p^{'}, p), \tag{41}$$

Because p and $p^{'}$ are two arbitrary values estimated by the investor on $[0, 1]$, Equation (40) and Equation (41) demonstrate that the corresponding disclosure contract is incentive-compatible and sufficiency is attained.

Q.E.D

Before proving Proposition 1, we first need to prove the arguments of the following Lemma.

Lemma 2. *For an arbitrary disclosure contract indexed by $k \in [0, 1]$, we have $p_{k1} < p_{k2}$ such that:*

$$\begin{aligned} p_{k1} &= min\{p \in [0,1], \int_{c+\bar{z}+\Sigma^k(p)-p}^{\infty} (p + u - c - \bar{z} - \Sigma^k(p))dQ_i(u) \geq D\} \\ p_{k2} &= max\{p \in [0,1], \int_{-\infty}^{c+\bar{z}+\Sigma^k(p)-p} (-p - u + c + \bar{z} + \Sigma^k(p))dQ_i(u) \geq D\}. \end{aligned} \tag{42}$$

In addition, p_{k1} and p_{k2} weakly increase in the contract index k with the boundary condition being given by $p_{01} = 0$ and $p_{12} = 1$, and the distribution function of the estimated value *satisfies:*

$$F(p_{k1}) \leq k \leq F(p_{k2}). \tag{43}$$

Proof of Lemma 2: Since the distribution function is continuous, we define $k = F(p_k)$. Moreover, the expression $\int_{c+\bar{z}+\Sigma^k(p)-p}^{\infty}(p+u-c-\bar{z}-\Sigma^k(p))dQ_i(u)$ is also continuous and strictly increases in p_{k1} owing to the continuity of the non-negative probability density function $f(p)$.

Suppose that $p_{k1} = p_k$. Then $\int_{c+\bar{z}+\Sigma^k(p)-p}^{\infty}(p+u-c-\bar{z}-\Sigma^k(p))dQ_i(u) = \int_{c+\bar{z}-p_k}^{\infty}(p_k + u - c)dQ_i(u) = DV(p_k, c, \bar{z}) + (p-c-\bar{z})I_{p-c-\bar{z}\geq 0}$, where the second equality is due to the expression of the social value of the voluntary disclosure defined by Equation (4). Furthermore the Assumption 2 implies that the social value of the voluntary disclosure is always higher than the cost of disclosure, i.e., $DV(p_k, c, \bar{z}) > D$. In addition, as the expression $\int_{c+\bar{z}+\Sigma^k(p)-p}^{\infty}(p+u-c-\bar{z}-\Sigma^k(p))dQ_i(u)$ is continuous and bounded for $p \in [0,1]$, we can obtain the corresponding minimum of the investor's estimated value p_{k1} with $p_{k1} \leq p_k$. Following the same procedure, we can show that $p_{k2} \geq p_k$.

Because the cumulative distribution function $F(\cdot)$ is weakly increasing in its argument, the inequality $F(p_{k1}) \leq k \leq F(p_{k2})$ is true because $p_{k1} \leq p_k \leq p_{k2}$. If we set $k = 0$ and $k = 1$, it is clear that $p_{01} = 0$ and $p_{12} = 1$.

Moreover, p_{k1} and p_{k2} are continuous in the index of the contract k according to their expression defined by Equation (42) since $\int_{c+\bar{z}+\Sigma^k(p)-p}^{\infty}(p+u-c-\bar{z}-\Sigma^k(p))dQ_i(u)$ is continuous in k and the corresponding minimum is attainable on the bounded interval $[0,1]$. We then derive $\int_{c+\bar{z}+\Sigma^k(p)-p}^{\infty}(p+u-c-\bar{z}-\Sigma^k(p))dQ_i(u)$ with respect to k:

