Executive Stock Options: Early Exercise Provisions and Risk-taking Incentives

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November 2004

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Abstract

Traditional Executive Stock Option plans typically allow fixed numbers of options to vest over a period of several years, independent of stock price performance. Such options may climb deep in-the-money long before the manager is permitted to exercise them, potentially making the manager more risk averse in project selection. When firms face risky-but-profitable growth opportunities, we show that by making the proportion of options that vest a gradually increasing function of the stock price achieved, the firm can ensure that appropriate numbers of options are retained when still providing risk-taking incentives, but exercised once they have lost their convexity. Our proposed '*progressive* performance vesting' can allow the firm more efficiently to rebalance risk-taking incentives for the manager.

JEL classification: G31, J33 .

Keywords: Executive compensation, stock options, vesting, early exercise, corporate investment decisions, risk taking.

I Introduction

During the course of the 1990s, the single largest component of CEO compensation became Executive Stock Options (ESOs). ESOs typically have a long-term (ten year) expiration date, yet the holder is permitted to exercise prior to maturity once the option has 'vested'. The most common ESO plan typically allows *fixed* numbers of options to vest at pre-determined calendar dates, independent of stock price performance. We label these 'calendar vesting' stock options, to stress that it is the mere passage of time that causes them to vest. In contrast, in 2001 the top executives of The Kroger Co. were granted options that vest conditional on stock price performance. As the proxy statement discloses:

> *"These performance based options vest during the first four years from the date of grant only if the Company's stock price has appreciated 78% from the option price. Thereafter, those options vest if the Company's stock price has achieved a minimum of a 15% appreciation per annum or 208% appreciation, whichever is less..."*

'Performance vesting' stock options vest contingent on achievement of a performance hurdle, often defined in terms of an accounting measure of profitability or stock price appreciation. In 2002, FW Cook & Co.'s report on the compensation practices of the 250 largest firms in the S&P index documented that, of the 99% that used stock options, 16% used performance vesting stock options.

In this paper we analyze the impact of ESO vesting conditions on the dynamic incentives for managers to select profitable risky projects. Issued at-the-money, ESOs can provide incentives for managers to take risks. Yet if traditional calendar vesting options move deep in-the-money, perhaps years prior to vesting, they lose their convexity in payoffs and may offer counter-productive incentives causing risk-averse managers to *reject* profitable risky projects. We address this problem and propose an alternative vesting schedule. We show that by making the proportion of options that vest a *gradually increasing* function of the stock price achieved, the firm can ensure that appropriate numbers of options are retained when still providing risk-taking incentives, but exercised once they have lost their convexity, thereby allowing the firm more efficiently to rebalance risk-taking incentives for the manager. We label this refinement '*progressive* performance vesting' to distinguish it from performance vesting plans such as that of The Kroger Co., where typically *all* the options vest upon achievement of a *single* performance hurdle.

While most traditional calendar vesting options have qualified for favorable accounting treatment and avoided expensing through the firm's income statement, some forms of performance vesting options have not enjoyed this advantage and may have been avoided for this reason. However, with the imminent prospect that *all* forms of ESO will be compulsorily expensed, a leveling of the accounting playing field and the ever-increasing pressures to improve corporate governance may lead more firms to consider adding performance related conditions to the vesting of ESOs. We contribute to the understanding of this important but little researched aspect of executive compensation. Our results suggest that firms wanting to ensure continued risk-taking should consider adopting vesting schedules that are contingent on stock price appreciation.

The previous literature has documented some variation in the vesting terms of options (Aboody (1996)) and sought to explain this by differences in investment horizon (Kole (1997) relates longer vesting dates to firms for which specialized knowledge is an important asset and for which the horizon for resolution of uncertainty is longer, namely firms with high R&D and innovation). We believe ours is the first paper to explain vesting in terms of risk-taking incentives. A large theoretical and empirical literature[1] finds that the convex payoff profile provided by stock options can be used to induce risk-averse managers to take more risky-but-profitable projects. Conversely, the theoretical work of Lambert, Larcker and Verrechia (1991) and Carpenter (2000) recognizes the potential risk *reduction* incentives caused as options move in-the-money. However, these two papers consider *European* options, exercisable at expiration only, and so do not contemplate the possibility of dynamically rebalancing incentives with the help of an early exercise provision. The idea that the vesting conditions of stock options can be used to help manage dynamically the convexity of the manager's incentive contract is the main contribution of our paper.

We construct a model in which a risk-averse manager is granted a number of long-term stock options. At a later date, the underlying stock price has evolved and the manager faces the choice of accepting or rejecting a risky project with positive expected net present value. Accepting the project is in the interests of the value-maximizing shareholders.[2] When the manager is *not* permitted to exercise any of his options early we demonstrate that if those options are sufficiently deep in-the-money the manager's risk aversion can cause him to reject the project. Effectively, the higher the stock price above the option strike price, the less is the convexity of these existing options and the more linear ('stock-like') become the payoffs. The exposure to downside risk on his unvested

[1] Amihud and Lev (1981), Smith and Stulz (1985), Hirshleifer and Suh (1992), Defusco, Johnson and Zorn (1990), Tufano (1996), Rajgopal and Shevlin (2002), Williams and Rao (2000).

[2] Numerous empirical studies (Smith and Watts (1992), Gaver and Gaver (1993), Mehran (1995), Guay (1999)) find that the use of stock options is associated with firms facing large growth opportunities and for this reason we model our manager facing a risky-but-profitable growth opportunity. We are interested in the design characteristics of options intended to *promote* risk-taking. Firms that face a problem of *over*-investment in *negative* NPV risky projects probably should not be using stock options in the first place.

options can therefore make the manager reject the profitable risky project. One way to overcome this problem and give the manager renewed risk-taking incentives is to grant 'new' at-the-money options. However, we show that this solution becomes rapidly more costly to the firm (requires many more new options), the deeper in-the-money the unexercised existing options are and the more of them that are outstanding. The firm's task of motivating risk-taking is made more expensive because the new options need to be sufficient in number to first overcome the counter-productive incentives provided by the existing options. However, we argue that permitting *early exercise*[3] of existing options (by allowing them to vest) can mitigate this problem and reduce the number of new options required to ensure sufficient risk-taking. Early exercise can remove managerial incentives that have become counter-productive, thereby rendering more effective the firm's attempts to re-establish risk-taking incentives via the issue of new options.

Early exercise of existing options can reduce the firm's costs in two ways. First, it dissipates the time value left in those options, were they to be held to maturity.[4] Second, early exercise can reduce the number of additional new options needed to provide sufficient risk-taking incentives. Hence, an appropriate amount of early exercise can be in the firm's interest and vesting conditions emerge as a natural device for the firm to permit and control early exercise. Our result contributes to answering Murphy's (1999) puzzle "Why are executives allowed, if not encouraged, to exercise options immediately upon vesting rather than holding them until expiration?" Moreover, we take a step further and ask "*How many* options should the executive be allowed to exercise early?" In our model, allowing some of the existing options to be exercised early can *reduce* the number of new options required to ensure risk-taking. However, unvested options do still contribute to satisfying

[3] Huddart and Lang (1996) find that option exercise usually involves immediate resale of the purchased shares. Consistent with this, whenever we write 'exercise', we shall mean 'exercise and resell the underlying stock'. Some firms encourage managers to hold the stock, indeed they make their option plans part of the mechanism by which the manager is expected to achieve his stock ownership target. However, such a policy may be inconsistent with generating the risk incentives that we study in the present paper. We also note that early exercise with the intention of holding on to the underlying stock is inconsistent with a risk aversion motive for early exercise (it is also inconsistent with an informed trading or liquidity needs motive).

[4] This alone might provide rationale for why options vest early (Hall and Murphy (2002)). However, if we believe that options play an incentive role, then we must recognize that early exercise of a ten-year option after, say, three years, leaves the firm with the imperative of re-instating incentives for the remaining period. 'Cheaper' they may be, but American options are likely to be much shorter lived than European options of the same maturity. It is exactly the contention of the present paper that this trade-off may be worthwhile to the granting firm because managers tend to end the life of an American option when it is deep in-the-money and has lost its risk inducing properties. Conversely, when the option remains close-to-the-money, its risk-inducing properties remain intact and it lives to create incentives for another day.

the manager's retention (participation) constraint for the following period, so allowing 'too many' of the existing options to be exercised early would *increase* the number of new options required in order to retain the manager. Given this trade-off we derive explicit expressions for the number of existing options that the firm should *permit* to be exercised early and show that the manager will rationally exercise these vested options early. The resulting expression is an *increasing* function of stock price. This suggests that firms should consider vesting schedules that allow *more* options to be exercised the *greater* the stock price gains achieved; '*progressive* performance vesting'.

Our research question could be interpreted as conceptually analogous to the problem of option 'repricing'. When the stock price *decreases* leaving executive options deep out-of-the-money, incentives may become counter-productive and one controversial solution has been for the firm to 'reprice' (reduce the exercise price of) these options. We consider the converse problem caused when the stock price *increases.* We concentrate on *ex post* incentives and highlight potential drawbacks of the most commonly observed contracts that allow *fixed* numbers of options to vest at pre-determined calendar dates. Aboody (1996) found that a typical grant of ten-year options would vest 25% after 1 year, 25% after 2 years, ... until all had vested after 4 years. Such a calendar vesting schedule may allow 'not enough' or 'too many' options to be exercised early. A manager who holds a large number of deep in-the-money (but unvested) options is likely to reject risky projects, unless the firm re-convexifies his incentives with a large new grant of at-the-money options. Conversely, a manager whose options have vested despite being only marginally in-the-money might exercise those options even though they still provide risk-taking incentives, leaving the firm the problem of reinstating incentives. When options have vested, undiversified risk-averse managers rationally exercise early *more* options the deeper they are in-the-money (Huddart (1994), Detemple and Sundaresan (1999), Heath, Huddart and Lang (1999)). Under calendar vesting, we conjecture that in some circumstances this exercise behavior might go some way towards approximating the outcomes of progressive performance vesting and we interpret this as a possible explanation for why the firm may be prepared to allow the manager discretion over the timing of his exercise decisions - paradoxically it can be his own risk aversion which causes the manager to remove those incentives which would otherwise stop him taking risks. However, when this alignment of objectives does not occur under calendar vesting, particularly when fewer options have vested than would be optimal to exercise from the manager's and firm's points of view, the firm may be vulnerable to missing out on valuable growth opportunities.

Our results on the role of vesting conditions in dynamic incentives have managerial implications for compensation committees, consultants and others concerned with the design of executive compensation contracts. These parties should be aware of how risk-taking incentives change as the stock price evolves; Careful design of option vesting conditions can provide the firm with a mechanism for adapting these incentives over time. Regulatory moves to expense all forms of stock options may lead to a reduced use of stock options in general, but give an opportunity for more thoughtful design of vesting terms where options continue to be used. Yet the playing field has not been perfectly leveled; FASB's proposal permits firms to deduct from expenses the cost of options that fail to vest due to the non-fulfillment of a performance condition. Strikingly, this accounting advantage is specifically *disallowed* when the performance criteria is related to the firm's stock price. Therefore, to the extent that the CEO's most pertinent performance measure is stock price appreciation, the FASB proposal may still provide some impediment to efficient contracting.

The remainder of the paper is organized as follows. The model is in Section 2, discussion of the results in Section 3. Section 4 summarizes and concludes. The Appendix contains a summary of notation, consideration of technical issues, proofs of all lemmas and propositions.

II The Model

A summary of all notation can be found in Appendix A.

We study a principal-agent model in which 'the manager' acts as an agent for 'the firm', by which we mean the shareholders, perhaps represented by a Board of Directors or by a Compensation Committee (we abstract from any conflicts of interests between such parties). There are three dates, t_0, t_1, t_2 and Figure 1 provides a timeline of events and decisions. At the initial date, t_0, the manager is granted N_0 call options, each with exercise price, X_0, and expiration date, t_2, representing the long-term horizon of these instruments. N_0 and X_0 are exogenously given and we do not model the t_0-environment which presumably led to their granting. Rather, we are interested in the effect that their vesting conditions have on incentives *at the interim date*, t_1, when the value of assets-in-place has evolved to V_1 and when the firm faces a risky-but-profitable growth opportunity with Net Present Value (NPV) $\varepsilon > 0$. The manager decides at t_1 whether or not to undertake this project. The firm affects his incentives to do so by choosing $N_1 \in [0, N_0]$, the number of options that remain unvested until t_2, allowing the remaining $N_0 - N_1$ options to vest 'early' at t_1. We shall derive N_1 as a function of the t_1-stock price and so this vesting schedule can be written into the option contract *ex ante* at t_0. The firm also chooses $n \geq 0$, the number of *new* options to grant at t_1. We proceed to specify in detail the preferences of the agents, the projects available, the option remuneration structure and the resulting objectives of the agents.

