Two Paradigms and Nobel Prizes in Economics: A Contradiction or Coexistence?*

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Abstract

Markowitz and Sharpe won the Nobel Prize in Economics for the development of Mean-Variance (M-V) analysis and the Capital Asset Pricing Model (CAPM). Kahneman won the Nobel Prize in Economics for the development of Prospect Theory. In deriving the CAPM, Sharpe, Lintner and Mossin assume expected utility (EU) maximization in the face of risk aversion. Kahneman and Tversky suggest Prospect Theory (PT) as an alternative paradigm to EU theory. They show that investors distort probabilities, make decisions based on change of wealth, exhibit loss aversion and maximize the expectation of an S-shaped value function, which contains a risk-seeking segment. Can these two apparently contradictory paradigms coexist? We show in this paper that although CPT (and PT) is in conflict to EUT, and violates some of the CAPM's underlying assumptions, the Security Market Line Theorem (SMLT) of the CAPM is intact in the CPT framework. Therefore, the CAPM is intact also in CPT framework.

Keywords: *asset pricing, cumulative prospect theory, capital asset pricing model, equilibrium.*

JEL Classification: *C62, D51, D52, G11, G12.*

1. Introduction

The Mean-Variance (M-V) analysis and the Capital Asset Pricing Model (CAPM) awarded Markowitz and Sharpe the Nobel Prize in Economics more than two decades ago. Kahneman won the Nobel Prize in Economics in 2002 for the development of Prospect Theory. By Prospect Theory, the revealed "actual person" is substantially different from the assumed "economic rational person" as actual investors' behaviour documented in experimental studies is much different from what is assumed in most economic models. Thus, Prospect Theory contradicts the expected utility theory in general and the classical assumptions of the CAPM in particular. However, Prospect Theory, does not provide any equilibrium pricing model which can substitute the existing expected utility model and in particular the CAPM. Accepting Prospect Theory as the correct description of investors' behaviour, can one save the CAPM? Can these two paradigms coexist? To this issue we address this article.

The Sharpe-Lintner-Mossin CAPM is derived by assuming that investors are risk-averse, that they maximize expected utility of total wealth, and that the returns are normally distributed with homogeneous expectations regarding these distributions.[1] Experimental studies cast doubt on the foundations of the CAPM. Based on the repeated experimental findings, Prospect Theory (PT) (see Kahneman and Tversky [1979]) and Cumulative Prospect Theory (CPT) (see Tversky and Kahneman [1992]), were developed as an alternative paradigm to expected utility paradigm. On the one hand, PT and CPT have become a cornerstone in economic research and are the foundation of behavioural finance and behavioural economics. Indeed, the Nobel Prize committee who awarded the prize in economics to Kahneman in 2002 has recognized this. On the other hand,

[1]The normality assumption can be relaxed by adding the assumption of quadratic utility functions. Because the quadratic utility has two severe drawbacks ($U'<0$ from some critical value, and increasing absolute risk aversion) researchers generally are not willing to assume this utility function. There are other justifications of the CAPM. Merton [1973] assumes continuous portfolio revisions, which leads to end of period lognormal distributions of returns and to an instantaneous CAPM. Levy [1973, 1977] assumes a discrete model of portfolio revisions with a lognormal distribution. Other cases under which the CAPM holds are discussed by Levy and Samuelson [1992]. Berk [1997] provides the general restrictions on all economic primitives that yield the CAPM. The CAPM can be obtained also as a special case of the Arbitrage Pricing Theory (APT), see Ross [1976]. In this paper we use the classical Sharpe-Lintner-Mossin CAPM assumptions, i.e., normal distribution is assumed.

the CAPM is still the most popular asset pricing model. Moreover, most popular textbooks nowadays devote one or two chapters to behavioural economics and the same textbooks devote a relatively large portion of the books to the M-V analysis and to the CAPM, which are apparently in contradiction to the behavioural economics approach. In addition, editors of journals accept for publication many papers, which rely on the CAPM, knowing that CPT reveal evidence that the assumptions corresponding to the rational investor's behaviour which underline this model are wrong[2] .Thus, it is of crucial importance to study whether these two models can coexist.

Levy [2010 and forthcoming] analyzes in detail some theoretical paradoxes arise with the employment of the variance as a measure of risk and suggests the changes needed in the M-V model to avoid such paradoxes. Moreover, he discuses the various empirical criticisms of the CAPM as well as the argument against these criticisms in defence of the CAPM. In this study we focus on the Expected Utility and Prospect Theory paradigms and discuss the implication of the contradiction between these two paradigms on the CAPM.

Let us begin by highlighting the following differences of PT to EUT. PT asserts that probabilities are distorted. This violates two assumptions of the CAPM: first, the normality assumption is violated, and secondly, as each investor has his/her subjective probability distortion, investors face heterogeneous probability distributions of returns, even if before the distortion they all face the same normal return distributions. Thus, the normality and the homogeneous expectation CAPM assumptions are violated. PT asserts that investors make decisions based on change of wealth, which violates EUT asserting that decision-making should be based on total wealth, rather than *change* of wealth. Moreover, PT claims that investors are loss averse, i.e. they are hurt by losses 2.25 times more than they derive utility from gains (see Kahneman and Tversky [1992]). Finally, PT assumes risk seeking in some range of returns, which contradicts the CAPM's risk aversion assumption.

[2] For more details on the experimental, empirical and theoretical criticisms of the CAPM , as well as on the defense of this model, see Levy [2010] and Levy [Forthcoming].

The purpose of this study is to re-examine the CAPM in light of the experimental evidence, which refutes expected utility theory as well as the CAPM.[3] To be more specific, we assume that PT and, alternatively, CPT, are intact, and examine the validity of the CAPM within each of these two frameworks.[4]

We show in this paper that the security market line theorem (SMLT) of the Sharpe-Lintner-Mossin *homogeneous* expectation CAPM is intact even in the CPT framework. Hence, as in the standard case of EUT, also with CPT the valuation of assets is given by a linear relation of their excess returns proportional to the excess return of the market portfolio. Like in the CAPM the proportionality factor, the beta, is as usual given by the covariance of the assets and the market portfolios returns divided by the variance of the market portfolio's return. This is a surprising result, in particular because with CPT the distributions of returns are subjectively distorted; hence investors face *heterogeneous* expectations of returns. While the equilibrium asset pricing under CPT may be deferent from the equilibrium pricing under EUT, in both cases a separation theorem is intact leading to the linear relationship between beta and expected return.

Our reasoning goes as follows: The SMLT is derived from Two Fund Separation, which in turn holds if investor's decisions can be described by the mean-variance-principle (MVP). We say that the MVP holds if investor decisions are solely based on the mean and variances of the portfolios and if the utility of the investor is increasing in mean. With normally distributed returns, the MVP is equivalent to first-order-stochastic dominance (FSD). Thus, our claims are made if we can show that PT contradicts FSD while CPT is consistent with FSD. As the choice by FSD is optimal also in CPT framework, the M-V and CPT are not in contradiction.

[3]This study is devoted to the CAPM. However, all results corresponding to the CAPM are intact also for the General CAPM (GCAPM) – known also as the segmented market equilibrium model, in which investors do not hold all available risky assets (see Levy [1978], Merton [1987], Markowitz [1990] and Sharpe [1991]). Thus, in the rest of the paper we focus on the CAPM, recalling that all the proofs are intact also for the GCAPM.

