TEV Sensitivity to Views in Black-Litterman Model

Maria Debora Braga[a;b], Francesco Paolo Natale[c;*]

[a]University of Valle d'Aosta, 2A Strada dei Cappuccini – 11100 Aosta, Italy
[b]SDA Bocconi School of Management, Via Bocconi 8 – 20136 Milano, Italy
[c]University of Milano – Bicocca, Piazza Ateneo Nuovo 1, 20123 Milano, Italy
[*]Corresponding author. E-mail address: *francesco.natale@unimib.it*

Abstract

In this paper we propose a new measure for the marginal contribution of each view to the ex-ante tracking error volatility (TEV).
The issue of the TEV sensitivity to the views is relevant for several purposes: 1) provide the asset managers with a method for revising the portfolio consistently with a given TEV constraint; 2) make the specialists responsible for the generation process of the views; 3) set a mechanism to connect the incentive fees not only to the excess return but also to the marginal contribution of each view to the TEV.
We provide also an empirical investigation in the Black-Litterman framework in order to modify the views to achieve a TEV goal.

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1. Introduction

In the context of active portfolio management the Black-Litterman model (1992, 1999, hereinafter BL-model) has been recognized as valuable tool to implement short-term forecasts on the expected returns. With the BL-model an asset manager can define the active portfolio using a formal and objective framework. These active positions yield an ex-ante return and risk. The former is the ex-ante tracking error (TE) while the latter is known as ex-ante tracking error volatility (TEV).

Since the introduction of the BL-model, few efforts have been done in order to investigate the effects of the BL-views on the TEV. Recently some researchers have begun to explore the implications of the views on the model output. Fusai and Meucci (2003) focused on a probability index inversely related to the Mahalanobis distance between BL combined returns and equilibrium returns and calculate the sensitivity of the probability to the views. Should the probability index be too low with respect to a selected threshold, Fusai and Meucci suggest to numerically modify the views starting with the 'boldest' one. Scherer (2000) shows a meaningful decomposition of risk as a prerequisite for a modern risk-budgeting technique. Particularly he proposes an enhanced version of the active risk using the trade matrix, i.e. a matrix with the active trades on the assets.

In this study we obtain a new measure of the marginal risk contribution to the TEV. Differently from the cited works, our measure directly relates the views to the active risk without encompassing the relationship between the probability distribution of the Mahalanobis distance and each single view. In addition we use the classical ex-ante TEV as measure of active risk which is familiar among practitioners. We also extend the Scherer's intuition to the formal BL framework, giving solid and elegant base to the final result.

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The rest of the paper proceeds as follows. Section 2 contains a brief description of the BL model. In Section 3 we describe the analytical derivation of the risk contribution measure we propose. An empirical investigation is detailed in Section 4 highlighting the practical contribution of our measure. Section 5 concludes the paper.

2. The Black-Litterman Model

The BL model shares the basic idea of Bayesian statistics which is the assessment of information from various sources and their combination in a single estimate.

The BL model derives the set of expected returns as inputs for the portfolio optimisation by combining (equilibrium) returns implied in the market and specialized views regarding the performance of the assets involved. The BL model allows to consider both absolute and relative views.

Equilibrium returns are extracted through reverse optimization, from observed market capitalizations weights, with a covariance matrix and a given risk aversion coefficient. Market portfolio is usually the neutral starting point in the BL framework. However other types of strategic allocation can be easily treated as reference portfolio.

Market views are probability statements about markets performance that are expression of the investment teams' research.

The equation for the expected return vector the optimizer will use, according to BL model, reflects the posterior mean of the posterior distribution jointly generated by the two distinct sources of information. In mathematical notation, the vector of the BL combined returns is given by:

$$R_{BL} = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1} \cdot \left[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q\right] \qquad (1)$$

where:

R_{BL} : vector of blended expected returns ($n \times 1$);

τ : scalar for weighting the variance-covariance matrix;

Σ : variance-covariance matrix of historical returns ($n \times n$);

P : matrix for the n assets involved in the k views ($k \times n$);

Ω : diagonal covariance matrix of error terms for the stated views ($k \times k$). This matrix is usually derived from the level of confidence in the views declared by the specialists;

Π : vector of implied equilibrium returns ($n \times 1$);

Q : vector with the q_j (for $j = 1, \ldots, k$) views as entries ($k \times 1$).

Equation (1) allows interpreting the posterior mean, i.e. BL combined returns, as a weighted average of equilibrium returns and subjective forecasts with weights that reflects the respective precision throughout the inverse of the respective variance given by $(\tau\Sigma)^{-1}$ and $\left(P^T \Omega^{-1} P\right)^{-1}$. The BL combined returns become then the new inputs for the investor objective function:

$$\max_{w_{BL}^*}\left(w_{BL}^T \cdot R_{BL} - \lambda\left(w_{BL}^T \cdot \Sigma \cdot w_{BL}\right)^{\frac{1}{2}}\right) \tag{2}$$

where w_{BL} denotes the $n \times 1$ vector of portfolio weights and λ is the investor's risk aversion.

