Prospect Theory and Option Prices:

Evidence from S&P500 Index Options

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Abstract

Using economics "laboratory" experiments, psychologists have demonstrated a variety of behavioural biases which affect individual investors. Investors are loss averse, do not aggregate individual positions but treat them separately, and use subjective probabilities which underweight likely outcomes and overweight unlikely ones (the favourite/longshot bias). Kahneman and Tversky (1979) developed prospect theory to take these features into account. We examine whether these biases are reflected in the prices of options on the S&P500 index. In particular, we would like to know whether cumulative prospect theory can explain the extraordinary steepness of the volatility smile in the loss domain, which is equivalent to a risk-neutral distribution with a fat left-hand tail. We begin by confirming with our data from 1990 to 2004 that there is a favourite/longshot bias in the option prices, such that some options are consistently overpriced and others are consistently underpriced. We then simultaneously estimate the parameters of the subjective probability weights and the prospect-theory pay-off function, under the assumption that there is a representative investor who is following a portfolio-insurance strategy. This allows us to make (to the best of our knowledge) the first systematic test of whether the parameters estimated in a market environment are similar to those found in laboratory experiments.

Introduction

It is widely accepted that individual investors do not always behave in a way which is consistent with the maximisation of expected utility. They face difficulties of information about the appropriate models and it can be costly to obtain the necessary data about market conditions. If professional investors are the dominant group, then the paradigm of rational agent behaviour may yet apply (Fama 1998). On the other hand professionals may not always be dominant in the short-run, as seems to have happened in the DotCom bubble (Ofek and Richardson 2003).

In this paper we examine whether the "biases" to which psychologists have drawn attention can help to explain peculiarities in the pricing of S&P500 index options. To do this we are going to apply prospect theory (Kahneman & Tversky 1974, 1979, Tversky & Kahneman 1992) to options pricing. To the best of our knowledge, this is the first paper to make a systematic test of prospect theory in this way.

The focus of our paper is on implementing cumulative prospect theory (Tversky and Kahneman, 1992, KT hereafter) which incorporates three particular features of investor behaviour which run counter to the neoclassical view. The first is that they are loss-averse, so that the pain of a losing position is greater than the joy from one which is winning. The second is that the individual does not consider his or her total wealth when making a decision but rather each individual investment is treated separately. There is no aggregation of positions into total wealth as there is in portfolio theory. The third feature is that individuals tend to make biased probability estimates; they underweight the probability of likely outcomes and overweight the probability of unlikely outcomes. This has become known as the "favourite/longshot bias", as it has been widely discussed in the context of horse-racing (Thaler and Ziemba, 1988).

In this paper we ask whether or not some of these biases are present in aggregate data on broad asset and investment classes, in particular using market data associated with financial derivatives on stock indices. This is a challenging environment in which to test these theories since these markets are heavily traded by a large number of market participants, so that any bias present would have to be common to a very wide number of participants who have the ability to lay off many aspects of their risks.

We have chosen to apply the theory to S&P500 options because they provide a very rich cross section of data points. For each day it is simple to infer a risk-neutral density and to be confident in the result. Our procedure begins by assuming that the objective (real-world) density is log-normal. We then use the data for that day to determine not only the prospect-theory parameters but also the location and volatility of the real-world density. The procedure minimises the squared errors of the observed prices from their model values. In effect, we are using prospect theory as a structural model for the transformation of real-world distributions to risk-neutral distributions. Several researchers have drawn attention to the peculiarities which exist in index-option prices, most notably to the steepness of the volatility smile (e.g. Bates 2000, Bollen and Whaley 2003, Branger and Schlag 2004). There has been much debate about whether the preferences underlying such a skewed distribution can be explained in a rational framework (Jackwerth 2000, Rosenberg and Engle 2002, Bliss and Panigirtzoglou, 2002, Brown and Jackwerth 2004). Our aim is to see whether cumulative prospect theory be used to explain this apparent mispricing.