[4]Barberis, Huang and Santos [2001] employ some, but not all, of the components of PT to determine asset pricing, with two assets, one risky and one riskless. In their study, the authors investigate asset pricing when investors care more about fluctuations in the value of their assets than is justified by a concern for consumption alone. While they do not analyze directly the one-period CAPM, they add an important dimension to the investment decision-making procedure by analyzing the dynamics of the investment process. A key feature in their analysis is that risk aversion changes over time and depends on the prior investment performance. In their model, the high volatility of returns generates large equity premiums. As in PT, also in their model the investor is much more sensitive to reduction in wealth than to increases, i.e., loss aversion prevails.

The structure of this paper is as follows: Section I provides a brief review of PT, CPT and the CAPM assumptions. Section II contrasts PT and EUT theory and explain why the CAPM collapses if PT is the correct framework of investors' behaviour. In addition, we show that if the modified version of PT, i.e., the CPT is adopted, then no contradiction exists between the CAPM and CPT. Thus, we demonstrate that if investors behave as suggested by CPT, the SMLT is intact even though CPT contradicts EUT. Concluding remarks are given in Section III.

2. The Two Competing Paradigms

The CAPM, developed by Sharpe [1964], Lintner [1965] and Mossin [1966] is no doubt one of the most influential contributions to modern finance. Yet, this model is controversial and has been criticized on theoretical as well as on empirical grounds. Despite the theoretical and empirical criticism, which will be discussed below, the Sharpe-Lintner-Mossin CAPM is still the most common risk-return equilibrium model and no other simple equilibrium model has yet been proposed in the literature as a challenge to the CAPM. In particular, PT and CPT, which raise objection directly to the EUT and indirectly to the CAPM, do not suggest an asset-pricing model to substitute for the CAPM.

Many empirical studies criticize the CAPM. The most comprehensive empirical study refuting the CAPM is probably the one conducted by Fama and French [1992]. Nevertheless, the CAPM also has some empirical and experimental supports (for example, see Fama and MacBeth [1973], Miller and Scholes [1972], Amihud, Christensen and Mandelson [1992], Jagannathan and Wang [1996], and Levy [1997]).[5] Moreover, recent studies have shown that the CAPM can not be empirically rejected when *ex-ante* rather than *ex-post* parameters are employed (see Levy and Roll [2010] and Levy [Forthcomimg]).

The CAPM is derived in the von Neumann-Morgenstern expected utility framework, and because EUT is experimentally criticized, the CAPM is indirectly also criticized. Let us elaborate.

[5]For the difficulties of testing the CAPM with *ex-post* data, see Roll [1977]. Yet, Levy [1997] experimentally tested the CAPM with *ex-ante* parameters, which is not exposed to Roll's criticism. With *ex-ante* parameters, Levy [1997] finds strong support for both the CAPM and the GCAPM.

The von Neumann-Morgenstern expected utility framework (as well as most other economic models) assumes that investors are rational, and that they maximize expected utility. However, not all agree with these "rational investor" assumptions. Allais [1953] presented the most well known paradox of expected utility maximization. Since the early fifties, psychologists have conducted experiments revealing evidence that individuals behave in a way, which contradicts the NM's expected utility. In particular, in making choices between alternative uncertain prospects, individuals tend to distort the objective probabilities in a systematic manner, which may lead to the choice of an inferior investment and to wealth destruction. In a very influential article, Kahneman and Tversky [1979] challenged the expected utility paradigm by suggesting Prospect Theory (PT) as an alternative descriptive paradigm. PT is based on experimental findings regarding subjects' behaviour and strictly contradicts the von Neumann-Morgenstern's expected utility. Although PT has several components, the following main elements as appear in Kahneman and Tversky's [1979] paper are:

a) In making choices with uncertainty, investors employ subjective decision weights, $\omega(p)$, rather than the objective probabilities, p.

b) Investors base their decisions on *change* of wealth, x, rather than on *total* wealth w+x. Thus, they maximize the expectation of a *value* function V(x) rather than of a utility function U(w+x).

c) The value function is S-shaped: $V' > 0$, for all $x \neq 0$, $V'' > 0$, for $x < 0$, and $V'' < 0$ for $x > 0$, where x is the change in wealth. Moreover, the value function exhibits *loss aversion*, i.e., at x=0 the derivative from the left is 2.25 bigger than the derivative from the right.[6].

The shape of the value function may change with wealth. Yet, the property of risk seeking for $x < 0$, and risk aversion for $x > 0$ holds for any initial wealth level.

[6]For some evidence regarding the investors' behavior in practice, in light of the S-shaped function, see Shefrin and Stateman [1985] and Odean [1998]. Benartzi and Thaler [1995] analyze the role of loss aversion on pricing of stocks and bonds and, in particular, on the risk premium which is too high to be explained with risk aversion alone; see also De Giorgi and Post [Forthcoming]. Their solution to the *equity-premium puzzle* is that people consider annual returns on bonds and stocks, and weight possible losses 2.5 times more heavily than possible gains of the same magnitude. However, recent experimental studies reveal a strong rejection of the S-Shape value function suggested by PT (see Levy and Levy 2002a, 2002b).

In pursuing PT, various researchers, including Kahneman and Tversky themselves, realize that a decision model where weights ω(p) rather than probabilities, p, are employed has three drawbacks:

1) It may contradict First-Degree Stochastic Dominance (FSD), i.e. the monotonicity axiom. This is unacceptable as it implies that one likes less rather than more money.
2) The sum of the subjective probabilities, ω(p), may add up to more or less than 1.Hence, one cannot interpret the decision weights as subjective probabilities.
3) The decision weights, ω(p), technically cannot be applied to continuous distributions.

To overcome these drawbacks, Quiggin [1982], Yaari [1987], Allais [1988], and Tversky and Kahneman [1992] themselves, suggest that the subjects conduct a transformation of the cumulative distribution, rather than a transformation of the probabilities.[7] Tversky and Kahneman [1992] suggest the Cumulative Prospect Theory (CPT) as a modification to PT, where the cumulative distribution functions are distorted. The other two components of PT mentioned above (basing decisions on change in wealth and the S-shaped value function) remain also in CPT.

In the next section, we show that PT and CPT contradict EUT. Thus, if one accept PT it casts doubt on the validity of virtually all the economic and finance models, which rely on expected utility theory. In particular, it questions the validity of the CAPM which is a model developed in the EUT framework. However, despite this contradiction, we show that the SMLT of the CAPM is surprisingly valid under CPT. This is an important result as CPT is the modified version of PT suggested by Tversky and Kahneman [1992].

3. Contrasting PT (and CPT) with the CAPM

We analyze in this section the effect of each of the main components of PT and CPT discussed above on the CAPM, and then analyze the combined effect of all three components of CPT on the equilibrium risk-return relationship. Let us first demonstrate that, with PT's decision weights, the

[7]See also Handa [1977].

monotonicity axiom may be violated and therefore the CAPM does not hold. Hence, PT may contradict the CAPM. Indeed, the monotonicity violation by PT motivates the introduction of cumulative probability distortions, which characterize the CPT.