A Preferences

The manager has exogenous initial wealth, W, and his only other source of income comes from his option compensation package. His utility function, U, is separable in total terminal t_2-wealth, x, and his cost, $F \in \{0, Q\}$, of participation in the second period.

$$U(x, F) = u(x) - F \tag{1}$$

The manager is risk averse, $x \geq 0$, $u' > 0$, $u'' < 0$, and his participation for the second period demands a personal non-monetary cost of $F = Q > 0$. If he quits the firm at t_1 then he 'saves' this cost and so $F = 0$. We can think of Q as a private benefit to the manager which he enjoys if he quits the firm at t_1. We do not formally introduce costly effort into the model, rather we follow Murphy (1999) "... it is widely acknowledged that the fundamental shareholder-manager agency problem is *not* getting the CEO to work harder, but rather getting him to choose actions that increase rather than decrease shareholder value." If the manager has in-the-money options that have vested at t_1, he rationally decides whether or not to exercise them based on the expected utility this will bring.[5] The proceeds from any option exercises at t_1 are held at the risk free interest rate of zero until t_2.

The firm is owned by shareholders who are either risk neutral or can diversify their portfolios sufficiently as to be indifferent to the riskiness of the firm's cashflows. There is no debt and we normalize the total number of shares in the firm to be 1. The shareholder objective is to maximize terminal cashflows, net of any compensation costs required to motivate the manager. We assume that shares are publicly traded on a stock market so that the t_1 stock price is observable and we give all of the bargaining power to the firm who can make a 'take-it-or-leave-it' offer to the manager, the latter accepting any offer at t_1 which gives expected utility at least as high as that from quitting the firm.

B Project selection

At t_1, the expected value of the firm's **assets-in-place** is denoted by V_1. We do not model any managerial actions taken at t_0 and so V_1 is just an exogenous parameter in our model whose

[5] The efficacy of ESOs as an incentive tool depends on their ability to expose the manager to the risks of the outcomes that his actions will provoke. If the manager were able to diversify or could somehow negate this risk then ESOs' utility as performance-based-pay would be undermined. For this reason, ESOs are inalienable; they cannot be sold nor transferred nor assigned to a third party, even when they have vested. Hedging via third part contracts is theoretically possible but there is little evidence of widespread and systematic hedging and very much evidence of early exercise involving substantial dissipation of option value (e.g., Huddart and Lang (1996)). This may be due to reputation and transaction cost reasons or the "potentially chilling tax consequences" of hedging options (Schizer (2000)).

importance is that it controls the degree to which the previously granted options are in-the-money at t_1. We shall consider values $V_1 > X_0$ to ensure that the existing options are indeed in-the-money. No manager would want to exercise an out-of-the-money option and so the question of how many options should vest is irrelevant to that scenario.[6]

We stylize the t_1-project selection decision as being a choice between a '**risky**' or a '**safe**' strategy. The risky strategy has a higher expected NPV than the safe strategy and so is preferable from the firm's point of view, but has higher variance in cashflows and so is potentially unattractive to a risk-averse manager compensated via performance related pay. In order to obtain transparent and tractable solutions, we assume that the safe strategy has a certain zero NPV and so the final payoff will maintain the interim asset value, V_1. The risky project yields a mean improving spread with positive NPV, ε, at the cost of some uncertainty in payoffs which have dispersion $2H$. If the manager stays with the firm, his project selection decision is unverifiable. If the manager decides to quit the firm, then asset value falls to zero - the manager's continued presence is essential for the realization of the assets-in-place (and their liquidation at t_1 is not possible). Quitting the firm is an observable verifiable event causing unexercised options to be forfeited.

We denote the manager's risky/safe project selection decision as choosing $q \in \{r, s\}$, his quit/participate decision as choosing $F \in \{0, Q\}$ and, depending on these t_1 decisions, the terminal t_2 gross firm payoffs are denoted as follows:

Manager action	**Project Payoffs**
'risky'	$\tilde{V}_{risky} = \begin{cases} V_1 + H + \varepsilon & \text{w.p.} \quad 1/3 \\ V_1 + \varepsilon & \text{w.p.} \quad 1/3 \\ V_1 - H + \varepsilon & \text{w.p.} \quad 1/3 \end{cases}$
'safe'	$\tilde{V}_{safe} = \; V_1 \quad \text{w.p.} \quad 1$
'quit'	$\tilde{V}_{quit} = \; 0 \quad \text{w.p.} \quad 1$

Thus, playing 'safe' locks into the existing t_1 asset value whereas going 'risky' has additional NPV of ε. We normalize this NPV to $\delta = \frac{\varepsilon}{H}$, a measure of the NPV relative to the size of the project.

[6] Out-of-the-money ('underwater') options do have incentive effects: they may make the manager take on excessive risk as he seeks to pull them back into-the-money or, conversely, they may lose all incentive value whatsoever as their value to a risk averse manager becomes insignificant. Recent papers (Brenner, Sundaram and Yermack (2000), Acharya, John and Sundaram (2000), Carter and Lynch (2001)) have considered repricing options and we do not intend to contribute to that here.

The variance of the risky project payoff is $\sigma_r^2 = \frac{2}{3}H^2$.

We assume that the NPV, ε, is large enough that the firm will always find it efficient to induce the manager to choose 'risky'. This is true providing the compensation cost of inducing 'risky' is not more than ε higher than the cost of inducing 'safe'. Moreover, we assume that the firm always finds it optimal to avoid the manager quitting the firm. We achieve this by assuming that the loss in firm value is unacceptably great should the manager depart e.g. $\tilde{V}_{quit} \equiv V_1 - L$, where L is 'large' (here, $L = V_1$) due to the costs of finding a replacement manager or the loss of vital knowledge held by the incumbent manager. Consideration of 'small' L would lead us into examination of management turnover and replacement, issues which are outside the risk-taking scope of the present paper.

With these assumptions, the firm's problem becomes one of motivating 'risky' at minimum compensation cost. Equivalently, it becomes a problem of minimizing compensation cost subject to the manager's incentive compatibility constraint and the participation constraint. Letting $\overline{U}$ represent the manager's expected utility from his course of action, these are:

Incentive compatibility (risk-taking)

$$\overline{U}\,(risky) \geq \overline{U}\,(safe) \tag{2}$$

Participation constraint

$$\overline{U}\,(risky) \geq \overline{U}\,(quit) \tag{3}$$

C Asset values and stock prices

We draw a distinction between the value of the firm's assets-in-place and the firm's stock price. This is because a rational market will impound into stock prices the value of assets-in-place *and* expected payoffs from future growth opportunities (providing the market believes these opportunities will be taken up). In our set-up we study the equilibrium where the market believes that the firm will offer the optimal contract to the manager and believes that he will take the risky project and so we denote the equilibrium stock price by $P_1 = V_1 + \varepsilon$. For completeness we consider (and reject) an alternative equilibrium in Appendix B. Furthermore, we invoke a 'small manager' assumption, namely that the variation of compensation costs in different final states of the world is small compared to the total value of the firm and so does not materially affect the stock price.[7]

[7] In support of this assumption we quote from Hall and Liebman (1998): "Even if the typical CEO had in wealth...almost fourteen times annual total compensation at the median - the CEO could purchase only about 0.9% of the firm." Relaxing this assumption would generate an analytical 'warrant problem' because, as option exercise leads to dilution, stock prices adjust slightly downwards to take account of this. In turn, this changes slightly the prospective option payoffs for the manager and complicates the analysis. We address this in Appendix C, but the slight quantitative adjustments necessary lead to the same qualitative conclusions.

D Option remuneration

The focus of our study is the design of the vesting characteristics of the options granted at t_0 - what proportion of these options should the firm allow the manager to exercise 'early' at t_1 and what proportion should remain unvested until t_2? Firms using calendar vesting would allow a fixed percentage, say 50%, of the options to vest at t_1. Firms using plain vanilla performance vesting might allow *all* options to vest at t_1, if and only if a stock price rise of 78% had been achieved. In this paper, we permit a more sophisticated performance vesting schedule, allowing the firm to condition fully upon the stock price gain achieved over the first period. The firm anticipates the incentives vesting will create for option exercise, participation and project selection decisions to be taken at t_1 and the firm takes into account the costs of option exercises and awards. Our vesting schedules will define, as a function of the t_1 stock price, the number, $N_0 - N_1$, of options which vest at t_1, leaving N_1 unvested until t_2. Equivalently, they will define the proportion, $k_1 = \frac{N_1}{N_0}$, of existing options remaining unvested at t_1. We shall assume that no options vest when they are out-of-the-money.

In order to re-establish risk-taking incentives, the firm may also choose to grant additional options at t_1. We denote their number, n, their exercise price, X_1, and this final grant of options we refer to as 'new' options. They have expiry date t_2. For clarity of exposition, we assume that the firm follows the almost universal practice of granting new options *at*-the-money and we solve for n. Thus we fix the exercise price, X_1, equal to P_1, the t_1-stock price. We show later that this assumption is not crucial to our analysis, indeed in our model the firm could generate the same risk-taking incentives at the same cost to the firm by issuing a greater number of out-of-the-money options ($X_1 > P_1$). Issuing in-the-money options would certainly be suboptimal since their downside exposure is counter-productive for risk-taking incentives.

Based on the incentives offered by the original and new option grants, the manager decides whether to exercise his vested options at t_1 and whether to remain with the firm for the second period. In equilibrium he exercises and stays and is faced with the project selection decision which will determine the distribution of terminal firm values.

We assume that all option exercises are settled in cash, so the number of outstanding shares remains unchanged and equal to 1. Option grant sizes N_0 and n will therefore be small fractions of 1, for example $N_0 = 0.01$ would represent the manager having a claim on 1% of the increases in firm value over the two periods. We assume that the firm uses only stock options to pay its manager. Clearly a richer model would include salary, bonus, restricted stock, pension rights etc. in the firm's armory of compensation tools, however we are primarily interested in the convex incentives offered by options which now represent the most significant component of US CEO remuneration.[8] Our approach is to combat managerial risk aversion by rewarding high performance. A theoretical contractual alternative would be to insure the manager against low outcomes by offering a simple fixed salary for the second period, indeed removing all exposure to performance based pay. We reject this alternative as unrealistic. Furthermore, Section 162(m) of the U.S. tax legislation caps the deductibility of non-performance related pay at \$1 million. Liquidity constrained firms often have little choice but to grant options in order to preserve cash. Also, salaries represent a fixed cost to the firm whereas ESOs allow the firm to reduce its financial risk by issuing equity.

In our model, the discrete payoff structure of the projects means that the terminal outcomes at t_2 reveal perfectly the manager's actions at t_1. Theoretically, this could make optimal a 'forcing' contract which rewards the manager for any of the risky outcomes $V_1+\varepsilon-H$, $V_1+\varepsilon$, and $V_1+\varepsilon-H$, but penalizes the manager for the 'safe' outcome V_1. Such a contract would be non-monotonic since the manager would be better rewarded for achieving $V_1 + \varepsilon - H$ than for achieving V_1 and we rule-out such contracts.

We denote the 'moneyness' of the existing options at t_1 as $M = P_1 - X_0$, the 'intrinsic' (immediate exercise) value of these options. This we normalize to define a measure of 'relative moneyness', $\lambda = \frac{M}{H}$, and this parameter plays a central role in our analysis. We have assumed that $V_1 > X_0$ which implies that $M > \varepsilon$ (equivalently $\lambda > \delta$) and so the options are in-the-money. We also assume that $M \leq H$; the options are not *so* deep in-the-money that they can never again fall out-of-the-money. In our set-up, options with $M > H$ would have no remaining 'time value', having lost all of their convexity. Thus, $\lambda \in (\delta, 1]$ and can be thought of as a measure of the convexity that the existing options have 'lost' due to stock price rises.

Any existing option which vests *and is exercised early* at t_1 gives a payoff of M. We can think of this as being paid out of V_1 funds (and hence out of V_2 funds). The exercise price of the new options we choose as $X_1 = P_1$ for notational economy and to accord with the widespread practice of granting options at-the-money. From the manager's t_1 perspective, prospective cash payoffs to

[8] Watson Wyatt's 2000/2001 Annual Survey of Top Management Compensation (conducted on a broad sample of 1545 US firms, all industries) gives the average CEO base salary as \$0.62 million compared to the average stock option compensation of \$3.98 million.

each option *held to maturity* are:

	existing options	**'new' options**
exercise price	X_0	X_1
'risky'	$\begin{cases} M+H & \text{w.p.} \quad 1/3 \\ M & \text{w.p.} \quad 1/3 \\ 0 & \text{w.p.} \quad 1/3 \end{cases}$	$\begin{cases} H & \text{w.p.} \quad 1/3 \\ 0 & \text{w.p.} \quad 2/3 \end{cases}$
Expected Value	$M + \frac{1}{3}(H - M)$	$H/3$
Intrinsic Value	M	0
Time Value	$\frac{1}{3}(H - M)$	$H/3$
'safe'	$M - \varepsilon \quad \text{w.p.} \quad 1$	$0 \quad \text{w.p.} \quad 1$
'quit'	forfeited	forfeited

Each new option is worthless in the 'safe' project and provides a payoff of H only in the upstate of the 'risky' project. Therefore the risk-taking incentives of n such at-the-money options could be exactly replicated by instead granting $\frac{H}{H+P_1-X_1}n$ out-of-the-money options $(X_1 > P_1)$, each giving a payoff $H + P_1 - X_1$ in the upstate. To this extent, the choice of exercise price is arbitrary in our model and we choose $X_1 = P_1$ for notational simplicity and to accord with industry practice. The incentives are more complex when project outcomes follow a continuous distribution, however Hall and Murphy (2000) present an analysis of why granting options at-the-money may be optimal.