Provided that in Equation (2) neither short-selling nor budget constraints are imposed, the solution for the BL weights w_{BL}^* to the maximization problem is straightforward.

$$w_{BL}^* = \left(\frac{1}{\lambda}\Sigma^{-1} \cdot R_{BL}\right) \tag{3}$$

Portfolio allocations resulting from (3) could diverge from the market weights. The stated views and the level of confidence in these views will determine the extent of these deviations. We call them active weights and store them in the w_{ACT} vector ($n \times 1$) defined as follows:

$$w_{ACT} = w_{BL}^* - w_{MKT} \tag{4}$$

where w_{MKT} denotes the market capitalization weights already used in the reverse optimization.

It is well known that active positions generate active risk. The measure of this risk, known as ex-ante tracking error volatility (TEV), is calculated using the familiar quadratic form in (5):

$$TEV = \left(w_{ACT}^T \cdot \Sigma \cdot w_{ACT}\right)^{\frac{1}{2}} \tag{5}$$

3. TEV sensitivity to views

Once ex-ante TEV has been defined in terms of active weights, we can calculate the ($k \times 1$) vector of the marginal contributions of each single view to the TEV.

It is worth noting that we need to calculate the derivative of the composite of two functions. The first is $TEV = f(w_{ACT;1}, w_{ACT;2}, \ldots, w_{ACT;n})$ whereas the second is $w_{ACT;i} = h(q_1, q_2, \ldots, q_k)$. If these functions are all differentiable the chain rule can be applied in order to determine the partial derivative of TEV to each single weight ($w_{ACT;i}$; for $i = 1 \ldots, n$) and the partial derivative of each active weight to each view (q_j for $j = 1, \ldots k$) as follows:

$$\frac{\partial TEV}{\partial q} = \begin{vmatrix} \sum_{i=1}^{n} \frac{\partial TEV}{\partial w_{ACT;i}} \cdot \frac{\partial w_{ACT;i}}{\partial q_j} & (j = \mathbf{1}) \\ \ldots\ldots\ldots\ldots & \ldots\ldots \\ \ldots\ldots\ldots\ldots & \ldots\ldots \\ \sum_{i=1}^{n} \frac{\partial TEV}{\partial w_{ACT;i}} \cdot \frac{\partial w_{ACT;i}}{\partial q_j} & (j = k) \end{vmatrix} \qquad (6)$$

In Equation (6) $\frac{\partial TEV}{\partial q}$ is decomposed in two marginal contributions which admit a financial interpretation:

$$\frac{\partial TEV}{\partial q} = \frac{\partial \sqrt{w_{ACT}^T \cdot \Sigma \cdot w_{ACT}}}{\partial w_{ACT}} \cdot \frac{\partial w_{ACT}}{\partial q} \qquad (7)$$

The first element in Equation (7) can be thought as marginal contribution of each active weight to the TEV while the second can be interpreted as marginal contribution of each view to the single active weight.

After some algebraic manipulation the first element is defined by:

$$\frac{\partial \sqrt{w_{ACT}^T \cdot \Sigma \cdot w_{ACT}}}{\partial w_{ACT}} = \frac{\Sigma \cdot w_{ACT}}{\sqrt{w_{ACT}^T \cdot \Sigma \cdot w_{ACT}}} \qquad (8)$$

Equation (8) represents the $n \times 1$ gradient of the TEV with the entries given by the partial derivatives with respect to the active weights. More specifically TEV can be decomposed with Euler equation into the summation of all marginal contributions to TEV multiplied by active weights.

$$TEV = \sum_{i=1}^{n} w_{ACT;i} \cdot \frac{\partial \sqrt{w_{ACT}^{T} \cdot \Sigma \cdot w_{ACT}}}{\partial w_{ACT;i}} = w_{ACT}^{T} \cdot \nabla TEV \tag{9}$$

The second element of Equation (7) requires the calculation of the contribution of each view to each BL return. This marginal contribution, in the case of optimisation without constraints (see Equation (3)), can be derived as follows:

$$\frac{\partial w_{ACT}}{\partial q} = \frac{\partial\left(\frac{1}{\lambda}\Sigma^{-1}\left(R_{BL} - \Pi\right)\right)}{\partial q} = \frac{1}{\lambda}\Sigma^{-1}\left(\frac{\partial R_{BL}}{\partial q}\right) = \frac{1}{\lambda}\Sigma^{-1}\left[\left(\tau\Sigma^{-1}\right) + P^{T}\Omega^{-1}P\right]^{-1} P^{T}\Omega^{-1} \tag{10}$$

Equation (10) identifies the *k* gradients of each single active weight (or BL-return) with respect to each view[1]. Therefore we obtain an $n \times k$ matrix with these partial derivatives.