$$\begin{aligned}
&-\frac{\Sigma^k(p)Q_i'(c+\bar{z}+\Sigma^k(p)-p)}{f(p)} - \Sigma^k(p)(-Q_i'(c+\bar{z}+\Sigma^k(p)-p)\frac{1}{f(p)}) - \frac{1}{f(p)}(1-Q_i(c+\bar{z}+\Sigma^k(p)-p)) \\
&= -\frac{1}{f(p)}(1-Q_i(c+\bar{z}+\Sigma^k(p)-p)) \leq 0, \forall p \in [0,1],
\end{aligned} \tag{44}$$

where the inequality arises because $f(p)$ and $Q_i(u)$ are the probability density function and cumulative distribution function, respectively. Thus, $\int_{c+\bar{z}+\Sigma^k(p)-p}^{\infty}(p+u-c-\bar{z}-\Sigma^k(p))dQ_i(u)$ is decreasing in k, implying that p_{k1} is weakly increasing in k owing to the minimum operator in the definition of p_{k1} according to Equation (42).

By the same procedure, we can show that $\int_{-\infty}^{c+\bar{z}+\Sigma^k(p)-p}(-p-u+c+\bar{z}+\Sigma^k(p))dQ_i(u)$ is increasing in k, and the maximum operator in the definition of p_{k2} based on Equation (42) demonstrates that p_{k2} is also weakly increasing in k.

Q.E.D

Proof of Proposition 1: We split the proof into three parts based on the boundary condition of the investor's individual rationality.

The left-bounded individual rationality: Since we are considering the fully informative case, Equation (15) is a sufficient and necessary condition for the incentive compatibility and $k = 1$. Moreover, the inequality of Equation (15) implies that $p_{11} \leq c + \bar{z}$. Meanwhile, $p - \Sigma^1(p)$ increases in the investor's estimated value p. Therefore, we have $p - c - \bar{z} - \Sigma^1(p) < 0$ when $p \leq p_{11}$.

Regarding $p \in [0, p_{11})$, the expression $\max\{p - c - \bar{z} - \Sigma^1(p), 0\}$ equals zero since $p_{11} \leq c + \bar{z}$. Furthermore, the integral on the left-hand side of Equation (12) is weakly positive owing to the definition of p_{11} in Equation (42). Thus, $\int_{c+\bar{z}+\Sigma^k(p)-p}^{\infty}(p+u-c-\bar{z}-\Sigma^k(p))dQ_i(u)$ is strictly less than D if $p < p_{11}$, leading to the result that $\int_U max\{p+u-c-\bar{z}-\Sigma^1(p), 0\}dQ_i(u) - D < max\{p - c - \bar{z} - \Sigma^1(p), 0\}$. Hence, the disclosure functions are either $\beta(p) = 0$ or $\alpha(p) = 0$ for $p \in [0, p_{11}]$. Consider $p \in (p_{11}, c + \bar{z})$. $\max\{p - c - \bar{z} - \Sigma^1(p), 0\}$ is still 0, whereas the integral on the left-hand side of Equation (12) is strictly positive, since the corresponding integrand is strictly increasing in p. Then, the disclosure functions are $\beta(p) = \bar{z}$ and $\alpha(p) = 1$ for $p \in (p_{11}, c + \bar{z})$. Finally, $p \in [c + \bar{z}, 1]$ implies that the inequality of Equation (12) is always true, since with the fixed c and $\bar{z}$, the social value of the disclosure $DV(p, c, \bar{z})$ is always larger than the disclosure cost for any $p \in [0, 1]$. Thus, the disclosure functions are characterized by $\beta(p) = \bar{z}$ and $\alpha(p) = 1$ for $p \in [c + \bar{z}, 1]$.

Combining the analysis above, we have proved the features of the disclosure contract when the investor's individual rationality is left-bounded.

*The right-bounded individual rationality:*In the case of perfect signalling, Equation (20) implies that incentive compatibility holds when $k = 0$. Moreover, the right-hand side of Equation (20) is non-negative, implying that $p_{02} \geq c + \bar{z}$. Meanwhile, $p - \Sigma^0(p)$ increases in the investor's estimated value p. Therefore, $p - c - \bar{z} - \Sigma^0(p) > 0$ when $p \geq p_{02}$.