E Agent Objectives

We have seen that the **firm** wishes to motivate the choice of 'risky' at the lowest possible compensation cost to shareholders. From the above payoffs, we see that if the manager exercises $N_0 - N_1$ vested options at t_1, holds N_1 unvested options until t_2 and is granted n new options at t_1 then the expected total compensation cost to the firm is

$$(N_0 - N_1) M + N_1 \left(M + \frac{1}{3}(H - M) \right) + n\frac{H}{3} \tag{4}$$

whereupon the firm's programme can be written

$$\min_{N_1 \in [0, N_0], n \geq 0} N_0 M + N_1 \left(\frac{1}{3}(H - M) \right) + n\frac{H}{3} \tag{5}$$

The minimand is increasing in both N_1 and n. Subject to satisfying incentive compatibility and participation constraints, the firm prefers to encourage strictly *higher* early exercise (early exercise reduces the expected cost of these options) and to grant strictly *fewer* new options. Later, Corollary 5 shows that these two goals are congruent and equate to reducing as far as possible the proportion $k_1 = \frac{N_1}{N_0}$ of existing options remaining unexercised at t_1, subject to fulfilling constraints (2) and (3).

Given the terms of his incentive contract, the **manager** will choose his actions at t_1 to maximize his expected utility. We assume that when the manager is indifferent between two courses of action, he chooses that which shareholders prefer.

$$\max_{q \in \{r,s\}, F \in \{0,Q\}} \overline{U}\left(\tilde{x}\left(q,F\right),F\right) = E\left[u\left(\tilde{x}\right)\right] - F \tag{6}$$

F Solving the vesting problem

Now that we have reduced the firm's problem to one of using options to satisfy the incentive compatibility constraints in the most efficient manner, we consider each constraint in turn. We wish to focus on options as an antidote to managerial risk aversion and so we first consider that incentive compatibility constraint. We derive the conditions for there to exist a risk avoidance problem and show how different degrees of early vesting can be used in conjunction with the grant of new options to solve this problem. We then optimize subject to satisfying the participation constraint. The result is that the firm monotonically reduces the expected cost of the vesting/granting solution to the risk avoidance problem until the participation constraint binds.

To make the solutions tractable we give the manager a logarithmic utility function over his final wealth, $u(x) = \ln x$ which exhibits Decreasing Absolute Risk Aversion (DARA)[9] and Constant Relative Risk Aversion (CRRA). His exogenous initial wealth, $W > 0$, ensures that this is defined even if all of his options expire worthless. When using the normalizations $\lambda = \frac{M}{H}$ and $\delta = \frac{\varepsilon}{H}$ we shall often find it convenient to normalize the manager's wealth to $\omega = \frac{W}{N_0 H}$, his wealth relative to the maximum potential further option gain from holding his initial options through the second period.

As a benchmark case, we consider first the situation if *none* of the existing options are permitted to be exercised at t_1. The risk-taking incentive compatibility constraint (2) can then be written

$$\frac{1}{3}\ln\left(W + N_0\left(M+H\right) + nH\right) + \frac{1}{3}\ln\left(W + N_0 M\right) + \frac{1}{3}\ln W \geq \ln\left(W + N_0\left(M - \varepsilon\right)\right) \tag{7}$$

or

$$\rho\left(n, N_0, W, M\right) = \frac{\left(W + N_0 M\right)\left(W + N_0\left(M+H\right) + nH\right)W}{\left(W + N_0\left(M - \varepsilon\right)\right)^3} \geq 1 \tag{8}$$

[9] Whilst we do not vary the utility function, we can change the manager's degree of absolute risk aversion by varying W.

We interpret $\rho(n, N_0, W, M)$ as a measure of the 'relative attractiveness of the risky project', (compared to the safe project).

The following lemma gives conditions under which the manager will not choose the risky project unless existing options are exercised and/or new options granted.

Lemma 1 *, **In the absence of early exercise and when no new options are granted***

i) There exists a threshold number, $\underline{N}_0(\lambda)$ of existing options such that constraint (8) holds at t_1 if and only if

$$N_0 \leq \underline{N}_0(\lambda) \tag{9}$$

where $\underline{N}_0(\lambda)$ is explicitly defined in the proof.

ii) $\underline{N}_0(\lambda)$ is decreasing in λ.

This lemma and the following corollary illustrate the potential problem of options; options which gave optimal risk-taking incentives when they were granted at t_0 may give decidedly counter-productive incentives at t_1. They are more likely to incite risk avoidance the deeper in-the-money they go because they lose their convexity and become more linear and stock-like in their payoffs. Options certainly impose risk on the manager, but once they have moved deep in-the-money they may impose the 'wrong kind of risk' on him, failing to convexify enough his concave preferences and exposing him to painful downside risk.

Corollary 2 *If $N_0 > \underline{N}_0(1)$ then*

i) there exists a threshold moneyness, $\underline{\lambda}(N_0) \in (\delta, 1)$ such that constraint (8) holds if and only if

$$\lambda \leq \underline{\lambda}(N_0) \tag{10}$$

where $\underline{\lambda}(N_0)$ is explicitly defined in the proof.

ii) $\underline{\lambda}(N_0)$ is decreasing in N_0.

Figure 2 illustrates our result that 'too many options, too far in-the-money, cause the manager to choose 'safe' instead of 'risky'. The following lemma shows that granting new options as a 'cure' for this problem becomes progressively more expensive for the firm the deeper in-the-money are the existing options.

Lemma 3 *, **In the absence of early vesting.***

When $N_0 \geq \underline{N}_0$,

i) there exists a threshold number, n_r', of new options such that constraint (8) can be satisfied by granting any $n \geq n_r'$ new options where

$$n_r' = n_r'\left(N_0, \lambda\right) \tag{11}$$

is explicitly defined in the proof.

ii) n_r' is increasing and convex in N_0

iii) n_r' is increasing and convex in λ.

Note that, by construction, for each moneyness, λ, n_r', becomes positive once $N_0 > \underline{N}_0$.

We now consider the firm's position if it *reduces* the number of outstanding existing options to N_1 by allowing $N_0 - N_1$ of them to be exercised by the manager. This also increases the manager's cash wealth by $(N_0 - N_1) M$.

Proposition 4 , ***Allowing early exercise.***

If $N_0 \geq \underline{N}_0$ then

i) constraint (8) can be satisfied by letting $N_0 - N_1$ old options be exercised and granting $n \geq n_r$ new options where

$$n_r = \begin{cases} n_r\left(N_0, N_1, W, M\right) & if \quad N_1 > \underline{N}_1 \\ 0 & if \quad N_1 \leq \underline{N}_1 \end{cases} \tag{12}$$

where $n_r\left(N_0, N_1, W, M\right)$ and $\underline{N}_1$ are given explicitly in the proof. $\underline{N}_1$ represents a threshold below which no new options are required to satisfy constraint (8).

Putting $k_1 = \frac{N_1}{N_0}$ and expressing the number of new options as a proportion of the number of existing options, this expression can be written explicitly

$$\frac{n_r\left(k_1\right)}{N_0} = \begin{cases} \frac{k_1\left(-k_1^2\delta^3 + \left(3\delta^2+\lambda\right)(\omega+\lambda)k_1 - (\omega+\lambda)^2(1+3\delta-\lambda)\right)}{(\omega+(1-k_1)\lambda)(\omega+\lambda)} & if \quad k_1 > \underline{k}_1 \\ 0 & if \quad k_1 \leq \underline{k}_1 \end{cases} \tag{13}$$

where

$$\begin{aligned} \underline{k}_1 &= \frac{\underline{N}_1}{N_0} \\ &= \frac{\omega+\lambda}{2\delta^3}\left(3\delta^2 + \lambda - \sqrt{\lambda^2 + \delta^2\left(6+4\delta\right)\lambda - \delta^3\left(4+3\delta\right)}\right) \end{aligned} \tag{14}$$

ii) n_r is increasing and convex in k_1

iii) n_r is increasing in λ

iv) $\underline{k}_1$ is decreasing in λ and increasing in δ

v) $\underline{N}_1 > \underline{N}_0$.

Proposition 4 gives us a recipe for solving the risk avoidance problem. If a proportion $1 - k_1$ of the existing options are exercised, then the firm needs to issue exactly $n_r(k_1)$ new options to restore risk-taking incentives. Part (ii) says that n_r is convex, increasing with the proportion of unexercised existing options as illustrated in Figure 3. Part (iii) says that deeper moneyness increases the amount of existing options that would need to be exercised in order to negate the need for *any* new options to ensure risk-taking. Part (v) says that if $N_0 > \underline{N}_0$, then in order to negate the risk induced need for any new options, the firm does *not* need to reduce the existing options outstanding down to the minimum level $\underline{N}_0$ that would have originally caused a problem. This is because of the wealth effect of cashed-in options on the DARA Manager.

As shown in Equation (5), the firm's compensation costs are decreasing in the exercise of existing options and increasing in the granting of 'new' options. The firms two objectives are therefore to minimize k_1 and to minimize n. These two objectives do not conflict - from Proposition 4 (ii), n_r is increasing in k_1 and so minimizing k_1 also minimizes the expected cash cost of new options. Thus we have

Corollary 5 *When risk-taking constraint (8) is binding and the firm allows $1 - k_1$ options to vest and grants $n_r(k_1)$ new options, the firm's objective is*

$$\min k_1 \tag{15}$$

subject to satisfying the participation constraint (3).

We see now that the firm can motivate risk-taking incentives most cost effectively by sliding down and leftwards as far as possible along the curve in Figure 3. Indeed, were risk-taking incentives the only concern, the firm could simply choose $k_1 = 0$, $n_r = 0$. However we must not forget the participation constraint to which we now turn. Since the firm will choose to satisfy the risk-taking constraint at the lowest possible cost, we consider the satisfaction of the participation constraint (3) *along the locus* $(k_1, n_r(k_1))$ which satisfies the risk-taking constraint. We show that participation incentives provided are monotonic along this locus and so for any particular level of moneyness, the firm will reduce k_1 (and hence $n_r(k_1)$) until the participation constraint binds.

If the manager decides not to continue with the firm for the second period (to 'quit') then he 'saves' the personal cost of participation but forfeits any unvested options. His cash payoffs are then only those from exercising at t_1 any options which have already vested. To ensure that he prefers to stay and choose 'risky' we note that at t_1, if the manager exercises $N_0 - N_1$ existing options, retains the remaining N_1 existing options and is granted n new ones then his participation constraint (3) can be written

$$\begin{aligned} &\tfrac{1}{3}\ln\left(W+M\left(N_0-N_1\right)+N_1\left(M+H\right)+nH\right)\\ &\quad+\tfrac{1}{3}\ln\left(W+M\left(N_0-N_1\right)+N_1M\right) \qquad \geq \ln\left(W+M\left(N_0-N_1\right)\right)+Q\\ &\qquad+\tfrac{1}{3}\ln\left(W+M\left(N_0-N_1\right)\right)\end{aligned} \tag{16}$$

Simplifying,

$$\frac{\left(\omega+\lambda+k_1+\frac{n}{N_0}\right)\left(\omega+\lambda\right)}{\left(\omega+\lambda\left(1-k_1\right)\right)^2}\geq \exp 3Q \tag{17}$$

From which we get

Lemma 6 *Along the locus $(k_1, n_r(k_1))$ required to satisfy risk-taking constraint (8)*

i) the participation constraint becomes

$$\beta\left(k_1, n_r\left(k_1\right)\right)\geq \underline{\beta}\left(Q\right) \tag{18}$$

where $\underline{\beta}(Q)=\exp 3Q$ and

$$\beta\left(k_1, n_r\left(k_1\right)\right)=\begin{cases}\frac{\left(\omega+\lambda-k_1\delta\right)^3}{\left(\omega+\lambda\left(1-k_1\right)\right)^3} & if \quad k_1\geq \underline{k}_1\\ \frac{\left(\omega+\lambda+k_1\right)\left(\omega+\lambda\right)}{\left(\omega+\lambda\left(1-k_1\right)\right)^2} & if \quad k_1<\underline{k}_1\end{cases} \tag{19}$$

ii) $\beta(k_1)$ is increasing in k_1.

iii) $\beta(k_1)$ is increasing in λ, $\forall\omega\geq 0.5$

iv) If $\omega<0.5$, then $\beta(k_1)$ is increasing in λ $\forall k_1\geq \min\left\{\underline{k}_1, \frac{(1-2\omega)(\omega+\lambda)}{(2\omega+\lambda)}\right\}$ but decreasing in λ for $k_1<\min\left\{\underline{k}_1, \frac{(1-2\omega)(\omega+\lambda)}{(2\omega+\lambda)}\right\}$.