Rearranging equations (8) and (10), the marginal contributions of each view to the TEV is given below:

$$\frac{\partial TEV}{\partial q} = \left[\frac{\mathbf{1}}{\lambda}\Sigma^{-\mathbf{1}}\left[\left(\tau\Sigma^{-\mathbf{1}}\right) + P^{T}\Omega^{-\mathbf{1}}P\right]^{-\mathbf{1}} P^{T}\Omega^{-\mathbf{1}}\right]^{T} \cdot \frac{\Sigma \cdot w_{ACT}}{\sqrt{w_{ACT}^{T} \cdot \Sigma \cdot w_{ACT}}} \tag{11}$$

The result is a vector $k \times 1$ whose elements indicate the specific view with both the highest and the lowest impact on the ex-ante TEV.

We could adjust the highest contributing view if our aim is to reduce the ex-ante TEV. Likewise, an intervention on the lowest contributing view allows to achieve a specific ex-ante TEV if the TEV implied in the BL model is below a definite threshold. The last case is typical of an asset manager with a risk budget to exploit.

It is worth emphasizing that to adjust the q_j views a numerical goal-attainment procedure is required. A slight shift in q_j, coherently with the signs of the partial derivatives in

[1] Notice that the numerator in Equation (10) corresponds to the definition of “active weights” in the case of optimisation without constraints both for BL returns (R_{BL}) and equilibrium returns (Π).

Equation (11), produces a change in the ex-ante TEV, in accordance with the asset manager expectations.

In the real world we deal with k views, thus we need to expand the numerical goal-attainment procedure to the k-space by analyzing all possible combinations of q_j ($j=1,\ldots,k$). Several solutions (q_j^*) might be achievable, i.e. we could find many combinations of q_j^* which allow us to obtain ex-ante TEV-constraint. Suppose we find R solutions to the ex-ante TEV-constraint problem. In order to choose the best one a selection criterion must be set.

We propose the use of Minkowski metric. This statistics is based on the summation of the distances between the initial q_j and q_j^*, considered in absolute value. In mathematical notation we could find a matrix $R \times k$ whose R rows are given by the r vectors of optimal $[q_{rj}^*]$ (with $j = 1,\ldots,k$ and $r = 1,\ldots, R$) . The best vector of k views $[q_1^{**}, \ldots; q_k^{**}]$ is obviously the specific r vector minimizing the Minkowski metric.

$$\underset{(j=1,\ldots,k)}{\left[q_j^{**}\right]} = \min \left| \begin{matrix} \left\{ \sum_{j=1}^{k} \left| q_j - q_{rj}^* \right|^p \right\}^{\frac{1}{p}} & (r=1) \\ \ldots\ldots\ldots\ldots & \ldots\ldots \\ \ldots\ldots\ldots\ldots & \ldots\ldots \\ \left\{ \sum_{j=1}^{k} \left| q_j - q_{rj}^* \right|^p \right\}^{\frac{1}{p}} & (r=R) \end{matrix} \right| \qquad (12)$$

4. Empirical investigation

We illustrate how Equation (11) works with real data. Our investment universe is built up on eighteen asset classes constituting the DJ STOXX 600. Each asset class represents a Supersector as defined by the Industry Classification Benchmark (ICB). Appendix A shows labels and descriptive statistics of the historical returns.

The sample period is from 1/1997 to 1/2007 of monthly euro-denominated returns whereby our risk-free rate is assumed to be constant and equal to 3% per year. Appendix A details the descriptive statistics of our asset classes.

Based on this data set we calculate the implied returns of our asset classes. This step requires the definition of an appropriate coefficient of risk aversion (λ) and the vector of the market neutral weights.

Following Best and Graurer (1985), the former is defined as the ratio of the excess market return over the market portfolio variance (λ=1.6118). The latter includes the market capitalisation of each sector in the DJ STOXX 600 at December 31, 2006.

In order to coherently build our active portfolio we introduce three views expressed, in the format of Black-Litterman, as follows:

1) View 1: Utilities (UTIL) will have an absolute return of 7% per year with a Confidence Level 80% of experiencing a final return in the range [6%; 8%];

2) View 2: Chemical (CHEM) and Industrial Goods (INDS) will outperform Oil & Gas (OIL) of 1% per year with a Confidence Level equals to 90% to register a final relative return in the interval [0%; 2%];

3) View 3: Banks (BANK) and Financial Services (FISV) will outperform Media (MED) and Technology (TECH) of 2.5% per year with a Confidence Level 70% of recording a final relative return in the range [1.5%; 3.5%];

Therefore we have both absolute and relative views. For the latter we follow the weighting scheme proposed by He and Litterman (1999) where the relative weightings of the assets entering Matrix P are proportional to their market capitalisation[2].