Data

We use CBOE data for (SPX) options on the spot S&P500 value, these are European style. For the period 1990 to 2004 our dataset contains options on a monthly as well as quarterly cycle i.e. up to 12 expiry months in a year (expiry is on the third Friday of the month). However the off quarter months, Jan, Feb, Apr etc, only trade closer in time to their expiry than Mar, Jun, Sep, Dec. Many strikes are traded within each series, strikes have a minimum separation of 5 index points although the initial interval is larger when the contract is first introduced. It does however provide a large dataset of option prices (almost one million) across approximately 2,500 trading days. A typical cross section of first nearby contracts may contain a total of more than 150 call or put contracts – the second nearby etc, contain fewer[1].

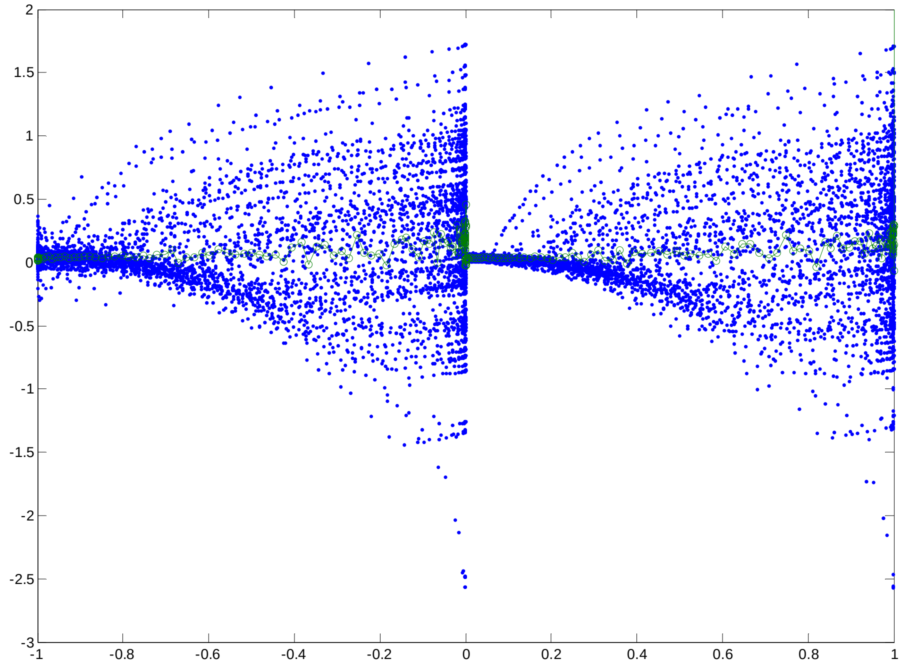
[1] Thanks go to Fergal O'Brien for his help with this data.

Our paper is most closely related to Rasiel (2003) and Hodges, Tompkins and Ziemba (2003), both of which also try and account for option-pricing anomalies through the use of prospect theory. However, in this paper we are careful to use methods (for preferences) which although inconsistent with marginal utilities (which should be monotonic) are not inconsistent with aggregation and other pricing (no arbitrage) techniques.

We also have a more comprehensive dataset; by way of example (the following) Figure 1 which uses a large number of option returns to illustrate some of the features discussed. For holding periods of 21 days, the (annualised) returns from portfolio insurance strategies are shown. The panel on the left constructs these with the underlying index and a protective put (S+P) while the panel on the right uses cash plus a call (C+X), both do so for different levels of X i.e. insurance level. The returns are plotted agains the ex-ante probability estimated using risk neutral Black Scholes measure.

A can be seen from the graph, either the risk premium is low, the probabilities are mis-estimated or the options themselves are subject to pricing biases, since the realised returns and their bin averages (in circles) do not appear to increase much across the probability range. A combination of the factors could also contribute to this feature.

Figure 1: Returns and average returns (circles) to portfolio insurance strategies of different probability



In the sections that follow, we will outline some asset pricing theory which is needed to show how we adapt the KT preference into a pricing function. We will also outline which subjective probability transformations are appropriate in this setting. We then use these to fit observed option prices for many days over the period 1990 to 2004. Finally, we will look at the sample historical realisations and test their distribution against those implied from density and option estimates.