We interpret $\beta(k_1)$ as a measure of the participation incentives generated for the manager when a proportion k_1 of the existing options remain unexercised and are augmented by the number $n_r(k_1)$ of new options required to give sufficient risk-taking incentives. Part (i) says that these incentives must be at least as high as a constant threshold level. Part (ii) expresses the intuition that the manager is more likely to stay with the firm the more of his option wealth remains unvested until t_2. The analytical importance of this is that the set $\{(k_1, n_r(k_1))\in[0,1]\times[0,\infty):\beta(k_1, n_r(k_1))\geq \underline{\beta}(Q)\}$ is continuous and connected. Thus, to satisfy the risk-taking and participation constraints at minimum cost, the firm need just reduce the value of k_1 until the participation constraint binds. Part (iii) says that participation incentives increase with the moneyness of options and again corresponds to the intuition that the manager is more likely to stay with the firm the more valuable his unvested option wealth. The caveat to this is in Part (iv) when the manager is at particularly low relative exogenous wealth levels ($\omega<0.5$), then the wealth effect of moneyness on his *vested* option wealth can reduce his marginal utility for wealth sufficiently that the participation incentives are decreasing in moneyness. However even this effect can only dominate for 'small' k_1, not high enough to trigger a grant of new options.

Proposition 7 ***The firm's optimal vesting schedule and granting of new options.***

The firm's optimal proportion, k_{part}, of unexercised existing options (i.e., allowing $1-k_{part}$ to be exercised at t_1) and corresponding number, n_{part}, of new options together ensure that the manager does not quit and does choose the risky project. These optima depend on the manager's total cost of participation, Q, as follows:

i) If $\underline{\beta}(Q) > \beta(1)$

Then

$$k_{part} = 1 \tag{20}$$

And $n_{part} > n_r(1)$ and can be written explicitly

$$n_{part} = \left(\frac{\omega^2}{(\omega+\lambda)} \underline{\beta} - (\omega+\lambda+1) \right) \tag{21}$$

ii) If $\underline{\beta}(Q) \in (\beta(\underline{k}_1), \beta(1))$

Then $k_{part} \in (\underline{k}_1, 1)$ and can be written explicitly

$$k_{part} = \frac{(\omega+\lambda)\left(e^Q - 1\right)}{e^Q \lambda - \delta} \tag{22}$$

and $n_{part} = n_r(k_{part}) > 0$ which can be written explicitly

$$\frac{\left(e^{2Q}\lambda^2 - e^Q\lambda - 2\lambda\delta e^{2Q} - 2e^Q\lambda\delta + \delta^2 e^{2Q} + \delta + \delta^2 e^Q + \delta^2\right)(\omega+\lambda)\left(e^Q - 1\right)}{\left(e^Q\lambda - \delta\right)^2} N_0 \tag{23}$$

iii) If $\underline{\beta}(Q) \leq \beta(\underline{k}_1)$

Then $k_{part} \in (0, \underline{k}_1]$ and can be written explicitly

$$k_{part} = \frac{(\omega+\lambda)}{2\underline{\beta}\lambda^2} \left(1 + 2\underline{\beta}\lambda - \sqrt{\left(1 + 4\underline{\beta}\lambda(1+\lambda)\right)} \right) \tag{24}$$

and

$$n_{part} = n_r(k_{part}) = 0 \tag{25}$$

This proposition uses the recipe of Proposition 4 to ensure risk-taking whilst satisfying the participation constraint (8) at minimum cost. Part (i) corresponds to the manager's participation requirements being so high that even keeping all existing options unvested and giving him $n_r(1)$ new ones would be insufficient to make him stay. Thus, nothing vests and an even higher number of new options are required (though fewer are required the higher is λ because moneyness 'helps' the existing options provide participation incentives). Risk-taking is not even a binding constraint in this case and so we shall pursue it no further. Risk-taking is binding in parts (ii) and (iii) where we have found explicit solutions for the vesting schedule and the number of new options that should be granted. These solutions have the following properties.

Proposition 8 *i) k_{part} is decreasing and convex in λ.*

ii) n_{part} is increasing in λ whenever $\beta(1) < \underline{\beta}$.

Part i) is the central result of our paper: the deeper in-the-money are the manager's existing options, the more the firm will want him to exercise and so the more it should allow to vest. This suggests that the firm should consider a vesting schedule which is increasing in stock price attained at t_1, a policy we term 'progressive performance vesting'. Re-optimizing risk incentives is made easier by the fact that the more valuable are the unvested options, the fewer are required to bind the manager to the firm. Given that we have painted the existing options as the risk avoiding menace which needs to be removed, this result is quite intuitive. Part ii) states how the size of the new option grant can be related: Increased moneyness increases the problem of existing options (tending to increase n_{part}) but also increases the vesting of existing options (tending to decrease n_{part}). Our result shows that the former effect can dominate - when the risk-taking constraint also binds, (parts (ii) and (iii) of Proposition 7) then the number of new options granted is increasing as a function of the moneyness of existing options. Thus the exercise of existing options may be positively associated with the granting of new options.

Figure 4 illustrates the result that the higher the moneyness, $(\lambda_2 > \lambda_1)$, the fewer existing options should be left unvested $(k_{part2} < k_{part1})$ because the required participation incentives are achieved with more vesting. Figure 5 shows that despite this, higher moneyness still implies the granting of more new options, i.e., $n_r(k_{part2}) > n_r(k_{part1})$.

To complete this section, we note that our derivations of optimal vesting schedules have been made on the assumption that the manager exercises options when they vest. The following lemma confirms that this is indeed the case.

Lemma 9 *When the firm follows the optimal vesting strategy for existing options and granting new options in accordance with Proposition 7, the manager does indeed maximize his own utility by exercising the whole $1 - k_{part}$ proportion of options that the firm has allowed to vest.*

If the manager were to leave some vested options unexercised (i.e. holding some fraction $k_1 > k_{part}$) then he would not have correct incentives to choose 'risky' so would choose 'safe'. But this would mean he would want to exercise as many options as possible at the t_1 stock price because the effect of this out-of-equilibrium project decision would be a fall in stock price, reducing the moneyness of any unexercised options. Similarly, choosing 'quit' he would want to exercise as many options as possible at t_1 because he will forfeit any unexercised options on departure.

III Executive Stock Option Plans

In this section, we interpret our results and discuss their empirical and normative implications for the design of option schemes. We point out what can 'go wrong' with calendar vesting and outline the empirical and economic implications. We discuss the use of performance vesting options and show how their design may be refined, then we discuss briefly the use of reload options.

Calendar vesting Having derived our vesting schedules, it is now clear that traditional calendar vesting schedules are unlikely to offer 'correct' incentives for risk-taking. If the calendar vesting schedule permits a fixed proportion $1 - k_c$ to vest at t_1, leaving k_c unvested until t_1 then the following scenarios are possible:

i) $k_c > k_*$ and the firm has allowed 'not enough' options to vest. From Lemma 4 and Figure 5 we can see that to re-establish risk-taking incentives, the firm needs to grant an unnecessarily high number of new options, $n_r(k_c) > n_r(k_*)$.

ii) $k_c < k_*$ and the firm has allowed 'too many' options to vest. If the manager exercises all of the $1-k_c$ vested options then the firm needs to grant an unnecessarily high number of new options, not in order to ensure risk taking but in order to satisfy the binding participation constraint. It may be that the manager chooses rationally to exercise less than $1-k_c$ options and this may mitigate the firm's failure to implement the optimal vesting schedule. In Appendix D we show that the manager rationally exercises more options the deeper they are in-the-money. We conjecture that under some circumstances this exercise behavior might enable calendar vesting to approximate the outcomes of progressive performance vesting. This is a possible explanation for why the firm may be prepared to allow the manager discretion over the timing of his exercise decisions - paradoxically it can be his own risk aversion which acts to remove those incentives which would otherwise stop him taking risks. However, when this alignment of objectives does not occur with calendar vesting, particularly when fewer options have vested than would be optimal to exercise from the manager's and firm's points of view, the firm may be vulnerable to missing out on valuable growth opportunities.

Empirically, it would be interesting to document how risk-taking behavior and subsequent option grants are affected dynamically, as options move in-the-money. We know that managers rationally exercise more, the deeper in-the-money and Heath, Huddart and Lang (1999) find evidence of particularly intense exercise activity immediately following vesting dates. This suggests the existence of a pent-up desire to exercise and is evidence that vesting restrictions are often binding. We have seen no study that examines the effect on risk-taking when managers are constrained by vesting requirements. Our results predict that when this is the case, risk-taking will decrease unless the firm issues large numbers of new options to reconvexify the incentives. Numerous studies have shown how options (but not restricted stock) raise volatility. We conjecture that after a period of increasing stock prices, the risk-taking incentives of options may weaken (and even become disincentives) unless renewed and reinvigorated by the mechanism of early exercise and granting of further options that we analyze in this paper. Future research should examine the dynamic portfolio incentives held by their managers and its interaction with temporal volatility evolution.

Our Proposition 8 implies that stock price run-ups leading to high levels of option exercise may be followed by particularly large grants of new options. This result has echoes of "You can pay me now and you can pay me later..." (Boschen and Smith (1995)). Their results on pre-1990 data show how managers reap the rewards for good performance not only contemporaneously, but also for up to five years following an innovation in firm performance. Our results provide further support for the notion that options are granted both as a reward for past success *and* to provide future incentives. Empirically, Ofek and Yermack (2000) find that in years when boards grant equity-based compensation to managers, the latter tend to reduce their previously owned equity holdings, thereby counteracting the board's attempts to increase their personal exposure to firm value. Our results suggest that another interpretation may be that there is an implicit recognition of the mutual benefit to manager and firm of early option exercise, sale of stock and granting of new options to reoptimize incentives.

Performance Vesting Options The normative conclusion of the present paper is that when risk-taking incentives are important, the firm might be advised to use progressive performance vesting rather than calender vesting options. A justification sometimes given for performance vesting options is that they strengthen the link between pay and performance, addressing the concerns of shareholder groups who resent managers receiving large option payouts for often mediocre performance. With a performance target, vesting becomes a reward for increasing stock price significantly rather than just an inevitable consequence of the passage of time. Performance targets are usually simple step functions of stock price e.g., no options vest unless a 78% price gain is achieved, whereupon 100% of the options vest. Yet theoretically, there is no limit to the complexity that could be built into a vesting schedule. We would not necessarily advise practitioners to adopt our equation (22) in the design of their vesting schedules, but the principle of 'the higher the stock price, the more options vest' might be incorporated more imaginatively into option contracts. Figure 6 shows qualitatively how our proposed progressive performance vesting differs from typical calendar vesting and plain vanilla performance vesting schemes.

We do not model any decisions and agency problems existing at t_0. However, we can say that the prospect of progressive performance vesting would increase pay/performance incentives *ex ante* at t_0. Managers would know that stock price appreciation would be rewarded in three ways: currently held options would each be more valuable (explicit option value incentive), a greater number would vest enabling them to be cashed in (explicit performance vesting incentive) and they would be further rewarded by the granting of a larger number of new options (implicit 'repeat award' incentive).

Reload Options The focus of our study has been on the way that firms may seek to manage the convexity of the incentives offered to management. In the context of risk-taking incentives we have seen the problem of ensuring sufficient convexity in the manager's incentives at the current share price. During the 1990's, compensation consultants popularized 'Reload Options' which have the characteristic that they involve the sequential exercise and 'reloading' of executive options. Each time that the manager exercises reload options, the firm issues him new options at-the-money. At first glance, this mechanism appears to perfectly address our identified problem of 'lost convexity' - absent large stock price falls, they enable the manager to keep his options always close-to-the-money. However, the reload feature only works if the manager 'pays' the exercise price of the option with previously held shares. Firms tend to use reloads to encourage managers to hold stock and we have already shown what risk-reducing incentives this may create.

IV Conclusion

This paper highlights the importance of setting appropriate risk-taking incentives for management as stock prices evolve over time. A firm that cares about risk-taking must care about the convexity of the compensation contracts offered to managers. In particular, when options become deep in-the-money, they lose their convexity and this may dramatically reduce risk-taking as managers concentrate on preserving the paper gains they have made. To firms with many valuable-but-risky growth opportunities, the dangers of overly-conservative management are obvious. The deeper in-the-money are the options, the more they may dissuade managers from selecting risky projects. We suggest this is one reason for allowing partial early vesting of stock options. Traditional calendar vesting may serendipitously achieve this - the deeper in-the-money are the options, the more of them the risk averse manager will wish to exercise. This can act in the interests of the firm, to the extent that it can rely on the manager to voluntarily remove incentives when they have become counter-productive. However, this cannot occur when fixed calender vesting schedules leave the manager holding large amounts of unvested deep in-the-money options. We propose as an alternative 'progressive performance vesting'. We derive vesting schedules as a function of stock price gains achieved, allowing the manager to remove incentives when and only when they have lost their risk-inducing properties and making more efficient the firm's attempts to re-establish risk-taking incentives. Progressive performance vesting costs the firm less in terms of dilution - forcing the manager to bear risk is costly because he demands a risk premium, forcing the manager to bear the 'wrong kind' of risk is doubly expensive. Permitting early exercise and re-optimizing incentives can reduce this cost because the firm needs to give the manager less dollar value but succeeds in rewarding him with reduced uncertainty. When the firm is keen to reduce media and shareholder perceptions of inflated option awards, this may be one step towards achieving that objective.

