

The Performance of Fundamentally Weighted Indices¹²

Noël Amenc³
Felix Goltz
Véronique Le Sourd

October 10th, 2007

¹ We would like to thank Robert Arnott for useful comments and John Southard and Aaron Toly (Intellidex), Steven Tischer and Kevin Heckert (Mergent), Vincent Lowry and Michael Gompers (VTL Associates), and Jeremy Schwarz (Wisdom Tree) for providing us with data for their indices.

² The different index providers own trademarks and copyrights for the names of the indices mentioned in this paper. Also, Research Affiliates owns the trademark and copyright on the term “Fundamental Index®” and many variations on that label, which has led us to employ the term “characteristics-based” indices throughout the paper, which we also hold to be more precise. Furthermore, there are patents pending in the area of the indices mentioned in this paper. The authors take no view on whether these patents are being honoured or violated by the indices analysed in this paper, the objective of which is merely to analyze and compare the commercially available indexes.

³ All authors are at EDHEC Risk & Asset Management Research Centre, 400 promenade des Anglais, 06200 Nice France, Email : research@edhec-risk.com

Abstract

This paper analyses a set of characteristics-based indices that have been recently launched on the U.S. market and have been argued to display considerable outperformance of standard market cap-weighted indices over particular backtest samples. We analyse the performance of an exhaustive list of such indices and show that i) the outperformance over value-weighted indices may be negative over long time periods, and ii) there is no significant outperformance over simple equal-weighted indices. Furthermore, an analysis of both the style exposures and the sector exposures of characteristics-based indices reveals a significant value tilt. When properly adjusting for this tilt, these indices do not show any abnormal performance. Therefore, we argue that the main value added of such indices may be to provide investors with a liquid, systematic and relatively cheap alternative to other value-tilted strategies. However, it should also be noted that if one recognises the possibility to implement tilts of exposures to sector or style factors, it is straightforward to construct factor portfolios that beat the characteristics-based indices in the sense of mean-variance efficiency.

1. Introduction

While an ever increasing part of equity assets is invested in indexing strategies, the standard practice of using a capitalisation weighting scheme for the construction of stock market indices has been subject to severe criticism. A number of papers (see e.g. Haugen and Baker (1991), Amenc, Goltz, and Le Sourd (2006), or Hsu (2006)) point out that the mechanics of capitalisation weighting lead to trend-following strategies that provide an inefficient risk-return trade-off.

As an answer to such critiques, equity indices with different weighting schemes have emerged. A particular type of weighting scheme is implemented by indices that use firm characteristics to weight the component stocks (see Arnott, Hsu and Moore (2005)). The idea behind such indices is that market capitalisation actually does not convey much information about a stock, and it would be preferable to use other indicators of company size, such as the book value, sales, or dividends. Some index providers also underline that the value-appreciation potential of a stock may be conveyed by such firm characteristics, making it preferable to overweight stocks with high value-appreciation potential.

Such characteristics-based indices in principle deviate from capitalisation-weighting in two aspects: i) components may be chosen by a characteristics-based selection process, meaning that some stocks will be excluded from the index (though the attribution of weights is not necessarily different from capitalisation-weighting); and/or ii) the weights in the index are not determined by the stocks' capitalisation (though the components may be the stocks with the highest capitalisations).

Over the past years, the market for characteristics-based indices has grown tremendously, with more and more providers launching an offer of such indices. On the demand side, institutional investors have allocated significant amounts to such alternatives to value-weighted indices. Likewise, a wide range of exchange-traded funds on these new indices is now available.

A common claim of providers of characteristics-based indices is that their weighting mechanism allows constructing index portfolios that outperform their market cap-weighted counterpart. The use of certain metrics for selecting stocks and/or attributing weights to these stocks is supposed to create some value in terms of the average returns of the resulting portfolio. Most index providers calculate performance indicators for their alternative weighted indices and compare the results to a value-weighted index in a similar range of capitalization. As shown in table 1a, the overall conclusion is that the characteristics-based indices outperform value-weighted indices. The characteristics-based indices are compared to the value-weighted indices in terms of average returns, Sharpe ratio, or in terms of information ratio. However, providers of characteristics-based indices do not consider indices with other weighting schemes, such as equal-weighted indices as benchmarks, though they constitute an obvious alternative that is not value weighted but does not rely on proprietary weighting mechanisms. In addition, the performance measures used, such as the CAPM alpha, do not take into account the exposure to risk factors such as the value premium. Obviously, for indices that are precisely constructed to capture a value premium, these comparisons will give an overly positive account of performance⁴.

⁴ Note that Arnott, Hsu and Moore (2005) mention both the Fama French three factor model (noticing that their weighting have a tilt to the value factor and thereby earn a value premium relative to a capitalization-weighted equity market index) and comparisons with equal weighted indices but do not provide any results for these.

Table 1 sums up the indicators used by the providers to test their indices and the results obtained.

Table 1: Index Performance results according to the providers

| Index Name | Provider | Source | Indicators | Results |
|--|----------------------------|--|---|---|
| Dow Jones US Select Dividend | Dow Jones | Web site | Average returns compared to value weighted indices | Overperformance compared to large-cap US indices |
| Dynamic Market Intellidex | AMEX | Southard & Bond (2003) | Sharpe ratio compared to value weighted indices CAPM alpha | Overperformance relative to S&P 500 |
| WisdomTree Earnings-Weighted Indexes and Dividend-Weighted Indexes | WisdomTree | Siracusano III & Schwartz (2007) Siegel, Schwartz & Siracusano III (2007) | Differential vs. value weighted index Sharpe ratio compared to value weighted indices CAPM alpha | Overperformance relative to S&P and Russell indices |
| FTSE RAFI 1000 | FTSE / Research Affiliates | Arnott, Hsu & Moore (2005) | Differential vs. S&P 500 Sharpe ratio compared to S&P 500 Information ratio with respect to S&P 500 CAPM alpha | Overperformance for the January 1962-December 2003 whole period and sub-periods |
| Mergent's Broad Dividend Achievers | Mergent / AMEX | Web site | Average returns compared to S&P 500 Sharpe ratio compared to S&P 500 | Overperformance relative to S&P 500 |

The practice of providers of characteristics-based indices to assess performance with respect to value weighted indices for the broad market or for large cap stocks does not take into account that multiple priced risk factors exist in the cross section of expected stock returns. In fact, there is a consensus in empirical finance that using only the value weighted market portfolio as a risk factor does not provide a full characterisation of systematic risk. Rather, multiple risk factors are priced in the cross section of expected stock returns and the existence of such risk factors will have an impact on optimal portfolio holdings (see e.g. Cochrane (1999) or Fama (1996)).

No study has yet assessed the performance of characteristics-based indices in a multifactor framework. In addition, despite their recent success, no extensive assessment or comparison involving indices from different providers has been conducted so far. This paper's objective is to fill this void by providing a study of the risk and return properties of these new indices, using an extensive database that includes all available indices with sufficient data. In particular, we provide an insight into the performance of a set of 14 characteristics-based indices that are calculated by seven different providers for the US stock market.

Our results confirm that characteristics-based indices outperform value-weighted indices, though the return difference is not statistically significant for most indices. With respect to equal-weighted indices, none of the indices achieves significant outperformance while two of them show significant underperformance. An analysis of the characteristics-based indices' exposures to style and industry portfolios shows that characteristics-based indices display significant value tilts, which explain their outperformance over value-weighted indices. Furthermore, minimum variance portfolios constructed from these style or industry portfolios generally dominate the characteristics-based indices in terms of mean-variance efficiency.

The remainder of the paper is organised as follows. Section two gives an overview of existing indices and describes the data set. The analysis of return differences with standard indices is found in section three, while section four and five proceed to the static and dynamic factor

analysis. Section six conducts the efficiency comparison with minimum variance portfolios and a final section concludes.

2. Data

2.1. Description of Alternative Weighting Mechanisms.

When constructing indices, the main two steps are the inclusion criteria for stocks and the weighting scheme. The choice of the index sample universe concerns the choice of the number and type of assets to include in the index. The weighting mechanism is the second important factor in constructing indices. Value-weighted indices use the same criterion, relative market capitalisation of a stock, for both these tasks. Alternative construction methodologies for either selecting or weighting stocks may outperform capitalisation weighted market indices for a number of reasons. These include:

- (i) Use of better allocation techniques;
- (ii) Access to additional risk premia;
- (iii) Exposure to undervalued securities and exploitation of market inefficiencies.

The possible differences to capitalisation weighting are twofold:

- (i) The weighting criterion is different from the market capitalisation;
- (ii) Strategies may not be buy-and-hold.

The different alternatives to constructing indices have to be seen in the light of the solution(s) they bring to the shortcomings of capitalisation-weighted indices. We have to assess if they are attractive investment alternatives or not. Therefore, we undertake an empirical study of their returns in order to conclude on this latter point. However, before proceeding to the empirical study, we will give a detailed overview over the indices that we study.

2.2. Overview of Providers of New Indices

The focus of this paper is exclusively on U.S. equity indices, though indices that deviate from the market capitalisation criterion in either in the weighting or in the selection of components have been launched for other regions as well. While it is beyond the scope of this paper to describe the construction method of each provider in detail, we provide an overview of the indices studied. Table 2a gives a broad overview of indices studied.

Table 2a: Broad Overview of Index Providers

| <i>Index Family</i> | <i>FTSE GWA US</i> | <i>DJ US Select Dividend</i> | <i>Intellidex</i> | <i>Wisdom Tree</i> | <i>FTSE RAFI 1000</i> | <i>Mergent Dividend Achievers</i> | <i>VTL Associates Revenue Indexes</i> |
|---------------------------------|----------------------------|--------------------------------------|-------------------|------------------------|---------------------------|---|---|
| Selection by Characteristics | | X | X | X | X | X | X |
| Characteristics-based Weighting | X | X | | X | X | X | X |

As can be seen from table 2a, most index providers choose to abandon the market capitalisation criterion completely, while the Intellidex index actually weighs components by market capitalisation and the FTSE GWA index selects the component stocks according to their market capitalisation. The following table contain the detailed characteristics of each

index described: the stock universe, weighting and selection mechanism, rebalancing in table 2b, and the index providers' websites in table 2c. As it can be seen, all indices have been created recently, the oldest ones dating back to 2003. Meanwhile, we have been provided with longer track records, including the backtest period prior to the launch date.

