Unifying Behavioral Biases Under a Market Probability Measure with an Application to Analysts' Forecasts*

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Abstract

We provide a methodology for examining the existence and magnitude of behavioral biases in an asset's return distribution. The influence of behavioral biases is modeled using a firm-specific market probability measure which represents the market's beliefs regarding the possibility of a price increase. Behavioral biases which distort this market probability create predictable abnormal returns. Distortions of the market probability also cause realized returns to be excessively volatile, although ex-ante return volatility is underestimated. Using analyst forecasts and revisions, proxies for representativeness, conservatism, overconfidence, and biased self-attribution are constructed. A trading strategy derived from their joint influence yields an annual return of 13.56% after accounting for size, book-to-market, and momentum characteristics. Furthermore, the susceptibility of individual stock returns to behavioral biases is industry-specific and time-varying.

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1 Introduction

Despite the preponderance of psychological evidence that individuals exhibit systematic errors when making decisions, one cannot immediately conclude that behavioral biases influence stock returns since the market may be superior to any individual when processing information. Indeed, even if the majority of investors commit behavioral biases, the tenets of market efficiency are preserved whenever the marginal investor responsible for determining prices is *rational*.[1]

Equally as important, provided behavioral biases are detected in stock returns, determining their relative magnitude is essential. Several biases have been proposed in the behavioral finance literature to explain anomalies such as momentum (Jegadeesh and Titman (1993)). Using representativeness and conservatism, Barberis, Shleifer, and Vishny (1998) generate over-reaction and under-reaction in stock returns. Daniel, Hirshleifer, and Subrahmanyam (1998) obtain similar return patterns for an overconfident investor exhibiting biased self-attribution. Therefore, it is critical to ascertain whether investor psychology influences stock returns, and if so, which behavioral biases are most important. This paper provides a methodology for examining these two central issues in behavioral finance.

The cornerstone of our framework is a firm-specific market probability measure which establishes a general relationship between behavioral biases and the distribution of an individual asset's return. This probability represents investor beliefs regarding price movements next period. Under binomial price dynamics, the market probability is summarized by the probability of a price increase. With regards to behavioral finance, this probability is distorted when the market conditions on superfluous information which has no bearing on the true price (return) process. As an illustration, consider a fair coin whose probability of heads is thought to differ from one-half because the previous three realizations have all been heads. In a financial context, a sequence of positive earnings surprises could improperly influence the market probability measure due to representativeness if investors falsely perceive a trend in this firm's earnings. In general, any behavioral bias has the potential to distort the market probability.

[1] Although limits to arbitrage can prevent certain market participants from exploiting mispricings, the assumption of no-arbitrage only requires a small number of patient well-funded investors to exist. Therefore, it is difficult to reject market efficiency by studying a subset of investors unless their trading behavior can be generalized to all market participants.

There are several theoretical implications of a biased market probability measure. Fluctuations in the market probability across time cause realized returns to be excessively volatile, although ex-ante return volatility is underestimated. Distortions of the market probability also generate predictable abnormal returns. However, our methodology does not assume investors commit *specific* psychological biases. Instead, we study whether abnormal returns documented in the empirical asset pricing literature are explained by behavioral biases. Thus, our methodology enables researchers to investigate the link between psychology and asset returns. The market probability is also capable of incorporating investor preferences such as loss aversion and Shefrin and Statman (1985)'s disposition effect. Furthermore, a slight generalization of the market probability measure relaxes the assumption of risk neutrality.

Market probabilities can be estimated experimentally, as in Bloomfield and Hales (2002), or with historical return data.[2] To facilitate our empirical study, we construct proxies for representativeness, conservatism, overconfidence, and biased self-attribution using analysts' earnings forecasts and revisions for 2,087 firms from 1986 to 2004. Calibrating the market probability is accomplished via a non-linear regression involving firm-specific returns and volatilities.

Linear regression is a restrictive special case of our probability approach, in which several important economic implications and parameter interpretations are lost. Furthermore, the statistical relationship between behavioral biases and stock returns depends on an individual stock's return volatility at various points in time. However, a linear regression of stock returns on the behavioral bias proxies ignores variation in a firm's return volatility over time, and further assumes this parameter is identical across different stocks. These misspecifications are corrected by our market probability framework.

As a direct "out-of-sample" test of our ability to exploit behavioral biases, we sort firms according to their estimated market probabilities in calendar-time. Trading strategies which buy (sell) stocks whose estimated market probabilities are above (below) certain thresholds are implemented. These strategies attempt to profit from the behavioral biases underlying our market probability estimates for individual stocks. Market probabilities arising from a combination of representativeness and conservatism as well as overconfidence and biased self-attribution are studied, in conjunction with a past return control variable. The inclusion of past returns enables us

[2] Asparouhova, Hertzel, and Lemmon (2005) re-examine and extend the experimental market of Bloomfield and Hales (2002).

to ascertain the marginal impact of behavioral biases on stock returns.

Empirically, when all behavioral biases are incorporated into a firm-specific market probability, the corresponding trading strategy yields an average cross-sectional return of 13.56% per annum after adjustments for book-to-market, size, and momentum characteristics. A trading strategy that focuses on representativeness and conservatism generates an even larger annual return of 14.45% on average, while a strategy derived from overconfidence and biased self-attribution produces positive but insignificant trading profits. In contrast, cross-sectional returns derived from linear regression models are always much smaller and insignificant. Overall, risk proxies and alternative return benchmarks cannot explain the significance of our trading profits.

We also implement a trading strategy when market probabilities are conditioned on the underlying earnings surprises and revisions of each firm. The insignificant trading profits from this strategy confirm the importance of our behavioral bias proxies since analysts' forecasts and revisions alone cannot generate significantly positive cross-sectional returns.

Furthermore, fluctuations in the market probability measure create excess return volatility averaging 4.75% per annum across the stocks in our sample. These fluctuations also yield positive as well as negative return autocorrelation with return predictability being induced on average. The above empirical results originate from a cross-sectional calibration within 66 different industries over four-year subperiods. Empirically, the susceptibility of individual stock returns to behavioral biases appears to vary across time and between industries.

To summarize, our market probability methodology is designed for empirical studies regarding the influence of investor psychology on stock returns. Furthermore, the estimation of firm-specific market probabilities facilitates trading strategies capable of exploiting behavioral biases.

The remainder of this paper begins with the introduction of the market probability framework and its return implications in Section 2. Section 3 describes the implementation of our methodology, while endogenous and exogenous specifications for the behavioral biases we investigate are the subject of Sections 4 and 5 respectively. Empirical results are reported in Section 6, with Section 7 offering our conclusions and suggestions for future research.

2 Market Probability Measures and Return Distributions

Price dynamics are described in discrete-time since our methodology is intended for empirical implementations. The binomial approximation to the lognormal Brownian motion process (Cox, Ross and Rubinstein (1979)), defines up and down price movements as

$$\begin{aligned} U_t &= \exp\{\mu_t + \sigma_t\} \\ D_t &= \exp\{\mu_t - \sigma_t\} , \end{aligned} \tag{1}$$

where μ_t and $\sigma_t > 0$ represent the asset's true expected return and its volatility. More formally, these coefficients equal $\mu_t\Delta$ and $\sigma_t\sqrt{\Delta}$ where Δ denotes the time interval between observations. For notational simplicity, this horizon is incorporated into the μ_t and σ_t parameters by setting $\Delta = 1$ until the model's estimation.

The market probability $\left\{P^U(t), P^D(t)\right\}$ below defines the probabilities assigned to the U_t and D_t price movements in equation (1)

$$\begin{aligned} P^U(t) &= \frac{1}{1+e^{-\alpha z_t}} \\ P^D(t) &= 1 - P^U(t) , \end{aligned} \tag{2}$$

which are both elements of the $[0, 1]$ interval and sum to one. Non-zero α coefficients imply the market probability conditions on a superfluous information set denoted z_t. For example, an element of z_t may indicate a trend in a firm's earnings surprises which enables representativeness to alter its market probability. The role of z_t is elaborated on in the next subsection.[3] Furthermore, other functions besides equation (2) yield identical economic conclusions. In particular, the results of Propositions 1 and 2 introduced later in this section are valid for any $P^U(t)$ specification. For emphasis, equation (2) represents an a firm-specific market probability that aggregates over the beliefs of individual market participants. Thus, individual investors are able to commit behavioral

[3]Shefrin (2000) examines the role of heterogeneity among investor beliefs in a multiperiod economy whose implications are illustrated using the binomial model. This paper also provides a formal definition of investor sentiment as a log ratio of two probability measures.

biases without distorting the market probability if their biases offset one another. For example, the beliefs of a contrarian and momentum trader can neutralize one another.[4]

Two probability measures exist simultaneously in our framework, both of which are indexed by their respective α vector. The first is referred to as the *reference* probability which has $\alpha = 0$. By definition, the reference probability is not influenced by superfluous information. Therefore, if returns are influenced by behavioral biases, they cannot be generated by the reference probability. Instead, the *market* probability with $\alpha \neq 0$ describes their distribution.

Each probability measure implies a conditional expected return $E\left[\text{Return} \middle| P^U(t)\right]$ denoted y_t, along with its conditional variance $Var[y_t]$. Combining equation (1) and equation (2) with $\alpha = 0$ implies the conditional expected return under the reference probability measure equals

$$\begin{aligned} y_t\big|_{P^U(t)=\frac{1}{2}} &= \ln(U_t)\,\frac{1}{2} + \ln(D_t)\,\frac{1}{2} \\ &= \frac{1}{2}\left[\mu_t + \sigma_t + \mu_t - \sigma_t\right] \\ &= \mu_t\,. \end{aligned} \tag{3}$$

Thus, under the reference probability, the asset's true expected return is recovered. In contrast, for $\alpha \neq 0$, the market probability measure implies the following conditional expected return

$$\begin{aligned} y_t\big|_{P^U(t)\neq\frac{1}{2}} &= \ln(U_t)\,P^U(t) + \ln(D_t)\,P^D(t) \\ &= \left[\mu_t + \sigma_t\right]P^U(t) + \left[\mu_t - \sigma_t\right]P^D(t) \\ &= \mu_t + 2\sigma_t\left[P^U(t) - \tfrac{1}{2}\right]\,, \end{aligned} \tag{4}$$

since $P^U(t) - P^D(t) = 2\left[P^U(t) - \frac{1}{2}\right]$. The return decomposition in equation (4) is interpreted as

$$\text{market's expected return} \;=\; \text{true expected return} + \text{distortion when } \alpha \neq 0\,.$$

Thus, the market's conditional expected return y_t is determined by the μ_t and σ_t parameters in equation (1) as well as $P^U(t)$ in equation (2). For ease of exposition, y_t refers to the conditional expected return under the market probability throughout the remainder of this paper, while μ_t corresponds to the reference probability.

[4]In addition, a non-zero α vector could produce $P^U(t) = \frac{1}{2}$ if the inner product αz_t in equation (2) is zero. Thus, even if every investor commits an identical set of behavioral biases, the biases themselves could offset one another to eliminate any distortion in the market probability.