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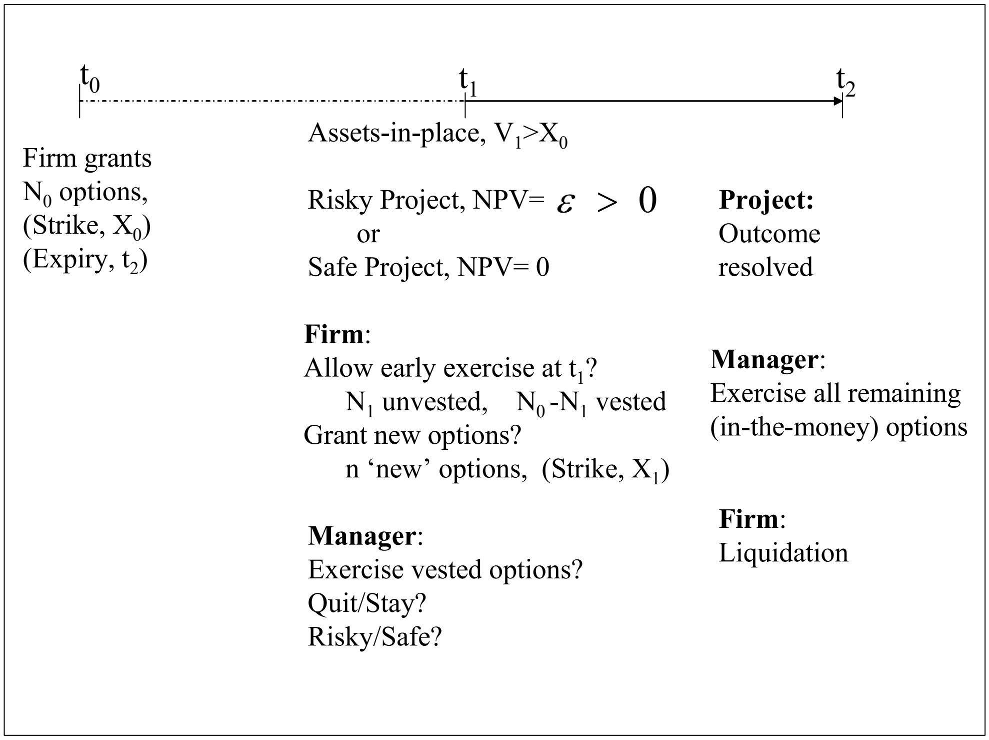

Figure 1: Timeline of events

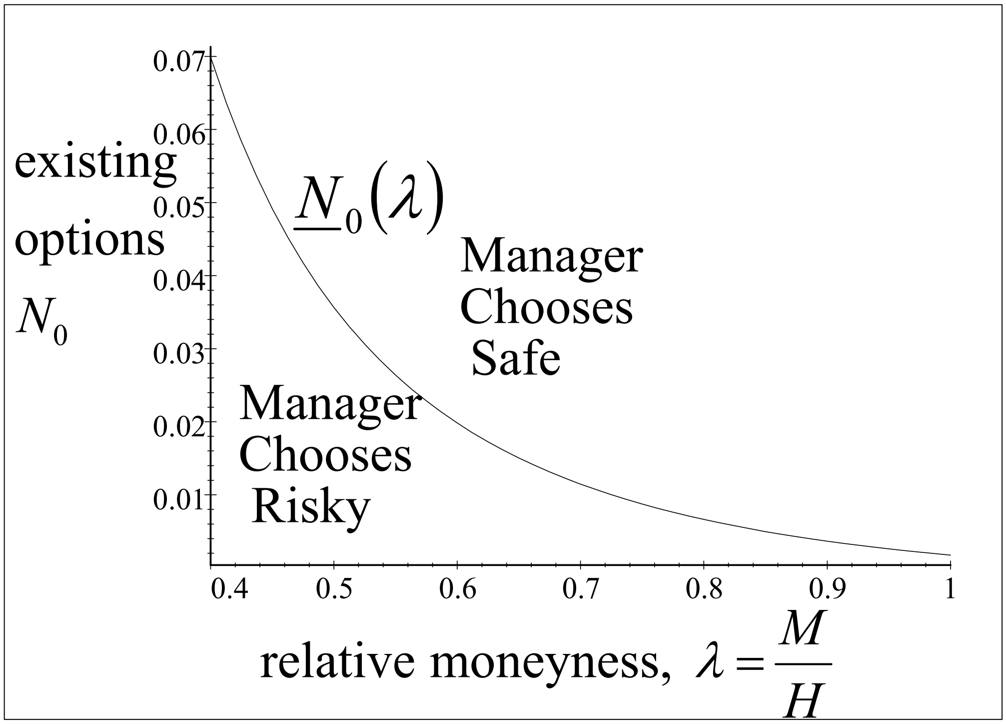

Figure 2: Graph of $\underline{N}_0(\lambda)$, the threshold number of existing options, above which the manager will reject the risky project (in the absence of any early exercise or granting of new options).

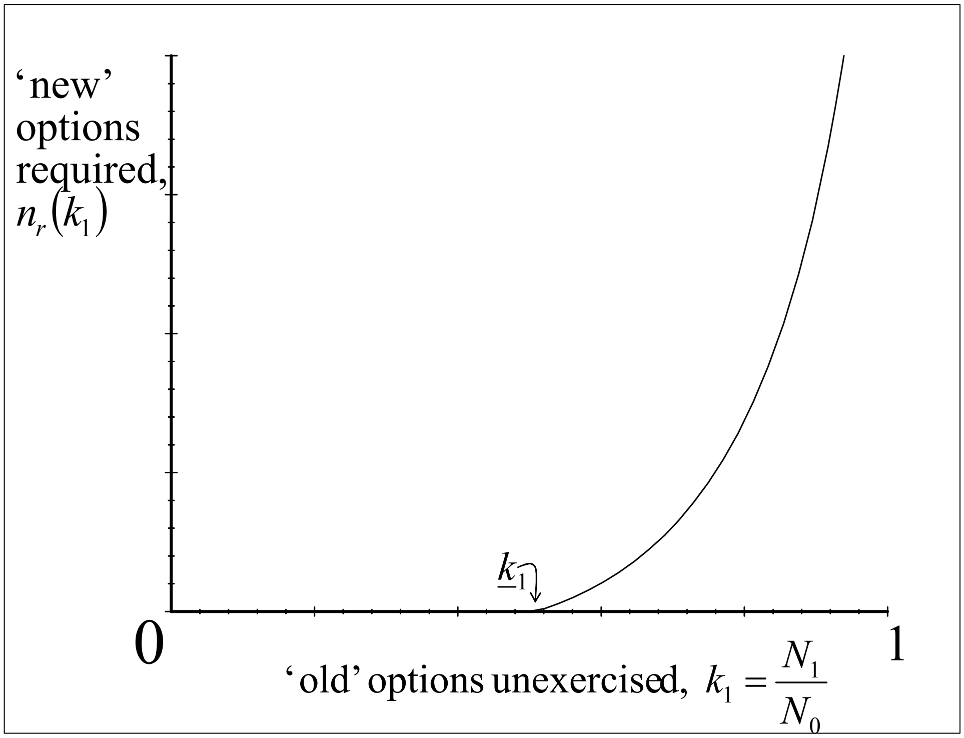

Figure 3: Number of new options required to solve risk avoidance problem, as function of proportion of old options unexercised.

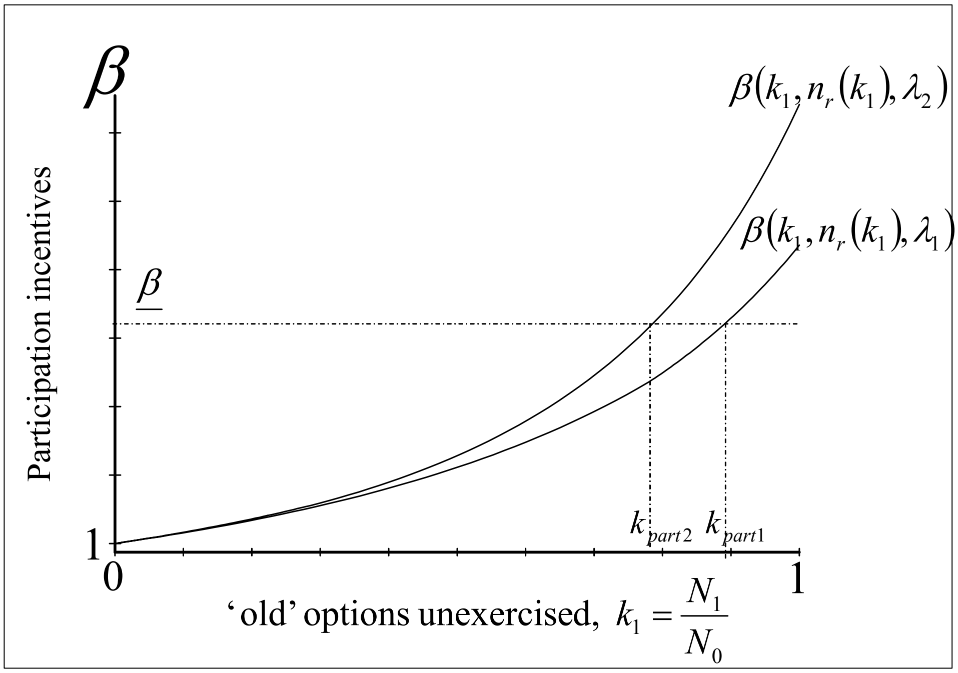

Figure 4: Participation (retention) incentives provided when proportion k_1 of old options remain unexercised and number $n_r(k_1)$ of new options are granted. Shown for two levels of moneyness, $\lambda_2 > \lambda_1$.

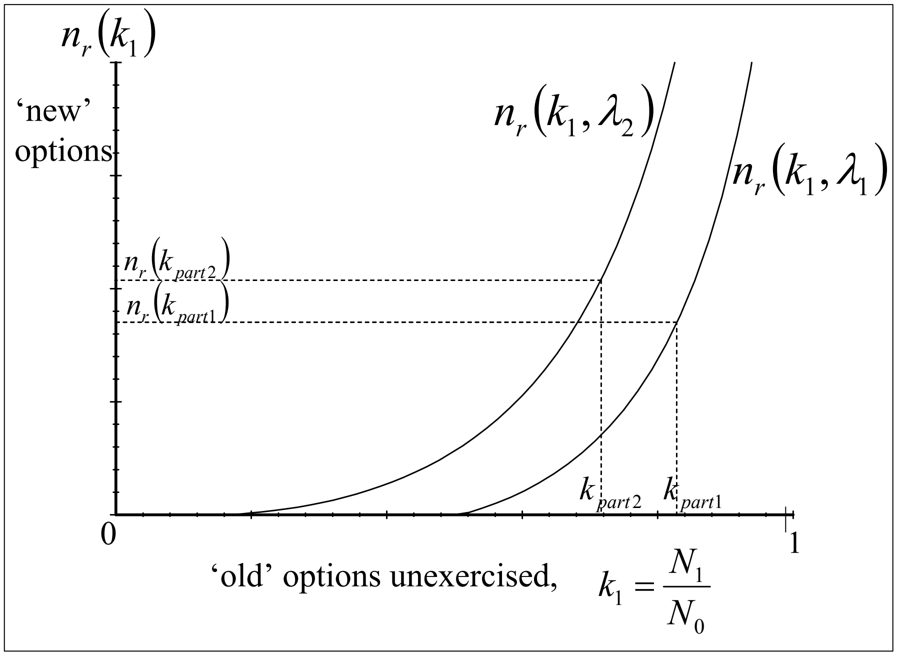

Figure 5: Optimal vesting fraction, $1 - k_{part}$, and number of new options, $n_r\left(k_{part}\right)$ shown for two different levels of moneyness, $\lambda_2 < \lambda_1$.