Table 2b: Detailed Overview of Index Providers

| Family Name / Index Name | Provider | Launching Date | Index Universe/ Nb Constituents | Weighting | Rebalancing |
|--|--------------------------------|-------------------|---|--|--|
| FTSE GWA Index Series: ◦ FTSE GWA US Index | FTSE / GWA | October 17, 2005 | FTSE All World Index / All constituents of the underlying index | Weighting according to the ability to create wealth measured by: ◦ Net profit ◦ Cash Flow ◦ Book Value | Quarterly (March, June, September, December) |
| Dow Jones US Select Dividend | Dow Jones | November 2003 | All dividend paying companies in the DJ US Total Market Index / 100 constituents (higher dividend yield) | Dividend-weighted | Annually December |
| Dynamic Intellidex ◦ Dynamic Market Intellidex | AMEX | May 12, 2001 | 2000 largest US NYSE, AMEX and NASDAQ stocks/ 100 constituent (selected according to growth potential, evaluated with 25 factors) | Equal weighted | Quarterly |
| WisdomTree Domestic Earnings-Weighted Indexes ◦ WisdomTree Earnings Index ◦ WisdomTree Low P/E Index ◦ WisdomTree Earnings 500 Index ◦ WisdomTree Earnings Top 100 Index | WisdomTree / Standard & Poor's | February 2, 2007 | NYSE, AMEX and NASDAQ US stocks generating earnings ◦ 2450 constituents ◦ 700 constituents (30% lowest P/E ratios) ◦ 500 constituents (largest market capitalizations) ◦ 100 constituents (highest earnings yield) | ◦ Aggregate earnings weighted ◦ Aggregate earnings weighted ◦ Aggregate earnings weighted ◦ Earnings yield weighted | Annually December |
| WisdomTree Domestic Dividend-Weighted Indexes ◦ WisdomTree Dividend Index ◦ WisdomTree High Yielding Equity Index ◦ WisdomTree LargeCap Dividend Index ◦ WisdomTree Dividend Top 100 Index | WisdomTree | June 1, 2006 | NYSE, AMEX and NASDAQ US stocks paying cash dividends ◦ 1500 constituents ◦ 400 constituents (30% highest dividend yield) ◦ 300 constituents (largest market capitalizations) ◦ 100 constituents (highest dividend yield) | ◦ Projected cash dividends weighted ◦ Projected cash dividends weighted ◦ Projected cash dividends weighted ◦ Dividend-yield weighted | Annually December |
| WisdomTree Domestic Hypothetical Net Income-Weighted Indexes | WisdomTree | In test | | | Annually December |
| FTSE RAFI ◦ FTSE RAFI 1000 | FTSE / Research Affiliates | November 28, 2005 | FTSE USA All 1000 Cap Index | Weighted according to: Sales Cash Flow Book Value Dividends | Annually February |
| Mergent's Dividend | Mergent / AMEX | | NYSE, AMEX and | Market | Annually |

| | | | | | |
|--|---------------------------------------|--------------------|---|------------------|----------------------|
| Achievers ◦ Mergent's Broad Dividend Achievers | | ◦ January 17, 2003 | NASDAQ stocks having increased their annual regular dividends for at least the past 10 consecutive years and meeting specific liquidity screening criteria. 330 constituents (April 30, 2007) | Capitalisation | |
| VTL Revenue Weighted Indices: | VTL Associates / Standard & Poor's | | All constituents of their respective underlying index: | Company revenues | Annually December |
| ◦ VTL Revenue Weighted Large Cap Index | | ◦ March 30, 2006 | ◦ S&P 500 | | |
| ◦ VTL Revenue Weighted Mid-Cap Index | | ◦ March 1, 2006 | ◦ S&P Mid Cap 400 | | |
| ◦ VTL Revenue Weighted Small- Cap Index | | ◦ August 4, 2006 | ◦ S&P Small Cap 600 | | |

Table 2c: Index Providers Websites

| Index Family | Website |
|--------------------------------|--|
| FTSE GWA Index Series | http://www.ftse.com/Indices/FTSE_GWA_Index_Series/Index_Rules.jsp |
| Dow Jones US Select Dividend | http://djindexes.com/mdsidx/?event=showSelectDiv |
| Dynamic Intellidex | http://powershares.com |
| WisdomTree indices | http://www.wisdomtree.com/home.asp |
| FTSE RAFI 1000 | http://www.researchaffiliates.com |
| Mergent Dividend Achievers | http://www.ftse.com/Indices/FTSE_RAFI_Index_Series/index.jsp http://www.mergent.com/Dividend_Achievers.asp www.dividendachievers.com |
| VTL Associates Revenue Indexes | http://www.vtlassoc.com/rwicomplete.asp |

2.3. Data Sources

In order to analyse the indices described, we collect returns data for all the indices mentioned⁵. In particular, we obtain the total returns (i.e. including dividends) on the US stock market index for the provider in question. There is generally one index per provider; only for Wisdom Tree, we conserve eight indices, since they calculate their US stock market indices in different versions.

In order to compare the characteristics-based indices to more straightforward weighting mechanisms, we also collect data for other US stock market indices. These include the equal- and value weighted indices of the S&P 500 components and the equal- and value-weighted total market indices of NYSE stocks collected from CRSP, as well as value-weighted US industry portfolios for 12 industry sectors collected from Kenneth French's website⁶. We change the sector labels slightly from the original ones for better comprehension. The sectors represented are Consumer Staples, Consumer Discretionary, Manufacturing, Energy, Chemicals, Technology, Telecom Services, Utilities, Sales (Retail & Wholesale), Health Care, Financial Services and Others. We also obtain returns for the Fama and French (1992, 1993) factors, i.e. the excess return on the total market index, the small cap premium and the value premium, as well as returns for the Jegadeesh and Titman (1993) momentum factor. For brevity, we will refer to these benchmark portfolios as the S&P 500, the Total Market Index

⁵ Data is either obtained directly from the index provider or from Datastream or Bloomberg.

⁶ <<http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>>

(TMI), the sector indices, and the style factors. The returns for these portfolios are also inclusive of dividends.

We choose monthly data frequency, as for some of the indices (e.g. Wisdom Tree, Mergent, GWA), the data is not available at daily frequency or only over an extremely short time period. Even with monthly data, the time period with available data is relatively short for some of the characteristics-based indices. This may be understandable, given their recent appearance. We impose the constraint of at least 100 monthly observations for inclusion of an index. As a result, the index with the shortest time-period that we retain starts in January 1998 (108 data points).

On the contrary, some providers have constructed track records ranging as far back as 1962. In order to not discard this data by looking at the shortest common time-period, we proceed as follows. First, we analyse each index for the time-period where data is available for the respective index. As a result, this long-horizon analysis yields results that may not be directly comparable, as the time-period analysed differs between different indices. This should be kept in mind when analysing the results. In particular, levels of statistical significance are naturally impacted by the number of observations, which differ across indices in this part of the analysis. Then, we analyse the indices over the period starting in January 1998. This analysis is based on a shorter time-period, but allows for a direct comparison across characteristics-based indices. Our sample for all indices stops in December 2006, as the CRSP data, which we use for the comparison to value- and equal-weighted indices ends at this point.

We refer to the two different datasets as “Long Term Data” and “Short Term Data” respectively. Table 3a and 3b show descriptive statistics for both time periods for the characteristics-based indices. The result for the value-weighted index of S&P 500 components is indicated in the rightmost column. The tables show the annualised mean of index returns, as well as various common risk and performance measures. The last two lines indicate the start date and the number of months used for each index. Downside risk is calculated as the semi-deviation of returns with respect to a threshold of zero. The Value-at-Risk measures have been calculated using a Cornish-Fisher extension as in Zangari (1996). For the case of table 3a, these lines indicate how much data is available for the respective index. Here, one can ascertain the important difference in availability of historical data which we mentioned above.

Table 3a: Descriptive Statistics for the Long Term Data

| | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellidex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent | S&P 500 Value Weighted |
|-------------------|-----------|-------|-------|--------------------|------------|---------------------|---------------------------|----------------------|-------------------------|--------------------|------------------------|----------------------------|------------------------|---------|------------------------|
| Ann. Mean | 13.8% | 14.9% | 14.3% | 16.8% | 17.2% | 13.0% | 12.4% | 14.7% | 14.2% | 14.3% | 14.1% | 16.6% | 16.2% | 14.9% | 11.9% |
| Ann. Std. Dev. | 14.7% | 16.9% | 14.1% | 13.7% | 13.3% | 13.2% | 13.2% | 12.7% | 13.4% | 13.1% | 13.2% | 14.6% | 14.5% | 13.7% | 14.7% |
| Downside Risk | 10.1% | 12.2% | 10.1% | 10.4% | 8.7% | 9.1% | 9.0% | 8.2% | 8.7% | 9.1% | 9.1% | 10.1% | 9.7% | 10.2% | 10.3% |
| 5% VaR | 6.1% | 7.5% | 6.1% | 5.6% | 5.3% | 5.4% | 5.4% | 4.8% | 5.1% | 5.5% | 5.5% | 6.0% | 5.9% | 5.7% | 6.3% |
| 1% VaR | 11.7% | 13.4% | 10.7% | 11.1% | 9.5% | 10.3% | 10.0% | 8.9% | 9.2% | 9.9% | 9.8% | 10.7% | 10.0% | 12.1% | 11.8% |
| Sharpe Ratio | 0.51 | 0.65 | 0.72 | 0.92 | 0.98 | 0.52 | 0.48 | 0.66 | 0.60 | 0.73 | 0.71 | 0.80 | 0.78 | 0.68 | 0.39 |
| Sortino Ratio | 0.74 | 0.91 | 1.00 | 1.21 | 1.49 | 0.75 | 0.70 | 1.03 | 0.92 | 1.05 | 1.03 | 1.16 | 1.17 | 0.91 | 0.56 |
| Corr with S&P 500 | 0.96 | 0.79 | 0.95 | 0.71 | 0.87 | 0.94 | 0.95 | 0.83 | 0.82 | 0.96 | 0.97 | 0.82 | 0.82 | 0.91 | 1.00 |
| Start Date | 1965. | 1998. | 1994. | 1992. | 1992. | 1964. | 1964. | 1964. | 1964. | 1989. | 1989. | 1989. | 1989. | 1983. | 1964. |

| | | | | | | | | | | | | | | | |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| | 11 | 01 | 01 | 02 | 12 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 02 | 01 |
| Number of months | 494 | 108 | 156 | 179 | 169 | 516 | 516 | 516 | 516 | 216 | 216 | 216 | 216 | 287 | 516 |

Table 3b: Descriptive Statistics for the Short Term Data

| | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellidex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | WisdomTree Income | WisdomTree Income 500 | WisdomTree Income Low P/E | WisdomTree Income 100 | Mergent | S&P 500 Value weighted |
|-------------------------------|-----------|-------|-------|--------------------|------------|---------------------|---------------------------|----------------------|-------------------------|-------------------|-----------------------|---------------------------|-----------------------|---------|------------------------|
| Ann. Mean | 11.4% | 14.9% | 10.1% | 12.5% | 13.5% | 9.9% | 9.3% | 13.3% | 12.4% | 10.2% | 9.6% | 14.0% | 13.0% | 8.1% | 7.5% |
| Ann. Std. Dev. | 14.4% | 16.9% | 15.2% | 15.6% | 14.3% | 12.2% | 12.7% | 12.6% | 13.1% | 13.9% | 14.0% | 16.0% | 15.5% | 12.4% | 15.2% |
| Downside Risk | 10.7% | 12.2% | 11.1% | 11.8% | 9.6% | 8.7% | 9.3% | 8.4% | 9.7% | 10.3% | 10.4% | 11.6% | 10.9% | 9.1% | 11.0% |
| 5% VaR (CF) | 6.5% | 7.5% | 6.9% | 6.8% | 6.1% | 5.4% | 5.7% | 5.2% | 5.6% | 6.3% | 6.4% | 6.8% | 6.6% | 5.4% | 7.2% |
| 1% VaR (CF) | 11.2% | 13.4% | 11.6% | 12.5% | 10.6% | 9.5% | 10.0% | 8.5% | 9.6% | 11.4% | 11.3% | 12.3% | 11.2% | 9.8% | 11.5% |
| Sharpe Ratio | 0.54 | 0.65 | 0.43 | 0.56 | 0.68 | 0.51 | 0.45 | 0.76 | 0.67 | 0.47 | 0.42 | 0.64 | 0.60 | 0.37 | 0.26 |
| Sortino Ratio | 0.73 | 0.91 | 0.58 | 0.74 | 1.01 | 0.72 | 0.61 | 1.13 | 0.89 | 0.64 | 0.57 | 0.89 | 0.85 | 0.50 | 0.35 |
| Start Date | 1998.01 | | | | | | | | | | | | | | |
| Number of months | 108 | | | | | | | | | | | | | | |
| Corr with RAFI 1000 | | 0.90 | 0.98 | 0.88 | 0.82 | 0.96 | 0.95 | 0.82 | 0.82 | 0.98 | 0.97 | 0.92 | 0.92 | 0.88 | 0.91 |
| Corr with VTL | | | 0.84 | 0.83 | 0.85 | 0.84 | 0.80 | 0.75 | 0.75 | 0.90 | 0.85 | 0.87 | 0.85 | 0.73 | 0.79 |
| Corr with GWA | | | | 0.83 | 0.80 | 0.94 | 0.94 | 0.77 | 0.77 | 0.98 | 0.98 | 0.89 | 0.89 | 0.89 | 0.94 |
| Corr with DJ Select Dividend | | | | | 0.62 | 0.92 | 0.89 | 0.90 | 0.92 | 0.84 | 0.82 | 0.91 | 0.93 | 0.78 | 0.66 |
| Corr with Intellidex | | | | | | 0.71 | 0.69 | 0.56 | 0.54 | 0.86 | 0.84 | 0.73 | 0.70 | 0.65 | 0.87 |
| Corr with WisdomTree Dividend | | | | | | | 0.99 | 0.90 | 0.91 | 0.93 | 0.93 | 0.92 | 0.94 | 0.93 | 0.82 |
| Corr with WT Large Div. | | | | | | | | 0.86 | 0.88 | 0.93 | 0.93 | 0.89 | 0.92 | 0.95 | 0.83 |
| Corr with WT HY Equity Index | | | | | | | | | 0.96 | 0.77 | 0.74 | 0.87 | 0.87 | 0.72 | 0.58 |
| Corr with WT Div. 100 | | | | | | | | | | 0.77 | 0.75 | 0.87 | 0.91 | 0.75 | 0.57 |
| Corr with WT Income | | | | | | | | | | | 0.99 | 0.92 | 0.90 | 0.88 | 0.94 |
| Corr with WT Income 500 | | | | | | | | | | | | 0.90 | 0.89 | 0.90 | 0.95 |
| Corr with WT Income Low P/E | | | | | | | | | | | | | 0.96 | 0.77 | 0.75 |
| Corr with WT Income Top 100 | | | | | | | | | | | | | | 0.82 | 0.73 |
| Corr with Mergent | | | | | | | | | | | | | | | 0.83 |