3.1 *PT and First Degree Stochastic Dominance (FSD)*

In the CAPM framework, all investors are assumed to have homogeneous expectations. As a result, in this framework, all risk averse investors, regardless of preferences, will mix the market portfolio m with the riskless asset. This result is well known as the Separation Theorem (see Figure 1). If investors employ subjective decision weights $\omega(p)$ rather than the objective probabilities p, it is possible that interior portfolios such as m_1 or m_2 will be selected (see Figure 1). Moreover, it will no longer be true that all investors select the same portfolio; hence, the separation theorem and the CAPM no longer hold. To see this, let us refer, once again, to Figure 1. Portfolio m is the market portfolio and under the CAPM, all investors hold some combinations of m and r (the separation property of the CAPM). However, with decision weights it is possible that portfolio m_1 or even portfolio m_2 will provide a higher expected utility than portfolio m. For example, suppose that the decision weights of the kth investor, $\omega_k(p)$, are defined so that portfolios m_1 and m_2 are subjectively shifted to the subjective points, say, $m_{1(s)}$ and $m_{2(s)}$ where the s-subscript indicates that subjective probabilities are employed . Furthermore, for simplicity only assume that the mean and variance of portfolio m remain unchanged. Then, each investor will have his/her best subjective portfolio, $m_{i(s)}$ (where m_i can be an interior portfolio before the probability distortion, see Figure 1), the Separation Theorem will not hold and hence the classic Sharpe-Lintner-Mossin CAPM will collapse.[8]

[Figure 1 here]

[8]In PT it remains unspecified whether the investors first mix the portfolio of risky assets with the riskless asset and only then distort the probabilities, or distort the probabilities of risky assets portfolio first and then mix the subjective portfolio with the riskless asset. In both cases, an interior portfolio may be selected as discussed in the text.

As mentioned in the introduction, using decision weights $\omega_k(p)$ for investor k has several drawbacks: the sum of the subjective probabilities may be less or more than 1, First Degree Stochastic Dominance (FSD) may be violated (see Fishburn [1982], Yaari [1987]), and decision weights cannot be employed in the continuous case. While the first and the third drawbacks are trivial, the second one can be illustrated with a simple example composed of two monetary values $y_1 < y_2$. With these two monetary values and with decision weights, it is possible that the mix $U(\omega(p)y_1 + \omega(1-p)\, y_2) > U(y_2)$ despite the fact that $y_2 > y_1$, and U is monotonic; hence employing decision weights may contradict the monotonicity axiom and FSD. For example, choose p = ½ and $\omega(½)$, = ¾, y_1 = \$50 and y_2 = \$100. Then, the investor may prefer the bet [(\$50, ½), (\$100, ½)] (subjectively perceived as [(\$50, ¾), (\$100, ¾)]) to \$100 with certainty, which is an unacceptable result. Thus, though y_2 dominates the distribution [(y_1, p), (y_2, 1−p)] by FSD, its subjective expected utility may be lower when PT's decision weights are employed. In terms of Figure 1, this means that investors may prefer portfolio A to portfolio B despite the fact that portfolio B dominates portfolio A by FSD. Note that the FSD dominance of portfolio B over portfolio A (see Figure 1) stems from the normality assumption. To see the relationship between M-V and FSD, recall that the cumulative distribution of portfolio A is located to the left of the cumulative distribution of portfolio B, because the density distributions of the two portfolios are identical, except for the fact that B is shifted to the right (recall that A and B are both normally distributed and have the same variance). Thus, portfolio B dominates portfolio A by FSD. If FSD is not kept, due to the probability weights transformation, investors may choose portfolio A; i.e., they will not hold a portfolio located on the objective efficient frontier. This, of course, violates the CAPM (see Sharpe [1964] and Roll [1977]) derived from EUT and normally distributed returns.[9&10]

[9] If the CAPM is not based on EUT then FSD need not hold. See Hens and Pilgrim [2003, chapter 7] for necessary conditions to guarantee FSD in this general case.

[10]The fact that the selected portfolio is not on the M-V frontier indicates that the CAPM does not hold (Sharpe [1964] and Roll [1977]). However, one may be tempted to believe that in such a case the segmented market equilibrium (the GCAPM) holds (See Levy [1978], Merton [1987], Markowitz [1990] and Sharpe [1991]). But this is an incorrect conclusion because the selected portfolio by the kth investor may be dominated by FSD by another portfolio also in the GCAPM framework; hence such a selection with decision weights contradicts NM's expected utility theory. In other words, the investor may select a

Noting that PT with decision weights may contradict the monotonicity axiom, it was suggested that the subjective probability distortion should be expressed as a transformation of the *cumulative probability function* F rather than a transformation of the raw probability p; hence the name Cumulative Prospect Theory (CPT) suggested by Tversky and Kahneman [1992]. To be more specific, according to CPT, investors make decisions based on $T_k(F)$ rather than on F, where F is the objective cumulative distribution, and T is some (non-decreasing) monotonic transformation. When returns are mixed this transformed function is not necessarily a cumulative probability function as the sum of the individual decision weights can be more or less than 1. With only positive returns or only negative returns the transformed function has the properties of cumulative probability function. For simplicity of the presentation and without loss of generality, in the rest of the paper we treat the transformed function as cumulative probability function. However, all the results are intact also for mixed returns (when the transformed function may be not a cumulative probability function) because all what we need for the various proofs is that if one portfolio dominates the other by FSD, the dominance is intact also after the CPT transformation takes place. Indeed, CPT probability transformation does not violate FSD also when returns are mixed.

The CPT decision weights method overcomes the deficiencies of PT's decision weights: the total probability is always 1 by construction, the transformed distribution is still a probability distribution function. The most important feature of the suggested decision weight model is that there is no contradiction to FSD, because if $F(x) \le G(x)$ for all values x, and T_k is a monotonic transformation (i.e., $T_k'(\cdot) \ge 0$), then $T_k(F(x)) \le T_k(G(x))$ for all x and all T_k (see Lemma below). Thus, if one prospect dominates the other by FSD with objective probabilities, all investors will accept this dominance even if they subjectively distort the cumulative probability function.

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portfolio located inside the CAPM or GCAPM efficient frontiers. Employing a direct utility maximization rather than selecting a portfolio by the mean-variance rule (see Levy & Markowitz [1979], and Markowitz [1991], is affected by decision weights in a similar way: an interior portfolio may be selected.

Finally, the transformation of the cumulative distribution can be employed with discrete and continuous random variables alike. [11]

The rest of the paper focuses on cumulative probability distribution distortion as suggested by CPT .Because the other components (change in wealth and value function) are common to both PT and CPT, we refer in the rest of the paper only to CPT. Let us turn now to the factor, which constitutes the main conflicting factor between CPT and EUT: by CPT, investors make decisions based on *change* of wealth while by EUT, decisions should be based on *total* wealth, i.e., the initial wealth plus the change of wealth.

3.2 *Change of Wealth, x, Versus Total Wealth, w+x*

In this section, we focus only on one component of CPT, namely the claim that investors make decisions based on change of wealth, x, rather than on total wealth, w+x. We show that by ignoring the initial wealth equilibrium prices are generally affected; yet the linear CAPM relationship (with different parameters) is intact.

It is easy to construct an example revealing that a maximization of expected utility of changes in wealth does not lead to the same choice as the maximization of the expected utility of total wealth.[12] Hence, by itself this component of CPT is sufficient to induce a contradiction between EUT and CPT. However, despite the fact that decision by EU(x) and EU(w+x) may lead to different choices, we will show that the separation theorem is intact and, therefore, the CAPM holds even when decisions are based on change in wealth. This claim seems to be paradoxical at first glance, but it is not. To see this, recall that by the separation theorem, all portfolios, which are located on the capital market line (CML), constitute the M-V efficient set. Each investor selects his/her optimum portfolio from the efficient set. The various choices from the efficient set do not affect the separation theorem and the CAPM. The only crucial factor for the CAPM derivation is that all investors choose from the M-V efficient set and, by making the investment based on

[11]For the effect of various transformations on the efficient set, derived under various assumptions regarding preference, see Levy and Weiner [1998].