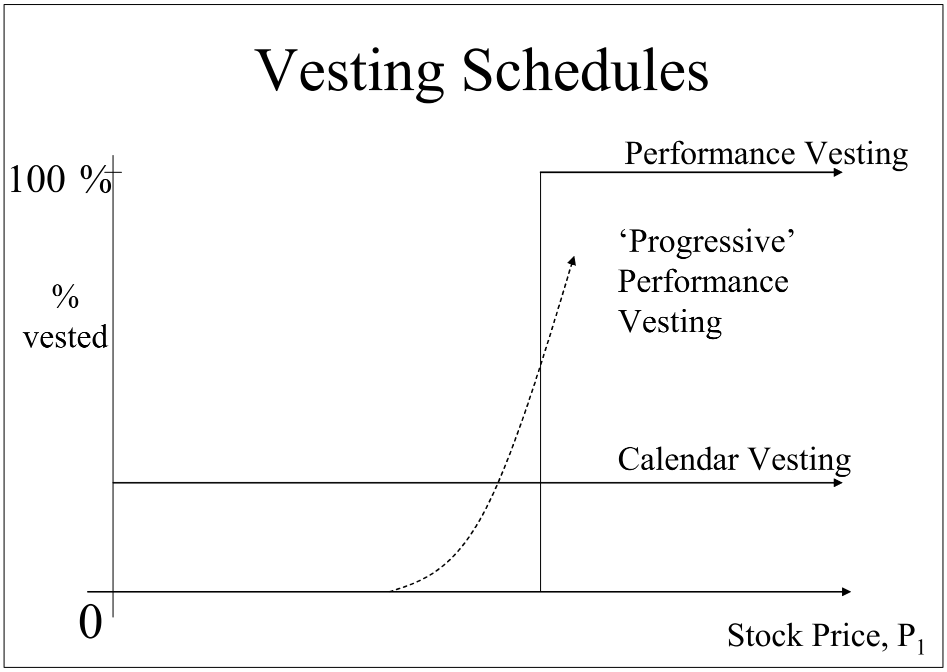

Figure 6: Comparison of number of options vested, as a function of stock price, for 'calendar vesting', plain vanilla performance vesting and 'progressive' performance vesting.

Appendix

A Summary of Notation

Dates, Asset Values and Projects

t_0	initial date
t_1	interim decision date
t_2	terminal liquidation date
P_1	equilibrium stock price at t_1
V_1	asset value at t_1 (parameter)
$\tilde{V}_{qF}$	liquidation asset value at t_2, (random variable depending on manager's decisions at t_1)
q	$\in \{r, s\}$, Manager Project choice, 'risky' or 'safe'
F	$\in \{0, Q\}$, Manager's personal cost of choosing, 'quit' or 'stay'

$$\tilde{V}_{risky} = \left\{ \begin{array}{cc} V_1 + H + \varepsilon & \text{w.p. } \frac{1}{3} \\ V_1 + \varepsilon & \text{w.p. } \frac{1}{3} \\ V_1 - H + \varepsilon & \text{w.p. } \frac{1}{3} \end{array} \right| \quad \text{payoffs to 'risky'}$$

$\tilde{V}_{safe} = V_1$ w.p. 1 , certain payoff to 'safe'

$\tilde{V}_{quit} = 0$ w.p. 1 , certain payoff to 'quit'

ε	Net Present Value of 'risky' $\tilde{V}_{risky}$ project
H	half of the range of the 'risky' $\tilde{V}_{risky}$ project outcomes
δ	$= \frac{\varepsilon}{H} \in (0, 1)$, normalisation of NPV
σ_r^2	$= \frac{2}{3} H^2$, variance of the 'risky' $\tilde{V}_{risky}$ project payoffs

Options

	# of outstanding shares = 1
N_0	# of options granted at t_0 (exog.)
X_0	exercise price of options granted at t_0
N_1	# of these existing options kept throughout the second period to t_2
$N_0 - N_1$	# of these existing options exercised early at t_1
k_1	$\frac{N_1}{N_0}$, proportion of existing options retained to maturity, (normalisation of N_1)
M	$= P_1 - X_0$ the 'moneyness' at t_1 of options granted at t_0
λ	$= \frac{M}{H} \in [\delta, 1]$, relative moneyness, (normalisation of M)
n	# of 'new' options granted at t_1 (endog.)
X_1	exercise price of any options granted at t_1

Manager

$U(x, F)$	$= u(x) - F$ utility function of total t_2-wealth, x, and effort cost, F
$u(x)$	$= \ln x$, logarithmic utility for wealth
W	fixed outside wealth of manager (exog.)
ω	$= \frac{W}{HN_0}$ normalisation of W

Derived Formulae

$\underline{N}_0$	# of existing options *above which* there is a risk avoidance problem at t_1
n_r'	# of new options required to solve that problem, in *absence* of early exercise
n_r	# of new options required to solve risk avoidance problem, in *presence* of early exercise
$\underline{N}_1$	# of unexercised existing options (after exercise at t_1), below which *no* new options are required for risk-taking
$\underline{k}_1$	$= \frac{\underline{N}_1}{N_0}$, normalisation of $\underline{N}_1$
N_{part}	optimal # of unexercised existing options to solve participation and risk constraints
k_{part}	$= \frac{N_{part}}{N_0}$, optimal proportion of unexercised existing options, normalization of N_{part}
n_{part}	corresponding # of new options to solve participation and risk constraints

B An Alternative Equilibrium?

In the analysis of our model we have chosen to study the equilibrium in which the market *believes* the manager will take the positive NPV project (and so share price $P_1 = V_1 + \varepsilon$) and in which the manager does find it optimal to take the positive NPV project. Market and manager take the optimal contract as given since they know it will be in the firm's interests to grant it. It is possible that there exists an alternative equilibrium in which the market believes that the manager will *not* take the positive NPV project and in which the manager will *not* find it optimal to take the positive NPV project. If such an equilibrium persisted then the stock price would fall to V_1 at which price a purchaser could buy up all the shares and give the manager an appropriate contract to ensure risk-taking.

C The Warrant Problem

We have employed a 'small manager' assumption in order to simplify the analysis. We outline here the complications it would cause if we did not make this approximation.

Strictly speaking, the stock prices at t_2 will not exactly equal the final gross asset value, but will adjust downwards to take account of the share taken by the manager through his exercise(s) at t_2 and/or t_1. Furthermore, the stock price at t_1 must also take account of current and future dilution caused by manager exercise. Thus, if the interim gross asset value is V_1 and interim stock price is P_1 and if the manager exercises $N_0 - N_1$ options, leaves N_1 unexercised, receives n new options and takes the 'risky with effort' project, then at t_2, the rational final stock price *after* the market has seen the realization of V_2 and before (and after) the manager has exercised his outstanding options is given by

$$P_2 + n\,(P_2 - X_1)^+ + N_1\,(P_2 - X_0)^+ + (N_0 - N_1)\,(P_1 - X_0) = V_2 \tag{26}$$

$$\text{where } V_2 = \begin{cases} V_1 + \varepsilon + H & \text{in the 'Up' state} \\ V_1 + \varepsilon & \text{in the 'Middle' state} \\ V_1 + \varepsilon - H & \text{in the 'Down' state} \end{cases}$$

Denoting P_{2U}, P_{2M}, P_{2D}, as the values of P_2 in the Up, Middle and Down terminal states respectively

$$P_1 = \frac{1}{3}\,(P_{2U} + P_{2M} + P_{2D}) \tag{27}$$

and assuming $(P_2 - X_1)^+ = 0$ in the Middle state (which we can do by choosing X_1 'high enough'), then

$$
\begin{aligned}
P_{2U}\left(1+n+N_1\right) &= V_1+\varepsilon+H+nX_1+X_0N_0-\left(N_0-N_1\right)P_1 \\
P_{2M}\left(1+N_1\right) &= V_1+\varepsilon+X_0N_0-\left(N_0-N_1\right)P_1 \\
P_{2D} &= V_1+\varepsilon-H+X_0\left(N_0-N_1\right)-\left(N_0-N_1\right)P_1
\end{aligned} \tag{28}
$$

giving

$$
P_1=\frac{\left(\frac{1}{(1+n+N_1)}+\frac{1}{(1+N_1)}+1\right)\left(V_1+\varepsilon+X_0N_0\right)+\frac{nX_1-(n+N_1)H}{(1+n+N_1)}-X_0N_1}{3+\left(\frac{1}{(1+n+N_1)}+\frac{1}{(1+N_1)}+1\right)\left(N_0-N_1\right)} \tag{29}
$$

from which we can recover P_{2U}, P_{2M} and P_{2D} each in terms of V_1, ε, H, X_0, N_0, n and N_1. These solutions themselves determine the payoffs to the manager's options which is an input to the derivation of the optimal numbers of options to grant and to vest...Effectively the above system of equations would represent yet another condition on the derivations of N_* and n_*.

D Rational Early Exercise

In equilibrium, the manager's expected utility can be written

$$
\overline{U}\left(risky\right)=\ln HN_0+\frac{1}{3}\ln\left(\left(\omega+\lambda+k_*+\frac{n_*}{N_0}\right)\left(\omega+\lambda\right)\left(\omega+\lambda\left(1-k_*\right)\right)\right)-Q \tag{30}
$$

We have shown in Lemma 9 the circumstances in which the manager will exercise the options that the firm allows to vest. This exercise decision is partly driven by the fact that the manager knows that to *not* exercise would change his incentives, leading him to supply the 'wrong' decisions and making it optimal to exercise. We have, quite properly, ensured that our manager make his exercise/project decisions simultaneously. However, to show the effect of the manager's risk aversion in isolation, we now take the 'risky' decision as given and ask how many options the manager would exercise *abstracting from its effects on his incentives within the firm.* This is the approach taken in Huddart (1994) and Detemple and Sundaresan (1999).

Taking n_* as given, the manager's exercise decision would be to choose k to maximise the following objective

$$
\left(\omega+\lambda+k+\frac{n_*}{N_0}\right)\left(\omega+\lambda\left(1-k\right)\right) \tag{31}
$$

Expanding, the maximand is quadratic in k,

$$
-\lambda k^2+\left(-\omega\lambda+\lambda-\lambda^2+\omega-\frac{n_*}{N_0}\lambda\right)k+\omega^2+2\omega\lambda+\lambda^2+\frac{n_*}{N_0}\omega+\frac{n_*}{N_0}\lambda \tag{32}
$$

This function has a maximum at

$$
k_{manager}=\frac{1}{2}\left(\left(\frac{1-\lambda}{\lambda}\right)\left(\omega+\lambda\right)-\frac{n_*}{N_0}\right) \tag{33}
$$

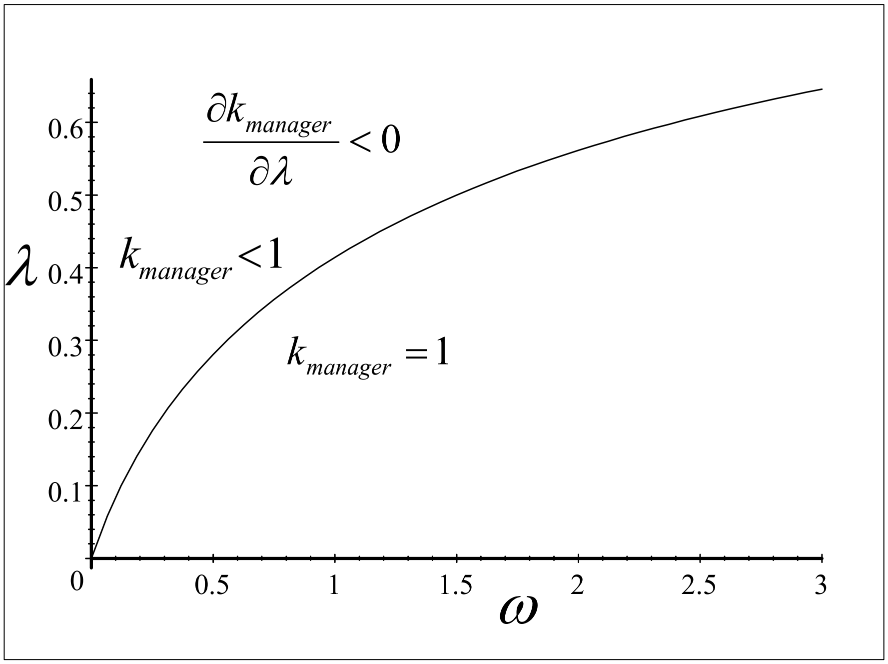

Figure 7:

and we proceed to examine whether this solution is interior to the range of $k \in (0, 1)$.

First take $n_* = 0$, then $k_{manager} = \frac{1}{2}\left(\frac{1-\lambda}{\lambda}\right)(\omega + \lambda) \geq 0$ and $k_{manager} < 1$ if and only if $\lambda + \omega\lambda + \lambda^2 - \omega > 0$ i.e.,

$$\lambda > \frac{1}{2}\left(\sqrt{(1 + 6\omega + \omega^2)} - (1 + \omega)\right) \tag{34}$$

Otherwise there is a corner solution at $k = 1$.

Also $\frac{\partial k_{manager}}{\partial \lambda} = -\frac{1}{2}\frac{\lambda^2+\omega}{\lambda^2}$ which is strictly negative so the manager exercises more options, the deeper they are in-the-money

$$k_{manager} = \begin{cases} \frac{1}{2}\left(\frac{1-\lambda}{\lambda}\right)(\omega + \lambda) & if \qquad \lambda > \frac{1}{2}\left(\sqrt{(1 + 6\omega + \omega^2)} - (1 + \omega)\right) \\ 1 & otherwise \end{cases} \tag{35}$$

So, for low λ he exercises nothing, exercising more as λ increases. Intuitively, higher wealth reduces the risk aversion of our DARA manager, increasing the threshold moneyness at which the manager will start to exercise early.

