Portfolio Construction with Downside Risk*

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November 1, 2007

*We are grateful to Isabelle Bajeux-Besnainou, Didier Maillard and the seminar participants at the FMA 2007 Conference in Barcelona for helpful comments and suggestions. Note that this paper expresses the authors' views that do not have to coincide with those of Union Investment or HSH Investment Management S.A..

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ABSTRACT

Portfolio construction seeks an optimal trade-off between a portfolio's mean return and its associated risk. Given that risk may not be properly described by return volatility we examine alternative measures that account for the asymmetric nature of risk. In particular, we optimize portfolios in an empirical out-of-sample setting with respect to various measures of downside risk. These optimization tasks are successful for most of the investigated measures when assuming perfect foresight of expected returns. While the latter assumption is a strong one we also show that our findings still hold when using more naïve return estimates. The reductions in downside risk are most convincing for semivariance, semideviation, maximum drawdown and loss penalty while value at risk and measures related to skewness appear rather useless for portfolio construction purposes.

Keywords: Portfolio Optimization, Downside Risk

JEL Classification: G11, G12, D81

Since Markowitz (1952) has lifted portfolio theory to a scientific level several shortcomings of the classic mean-variance optimization approach have been discussed controversially in literature. The main issues concern its static one period character, the high sensitivity of optimization outcomes with respect to small changes in the inputs and the use of volatility as a risk measure. In this paper we focus on the latter. In fact, being a symmetric risk measure, volatility does not seem to fit most investors' definition of risk as the peril of ending up with less than expected. By conceiving negative and positive surprises as risk volatility does not distinguish between cold and warm rain.

Tobin (1958) was one of the first to show that volatility can only be the risk measure of choice for quadratic utility functions or, equivalently, for normally distributed returns. Both assumptions are unsustainable, for instance, see Mandelbrot (1963) and Fama (1965) for non-normality in returns and Rubinstein (1973) or von Neumann and Morgenstern (1947) for properties of utility functions. Of course, there are a number of proposals to account for the asymmetric nature of risk. For example, Roy (1952) adds a criterion to Markowitz' efficient frontier which selects the efficient portfolio with the lowest probability to fall short of a given target return. Markowitz (1959) proposes a portfolio optimization procedure based on the semivariance measure. Fishburn (1977) employs a utility function model that translates downside risk depending on a risk aversion parameter and a target return. His findings indicate that investors' risk perception significantly changes below this individual threshold. Hence, there is a broad consensus among both researchers and practitioners that asymmetry is a reasonable property for any risk measure. Although this questions the use of volatility, the mean-variance approach is still prevalent.

Whatever the choice of risk measure, any optimizer will certainly find risk-minimized portfolios given perfect information. In practice, however, portfolio construction takes place in an ex-ante context relying on forecasts of the respective risk measure that have to be derived from historical data. Therefore, forecastability of a risk measure is a prerequisite in practice. This is an important challenge because risk structures—think of correlations—are known to be unstable over time. For instance, the instability of beta factors and, as a consequence, covariance matrices and other correlation-based risk measures has been documented by numerous studies.[1] Although

[1] See, among others, Fabozzi and Francis (1978), Bos and Newbold (1984) and Schwert and Seguin (1990) for the U.S., Wells (1994) for Sweden, Faff, Lee and Fry (1992) for Australia and finally Ebner and Neumann (2005) for Germany.

it is important to consider alternative risk measures in an empirical out-of-sample setting this has not yet been addressed intensively in the literature. This paper adds to the existing evidence by empirically examining whether asymmetric risk measures are more useful to define risk in the context of portfolio construction. Therefore, we substitute volatility as the objective function of portfolio optimization various alternative risk measures. We compare the out-of-sample performance characteristics of the mean-variance based investment strategy to those of other risk philosophies in an empirical backtest. To assess whether downside risk measures are of any value in portfolio optimization we first consider the case of perfect foresight with respect to returns. Since mean-variance optimization is typically confounded by estimation risk, especially the one embedded in estimates of expected returns, we do so to isolate portfolio construction from the return forecasting problem. Given perfect foresight minimizing downside risk is promising for six out of eight investigated measures. To also judge whether these risk reductions can be achieved in absence of a crystal ball we then imitate a more naïve portfolio manager that employs historical means to estimate expected returns. While the results are more noisy they qualitatively confirm the prior findings. Hence, downside risk measures can successfully be used for portfolio optimization as long as they are sufficiently persistent over time.

The paper is structured as follows. Section I presents various measures of downside risk and discusses their strengths and weaknesses with respect to portfolio optimization. Section II motivates the methodology of the empirical backtest that is pursued in Section III. Therein, we thoroughly describe the portfolio characteristics of the various downside risk strategies. Section IV concludes.

I. Measuring Downside Risk

Positive and negative deviations contribute equally to risk when measured by volatility. However, investors' risk perception is typically more affected by downside events. In a world of symmetric returns control of volatility coincides with downside risk control, however, it is well known that returns exhibit asymmetry. Especially, return distributions with equal volatility may entail different downside risk. In the following we discuss appropriate measures that will be used in the subsequent portfolio optimization study. All of the measures capture downside risk by considering negative deviations as the sole source of risk.

A. Value at Risk and Conditional Value at Risk

Given the random return R of a portfolio for a certain holding period the value at risk (VaR) determines the loss corresponding to the lower quantile of its distribution for a given (small) probability. Let the probability be $p = 0.01$, then

$$\mathrm{VaR}_p(R) = -F_R^{-1}(0.01), \tag{1}$$

where F_R is the cumulative distribution function of R. Thus, VaR represents a loss value that is not breached with a certain (high) probability. However, VaR ignores extreme events below the specified quantile. If optimization is not limited to certain distribution families the optimizer may sweep the dirt under the rug by providing optimized portfolios with a low VaR but possibly fat tails beyond that threshold. Alexander and Baptista (2002) also point out that mean-VaR optimization does not necessarily improve upon mean-variance, in fact, one may end up with more volatile portfolios when switching from variance to VaR as a measure of risk. Finally, VaR is not a coherent risk measure since the subadditivity property is not satisfied, i.e., the VaR of a portfolio may be larger than the sum of the VaRs of its constituents.[2]

These drawbacks of VaR can be mitigated by a related risk measure, the conditional VaR (CVaR). While VaR basically is a quantile CVaR simply is a conditional expectation that gives the expected loss beyond a specific threshold, e.g., beyond the VaR. Therefore, it is often referred to as expected shortfall. We define CVaR as follows

$$\mathrm{CVaR}_p(R) = -\mathbb{E}[R \mid R < F_R^{-1}(0.01)]. \tag{2}$$

Bertsimas, Lauprete and Samarov (2004) show that CVaR exhibits convexity with respect to portfolio weights under fairly weak continuity assumptions which facilitates mathematical opti-

[2] The concept of coherent risk measures encompasses reasonable properties of a given risk measure. Following Artzner, Delbaen, Embrechts and Heath (1999) these properties are subadditivity, monotonicity, homogeneity and translation invariance.

mization and contributes to the risk measure's coherence. Also, Alexander and Baptista (2004) theoretically compare CVaR and VaR in a portfolio selection context and contend that imposing a CVaR constraint is more effective than a VaR constraint. However, optimization with respect to CVaR requires either an assumption about the return distribution or a considerable amount of observations below the target.