[12]Take x= \$10 or x = \$1,000 with an equal probability and y = \$300 with certainty. Assume an initial wealth of \$9,000. For a square root utility function we have $EU(x) \cong 15.81 < EU(y) \cong 17.32$ and $EU(w+x) \cong 97.43 > EU(w+y) \cong 96.44$

change of wealth rather than total wealth, the efficient set does not change. To show this point, simply note that the following trivial conditions hold:

$$E(w+x) \geq E(w+y) \quad \Leftrightarrow \quad E(x) \geq E(y)$$

$$\sigma^2 (w+x) \leq \sigma^2(w+y) \Leftrightarrow \quad \sigma^2(x) \leq \sigma^2(y)$$

where w, the initial wealth is a constant.

Similarly the FSD efficient set is independent of the initial wealth because

$$F(w+x) \leq G(w+x) \Leftrightarrow F(x) \leq G(x).$$

Graphically, by adding w, the two cumulative distributions under consideration are simply shifted to the right by a constant.[13]

Figure 2 illustrates the effect of the initial wealth, w, on the M-V efficient set, on the separation theorem and on the optimal portfolio of assets. Line rr' contains the portfolio compositions (for $1 investment, or when the expected values and standard deviations are measured in percent) of all the efficient M-V portfolios. By the argument above it is clear that rr' is unaffected by whether w is included or omitted from the decision.[14] Therefore, without loss of generality, in deriving the efficient set, we can assume that all investors invest $1, and x measures the rate of return on this one dollar, i.e. the change in wealth per one dollar of investment.[15]

[Figure 2 here]

It is important to emphasize that making portfolio investment decisions based on EU(x) rather than EU(w+x) *does* affect equilibrium prices of risky assets. Take the extreme case where all investors have preferences U(w+x) and U(x) as illustrated in Figure 2. For given fundamentals,

[13]This assertion is intact also for second and third degree stochastic dominance (SSD and TSD, respectively) as well as for prospect stochastic dominance (PSD). In the last case, the proof is less trivial but as we do not explicitly need it for this paper, we do not give the proof here. We refer the interested reader to Levy and Wiener [1998], and Levy [1998].

[14] Levy and Levy [2004] analyze the impact of decision weights on the efficient frontier and on FSD violation in the absence of the riskless asset. In this paper we add the riskless asset which allow us to examine the CAPM validity in CPT framework.

[15]Note that if we measure the portfolio expected return and the standard deviation in dollars rather than in percents (as required by EUT), the line rr' will change as a function of the initial wealth w. However, the portfolio compositions described by line rr' are unaffected by the initial wealth, which allows all standard mean-variance analysis to be conducted in percent rather than the dollar terms.

i.e., future distributions $\tilde{V}_{i1}$ (where $\tilde{V}_{i1}$ stands for the future value of the ith firm), with U(x) rather than U(w+x) the demand for risky assets will be lower and the equilibrium prices V_{i0} may be lower. Therefore, the mean rate of return on the ith asset μ_i may be higher. Thus, we may have a different Security Market Line (SML) with U(x) and U(w+x), but still we get the same general $\mu - \beta$ *linear* relationship as advocated by the CAPM. In other words, the parameters of the SML change but the linearity is intact. Thus, in the EUT framework one cannot ignore initial wealth, w. As we are concerned with the CAPM (and not EUT), in the rest of the paper we can safely ignore the initial wealth, w, i.e., switch from w+x to x.

3.3 *Transformation of the cumulative probability distribution*

In analyzing the effect of the transformation T(·) on the equilibrium risk-return relationship, we assume, as in the CAPM, that investors face the riskless asset, the distributions of all individual assets, and the distributions of all unlevered as well as levered available portfolios and the distribution of mutual funds, and, in particular, index funds, which mimic the market portfolio.[16] Considering all these portfolios, the investors first distort all these available probability distributions, as suggested by CPT, and then make a choice from the distorted distributions. Thus, portfolios that include risky and the riskless asset, i.e., portfolios located on the Capital Market Line (CML) are also distorted.

Figure 3 demonstrates the mean-variance efficient frontier (before the probability distortion), the capital market line rr', and the curve of all efficient distributions after the probability distortion, denoted by rr'_1. The CAPM efficient frontier (before the returns are distorted) is given by curve AmA_1 with portfolio m as the tangential portfolio. T_1 is a hypothetical subjective efficient frontier where a transformation T_1 (·) was conducted, corresponding to investor k = 1. The same results obviously hold for all transformations $T_k(\cdot)$ as long as $T'_k(\cdot) \geq 0$.

[16]However, the existence of the riskless asset is not crucial because if it does not exist the zero-beta equilibrium of Black [1972] rather than Sharpe-Lintner CAPM is intact. Investors do not have to directly construct their portfolios from the thousands of stocks available but rather look at the distribution of returns of mutual funds (and, in particular, index funds) and then distort these distributions.

We have no knowledge regarding the shape of the distorted distributions frontier in the mean-variance space: this subjective frontier can be above, below or even intersect line rr' depending on the particular transformation $T_1(\cdot)$. In addition, the shape of T_1 depends on the specific selected transformation. Moreover, note that distorted distributions are not normal anymore; hence, one cannot employ the mean-variance rule to select the tangential portfolio.

[Figure 3 here]

Although each investor has his/her subjective efficient frontier, *a priori*, the curve rr'_1 may include inefficient portfolios with objective probabilities like portfolios A', d, etc. (see Figure 3). We will show below that this is not the case and that rr'_1 (the subjective efficient frontier) is composed solely of combinations of portfolio m and the riskless asset. Without a probability distortion, for any asset below line rr' there is a portfolio (mutual fund) on line rr' which dominates it by FSD (see Figure 3). After the distortion, portfolio a is shifted to a', b to b', c to c', etc. Note, that a' does not have to be vertically above a, because the parameters are distorted (the same holds for c and c', b and b' and d and d'). We should ask the question, whether there are portfolios which are located below rr' on the rr'_1 efficient frontier and whether for each portfolio d located below line rr', there is a portfolio d'' on the rr'_1 efficient frontier, which is composed solely of m and r and dominates it by FSD. We prove in the Theorem below that these two questions can be positively answered. Thus, it follows that only portfolios located on rr_1 are contained in the rr'_1 efficient portfolio set, hence, the SMLT is intact![17] This is a very strong result, because the distorted distributions, $T_k(\cdot)$, are not normal even though the objective distributions (before the transformations have been conducted) are assumed to be normal. Moreover, each investor has his/her subjective transformation $T_k(\cdot)$; hence investors face different subjective efficient frontiers. We point out the following Lemma:

[17] Similarly, the GCAPM (see Levy [1978], Merton [1987], Markowitz [1990] and Sharpe [1991]) follows in cases where portfolios composed of a relatively small number of assets are held and the transformations $T(\cdot)$ are conducted on these portfolios.