According to equation (4), abnormal returns are defined as

$$y_t - \mu_t \;=\; 2\sigma_t \left[P^U(t) - \tfrac{1}{2}\right] , \tag{5}$$

under the $P^U(t)$ specification in equation (2). The inequalities

$$-\sigma_t \leq \;\; y_t - \mu_t \;\; \leq \sigma_t \tag{6}$$

arise from equation (5) since $P^U(t)$ is contained in the [0,1] interval. Thus, abnormal returns are bounded by the true volatility of the asset's nominal return (usually 30% to 60% per annum for stocks) which is independent of all behavioral biases. The definition of abnormal returns in equation (5) is modified slightly when a reference probability measure different from $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ is studied in Appendix A but the model's underlying economics are preserved. Equation (5) illustrates that the α coefficients underlying $P^U(t)$ in equation (2) capture the source of abnormal returns. Moreover, non-zero values of $P^U(t) - \frac{1}{2}$ in equation (5) are scaled by twice the asset's return volatility. Consequently, for a given distortion in the market probability, more volatile stocks generate larger abnormal returns. This relationship serves an important role in our estimation of the α coefficients but undermines the appropriateness of simply regressing the $y_t - \mu_t$ deviations on z_t as illustrated in the next section.

Besides generating abnormal returns, non-zero α coefficients also cause ex-ante return volatility to be underestimated. These assertions are formalized in the next proposition.

Proposition 1. *Non-zero α coefficients imply:*

1. *Abnormal expected returns.*

2. *Underestimation of ex-ante return volatility.*

Proof: From equation (5), abnormal returns $y_t - \mu_t$ are positive (negative) when $P^U(t) > \frac{1}{2}$ $\left(P^U(t) < \frac{1}{2}\right)$, while the variance of the conditional expected return equals[5]

$$\begin{aligned} Var[y_t] &= P^U(t)\;[\mu_t + \sigma_t - y_t]^2 + P^D(t)\;[\mu_t - \sigma_t - y_t]^2 \\ &= P^U(t)\;\left[\sigma_t - 2\,\sigma_t\;\left[P^U(t) - \tfrac{1}{2}\right]\right]^2 + P^D(t)\;\left[-\sigma_t - 2\,\sigma_t\;\left[P^U(t) - \tfrac{1}{2}\right]\right]^2 \\ &= \sigma_t^2 + 4\,\sigma_t^2\;\left[P^U(t) - \tfrac{1}{2}\right]^2 + \left[P^D(t) - P^U(t)\right]\;4\,\sigma_t^2\;\left[P^U(t) - \tfrac{1}{2}\right] \\ &= \sigma_t^2 - 4\,\sigma_t^2\;\left[P^U(t) - \tfrac{1}{2}\right]^2 . \end{aligned} \tag{7}$$

[5]The statistical intuition behind equation (7) follows from $p = \frac{1}{2}$ being the minimum of $p\,(1-p)$ which equals the variance of a Bernoulli distribution.

Thus, the asset's conditional return volatility is underestimated when $\alpha \neq 0$. □

Observe that an upward bias in the market probability, $P^U(t) > \frac{1}{2}$, causes y_t to exceed μ_t. Intuitively, positive abnormal returns are created by this "optimism" regarding next period's price movement. Next period, $P^U(t+1)$ may continue to increase or revert towards one-half. Thus, expected returns in the future remain dependent on the market probability measure.

Furthermore, although non-zero α coefficients cause the market to underestimate return volatility, this property is not identical to overconfidence. The market probability represents the aggregate beliefs of the market, while overconfidence stems from the beliefs of individual investors regarding their ability to access and interpret information.

2.1 Role of Superfluous Information

We define *irrelevant* information as anything which has no bearing on an asset's true expected return. In contrast, the asset's true expected return μ_t is defined by *fundamental* information. The return decomposition in equation (4) illustrates the economic repercussions of non-zero α coefficients, regardless of whether z_t contains irrelevant or fundamental information.

In particular, non-zero α coefficients imply

1. The market conditions on *irrelevant* information.

2. Fundamental information is *incorrectly processed* by the market.

In the first scenario, irrelevant information in z_t which alters the market's expected return y_t, but not μ_t, distorts $P^U(t)$ according to equation (5). As a result, non-zero α coefficients reveal the market improperly conditions on irrelevant information when forming its beliefs regarding future price movements.

The market's ability to correctly process fundamental information is evaluated in the second scenario. Intuitively, after being incorporated into μ_t, fundamental information cannot alter the market probability. Thus, all information becomes superfluous after being correctly incorporated into the asset's true expected return. Consequently, provided μ_t reflects the proper processing of available fundamental information, only $\alpha = 0$ is consistent with its correct interpretation by the market.[6] Otherwise, the market is overreacting or underreacting to the release of fundamental information.

The behavioral literature often distinguishes between biases in *beliefs* versus *preferences* with examples of the later being loss aversion and investor disposition. Loss aversion consistent with Tversky and Kahneman (1992)'s prospect theory is documented by Coval and Shumway (2005). Empirical evidence of Shefrin and Statman (1985)'s disposition effect is reported in Barber, Odean, and Zhu (2003) as well as Shumway and Wu (2005) who demonstrate the predictive power of unrealized gains for returns. When the superfluous information set has z_t elements representing unrealized gains or prior losses, the market probability infers the influence of biased preferences on returns. Therefore, our probability approach is able to examine biased beliefs as well as preferences.[7]

For clarification, the superfluous information set is not necessarily comprised of residuals which are zero on average. Regardless of $|z_t|$'s magnitude, $\alpha = 0$ prevents superfluous information from distorting the market probability.

2.2 Return Predictability and Excess Volatility

The market probability approach also enables us to analyze the time series properties of returns. In particular, the second term of equation (4) may generate predictability and excess volatility (Shiller (1981)) is realized returns. In Proposition 2 below, we examine the contribution of non-zero α coefficients to these properties.

For notational simplicity, define B_t as $2\sigma_t\left[P^U(t) - \frac{1}{2}\right]$, and consider a sequence of returns $y_1, \ldots, y_t$ conditioned on $P^U(1), \ldots, P^U(t)$ respectively. The next proposition examines the time series characteristics of returns attributable to the $B_1, \ldots, B_t$ sequence which arises from non-zero α coefficients.

[6] If the market ignores fundamental information, then μ_t is altered while y_t remains unchanged. This scenario also requires a bias in the market probability measure, and therefore non-zero α coefficients.

[7] Barberis and Huang (2004) examine security pricing when objective probabilities are transformed via a weighting function into a probability distribution whose tails are overweighted. Their approach focuses on a *permanent transformation* of objective probabilities. In contrast, we examine time-varying distortions in a probability that result from conditioning on superfluous information.

Proposition 2. *Within a sequence of returns, non-zero α coefficients imply:*

1. *Excess return volatility.*

2. *Autocorrelated returns.*

Proof: Denote the sample variance for the time series of true expected returns $\mu_1, \ldots, \mu_t$ as σ^2_μ. According to equation (4), the sample variance for the corresponding time series of market returns $y_1, \ldots, y_t$ equals

$$\begin{aligned} & Var\left[y_1, \ldots, y_t \middle| P^U(1), \ldots, P^U(t)\right] \\ = \; & Var\left[\mu_1, \ldots, \mu_t\right] + Var\left[B_1, \ldots, B_t\right] + 2\,Cov\left[\{\mu_1, \ldots, \mu_t\}, \{B_1, \ldots, B_t\}\right] . \end{aligned} \tag{8}$$

When the covariance term above is negative, excess return variability remains positive provided $Var\left[B_1, \ldots, B_t\right]$ is larger than twice the absolute value of $Cov\left[\{\mu_1, \ldots, \mu_t\}, \{B_1, \ldots, B_t\}\right]$. Otherwise, behavioral biases reduce return volatility. A negative covariance in equation (8) is consistent with the market underreacting to fundamental information. Conversely, a positive covariance in equation (8) indicates the market overreacts to fundamental information. When $\alpha = 0$, the asset's return variance reduces to $Var\left[\mu_1, \ldots, \mu_t\right]$ since the $B_1, \ldots, B_t$ sequence is identically zero. The variance decomposition in equation (8) is interpreted as

$$\text{realized variance} \;=\; \text{true variance} + \text{excess variability attributable to } \alpha \neq 0\,.$$

Similarly, return autocorrelation in the $y_t, \ldots, y_1$ sequence is defined as

$$Corr\left[\{\mu_t + B_t, \ldots, \mu_2 + B_2\}, \{\mu_{t-1} + B_{t-1}, \ldots, \mu_1 + B_1\}\right] \tag{9}$$

which equals

$$\begin{aligned} & Corr\left[\{\mu_t, \ldots, \mu_2\}, \{\mu_{t-1}, \ldots, \mu_1\}\right] \\ + \; & Corr\left[\{\mu_t, \ldots, \mu_2\}, \{B_{t-1}, \ldots, B_1\}\right] + Corr\left[\{\mu_{t-1}, \ldots, \mu_1\}, \{B_t, \ldots, B_2\}\right] \\ + \; & Corr\left[\{B_t, \ldots, B_2\}, \{B_{t-1}, \ldots, B_1\}\right] . \end{aligned} \tag{10}$$

When $\alpha = 0$, equation (10) reduces to the autocorrelation of $\mu_t, \ldots, \mu_1$ in the first term, while the second and third lines of equation (10) result from non-zero α coefficients. □

To clarify, the second implication of Proposition 1 concerns the ex-ante underestimation of next period's return volatility while Proposition 2 applies to a sequence of returns. Non-zero α coefficients can also generate skewness and kurtosis in return data although these higher order moments are not investigated in this paper.

3 Calibrating the Market Probability Measure

This section demonstrates that the α coefficients in equation (2) can be calibrated from a non-linear regression involving historical returns. The estimation of alternative reference probabilities is also addressed as well as the advantages of our probability approach over linear regression.

3.1 Non-Linear Calibration

Each α coefficient determines the relative influence of a behavioral bias on an asset's conditional return distribution. The main result of this section is the following corollary of Proposition 1 which inserts $P^U(t)$ from equation (2) into the return decomposition of equation (4).

Corollary 1. *The α coefficients may be estimated by the following non-linear regression*

$$y_t - \mu_t = 2\sigma_t \left(\frac{1}{1+e^{-\alpha z_t}} - \frac{1}{2} \right) + \epsilon_t \, , \tag{11}$$

where ϵ_t are i.i.d. error terms. Equation (11) is equivalent to

$$\frac{y_t - \mu_t}{2\sigma_t} + \frac{1}{2} = \frac{1}{1+e^{-\alpha z_t}} + \epsilon_t \, , \tag{12}$$

once σ_t is specified for an individual firm.