The definition of p_{k2} according to Equation (42) implies the following inequality when $k = 0$:

$$\begin{aligned} &\int_{-\infty}^{c+\bar{z}+\Sigma^0(p)-p} (-p - u + c + \bar{z} + \Sigma^0(p)) dQ_i(u) \geq D \\ \rightarrow &\int_{c+\bar{z}+\Sigma^0(p)-p}^{\infty} (p + u - c - \bar{z} + \Sigma^0(p)) dQ_i(u) - D \geq p - c - \bar{z} + \Sigma^0(p). \end{aligned} \tag{45}$$

For $p \in (p_{02}, 1]$, the inequality of Equation (18) is equivalent to Equation (45), since $p_{02} \geq c + \bar{z}$. Furthermore, the definition of p_{02} by Equation (42) implies that Equation (45) is false, leading to either $\beta(p) = 0$ or $\alpha(p) = 0$ for $p \in (p_{02}, 1]$.

If $p \in [c + \bar{z}, p_{02}]$, the right-hand side of Equation (18) is equal to $p - c - \bar{z} + \Sigma^0(p)$. In addition, the difference between the left- and right-hand sides of Equation (45) is decreasing in p by calculating the corresponding first-order derivative with respect to p. Note that Equation (45) is true for $p \in [c + \bar{z}, p_{02}]$. Thus $\beta(p) = 1$ and $\alpha(p) = \bar{z}$ when $p \in [c + \bar{z}, p_{02}]$.

Finally, Assumption 2 indicates that the social value of disclosure conditional on c and $\bar{z}$, $DV(p, c, \bar{z})$ is always larger than the cost of disclosure D for all $p \in [0, 1]$. Hence we have either $\beta(p) = 0$ or $\alpha(p) = 0$ for $p \in [0, c + \bar{z})$.

The two-side-bounded individual rationality: For the case of two-side-bounded individual rationality, we assume that the index of the disclosure contract is $k \in (0, 1)$ such that the corresponding boundary of the disclosure contract satisfies:

$$p_{k2} - c - \bar{z} - \int_{p_{k1}}^{p_{k2}} [1 - Q_i(c + \bar{z} + \Sigma^k(p) - p)]dp = 0. \tag{46}$$

Notable from this condition is that the p_{k1} type investor and the p_{k2} type investor are both indifferent to either accepting or rejecting the disclosure contract offered by the firm, and receive the same level of outside options. The existence of such k can be proved by an intermediate theorem. Since we are now examining the case of two-side-bounded individual rationality, Equation (15) and Equation (20) are both false, which implies that the left-hand side of Equation (46) is negative when $k = 0$ and positive when $k = 1$. Given that the left-hand side of Equation (46) is continuous, the existence of such $k \in (0, 1)$ inducing Equation (46) can be obtained by an intermediate theorem.

With the disclosure contract indexed by k and the corresponding disclosure interval being $[p_{k1}, p_{k2}]$, the investor's expected probability of investment is equal to:

$$\Xi(p) = \begin{cases} 0, p \in [0, p_{k1}) \\ 1 - Q_i(c + \bar{z} + \Sigma^k(p) - p), p \in [p_{k1}, p_{k2}] \\ 1, p \in (p_{k2}, 1]. \end{cases} \tag{47}$$

The reservation payoff $\Sigma^k(p)$ weakly decreases in the investor's estimated value p because the probability density function $f(p)$ is log-concave. Thus, the investor's incentive compatibility is true due to Lemma 1.