B. Lower Partial Moments

Next, we consider the semideviation risk measure which is related to CVaR since it also entails a linear evaluation of below-target returns. Semideviation belongs to the broader class of lower partial moments (LPM) risk measures that only take downside events below a given target into account. Returns below this threshold are evaluated by a penalty function that increases polynomially with the distance from the target return.

An LPM risk measure is determined by two parameters. One is the exponent or degree k and the other is the target return τ. Given the return distribution R of a portfolio its lower partial moment is defined by

$$LPM_{\tau,k}(R) = \mathbb{E}\left((\tau - R)^k \mid R < \tau\right) \cdot P(R < \tau). \tag{3}$$

Common parameter choices are $k = 1$ or $k = 2$ and $\tau = \mathbb{E}(R)$, yielding the semideviation

$$\mathrm{SD}(R) = \mathbb{E}\left((R - \mathbb{E}R) \mid R < \mathbb{E}R\right) \cdot P(R < \mathbb{E}R), \tag{4}$$

and the semivariance of R,

$$\mathrm{SV}(R) = \mathbb{E}\left((R - \mathbb{E}R)^2 \mid R < \mathbb{E}R\right) \cdot P(R < \mathbb{E}R). \tag{5}$$

Other common values of τ are the risk-free rate or a zero return. These choices refer to the risk of falling behind opportunity costs or to the risk of realizing an absolute loss, respectively. The parameter k serves as a risk aversion parameter—losses are penalized with power k. Porter (1974) and Bawa (1975) show that the semivariance measure is consistent with the concept of stochastic dominance and an analogous proof for semideviation is given by Ogryczak and Ruszczynski (1998). For a detailed discussion of further LPM properties see, among others, Harlow and Rao (1989).

LPMs entail considerable computational difficulties, for instance, a portfolio's LPM cannot be expressed as a function of security LPMs as it is the case for volatility. This shortcoming contributes to the limited application of these risk measures in practice, however, we include both, semideviation and semivariance, in our empirical investigation. Let $(R_t)_{t=1,\ldots,T}$, $R_t \in \mathbb{R}^N$ denote the sample of return (vector) realizations and let $\boldsymbol{x}$ be the vector of portfolio weights. Then the vector of portfolio returns obtains as $(X_t)_{t=1,\ldots,T}$ where $X_t = \boldsymbol{x}^T R_t$. Semideviation and semivariance of this portfolio are computed as follows:

$$f(\boldsymbol{x}) = \mathrm{SD}(X) = \frac{1}{T}\sum_{t=1}^{T} \min\left\{(X_t - \bar{X}), 0\right\}$$

$$f(\boldsymbol{x}) = \mathrm{SV}(X) = \frac{1}{T}\sum_{t=1}^{T} \min\left\{(X_t - \bar{X}), 0\right\}^2$$

where $\bar{X}$ denotes the mean of X estimated by $\bar{X} = \frac{1}{T}\sum_t X_t$.[3]

C. Loss Penalty

Mean-variance optimization is often set in a utility maximization framework. The investor maximizes the expected utility u of his portfolio return distribution by solving

$$f(\boldsymbol{x}) = \mathbb{E}(u(\boldsymbol{x}^T \boldsymbol{R})) \rightarrow \max \tag{6}$$

for a given random vector of market returns $\boldsymbol{R}$. The approach builds on the work of von Neumann and Morgenstern (1947). For ease of deduction, the utility function is assumed to be quadratic (or fitting its quadratic approximation very well) giving rise to consistency of mean-variance optimization and utility maximization, see Tobin (1958). Again, this approach addresses downside risk only in a world of symmetric returns, however, the properties of u allow definitions of utility

[3] Alternative ways of computing semideviation and semivariance have been proposed by Bawa and Lindenberg (1977) and Markowitz (1959) that save computational efforts at the cost of a certain inaccuracy. Given today's computing power we neglect these procedures.

functions that match our intuition of an investor with a downside risk attitude. Therefore, we choose the negative exponential as utility function given by

$$u(\boldsymbol{x}^T\boldsymbol{R}) = -\exp(-(\boldsymbol{x}^T\boldsymbol{R} - \mathbb{E}\boldsymbol{x}^T\boldsymbol{R})). \tag{7}$$

Adjusting for the mean gives a reasonable threshold below which the outcomes are penalized more as compared to positive deviations, therefore, we will further refer to this measure as *loss penalty*. Note that we do not claim to know the investor's utility function. We rather optimize a function having returns as arguments that fits our goal of mitigating downside risk and can coincidentally be interpreted as one possible utility function.

Again, the computation of investor utility is not as convenient as for volatility. For a given time series vector $(R_t)_{t=1,\ldots,T}$ and a specific vector $\boldsymbol{x}$ of portfolio weights we determine

$$X = (X_t)_{t=1,\ldots,T}, \qquad X_t = \boldsymbol{x}^T R_t \tag{8}$$

$$\mathbb{E}(u(X - \bar{X})) = \frac{1}{T}\sum_{t=1}^{T} - \exp(-X_t + \bar{X}). \tag{9}$$

D. Skewness

The skewness of a return distribution R captures its deviation from symmetry and is defined as

$$\gamma(R) = \frac{\mathbb{E}\left((R - \mathbb{E}R)^3\right)}{\sigma(R)^{3/2}}. \tag{10}$$

The more positive the skewness measure is, the heavier are the upper tails compared to the lower tails of a distribution. Figure 1 shows how downside risk decreases with positive skewness. This is because the positively skewed (or right-skewed) distribution has less events at the far lower tail at the cost of more events slightly below the mean, when compared to a symmetric distribution.