Asset pricing and Prospect Theory

Non-normal returns pose particular challenges and opportunities to asset pricing theory. In particular the potential existence and direction of skewness in the distribution of market returns (accompanied with a signed skewness preference) greatly increases the range of phenomena that asset pricing can tackle (see Barberis, Huang & Santos 2001 and Barberis & Huang 2004).

This is particularly important in the context of option pricing since the main, and most persistent, feature derived from option prices is the degree of negative skewness implied by the risk neutral density. This is to say that when option prices are used to infer an implied risk neutral density (RND), the result is a distribution which is more negatively skewed than can be justified by subsequent historical realisations.

One possible explanation of this artefact is that ever since October 1987 (Bates 2000), market participants have anticipated the possibility of a crash of similar magnitude even though it has not occurred in almost 20 years. Clearly the *possibility* of another such crash raises the question of the magnitude of its *probability,* which may have been previously been considered zero.

This negative skewness is difficult to model and price using (risk neutral) diffusion dynamics. The Black Scholes model itself has long been known to possess a smiling, not constant, implied volatility as a function of future asset price states (other models such as Heston's which include stochastic volatility are also difficult to calibrate with one parameter set). Furthermore using utility transformations alone (see Jackwerth 2000, Rosenberg and Engle 2002, Liu et al 2003), it is difficult to account for the difference in skewness across each of the following (three) density types; those implied from options, those from modelled diffusions and finally those sampled from historical data. This is why we believe that a prospect theory approach that embraces subjective probability distributions and (non utility) decision weights will be more successful.

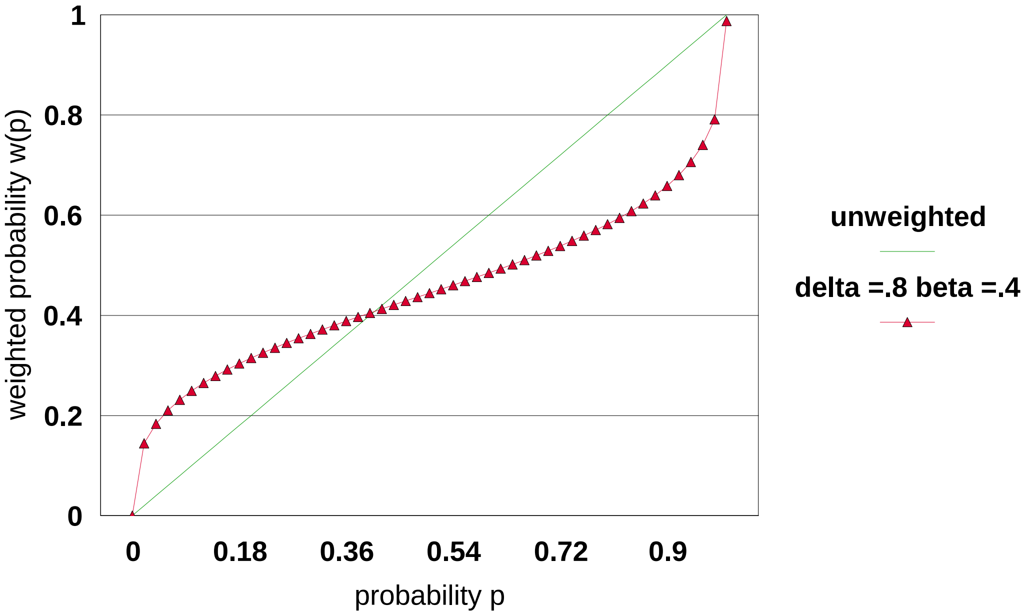
<u>Subjective probability densities (favourite/longshot bias)</u>

Within prospect theory there are decisions to be made at two levels. First is a weighting $w(P)$ function that the (true and cumulative P) probability from a distribution will receive when a (biased) individual forms an opinion. Many such weighting functions w exist (see Prelec 1998 and Stott 2003) but we use the so called (Goldstein Einhorn) weighting function. These are tractable having the advantage of being linear in log odds. The uncertain object is the stock index level S_T at some distant time T.

$$w(P(S_T)) = \frac{\delta P(S_T)^{\beta}}{\delta P(S_T)^{\beta} + (1 - P(S_T))^{\beta}}$$

The two parameters, beta and delta control the relative moments of the derived distribution. Figure 2 shows an example of a transformation with beta=0.4 and delta=0.8.