Researchers are still in search for an optimal definition of τ. We decide to give relevance to the sample length assuming for τ the inverse of the number of the historical observations, i.e. 120. With this choice we use an objective criterion for recognize the large sample effect.

Equilibrium returns and BL returns are reported in Appendix B Figure 1.B.

It is worth emphasizing that only three views have an important effect on the entire set of asset returns, due to the covariance matrix. However among our views q_2 and q_3 deserve more attention.

According to our view 2, INDS and CHEM should overperform OIL of 1% which is much less than the equilibrium relative overperformance, equal to 2.54%. Notwithstanding the role of OIL as underperforming asset its BL return increases for the above reason.

In view 3 both the lowest confidence level and the opposite sign of the view with respect to the equilibrium explain the weak difference between the equilibrium relative performance[3] (-1.44% per year) and the BL relative return of the grouped assets (-0.19%).

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[2] An alternative weighting scheme is the equally-weighting scheme suggested by Satchell and Scowcroft (2000). We selected the He and Litterman's to give importance to the relative market capitalisation of the outperforming and underperforming assets in each view.

[3]. The equilibrium relative performance is the difference between the performance of the overperfoming asset portfolio and the performance of the underperfoming asset portfolio. In our case this difference is determined as

Solving the maximization problem, the optimal BL portfolio weights are computed with Equation (3). Deviations from the equilibrium weights are shown in figure 2.B in Appendix B.

As expected OIL, UTIL and BANK gain importance in the optimised portfolio. The deviations for OIL and UTIL can be attributed to quite low correlations, while the positive deviation for BANK can be justified by its large market capitalisation.

Through Equation (5) we get the ex-ante TEV of 4.71%, per-annum.

In order to analyze the ex-ante TEV sensitivity we report the results of the components in Equations (7) and (10).

Table 1: marginal contributions

	$\partial TEV/\partial w_{ACT;i}$		$\partial w_{ACT}/\partial q_1$	$\partial w_{ACT}/\partial q_2$	$\partial w_{ACT}/\partial q_3$
OIL	0.0247	*OIL*	6.3980	-83.7992	-7.1934
CHEM	0.0095	*CHEM*	-1.6456	21.5532	1.8501
AUTO	0.0097	*AUTO*	0.0000	0.0000	0.0000
BANK	0.0174	*BANK*	1.3883	6.1359	33.3300
BRES	0.0032	*BRES*	0.0000	0.0000	0.0000
CONS	0.0078	*CONS*	0.0000	0.0000	0.0000
FISV	0.0147	*FISV*	0.2392	1.0574	5.7439
FBEV	0.0191	*FBEV*	0.0000	0.0000	0.0000
INDS	-0.0036	*INDS*	-4.7524	62.2460	5.3432
INSU	0.0125	*INSU*	0.0000	0.0000	0.0000
MED	-0.0259	*MED*	-0.7584	-3.3521	-18.2084
HEAL	0.0186	*HEAL*	0.0000	0.0000	0.0000
RTL	0.0086	*RTL*	0.0000	0.0000	0.0000
TECH	-0.0352	*TECH*	-0.8691	-3.8413	-20.8654
TELE	-0.0224	*TELE*	0.0000	0.0000	0.0000
UTIL	0.0208	*UTIL*	68.8961	-6.3980	1.6275
HOUS	0.0064	*HOUS*	0.0000	0.0000	0.0000
TRAV	0.0085	*TRAV*	0.0000	0.0000	0.0000

With the first column it is straightforward to verify through Equation (9) that the summation of all marginal contributions to TEV ($\partial TEV/\partial w_{ACT;i}$) multiplied by active weights yields ex-ante TEV.

By looking at the matrix $\partial w_{ACT}/\partial q_j$ ($j=1...k$) Table 1 not surprisingly highlights that the highest sensitivity of active weight for each asset comes from the view directly involving that asset. Interestingly a specific view affects not only the directly involved assets but also those addressed by other views. Seemingly each asset is affected by all views, provided that the asset is recalled at least in one view.

The product of the partial derivatives in Table 1 leads to the following TEV-sensitivities to each view: $\partial TEV/\partial q_1 = 1.6695$, $\partial TEV/\partial q_2 = -1.8741$, $\partial TEV/\partial q_3 = 1.7243$.