[Figure 1 about here.]

However, skewness alone does not add value and one needs to limit volatility as well. Otherwise optimized portfolios might exhibit favorable skewness and nonetheless fat downside tails due to an overall rise in volatility. This point is illustrated in Figure 2 by comparing two equally-skewed distributions with differing volatilities. Apparently, this effect also holds for left-skewed distributions. Therefore, portfolio optimization will additionally require a non-linear constraint that limits volatility to a fixed level while testing for the performance effects of alternative skewness levels.

[Figure 2 about here.]

As for skewness measures we consider two estimators. First we use the standard skewness estimator given by

$$\hat{\gamma}(X) = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{X_t - \hat{\mu}}{\hat{\sigma}(X)}\right)^3. \tag{11}$$

However, it is repeatedly mentioned in the literature that the straightforward estimator is not only biased but also highly sensitive with regard to sample outliers, see Kim and White (2004) and Harvey and Siddique (2000). A few more robust skewness estimators have been discussed and we choose the one proposed by Bowley (1920):

$$S_B(X) = \frac{Q_3(X) + Q_1(X) - 2Q_2(X)}{Q_3(X) - Q_1(X)}, \tag{12}$$

where $Q_i(X)$ is the ith quartile of the sample $X = (X_i)_{i=1,\ldots,T}$. This estimator should be less sensitive to outliers than the traditional one, which is of particular relevance in portfolio construction given that return data exhibit fat tails.

E. Maximum Drawdown

The maximum drawdown measure (MDD) defines risk as the maximum percentage loss an asset experienced from its top valuation to its bottom valuation within a given sample period. An important advantage of this risk measure is its independence from the underlying return distribution and its properties since it entirely relies on historical returns. For any given time series of (discrete) returns for a single security S, $(R_t^{(S)})_{t=1,\ldots,T}$, we can easily compute the corresponding time series of the portfolio value if it solely consists of security S. This value is given by

$$V_0^{(S)} = 1, \qquad V_t^{(S)} = V_{t-1}^{(S)}(1 + R_t^{(S)}/100), \qquad t = 1, \ldots, T.^4 \tag{13}$$

In order to retrieve the worst period to be invested in S we calculate the MDD for period $[0, T]$ recursively. Given the Maximum Drawdown for the period $[0, T-1]$, $\mathrm{MDD}_{[0,T-1]}(S)$, we obtain

$$\mathrm{MDD}_{[0,T]}(S) = \min\left\{\frac{V_T - V_{\hat{t}}}{V_{\hat{t}}}, \mathrm{MDD}_{[0,T-1]}(S)\right\}, \tag{14}$$

where $V_{\hat{t}}$ is the maximum value of the security held within $[0, T-1]$. Setting $\mathrm{MDD}_{[0,0]}(S)$ to a sufficiently large number we compute $\mathrm{MDD}_{[0,T]}(S)$ by stepping through the sample and applying equation (14). For ease of presentation we neglect the index for the time interval in the following. Figure 3 illustrates the MDD of a portfolio that has been optimized with respect to the MDD and compares it to the MDD one would have realized holding the benchmark.

[Figure 3 about here.]

Portfolio optimization techniques for drawdown constraints have been demonstrated by Checklov, Uryasev and Zabarankin (2003) by means of linear programming and by Karatzas and Cvitanic (1995) in a continuous time framework. For empirical results on maximum drawdown as a portfolio risk measure, see e.g. Hamelink and Hoesli (2004), Burghardt, Duncan and Liu (2003) or Johansen and Sornette (2001).

II. Empirical Methodology

A. Data and Predictability of Risk Measures

We aim at investigating the characteristics of the above portfolio construction approaches that arise from active benchmark-oriented management of a European equity portfolio. We examine discrete weekly return data of the Dow Jones EURO STOXX 50 universe from January 1993

[4]Typically, returns are deducted from values, so one can save the double computation.

to April 2006 exclusive of ten names that have missing data for parts of the period. For the remaining 40 stocks we adjust index weights proportionally to ensure that they sum to 100% and adjusted benchmark returns are computed accordingly.

Predictability is key for any risk measure to be of practical value in portfolio optimization procedures. To judge the persistence of downside risk measures we examine the stability of the respective stock rankings. In particular, we yearly rank the stock universe according to the respective risk measure and compute Spearman's rank correlation coefficient for all pairs of consecutive years in table IV. For instance, we first sort the stocks included in this study by their volatility observed in 1995, and we then sort the same set of stocks by their volatility observed in 1996. These two rankings exhibit a correlation of 0.67. Over all pairs of years, volatility exhibits a mean correlation of 0.592 suggesting considerable predictability. On the other hand, the VaR correlation is less stable—a property that is not translated to CVaR. Table IV also reveals that both skewness measures are rather instable suggesting that portfolio optimization with respect to skewness may also suffer. The results for loss penalty and maximum drawdown are more stronger—the latter contrasting the view of Acar and James (1997) who report poor predictability of the MDD. Also, the overall results in Table IV indicate that volatility, semideviation and semivariance exhibit high correlation over the years that one may well exploit when minimizing the respective risk dimensions.

B. Formulation of Portfolio Optimization Tasks

The empirical examination adopts the following investment strategy. Our objective is to minimize risk according to the respective definition of downside risk while requiring all optimization solutions to exceed the benchmark return expectation and to satisfy a certain maximum tracking error limit.

Imposing quarterly rebalancing the necessary inputs to the optimization are computed every three months as follows. The respective measures of downside risk are estimated using two years of historical data amounting to 104 weekly observations. As for the estimation of expected returns we consider two approaches. First, we assume perfect foresight of returns and, second, we consider a simple return forecast by computing the historical mean. At first glance, assuming perfect foresight seems inappropriate, however, it allows for isolating the portfolio construction problem from the return forecasting problem. This issue is important since it is well-known that mean-variance optimization typically suffers more from estimation risk embedded in expected returns as opposed to estimation risk in variances. Since we want to focus on the risk dimension perfect foresight of returns represents a natural base case for testing portfolio construction methods. Any method failing in this base case may be discarded immediately. Any method succeeding in this base case may be subjected to further test. A reasonable test is reflected in our second approach of estimating expected returns. Instead of using the perfect estimate we employ a simple one given by the historical mean of the 2–year estimation window.