Corollary 1 illustrates the explicit relationship between abnormal returns and superfluous information which results from the market probability. The need to normalize $y_t - \mu_t$ by $2\,\sigma_t$ before estimating the α coefficients is a crucial.[8]

Of interest to future research, a time series of probabilities $P^U(t)$ could be elicited from an experimental market. For example, Bloomfield and Hales (2002) as well as Asparouhova, Hertzel,

[8] The return implications of Proposition 2 could also facilitate the calibration of our market probabilities. However, equations (8) and (10) require sample estimates for the respective variance and autocorrelation of the $\mu_t, \ldots, \mu_1$ sequence. The accuracy of these sample moments is compromised when the α coefficients are computed cross-sectionally over short horizons as in Section 6. Therefore, our estimation procedure focuses on equation (4).

and Lemmon (2005) have subjects set prices equal to their estimated probability of an upward price movement. This experimental approach eliminates the need to specify $\frac{y_t - \mu_t}{2\sigma_t}$ in equation (11) when calibrating the α vector. Thus, the joint-hypothesis regarding abnormal returns and the correct model of market equilibrium is circumvented. Instead, α coefficients can be estimated directly from the $P^U(t)$ probabilities reported by participants in a laboratory market using equation (2).

3.2 Special Case of Linear Regression

We now contrast our proposed methodology with linear regression to highlight its economic and statistical advantages. Specifically, regressing the $y_t - \mu_t$ deviations on the superfluous information set

$$y_t - \mu_t = \alpha_0 + \alpha\, z_t + \epsilon_t\,, \tag{13}$$

is a special case of our market probability since inserting the linear formulation

$$P^U(t) \;=\; \frac{1}{2} - \frac{\lambda}{2\sigma_t} + \frac{\alpha}{2\sigma_t} z_t\,, \tag{14}$$

into the market's conditional expected return given by equation (4) yields

$$y_t - \mu_t = -\lambda + \alpha z_t\,, \tag{15}$$

which parallels the expectation of the linear regression in equation (13). Although the role of $-\lambda$ in equation (15) appears similar to the α_0 intercept in equation (13), the economic significance of λ is considerable and the subject of the next subsection.

3.3 Altering the Reference Probability

A non-zero λ parameter, independent of the superfluous information, can generalize equation (2) as follows

$$P^U(t) \;=\; \frac{1}{1 + e^{\lambda - \alpha z_t}}\,. \tag{16}$$

The λ intercept in equation (16) indexes the reference probability measure. Specifically, when $\lambda \neq 0$, the reference probability measure $\left\{P^U(t), P^D(t)\right\}\big|_{\alpha=0}$ equals $\left\{\frac{1}{1+e^{\lambda}}, \frac{e^{\lambda}}{1+e^{\lambda}}\right\}$ instead of $\left\{\frac{1}{2}, \frac{1}{2}\right\}$. Thus, a positive λ parameter implies the relationship $P^U(t) < \frac{1}{2} < P^D(t)$ when $\alpha = 0$, with larger $P^D(t) - P^U(t) > 0$ values for the reference probability indicating greater risk aversion. A negative estimate for λ implies the opposite inequalities for the reference probability, and may be interpreted as the investor being either risk-seeking or optimistic. A slight extension of equation (12) in Corollary 1 estimates the λ intercept in equation (16) using the following procedure

$$\frac{y_t - \mu_t}{2\sigma_t} + \frac{1}{2} \;=\; \frac{1}{1 + e^{\lambda - \alpha z_t}} + \epsilon_t \,. \tag{17}$$

Non-zero $y_t - \mu_t$ deviations in equation (17) may therefore be attributable to the λ intercept as well as the α coefficients. Further details regarding the reference probability measure are available in Appendix A which slightly adjusts the definition of abnormal returns in equation (5) for non-zero λ parameters.

3.4 Advantages of the Market Probability Approach

Besides the generality of the market probability, our approach has several advantages over a linear regression of stock returns on superfluous information.

First, the regression formulation in equation (13) is only a statistical exercise, while Propositions 1 and 2 provide economic consequences for non-zero α coefficients in the market probability. For example, linear regression cannot result in the underestimation of ex-ante return volatility nor excess realized volatility. Indeed, although the α coefficients are statistical estimates, their implications for the conditional return distribution are causal.

Second, the market probability provides an economic interpretation for the λ intercept and recognizes the crucial role of volatility when examining the relationship between stock returns and superfluous information. Unlike α_0 in equation (13), the λ intercept in equation (16) indexes the reference probability measure and therefore incorporates risk preferences. Furthermore, firm-specific volatility scales any distortion in the market probability measure attributable to non-zero α coefficients. In contrast, the linear regression in equation (13) ignores variation in σ_t across time for individual firms, and differences in σ_t between firms when calibrating the α coefficients cross-sectionally.[9]

Third, the market probability measure may be inferred experimentally, which eliminates the need to specify firm-specific dependent variables $\frac{y_t - \mu_t}{2\sigma_t}$ in equation (17). Thus, the joint-hypothesis

[9]The standard deviation for the time series of ϵ_t residuals in equation (13) is unrelated to σ_t in equation (14) since the asset's true volatility is independent of α and z_t. Therefore, even if equation (15) describes the true relationship between $y_t - \mu_t$ and z_t, the asset would not be riskless despite the ϵ_t residuals being identically zero.

confounding the estimation of expected returns and a model of market equilibrium is circumvented. Indeed, by providing the return implications of distorted market probabilities that summarize investor beliefs, our methodology facilitates further studies into the relationship between psychology and stock returns.

Fourth, from an econometric perspective, the "linear probability" in equation (14) is inappropriate since its right side is not constrained to lie within the $[0, 1]$ interval. To avoid misspecification errors, additional constraints on the α coefficients must be imposed by the estimation procedure. Although the $P^U(t)$ probability in equation (17) can be approximated as

$$\frac{1}{1+e^{\lambda-\alpha z_t}} \approx \frac{1}{2} - \frac{\lambda}{4} + \frac{\alpha z_t}{4}, \tag{18}$$

using equation (39) in Appendix B, this linear approximation does not imply that linear regression is appropriate for examining the relationship between abnormal returns and superfluous information. Indeed, the ability of equation (18) to depart from the [0,1] interval subjects this linear probability to the same criticism as equation (14).

More importantly, equation (12) and its generalization in equation (17) are implications of our market probability approach since they normalize $y_t - \mu_t$ by $2\sigma_t$ and center this ratio around one-half. Therefore, the linear approximation in equation (18) is not equivalent to a linear regression of abnormal returns on superfluous information. In contrast, our market probability is designed to examine whether abnormal returns are explained by superfluous information influencing investor beliefs. Our empirical results in Section 6 attest to the inferiority of regressing $y_t - \mu_t$ on z_t when attempting to profit from behavioral biases.

To clarify, standard linear regression is inappropriate for studying the relationship between behavioral biases and abnormal returns. However, linear multifactor models can define the abnormal returns underlying the dependent variable in equation (17).

4 Four Common Behavioral Biases

Barberis, Shleifer, and Vishny (1998) consider a single asset model with one risk neutral investor. Although earnings, hence prices, evolve as a random walk, representativeness causes investors to perceive a nonexistent trend in these sequences. When the pattern reverses, conservatism emerges as the investor reacts timidly to the arrival of recent observations. Consequently, underreaction results from the perception that earnings are generated by a mean reverting model, while overreaction occurs when investors perceive a trend in earnings.

Daniel, Hirshleifer, and Subrahmanyam (1998) assume an overconfident investor's processing of private information exhibits biased self-attribution. Specifically, instances where public information confirms a private signal is attributed to skill while disconfirming events are discounted. This asymmetry results in short-term momentum with long-term reversal occuring when the true value of the risky security is revealed.

In the next two subsections, we describe representativeness, conservatism, overconfidence, and biased self attribution using previous price movements and their respective market probabilities. This analysis motivates our proxies for these biases constructed from earnings data in the next section. For expositional simplicity but without loss of generality, we assume $\lambda = 0$ in equation (16) and employ the $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ reference probability for the remainder of this section.

4.1 Representativeness and Conservatism

Under the reference probability measure $\left\{\frac{1}{2}, \frac{1}{2}\right\}$, price movements follow a random walk. Therefore, three consecutive upward price movements occurs with probability $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$. However, the market may become optimistic and increase $P^U(t)$ beyond $\frac{1}{2}$ after the occurrence of this sequence. Therefore, when z_t indicates a trend in a stock's price or the firm's underlying earnings, the potential for representativeness to influence the market probability is captured.

Conditional on the appearance of a trend in a firm's earnings or its associated price movements, the $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ reference measure implies that its continuation and reversal are equally-likely. When z_t indicates a reversal in a sequence of realized earnings or price movements, the impact of conservatism on the market probability may be assessed.

4.2 Overconfidence and Biased Self-Attribution

Denote the *private* probability measure of an individual investor as $\left\{P^U_{pr}(t), P^D_{pr}(t)\right\}$ versus the reference probability $\left\{\frac{1}{2}, \frac{1}{2}\right\}$. Equation (7) details the extent of an individual investor's overconfidence when $4\sigma_t^2 \left[P^U_{pr}(t) - \frac{1}{2}\right]^2$ results from private information manifested in $P^U_{pr}(t)$. Thus, overconfidence arises whenever $P^U_{pr}(t)$ and $\frac{1}{2}$ diverge due to an investor's access to private information or their interpretation of public information.

The incorporation of biased self-attribution into the our framework compares $P_{pr}^{U}(t)$ with the realized price movement at $t+1$. A confirming signal occurs when $P_{pr}^{U}(t) > \frac{1}{2}$ and the subsequent price movement is up, while a disconfirming signal has a downward price movement following $P_{pr}^{U}(t) > \frac{1}{2}$. Similarly, a confirming (disconfirming) signal results when a downward (upward) price movement follows $P_{pr}^{D}(t) < \frac{1}{2}$.

5 Proxies for Behavioral Biases Using Analysts' Forecasts

We construct proxies for behavioral biases from analysts' forecasts of quarterly earnings, which are particularly well-suited to this purpose for at least three reasons. First, there is ample evidence in the literature that the market reacts improperly to realizations of fundamental information such as earnings. For example, Barth, Elliot, and Finn (1999) find that firm-specific price-earnings multiples have dynamics consistent with both representativeness and conservatism, while Chan, Frankel, and Kothari (2004) detect conservatism but not representativeness using portfolios constructed from accounting variables.[10]

Second, analysts' forecasts allow us to gauge market expectations regarding earnings. This feature is important since stock returns are driven by changes in expectations about future fundamentals. For example, Chan, Jegadeesh, and Lakonishok (1996) attribute a significant fraction of momentum profits to earnings momentum as measured by earnings forecast revisions. Daniel and Titman (2005) also document that tangible information derived from realized earnings growth is unable to predict stock returns. In contrast, intangible information, which is presumably comprised of expectations regarding future earnings, has predictive ability.

Third, the possibility that analysts' forecasts are biased is immaterial to our methodology. Richardson, Teoh, and Wysocki (2004) find that analysts walk-down their earnings forecasts to allow firms to exceed their final forecast. However, if the market is processing information efficiently, the market probability should not react to trends in earnings surprises.

In forming the behavioral bias proxies, we deliberately avoid using returns (price movements) since returns are already included in the dependent variable of equations (12) and (17). Instead,

[10] The α coefficients easily ascertain the marginal influence of various behavioral biases on stock returns, and may be conditioned on cross-sectional or industry characteristics. This flexibility and level of precision cannot be duplicated by sorting stocks into portfolios.

a stock's nominal return from the prior quarter is included in the superfluous information set to examine its role in generating future returns versus the marginal contribution of our behavioral bias proxies. We also refrain from utilizing returns to define these proxies due to the potential release of fundamental information around the earnings announcement date. For example, a negative earnings revision coinciding with a positive stock return (or vice-versa) suggests that information besides earnings may have become public. Therefore, we limit our attention to the market's processing of analysts' earnings forecasts by having positive (negative) earnings revisions and surprises constitute good (bad) private and public signals respectively.