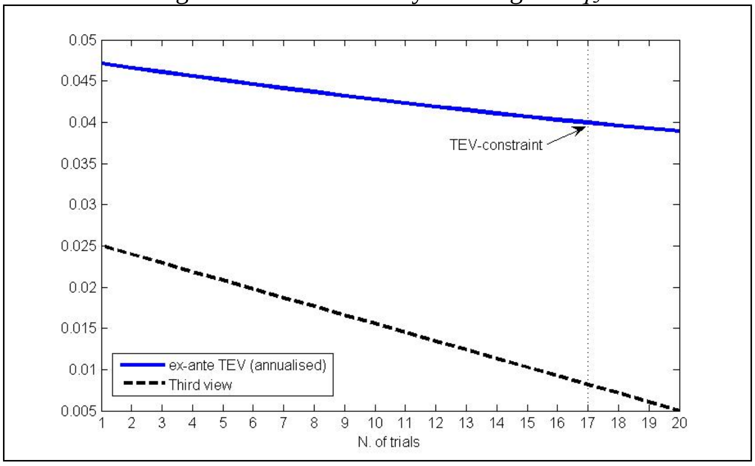
follows: [w_{BANK} /(w_{BANK} + w_{FISV}) × BANK + w_{FISV} /(w_{BANK} + w_{FISV}) × FISV)]– [w_{MED} /(w_{MED} + w_{TECH}) × MED + w_{TECH} /(w_{MED} + w_{TECH}) × TECH].

To prove the usefulness of Equation (11) we suppose to manage the portfolio under a TEV-constraint of 4.00 % per annum. Thus, we need to identify which view is mostly contributing to the TEV in order to fix the specific view accordingly.

For the sake of simplicity we modify only q_3 which has the strongest contribution to increase the ex-ante TEV. Therefore a slight reduction in view 3 is expected to decrease the overall ex-ante TEV. To exactly attain our goal, a numerical procedure is implemented to repeat the last step until we get the desired TEV-threshold. We also see that each slight reduction of q_3 impacts the active weights of the assets involved in view 1, view 2 and view 3, in accordance with both the sign and the magnitude into the partial derivative vector $\partial w_{ACT}/\partial q_3$ (see Table 1).

In figure 1 we see the effect on the ex-ante TEV of progressively decreasing q_3.

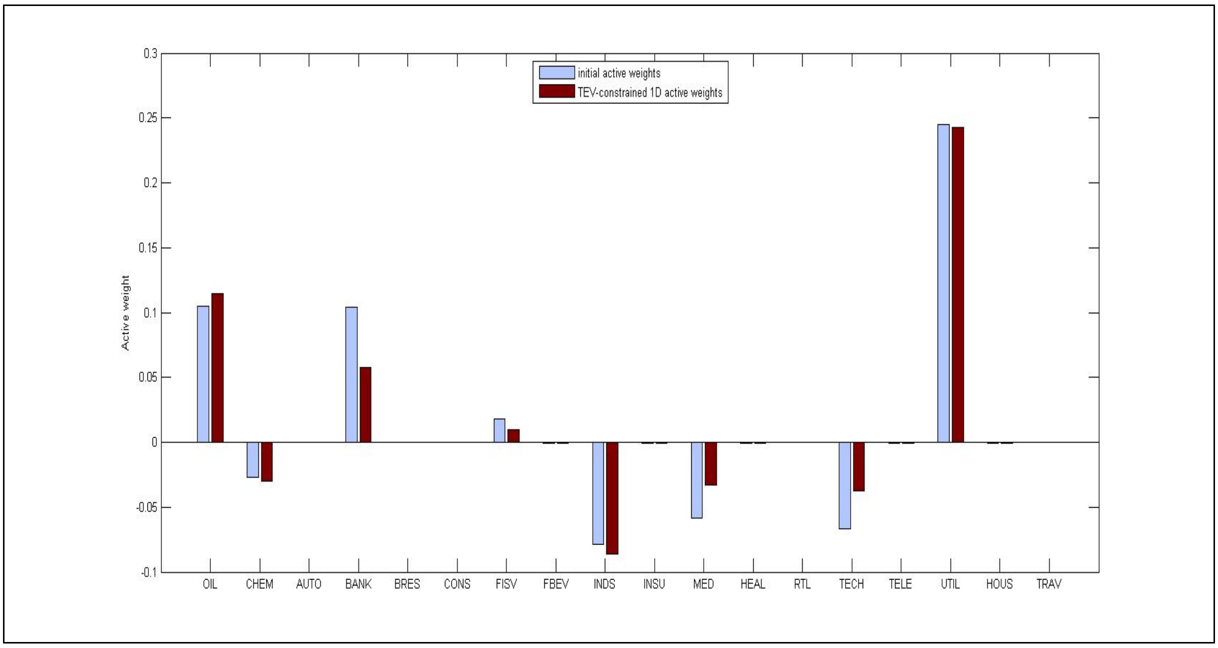
Figure 1: TEV sensitivity to changes in q_3



We display by the dashed line the different values for q_3 starting from the initial view (2.5%) to 0.5 %. Our goal of 4% per annum ex-ante TEV is achieved on the bolded line with q_3 = 0.82%.