All in all, we examine eight portfolio strategies based on the objective functions introduced in the last section, together with the traditional mean-variance approach. Setting $X = \boldsymbol{x}^T \boldsymbol{R}$ the optimization tasks are the following:

1. Volatility: $f(\boldsymbol{x}) = \sigma(X) = (\boldsymbol{x}^T C \boldsymbol{x})^{1/2} \to \min,$
2. VaR: $f(\boldsymbol{x}) = \mathrm{VaR}_p(X) \to \min,$
3. CVaR: $f(\boldsymbol{x}) = \mathrm{CVaR}_p(X) \to \min,$
4. Semideviation: $f(\boldsymbol{x}) = SD(X) = \mathbb{E}(X - \mathbb{E}X \mid X < \mathbb{E}X) \cdot P(X < \mathbb{E}X) \to \min,$
5. Semivariance: $f(\boldsymbol{x}) = SV(X) = \mathbb{E}\left((X - \mathbb{E}X)^2 \mid X < \mathbb{E}X\right) \cdot P(X < \mathbb{E}X) \to \min,$
6. Loss Penalty: $f(\boldsymbol{x}) = \mathbb{E}(u(X - \mathbb{E}X)) \to \max,$
7. Standard skewness: $f(\boldsymbol{x}) = \hat{\gamma}(X) = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{X_t - \hat{\mu}}{\hat{\sigma}(X)}\right)^3 \to \min,$
8. Robust skewness: $f(\boldsymbol{x}) = S_B(X) = \dfrac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1} \to \max,$
9. Maximum Drawdown: $f(\boldsymbol{x}) = \mathrm{MDD}(X) \to \min.$

To complete the formulation of the optimization tasks we add the following constraints:

First, we restrict the tracking error τ. For a given vector of benchmark portfolio weights $\boldsymbol{x}_{BM}$ and (optimized) weights $\boldsymbol{x}$ we constrain the tracking error by

$$\tau(\boldsymbol{x}) = \sqrt{(\boldsymbol{x} - \boldsymbol{x}_{BM})^T C (\boldsymbol{x} - \boldsymbol{x}_{BM})} \le \tau_{\max}. \tag{15}$$

The $N \times N-$matrix C is the sample covariance matrix of the random vector of portfolio returns. We set an annual tracking error limit of 5%[5] which provides the optimizer with a fairly large discretion in determining the weights and allows for a clear picture about the impact of the optimization process.

Second, we restrict the expected portfolio return to reach at least benchmark level, i.e.

$$\boldsymbol{\mu}^T \boldsymbol{x} \geq \boldsymbol{\mu}^T \boldsymbol{x}_{BM}. \tag{16}$$

The estimated mean return of the optimized portfolio (which coincides with the realized return in the case of perfect foresight) is therefore greater or equal to the benchmark return. On the one hand, this constraint avoids optimization solutions that reduce risk at the cost of return. On the other hand, this constraint sets no incentive for the optimizer to go for higher returns.

Third, we additionally require the following linear (in)equalities to be met:

$$\sum_{i=1}^{N} x_i = 1 \quad \text{and} \quad x_i \geq 0 \quad \forall i, \tag{17}$$

representing the full investment and non-negativity constraints, respectively.

As for optimization problems 2 and 3 p equals 0.5 because lack of data may infer the optimization for low values of p. Hence, the median of the return distribution is the VaR quantile of choice.

When skewness is the objective we also restrict volatility, thus, we add the following constraint for optimization problems 7 and 8:

$$\sigma(\boldsymbol{x}) = \sqrt{\boldsymbol{x}^T C \boldsymbol{x}} \leq \sigma_{BM}. \tag{18}$$

Summing up, these assumptions and restrictions imitate a portfolio manager who adjusts his portfolio every three months for a certain risk objective considering data of a revolving two-year-period. Regarding the optimization process our approach is quite pragmatic. We employ

[5]We therefore apply the rule for log-returns as a rule of thumb for discrete returns: $\tau_{\text{annual}} = \tau_{\text{weekly}} \cdot \sqrt{52}$. Since we optimize using weekly return and volatility data, the implemented tracking error limit is $\tau_{\max} = 5\% \cdot \sqrt{(}52) = 0.69\%$.

optimization routines embedded in standard software packages, especially we do not spend much effort on streamlining the optimization itself but focus on setting the objectives and interpreting the outcome.[6]

III. Minimizing Downside Risk

A. Optimizing with perfect foresight of returns

If there is any value to downside risk minimization it should reveal when optimizing given perfect foresight of returns. To investigate the degree to which the different optimization approaches succeed in mitigating downside risk we compare the optimized strategy with two competing strategies. First, we check whether we are able to outperform the benchmark in terms of the risk dimension that is to be minimized. Second, we also compute the risk measure of the optimal strategy that obtains having perfect foresight of risk and return thus providing an upper boundary that allows for judging our optimization efforts.

Consider for example the traditional mean-variance optimization in table II. The average weekly volatility of the benchmark across all holding periods is 2.538%, while the optimized portfolio exhibits a lower volatility of 2.264%. Given perfect information one would have even been able to achieve an optimal volatility as low as 1.854% which corresponds to an annual volatility of 13.369%.

The column headed "Gain" indicate the degree to which the maximum risk spread between benchmark and optimal strategy is translated into the optimized portfolios. In case of volatility, the gain is (2.538%-2.264%) over (2.538%-1.854%) which equals 39.96%.

While scaling the average gain to percentages allows for a comparison across measures one should be very careful interpreting differences since it is undue to compare percentage reductions in risk measures with different growth characteristics. More precisely, semideviation grows linearly, semivariance exhibits a quadratic increase, while loss penalty grows exponentially. Finally, robust skewness is bounded by the interval $[-1, 1]$. Instead, we compute a hit rate that indicates the

[6]We employ Matlab's flexible *fmincon* function for all alternatives except volatility for which we use the *quadprog* function. The function *fmincon* offers a trust region method intended to tackle large scale problems and a line search algorithm for medium scale tasks. After adjusting for the number of iterations and error tolerance, the medium scale method performs satisfactorily.

fraction of optimization periods in which the optimized portfolio exhibits more favorable risk figures than the benchmark. For volatility, the last column of Table II gives a hit rate of 89.13% since 41 out of 46 holding periods are successful in that respect. Of course, the hit rate neglects the accuracy of the predicted risk measures in absolute terms. For the hit rate to be significant at the 5% (1%) level, Y needs to be greater or equal to 29 (31).[7] Therefore, the hit rate for volatility is highly significant.