Lemma: *Let F and G be the cumulative distributions of two distinct prospects. Denote by* $\mathbf{U}_1$*, the set of all non-decreasing utility functions, and by* $\mathbf{V}$ *the set of all S-shaped utility (value) functions suggested by PT and CPT. Then:*

a) *FSD:* $F(x) \le G(x)$ *for all* $x \Leftrightarrow E_F\, U(x) \ge E_G U(x)$ *for all* $U \in \mathbf{U}_1$.

b) *PSD (Prospect Stochastic Dominance):*

$\int_y^x [G(t) - F(t)]\, dt \ge 0$ *for all* $y<0$ *and* $x>0 \Leftrightarrow E_F\, V(x) \ge E_G V(x)$ *for all* $V \in \mathbf{V}$.

c) $F(x) \le G(x)$ *for all* $x \Leftrightarrow T_k(F(x)) \le T_k(G(x))$ *for all x and all transformations,* $T_k(\cdot)$*, as long as* $T'_k(\cdot) \ge 0$.

d) *Suppose that x and y are normally distributed with means and variances* (μ_x, σ_x^2) and (μ_y, σ_y^2), *respectively. If* $\mu_x > \mu_y$ *and* $\sigma_x = \sigma_y$*, then x dominates y by FSD. Namely,* $EU(x) \ge EU(y)$ *for all utility functions with* $U' \ge 0$*. It follows from b) above that in this case x dominates y also by PSD, because obviously FSD* $\Rightarrow$ *PSD.*

Proof: For the proof of (a), see Hanoch and Levy [1969], Hadar and Russell [1969], and Levy (1992). For the proof of (b) and (c) see Levy and Weiner [1998] and Levy [2006]. For proof of (d) see Hanoch and Levy [1969].

Using the above Lemma we are able to prove our main result, which asserts that the separation theorem and the SMLT hold, even with the transformation $T_k(\cdot)$ which varies across investors and where $T_k(F(x))$ and $T_k(G(x))$ are obviously not normal distributions anymore.

Theorem: *Suppose that before the transformations* $T_k(\cdot)$ *are conducted, the rates of return are normally distributed. When the riskless asset exists, then for any mix of a portfolio of risky assets with the riskless asset, there is a mix of the market portfolio m (see Figure 1), with the riskless asset which dominates it by FSD. This statement is valid also after the transformation* $T_k(\cdot)$ *is conducted and the normality is violated, as long as* $T'_k(\cdot) > 0$*. Hence, all investors*

maximizing a subjective expected utility, as for example in CPT, hold the mix of portfolio m and the riskless asset, implying that the separation theorem and the SMLT are intact.[18]

Proof: Investors can mix any portfolio, efficient or inefficient, mutual funds and individual assets with the riskless asset. Suppose that after the probability distortion an investor selects to mix portfolio m_2 with the riskless asset, e.g., selecting point A (see Figure 1). This could not be an optimal investment policy. In fact, recall first that before the distortion takes place, there is a portfolio B composed of m and r, which dominates portfolio A by the M-V rule. However, because A and B are normally distributed and by construction both have the same variance and B has a higher mean, we can use the Lemma d), to conclude that portfolio B dominates portfolio A by FSD. (The same argument holds for lognormal distribution, see footnote 18).

In the CPT framework, an investor makes investment decisions based on $F_A^*(x) \equiv T_k(F_A(x))$ and $F_B^*(x) \equiv T_k(F_B(x))$ rather than on $F_A(x)$ and $F_B(x)$. Because these transformed distributions, $F_A^*(x)$ and $F_B^*(x)$ may not be normal anymore, one cannot employ the M-V rule for investment decision-making. However, by the Lemma c), $F_B(x) \leq F_A(x) \Leftrightarrow T_k(F_B(x)) \leq T_k(F_A(x))$; hence we conclude that portfolio B dominates portfolio A by FSD also after the probability distortion, for any transformation T_k as long as that $T_k'(\cdot) > 0$.

The same procedure can be employed to prove that any portfolio below line rr' is dominated by some portfolio located on line rr' with and without the probability distortion. Moreover, the dominance is by FSD, hence, all expected utility maximizers will choose to mix portfolio m with r (see Figure 3).

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[18] The same claim holds also if F and G are lognormally distributed. The reason is that $\sigma_F = \sigma_G$ and $E_F > E_G$ are also necessary and sufficient conditions for FSD dominance of F over G for lognormal distributions. (see Levy [1973] [1991]). Therefore, also in the lognormal case, dominance by M-V rule implies dominance by FSD, which is intact also after the probability distortion takes place. The advantage of the lognormality assumption is that the returns are bounded from below; i.e., $R \geq 0$. The disadvantage of the lognormality assumption is that a distribution of a mix of two lognormal random variables is distributed only approximately but not precisely as lognormal distribution (see Lintner [1972]).

Yet, by the Theorem *all* investors will diversify between m and r, because for any other combination (say of m_2 and r, see Figure 1) there is at least one combination of m and r which dominates it by FSD, before as well as after the transformation $T_k(\cdot)$ is employed. Because FSD corresponds to all $U \in \mathbf{U}_1$, it allows us to obtain the separation theorem for all investors regardless of their preference, despite the fact that the normality (or lognormality, see footnote 18) is violated as a result of the transformation $T_k(\cdot)$. Therefore, the SMLT with homogeneous expectation is intact despite the fact that normality is violated and the transformation $T_k(\cdot)$ varies across investors Namely, the *homogeneous* expectation Sharpe and Lintner's SMLT is valid even with *heterogeneous* (subjective) expectations.[19] Finally, note that investors do not have to directly mix portfolios of risky assets with the riskless asset because a mutual fund (index funds) with assets represented by point B may be purchased.[20]

In the derivation of the SMLT, risk-aversion is assumed to avoid infinite borrowing. The S-shape PT value function has a risk-seeking segment in the negative domain. In the next section,

[19]The Sharpe-Lintner-Mossin CAPM holds also under quadratic utility function, or for concave utility functions with a quadratic approximation (see Levy & Markowitz [1979]). However, probability distortion with quadratic preferences affects the CAPM. As the normality assumption is relaxed, portfolio B dominates portfolio A by the M-V rule but not necessarily by FSD (see Figure 1). Therefore, Portfolio A may dominate Portfolio B by the M-V rule after the transformation is done, hence the Separation Theorem does not hold and the CAPM collapses. However, it can be shown that in such a case the following modified CAPM is intact

$$\frac{\sum_k W_k \mu_{ik}}{\sum W_k} = r + \frac{\sum_k W_k (\mu_k - r)\beta_{ik}}{\sum W_k}$$

where W_k is the wealth of the kth investor, μ_k is the mean return of the portfolio selected by the kth investor, β_{ik} is the beta of the ith asset calculated with the portfolio selected by the kth investor and μ_{ik} is the mean of the ith asset after the distortion of probability takes place, hence the index k. Thus, apart from W_k and r all figures are affected by the probability distortion. This equation is similar to Levy's GCAPM equilibrium [1978]. Note that the equilibrium mean is affected by the probability distortion as well as by the wealth of each investor. Also note that if probabilities are not distorted $\mu_k = \mu_m, \beta_{ik} = \beta_i$ and this equation is reduced to the security market line of the CAPM.

[20]Another possibility is that investors first distort all possible risky portfolios and then mix the distorted portfolio with the riskless asset. The SMLT is intact also in this case. Moreover, suppose even that the riskless asset does not exist. In such a case, all investors will choose a risky portfolio from segment MM' (see Figure 1), because for any other portfolio located below MM' there is a portfolio located on this segment with the same variance and a higher mean. This guarantees not only M-V dominance but also FSD dominance (recall the normality assumption). Thus, each investor selects a portfolio from the efficient frontier MM' (though not all will select the same portfolio), a case when the Sharpe-Lintner CAPM does not hold but the zero-beta equilibrium model of Black (1972) is intact.

we show that the S-shape value function is consistent with the SMLT, despite its risk-seeking segment.