5.1 Data

Our data consists of analysts' earnings forecast data for earnings announced from January 1^{st}, 1986 to September 30^{th}, 2004 from the Institutional Brokers Estimate System (I/B/E/S) Summary unadjusted file. The unadjusted file addresses the problem of imprecise forecasts caused by I/B/E/S's practice of rounding to the nearest cent when adjusting historical forecasts after stock splits (see Diether, Malloy, and Scherbina (2002)).

The earnings surprise S_t is defined as the actual announced earnings less the consensus mean earnings forecast immediately prior to the earnings announcement denoted F_t, scaled by the stock price from the Center for Research in Security Prices (CRSP) at day -2. Day 0 is defined as the earnings announcement date or the next trading day whenever this date is a non-trading day. We define the forecast revision R_t as F_t less the consensus forecast two months before the announcement date, scaled by price in the same manner as S_t. Following Clement and Tse (2005), we remove forecast errors greater than 40% of price, and forecast revisions greater than 10% of price, to guard against data input errors by I/B/E/S. As in Diether, Malloy, and Scherbina (2002), the dispersion of analyst forecasts denoted D_t is the reported standard deviation of earnings forecasts in I/B/E/S divided by the absolute value of the mean estimate. If the mean estimate is zero, the firm's dispersion for that quarter is coded as the maximum dispersion reported throughout the firm's history.[11] We also remove firm-quarter observations when the consensus forecast involves fewer than two analysts.

We obtain each firm's previous fiscal year-end book-to-market ratio (B/M) from the CRSP /

[11]We also scaled dispersion by the firm's stock price at day -2 and obtained similar empirical results.

Compustat Merged Industrial Annual File. This ratio is constructed as the book value of common equity (item #60) divided by the market capitalization from CRSP. Firm-quarter observations where the previous fiscal year-end B/M ratio is zero or negative are removed. Two-digit SIC codes are also obtained for each stock with a total of 66 different industries being represented in our sample.

After applying the above screens, we select firms having at least 24 observations for every element of the superfluous information set defined in this section. In total, the final sample contains 2,087 unique firms providing 94,242 firm-quarter observations. 34.9% of earnings surprises are negative, 51.2% are positive, and the remaining 13.9% are zero. In contrast, 29.4%, 16.9%, and 53.7% of earnings revisions are negative, positive, and zero respectively. These percentages are consistent with walk-downs in earnings forecasts.

5.2 Proxies for Representativeness and Conservatism

When constructing proxies for representativeness and conservatism, we consider three consecutive earnings surprises. Besides maintaining model parsimony, this horizon approximates the nine-month horizon typical of return anomalies such as momentum and post-earnings announcement drift. As a consequence, we focus on *trends* consisting of consecutive positive or negative earnings surprises over three periods as well as a *reversal* for the third realization. Furthermore, the evolution of firm-specific earnings surprises resembles a random walk, which is the true process underlying Barberis, Shleifer, and Vishny (1998)'s theoretical model.[12]

The z_t element pertaining to representativeness involves a *trend* in earnings surprises

$$z_t^{rep} = \begin{cases} \frac{S_t + S_{t-1} + S_{t-2}}{3} & \text{If all three surprises are positive or negative} \quad \text{Trend} \\ \\ 0 & \text{Otherwise} \quad \text{No Trend}\,. \end{cases}$$

The magnitude of the trend is summarized by the average earning's surprise over the last three

[12]Across the 2,087 firms in our sample, the average autocorrelation in the sequence of firm-specific earnings surprises is 0.118, with 21.3% and 35.1% of these autocorrelations being significant at the 1% and 5% levels respectively. These autocorrelations are positive for 70.44% of the firms in our sample and negative for the remaining 29.56%. In addition, slight negative skewness of -1.49 on average is detected in the earnings surprises.

quarters. This definition of representativeness is expressed more succinctly as

$$z_t^{rep} = \left(\frac{S_t + S_{t-1} + S_{t-2}}{3}\right) 1_{\{\text{All three surprises are positive or negative.}\}} \cdot \tag{19}$$

A positive α coefficient for z_t^{rep} is consistent with the market extrapolating further positive (negative) earnings surprises when the representativeness proxy is positive (negative). Conversely, a negative coefficient for z_t^{rep} indicates the market expects an earnings trend to reverse rather than continue. Asparouhova, Hertzel, and Lemmon (2005) find empirical evidence consistent with individuals anticipating reversals after short trends.

In contrast to z_t^{rep}, our conservatism proxy considers an earnings surprise that contradicts the previous two surprises

$$z_t^{con} = \begin{cases} \left[S_t - \left(\frac{S_{t-1}+S_{t-2}}{2}\right)\right] & \text{If } S_t \text{ is of the opposite sign as } S_{t-1} \text{ and } S_{t-2} \quad \text{Reversal} \\ 0 & \text{Otherwise} \quad \text{No Reversal} \end{cases}$$

which is equivalent to

$$z_t^{con} = \left[S_t - \left(\frac{S_{t-1} + S_{t-2}}{2}\right)\right] 1_{\{S_t \text{ is of the opposite sign as } S_{t-1} \text{ and } S_{t-2}.\}} \cdot \tag{20}$$

The conservatism proxy has an identical sign as the most recent earnings surprise, and coincides with a reversal from the previous two surprises. When the α coefficient associated with z_t^{con} is negative, a positive (negative) conservatism proxy leads to negative (positive) abnormal returns in the absence of other biases. Intuitively, this relationship is consistent with the market focusing on the firm's previous two earnings surprises rather than the most recent surprise. In contrast, a positive coefficient for z_t^{con} suggests the market responds more to the actual reversal. By construction, z_t^{rep} and z_t^{con} are not simultaneously non-zero since S_t cannot have the identical and opposite sign as the firm's previous two earnings surprises.

5.3 Proxies for Overconfidence and Biased Self-Attribution

With overconfidence defined as market participants overestimating the precision of their private information, forecast revisions and analyst dispersion reflect the magnitude and variability of private information respectively. Specifically, the arrival of good (bad) private information is indicated by an upward (downward) earnings revision.

Higher earnings dispersion is conducive to overconfidence. Barron, Kim, Lim, and Stevens (1998) demonstrate that earnings dispersion captures the level of disagreement between analysts as well as their individual forecast uncertainty. However, decomposing a firm's forecast dispersion $\sigma^2_{pr}(t)$ into these separate components is unnecessary since both of these elements are subject to overconfidence. Our overconfidence proxy z^{oc}_t is defined as

$$z^{oc}_t = \begin{cases} R_t\, \sigma^2_{pr}(t) & \text{If } |R_t| > 0 \text{ and } \sigma^2_{pr}(t) \neq 0 \quad \text{Disagreement over private information} \\ \\ 0 & \text{If } R_t = 0 \text{ or } \sigma^2_{pr}(t) = 0 \quad \text{No Disagreement} \end{cases}$$

which simplifies to

$$z^{oc}_t = R_t\, \sigma^2_{pr}(t)\,. \tag{21}$$

Clement and Tse (2005) report that *bold* revisions, defined as large deviations from the concensus forecast, have a greater likelihood of originating from private information. Conversely, forecasts which result from herding contain less private information and reduce dispersion.[13] The results of Clement and Tse (2005) are consistent with our overconfidence proxy in equation (21) as bold revisions arising from private information increase $|R_t|$ as well as $\sigma^2_{pr}(t)$. The coefficient for z^{oc}_t may be negative since disagreement over the return implications of private signals, namely the sign of R_t, is easier to detect as overconfidence.

Observe that the sign of an earnings revision versus the subsequent surprise is irrelevant to overconfidence. In contrast, the biased self-attribution signal evaluates the consistency between private and public information since positive (negative) surprises indicate the realization of good (bad) public signals.

A disconfirming signal occurs when an earnings surprise follows a non-zero revision. In contrast, a confirming signal begins with a non-zero forecast revision, signifying the release of private information, but is not followed by an earnings surprise. Consequently, the proxy for biased

[13] In contrast to Diether, Malloy, and Scherbina (2002), our overconfidence proxy in equation (21) supplements earnings dispersion by multiplying this measure for differences of opinion by earnings revisions.

self-attribution equals

$$z_t^{bsa} = \begin{cases} R_t & \text{If } |R_t| > 0 \text{ and } S_t = 0 & \text{Confirming Signal} \\ \\ \gamma R_t & \text{If } |R_t| > 0 \text{ and } |S_t| > 0 & \text{Disconfirming Signal} \\ \\ 0 & R_t = 0 & \text{No Signal} \end{cases}$$

for $0 < \gamma < 1$ which represents the extent to which disconfirming signals are underweighted by the market. Hence, disconfirming public signals are underweighted in comparison to their confirming counterparts which implicitly have γ equal to one. This definition of biased self-attribution is equivalent to

$$z_t^{bsa} \;\; = \;\; R_t \left[1_{\{S_t = 0\}} \, + \, \gamma \, 1_{\{|S_t| > 0\}} \right] \, . \tag{22}$$

In our empirical implementation, the z_t^{bsa} entries are defined with γ equaling one-half. However, our empirical results are robust to different values of this parameter within the [0,1) interval. Thus, our choice of $\gamma = \frac{1}{2}$ is without loss of generality.

For our sample period, the percentage of confirming and disconfirming signals implied by equation (22) are 9.41% and 90.59% respectively. An alternative definition for biased self-attribution considers the interval [-0.002, 0.002] surrounding S_t when defining the confirming, hence disconfirming, public signals

$$\begin{cases} R_t & \text{If } |R_t| > 0 \text{ and } S_t \in [-0.002, 0.002] & \text{Confirming Signal} \\ \\ \gamma R_t & \text{If } |R_t| > 0 \text{ and } S_t \notin [-0.002, 0.002] & \text{Disconfirming Signal} \\ \\ 0 & R_t = 0 & \text{No Signal} \, . \end{cases}$$

Under this weaker definition for a confirming signal, 58.09% of the signals are confirming with the remaining 41.91% being disconfirming. However, our empirical results are not sensitive to the demarcation between confirming versus disconfirming signals. In particular, the economic role of the biased self-attribution proxy is insensitive to the magnitude of $|S_t|$ used in its construction.

5.4 Summary of Superfluous Information

The last element of the superfluous information set, z_t^{pr} is the firm's return from the previous quarter defined over the $[-63, -2]$ interval. As alluded to earlier, this variable is included to examine the marginal explanatory power of representativeness, conservatism, overconfidence, and biased self-attribution in generating abnormal returns beyond simple return extrapolation.

In summary, the z_t vector representing the set of superfluous information we study equals

$$z_t \quad = \quad \left\{ z_t^{rep}, z_t^{con}, z_t^{oc}, z_t^{bsa}, z_t^{pr} \right\} \, , \tag{23}$$

which has five associated α coefficients. The importance of the behavioral bias proxies is confirmed by our empirical results in the next section.