Table 3b for the short term data also shows the matrix of correlation coefficients between index returns. In fact, given the variety of indices available, one may ask if there is a clear difference between these indices or if they simply represent the same investment strategy. Clearly, from the coefficient values in table 3b, one can see that the different indices are highly correlated overall, with most correlation coefficients around or above 0.9. On the other hand, some indices seem to display quite different behaviour, taking for example the correlation of 0.54 between the Intellidex index and the Wisdom Tree Dividend Top 100. It should also be noted that Intellidex has relatively low correlation with most other characteristics-based indices. Furthermore, the correlation with the S&P 500 value-weighted index is quite high for most of these indices over the short period, while correlations are even higher over the long period, as indicated in table 3a.

3. Out-Performance of Standard Indices

3.1. Overall Out-Performance

In order to assess whether characteristics-based indices do yield higher returns than standard stock market indices, we compare the annualised mean returns of the respective characteristics-based index to those of the standard index over the same time-period. In addition to the value-weighted S&P 500 index, we also consider the equal-weighted index of S&P components, as well as the equal-weighted and value-weighted total market index for this comparison. Table 4a shows the difference in terms of annual mean returns, as well as the p-value for the corresponding t-statistic⁷. The p-values for mean return differences that are significant at the 5% level are indicated in bold typeface.

Table 4a: Return Differences with Standard Indices for the Long Term Data

| Return Difference | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellindex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent |
|-----------------------------|-------------|-------------|--------|--------------------|-------------|---------------------|---------------------------|----------------------|-------------------------|--------------------|------------------------|----------------------------|------------------------|---------|
| over S&P 500 Value Weighted | 1.88% | 6.93% | 1.83% | 4.23% | 4.58% | 1.04% | 0.51% | 2.56% | 2.09% | 1.07% | 0.84% | 3.06% | 2.70% | 0.52% |
| p-value for difference | 0.4% | 6.0% | 15.5% | 12.3% | 1.4% | 16.5% | 48.1% | 4.6% | 11.6% | 25.3% | 31.1% | 13.5% | 19.0% | 67.4% |
| over S&P 500 Equal Weighted | -0.96% | 2.09% | -0.61% | 1.53% | 1.88% | -1.85% | -2.36% | -0.36% | -0.82% | -0.91% | -1.14% | 1.04% | 0.69% | -1.42% |
| p-value for difference | 15.3% | 39.2% | 61.0% | 46.4% | 36.9% | 4.8% | 2.7% | 78.1% | 50.8% | 35.6% | 32.6% | 48.5% | 65.3% | 37.3% |
| over TMI Value Weighted | 1.72% | 5.90% | 1.64% | 4.03% | 4.33% | 0.88% | 0.36% | 2.41% | 1.93% | 1.03% | 0.80% | 3.02% | 2.66% | 0.63% |
| p-value for difference | 0.0% | 4.2% | 7.1% | 5.2% | 1.4% | 10.2% | 54.2% | 3.2% | 8.8% | 4.1% | 12.7% | 5.4% | 8.2% | 51.9% |
| over TMI Equal Weighted | -0.91% | 1.39% | 0.19% | 2.24% | 2.31% | -1.83% | -2.34% | -0.34% | -0.81% | 0.10% | -0.13% | 2.08% | 1.72% | 0.04% |
| p-value for difference | 40.1% | 54.4% | 93.2% | 32.5% | 24.6% | 14.0% | 9.8% | 81.4% | 57.2% | 94.7% | 94.5% | 22.7% | 38.5% | 98.2% |

Table 4b reports the same results for the short term data for the recent time period. The p-values for mean return differences that are significant at the 5% level are indicated in bold typeface.

For the long term data, while all characteristics-based indices outperform the S&P 500 value-weighted, only 3 of the 14 indices show a difference that is statistically significant. Nevertheless, it should be noted that the return difference in terms of annualised means is economically important for all indices, ranging from 0.51% to 6.93%. The picture changes however drastically when replacing the value-weighted S&P 500 by its equal weighted counterpart. Compared to this index, the mean returns of characteristics-based indices are lower for most of the 14 indices, two of them being significantly lower than the returns of the equal-weighted S&P 500 index. The results are somewhat comparable for the return differences with respect to the value-weighted and equal-weighted versions of the total market index. It should also be noted that using a broader index (i.e. the total market index rather than the S&P 500 index) does not diminish the outperformance of the characteristics-based indices. Rather, changing the weighting mechanism to equal-weighted leads to the result that the outperformance is no longer there. This seems to confirm that the main difference actually is the weighting mechanism, which also seems to be consistent with the way characteristics-based index providers describe their value-added. However, one may also asked why elaborate and sometimes non-transparent weighting procedures are necessary if a naïve equal weighting scheme can achieve the same or higher outperformance over value-weighted indices.

⁷ More precisely, we conduct a (two-sided) paired t-test for the null hypothesis that the difference in the mean of the return series is zero.

Table 4b: Return Differences with Standard Indices for the Short Term Data

| Return Difference | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellindex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent |
|-----------------------------|-----------|-------|--------|--------------------|-------------|---------------------|---------------------------|----------------------|-------------------------|--------------------|------------------------|----------------------------|------------------------|---------|
| over S&P 500 Value Weighted | 3.73% | 6.93% | 2.49% | 4.72% | 5.62% | 2.28% | 1.74% | 5.46% | 4.63% | 2.60% | 1.97% | 6.15% | 5.22% | 0.61% |
| p-value for difference | 8.5% | 6.0% | 17.1% | 27.8% | 3.3% | 44.3% | 54.4% | 22.0% | 30.6% | 14.1% | 21.4% | 10.6% | 17.4% | 83.2% |
| over S&P 500 Equal Weighted | -0.97% | 2.09% | -2.16% | -0.03% | 0.84% | -2.36% | -2.88% | 0.68% | -0.11% | -2.06% | -2.65% | 1.35% | 0.46% | -3.96% |
| p-value for difference | 49.5% | 39.2% | 15.9% | 99.3% | 78.3% | 35.8% | 28.2% | 84.9% | 97.6% | 21.5% | 16.1% | 61.2% | 86.3% | 21.8% |
| over TMI Value Weighted | 2.74% | 5.90% | 1.50% | 3.71% | 4.61% | 1.30% | 0.76% | 4.45% | 3.63% | 1.61% | 0.99% | 5.13% | 4.21% | -0.36% |
| p-value for difference | 2.2% | 4.2% | 23.1% | 24.7% | 6.5% | 46.8% | 67.0% | 18.5% | 27.7% | 6.4% | 26.2% | 6.6% | 11.5% | 85.7% |
| over TMI Equal Weighted | -1.65% | 1.39% | -2.84% | -0.72% | 0.15% | -3.04% | -3.55% | -0.01% | -0.80% | -2.73% | -3.33% | 0.65% | -0.24% | -4.63% |
| p-value for difference | 47.4% | 54.4% | 30.6% | 82.7% | 95.7% | 27.9% | 26.0% | 99.7% | 81.7% | 22.2% | 20.8% | 80.6% | 93.7% | 21.2% |

The results for the short term data in table 4b confirm the results for the long-term period, notably in the sense that outperformance can be found with respect to value-weighted indices but disappears when comparing the characteristics-based indices to equal-weighted indices. It is not surprising that none of the differences are significant for this short time period with relatively few observations.

3.2. Drawdown Analysis

While the results above confirm the outperformance of characteristics-based indices with respect to value weighted indices, it is not clear whether this outperformance on average is robust through time. In particular, there may be periods in which value-weighting underperforms, but these may alternate with periods of overperformance of value-weighting with respect to the characteristics-based indices.

Therefore, we form portfolios that go long the characteristics-based index and short the value-weighted index. For brevity, we only consider the S&P 500 index as the value-weighted index of reference. From the results above, we know that such a portfolio will have positive returns on average, but periods of underperformance may also exist. The risk an investor is taking is that this portfolio may actually yield losses over sustained periods of time. The cumulative returns of this investment strategy can simply be interpreted as the relative cumulative returns of the characteristics-based index with respect to the value weighted S&P. In order to analyse if the investor has to support sustained periods of underperformance, we simply look at the drawdown of this investment strategy. This indicates how much loss (in terms of a relative loss) an investor may have accumulated over the past. We also indicate the time period where this long/short portfolio was “under water” for the longest time. The time under water of the long/short strategy simply corresponds to the longest time period of cumulative underperformance of the characteristics-based index. It indicates how long an investor who has chosen to invest in the characteristics-based index rather than the value-weighted index had to wait in order to recover the underperformance of the characteristics-based index.

Table 5a and 5b show the maximum drawdown of the characteristics-based index with respect to the value weighted index. It can be seen that drawdowns are quite substantial, being in the order of a 30% loss for most characteristics-based indices. In addition, the longest time during which the characteristics-based index underperforms the value-weighted index ranges from 37 to 189 months. In other words, though an investor holding the characteristics based index for the entire period with available data would have outperformed the value-weighted index on

average, he will have suffered from periods of underperformance lasting approximately between 3 to 16 years, depending on the characteristics-based index at hand.