3.4 *The S-shaped Value Function, V(x)*

In this section we assume no distortion in probabilities and focus on the S-shaped value function advocated by CPT. We analyze the role that the risk aversion plays in the CAPM derivation and show that the SMLT is also intact when S-shaped value functions are assumed. To show this claim, first note that FSD can be stated in terms of w+x or x and that in the SMLT derivation the initial wealth can be ignored. To prove that the SMLT holds also for all S-shaped V(x) value functions we use Figure 4 and the previous results. We have shown before that because of the normality assumption, for any portfolio like portfolio Q, there is a portfolio Q' (see Figure 4) which dominates it by FSD. However, as the set of all S-shaped functions is a subset of all non-decreasing utility functions, it is obvious that FSD $\Rightarrow$ PSD (but not vice-versa). Therefore, portfolio Q' dominates portfolio Q also by PSD, i.e., for all S-shaped value functions. Thus, the separation theorem and the CAPM hold also with all V(x) functions (see Figure 4).

[Figure 4 here]

Because we employ in the above proof FSD, i.e., a decision rule corresponding to preferences $U \in \mathbf{U}_1$, one may be tempted to believe that in the derivation of the asset equilibrium prices (CAPM) the risk-seeking preference in the whole range is also allowed. While this is true for achieving the separation property, it is not true for the CAPM to have equilibrium. To see this, consider a risk seeking investor, e.g., $U(x) = e^x$, hence $U \in \mathbf{U}_1$. This investor still prefers portfolio Q' to Q. However, as he/she is a risk-seeker, increasing leverage (moving along rr' in the direction of the arrows (in Figure 4) increases expected utility ($U_3 > U_2 > U_1$), because both expected return and variance increases simultaneously, and both are desired by a risk-seeker investor (recall that normality is assumed). Thus, if unlimited borrowing is allowed (which is not the case in practice),

it is sufficient that there will be one risk-seeking investor whose demand for portfolio m will be infinite (financed with an infinite borrowing) in order to induce infinite prices, which contradicts equilibrium. Of course, the infinite demand for portfolio m does not occur with risk aversion in the whole range; see indifference curves V_1, V_2, and V_3 in Figure 4.

As the S-shaped value function has concave as well as convex sections, it is ambiguous whether an interior solution like the one demonstrated by the U_1, U_2, U_3 (see Figure 4) will take place. A necessary condition (but not sufficient) for a finite optimum borrowing is that to the right of a given point the value function (or utility) function must be concave. The Kahneman and Teversky [1979] value function fulfils this necessary condition. However, not all possible value functions will yield a finite optimum borrowing. The optimum borrowing (finite or infinite) depends on the speed of the reduction in V' as we shift to the right and to the left of x = 0. Thus, to have not only the SML but also CAPM equilibrium under CPT, one may need to add the following two alternate procedures.

a) To impose a constraint on the S-shaped functions, because not all guarantee a finite borrowing. Thus, not all S-shape functions are allowed.

b) To impose a constraint on the amount borrowed.

3.5 *CPT and the CAPM: the simultaneous effect of change of wealth, transformation of the probability distribution and S-shaped value function on the CAPM*

So far, we have analyzed the effect of each of the three components of PT and CPT on the CAPM. Now we will analyze their simultaneous effect on the separation theorem and on the CAPM. We compare the following two alternative paradigms:

I. EUT: Suppose initially that the conditions of the CAPM holds: distributions F(w+x), G(w+x), etc. are normal and investors maximize EU(w+x) where U is concave (U' > 0, U" < 0) and w+x is the total wealth. Under these conditions, the separation theorem and the Sharpe-Lintner SMLT follow.

II. CPT: Suppose now that F(w+x) and G(w+x) are normally distributed, the k^{th} investor looks at F(x) and G(x), where x is the change in wealth, makes subjective transformations

$F_k^* = T_k(F), G_k^* = T_k(G)$, etc., and then chooses the portfolios which maximize EV(x) where V(x) is an S-shaped value function with a risk-seeking segment.

Frameworks (I) and (II) are quite different, and do not lead to the same *optimal* levered portfolio choice, hence may lead to different equilibrium prices. Yet both lead to the separation theorem and the SMLT. We demonstrate the simultaneous effect of the three factors of CPT, once again, by means of Figure 4.

Because of the normality assumption, portfolio Q' with corresponding cumulative distribution F(w+x) dominates portfolio Q with corresponding cumulative distribution G(w+x) by FSD (see Figure 4). Because the FSD relationship is unaffected by the initial wealth, we also conclude that distribution F(x) dominates G(x) by FSD even if stated in terms of change of wealth rather than terminal wealth. Because FD_1G implies $T_k(F)D_1T_k(G)$ (where D_1 means dominance by FSD), portfolio Q' dominates portfolio Q even with a subjective monotone transformation T_k as long as $T_k' > 0$, despite the fact that the distributions $T_k(F)$ and $T_k(G)$ are not normal anymore. In addition, T_k varies across investors; hence, the homogeneous expectation assumption is violated. Finally, because FSD $\Rightarrow$ PSD, where PSD corresponds to all S-shaped value functions V(x), we conclude that portfolio Q' dominates portfolio Q for all value functions V(x). Thus, with non-normality of $T_k(F)$ and $T_k(G)$, and no-risk aversion prevalence everywhere (as characterizes the value functions, V(x)), for every portfolio located below the CML (such as portfolio Q), there is a portfolio located on the CML (such as portfolio Q') which dominates it for all CPT investors. Therefore, even in CPT framework, all investors will choose to mix portfolio m with the riskless asset, and the separation theorem and the SMLT is valid in the CPT framework.

4. Conclusions

Mean-variance analysis, the CAPM and Prospect Theory (and Cumulative Prospect Theory) were the innovations of Markowitz, Sharpe, Lintner, Mossin and Kahneman and Tversky, for which Markowitz, Sharpe and Kahneman won the Nobel Prize in economics. The CAPM and PT seem

to contradict each other. Surprisingly, we show in this paper that the SMLT (the security market line theorem) is intact in the PT framework.

Since 1979 there has been a direct and strong attack on von Neumann-Morgenstern's expected utility led by Kahneman and Tversky's Prospect Theory (PT). Experimental findings reveal that investors make decisions based on *change* of wealth, x, rather than *total* wealth w+x, subjectively distort probabilities, and maximize the expected value of an S-shaped value function V(x).

The fact that investors base decisions on x rather than on w+x is sufficient to contradict NM's expected utility paradigm, because the optimum portfolio choice generally depends on the initial wealth, w. We use in this paper the First Degree Stochastic Dominance (FSD) and recently developed Prospect Stochastic Dominance (PSD) criteria to show that the Separation Theorem and the CAPM are intact in the CPT framework. This strong result is intact despite the violation of normality, the violation of risk-aversion as implied by the S-shape value function, and the violation of von Neuman and Morgenstern's expected utility as implied by basing decisions on change of wealth rather than on total wealth.

While it is true that under CPT the *optimum* selected levered portfolio of the k^{th} individual is not the same as under the CAPM, all investors will still choose a portfolio located on the Sharpe-Lintner CML. Therefore, the CAPM separation theorem is intact in the CPT framework. It is important to emphasize that equilibrium prices in the CPT framework are not identical to the CAPM equilibrium prices. Similarly, the $\mu - \beta$ security line may have different parameters under these two frameworks. Yet, the general form of the SMLT still holds under CPT and beta is the risk index, though the SML may have a different slope under CPT than under the CAPM. Finally, to guarantee equilibrium, one may need to add a constraint either on the CPT value function or on the amount of borrowing, otherwise it could be optimal to borrow an infinite amount of money , which of course is not consistent with the notion of equilibrium.

To sum up, the CPT challenges the von Neumann-Morgenstern's expected utility paradigm but the valuation formula of the CAPM and CPT coexist – quite a surprising result!