6 Estimation Methodology and Empirical Results

After an earnings announcement, abnormal returns are defined over the subsequent $[\delta_1, \delta_2] = [6{,}26]$ day interval. The $\delta_1 > 0$ parameter allows market participants to update μ_t with respect to the fundamental information contained in the earnings announcement before the $y_t - \mu_t$ deviations are computed. In an efficient market, the release of earnings information should have no bearing on returns after five days. Moreover, expected returns are unrelated to the z_t elements in equation (23). With regards to earlier material after equation (1), the time increment Δ is defined as $\delta_2 - \delta_1$ which equals 21 days in our empirical study.

Table 1 provides summary statistics for the behavioral bias proxies. The reported averages are computed for the non-zero elements of z_t^{bias}, along with their frequency. Recall that representativeness and conservatism cannot simultaneously be non-zero by construction, while their combined frequency equals 44.5% on average. This percentage is comparable to 42.4% for overconfidence and 44.0% for biased self-attribution. The percentage of periods in which the superfluous information set has 1, 2, 3, and 4 non-zero entries is 31.3%, 23.4%, 23.4%, and 21.9% respectively.

6.1 True Expected Returns and Volatilities

Estimating the true expected return μ_t involves adjusting a firm's nominal return by its counterpart for the market. We begin by regressing the daily returns y_i of each firm on the corresponding market return denoted RM_i over the $[-200, -11]$ horizon as in Bailey, Karolyi, and Salva (2005)

$$y_i = \beta_0 + \beta_1 \; RM_i + \epsilon_i \quad \text{for } i = -200, \ldots, -11 \, . \tag{24}$$

The β_1 coefficient estimated in equation (24) computes a firm's daily expected return during the subsequent 21 day horizon as[14]

$$\hat{\mu}_k = \hat{\beta}_1 \; RM_k \quad \text{for } k = 6, \ldots, 26 \, . \tag{25}$$

The product $(1 + \hat{\mu}_6) \cdots (1 + \hat{\mu}_k) \cdots (1 + \hat{\mu}_{26}) - 1$ obtained from equation (25) is denoted $\hat{\mu}_t$, and comprises a firm's expected return in quarter t.

The volatility for each firm-quarter is estimated as the daily standard deviation of returns over the previous quarter which corresponds to the [-63,-2] interval. This standard deviation is then scaled by $\sqrt{21}$ for compatibility with the calibration period of $\hat{\mu}_t$ to form the return volatility estimate denoted $\hat{\sigma}_t$.

Once $\hat{\mu}_t$ and $\hat{\sigma}_t$ are calibrated, estimates for the α coefficients as well as the λ intercept are estimated via equation (17) as follows

$$\hat{L}_t = P^U(t) + \epsilon_t = \frac{1}{1 + e^{\lambda - \alpha z_t}} + \epsilon_t \, , \tag{26}$$

where $\hat{L}_t$ is defined as

$$\hat{L}_t = \frac{y_t - \hat{\mu}_t}{2\hat{\sigma}_t} + \frac{1}{2} \, , \tag{27}$$

with summary statistics for this dependent variable reported in Table 1. Starting values for the non-linear estimation of λ and α are zero, although our empirical results are insensitive to these initial settings. The market model is sufficient for estimating abnormal returns since the inclusion of additional factors could only explain non-zero α estimates if they are correlated with elements of the superfluous information set in equation (23). Nonetheless, we invoke the Fama and French (1993) three factor model augmented with Carhart (1997)'s momentum factor to adjust the cross-sectional returns generated by trading strategies derived from our estimated market probabilities.

In our empirical implementation, violations of the upper (lower) bound in equation (6) imply

[14] The $\hat{\mu}_k$ estimate in equation (25) only reflects publically available information. Semi-strong and strong forms of market efficiency can be tested after incorporating private information.

equation (26) produces a positive (negative) $\hat{\epsilon}_t$ residual.[15] However, equation (6) is an ex-ante relationship, while $\hat{L}_t$ in equation (27) is constructed from ex-post estimates of the asset's true expected return and volatility. Hence, violations of the inequalities in equation (6) may stem from parameter uncertainty. Therefore, although equation (16) prevents superfluous information from generating market probabilities below zero or above one, dependent variables $\hat{L}_t$ outside the [0,1] interval are not removed from the sample.[16] Instead, we verify that the residuals from our non-linear estimation procedure are symmetric and sum to zero.

Finally, when ex-post historical returns are contaminated by behavioral biases, their role in calibrating $\hat{\mu}_t$ in equation (25) and $\hat{\sigma}_t$ may be suspect. Indeed, any empirical study that attempts to explain abnormal returns confronts this limitation. This concern motivates an experimental study to elicit the market probabilities which avoids estimating the moments of an asset's true return distribution.

6.2 Pooled Coefficient Estimation

We first determine if behavioral biases affect stock returns in our sample by estimating the market probability coefficients over the entire sample period. To account for cross-correlation across firms, we follow the Fama-MacBeth (1973) procedure by calibrating equation (17) each quarter using all earnings announcements that fall within a calendar quarter. Each parameter's significance is then determined from the empirical distribution of its time series using Newey-West t-statistics.

Table 2 reports strong evidence in favor of representativeness, with conservatism, overconfidence, and biased self-attribution also detected at the 10% significance level in our joint estimation. In contrast, the contribution of past returns is insignificant. When the effects of the biases are estimated individually, the coefficients for representativeness and conservatism remain significant while the other z_t elements have an insignificant role in creating abnormal returns.[17]

[15]Equation (6) implies $0 \leq y_t - \mu_t + \sigma_t \leq 2\sigma_t$ after adding σ_t to both inequalities. Dividing this result by $2\sigma_t$ yields $0 \leq \frac{y_t-\mu_t}{2\sigma_t} + \frac{1}{2} \leq 1$. Thus, when μ_t and σ_t are known parameters, equation (6) implies $\frac{y_t-\mu_t}{2\sigma_t} + \frac{1}{2}$ is contained in the [0,1] interval.

[16]Small fluctuations in $\hat{L}_t$ could result from fundamental information released after the earnings announcement date. However, this effect cannot bias our estimates for α unless the z_t elements, which are known five days prior to the calibration of these coefficients, constitute "unprocessed" fundamental information.

[17]The limited empirical support for overconfidence and biased self-attribution may be overcome in future research by constructing their proxies from long-term growth projections (LaPorta (1996)) or price targets (Brav and Lehavy

We confirm that our results are qualitatively similar with or without the estimation of the λ intercept, implying a reference probability different from one-half is unnecessary. Consequently, for expositional simplicity, we report our empirical findings under the market probability in equation (2). Interestingly, when elements of z_t are studied individually, the $\hat{\lambda}$ estimates are identical to two decimal places. This commonality is important since the reference probability is required to be independent of any psychological bias.

The results in Table 2 assume the true α coefficients are constant and identical across all 2,087 stocks. Therefore, this assumption ignores a stock's investor clientele. However, even if the behavioral biases committed by individual investors are time-invariant, transactions can induce time-varying coefficients when buyers and sellers are not equally as susceptible to behavioral biases. For example, the investors who owned Internet stocks during and after the bubble period may differ. This time-variation is formalized in Appendix C for an individual stock (but not an individual investor). In the next subsection, we overcome the deficiencies of our pooled procedure by estimating time-varying coefficients during non-overlapping four-year subperiods. We also calibrate industry-specific coefficients using subsamples constructed from two-digit SIC codes.[18]

Finally, in an earlier version of the paper, firm-specific $\hat{\alpha}$ coefficients are calibrated using time series data for individual stocks. However, this estimation procedure is complicated by the percentage of zero elements in the superfluous information set. In particular, requiring a firm to have a minimum of ten non-zero entries for each element of equation (23) reduces our sample size significantly.

6.3 Trading Strategies

To ascertain the cross-sectional return implications of behavioral biases, calendar-time portfolios from January 1^{st}, 1990 to December 31^{st}, 2004 are formed from firm-specific estimated market probabilities each firm-quarter. Buy and sell portfolios in calendar-time are formed from the estimated market probabilities $\hat{P}^U(t) = \frac{1}{1+e^{-\hat{\alpha} z_t}}$ of individual stocks after their earnings announcement

(2003)) which are closer to Daniel and Titman (2005)'s notion of intangible information.

[18]In contrast to industry-specific coefficients, conditioning on B/M and size characteristics does not capture a stock's susceptibility to behavioral biases. However, the 2,087 firms we study are recorded in the I/B/E/S database which is orientated towards large firms. Thus, caution should be exercised when generalizing our findings to a wider universe of stocks.

date in quarter t. The first superfluous information set examined is the z_t vector in equation (23). The non-linear model in equation (12) provides time-varying $\hat{\alpha}$ estimates from the prior four-year non-overlapping period. These estimated coefficients are also industry-specific. For example, the first estimation period is 1986 to 1989, with these estimates applied to firms in the same industry during the subsequent 1990 to 1993 horizon.

A stock is bought (sold) for 1-month, specifically the [6,26] day interval, in quarter t whenever its estimated market probability is above $x_{mp} = 0.54$ (below $1 - x_{mp}= 0.46$). A threshold x_{mp} higher than 0.50 accounts for estimation error in the market probabilities. Daily returns for the buy and sell portfolios are computed as the value-weighted (using the lagged market capitalization) return of all stocks in the respective portfolio. We exclude firm-days where the lagged price is less than $5.

Daily portfolio returns are then compounded to monthly returns which are regressed against the four-factor model. Data on these four-factors (MKTRF, SMB, HML, and UMD) are obtained from Kenneth French's website. The buy, sell, and cross-sectional returns generated by our trading strategies are recorded in Table 3 after adjustments by the four-factor model. The average returns in Panel A result from estimated market probabilities using the entire set of superfluous information in equation (23). A trading strategy that buys (sells) stocks whose estimated market probabilities are above $x_{mp} = 0.54$ (below $1 - x_{mp}= 0.46$) yields significantly positive returns. Specifically, the combined influence of superfluous information generates an annual return of 13.56% on average after adjustments for book-to-market, size, and momentum.

The results in Panels B and C involve estimated market probabilities for a combination of representativeness and conservatism as well as overconfidence and biased self-attribution, in conjunction with past returns. The inclusion of past returns enables us to investigate the marginal contribution of behavioral biases on stock returns after accounting for return extrapolation.[19] A trading strategy that focuses on representativeness and conservatism generates an annual return of 14.45% on average, while the combination of overconfidence and biased self-attribution produces positive but insignificant trading profits.

We also implement a separate trading strategy focusing exclusively on past returns for compar-

[19]Econometrically, having past returns in the superfluous information set also stabilizes the $\hat{\alpha}$ estimates given the percentage of zero entries for the representativeness, conservatism, overconfidence, and biased self-attribution proxies.

ison. As recorded in Panel D, this strategy has the worst return profile, a property which attests to the importance of our behavioral bias proxies. As a further robustness test, we replace the z_t vector in equation (23) with analysts' forecasts and revisions, $\{S_t, R_t\}$. Once again, the insignificant trading profits arising from this strategy in Panel E confirm the importance of constructing our behavioral bias proxies. Indeed, analysts' forecasts and revisions alone are not responsible for our earlier trading profits. Instead, their arrangement into proxies for behavioral biases enables them to generate positive cross-sectional returns.