Having examined the traditional mean-variance case let us now investigate the results for the optimization with respect to downside risk. All of the optimization tasks return optimized portfolios that exhibit a more favorable downside risk figure than the benchmark. The average values of CVaR, volatility, VaR, semideviation, semivariance, loss penalty and maximum drawdown of optimized portfolios are far below those of the benchmark. For instance, the weekly maximum drawdown of the benchmark is 7.3% while the optimized portfolio only exhibits 5.4%, thereby exploiting almost half of the spread to the optimal strategy which has a maximum drawdown of 3.2%. Regarding loss penalty, we state the best performance with an average gain of 60.84%. Given the low persistence in skewnness it is not surprising that the average improvement for the two skewness related risk measures is close to zero. This observation indicates that skewness is an unreliable property for the assets under scrutiny. Evidently, the sensitivity towards sample outliers does not account for the lacking reliability of standard skewness. This can be deducted from the paltry performance of the robust skewness measure on the other hand. Despite the obvious effect of skewness on downside risk, this result adds to the finding that the empirical evidence for a skewness premium in asset prices, as reported by Kraus and Litzenberger (1976), is rather weak, see the empirical study of Post and Vliet (2003).

For six out of eight investigated downside risk measures we find hit ratios far greater than 50%, reinforcing the evidence given by the average gain figures. Also, the improvement obtained from the examined optimization procedures is significant for all risk measures under examination except for the ones related to skewness. The maximum hit ratio is 91.30% and obtains for the CVaR.

[7] For significance testing of the hit rate we assume that the number of intended objective changes results from a random guess, or in statistical terms, a realization of a binomially distributed variable $Y \sim B(Z, p)$ with $p = 0.5$ and $Z = 46$.

Given quarterly rebalancing we investigate the resulting 46 optimization outcomes by plotting the optimized strategy's risk measure over time—together with the risk measures obtaining for the benchmark and for the optimal strategy. Thereby, we provide intuition about the time-varying characteristics of the portfolios' downside risk and visualizes the degree to which we are able to exploit the spread to the optimal downside risk strategies.

In particular, figure IV illustrates the spread between the risk measure of the benchmark and of the optimal portfolio (given perfect foresight of risk and return) as a lightshaded area—giving a general idea of the feasible risk levels that can be expected from a given optimization routine. For example, the weekly volatility ranges between 1% to 6% over time usually exhibiting a spread of bandwidth 1%. The realized volatility of the optimized mean-variance strategy is given by a solid line. Note that this line is not confined to the shaded spread area since it may be that the optimized strategy is more risky than the benchmark at a given time. Below, we additionally plot the evolution of the according gain figures over time. For instance, if the optimized strategy's VaR equals the one of the optimal strategy the gain is 100% and if the former equals the VaR of the benchmark the gain is 0%. As a consequence, the gain may also turn negative when the optimized strategy's VaR is higher than the one of the benchmark. The latter behavior most often applies to both skewness strategies, moreover, the benchmark's skewness is hardly worse than the optimized strategy. As a result, the gain figures are rather disappointing. Considering the case of mean-variance the results are more convincing with little periods of negative gain. We note that the gain profiles of volatility and semivariance almost coincide while semideviation has a more volatile one. Note that the gain profile of VaR is especially desirable being consistently above zero, closely followed by the ones of CVaR and Maximum Drawdown.

[Figure 4 about here.]

B. Optimizing with forecasted returns

Lacking a crystal ball the practical value of the above results may be rather limited. Therefore, we examine next whether a less skilled investor is also able to significantly reduce his exposure to downside risk. To imitate such an investor we employ a simple estimate of expected returns represented by the historical mean.

Table III gives the results that obtain in this setting and, of course, the degree of risk reduction is muted compared to the base case. The observed setback is most severe for VaR and CVaR. The latter exhibits a reduced gain figure of 5.83% and VaR even has a negative average gain. Obviously, the according hit rates are no longer significant. Given that the skewness measures have already failed in the base case the conclusions do not change when using less perfect information. However, the hit rates still prove to be significant for volatility and 4 downside measures, i.e., semideviation, semivariance, loss penalty and maximum drawdown. Among these four measures semivariance performs similarly to the case of perfect foresight.

In figure IV the corresponding 46 optimization outcomes are again evaluated by inspecting the optimized strategy's risk measure over time—together with the risk measures obtaining for the benchmark and for the optimal strategy. In line with the above average gain figures we find the according gain profiles for volatility, semideviation, semivariance, and loss penalty to be similar to the case of perfect foresight. However, the VaR gain profile is almost completely neutralized for the whole sample period, and the CVaR strategy is only useful in recent years. Note that the weak performance of the CVaR at the end of the nineties fits with the observation of poor predictability, see the according rank correlations in table IV. This observation also applies to the Maximum Drawdown strategy for which the underperformance in the first half of the sample period is more severe.

[Figure 5 about here.]

To visualize the characteristics of portfolios optimized with respect to downside risk we plot the corresponding performance time series in figure IV. The light-shaded area indicate the spread between the strategies' best and worst cumulated return for a given month. We note that this spread is quite narrow highlighting that downside risk control does not necessarily come at the cost of return, in fact, some strategies, e.g. maximum drawdown or loss penalty, do outperform the benchmark. Nevertheless, in our setting downside risk control is not identical to hedging market risk, thus, one is not immune to sudden market shocks. We learn that the VaR strategy is the worst in terms of return while the CVaR strategy performs quite convincing—at least until the end of the mineties. This observation extends to the strategies driven by volatility, semideviation and semivariance. Surprisingly, standard skewness exhibits a quite favorable return path whereas robust skewness completely disappoints.

[Figure 6 about here.]

C. Comparison of Downside Strategies

Given four measures succeeding in significantly reducing downside risk one may wonder what measure to choose. In particular, it may well be that the different measures of risk ultimately lead to very similar portfolios. As a metric of strategy similarity we compute mutual tracking errors of the various strategies displayed in table V. Along its diagonale we give tracking errors between the downside risk strategy with perfect foresight versus the one with forecasted returns. As expected, these figures are highest for VaR, CVaR, and maximum drawdown, all of which suffer the most when dropping the assumption of perfect foresight. Note that we give tracking errors for the case of perfect foresight above the diagonale and for the case of forecasted returns below the diagonale. While the latter figures appear more noisy the results are quite similar in both cases, thus, we will focus on the case of forecasted returns. First, we consider the first column that gives the realized tracking errors to the benchmark. While the ex ante tracking error has been restricted to 5%, the ex post figures only slightly exceed this boundary for most of the strategies. However, we note that the CVaR and the maximum drawdown strategy are most different to the benchmark. Concerning the remaining strategies one may identify one cluster consisting of strategies driven by volatility, semivariance, and semideviation. The remaining strategies exhibit tracking errors in excess of 4%, interestingly, the two skewness strategies are not closely related given a mutual tracking error of 6.852%—paralleling the difference observed in the cumulated returns of both strategies.