The integrand on the left-hand side of Equation (46) is on the interval $(0, 1)$, meaning that the value of the corresponding integral is in $(0, p_{k2} - p_{k1})$. Furthermore, Lemma 1 demonstrates that $R(p) = 0$ for $p \in [0, p_{k1}]$, which further indicates that $R(p_{k2}) = p_{k2} - c - \bar{z}$ by combining Equation (8) with Equation (47). Therefore $R(p) = p - c - \bar{z}$ if $p \in [p_{k2}, 1]$. Moreover, the individual rationality is also true for $p \in (p_{k1}, p_{k2})$, since the corresponding expected probability of investment $\Xi(p)$ is in $(0, 1)$

$$R(p) > \max\{p - c - \bar{z}, 0\}, p \in (p_{k1}, p_{k2}). \tag{48}$$

The disclosure contract is optimal, following the scheme for left- and right-bounded individual rationality on intervals $[0, \hat{p}]$ and $(\hat{p}, 1]$, respectively. The remaining task is to determine $\hat{p}$. Note that $\hat{p}$ is defined to ensure that $F(\hat{p}) = \hat{k}$, and $\hat{p}$ is chosen to guarantee that the expected payoffs from following the contracts for $p \in [0, \hat{p})$ and $p \in [\hat{p}, 1]$ are the same for the firm. Following the same procedure as in the previous two cases, we can show that $\beta(p) = 1$ and $\alpha(p) = \bar{z}$ for investors with estimated values of $p \in [p_{k1}, p_{k2}]$. For investors of type $p \notin [p_{k1}, p_{k2}]$, the optimal disclosure functions are characterized by either $\beta(p) = 0$ or $\alpha(p) = 0$.

By the definition of p_{k1} and p_{k2} and the property of the cumulative distribution function, we have $F(p_{k1}) \leq \hat{k} \leq F(p_{k2})$ due to Lemma 2. In addition, as $F(\hat{p}) = \hat{k}$, the following inequality is true:

$$p_{k1} \leq \hat{p} \leq p_{k2}. \tag{49}$$

Moreover the corresponding reservation payoff induced by the disclosure contract is:

$$\Sigma^k(p) = \frac{k - F(p)}{f(p)} = \begin{cases} \dfrac{1 - \underline{F}(p)}{\underline{f}(p)}, p \in [0, \hat{p}) \\ \dfrac{-\bar{F}(p)}{\bar{f}(p)}, p \in [\hat{p}, 1]. \end{cases} \tag{50}$$

Since $R(p_{k1}) = 0$ and $R(p_{k2}) = p_{k2} - c - \bar{z}$ as calculated above, Equation (46) demonstrates that the expected payoff of a type $\hat{p}$ investors is the same no matter they chooses the optimal disclosure contract for $p \in [0, \hat{p})$ or for $p \in [\hat{p}, 1]$, as characterized by the following equation:

$$\int_{p_{k1}}^{\hat{p}} [1 - Q_i(c + \bar{z} + \Sigma^k(p) - p)]dp = p_{k2} - c - \bar{z} - \int_{\hat{p}}^{p_{k2}} [1 - Q_i(c + \bar{z} + \Sigma^k(p) - p)]dp. \tag{51}$$

where the right-hand side is the type $\hat{p}$ investor's expected payoff when choosing the optimal contract for $p \in [0, \hat{p})$, and the left-hand side is the expected payoff conditional on the optimal contract for $p \in [\hat{p}, 1]$.

Q.E.D

Proof of Proposition 2: The additional transfer is defined by Equation (28). Moreover, the "reservation payoff" under the contract indexed by k is $\Sigma^k(p)$ for $p \in [p_{k1}, p_{k2}]$. To ensure the investor is incentive compatible, Lemma 1 implies that the investor's truth-telling payoff satisfies:

$$R(p^{'}) = R(p) + \int_{p}^{p^{'}} \Xi(u)du, \tag{52}$$

where $\Xi(p)$ for $p \in [0, 1]$ is an investor's expected probability of investing with an estimated payoff of p. In particular, it equals the following expression under disclosure contract k.