Regarding transaction costs, the long and short portfolios underlying our trading strategy are not populated by large numbers of stocks. Nonetheless, idiosyncratic risk is likely mitigated by having an average of 57.5 and 43.0 stocks in the long and short portfolio respectively. Therefore, our strategies perform well with relatively few stocks, approximately 50 on average. In addition, our cross-sectional returns are not driven by short-selling. Instead, our market probabilities are identifying stocks with high future returns as a result of behavioral biases. Finally, three-year non-overlapping windows produce similar cross-sectional although they become insignificant when a shorter two-year window is considered. Thus, a longer estimation window reduces the noise in our calibration procedure, resulting in more precise market probabilities that consistently generate positive trading profits.

To clarify, the holding period of our trading strategy is not required to coincide with the $[\delta_1, \delta_2] = [6, 26]$ estimation interval. Indeed, cross-sectional returns are similar when the trading strategy purchases or sells stocks on the actual earnings announcement day, which coincides with the [1,26] horizon. Therefore, our reported trading profits are not driven by short-term effects that disappear once the market has been given sufficient time to interpret the firm's earnings announcement and update its expected return. Moreover, a longer 42-day holding period over the [6,47] interval produces nearly identical trading profits as those reported in Table 3. Consequently, the cross-sectional returns from our trading strategies are robust to different holding periods within the quarter.

Note that our buy and sell portfolios arise entirely from non-zero $\hat{\alpha}$ estimates which are not sources of risk. Indeed, the buy and sell portfolios select stocks according to their $\hat{\alpha} z_t$ elements, not B/M nor size characteristics. The only caveat is the dependence of these coefficients on $\hat{\mu}_t$, hence an assumed model of market equilibrium. However, our market probability approach is motivated by previous empirical studies that document abnormal returns under the joint-hypothesis. In addition, an omitted risk factor would have to be correlated with an element of z_t to explain its non-zero $\hat{\alpha}$ coefficient. Furthermore, when implementing our trading strategies, stocks are sorted according to their estimated market probabilities rather than $2\hat{\sigma}_t \left[\hat{P}^U(t) - \frac{1}{2}\right]$ in equation (5) to avoid any potential association between their profitability and idiosyncratic volatility.

6.4 Return Benchmarking

For benchmarking the cross-sectional returns from our market probability, we replicate the methodology in the previous section with a linear regression of individual stock returns on the z_t elements comprising our superfluous information set

$$y_t - \hat{\mu}_t \;=\; \phi_0 + \phi z_t + \epsilon_t \,, \tag{28}$$

as in equation (13). We then form buy and sell portfolios based on predicted abnormal returns that are attributable to behavioral biases. These predicted abnormal returns are computed as

$$\widehat{y_{t+1} - \mu_{t+1}} - \hat{\phi}_0 \;=\; \hat{\phi} z_{t+1} \,, \tag{29}$$

using the $\hat{\phi}$ estimates. Although this methodology appears to simply replace equation (17) with equation (28), natural thresholds for the buy and sell portfolios are unavailable in the linear regression analysis. Therefore, stocks are placed into buy (sell) portfolios whenever their predicted abnormal return $\hat{\phi} z_{t+1}$ from the linear regression in equation (29) is above (below) $x_{lr} = 0.03$ ($-x_{lr}$ = -0.03). The x_{lr} threshold forms portfolios containing an average of about 50 stocks per day, which is comparable to the market probability strategy in Panel A of Table 3. This consistency is important to ensure return differences are not the result of idiosyncratic risk arising from having fewer stocks in the buy and sell portfolios.

Recall from Section 3 that linear regression is a special case of our market probability which assumes no cross-sectional differences in return volatility across stocks, and further assumes this volatility is constant across time for individual stocks. Moreover, the market probability and linear regression procedures would not select identical stocks for their respective buy and sell portfolios.[20] In particular, stocks whose market probability at time t exceeds the threshold $x_{mp} = 0.54$ are not

[20] Sorting stocks according to whether their z_t values are positive or negative cannot replicate the trading strategies derived from our market probabilities, or the linear regression in equation (28), since this technique assumes the true α and ϕ coefficients are identically one for every element of the superfluous information set.

necessarily the same as those whose predicted returns $\hat{\phi} z_{t+1}$ are above $x_{lr} = 0.03$. This disparity holds for any pair of x_{mp} and x_{lr} values. Empirically, we find that linear regression produces smaller and insignificant cross-sectional returns.

6.5 Return Predictability and Excess Volatility

Our market probabilities enable us to investigate the excess volatility and return predictability attributable to non-zero $\hat{\alpha}$ coefficients. The time series of firm-specific $P^U(t)$ probabilities is identical to those underlying our calendar-time trading strategy. Proposition 2 implies the excess volatility attributable to the $\hat{\alpha}$ coefficients equals

$$\sqrt{4Var\left(\hat{\sigma}_t \left[\hat{P}^U(t) - \tfrac{1}{2}\right] \middle| t = 1, \ldots, n\right) + 2\,Cov\left(\hat{\mu}_t\,,\, 2\,\hat{\sigma}_t \left[\hat{P}^U(t) - \tfrac{1}{2}\right] \middle| t = 1, \ldots, n\right)}\,, \tag{30}$$

while the return predictability induced by these biases is

$$\begin{aligned} & Corr\left(\hat{\mu}_t\,,\, \hat{\sigma}_{t-1} \left[\hat{P}^U(t-1) - \tfrac{1}{2}\right] \middle| t = 2, \ldots, n\right) + Corr\left(\hat{\mu}_{t-1}\,,\, \hat{\sigma}_t \left[\hat{P}^U(t) - \tfrac{1}{2}\right] \middle| t = 2, \ldots, n\right) \\ + \;\; & Corr\left(\hat{\sigma}_t \left[\hat{P}^U(t) - \tfrac{1}{2}\right]\,,\, \hat{\sigma}_{t-1} \left[\hat{P}^U(t-1) - \tfrac{1}{2}\right] \middle| t = 2, \ldots, n\right). \end{aligned} \tag{31}$$

Table 4 contains summary statistics for the time series properties of the market probability measure. We document substantial excess return volatility resulting from the market having conditioned on superfluous information. In economic terms, the average excess return volatility attributable to behavioral biases is 4.75% per annum. Furthermore, these biases induce negative and positive serial return correlation with return predictability on average. In particular, return predictability is more pronounced than return reversals.

Finally, we examine the extent to which the market underestimates ex-ante return volatility in equation (7) by calculating

$$2\,\hat{\sigma}_t \sqrt{\left[\hat{P}^U(t) - \tfrac{1}{2}\right]^2} \;=\; 2\,\hat{\sigma}_t \left|\hat{P}^U(t) - \tfrac{1}{2}\right| \quad \text{for } t = 1, \ldots, n \tag{32}$$

for each firm-quarter. Firm-specific summary statistics are then computed with their average values across the firms in our sample presented in Table 4. On average, return volatility is underestimated by 1.62% per annum due to behavioral biases.

7 Conclusion

We introduce a market probability measure to unify the influence of psychological biases on returns. This probability represents a reduced-form expression for the market's beliefs regarding future price movements. Every behavioral bias can potentially distort the market probability which facilitates empirical investigations into the origin and magnitude of abnormal returns. Therefore, our methodology provides greater resolution on whether behavioral biases influence stock returns, and if so, which biases are the most prevalent.

There are several theoretical implications associated with distortions in the market probability attributable to investor psychology. Besides abnormal returns, they cause ex-ante return volatility to be underestimated. Furthermore, fluctuations in the market probability induce return predictability and excess return volatility.

After constructing proxies for representativeness, conservatism, overconfidence, and biased self-attribution from earnings forecasts and revisions, we find empirical evidence supporting their ability to explain abnormal returns. Furthermore, sorting stocks into buy and sell portfolios according to the combined influence of all behavioral biases on their estimated market probabilities generates a 13.56% annual return after adjustments for book-to-market, size, and momentum factors. An even higher 14.45% return is produced when the trading strategy focuses on representativeness and conservatism in conjuction with past returns, while overconfidence and biased self-attribution yield positive but insignificant trading profits. Furthermore, a trading strategy derived from analysts' forecasts and revisions does not generate significant trading profits. This result confirms the importance of arranging these earnings realizations into behavioral bias proxies.

In addition, fluctuations in the market probability measure attributable to behavioral biases yield excess annualized return volatility of 4.75% on average. These market probability distortions also induce positive as well as negative return autocorrelation, with return predictability occuring on average.

Examining additional behavioral biases and alternative methods for constructing their proxies is an important topic for future empirical research. For example, long-term growth rate projections (LaPorta (1996)) or price targets (Brav and Lehavy (2003)) could replace analysts' earnings forecasts and revisions over quarterly horizons. Biases such as overconfidence may become more salient when their proxies are defined by these variables whose properties are closer to Daniel and Titman (2005)'s notion of intangible information. The superfluous information set could also be expanded to include media coverage whose role in financial markets is studied by Barber and Odean (2005) as well as Bhattacharya, Galpin, Ray, and Yu (2004).

Another avenue for future study would extract market probabilities from subjects in an laboratory experiment to circumvent the estimation of an asset's true return distribution. These market probabilities could then classify stocks trading on financial exchanges into buy and sell portfolios. Provided these two approaches examine common superfluous information, the experimental estimation of market probabilities and the implementation of trading strategies on historical return data are separated. The profitability of trading strategies conditioned on experimental parameter estimates gauges the relevance of laboratory experiments to financial market.

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Appendices

A Altering the Reference Probability Measure

Instead of equation (2), consider the market probability $P^U(t)$ in equation (16). A non-zero λ intercept in equation (16) alters the reference probability from $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ to

$$\left\{P^U(t), P^D(t)\right\}\Big|_{\alpha=0} \;=\; \left\{\tfrac{1}{1+e^{\lambda}}, \tfrac{e^{\lambda}}{1+e^{\lambda}}\right\} . \tag{33}$$

By being independent of z_t, the λ parameter indexes the reference probability in equation (33), while the α coefficients continue to indicate whether the market conditions on superfluous information. The λ intercept and the α coefficients define the following probability measures

1. **Risk Neutral Reference Probability** with $\lambda = 0$ and $\alpha = 0$: This $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ measure refers to a risk neutral market in the absence of any behavioral biases.

2. **General Reference Probability** with $\lambda \neq 0$ and $\alpha = 0$: This probability remains independent of behavioral biases and replaces $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ with equation (33).

   The two reference probabilities above are mutually exclusive since λ is either zero or non-zero while $\alpha = 0$ by definition.

3. **Market Probability** with $\alpha \neq 0$: Regardless of the λ intercept, superfluous information distorts the market probability away from the appropriate reference measure.