The similarity of strategies can also be judged by computing the mutual downside risk measures of the various downside strategies. Panel A of table IV gives the mutual risk measures obtaining in the optimal case, whereas panels B and C cover the results for the portfolios optimized with perfect foresight and forecasted returns. In each row we compute one specific risk measure over the whole sample period for all of the downside strategies together with the mean-variance one and the benchmark. For instance, in panel A the benchmark volatility is 2.85% while the maximum value is 2.99% on behalf of the VaR. As expected, the most favorable volatility figure is exhibited by the mean-variance strategy, therefore, the outcome of 2.29% volatility is in bold face labelling the strategy with the lowest volatility. Only in two cases do these figures not belong to the downside strategy that seeks to minimize the respective risk dimension. To judge the overall performance of strategies we compute their mean ranking which equals 1 if the strategy always entails the least downside risk among the alternatives and which equals 10 if the strategy always entails the most downside risk. Semivariance and maximum drawdown rank first with a mean ranking of 3.44 closely followed by volatility with 3.56. Notably the benchmark is worst with a mean ranking of 8.11.

Stepping on to panel B containing the case of perfect foresight we state that the maximum drawdown strategy is most robust with respect to its mean ranking which is almost unchanged to the optimal case. Even though the semivariance strategy loses some ground it has still a mean ranking of 4.33 and, moreover, it has the lowest values for volatility, semideviation and semivariance.

Finally, panel C gives the results for the case of forecasted returns. First of all, we remark that the benchmark ceases to be the worst strategy since its mean ranking exceeds those of VaR and Standard Skewness. Semivariance ranks first with a mean ranking of 3.44, now exhibiting the lowest values for four downside dimensions. Note that the mean-variance strategy has only slightly less favorable downside figures giving rise to mean ranking of 4.56. Also, the loss penalty strategy exhibits consistent performance across various measures.

Summarizing, we find that it is possible to optimize portfolios with respect to certain downside risk objectives like semivariance, loss penalty and maximum drawdown. However, given little asymmetry in returns one may also resort to classical mean-variance for achieving decent downside risk control.

IV. Conclusion

Since investors are mostly concerned about downside risk volatility may not be a reasonable choice of risk measure. Therefore, we have empirically examined the characteristics of asymmetric risk measures in an out-of-sample context. All in all, our results indicate that portfolio optimization techniques can successfully reduce asymmetric risk when compared to a strategy of buying and holding a benchmark portfolio. This is in particular true for semideviation, semivariance, loss penalty as well as maximum drawdown. While this result is first derived assuming perfect foresight of returns it is comforting that it remains valid when using forecasted estimates of expected return. However, skewness risk is the hardest one to control, even when using a more robust estimator. Given the instability of the associated measures skewness risk cannot be predicted properly and optimization fails to reduce skewness risk ex post. Of course, predictability of risk measures is key for their successful implementation in a portfolio optimization processes. Furthermore, given limited asymmetry in equity returns the added value of alternative risk measures compared to the traditional volatility measure might also be limited. However, considering our empirical study we conclude that some alternative risk measures do a good job in portfolio optimization. For additionally demonstrating their added value over traditional approaches future research might improve the forecasting techniques for these asymmetric measures and might examine their characteristics in the presence of asset returns facing stronger asymmetry.

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Table I
Rank Correlation of Various Risk Measures

The Table reports Spearman rank correlation coefficients of estimated risk measures using pairs of consecutive years.

Years	Volatility	VaR	CVaR	Semi-		Loss	Skewness		MDD
				Deviation	*Variance*	Penalty	*Standard*	*Robust*	
93 vs 94	0.717	-0.013	0.464	0.758	0.759	0.366	0.428	-0.005	0.256
94 vs 95	0.841	0.185	0.720	0.870	0.839	0.439	-0.125	-0.055	0.221
95 vs 96	0.670	0.417	0.640	0.657	0.661	0.468	-0.158	-0.025	0.378
96 vs 97	0.431	0.171	0.273	0.527	0.426	-0.012	-0.079	-0.014	0.167
97 vs 98	0.473	0.170	-0.025	0.398	0.380	0.093	-0.017	0.057	0.152
98 vs 99	0.386	-0.039	-0.041	0.415	0.373	0.267	-0.150	0.085	-0.138
99 vs 00	0.652	-0.029	-0.043	0.660	0.677	0.060	0.314	0.193	-0.091
00 vs 01	0.399	0.186	0.477	0.463	0.400	0.141	-0.209	0.143	0.296
01 vs 02	0.651	0.104	0.738	0.752	0.657	0.338	-0.176	-0.001	0.566
02 vs 03	0.710	-0.332	0.707	0.759	0.778	0.530	0.292	-0.036	0.457
03 vs 04	0.733	0.157	0.647	0.767	0.688	0.309	0.112	-0.081	0.368
04 vs 05	0.440	-0.139	0.403	0.528	0.376	0.119	0.022	-0.196	0.130
Mean ρ	0.592	0.070	0.413	0.630	0.584	0.260	0.021	0.005	0.230
std. dev ρ	0.156	0.193	0.305	0.158	0.179	0.177	0.217	0.104	0.207

Table II
Downside Risk Reduction with Perfect Foresight of Returns

The first column contains the average value of the respective risk measure obtained from holding the benchmark portfolio. The second column gives the respective value according to a portfolio optimized with respect to that measure given perfect foresight of returns. The optimal values obtain when optimizing with respect to that measure given perfect foresight of risk and return. The fourth column gives the average gain of the optimization efforts, that is the degree to which the maximum risk spread between benchmark and optimal strategy is translated into the optimized portfolios. The last column gives the hit rate which is computed as the percentage of holding periods that exhibit more favorable risk terms than the benchmark; hit rates significant at the 5%-level are in bold face.