Table 5a: Relative Drawdown for the Long Term Data

| | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellindex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent |
|-------------------------------|-----------|---------|---------|--------------------|-------------|---------------------|---------------------------|----------------------|-------------------------|--------------------|------------------------|----------------------------|------------------------|---------|
| Max. Drawdown | 21.7% | 30.0% | 17.3% | 43.2% | 16.2% | 36.2% | 31.5% | 48.2% | 51.6% | 24.8% | 20.0% | 38.6% | 39.2% | 26.6% |
| Maximum time under water | 88 | 37 | 42 | 41 | 41 | 179 | 178 | 94 | 189 | 151 | 146 | 89 | 65 | 115 |
| Beginning of longest drawdown | 1993.10 | 1998.01 | 1994.06 | 1998.01 | 1994.03 | 1986.10 | 1986.10 | 1993.10 | 1986.09 | 1989.01 | 1989.01 | 1993.10 | 1995.10 | 1992.01 |
| End of longest drawdown | 2001.02 | 2001.02 | 1997.12 | 2001.06 | 1997.08 | 2001.09 | 2001.08 | 2001.08 | 2002.06 | 2001.08 | 2001.03 | 2001.03 | 2001.03 | 2001.08 |

Moreover, when looking at the start and the end date of the longest period of underperformance, one can see that for most indices, the longest relative drawdown period occurred in the 1990s and ended in 2001. This confirms that the different characteristics-based indices provide portfolios that follow the same type of regimes of over- and underperformance with respect to value-weighted indices.

Table 5b: Relative Drawdown for the Short Term Data

| | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellindex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent |
|-------------------------------|-----------|---------|---------|--------------------|-------------|---------------------|---------------------------|----------------------|-------------------------|--------------------|------------------------|----------------------------|------------------------|---------|
| Max. Drawdown | 18.9% | 30.0% | 17.3% | 43.2% | 15.1% | 30.9% | 27.5% | 40.7% | 41.7% | 21.7% | 17.8% | 37.9% | 39.0% | 22.5% |
| Max. time under water | 35 | 37 | 36 | 41 | 27 | 42 | 46 | 37 | 42 | 38 | 37 | 37 | 38 | 49 |
| Beginning of longest drawdown | 1998.01 | 1998.01 | 1998.01 | 1998.01 | 1998.05 | 1998.01 | 2002.09 | 1998.01 | 1998.01 | 1998.01 | 1998.01 | 1998.01 | 1998.01 | 2002.10 |
| End of longest drawdown | 2000.12 | 2001.02 | 2001.01 | 2001.06 | 2000.08 | 2001.07 | 2006.07 | 2001.02 | 2001.07 | 2001.03 | 2001.02 | 2001.02 | 2001.03 | 2006.11 |

The analysis with the short term data leads to relative drawdowns of roughly the same order. In some cases, the maximum drawdown or longest drawdown period are actually identical with the results for the long term data, since the late 1990s tended to be unfavourable to characteristics-based indices. However, the results are often different from the long-term analysis, simply because part of the data is excluded now.

4. Static Factor Analysis

Thus far, we have ascertained that the characteristics-based indices analysed here do outperform value-weighted indices over the past years on average, though investors may be faced with long periods of underperformance with respect to value weighted indices. In addition, simple equal-weighted indices outperform most of the characteristics-based indices. The analysis of outperformance above however completely ignores the aspect of risk. Thus, the higher average returns of characteristics-based indices may simply be due to taking on higher risk. On the contrary, it may be the case that characteristics-based indices achieve outperformance with lower risk, thus dominating value weighted indices from a risk-return perspective.

In order to address the question of risk-adjusted performance, we assess the risk factor exposure and estimate the abnormal return or alpha from standard linear factor models. This

analysis will provide us with an insight on potential factor or style biases of the characteristics-based indices, as well as with a clear idea of their risk-adjusted performance.

4.1 CAPM and Four Factor Model

A standard measure of abnormal returns is Jensen's alpha, which is defined as the differential between the return on a portfolio or asset i in excess of the risk-free rate and the return explained by the market model, or:

$$E(R_i) - R_F = \alpha_i + \beta_i(E(R_M) - R_F)$$

where

- $E(R_i)$ denotes the expected return of asset i ;
- R_F denotes the rate of return of the risk-free asset;
- $E(R_M)$ denotes the expected return of the market portfolio;

It is calculated by carrying out the following regression:

$$R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + \varepsilon_{it}$$

The Jensen measure is based on the CAPM. The term $\beta_i(E(R_M) - R_F)$ measures the return on the portfolio forecast by the model. α_i measures the share of additional return that is due to the manager's choices.

However, there is now a consensus in academic finance and also among practitioners that the simple one factor model does not do a good job in capturing the cross section of expected stock returns. This insight has led to the development of multifactor models that account for a range of priced risk factors, in addition to the single market factor used in the CAPM.

Fama and French have carried out several empirical studies to identify the fundamental factors that explain average asset returns, as a complement to the market beta. They emphasize two factors that characterise a company's risk: the book-to-market ratio and the firm size measured by its market capitalisation. Carhart (1997) proposes an extension to the Fama and French three-factor model. The additional factor is momentum, which was added to take the anomaly revealed by Jegadeesh and Titman (1993) into account. This model is written as:

$$E(R_i) - R_F = b_{i1}(E(R_M) - R_F) + b_{i2}E(SMB) + b_{i3}E(HML) + b_{i4}(WML)$$

where

- SMB (*small minus big*) denotes the difference between returns on two portfolios: a small-capitalisation portfolio and a large-capitalisation portfolio;
- HML (*high minus low*) denotes the difference between returns on two portfolios: a portfolio with a high book-to-market ratio and a portfolio with a low book-to-market ratio;
- WML (*winners minus losers*) denotes the difference between the average of the highest stock returns and the average of the lowest stock returns over the previous year.

b_{ik} denotes the factor loadings.

The b_{ik} are calculated by regression from the following equation:

$$R_{it} - R_{Ft} = \alpha_i + b_{i1}(R_{Mt} - R_{Ft}) + b_{i2}SMB_t + b_{i3}HML_t + b_{i4}WML_t + \varepsilon_{it}$$

This model has the advantage of incorporating the aspect of the investment style of an equity portfolio. Practitioners are widely aware of the fact that a value or small cap tilt of a stock portfolio will yield to enhanced returns. This is nothing but capturing the HML and SMB factor of the Fama French model. In addition, we account for the profits to systematic momentum strategies by introducing the fourth factor.

We will consider both the alpha from the one factor model and from the four factor model. The results from the one factor model should obviously be interpreted with extreme caution. The CAPM alpha does not take into account the exposure to the value premium, the small cap premium or momentum profits of a given portfolio. Therefore, the abnormal returns of a portfolio with for example a strong growth (or value) tilt, and/or with a strong large cap (small cap) tilt may be severely underestimated (overestimated). In other words, a portfolio that loads heavily on additional risk factors will show high abnormal returns in the one-factor model though these returns can be explained by additional risk exposure. Multi-factor models like the four factor model are able to pick up these risk exposures and thus alpha from such models is better suited to represent returns that are generated irrespective of risk factor exposure.

In order to evaluate the statistical significance of alpha, we calculate the t-statistic of the coefficient, which is equal to the coefficient estimate divided by its standard error. The standard errors are obtained from the Newey West (1987) serial correlation and heteroscedasticity consistent covariance estimator. This is done in order to avoid erroneous inference due to these two effects, which are commonly admitted to exist in stock returns data. Rather than indicating the t-statistic, we directly indicate the p-value for the null hypothesis that the alpha is zero, under the alternative hypothesis that alpha is different from zero (i.e. the p-value for a two-sided t-test). We also indicate the factor exposures (the b_{ik} in the three factor model and β_i in the one-factor model) as well as the corresponding p-values. In order to give an idea of the overall model fit, we also indicate the adjusted R-square or R-bar of the regression.

Table 6a indicates the results using the long term data. Significant p-values (smaller or equal to 5%) are indicated in bold typeface. The results from the one factor model, show that the (monthly) alpha generated by all characteristics-based indices is positive. However, only five out of 14 indices have an alpha that is also significantly different from zero. We can also see from the R-bar that the fit of the one factor model for some of the indices is rather poor (e.g. 0.43 for the Dow Jones Select Dividend), while other indices obtain a good model fit (e.g. the RAFI 1000 with 0.91). Overall, the alpha generated by the different characteristics-based indices takes on a quite impressive amount, averaging 31 basis points per month across all indices considered.

Table 6a: Factor Models for the Long Term Data

| | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellindex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent |
|---------------------|-----------|-----------|-----------|--------------------|-------------|---------------------|---------------------------|----------------------|-------------------------|--------------------|------------------------|----------------------------|------------------------|-----------|
| One Factor Model | | | | | | | | | | | | | | |
| R bar | 0.91 | 0.61 | 0.80 | 0.43 | 0.81 | 0.86 | 0.84 | 0.66 | 0.65 | 0.87 | 0.87 | 0.63 | 0.61 | 0.74 |
| Alpha | 0.19% | 0.57% | 0.26% | 0.58% | 0.48% | 0.18% | 0.14% | 0.36% | 0.31% | 0.19% | 0.17% | 0.38% | 0.37% | 0.22% |
| p-value | 1% | 17% | 13% | 8% | 0% | 4% | 9% | 1% | 1% | 18% | 17% | 17% | 17% | 10% |
| Beta R _M | 0.91 | 0.83 | 0.86 | 0.65 | 0.84 | 0.81 | 0.79 | 0.68 | 0.71 | 0.87 | 0.87 | 0.82 | 0.80 | 0.79 |
| p-value | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Four Factor Model | | | | | | | | | | | | | | |
| R bar | 0.98 | 0.82 | 0.97 | 0.81 | 0.84 | 0.95 | 0.95 | 0.86 | 0.86 | 0.96 | 0.96 | 0.87 | 0.86 | 0.86 |
| Alpha | 0.07% | 0.10% | 0.19% | 0.04% | 0.25% | 0.03% | 0.04% | 0.13% | 0.05% | 0.08% | 0.09% | 0.13% | 0.05% | 0.07% |
| p-value | 4% | 60% | 2% | 79% | 3% | 42% | 29% | 8% | 43% | 34% | 26% | 44% | 69% | 46% |
| Beta RM | 1.02 | 1.07 | 0.98 | 0.97 | 0.93 | 0.93 | 0.93 | 0.84 | 0.89 | 0.98 | 0.98 | 1.05 | 1.06 | 0.91 |
| p-value | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Beta SMB | -0.07 | 0.23 | -0.18 | -0.03 | 0.10 | -0.16 | -0.27 | -0.08 | -0.09 | -0.10 | -0.19 | 0.01 | -0.09 | -0.37 |
| p-value | 0% | 1% | 0% | 59% | 3% | 0% | 0% | 2% | 1% | 0% | 0% | 84% | 8% | 0% |
| Beta HML | 0.35 | 0.70 | 0.29 | 0.76 | 0.21 | 0.35 | 0.30 | 0.56 | 0.61 | 0.28 | 0.22 | 0.64 | 0.65 | 0.15 |
| p-value | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 7% |
| Beta WML | -0.08 | -0.09 | -0.14 | -0.10 | 0.06 | -0.04 | -0.04 | -0.09 | -0.10 | -0.06 | -0.05 | -0.15 | -0.09 | 0.05 |
| p-value | 0% | 2% | 0% | 0% | 17% | 15% | 13% | 0% | 0% | 2% | 1% | 1% | 2% | 29% |

When considering the four factor model, the conclusion changes in a very pronounced manner, due to the fact that the exposure to value, small cap, and momentum risk is now taken into account. It can be seen that the exposure to the value premium (i.e. the HML factor in the four factor model) is positive for all and significant for all but one index. The exposures to the value premium range from 0.15 to 0.76. Exposures to the small cap factor (SMB) are also significant for most indices, though the picture is somewhat weaker, with some exposures being positive and most of them being negative. In addition, the magnitude of exposures is smaller than for the value factor. Exposures to the momentum factor are mostly negative and also significant for most indices, though most exposures are below 0.1 in absolute magnitude. From the factor exposures, one can clearly see that all of the characteristics-based indices contain an important value-tilt. When adjusting for this value tilt and for the exposures to the small cap and momentum factors in addition to the market factor, the impressive abnormal returns from the one factor model reduce drastically. Only three indices show a significant positive alpha. On average, the monthly alpha of the characteristics-based indices amounts to 9 basis points per month, compared to 31 basis points using the one factor model. From the analysis of the long term data, one may conclude that the strong value tilt explains most of the outperformance of the characteristics-based indices. It should also be noted that the negative exposures to the momentum factor show that characteristics-based indices contain an element of contrarian investing, i.e. they have a tendency to reduce weights in winning stocks and increase weights in losing stocks, the opposite of the strategy underlying the momentum factor.