For $n_* > 0$, we know that n_* is increasing in λ, so $\frac{\partial k_{manager}}{\partial \lambda}$ is still negative and manager chooses to exercising more, the deeper in-the-money. Substituting in the solution for n_* from equation 23,

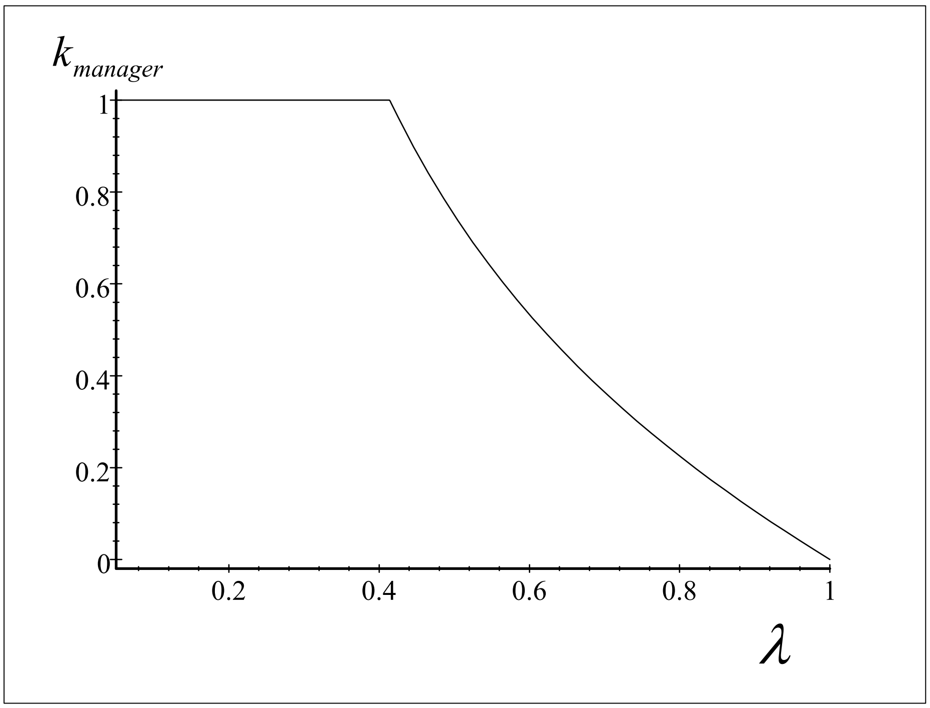

Figure 8: with $\omega = 1$, $k_{manager} < 1$ once $\lambda > \sqrt{2} - 1$. Note also that $k_{manager} = 0$ once $\lambda = 1$ since then the options have lost all time value and so lose nothing in early exercise.

and the expression for k_* from equation 22

$$k_{manager} = k_{part}\left[1 - \frac{1}{2}\frac{(\lambda-\delta)\left(e^{3Q}\lambda^2 - \left(\delta e^{2Q}+1\right)e^{Q}\lambda+\delta\right)}{\lambda\left(e^{Q}-1\right)\left(e^{Q}\lambda-\delta\right)}\right] \tag{36}$$

To the extent that the term $\frac{1}{2}\frac{(\lambda-\delta)\left(e^{3Q}\lambda^2-\left(\delta e^{2Q}+1\right)e^{Q}\lambda+\delta\right)}{\lambda(e^{Q}-1)(e^{Q}\lambda-\delta)}$ is 'small', the manager may exercise approximately the 'right number' of options from the firm's point of view.

E Proofs of Lemmas and Propositions

Proof of Lemma 1. i) Constraint (8) binds when

$$\frac{(W+NM)\left(W+N\left(M+H\right)\right)W}{\left(W+N\left(M-\varepsilon\right)\right)^3} = 1 \tag{37}$$

Simplifying

$$\left(W+N\left(M-\varepsilon\right)\right)^3 - (W+NM)\left(W+N\left(M+H\right)\right)W = 0 \tag{38}$$

Which gives the following equation in N

$$\left[(M-\varepsilon)^3 N^2 + \left(2M^2 - 6M\varepsilon + 3\varepsilon^2 - MH\right)WN - (H-M+3\varepsilon)W^2\right]N = 0 \tag{39}$$

which has solutions

$N=0$, and $\frac{W\left(MH-2M^2+6M\varepsilon-3\varepsilon^2\pm\sqrt{(M^2H^2+6MH\varepsilon^2+4M\varepsilon^3-3\varepsilon^4-4\varepsilon^3H)}\right)}{2(M-\varepsilon)^3}$

The two non-zero roots are respectively positive and negative. We choose the positive one, namely

$$\underline{N}_0 = \frac{W\left(MH-2M^2+6M\varepsilon-3\varepsilon^2+\sqrt{(M^2H^2+6MH\varepsilon^2+4M\varepsilon^3-3\varepsilon^4-4\varepsilon^3H)}\right)}{2\left(M-\varepsilon\right)^3} \tag{40}$$

equivalently

$$\underline{N}_0(\lambda) = \frac{W\left(-2\lambda^2+\lambda\left(1+6\delta\right)-3\delta^2+\sqrt{\lambda^2+2\lambda\delta^2(3+2\delta)-\delta^3\left(4+3\delta\right)}\right)}{2H\left(\lambda-\delta\right)^3} \tag{41}$$

Now equation 39 has positive leading coefficient, so for $N_0 > \underline{N}_0(\lambda)$ we have

$$\left(W+N\left(M-\varepsilon\right)\right)^3 - (W+NM)\left(W+N\left(M+H\right)\right)W > 0 \tag{42}$$

i.e.,

$$1 > \frac{(W+NM)\left(W+N\left(M+H\right)\right)W}{\left(W+N\left(M-\varepsilon\right)\right)^3} \tag{43}$$

breaking the risk-taking constraint (8).

ii)To find the sign of $\frac{\partial}{\partial\lambda}\underline{N}_0(\lambda)$, we transform equation (41) using $\lambda=\delta+y$ and calculate

$\frac{\partial}{\partial y}\left(\frac{\delta^2+2\delta y-2y^2+\delta+y+\sqrt{\left(\delta^2+2\delta y+y^2+2\delta^3+6\delta^2 y+\delta^4+4\delta^3 y\right)}}{y^3}\right) =$

$= -\left(\frac{\left(4y\delta+2y(1-y)+3\delta^2+3\delta\right)}{y^4} + \frac{5\delta y+2y^2+15\delta^2 y+10\delta^3 y+3\delta^2+6\delta^3+3\delta^4}{\sqrt{\left(\delta^2+2\delta y+y^2+2\delta^3+6\delta^2 y+\delta^4+4\delta^3 y\right)}y^4}\right)$ which is certainly negative.

Proof of Corollary2. i) The condition says that there is a risk avoidance problem at least when the options are 100% in-the-money, $\underline{N}_0(1) < N_0$.

However, we have shown that $\underline{N}_0(\lambda)$ is decreasing in λ and we can see that $\underline{N}_0(\lambda) \nearrow \infty > N_0$ as $\lambda \searrow \delta$.

Thus, $\exists \underline{\lambda} \in (\delta, 1)$ s.t. $\underline{N}_0(\underline{\lambda}) = N_0$ whereupon $\underline{N}_0(\lambda) < N_0$, $\forall \lambda > \underline{\lambda}$.

$\underline{\lambda}$ is the solution to

$$\frac{\left(-2\lambda^2 + \lambda(1+6\delta) - 3\delta^2 + \sqrt{\lambda^2 + 2\lambda\delta^2(3+2\delta) - \delta^3(3\delta+4)}\right)}{2(\lambda-\delta)^3} = \frac{1}{\omega} \tag{44}$$

i.e., to

$$\lambda^3 + (2\omega - 3\delta)\lambda^2 + \left(-6\omega\delta + 3\delta^2 - \omega + \omega^2\right)\lambda - \delta^3 - 3\omega^2\delta + 3\omega\delta^2 - \omega^2 = 0 \tag{45}$$

ii) We have shown that the LHS of equation 44 is monotonic decreasing in λ. Increasing N_0 causes the RHS to increase and so reduces the $\underline{\lambda}$ at which equality is achieved.

Proof of Lemma 3. From equation 8, without early exercise, if there is a risk avoidance problem when $n = 0$, then we need to increase n until equation 8 is satisfied, whereupon $\rho(n'_r, N_0, W, M) = 1$ which gives

i)

$$n'_r = \left(\frac{(\omega + (\lambda - \delta))^3}{\omega(\omega + \lambda)} - (\omega + \lambda + 1)\right) N_0 \tag{46}$$

ii) $\frac{\partial}{\partial N_0}\left(\frac{(W+N_0(M-\varepsilon))^3}{WH(W+N_0 M)} - \frac{(W+N_0(M+H))}{H}\right) =$

$$-(\lambda+1) + \frac{(W + N_0(M-\varepsilon))^2}{(W+N_0 M)^2}\frac{W(2M-3\varepsilon) + 2N_0 M(M-\varepsilon)}{WH} \tag{47}$$

which is positive for $N_0 = \underline{N}_0$

And $\frac{\partial^2}{\partial N_0^2}\left(\frac{(W+N_0(M-\varepsilon))^3}{WH(W+N_0 M)} - \frac{(W+N_0(M+H))}{H}\right)$

$= 2(W + N_0(M-\varepsilon))\frac{M^2(M-\varepsilon)^2 N_0^2 + MW(2M-3\varepsilon)(M-\varepsilon)N_0 + W^2\left(\left(M-\frac{3\varepsilon}{2}\right)^2 + \frac{3\varepsilon^2}{4}\right)}{WH(W+N_0 M)^3}$ which is positive

iii) transforming equation 46, with $\lambda = y + \delta$, $y > 0$

$$
\begin{aligned}
&\frac{\partial}{\partial y}\left(\frac{(\omega+y)^3}{\omega(\omega+\delta+y)}-\omega-\delta-y-1\right)\\
=\ &\frac{\omega^3+\omega^2\delta+4\omega^2 y+4\omega y\delta+5\omega y^2+3y^2\delta+2y^3-\omega\delta^2}{\omega(\omega+\delta+y)^2}>0
\end{aligned}
\tag{48}
$$

and

$$
\begin{aligned}
&\frac{\partial^2}{\partial y^2}\left(\frac{(\omega+y)^3}{\omega(\omega+\delta+y)}-\omega-\delta-y-1\right)\\
=\ &2(\omega+y)\frac{\omega^2+3\omega\delta+2\omega y+3\delta^2+3\delta y+y^2}{\omega(\omega+\delta+y)^3}>0
\end{aligned}
\tag{49}
$$

Proof of Proposition 4. With the cash-in of (N_0-N_1) existing options, each of moneyness M, and the granting of n new options, condition (2) becomes

$$
\begin{aligned}
&(W+MN_0+N_1H+nH)(W+MN_0)(W+M(N_0-N_1))\\
\geq\ &(W+MN_0-\varepsilon N_1)^3
\end{aligned}
\tag{50}
$$

i.e.,

$$\left(\omega+\lambda+k_1+\frac{n}{N_0}\right)(\omega+\lambda)(\omega+\lambda(1-k_1))\geq(\omega+\lambda-\delta k_1)^3 \tag{51}$$

i) When $n=0$, if this condition is satisfied then no new options need be granted.

The inequality is breached when

$$(\omega+\lambda+k_1)(\omega+\lambda)(\omega+\lambda(1-k_1))<(\omega+\lambda-\delta k_1)^3 \tag{52}$$

i.e.,

$$k_1\left[k_1^2\delta^3-\left(\lambda+3\delta^2\right)(\omega+\lambda)k_1+(\omega+\lambda)^2(1-\lambda+3\delta)\right]<0 \tag{53}$$

The quadratic in k_1 has solutions

$k_1=\frac{(\omega+\lambda)}{2\delta^3}\left(3\delta^2+\lambda\pm\sqrt{\left(-3\delta^4+6\lambda\delta^2+\lambda^2+4\delta^3\lambda-4\delta^3\right)}\right)$

which are both positive so we take the smaller and our inequality is breached when

$$k_1>\underline{k}_1=\frac{(\omega+\lambda)}{2\delta^3}\left(3\delta^2+\lambda-\sqrt{\lambda^2+\delta^2(6+4\delta)\lambda-(4+3\delta)\delta^3}\right) \tag{54}$$

whereupon then increasing n achieves equality when

$$
\begin{aligned}
\frac{n}{N_0} &= \frac{(\omega+\lambda-\delta k_1)^3}{(\omega+\lambda)(\omega+\lambda(1-k_1))}-(\omega+\lambda+k_1)\\
&= \frac{k_1\left(-\frac{\delta^3}{(\omega+\lambda)}k_1^2+\left(3\delta^2+\lambda\right)k_1-(\omega+\lambda)(1-\lambda+3\delta)\right)}{(\omega+\lambda(1-k_1))}
\end{aligned}
\tag{55}
$$

ii)

The first derivative

$$\frac{\partial}{\partial k_1}\left(\frac{n}{N_0}\right)=\frac{2\delta^3k_1^3\lambda-\left(3\delta^3+3\delta^2\lambda+\lambda^2\right)(\omega+\lambda)\,k_1^2+2\,(\omega+\lambda)^2\left(3\delta^2+\lambda\right)k_1-(\omega+\lambda)^3\,(1-\lambda+3\delta)}{(\omega+\lambda)\,(-\omega-\lambda+\lambda k_1)^2} \tag{56}$$