To clarify, there is only one underlying binomial tree with two probabilities indexed by α and λ existing on this lattice. When $\alpha \neq 0$, each probability implies a distinct conditional expected return and variance.[21] Furthermore, the return decomposition in equation (4) is valid for any $P^U(t)$ probability. In particular, inserting equation (16) with $\alpha = 0$ and $\lambda > 0$ into equations (4)

[21] Unlike changes of probability in the derivatives pricing literature, μ_t does not equal the riskfree interest rate since a riskless portfolio is not constructed through hedging. Furthermore, since the U_t and D_t magnitudes are identical under the reference and market probability measures, the change of probability induced by non-zero α coefficients alters the risky asset's conditional expected return and its variance.

and (7) implies the following relationships[22]

$$\mu_t + \sigma_t \left(\frac{1 - e^{\lambda}}{1 + e^{\lambda}} \right) \quad < \quad \mu_t \tag{34}$$

$$\frac{2\sigma_t^2}{1 + \cosh(\lambda)} \quad < \quad \sigma_t^2 \,. \tag{35}$$

Thus, a more "risk-averse" market lowers its conditional expected return and variance by increasing the λ intercept. According to equation (33), positive λ parameters imply $P^U(t) < \frac{1}{2} < P^D(t)$ when $\alpha = 0$, with the disparity $P^D(t) - P^U(t) > 0$ in the reference probability increasing with λ. A negative estimate for λ implies the opposite inequalities for the reference probability, and may be interpreted as the investor being either risk-seeking or optimistic.

For $\alpha = 0$, the conditional expected return under the reference probability measure is

$$y_t \quad = \quad \mu_t + \sigma_t \left(\frac{1 - e^{\lambda}}{1 + e^{\lambda}} \right) \,, \tag{36}$$

which equals μ_t if the asset is riskless ($\sigma_t = 0$) or the market is risk neutral ($\lambda = 0$). When $\lambda \neq 0$ and $\alpha \neq 0$, abnormal returns are defined according to equation (36) as

$$y_t - \left[\mu_t + \sigma_t \left(\frac{1 - e^{\lambda}}{1 + e^{\lambda}} \right) \right] \tag{37}$$

instead of equation (5), while ex-ante return volatility is compared with $\frac{2\sigma_t^2}{1+\cosh(\lambda)}$ in equation (35) instead of σ_t^2.

The impact of αz_t on the asset's conditional expected return and its variance is plotted in Figure 1 for both a positive and negative λ parameter. Observe that over the $(0, 2\lambda)$ interval for $\lambda > 0$ and the $(2\lambda, 0)$ interval for $\lambda < 0$, the variance of the conditional expected return under the market probability measure is overestimated. Intuitively, the effects of behavioral biases summarized by αz_t offset the non-zero λ intercepts in these intervals to yield conditional variances higher than $\frac{2\sigma_t^2}{1+\cosh(\lambda)}$ in equation (35). The 2λ boundaries result from $\cosh(\lambda - \alpha z_t)$ being equal to $\cosh(-\lambda)$, hence $\cosh(\lambda)$, when $\alpha z_t = 2\lambda$. This minor modification to the second implication of Proposition 1 is the only theoretical difference arising from a non-zero λ parameter in equation (16). Indeed, when λ is constant, this parameter cannot influence the variance, covariance, and correlation terms in Proposition 2.

[22]The function $\cosh(x)$ is defined as $\frac{\exp(x)+\exp(-x)}{2}$ with the properties $\cosh(0) = 1$ and $\cosh(x) = \cosh(-x)$.

If time-varying λ_t parameters are estimated using equation (17), the results of Proposition 2 remain valid. In particular, equation (38) in Appendix B with $x = \lambda_t - \alpha_t z_t$ implies abnormal returns are approximately $2\,\sigma_t\left[\frac{\alpha_t z_t}{4}\right] = \frac{\sigma_t \alpha_t z_t}{2}$, with this term inducing excess realized return volatility and return autocorrelation.

B Taylor Series Expansion of Probability

To obtain a linear approximation of $P^U(t)$ in equation (16), consider the Taylor series expansion of $f(x) = \frac{1}{1+e^{-x}} = (1+e^{-x})^{-1}$ whose first and second derivatives evaluated at $x = 0$ are

$$\begin{aligned} f'(x=0) &= -(1+e^{-x})^{-2}(-e^{-x})\Big|_{x=0} = \frac{1}{4} \\ f''(x=0) &= 2(1+e^{-x})^{-3}(-e^{-x})^2 - (1+e^{-x})^{-2}(e^{-x})\Big|_{x=0} = \frac{2}{8} - \frac{1}{4} = 0\,, \end{aligned}$$

which implies

$$\frac{1}{1+e^{-x}} = \left(\frac{1}{0!}\cdot\frac{1}{2}\right) + \left(\frac{1}{1!}\cdot\frac{1}{4}\right)x + \left(\frac{1}{2!}\cdot 0\right)x^2 + \text{h.o.t.} \approx \frac{1}{2} + \frac{x}{4}\,. \tag{38}$$

For completeness, the coefficient for the cubic term equals $\frac{1}{3!}f'''(0)\,x^3 = -\frac{x^3}{48}$. Therefore, with $x = -\lambda + \alpha z_t$, the probability $P^U(t)$ in equation (16) has the following linear approximation

$$P^U(t) \approx \frac{1}{2} - \frac{\lambda}{4} + \frac{\alpha z_t}{4}\,, \tag{39}$$

whose accuracy extends to the third order.

C Aggregating Individuals into a Market Probability

This appendix extends our previous analysis by considering two types of investors whose probabilities are $P_1^U(t)$ and $P_2^U(t)$ respectively. Generalizing the economy to contain $N > 2$ traders is straightforward. Aggregating multiple probability measures into the market probability measure allows our methodology to obtain time-varying α coefficients. In addition, investor-specific probabilities regarding future price increases that are obtained from experimental markets can be aggregated into market probabilities.

Denote the fraction invested by the two types of traders in the asset as $f_1(t) \in [0,1]$ and $f_2(t) = 1 - f_1(t)$. Assumptions are not imposed on the wealth of the traders nor the fraction of their wealth invested in the asset since the $f_j(t)$ functions for $j = 1, 2$ are subsumed by time-varying coefficients for the market probability.[23]

In particular, conditional on any $f_j(t)$ allocation, the market probability measure equals

$$P^U(t) \;=\; f_1(t)\, P_1^U(t) + f_2(t)\, P_2^U(t)\,, \tag{40}$$

since a convex combination of two probability measures forms another probability. Time-variation in the $f_j(t)$ allocation causes the α and λ coefficients of $P^U(t)$ in equation (40) to be time-varying.

As an illustration, equation (39) in Appendix B enables equation (40) to be approximated as

$$\begin{aligned}
P^U(t) \;&\approx\; f_1(t)\left(\frac{1}{2} - \frac{\lambda_1}{4} + \frac{1}{4}\alpha_1 z_t\right) + f_2(t)\left(\frac{1}{2} - \frac{\lambda_2}{4} + \frac{1}{4}\alpha_2 z_t\right) \\
&=\; \frac{1}{2} - \frac{1}{4}\left[f_1(t)\,\lambda_1 \;+\; f_2(t)\,\lambda_2\right] + \frac{1}{4}\left[f_1(t)\,\alpha_1 \;+\; f_2(t)\,\alpha_2\right]\, z_t \\
&=\; \frac{1}{2} - \frac{\lambda_t}{4} + \frac{\alpha_t\, z_t}{4} \\
&\approx\; \frac{1}{1 + e^{\lambda_t - \alpha_t z_t}}\,,
\end{aligned} \tag{41}$$

since $f_1(t) + f_2(t) = 1$. Consequently, time-varying α_t coefficients defined as $f_1(t)\,\alpha_1 \;+\; f_2(t)\,\alpha_2$ in equation (41) capture fluctuations in the positions of the traders. Thus, the composition of the investor clientele for an asset may change over time when traders with different beliefs regarding future price increases transact with one another. However, it is unnecessary to calibrate the individual α_1, α_2, $f_1(t)$, and $f_2(t)$ components separately. Instead, estimating time-varying α_t coefficients is sufficient. An identical property follows for the λ_t intercept.

[23]Feedback trading may occur when the $f_j(t)$ functions are elements of the superfluous information set z_t.

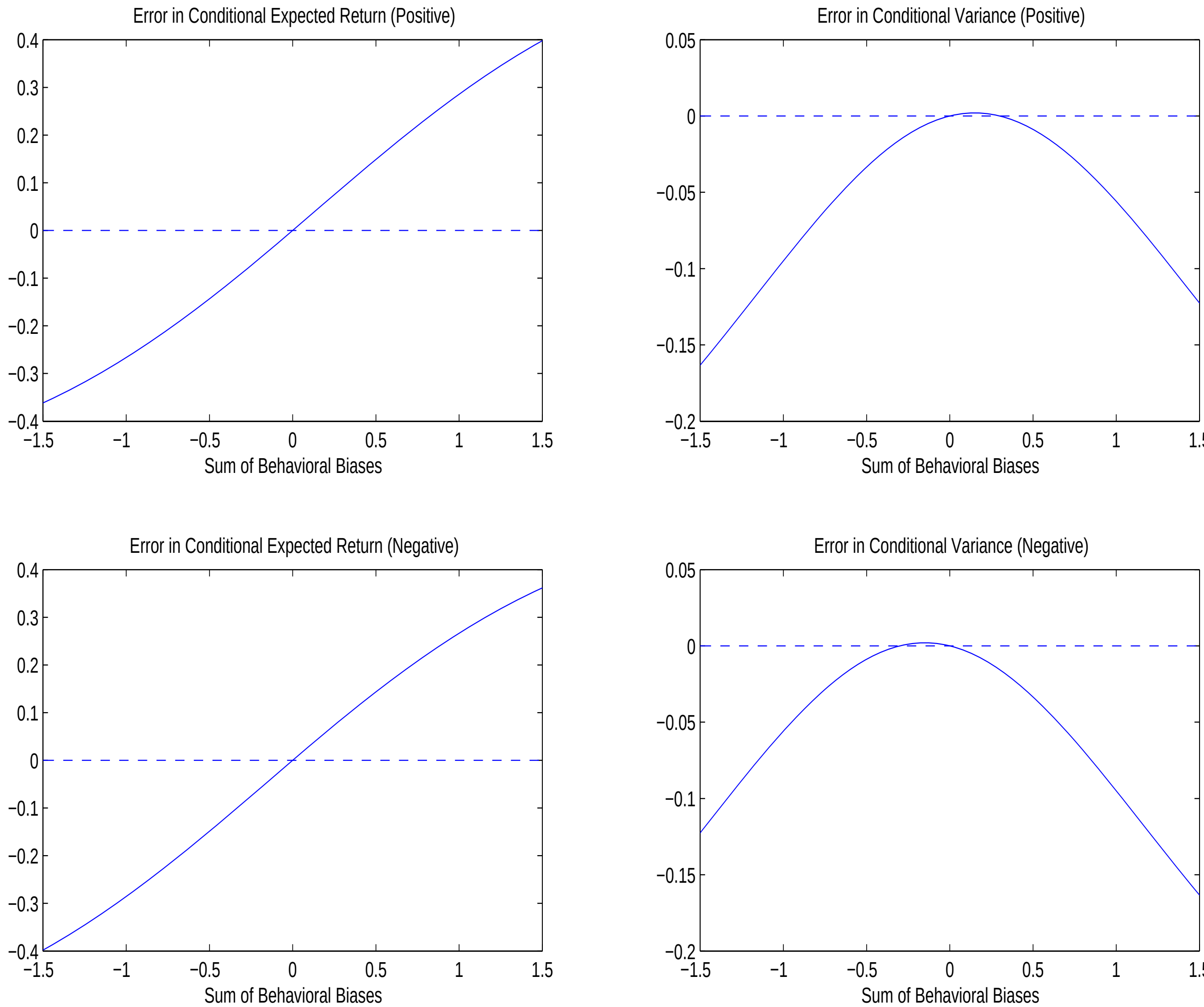

Figure 1: Errors in the market's conditional expected return equal the difference between equation (4) and equation (34), while errors in the variance of this conditional expectation are computed as the difference between equation (7) and equation (35). The σ_t parameter equals 0.60 in all four plots, while all errors are independent of μ_t. The *sum of behavioral biases* along the x-axis is defined as αz_t in equation (16). The two plots in the top row (Positive) have $\lambda = 0.15$ while those in the bottom row (Negative) have $\lambda = -0.15$. Thus, the variance of the market's conditional expected return is slightly overestimated when αz_t lies in either the $(0, 2\lambda)$ or $(2\lambda, 0)$ interval.