The results for the short term data allows comparing factor exposures and alphas across different characteristics-based indices as the estimates are obtained over the same time period. Qualitatively, the results do not differ from the long term data. In particular, the strong value exposure and the high alpha in the one-factor model, which is greatly reduced in the four factor model are confirmed. The fact that none of the alphas is significant can be explained with the low number of observations over this short time period.

Table 6b: Factor Models for the Short Term Data

| | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellindex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent |
|---------------------|-----------|-----------|-----------|--------------------|-------------|---------------------|---------------------------|----------------------|-------------------------|--------------------|------------------------|----------------------------|------------------------|-----------|
| One Factor Model | | | | | | | | | | | | | | |
| R bar | 0.74 | 0.61 | 0.77 | 0.36 | 0.80 | 0.55 | 0.55 | 0.26 | 0.24 | 0.82 | 0.81 | 0.50 | 0.47 | 0.53 |
| Alpha | 0.34% | 0.57% | 0.22% | 0.49% | 0.48% | 0.30% | 0.25% | 0.61% | 0.55% | 0.24% | 0.19% | 0.56% | 0.50% | 0.16% |
| p-value | 19% | 17% | 31% | 31% | 0% | 28% | 34% | 16% | 15% | 28% | 33% | 22% | 23% | 44% |
| Beta R _M | 0.77 | 0.83 | 0.83 | 0.59 | 0.80 | 0.56 | 0.59 | 0.41 | 0.41 | 0.79 | 0.79 | 0.71 | 0.66 | 0.56 |
| p-value | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Four Factor Model | | | | | | | | | | | | | | |
| R bar | 0.96 | 0.82 | 0.97 | 0.81 | 0.83 | 0.91 | 0.91 | 0.79 | 0.84 | 0.96 | 0.96 | 0.87 | 0.87 | 0.83 |
| Alpha | 0.15% | 0.10% | 0.19% | 0.00% | 0.23% | 0.06% | 0.08% | 0.23% | 0.13% | 0.08% | 0.10% | 0.17% | 0.07% | 0.07% |
| p-value | 18% | 60% | 7% | 100% | 12% | 48% | 36% | 14% | 18% | 48% | 34% | 45% | 64% | 60% |
| Beta RM | 0.96 | 1.07 | 0.96 | 0.95 | 0.91 | 0.80 | 0.82 | 0.68 | 0.73 | 0.94 | 0.94 | 0.98 | 0.99 | 0.77 |
| p-value | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Beta SMB | -0.06 | 0.23 | -0.16 | -0.01 | 0.12 | -0.13 | -0.23 | 0.07 | -0.01 | -0.06 | -0.16 | 0.07 | -0.03 | -0.33 |
| p-value | 4% | 1% | 0% | 91% | 4% | 0% | 0% | 9% | 69% | 8% | 0% | 26% | 57% | 0% |
| Beta HML | 0.45 | 0.70 | 0.30 | 0.83 | 0.25 | 0.49 | 0.42 | 0.72 | 0.77 | 0.35 | 0.28 | 0.74 | 0.76 | 0.26 |
| p-value | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Beta WML | -0.12 | -0.09 | -0.15 | -0.10 | 0.05 | -0.07 | -0.07 | -0.15 | -0.13 | -0.08 | -0.07 | -0.17 | -0.11 | 0.01 |
| p-value | 0% | 2% | 0% | 0% | 22% | 0% | 0% | 0% | 0% | 0% | 0% | 1% | 0% | 84% |

4.2 Sector Benchmark

Implicitly, the linear factor models used above construct a benchmark (given by the multiplication of the factor exposures with the factor risk premium) for the excess returns of a portfolio, the characteristics-based index in our case. In practice, it is common to choose a benchmark made up of specific indices that represent the holdings of the benchmark portfolio. A natural candidate for inclusion in this benchmark are sector indices. Thus, we may ask what the abnormal performance of a portfolio is, given its exposure to industry sectors such as utilities, telecom etc.

Sharpe (1992) developed a framework for constructing a benchmark by comparing the returns on a portfolio with those of a certain number of selected indices. Sharpe's method finds the combination of indices which gives the highest R^2 with the returns on the portfolio being studied.

The Sharpe model is a linear multifactor model, applied to K asset classes. The model is written as follows:

$$R_{it} = b_{i1}F_{1t} + b_{i2}F_{2t} + \dots + b_{iK}F_{Kt} + e_{it}$$

where F_{kt} denotes the return on index k ;

b_{ik} denotes the sensitivity of the portfolio i to index k and is interpreted as the weighting of class k in the portfolio;

e_{it} represents the residual return term for period t .

In this model the factors are the asset classes, but unlike ordinary multifactor models, where the values of the coefficients can be arbitrary, here they represent the allocation of the

different asset groups in the portfolio, without the possibility of short selling, and must therefore respect the following constraints:

$$0 \leq b_{pk} \leq 1$$

and

$$\sum_{k=1}^K b_{pk} = 1$$

The introduction of these constraints is the main difference with respect to the factor models used above. The weightings are determined by a quadratic program, which consists of minimising the variance of the portfolio's residual return.

The style analysis thus carried out allows us to construct a customised sector benchmark for the characteristics-based indices. To do so, we simply take the weightings obtained for each sector index to obtain the Sharpe benchmark or sector benchmark for each index. Sharpe (1992) also proposes to calculate the alpha as the return differential between the portfolio and the benchmark. Table 7a reports the results obtained for the sector benchmarks for each of the characteristics-based indices for the long term data.

Table 7a: Sector RBSA for the Long Term Data

| | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellindex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent | S&P Value Weighted | Average of characteristics-based indices | Difference |
|---------------------|-------------|-------|-------|--------------------|-------------|---------------------|---------------------------|----------------------|-------------------------|--------------------|------------------------|----------------------------|------------------------|---------|--------------------|--|------------|
| Cons. Staples | 8% | 8% | 6% | 10% | 11% | 15% | 12% | 17% | 11% | 13% | 13% | 10% | 5% | 24% | 9% | 12% | 3% |
| Cons. Discretionary | 15% | 7% | 11% | 11% | 7% | 10% | 10% | 14% | 12% | 7% | 7% | 20% | 9% | 0% | 2% | 10% | 8% |
| Manufacturing | 9% | 32% | 1% | 0% | 17% | 4% | 0% | 0% | 0% | 11% | 8% | 7% | 9% | 0% | 10% | 7% | -3% |
| Energy | 12% | 10% | 8% | 10% | 8% | 13% | 15% | 10% | 3% | 8% | 8% | 11% | 8% | 2% | 7% | 9% | 2% |
| Chemicals | 6% | 0% | 7% | 5% | 0% | 11% | 14% | 2% | 7% | 3% | 6% | 0% | 4% | 12% | 8% | 6% | -2% |
| Technology | 3% | 1% | 9% | 0% | 19% | 0% | 0% | 0% | 0% | 8% | 8% | 0% | 0% | 0% | 17% | 3% | -14% |
| Telecom | 10% | 0% | 12% | 0% | 0% | 11% | 15% | 12% | 3% | 7% | 10% | 0% | 0% | 8% | 15% | 6% | -8% |
| Utilities | 11% | 10% | 6% | 16% | 13% | 16% | 14% | 34% | 43% | 7% | 5% | 9% | 16% | 6% | 2% | 15% | 13% |
| Retail & Wholesale | 6% | 21% | 3% | 0% | 10% | 1% | 0% | 0% | 0% | 6% | 3% | 2% | 0% | 6% | 4% | 4% | 1% |
| Health Care | 0% | 0% | 3% | 0% | 7% | 3% | 6% | 0% | 0% | 4% | 6% | 0% | 0% | 21% | 11% | 4% | -7% |
| Finance | 17% | 11% | 33% | 48% | 7% | 16% | 14% | 9% | 21% | 24% | 26% | 40% | 48% | 20% | 15% | 24% | 9% |
| Other | 4% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 3% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Alpha | 0.08% | 0.27% | 0.06% | 0.13% | 0.25% | 0.02% | -0.02% | 0.18% | 0.14% | -0.01% | -0.04% | 0.13% | 0.06% | -0.07% | -0.05% | | |
| p-value | 0.2% | 15.5% | 36.5% | 22.5% | 5.0% | 48.2% | 57.6% | 0.2% | 0.5% | 86.4% | 44.9% | 14.9% | 45.3% | 17.6% | 16.7% | | |
| Rsqr | 0.98 | 0.85 | 0.97 | 0.87 | 0.82 | 0.97 | 0.96 | 0.88 | 0.91 | 0.98 | 0.97 | 0.90 | 0.93 | 0.95 | 0.98 | | |

The first twelve lines of the table show the weights of the different sectors in the Sharpe benchmark for each of the characteristics-based indices. Furthermore, the sector weights of the Sharpe benchmark for the S&P 500 value weighted index is indicated, as well as the

average weights for the characteristics-based indices. The last column indicates the average weight for the 14 indices and the weight for the S&P 500 index. This difference directly shows how much a sector is overweighted on average in the characteristics-based indices (if the percentage is positive) or how much it is underweighted (if the percentage is negative).

The average sector weight for the characteristics-based indices shows a strong underweighting of Technologies, Telecom and Healthcare and a strong overweight of the Utilities, Consumer Discretionary and Finance sectors with respect to the weights of the value-weighted S&P 500. These sector weights confirm the value-tilt found in the four factor model, in the sense that the sectors that are underweighted are typical growth sectors (with high valuation ratios) and the overweighted sectors are typical value sectors (with low valuation ratios). The low weight on the Telecom and Technology sectors and the high weight on the Utilities sector is the most robust finding across different characteristics-based indices. In fact, none of the indices overweighted Telecom or Technology with respect to the value-weighted S&P 500 and none of the indices underweights the Utilities sector.

The table also shows the alpha for each index with respect to its Sharpe benchmark of sector indices. The alpha indicates the returns above those of the Sharpe sector benchmark, i.e. the abnormal returns after adjusting for the sector exposure of each characteristics-based index. Table 7a shows that the alphas are quite close to zero, with five of them being negative, nine of them being positive, and the average value amounting to 8 basis points monthly. Furthermore, four of the characteristics-based indices have positive alphas that are significant at the 5% level.

It should also be noted that the Sharpe sector benchmarks provide quite a good fit to the returns of the characteristics-based indices. Most R-squared are higher than 0.95 indicating that almost all of the variability of the returns of these indices can be explained by the variability of the returns of the sector benchmark. This may be surprising, given that the providers of these indices insist on the originality of their weighting method. The sector indices used are value weighted portfolios of stocks, and apparently, a Sharpe benchmark of such value weighted portfolios replicates quite closely the returns of the characteristics-based indices.