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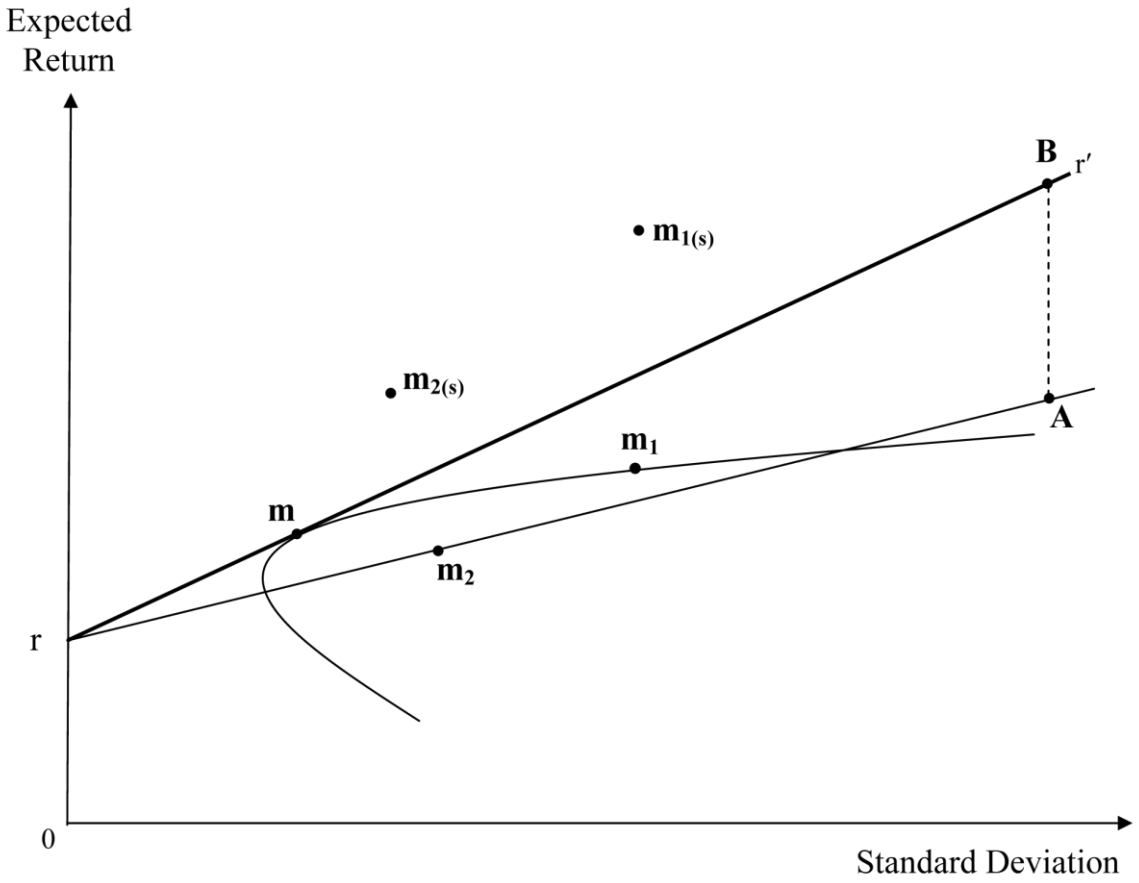

Fig. 1. Decision Weights in Prospect Theory

This Figure shows the effect of decision weights in prospect theory. The line rr′ is the mean-variance efficient frontier, m is the market portfolio, and m_1 and m_2 are two inefficient portfolios. The points $m_{1(s)}$ and $m_{2(s)}$ are the subjective evaluation of portfolios m_1 and m_2 under subjective decision weights $\omega(p)$. We also report two portfolios A and B, where B dominates A by FSD under the assumption of normally distributed returns.

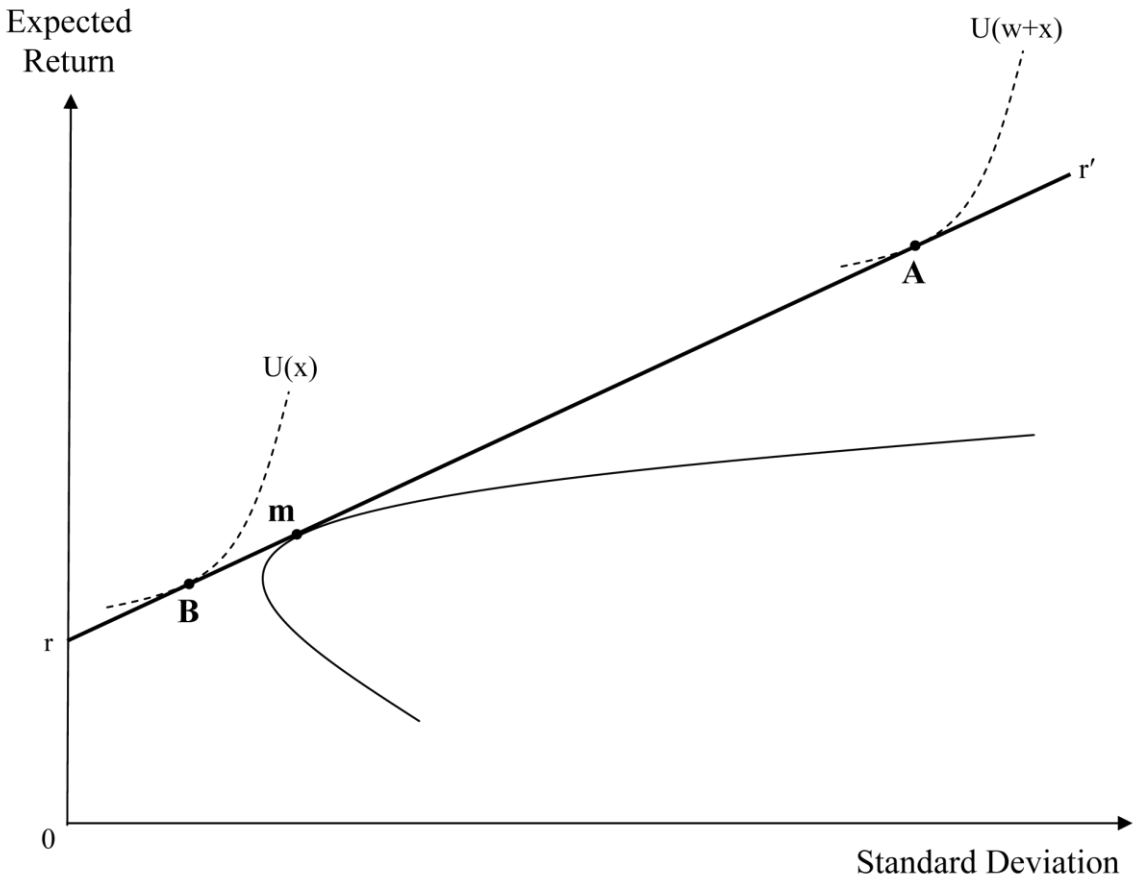

Fig. 2. Utility on Change of Wealth U(x) versus Total Wealth U(w+x)

This Figure shows the effect of having utility U on change of wealth x, rather than on total wealth w+x. The line rr′ is the mean-variance efficient frontier, m is the market portfolio, A and B are two efficient portfolios, and the dotted lines are indifference curves under the assumption that utility is on change of wealth (U(x)) and on total wealth (U(w+x)), respectively. Portfolio A and portfolio B maximize investor's utility under the assumption that utility is on change of wealth (U(x)) and on final wealth (U(w+x)), respectively.