The usefulness of our sensitivity measure is also well shown by the active weights before and after the application of the numerical procedure. They are reported in figure 2.

Figure 2: active weights before and after the one dimensional view-adjustment



Easily can be seen that after the view-adjustment our active bets lead to a less extreme portfolio. Particularly the assets involved in view 3 reduce the absolute value of their active weights. Also the active weights of the assets not involved in view 3 are changed due to the covariance transmission effect.

Next we extend the study on the marginal contributions of each view to the multi-dimensional case. Particularly we study the effect of all possible triples. For each q_j we generate h possible views with one extreme given by the initial view. The span must be chosen appropriately, such that the number of points are sufficient but not excessive.

Since our aim is reducing ex-ante TEV the possible triples are generated in accordance with the sign of $\partial TEV/\partial q_j$. View 1 positively contributes to the ex-ante TEV ($\partial TEV/\partial q_j > 0$). Consequently we examine the vector of possible q_1 ranging from 7% to 4.5% with a span of 0.005%. View 2 has a negative contribution to the ex-ante TEV ($\partial TEV/\partial q_j < 0$). Therefore a vector of possible q_2 ranging from 1% to 3% with a span of 0.005% is tested. Finally for view 3, the marginal contribution of an increase in q_3 on ex-ante TEV is positive ($\partial TEV/\partial q_3 > 0$). Consequently we calculate all possible ex-ante TEVs for a vector of q_3 with elements ranging from 2.5 % to -0.5% with a span of 0.005%[4].

We examine all possible triples q_1, q_2, q_3 in accordance with the sign of the partial derivatives in Equation (11). Therefore we derive the matrix $R \times k$ of R triples.

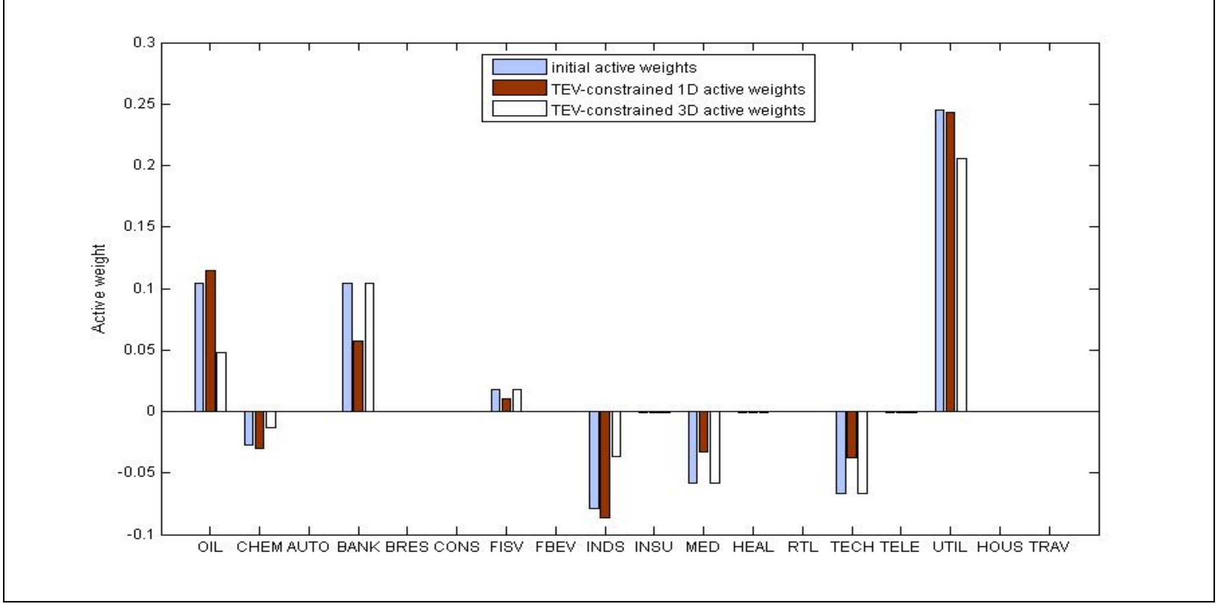
[4] We choose these extremes because, under normality assumption for each view, the probability of returns lower (for views 1 and 3) and higher (for view 2) than these extreme returns is below 0.001, then can be considered negligible.