Table 7b contains the results for the sector analysis with the short term data. These results confirm the sector exposures from the analysis with long term data in Table 7a, notably the strong underweighting of the Telecom and Technology sectors and the overweighting of the Utilities sector. The only notable difference is that Consumer Staples now has a more pronounced overweighting with respect to the value weighted S&P 500 index. The alphas estimated from the short term data and the values for R-squared are also in the same order of magnitude as the ones presented in Table 7a.

Table 7b: Sector RBSA for the Short Term Data

| | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellidex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent | S&P Value Weighted | Average Characteristics-based | Difference Avg. vs. S&P |
|---------------|-------------|-------|-------|--------------------|------------|---------------------|---------------------------|----------------------|-------------------------|--------------------|------------------------|----------------------------|------------------------|---------|--------------------|-------------------------------|-------------------------|
| Cons. Staples | 9% | 8% | 5% | 14% | 10% | 17% | 17% | 29% | 15% | 15% | 12% | 13% | 7% | 16% | 5% | 13% | 8% |
| Cons. Disc. | 13% | 7% | 12% | 11% | 6% | 7% | 6% | 12% | 10% | 10% | 10% | 27% | 16% | 0% | 1% | 11% | 10% |
| Manuf. | 8% | 32% | 1% | 0% | 17% | 0% | 0% | 0% | 0% | 7% | 4% | 0% | 2% | 0% | 10% | 5% | -5% |
| Energy | 6% | 10% | 8% | 10% | 9% | 7% | 7% | 1% | 3% | 7% | 7% | 14% | 12% | 4% | 4% | 8% | 3% |
| Chemicals | 2% | 0% | 6% | 1% | 0% | 8% | 7% | 0% | 8% | 0% | 2% | 0% | 1% | 10% | 6% | 3% | -3% |
| Technology | 3% | 1% | 8% | 0% | 18% | 0% | 0% | 0% | 0% | 7% | 7% | 0% | 0% | 0% | 18% | 3% | -15% |
| Telecom | 9% | 0% | 13% | 0% | 0% | 9% | 12% | 4% | 0% | 6% | 10% | 0% | 0% | 6% | 13% | 5% | -8% |
| Utilities | 11% | 10% | 6% | 17% | 14% | 16% | 12% | 37% | 42% | 6% | 4% | 9% | 17% | 3% | 2% | 15% | 13% |
| Sales | 10% | 21% | 4% | 0% | 17% | 4% | 0% | 0% | 0% | 8% | 5% | 2% | 0% | 12% | 8% | 6% | -2% |
| Health Care | 2% | 0% | 4% | 0% | 9% | 6% | 9% | 0% | 0% | 2% | 6% | 0% | 0% | 24% | 12% | 4% | -8% |
| Finance | 22% | 11% | 32% | 47% | 0% | 25% | 28% | 16% | 21% | 24% | 28% | 36% | 45% | 24% | 19% | 26% | 7% |
| Other | 5% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 8% | 5% | 0% | 0% | 0% | 2% | 1% | -1% |
| Alpha | 0.15% | 0.27% | 0.06% | 0.14% | 0.22% | 0.05% | 0.02% | 0.30% | 0.19% | 0.06% | 0.03% | 0.29% | 0.17% | -0.05% | -0.13% | | |
| p-value | 2.1% | 15.5% | 45.2% | 38.0% | 20.6% | 55.8% | 81.5% | 6.7% | 17.4% | 33.2% | 70.2% | 5.2% | 14.4% | 58.4% | 2.8% | | |
| Rsqr | 0.97 | 0.85 | 0.96 | 0.86 | 0.81 | 0.94 | 0.94 | 0.78 | 0.86 | 0.98 | 0.97 | 0.89 | 0.93 | 0.93 | 0.98 | | |

4.3. Overall Assessment of the Value-Added

A brief summary of the results we obtained for the value-added in terms of alpha for the different characteristics-based indices is in order. As a convenient overview, we provide table 8 which shows the results for the three different benchmarks, for both periods, and across different indices in one table. In order to ensure readability, we indicate the minimum, median and maximum value across the 14 indices, rather than indicating the results for all 14 indices. In addition, we annualise the values by multiplying the monthly values by twelve and show percentage values. This obviously does not have any impact on the results but may provide magnitudes that are more straightforward than the monthly values above.

Table 8: Annualised Alpha in Percent with Different Factor Models

| | Long Term Data | | | Short Term Data | | |
|----------------------------|----------------|-----|--------|-----------------|-----|--------|
| | Min | Max | Median | Min | Max | Median |
| Alpha One Factor Model | 1.6 | 6.9 | 3.4 | 1.9 | 7.4 | 4.9 |
| Alpha Four Factor Model | 0.4 | 3.0 | 0.9 | 0.0 | 2.8 | 1.2 |
| Alpha vs. Sector Benchmark | -0.8 | 3.2 | 0.8 | -0.6 | 3.6 | 1.8 |

Table 8 shows that the median alpha is tremendous when using the one factor CAPM as a model of performance evaluation. However, when properly taking into account the exposure to multiple risk factors (in the four factor model or through industry sectors using a Sharpe sector benchmark), the median alpha is very much reduced. However, it should also be noted that the 14 indices display pronounced differences, as can be seen for example by the fact that the alpha with respect to the four factor benchmark takes on values from -0.8 percent annual to +3.2 percent annual.

It should be noted that in practice, there are three additional considerations that may reduce the value-added of characteristics-based indices. These are management fees, transaction costs, and out of sample performance. First, note that there is an important difference in fees between simple capitalisation-weighted indices and characteristics-based indices. For example, the total expense ratio (TER) for exchange-traded funds (ETFs) on indices mentioned in this study typically vary from 60 to 76 basis points annual, while ETFs on the S&P 500 or the Russell 1000 have TERs of 10 to 20 basis points⁸. This fee increment of 50 basis points means that an alpha of at least this magnitude is needed in order to compensate for higher fees. Second, the index returns do not include transaction costs that occur for investors or asset managers trying to replicate these indices⁹. While transactions are limited to constituent changes for value weighted indices, a portfolio tracking characteristics-based indices will incur transaction costs in order to re-adjust asset holdings that evolve with price movements to their characteristics-based weight. Finally, it is necessary to recall that we deal with ex post track records. Such ex post track records include the possibility of data snooping, i.e. it is possible that the weighting mechanisms that generate good performance have been retained with hindsight. However, this does not necessarily mean that this good performance will reproduce in the future. Therefore, the robustness test for performance of characteristics-based indices will lie in the out-of-sample results, i.e. the future performance of these indices.

5. Dynamic Factor Analysis

The static analysis conducted above has its limit in the fact that the factor exposures are assumed to remain constant over the entire time period. The characteristics-based indices may actually change their factor exposures over time, though one may expect the systematic construction method of these indices to lead to a factor exposure that is more stable than that of actively managed portfolios that may be sudden subject to sudden style or factor drifts. A simple method of introducing time varying factor loadings would be to conduct a rolling window analysis using the static models from above, but using a fixed calibration window of past data and rolling this window forward at each time series observation. However, this rolling window analysis is faced with the problem of being inconsistent theoretically and of relatively low reactivity to changes in the factor exposures. In order to analyse the factor exposures of the characteristics-based indices with a reactive tool, we use state space models.

Such models have been proposed for style analysis of mutual funds in Annaert and Van Campenhout (2007) and Swinkels and Van der Sluis (2006). We prefer the Kalman filter employed in Swinkels and Van der Sluis (2006) combined with a Kalman Smoother over the structural change method used by Annaert and Van Campenhout (2007) for the following

⁸ According to Information on ETF TERs published by Morningstar. See <<http://www.morningstar.com>>

⁹ For example, Arnott, Hsu and Moore (2005) explicitly specify that the historical portfolio performance results presented are not adjusted for any transaction costs associated with maintaining the strategy.

reasons. First, their method requires too many computational resources in the context of our data set. Second, the structural change was created for mutual fund holdings, where the logic of structural breaks in the allocation is more convincing than in the case of characteristics-based indices, where – by construction – no abrupt allocation shifts occur. By applying the Kalman filter combined with a Kalman Smoother, we can obtain the factor exposure over the entire period. In addition, the use of the Kalman Smoother enables us to filter out the noise in the changes of factor exposure and show only the smooth evolution of style weights. Methodological details can be found in the papers cited above.

Table 10a reports the alpha and p-value obtained when estimating the dynamic versions of the one factor, the four factor, and the sector model. The benchmark is now a portfolio of the factors where the weights vary over time. In order to get a summary measure for how much variation in factor exposures there is, we compute the style drift score proposed by Idzorek and Bertsch (2004), which is defined as

$$SDS_i = \sum_{k=1}^K \sigma_{ik}^2$$

where σ_{ik}^2 denotes the variance of the exposure b_{ik} to the k -th factor for index i over time. Idzorek and Bertsch (2004) propose to use this measure in order to assess the style drift of actively managed portfolios, and the same measure may obviously be used in the context of assessing variability of factor exposures of the indices studied in this paper.

Table 9a: Dynamic Factor Models for the Long Term Data

| | | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellindex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent |
|-----------|---------|-----------|-------|-----------|--------------------|-------------|---------------------|---------------------------|----------------------|-------------------------|--------------------|------------------------|----------------------------|------------------------|---------|
| w.r.t. | Alpha | 0.18% | 0.40% | 0.20% | 0.42% | 0.37% | 0.16% | 0.12% | 0.35% | 0.30% | 0.12% | 0.11% | 0.28% | 0.24% | 0.14% |
| Market | p-Value | 0% | 14% | 12% | 4% | 0% | 0% | 5% | 0% | 0% | 13% | 19% | 8% | 13% | 16% |
| Portfolio | SDS | 7% | 19% | 11% | 18% | 10% | 12% | 10% | 15% | 16% | 10% | 10% | 14% | 17% | 19% |
| w.r.t. | Alpha | 0.07% | 0.17% | 0.16% | 0.08% | 0.25% | 0.06% | 0.05% | 0.14% | 0.07% | 0.07% | 0.07% | 0.08% | 0.05% | 0.08% |
| Factor | p-Value | 0% | 25% | 0% | 45% | 1% | 1% | 5% | 0% | 18% | 3% | 4% | 27% | 53% | 17% |
| Benchmark | SDS | 8% | 26% | 7% | 21% | 14% | 11% | 10% | 17% | 16% | 12% | 10% | 22% | 23% | 18% |
| w.r.t. | Alpha | 0.06% | 0.10% | 0.04% | 0.11% | 0.11% | 0.04% | 0.00% | 0.17% | 0.13% | 0.00% | -0.02% | 0.08% | 0.01% | -0.07% |
| Sector | p-Value | 0% | 38% | 18% | 9% | 14% | 1% | 87% | 0% | 0% | 93% | 38% | 13% | 88% | 3% |
| Benchmark | SDS | 11% | 26% | 13% | 21% | 21% | 14% | 18% | 24% | 18% | 12% | 14% | 22% | 17% | 21% |

The results in terms of alphas from the dynamic analysis are consistent with those from the static analysis. Alphas show a considerable reduction in magnitude when going from the one factor model to the multifactor models. It should however be noted that the alphas for a range of indices are significant using the dynamic model specification.

Furthermore, the style drift scores for all three models show an important variation in the factor exposures. They lie in the order of 10% to 30%, indicating important variations in the factor exposures. Thus the investor holding one of these characteristics-based indices will be faced with changes of his allocation to say value and small cap risk that are quite high in magnitude.

Table 10b reports the alpha and style drift score for the short term data. The results are similar to the long term analysis. In particular, the style drift scores confirm important changes in the style exposure over this short period.