Now when $k_1=\underline{k}_1$, this has the sign of

$$\begin{aligned}&\left(\lambda^2+3\delta^2\lambda-\delta^3\right)\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)\\&+\left(-\lambda^3-3\lambda\delta^4-2\lambda^2\delta^3+3\delta^3\lambda-6\lambda^2\delta^2+3\delta^5\right)\sqrt{\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)}\end{aligned} \tag{57}$$

To show this is positive, we need

$$\begin{aligned}&\left(\left(\lambda^2+3\delta^2\lambda-\delta^3\right)\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)\right)^2\\>\;&\left(-\lambda^3-3\lambda\delta^4-2\lambda^2\delta^3+3\delta^3\lambda-6\lambda^2\delta^2+3\delta^5\right)^2\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)\end{aligned} \tag{58}$$

i.e.

$$\begin{aligned}0\;<\;&\left(\left(\lambda^2+3\delta^2\lambda-\delta^3\right)\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)\right)^2\\&-\left(-\lambda^3-3\lambda\delta^4-2\lambda^2\delta^3+3\delta^3\lambda-6\lambda^2\delta^2+3\delta^5\right)^2\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)\end{aligned} \tag{59}$$

but the RHS $=4\delta^6\,(1-\lambda+3\delta)\left(\lambda^2+6\delta^2\lambda+4\delta^3\lambda-4\delta^3-3\delta^4\right)(\lambda-\delta)^3$ which is indeed positive

Furthermore, the second derivative, $\frac{\partial^2}{\partial k_1^2}\left(\frac{n}{N_0}\right)$ has the sign of $\left(\delta\lambda k_1+\frac{(\lambda-3\delta)(\omega+\lambda)}{2}\right)^2+\frac{3}{4}(\lambda-\delta)^2(\omega+\lambda)^2$ which is positive.

iii) $\frac{\partial}{\partial\lambda}\left(\frac{n}{N_0}\right)$ can be found as an expression which has the sign of

$$\begin{aligned}&\delta^2k_1^2\,(\omega+\lambda)\,(\lambda+2\omega+2\delta)+\delta^2k_1^2\,(2\lambda+\omega)\,(\lambda-\delta k_1+\omega)\\&+(\omega+\lambda)^2\left(\omega\,(\omega+2\lambda-3\delta)-3\delta^2\right)+(\omega+\lambda)^2\left(\lambda^2+3\delta^2+3\delta\omega\right)(1-k_1)\end{aligned} \tag{60}$$

which is positive.

Proof of Lemma6 i) substituting equation 13 into inequality 17, we see that

If $k_1\le\underline{k}_1$, then $\beta(k_1)=\frac{(\omega+\lambda+k_1)(\omega+\lambda)}{(\omega+\lambda(1-k_1))^2}$

and if $k_1 > \underline{k}_1$, then

$$\begin{aligned}\beta(k_1) &= \frac{\left(\omega+\lambda+k_1+\frac{k_1\left(-k_1^2\delta^3+\left(3\delta^2+\lambda\right)(\omega+\lambda)k_1-(\omega+\lambda)^2(1+3\delta-\lambda)\right)}{(\omega+(1-k_1)\lambda)(\omega+\lambda)}\right)(\omega+\lambda)}{(\omega+\lambda(1-k_1))^2} \\ &= \frac{(\omega+\lambda-\delta k_1)^3}{(\omega+\lambda(1-k_1))^3} = \left(1+\frac{(\lambda-\delta)k_1}{\omega+\lambda(1-k_1)}\right)^3 \end{aligned} \tag{61}$$

ii) If $k_1 \leq \underline{k}_1$, then

$$\frac{\partial}{\partial k_1}\beta(k_1) = (\omega+\lambda)\frac{\omega+\lambda+\lambda k_1+2\lambda\omega+2\lambda^2}{(\omega+\lambda-\lambda k_1)^3} \tag{62}$$

which is positive.

If $k_1 > \underline{k}_1$, then

$$\frac{\partial}{\partial k_1}\beta(k_1) = 3(\omega+\lambda-\delta k_1)^2\frac{(\lambda-\delta)(\omega+\lambda)}{(\omega+\lambda-\lambda k_1)^4} \tag{63}$$

which is also positive

iii) and iv) If $k_1 > \underline{k}_1$, then

$$\frac{\partial}{\partial \lambda}\beta(k_1) = 3(\omega+\lambda-\delta k_1)^2 k_1\frac{\omega+\delta-\delta k_1}{(\omega+\lambda-\lambda k_1)^4} \tag{64}$$

which is positive

If $k_1 \leq \underline{k}_1$, then

$$\frac{\partial}{\partial \lambda}\beta(k_1) = k_1\frac{2\lambda\omega-\omega-\lambda+\lambda k_1+2\omega^2+2\omega k_1}{(\omega+\lambda-\lambda k_1)^3} \tag{65}$$

which is positive if and only if

$$k_1 > \frac{(1-2\omega)(\omega+\lambda)}{(\lambda+2\omega)} \tag{66}$$

Aside: $\beta(1)$ and $\beta(\underline{k}_1)$

$$\beta(1, n_r(1)) = \frac{(\omega+\lambda-\delta)^3}{\omega^3} \tag{67}$$

and

$$\beta(\underline{k}_1) = \frac{\left(1+\frac{1}{2\delta^3}\left(3\delta^2+\lambda-\sqrt{\lambda^2+\delta^2(6+4\delta)\lambda-\delta^3(4+3\delta)}\right)\right)}{\left(1-\frac{\lambda}{2\delta^3}\left(3\delta^2+\lambda-\sqrt{\lambda^2+\delta^2(6+4\delta)\lambda-\delta^3(4+3\delta)}\right)\right)^2} \tag{68}$$

Note that whilst $\beta(k_1)$ is increasing in λ, $\beta(\underline{k}_1)$ is decreasing in λ (because $\underline{k}_1$ is decreasing in λ).

Proof of Proposition 7 *i)* If $\exp 3Z > \beta(1)$, then $k_{part} = 1$ and we need to grant strictly more than $n_r(1)$ new options to satisfy

$$\frac{\left(\omega+\lambda+1+\frac{n}{N_0}\right)(\omega+\lambda)}{\omega^2} = \exp(3Q) \tag{69}$$

which gives directly equation 21.

ii) If $\exp 3Q \in (\beta(\underline{k}_1), \beta(1))$

Then $k_{part} \in (\underline{k}_1, 1)$ and $\beta(k_1) = e^{3Q}$ yields the cubic

$$\begin{aligned} &\left(e^{3Q}\lambda^3 - \delta^3\right) k_1^3 - 3\left(e^{3Q}\lambda^2 - \delta^2\right)(\omega+\lambda) k_1^2 \\ &+3(\omega+\lambda)^2\left(e^{3Q}\lambda - \delta\right) k_1 - (\omega+\lambda)^3\left(e^{3Q}-1\right) \quad = 0 \end{aligned} \tag{70}$$

to which $k_{part} = \left(\frac{(\omega+\lambda)\left(e^{Q}-1\right)}{e^{Q}\lambda-\delta}\right)$ is the unique real solution.

Furthermore, $\frac{n_{part}}{N_0} = \frac{n_r(k_{part})}{N_0}$

$= \dfrac{\left(\frac{(\omega+\lambda)\left(e^{Z}-1\right)}{e^{Z}\lambda-\delta}\right)\left(-\left(\frac{(\omega+\lambda)\left(e^{Z}-1\right)}{e^{Z}\lambda-\delta}\right)^2\delta^3+\left(3\delta^2+\lambda\right)(\omega+\lambda)\left(\frac{(\omega+\lambda)\left(e^{Z}-1\right)}{e^{Z}\lambda-\delta}\right)-(\omega+\lambda)^2(1+3\delta-\lambda)\right)}{\left(\omega+\left(1-\left(\frac{(\omega+\lambda)\left(e^{Z}-1\right)}{e^{Z}\lambda-\delta}\right)\right)\lambda\right)(\omega+\lambda)}$

giving $n_{part} = \frac{\left(e^{2Z}\lambda^2-e^{Z}\lambda-2\lambda\delta e^{2Z}-2e^{Z}\lambda\delta+\delta^2 e^{2Z}+\delta+\delta^2 e^{Z}+\delta^2\right)(\omega+\lambda)\left(e^{Z}-1\right)}{\left(e^{Z}\lambda-\delta\right)^2} N_0$

iii) If $\exp 3Z \leq \beta(\underline{k}_1)$, then $n_{part} = 0$ and we need to retain only enough existing options to satisfy

$$\frac{(\omega+\lambda+k_1)(\omega+\lambda)}{(\omega+\lambda(1-k_1))^2} = \exp 3Q \tag{71}$$

the quadratic

$$e^{3Q}\lambda^2 k_1^2 + \left(-2e^{3Q}\omega\lambda - 2e^{3Q}\lambda^2 - \omega - \lambda\right) k_1 + e^{3Q}\omega^2 + e^{3Q}\lambda^2 + 2e^{3Q}\omega\lambda - 2\omega\lambda - \lambda^2 - \omega^2 = 0 \tag{72}$$

Then $k_{part} \in (0, \underline{k}_1]$ and can be written explicitly

$$k_{part} = \frac{(\omega+\lambda)}{2e^{3Q}\lambda^2}\left(1 + 2e^{3Q}\lambda - \sqrt{\left(1+4e^{3Q}\lambda(1+\lambda)\right)}\right) \tag{73}$$

Proof of Proposition 8 i)$\frac{\partial}{\partial\lambda}(k_{part}) = -\left(e^{Q}-1\right)\frac{\delta+e^{Z}\omega}{\left(e^{Q}\lambda-\delta\right)^2}$ which is certainly negative

so $(1-k_{part})$ is increasing

$\frac{\partial^2}{\partial\lambda^2}(k_{part}) = \frac{\partial^2}{\partial\lambda^2}\left(\frac{(\omega+\lambda)\left(e^{Q}-1\right)}{e^{Q}\lambda-\delta}\right) = 2\left(e^{Q}-1\right)e^{Q}\frac{\delta+e^{Q}\omega}{\left(e^{Q}\lambda-\delta\right)^3}$ which is certainly positive

so $(1-k_{part})$ is concave

ii)

$\frac{\partial}{\partial\lambda}(n_{part})$ has the sign of the expression

$$\begin{aligned} &e^{3Q}(\lambda-\delta)^3 + 3e^{2Q}\delta\left(e^{Q}-1\right)(\lambda-\delta)^2 + \\ &\left(\left(e^{Q}-1\right)2\delta^2 e^{2Q} + \left(\omega + 2e^{Q}\delta\omega - \delta^2\right)e^{2Q} + e^{Q}\delta(1+3\delta)\right)(\lambda-\delta) \\ &+\left(e^{Q}-1\right)\left(\delta^2+\delta+\left(\omega-\delta^2\right)e^{Q}\right)\delta \end{aligned} \tag{74}$$

which is positive.

Proof of Lemma 9 Imagine first that the manager does as is hoped of him and exercises exactly $N_0 - N_{part}$ existing options. He then has the correct incentives for 'risky' and looks forward to expected utility of

$$\begin{aligned} \overline{U}(risky) &= \frac{1}{3}\ln\left(W + M\left(N_0 - N_{part}\right) + \left(M+H\right)N_{part} + Hn_{part}\right) \\ &+\frac{1}{3}\ln\left(W + M\left(N_0 - N_{part}\right) + MN_{part}\right) \\ &+\frac{1}{3}\ln\left(W + M\left(N_0 - N_{part}\right)\right) \\ &-Q \end{aligned} \tag{75}$$

$$\begin{aligned} \overline{U}(risky) &= \ln HN_0 \\ &+\frac{1}{3}\ln\left(\left(\omega + \lambda + k_{part} + \frac{n_{part}}{N_0}\right)\left(\omega+\lambda\right)\left(\omega + \lambda\left(1 - k_{part}\right)\right)\right) - Q \end{aligned} \tag{76}$$

Now imagine the contrary, that he decides to keep some $N_1 > N_{part}$ existing options 'in play' for the second period. Since the IC Risky constraint was binding at $N_1 = N_{part}$, he will no longer have correct incentives to choose 'risky' so will choose an alternative course of action.

The 'safe' expected utility is

$$\overline{U}(safe) = \ln HN_0 + \ln\left(\omega + \lambda - \delta k_1\right) - Q \tag{77}$$

which is strictly decreasing in k_1 and so the manager will not choose to leave vested options unexercise (effectively he would be failing to exercise at the t_1 stock price when he knows that the stock price will fall if he takes the out-of-equilibrium decision 'safe'.

The 'quit' expected utility is

$$\overline{U}(quit) = \ln HN_0 + \ln\left(\omega + \lambda\left(1 - k_1\right)\right) \tag{78}$$

which is strictly decreasing in k_1 and so the manager will not do this (to resign leaving in-the-money vested options unexercised would be irrational).