Table 1

Summary Statistics for Abnormal Returns and Behavioral Bias Proxies

This table reports summary statistics for the dependent variable $\hat{L}_t$ and the behavioral bias proxies in z_t for the 2,087 firms (94,242 firm-quarter observations) during our 1986 to 2004 sample period. The dependent variable is defined as $\hat{L}_t = \frac{y_t - \hat{\mu}_t}{2\hat{\sigma}_t} + \frac{1}{2}$ where y_t is the firm's 1-month nominal return, while $\hat{\mu}_t$ denotes the stock's expected return from the market model. Volatility is measured as the annualized standard deviation of daily returns over the previous quarter. For each firm, averages for the components of the superfluous information set and the firm-specific dependent variables are computed for those entries which are non-zero, along with this percentage. Summary statistics for these averages and non-zero percentages are reported below across the 2,087 firms in our sample. The annualized volatility, abnormal return, dependent variable and past return are always non-zero.

Variable	Description	Mean	Median	Std. Dev.	25th Percentile	75th Percentile
Return Volatility	Annualized	0.3853	0.3386	0.1970	0.2459	0.4761
Abnormal Return (y_t - $\hat{\mu}_t$)	1-month	0.0069	0.0033	0.1006	-0.0457	0.0544
Dependent Variable ($\hat{L}_t$)	1-month	0.5352	0.5176	0.4479	0.2682	0.7860
Representativeness (z_t^{rep})	Average	-0.0007	0.0003	0.0051	-0.0014	0.0013
	% non-zero	32.0%	31.5%	14.1%	22.0%	40.0%
Conservatism (z_t^{con})	Average	0.0009	0.0002	0.0087	-0.0016	0.0027
	% non-zero	12.5%	12.1%	6.6%	7.7%	17.1%
Overconfidence (z_t^{oc})	Average	-0.0011	-0.0002	0.0045	-0.0008	0.0000
	% non-zero	42.4%	40.0%	20.6%	26.7%	56.4%
Biased Self-Attribution (z_t^{bsa})	Average	-0.0008	-0.0004	0.0012	-0.0010	-0.0001
	% non-zero	44.0%	41.5%	20.0%	29.2%	57.8%
Past Return (z_t^{pr})	Average	0.0468	0.0428	0.0335	0.0286	0.0618

Table 2

Evidence of Behavioral Biases from Pooled Estimation

This table reports on the presence of behavioral biases in stock returns using a pooled sample of quarterly earnings announcements from 1986 to 2004. The α estimates below arise from the non-linear regression in equation (12). Summary statistics for the dependent variable $\hat{L}_t$ and the z_t vector of behavioral bias proxies constructed from analyst forecasts are reported in Table 1. The estimates for α are obtained from a Fama-MacBeth (1973) procedure by first estimating coefficients using all firms that announce earnings in each calendar quarter, and then computing their time-series average. The significance of the time-series averages is determined by Newey-West *t*-statistics. Panel A records the α estimates from a joint estimation involving the entire superfluous information set in equation (23), while Panel B contains estimates from separate calibrations using single elements of the z_t vector. The asterices *, **, and *** denote a coefficient's statistical significance at the 10%, 5%, and 1% levels respectively.

	Fama - MacBeth Estimates		
	Time-Series Average	Newey-West t-statistic	p-value
Panel A: Joint Estimation of Biases			
Representativeness	9.714***	(2.74)	0.0078
Conservatism	7.782*	(1.98)	0.0519
Overconfidence	9.075*	(1.67)	0.0982
Biased Self-Attribution	-20.036*	(-1.77)	0.0807
Past Return	0.000	(0.01)	0.9959
Panel B: Separate Estimation of Biases			
Representativeness	9.215**	(2.27)	0.0261
Conservatism	8.090**	(2.13)	0.0361
Overconfidence	-15.593	(-1.50)	0.1386
Biased Self-Attribution	-13.924	(-1.38)	0.1703
Past Return	0.006	(0.07)	0.9451

Table 3

Returns from Calendar-Time Portfolios formed using Biased Market Probabilities

Calendar-time portfolios from 1990 to 2004 are formed from firm-specific market probabilities each firm-quarter. A stock is bought (sold) for 1-month, specifically the [6,26] day interval, in quarter t if the market probability $\hat{P}^U(t)$ is above 0.54 (below 0.46) where $\hat{P}^U(t)$ is defined in equation (2). The superfluous information z_t is the vector of behavioral bias proxies constructed from analyst forecasts and a stock's prior return during the previous quarter. The required estimates for α are obtained by applying the non-linear model in equation (12) to cross-sections of firms in different two-digit SIC industries over the *prior* four-year non-overlapping period. For example, the first four-year calibration period is 1986 to 1989, with these corresponding estimates for α resulting in market probabilities $\hat{P}^U(t)$ for firms in the same industry during the subsequent 1990 to 1993 subsample. Daily value-weighted (using the firm's lagged-day market capitalization) returns for these portfolios are compounded to monthly returns, excluding firm-days where the lagged price is less than $5. Monthly portfolio returns are then regressed against the four-factor model with their corresponding loadings reported below. Panel A reports the portfolio returns using the entire z_t vector to compute $\hat{P}^U(t)$ while Panels B to D focus on subsets of the z_t vector. The results in Panel E arise from a trading strategy which replaces the z_t vector with the underlying analysts' forecasts and revisions defining our behavioral bias proxies. The asterices *, **, and *** denote significance of the coefficients at the 10%, 5%, and 1% levels respectively with the absolute value of their t-statistics in parentheses. All t-statistics use zero as the null except for the MKTRF coefficient of the buy and sell portfolios which uses one.

Portfolio	Intercept (%)	MKTRF	SMB	HML	UMD	Adj-R^2	Average Firms per Day	Number of Months
			Panel A: Entire Superfluous Information Set					
Buy	0.787**	1.02	0.195**	0.083	-0.042	0.527	57.5	179
	(2.34)	(0.22)	(2.09)	(0.72)	(0.63)			
Sell	-0.343	1.169*	0.045	0.376***	-0.104	0.484	43.0	179
	(0.90)	(1.70)	(0.42)	(2.87)	(1.39)			
Buy-Sell	1.130**	-0.15	0.151	-0.293*	0.062	0.022	100.5	179
	(2.22)	(1.13)	(1.06)	(1.67)	(0.62)			

Table 3 (Continued)

Portfolio	Intercept (%)	MKTRF	SMB	HML	UMD	Adj-R^2	Average Firms per Day	Number of Months
				Panel B: Representativeness, Conservatism and Past Return				
Buy	0.781**	1.001	0.210*	0.074	0.002	0.451	46.0	179
	(2.04)	(0.01)	(1.97)	(0.56)	(0.02)			
Sell	-0.423	1.176*	0.128	0.418***	0.202***	0.506	37.6	179
	(1.11)	(1.77)	(1.21)	(3.19)	(2.70)			
Buy-Sell	1.204**	-0.175	0.082	-0.344**	0.204**	0.052	83.6	179
	(2.40)	(1.33)	(0.58)	(1.99)	(2.06)			
				Panel C: Overconfidence, Biased Self-Attribution and Past Return				
Buy	0.532	1.044	0.257**	0.132	-0.104	0.498	45.6	179
	(1.44)	(0.46)	(2.50)	(1.03)	(1.44)			
Sell	-0.116	1.186*	-0.142	0.379***	-0.067	0.450	34.8	179
	(0.29)	(1.75)	(1.26)	(2.72)	(0.85)			
Buy-Sell	0.648	-0.141	0.399**	-0.247	-0.037	0.051	80.5	179
	(1.16)	(0.97)	(2.58)	(1.29)	(0.34)			
				Panel D: Past Return				
Buy	0.323	0.98	0.316**	0.136	-0.032	0.303	32.3	179
	(0.62)	(0.15)	(2.19)	(0.76)	(0.31)			
Sell	0.078	1.09	-0.015	0.276*	-0.227**	0.389	27.3	179
	(0.17)	(0.75)	(0.12)	(1.75)	(2.52)			
Buy-Sell	0.245	-0.111	0.331*	-0.14	0.195	0.037	59.6	179
	(0.39)	(0.68)	(1.91)	(0.66)	(1.60)			
				Panel E: Analysts' Forecasts and Revisions				
Buy	0.526	1.210*	0.111	0.491***	-0.312***	0.501	22.2	179
	(1.29)	(1.97)	(0.97)	(3.49)	(3.88)			
Sell	0.116	1.201*	0.207	0.735***	-0.171*	0.397	19.3	179
	(0.25)	(1.67)	(1.61)	(4.63)	(1.88)			
Buy-Sell	0.411	0.009	-0.096	-0.244	-0.141	-0.001	41.5	179
	(0.71)	(0.06)	(0.60)	(1.23)	(1.25)			

Table 4

Excess Return Volatility and Autocorrelation

This table summarizes the implications of market probability fluctuations induced by behavioral biases on nominal return volatility and autocorrelation. The α coefficients underlying the market probabilities are identical to those employed in constructing the calendar-time trading profits in Table 3 when the superfluous information set consists of all five z_t elements in equation (23). The resulting estimates for $\hat{P}^U(t)$ are then used to compute the excess return volatility in equation (30) as well as the return autocorrelation in equation (31) attributable to behavioral biases. In addition, firm-specific summary statistics are computed for equation (32) which pertains to the underestimation of ex-ante return volatility. The average of each summary statistic across the individual firms is reported below in the bottom row.

Variable	Mean	Std. Dev.	Percentiles			Number of Firms
			25th	50th	75th	
Annualized Excess Volatility	0.0475	0.0205	0.0130	0.0350	0.0700	2,085
Autocorrelation	0.0728	0.0753	-0.1566	0.0769	0.3098	2,085
Underestimated Ex-Ante Volatility	0.0162	0.0165	0.0063	0.0114	0.0197	2,085