Table 9b: Dynamic Factor Models for the Short Term Data

| | | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellindex | Wisdom Tree Dividend | Wisdom Tree Large Dividend | Wisdom Tree HY Equity | Wisdom Tree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent |
|-----------|---------|-----------|-------|-----------|--------------------|-------------|----------------------|----------------------------|-----------------------|--------------------------|--------------------|------------------------|----------------------------|------------------------|---------|
| w.r.t. | Alpha | 0.23% | 0.40% | 0.16% | 0.35% | 0.38% | 0.16% | 0.13% | 0.45% | 0.38% | 0.17% | 0.13% | 0.45% | 0.36% | 0.03% |
| Market | p-Value | 19% | 14% | 35% | 27% | 2% | 42% | 54% | 9% | 18% | 27% | 38% | 13% | 22% | 88% |
| Portfolio | SDS | 15% | 19% | 13% | 17% | 11% | 16% | 15% | 21% | 22% | 9% | 8% | 14% | 17% | 15% |
| w.r.t. | Alpha | 0.13% | 0.17% | 0.19% | 0.11% | 0.27% | 0.10% | 0.11% | 0.27% | 0.17% | 0.11% | 0.11% | 0.16% | 0.12% | 0.09% |
| FF Factor | p-Value | 2% | 25% | 0% | 50% | 4% | 19% | 20% | 5% | 18% | 5% | 7% | 20% | 31% | 39% |
| Benchmark | SDS | 9% | 26% | 8% | 22% | 12% | 11% | 11% | 20% | 13% | 10% | 8% | 15% | 16% | 15% |
| w.r.t. | Alpha | 0.09% | 0.10% | 0.06% | 0.11% | 0.07% | 0.04% | 0.02% | 0.21% | 0.11% | 0.05% | 0.02% | 0.16% | 0.11% | -0.03% |
| Sector | p-Value | 1% | 38% | 15% | 21% | 44% | 41% | 76% | 1% | 11% | 17% | 52% | 3% | 10% | 55% |
| Benchmark | SDS | 10% | 26% | 13% | 24% | 24% | 13% | 14% | 31% | 21% | 12% | 12% | 22% | 17% | 16% |

Table 9c summarises the results for the dynamic factor models by reporting annualised percentage values for the alpha. Compared to the results for the static models reported in table 9, the magnitude of the annual percentage values is roughly equal. In particular, the dynamic analysis confirms the reduction of alpha in the multifactor models compared to the single factor.

Table 9c: Annualised Alpha in Percent with Different Dynamic Factor Models

| | Long Term Data | | | Short Term Data | | |
|----------------------------|----------------|-----|--------|-----------------|-----|--------|
| | Min | Max | Median | Min | Max | Median |
| Alpha One Factor Model | 1.3 | 5.0 | 2.6 | 0.4 | 5.4 | 3.5 |
| Alpha Four Factor Model | 0.6 | 3.0 | 0.9 | 1.1 | 3.3 | 1.5 |
| Alpha vs. Sector Benchmark | -0.8 | 2.1 | 0.6 | -0.4 | 2.6 | 1.0 |

Given the fluctuations of factor exposures that are evident from the style drift scores, a natural question is to ask whether the factor exposures implicit in the characteristics-based indices actually provide investors with an optimal risk/return trade-off. The next section turns to this aspect.

6. Optimal Factor Portfolios

The sections above come to the conclusion that the outperformance of characteristics-based indices greatly reduces when properly adjusting for the systematic risk-factor exposure and shows evidence of important fluctuations of this exposure to systematic risk. However, the analysis does not address the question whether or not the choice of risk factors inherent in the characteristics-based indices leads to an efficient risk return trade-off or not. In this section, we will treat this question by using an optimisation procedure to construct efficient portfolios from the different factors. We consider both an optimal allocation of the three Fama French factors (the market portfolio, the small cap factor and the value factor) and an optimal allocation of the different sector indices.

In what follows, we provide a very simple example of such an optimization procedure. Our goal here is not to introduce a state-of-the-art optimisation model, but rather to present evidence that even a basic and simple procedure can lead to substantial efficiency gains. The most widely quoted quantitative model in the strategic allocation literature is of course Markowitz's (1952) optimization model. The input data are the means and the variances, estimated for each asset class, and the covariances between the asset classes. The model provides the optimal percentage to assign to each asset class in order to obtain the highest return for a given level of risk, measured by portfolio volatility. The main drawback of the Markowitz model stems from the fact that the optimal proportions are very sensitive to the estimates of expected return values. What is more, the statistical estimates of expected returns are very noisy (see Merton (1980)). As a result, the model often allocates the most significant proportion to the asset class with the largest estimation error. While these problems probably explain why the Markowitz model is still not used very often in practice, there is a pragmatic approach that allows these problems to be avoided without abandoning the model.

This approach consists in focusing on the only portfolio on the mean variance frontier for which the estimation of mean returns is not necessary, namely the minimum-variance portfolio, which solves the following programme.

$$\text{Minimise } \text{var}(R_p) = \sum_{i=1}^n \sum_{j=1}^n x_i x_j \text{cov}(R_i, R_j)$$

s.t.

$$\sum_{i=1}^n x_i = 1$$

$$x_i \geq 0, i = 1, \dots, n$$

where x_i denotes the proportion of asset i held in the portfolio;

and $\text{cov}(R_i, R_j)$ denotes the covariance between asset i and asset j .

Since the future returns of assets are difficult to estimate precisely, it is preferable to obtain an efficient portfolio by minimising the risk rather than by optimising the risk/return combination. For more details on the minimum-variance approach, see for example Chan, Karceski and Lakonishok (1998) or Amenc and Martellini (2003). Though this approach avoids the problem of estimation risk for the expected returns, it is still faced with the estimation risk for the covariance matrix. It should be noted that our approach is simpler than the approach taken in these two papers in that we do not impose a model for the covariance matrix. Our forecasts for the covariance matrix are simply derived from the sample estimates. In practice, an investor may choose to implement the noise dressing techniques used by Amenc and Martellini and/or impose some structure by using constant correlations or factor models (see Chan, Karceski and Lakonishok and the references therein).

We choose to deal with the problem of estimation risk, not by imposing a model, but by imposing a maximum constraint of 50% for the weight of a given sector and imposing a maximum weight of one for a given Fama French factor. Imposing constraints has been shown to be a useful option as it increases the performance of asset allocation (see

Jagannathan and Ma (2003)). This procedure has the advantage that our conclusions may apply more generally rather than being limited to a certain type of model used for covariance forecasts.

In order to assess the performance of minimum-variance portfolios, we apply the following steps:

- The data used are daily returns for the sector indices, respectively for the four style factors¹⁰. The use of daily returns is useful in order to improve the precision of the covariance estimates.
- We compute the minimum-variance portfolio based on a calibration period of the daily return observations for the past year (12 calendar months). We obtain the optimal weights and hold this portfolio for the following three calendar months. We then repeat the analysis, rolling the sample three months forward and holding the new optimal portfolio for the following three months. Therefore, our analysis is purely out-of-sample.

We thereby obtain the monthly time series of returns for the minimum variance strategy over the period from January 1965 to December 2006 (the total period minus the calibration period). Since we implement the approach using the sector indices and four style factors, we obtain two minimum variance portfolios, a style factor portfolio and a sector portfolio.

Table 10a compares the performance of the different characteristics-based indices with that of the two minimum variance portfolios over the period of data availability for the respective characteristics-based index. In order to assess the efficiency of the characteristics-based index, its Sharpe ratio is compared to that of the two minimum variance portfolios and a p-value for the difference in the Sharpe ratio between the minimum variance portfolio and the characteristics-based index is calculated.

The p-value we report is based on a test of significance of differences in Sharpe ratio between two portfolios proposed by Jobson and Korkie (1981). Their test is based on the statistic

$$Z = \frac{\sigma_A(\mu_B - R_F) - \sigma_B(\mu_A - R_F)}{\sqrt{\Theta}}$$

where

$$\Theta = \frac{1}{T} \left(2\sigma_A^2\sigma_B^2 - 2\sigma_A\sigma_B\sigma_{AB} + \frac{1}{2}\mu_A^2\sigma_B^2 + \frac{1}{2}\mu_B^2\sigma_A^2 - \frac{\mu_A\mu_B}{2\sigma_A\sigma_B} [\sigma_{AB}^2 + \sigma_A^2\sigma_B^2] \right);$$

$\mu_i, \sigma_i^2, \sigma_{ij}$ are the estimates of the mean and variance of the returns of portfolio i and the covariance between returns of portfolio i and j respectively;

R_F is the riskfree rate;

T is the number of observations.

¹⁰ Since all four factors are zero investment (long-short) strategies, we add the riskless rate to the returns in order to have a strategy that requires an investment of total portfolio wealth. This can be thought of as an investment of total portfolio wealth in the riskless asset and a holding of the optimal weight in the Fama French factors.

Jobson and Korkie show that the Z statistic approximately follows a standard normal distribution for large samples, hence it is straightforward to calculate a p-value for the null hypothesis that the difference in Sharpe ratios is zero. Unfortunately, the test has low power, i.e. it is difficult to find statistical significant results even if the true difference in Sharpe ratios is not zero.

Table 10a: Efficiency Comparison for the Long Term Data

| | RAFI 1000 | VTL | GWA | DJ Select Dividend | Intellindex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent |
|--|-----------|-----------|-------|--------------------|-------------|---------------------|---------------------------|----------------------|-------------------------|--------------------|------------------------|----------------------------|------------------------|-----------|
| Diff. In SR w.r.t. Factor Portfolio | -0.68 | -0.74 | -0.67 | -0.64 | -0.54 | -0.70 | -0.74 | -0.55 | -0.62 | -0.64 | -0.67 | -0.57 | -0.59 | -0.68 |
| p-value for difference | 0% | 5% | 7% | 4% | 8% | 0% | 0% | 0% | 0% | 3% | 3% | 3% | 3% | 1% |
| Diff in SR w.r.t. Sector Portfolio | -0.44 | -0.09 | -0.33 | -0.21 | -0.14 | -0.44 | -0.48 | -0.30 | -0.36 | -0.35 | -0.37 | -0.28 | -0.30 | -0.48 |
| p-value for difference | 0% | 69% | 8% | 19% | 53% | 0% | 0% | 0% | 0% | 2% | 1% | 7% | 3% | 0% |
| Sharpe Ratio Index | 0.51 | 0.65 | 0.72 | 0.92 | 0.98 | 0.50 | 0.46 | 0.65 | 0.58 | 0.73 | 0.71 | 0.80 | 0.78 | 0.68 |
| Sharpe Ratio Factor Portfolio | 1.18 | 1.39 | 1.38 | 1.55 | 1.52 | 1.20 | 1.20 | 1.20 | 1.20 | 1.38 | 1.38 | 1.38 | 1.38 | 1.36 |
| Sharpe Ratio Sector Portfolio | 0.94 | 0.74 | 1.05 | 1.13 | 1.11 | 0.94 | 0.94 | 0.94 | 0.94 | 1.08 | 1.08 | 1.08 | 1.08 | 1.16 |
| Mean Excess Return Index | 7% | 11% | 10% | 13% | 13% | 7% | 6% | 8% | 8% | 10% | 9% | 12% | 11% | 9% |
| Mean Excess Return Factor Portfolio | 6% | 7% | 6% | 7% | 7% | 6% | 6% | 6% | 6% | 6% | 6% | 6% | 6% | 6% |
| Mean Excess Return Sector Portfolio | 12% | 8% | 11% | 12% | 12% | 12% | 12% | 12% | 12% | 12% | 12% | 12% | 12% | 13% |
| Std. Dev. Of Excess Returns Index | 15% | 17% | 14% | 14% | 13% | 13% | 13% | 13% | 14% | 13% | 13% | 15% | 15% | 14% |
| Std. Dev. Of Excess Returns Factor Portfolio | 5% | 5% | 4% | 4% | 4% | 5% | 5% | 5% | 5% | 4% | 4% | 4% | 4% | 4% |
| Std. Dev. Of Excess Returns Sector Portfolio | 12% | 11% | 11% | 10% | 10% | 12% | 12% | 12% | 12% | 11% | 11% | 11% | 11% | 12% |
| Start Date | 65.11 | 98.01 | 94.01 | 92.02 | 92.12 | 65.01 | 65.01 | 65.01 | 65.01 | 89.01 | 89.01 | 89.01 | 89.01 | 83.02 |
| Number of months | 494 | 108 | 156 | 179 | 169 | 504 | 504 | 504 | 504 | 216 | 216 | 216 | 216 | 287 |

The results in table 10a show quite clearly that none of the characteristics-based indices constitutes a viable alternative to the minimum variance portfolios. The Sharpe ratios of both the sector and the four-factor minimum variance portfolio are higher than the Sharpe ratios of the characteristics-based index for all of the 14 indices analysed here. The difference is of economic importance for all indices, and is statistically significant in most cases. It should be noted that the presence of some Sharpe ratios that are higher and statistically significant is strong evidence for the inefficiency of the respective characteristics-based indices, given that the Jobson-Korkie test has low power, i.e. given that it often does not reject a false null hypothesis.