	Strategy	Risk			Gain	Hit Rate
		benchmark	*optimized*	*optimal*	*average*	
1	Volatility	2.538	2.264	1.854	39.96%	**89.13%**
2	VaR	-0.460	-1.025	-2.165	33.13%	**84.78%**
3	Conditional VaR	1.702	1.095	0.553	52.83%	**91.30%**
4	Semideviation	2.106	1.877	1.345	30.01%	**73.91%**
5	Semivariance	3.679	2.868	2.095	51.16%	**84.78%**
6	Loss Penalty	33.427	17.908	7.918	60.84%	**76.09%**
7	Skewness standard	0.149	0.097	-3.437	1.46%	54.35%
8	Skewness robust	0.066	0.019	-0.919	4.74%	50.00%
9	Maximum Drawdown	0.073	0.054	0.032	46.98%	**84.78%**

Table III
Downside Risk Reduction with Forecasted Returns

The first column contains the average value of the respective risk measure obtained from holding the benchmark portfolio. The second column gives the respective value according to a portfolio optimized with respect to that measure. The optimal values obtain when optimizing with respect to that measure given perfect foresight of risk and return. The fourth column gives the average gain of the optimization efforts, that is the degree to which the maximum risk spread between benchmark and optimal strategy is translated into the optimized portfolios. The last column gives the hit rate which is computed as the percentage of holding periods that exhibit more favorable risk terms than the benchmark; hit rates significant at the 5%-level are in bold face.

	Strategy	Risk			Gain	Hit Rate
		benchmark	*optimized*	*optimal*	*average*	
1	Volatility	2.538	2.275	1.854	38.37%	**86.97%**
2	VaR	-0.460	-0.407	-2.165	-3.09%	39.13%
3	Conditional VaR	1.702	1.635	0.553	5.83%	58.70%
4	Semideviation	2.106	1.908	1.345	26.03%	**58.70%**
5	Semivariance	3.679	2.862	2.095	51.55%	**84.78%**
6	Loss Penalty	33.427	17.700	7.918	61.65%	**69.57%**
7	Skewness standard	0.149	0.029	-3.437	3.35%	58.70%
8	Skewness robust	0.066	0.016	-0.919	5.01%	63.04%
9	Maximum Drawdown	0.073	0.069	0.032	10.67%	**67.39%**

Table IV
Mutual Tracking Errors of Strategies

Mutual tracking errors of the implemented strategies. Above the diagonale we show tracking errors for the case of perfect foresight. The mutual tracking errors for the case of forecasted returns are noted below the diagonale. Along the diagonale we display the mutual tracking errors of strategies with perfect foresight versus forecasted returns in bold face. BM denotes benchmark and MDD is maximum drawdown.

	BM	Volatility	VaR	CVaR	Semi- *Deviation*	Semi- *Variance*	Loss Penalty	Skewness *Standard*	Skewness *Robust*	MDD
BM	**0.000**	4.972	6.852	7.275	4.742	5.151	5.161	5.133	5.014	6.338
Volatiltiy	5.191	**2.201**	7.889	6.852	2.796	2.109	3.303	7.038	6.062	5.827
VaR	5.200	6.833	**8.042**	6.102	7.701	8.180	8.141	8.670	7.886	7.240
CVaR	5.743	3.947	6.779	**8.127**	6.935	6.952	7.101	8.746	7.697	5.603
Semideviation	5.098	2.829	6.764	3.896	**3.131**	3.278	4.388	6.734	6.042	6.297
Semivariance	5.362	2.180	7.214	4.453	3.310	**2.112**	2.341	6.418	6.133	5.749
Loss Penalty	5.473	3.318	7.331	5.212	4.422	2.279	**1.918**	6.209	6.156	5.663
Skewness Std.	5.378	7.124	7.567	7.179	6.895	6.331	6.194	**3.866**	7.115	7.715
Skewness Rob.	5.018	6.165	7.343	6.587	6.115	6.201	6.510	6.852	**6.377**	7.349
MDD	5.982	5.540	7.461	5.618	5.919	5.474	5.441	7.104	7.288	**6.332**

Table V
Mutual Downside Risk

Mutual downside risk of the implemented strategies. Each row contains the risk measure as given in the first column for the downside strategies together with the mean-variance strategy and the benchmark. The downside measure of the best strategy according to the respective dimension is in bold face. BM denotes benchmark and MDD is maximum drawdown.

	BM	Volatility	VaR	CVaR	Semi-		Loss	Skewness		MDD
					Deviation	*Variance*	Penalty	*Standard*	*Robust*	
Panel A: Optimal										
BM										
Volatiltiy	2.85	**2.29**	2.99	2.54	2.44	2.36	2.40	2.96	2.89	2.49
VaR	-0.53	-0.55	**-1.57**	-1.13	-0.55	-0.37	-0.41	-0.20	-0.17	-0.92
CVaR	1.71	1.10	1.21	**0.66**	1.16	1.14	1.19	1.61	1.66	0.74
Semideviation	1.04	**0.78**	1.14	0.89	0.81	0.80	0.82	1.04	1.08	0.89
Semivariance	8.13	**5.22**	8.90	6.45	5.92	5.56	5.74	8.74	8.33	6.21
Loss Penalty	291.1	70.3	540.9	118.4	123.5	54.2	**46.0**	106.2	294.7	86.0
Skewness Std.	0.00	-0.09	0.12	-0.27	-0.19	-0.49	-0.51	**-0.79**	-0.11	-0.49
Skewness Rob.	0.07	0.06	0.33	-0.04	0.10	-0.09	-0.01	-0.05	**-0.32**	-0.09
MDD	50.48	34.63	29.73	21.61	38.56	35.85	36.11	44.98	43.63	**21.28**
Mean Ranking	8.11	3.56	7.89	4.33	5.56	3.44	4.22	6.78	7.67	3.44
Panel B: Perfect Foresight										
BM										
Volatiltiy	2.85	2.55	3.00	2.85	2.60	**2.52**	2.56	2.88	2.79	2.67
VaR	-0.53	-0.60	-0.99	**-1.09**	-0.53	-0.57	-0.55	-0.54	-0.59	-0.89
CVaR	1.71	1.45	1.22	**1.09**	1.48	1.44	1.47	1.73	1.55	1.14
Semideviation	1.04	0.94	1.08	1.05	0.95	**0.94**	0.95	1.07	1.03	1.00
Semivariance	8.13	6.47	8.98	8.12	6.77	**6.36**	6.54	8.28	7.79	7.11
Loss Penalty	291.1	83.4	1632.7	174.8	150.6	65.9	**52.7**	187.8	395.5	87.8
Skewness Std.	0.00	0.08	**-0.36**	-0.24	-0.04	0.07	0.04	0.02	-0.03	-0.05
Skewness Rob.	0.07	0.11	0.03	0.07	0.01	0.08	0.06	0.06	**-0.04**	0.01
MDD	50.48	40.97	23.82	**23.04**	40.72	38.84	37.82	49.55	38.74	23.18
Mean Ranking	8.22	5.11	5.89	4.44	5.00	4.33	4.67	8.11	5.67	3.56
Panel C: Forecasted										
BM										
Volatiltiy	2.85	2.54	2.90	2.67	2.59	**2.52**	2.55	2.85	2.81	2.65
VaR	-0.53	**-0.57**	-0.41	-0.50	-0.43	-0.55	-0.50	-0.52	-0.51	-0.53
CVaR	1.71	1.53	1.81	1.63	1.57	**1.50**	1.51	1.73	1.73	1.60
Semideviation	1.04	0.94	1.08	1.00	0.96	**0.94**	0.95	1.06	1.05	1.00
Semivariance	8.13	6.42	8.38	7.13	6.70	**6.32**	6.51	8.13	7.90	6.99
Loss Penalty	291.1	70.1	689.7	113.6	107.1	60.8	**53.8**	196.5	211.3	74.3
Skewness Std.	0.00	0.11	0.08	0.03	0.01	0.11	0.10	0.01	**-0.06**	0.10
Skewness Rob.	0.07	0.13	**0.02**	0.09	0.03	0.14	0.07	0.07	0.05	0.03
MDD	50.48	50.03	46.99	46.87	49.80	48.08	**45.89**	49.63	48.93	49.17
Mean Ranking	6.89	4.56	7.78	5.89	4.89	3.44	3.67	7.00	6.00	4.89