$$\Xi(p) \begin{cases} 0, p < p_{k1} \\ 1 - Q_i(c + \bar{z} + \Sigma^k(p) - p), p \in [p_{k1}, p_{k2}]. \\ 1, p > p_{k2} \end{cases} \tag{53}$$

Consider the investor's incentive compatibility. If an investor with estimated payoff p reports $p^{'} \notin [p_{k1}, p_{k2}]$, then the payoff equals $\max\{p - c - \bar{z}, 0\}$, which is less than or equal to the investor's truth-telling payoff $R(p)$. This is because the individual rational condition is always satisfied under the fully informative case, as stated in Proposition 1. Thus, this investor has no incentive to deviate to misreporting $p^{'} \notin [p_{k1}, p_{k2}]$.

Now suppose that the investor with estimated payoff p misreports $p^{'} \in [p_{k1}, p_{k2}]$. Note that the truth-telling payoff at $p^{'}$ is equal to the following expression according to Lemma 1 and the definition of $\Xi(p)$:[14]

$$R(p^{'}) = R(p) + \int_{p}^{p^{'}} (1 - Q_i(c + \bar{z} + \Sigma^k(u) - u))du. \tag{54}$$

[14]We use the result that $\int_a^b = -\int_b^a$ throughout the proof.

Therefore, the misreporting means that the investor only invests when $p+u-c-\bar{z} \geq \Sigma^k(p^{'})$ and the corresponding deviation payoff is:

$$r(p,p^{'}) = \int_U \max\{p+u-c-\bar{z}-\Sigma^k(p^{'}).0\}dQ_i(u) - e(p^{'}). \tag{55}$$

Next, we subtract $r(p,p^{'})$ from $R(p^{'})$:

$$\begin{aligned} R(p^{'}) - r(p,p^{'}) &= \int_U \max\{p^{'}+u-c-\bar{z}-\Sigma^k(p).0\}dQ_i(u) + e(p^{'}) - \int_U \max\{p+u-c-\bar{z}-\Sigma^k(p^{'}).0\}dQ_i(u) + e(p^{'}) \\ &= \int_U \max\{p+u-c-\bar{z}-\Sigma^k(p).0\}dQ_i(u) - \int_U \max\{p+u-c-\bar{z}-\Sigma^k(p^{'}).0\}dQ_i(u). \end{aligned} \tag{56}$$

In addition, the Lebniz theorem indicates that:

$$\max\{p^{'}+u-c-\bar{z}-\Sigma^k(p).0\} - \max\{p+u-c-\bar{z}-\Sigma^k(p^{'}).0\} = \int_p^{p^{'}} 1_{x+u-c-\bar{z}-\Sigma^k(p^{'})}dx. \tag{57}$$

Moreover, by plugging this expression into Equation (56), we obtain:

$$\begin{aligned} R(p^{'}) - r(p,p^{'}) &= \int_U \int_p^{p^{'}} 1_{x+u-c-\bar{z}-\Sigma^k(p^{'})}dxdQ_i(u) \\ &= \int_p^{p^{'}} [1 - Q_i(c+\bar{z}+\Sigma^k(p^{'})-p)]dp. \end{aligned} \tag{58}$$

where the second equality is obtained by changing the order of integration. To ensure that the investor has no incentive to report $p^{'}$, we need the following inequality:

$$R(p) \geq r(p,p^{'}) \to R(p^{'}) - r(p,p^{'}) \geq R(p^{'}) - R(p). \tag{59}$$

This is equivalent to showing the following condition according toEquation (54) and Equation (58):

$$\int_p^{p^{'}} [1 - Q_i(c+\bar{z}+\Sigma^k(p)-p)]dp \geq \int_p^{p^{'}} [1 - Q_i(c+\bar{z}+\Sigma^k(p)-p)]dp. \tag{60}$$

Note that the "reservation payoff" decreases in p owing to our assumption of the log-concavity of the probability density of p. Equation (60) holds, and the claim of Proposition 2 is proved.

Q.E.D