The triple q_j^{**} (for $j = 1,...,3$) minimizing Minkowski metric with $p = 1$ is [q_1^{**} = 6.38%, q_2^{**} = 1.77%, q_3^{**} = 2.37%]. However care should be taken in the choice of the span. We suggest a second-step optimization in order to find the span that minimizes Minkowski distance among all possible q_j^{**}.

Notice that the choice of this optimal span could require a huge computational burden.

Figure 3 highlights the changes provoked by the numerical goal-attainment procedure to active weights.

Figure 3: active weights before and after both one and three dimensional view-adjustments



It is worth noting that views 1 and 2 are significantly touched by this numerical procedure while view 3 is only slightly reduced. These effects can be attributed to the confidence levels and to the consequent assessment of the extremes in the numerical process.

5. Conclusions

We have proposed a new measure to determine ex-ante TEV sensitivity to views expressed on the performance of assets. Our measure has been derived analytically by exploiting the chain rule as the product of marginal contribution of each active weight to the TEV with the marginal contribution of each view to the single active weight.

We have proven the usefulness of our measure with reference to an active portfolio. It results from a comparison between an initial portfolio based on 18 sectors of the DJ Stoxx implied equilibrium returns and a portfolio based on Black-Litterman returns for an investor with a given risk aversion.

In particular we have shown that calculating the marginal contributions of views to ex-ante TEV by our measure can significantly enhance the interpretation and understanding of sources of active risk inherent in the active portfolio. Besides our measure can suggest the portfolio manager an objective way to refine the view in order to comply with a TEV constraint.

In our empirical investigation, at a first stage only the view with the highest marginal contribution to ex-ante TEV is set repeatedly to different values through a numerical procedure until the TEV bound is satisfied. This goal-attainment process illustrates the basic approach to exploit our TEV sensitivity measure. Furthermore we provide an extension to a more realistic case when k views are adjusted simultaneously to respect TEV constraint. We believe that Minkowski distance is a proper criterion to select the best mix of q_j values among many plausible solutions under TEV constraint. An argument for adopting a multidimensional approach is the control of the consistency of views provided by different investment teams.

The topic of TEV sensitivity to views with no short-selling and budget constraints in the optimisation is left for future investigations.

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Appendix A: Descriptive Statistics