<u>Figure 2: Weighted and Unweighted Probabilities</u>



<u>Objective densities</u>

The one thing all market participants can agree on are the option prices themselves and therefore their implied RNDs. Therefore flexible models are important since we would like to be able to fit RNDs at a first stage.

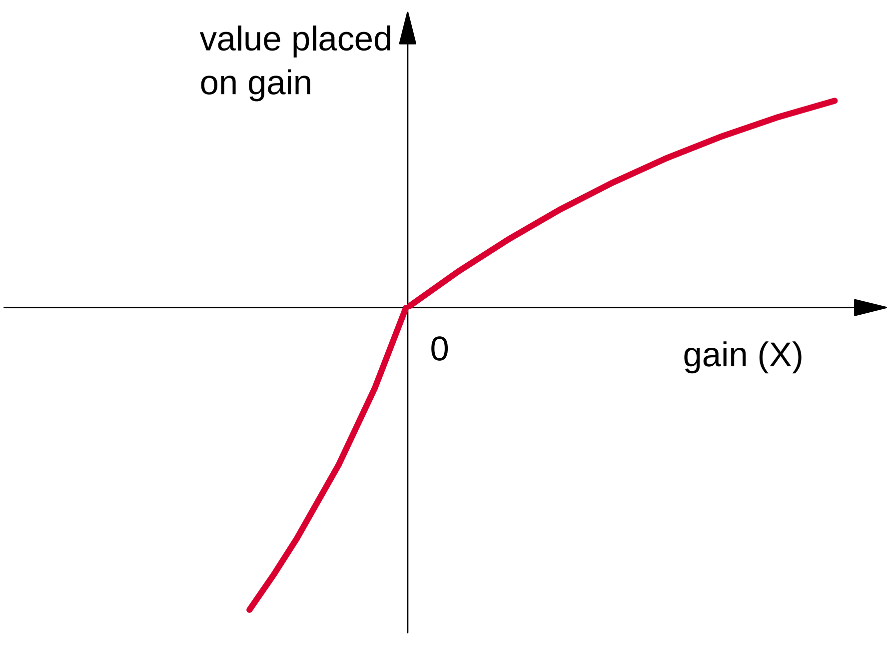
Modelling choices – Value function

A form of derived “utility” must be chosen. The most common form in prospect theory has a threshold above and below which behaviour is different. To the right (above) the threshold the value function is convex below it is concave, both positive and negative convexity captured through a parameter alpha. These so called gain and loss domains are also different in that the (local) slopes at the threshold can differ on either side. This is captured by a second parameter lambda.

$$KTV(S_T) = S_* + |S_T - S_*|^{\alpha}$$
$$KTV(S_T) = S_* - \lambda|S_T - S_*|^{\alpha}$$

The first line applies in the case where the terminal stock index level exceeds a critical threshold S_*, ($S_T > S_*$) whereas the second applies in the other case ($S_T < S_*$). There is a restriction on parameters S_* and lambda that prevents the derived KT value function from becoming negative. Figure 3 shows a typical pattern for a prospect theory value function, note its asymmetry in convexity and slope.

Figure 3: The Value Function in Prospect Theory



Note that if lambda = alpha = 1 the value function reverts to linear which is consistent with risk neutrality, thus the models we estimate encompass risk neutrality as a special case. This means that when fitting prices, we would expect a model with no lambda or alpha restrictions to do better than a (parameterised) fit of risk neutral densities and pricing.

We then use these two modelling features of prospect theory in a similar fashion to the change in probability measure between real (RW) and risk neutral (RN) worlds that is used in asset pricing along with the marginal utility measure that is also embedded in the asset pricing kernel.

We aim to account for the stylised facts from options markets using either a probability transformation or a behavioural (and potentially irrational) value function or a combination of both. Much of this can be done within the asset pricing framework by relaxing the monotonicity of marginal utility assumption alone.

Results & Conclusion

As well as presenting further results for returns derived from a portfolio insurance strategy, the remainder of the paper will show results of calibrations of the proposed models. These will then be tested to examine the extent to which they can account for some of the biases observed.

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