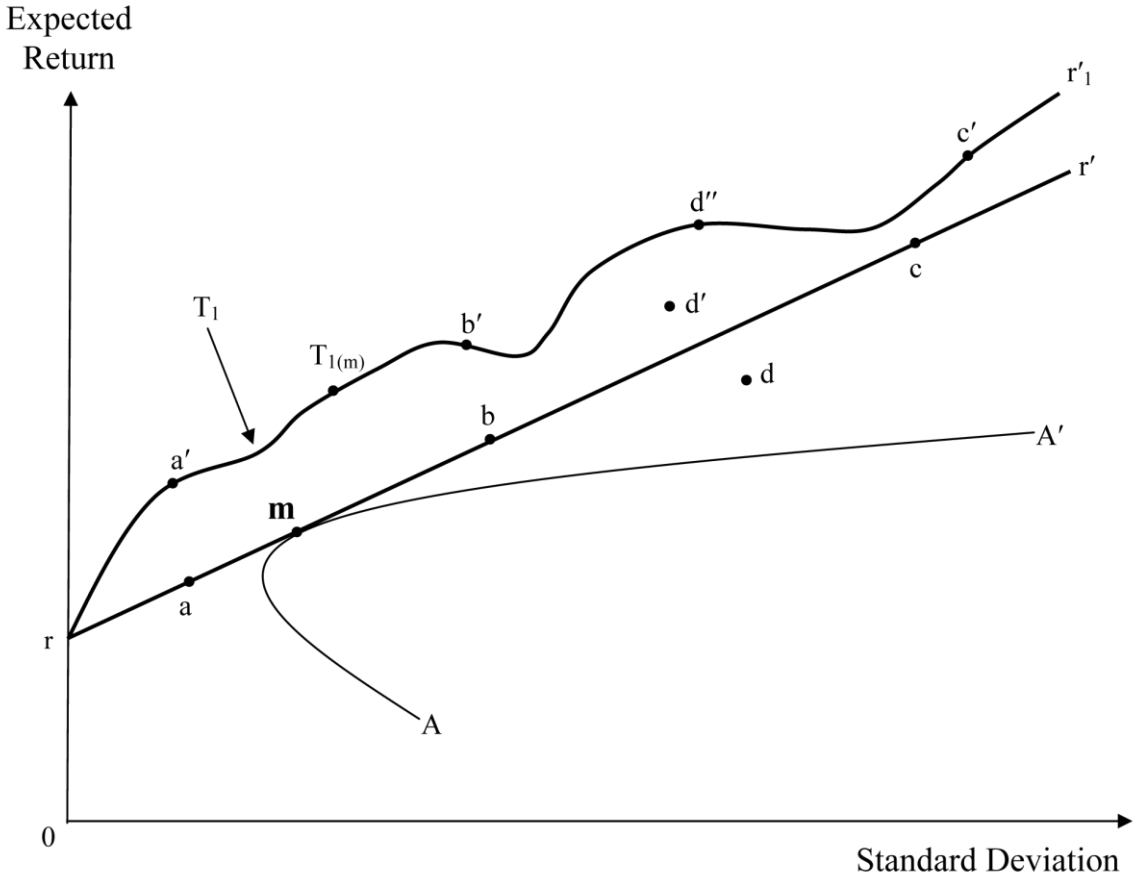

Fig. 3. Decision Weights and the Mean-Variance Efficient Frontier

This Figure shows the effect of decision weight on the mean-variance efficient frontier. The line rr′ is the mean-variance efficient frontier; m is the market portfolio, a, b, c and d portfolios. The points a′, b′, c′ and d′ are the subjective evaluation of portfolios a, b, c and d, respectively, under subjective decision weights ω(p). The curve rr'_1 is the subjective efficient frontier and portfolio d″ is subjectively evaluated portfolio which lies on the subjective efficient frontier rr'_1.

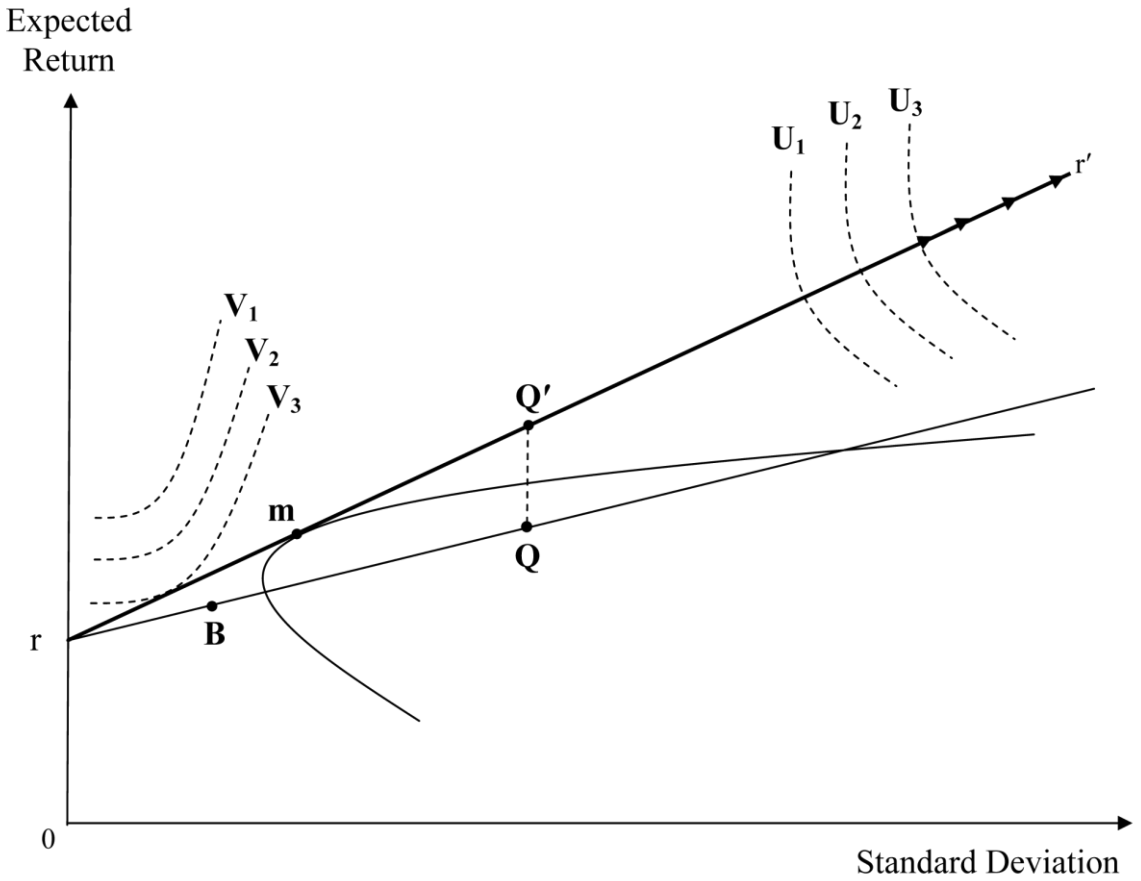

Fig. 4. Security Market Line Theorem (SMLT) with an Assumed S-shaped Value Function

This Figure shows the effect of risk seeking and risk averse preferences. The line rr′ is the mean-variance efficient frontier, m is the market portfolio, Q and Q′ are two portfolio, where Q′ dominates Q by FSD under the assumption of normally distributed returns. The dashed lines are indifference curves under risk aversion (V_1, V_2 and V_3) and risk seeking (U_1, U_2, U_3).