Note that the higher Sharpe Ratios for the global minimum variance portfolios mainly stem from the lower volatility. In particular, volatility is reduced dramatically when using the four factors to construct minimum variance portfolios. The results for the short term data in table 10b confirm the higher Sharpe ratios for the minimum variance portfolios based on the market, small cap and value factor. The Sharpe ratio of the sector-based minimum variance portfolio is higher for all but one index (the Wisdom Tree High Yield Equity index).

Table 10b: Efficiency Comparison for the Short Term Data

| | RAFI 1000 | VTL | GWA | DJ Select Dividend | IntelIndex | WisdomTree Dividend | WisdomTree Large Dividend | WisdomTree HY Equity | WisdomTree Dividend 100 | Wisdom Tree Income | Wisdom Tree Income 500 | Wisdom Tree Income Low P/E | Wisdom Tree Income 100 | Mergent |
|--|-----------|-----------|-----------|--------------------|------------|---------------------|---------------------------|----------------------|-------------------------|--------------------|------------------------|----------------------------|------------------------|-----------|
| Diff. In SR w.r.t. Factor Portfolio | -0.85 | -0.74 | -0.96 | -0.83 | -0.71 | -0.87 | -0.94 | -0.63 | -0.72 | -0.92 | -0.96 | -0.75 | -0.79 | -1.02 |
| p-value Jobson Korkie | 5% | 5% | 4% | 4% | 8% | 5% | 4% | 12% | 8% | 4% | 3% | 6% | 5% | 4% |
| Diff in SR w.r.t. Sector Portfolio | -0.20 | -0.09 | -0.31 | -0.18 | -0.06 | -0.23 | -0.29 | 0.02 | -0.07 | -0.27 | -0.32 | -0.10 | -0.14 | -0.37 |
| p-value Jobson Korkie | 33% | 69% | 18% | 39% | 81% | 20% | 13% | 93% | 66% | 23% | 18% | 64% | 48% | 12% |
| Sharpe Ratio Index | 0.54 | 0.65 | 0.43 | 0.56 | 0.68 | 0.51 | 0.45 | 0.76 | 0.67 | 0.47 | 0.42 | 0.64 | 0.60 | 0.37 |
| Sharpe Ratio Factor Portfolio | | | | | | | | 1.39 | | | | | | |
| Sharpe Ratio Sector Portfolio | | | | | | | | 0.74 | | | | | | |
| Mean Excess Return Index | 8% | 11% | 6% | 9% | 10% | 6% | 6% | 10% | 9% | 7% | 6% | 10% | 9% | 5% |
| Mean Excess Return Factor Portfolio | | | | | | | | 7% | | | | | | |
| Mean Excess Return Sector Portfolio | | | | | | | | 8% | | | | | | |
| Std. Dev. Of Excess Returns Index | 14% | 17% | 15% | 16% | 14% | 12% | 13% | 13% | 13% | 14% | 14% | 16% | 15% | 12% |
| Std. Dev. Of Excess Returns Factor Portfolio | | | | | | | | 5% | | | | | | |
| Std. Dev. Of Excess Returns Sector Portfolio | | | | | | | | 11% | | | | | | |
| Start Date | | | | | | | | 1998.01 | | | | | | |
| Number of months | | | | | | | | 108 | | | | | | |

Overall, it can be seen that the Sharpe ratio of the minimum-variance portfolios is almost always higher than that of the characteristics-based indices. Therefore, if an investor realises that she can conduct an allocation between factors other than the value-weighted market portfolio, she can obtain portfolios that are more efficient in a mean-variance sense than the characteristics-based indices by using a very straightforward asset allocation strategy.

7. Conclusion

This paper assesses the performance of characteristics-based indices. Based on returns data for U.S. indices from various providers, the properties of these indices are analysed and contrasted with well established indices, such as capitalisation and equal-weighted indices for the total stock market and for the S&P 500 universe.

The paper finds that all characteristics-based indices have higher returns than the capitalisation-weighted S&P 500 index though the return difference is not statistically significant for most indices. However, when compared to equal-weighted indices, most characteristics-based indices actually have lower returns. What is more, outperformance with respect to the value-weighted S&P 500 depends on market conditions and all characteristics-based indices face lengthy periods of underperformance with respect to the S&P 500.

When adjusting for the systematic risk factor exposure of characteristics-based indices, the abnormal returns generated by these indices are tremendous when only the market factor is taken into account. In the single factor model, the alpha of all characteristics-based indices is positive, though most of them are not significantly different from zero. However, when accounting for small cap, value, and momentum risk by employing the four factor model, the magnitude of alphas is greatly reduced and only three of them are significantly different from zero. These findings may be explained by the fact that the exposure to the value premium is positive and significant for all indices, and explains the outperformance of these indices. This value bias is confirmed in an analysis of factor exposures where one finds that typical value

sectors (e.g. utilities) are overweighed and growth sectors (e.g. technology) are underweighted.

However, the fact that the outperformance of characteristics-based indices disappears when adjusting for the systematic risk-factor exposure does not say anything about this choice of risk factors being optimal or not. Therefore, a simple asset allocation strategy based on the market, small cap, value and momentum factor in the first case, and based on sector indices in the second case is implemented. This strategy consists of an out-of sample minimum variance portfolio, which is compared to the characteristics-based indices in terms of mean variance efficiency (i.e. in terms of the Sharpe Ratio). The results show that the Sharpe ratios for both the sector and the factor portfolios are always higher than those of the characteristics-based indices, except for the case of one index who beats the sector portfolio over one subperiod. On average, the simple allocation approach yields Sharpe ratios that are 0.5 higher than the characteristics-based indices.

One can conclude that the characteristics-based indices are nothing but value-tilted active strategies. When properly adjusting for this tilt, they do not show any outperformance. The benefits of value investing are of course well known. The empirical findings of a value premium (e.g. Fama and French (1993)) confirm an investment approach that has long been advocated in textbooks of fundamental analysis (see e.g. Williams (1938) or Graham (1962)). While the indices studied here do not apply any new concepts, their value added may lie in the fact that they provide investors with a liquid, systematic and relatively cheap alternative to other value-tilted strategies. However, it should also be noted that if one recognises the possibility to take bets across factors, it is possible to construct portfolios that beat the characteristics-based indices in terms of mean-variance efficiency.

References

Amenc, N., F. Goltz, and V. Le Sourd, 2006, "Assessing the Quality of Stock Market Indices", *EDHEC Publication*

Amenc, N. and L. Martellini, "Portfolio Optimization and Hedge Fund Style Allocation Decisions", *Journal of Alternative Investments*, vol. 5, n° 2, autumn 2002.

Annaert, J., and G. Van Campenhout, 2007, Time Variation in Mutual Fund Style Exposures, *Review of Finance*, forthcoming

Arnott, R.D., J. Hsu, and P. Moore, 2005, "Fundamental Indexation", *Financial Analysts Journal* 60(2), 83-99.

Carhart, M., 1997, On Persistence in Mutual Fund Performance, *Journal of Finance*, 52(1)

Chan, L.K.C., J. Karceski and J. Lakonishok, 1998, "The Risk and Return from Factors", *Journal of Financial and Quantitative Analysis*, vol. 33, pp. 159-188.

Cochrane, J., Portfolio Advice for a Multifactor World, *Economic Perspectives, Federal Reserve Bank of Chicago*, 23(3), 59-61

Fama E., 1972, Components of Investment Performance, *Journal of Finance*, 17(3), p. 551-567.

Fama, E., 1996, Multifactor Portfolio Efficiency and Multifactor Asset Pricing, *Journal of Financial and Quantitative Analysis*, 31(4), 441-465

Fama, E., and French, K., 1992, The Cross-Section of Expected Stock Returns, *Journal of Finance*, Vol. 47, pp. 427-467

Fama E., and K. French, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics*, 33, p. 3-56.

Graham, Benjamin, 1962, Security analysis; principles and technique. Fourth edition, McGraw-Hill.

Haugen, R.A., and N.L. Baker, 1991, The Efficient Market Inefficiency of Capitalization-Weighted Stock Portfolios, *Journal of Portfolio Management*

Hsu, Jason, 2006, Cap-Weighted Portfolios are Sub-optimal Portfolios, *Journal of Investment Management*, 4(3), 1-10

Idzorek, T., and F. Bertsch, The Style Drift Score, *Journal of Portfolio Management*, 2004

Jagannathan, R. and T. Ma, "Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps", *Journal of Finance*, 2003.

Jegadeesh, N, and S. Titman, 1993, Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *Journal of Finance*, 48, 65-91

Jobson, I.D. and B.M. Korkie, 1981, Performance Hypothesis Testing with the Sharpe and Treynor Measures, *Journal of Finance*, 36 (September), 544-54.

La Porta, Rafael, Josef Lakonishok, Andrei Shleifer, Robert Vishny, 1997, Good News for Value Stocks: Further Evidence on Market Efficiency, *Journal of Finance*. Volume 52, Issue 2, 859-874.

Merton, R. C. 1980. On estimating the expected return on the market: An exploratory investigation, *Journal of Financial Economics* 8, 323–361.

Newey, Whitney K. , and Kenneth D. West, 1987, A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix , *Econometrica*, Vol. 55, No. 3, pp. 703-708

Sharpe, William, 1992, Asset Allocation: Management Style and Performance Measurement, *Journal of Portfolio Management*, 18 (2), 7-19

Siegel, J.J., J.D. Schwartz and L. Siracusano, The Unique Risk and return Characteristics of Dividend-Weighted Stock Indexes, WisdomTree White Papers, 2007.

Siracusano, L., and J.D. Schwartz, WisdomTree Earnings-Weighted Indexes – The Market at a Reasonable Price, WisdomTree White Papers, 2007.

Southard, John, and Bruce Bond, Intelligent Indexes – The Scientific Approach to Money Management, *Journal of Indexes*, Fourth Quarter 2003.

Swinkels, L., and P. Van Der Sluis, 2006, Return-based style analysis with time-varying exposures, *European Journal of Finance*

Williams, J.B., 1938, *The Theory of Investment Value*, Amsterdam: North Holland

Zangari, P. 1996, A VaR Methodology for Portfolios that include Options”, *Risk MetricsTM Monitor*, 1st quarter, pp. 4–12