Sectors	Oil & Gas	Chemicals	Automobiles & Parts	Banks	Basic Resources	Construction & Materials	Financial Services	Food & Beverages	Industrial Goods & Services	Insurance	Media	Healthcare	Retail	Technology	Telecommu-nication	Utilities	Persolnal & Household Goods	Travel & Leisure
Labels	***OIL***	***CHEM***	***AUTO***	***BANK***	***BRES***	***CONS***	***FISV***	***FBEV***	***INDS***	***INSU***	***MED***	***HEAL***	***RTL***	***TECH***	***TELE***	***UTIL***	***HOUS***	***TRAV***
Mean	0.0773	0.0695	0.0551	0.1126	0.1239	0.1133	0.1084	0.0774	0.0545	0.0562	0.0033	0.0716	0.0399	0.0448	0.0625	0.1000	0.0855	0.0523
Std dev	0.1927	0.2133	0.2640	0.2294	0.2673	0.1969	0.2274	0.1567	0.2290	0.2814	0.2769	0.1657	0.1622	0.4025	0.2677	0.1461	0.1851	0.2237
Market Cap.	0.0826	0.0255	0.0260	0.2023	0.0395	0.0320	0.0349	0.0527	0.0738	0.0660	0.0258	0.0699	0.0370	0.0296	0.0603	0.0798	0.0457	0.0168
Correlations																		
OIL	1.0000	0.5729	0.4783	0.4883	0.6354	0.5454	0.5267	0.4347	0.5262	0.3900	0.2978	0.3782	0.4667	0.3580	0.1430	0.4040	0.5501	0.5235
CHEM	0.5729	1.0000	0.7953	0.7927	0.7826	0.8735	0.7783	0.6851	0.8199	0.7851	0.5642	0.4869	0.7132	0.6220	0.4306	0.5727	0.8056	0.8056
AUTO	0.4783	0.7953	1.0000	0.7765	0.6904	0.7856	0.7584	0.5716	0.7787	0.7286	0.5438	0.4142	0.6643	0.6179	0.4973	0.5365	0.8049	0.7408
BANK	0.4883	0.7927	0.7765	1.0000	0.6618	0.7935	0.9251	0.6253	0.8064	0.8535	0.5238	0.5497	0.6844	0.6900	0.5471	0.5990	0.8252	0.8133
BRES	0.6354	0.7826	0.6904	0.6618	1.0000	0.7755	0.6929	0.4274	0.7377	0.5958	0.4887	0.2978	0.6102	0.5801	0.3515	0.3422	0.7045	0.6881
CONS	0.5454	0.8735	0.7856	0.7935	0.7755	1.0000	0.7932	0.5882	0.8818	0.7394	0.6254	0.3771	0.7187	0.6407	0.4970	0.6152	0.8016	0.8399
FISV	0.5267	0.7783	0.7584	0.9251	0.6929	0.7932	1.0000	0.6161	0.8066	0.8529	0.5647	0.5416	0.6986	0.7075	0.5388	0.5978	0.8499	0.8266
FBEV	0.4347	0.6851	0.5716	0.6253	0.4274	0.5882	0.6161	1.0000	0.5148	0.5623	0.2310	0.5688	0.5778	0.3323	0.1791	0.5585	0.6651	0.6383
INDS	0.5262	0.8199	0.7787	0.8064	0.7377	0.8818	0.8066	0.5148	1.0000	0.7657	0.7885	0.3962	0.7414	0.8281	0.6487	0.6291	0.8586	0.8254
INSU	0.3900	0.7851	0.7286	0.8535	0.5958	0.7394	0.8529	0.5623	0.7657	1.0000	0.5798	0.5859	0.6961	0.6999	0.5392	0.6298	0.7866	0.7665
MED	0.2978	0.5642	0.5438	0.5238	0.4887	0.6254	0.5647	0.2310	0.7885	0.5798	1.0000	0.2269	0.5811	0.8126	0.7425	0.5504	0.6367	0.6034
HEAL	0.3782	0.4869	0.4142	0.5497	0.2978	0.3771	0.5416	0.5688	0.3962	0.5859	0.2269	1.0000	0.4469	0.3587	0.2938	0.5308	0.4980	0.4233
RTL	0.4667	0.7132	0.6643	0.6844	0.6102	0.7187	0.6986	0.5778	0.7414	0.6961	0.5811	0.4469	1.0000	0.6036	0.4985	0.6592	0.7488	0.7006
TECH	0.3580	0.6220	0.6179	0.6900	0.5801	0.6407	0.7075	0.3323	0.8281	0.6999	0.8126	0.3587	0.6036	1.0000	0.8034	0.5044	0.7243	0.6672
TELE	0.1430	0.4306	0.4973	0.5471	0.3515	0.4970	0.5388	0.1791	0.6487	0.5392	0.7425	0.2938	0.4985	0.8034	1.0000	0.4551	0.5150	0.5141
UTIL	0.4040	0.5727	0.5365	0.5990	0.3422	0.6152	0.5978	0.5585	0.6291	0.6298	0.5504	0.5308	0.6592	0.5044	0.4551	1.0000	0.6070	0.5728
HOUS	0.5501	0.8056	0.8049	0.8252	0.7045	0.8016	0.8499	0.6651	0.8586	0.7866	0.6367	0.4980	0.7488	0.7243	0.5150	0.6070	1.0000	0.8187
TRAV	0.5235	0.8056	0.7408	0.8133	0.6881	0.8399	0.8266	0.6383	0.8254	0.7665	0.6034	0.4233	0.7006	0.6672	0.5141	0.5728	0.8187	1.0000

Appendix B

Figure 1.B Equilibrium and Black-Litterman returns

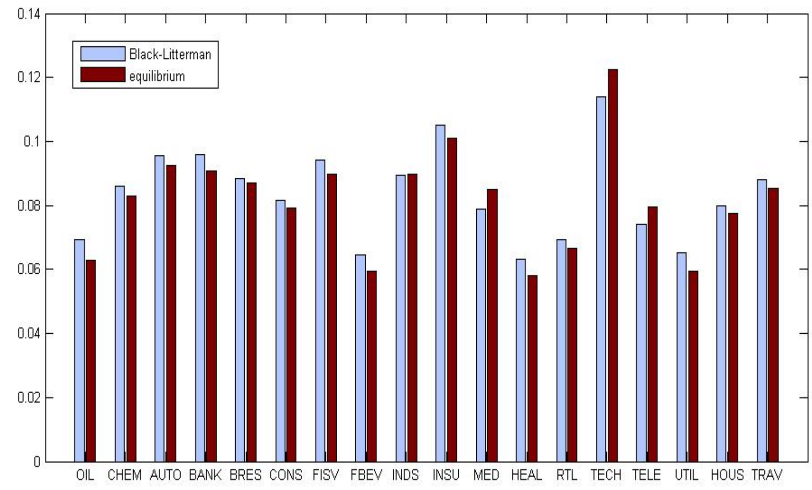

Figure 2.B Equilibrium and Black-Litterman portfolio weights