Figure 1: Some Density Functions of Skewed Distributions

The graph gives sample density functions for skewed distributions. All plotted density functions have zero mean and are scaled to a volatility equal to 1. The more rightskewed density functions refer to random variables that have significantly less mass at the lower tails and are represented by dashed lines. Accordingly, for a downside risk-oriented investor the dotted curves correspond to more risky assets.

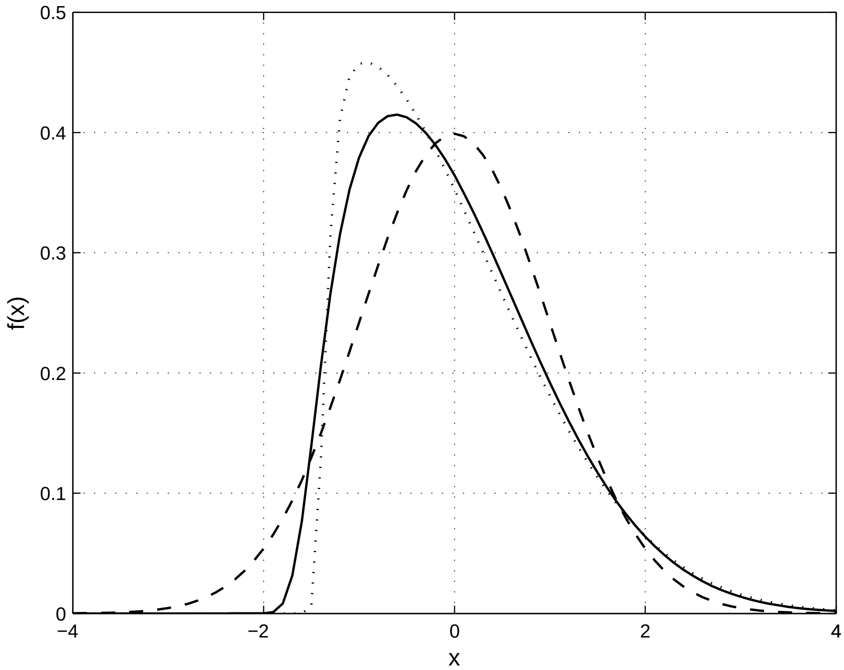

Figure 2: Skewness versus Volatility

The left graph gives two equally right-skewed distribution functions with volatility 1 (solid line) and volatility 2 (dotted line), compared to a symmetric distribution with volatility 1 (dashed line). The right graph compares two symmetric density functions with volatility 1 (solid line) to one with volatility 2 (dotted line) and to a leftskewed density function with volatility 1 (dashed line).

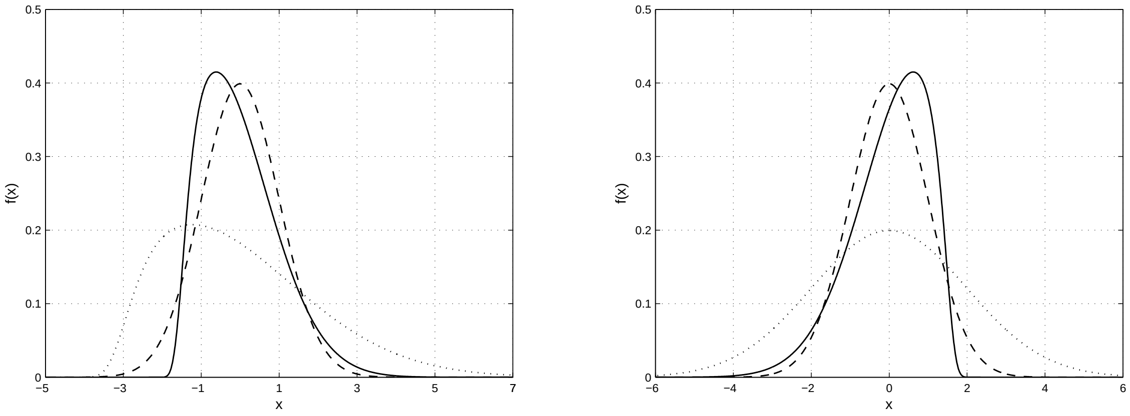

Figure 3: Visualizing Maximum Drawdown

The dashed line gives the time series of portfolio value that arises from a benchmark investment. The solid line represents the respective time series according to a portfolio optimized with respect to maximum drawdown (given perfect foresight of return). The arrows indicate the maximum drawdown, again, the dashed arrow is for the benchmark while the solid arrow is for the optimized portfolio.

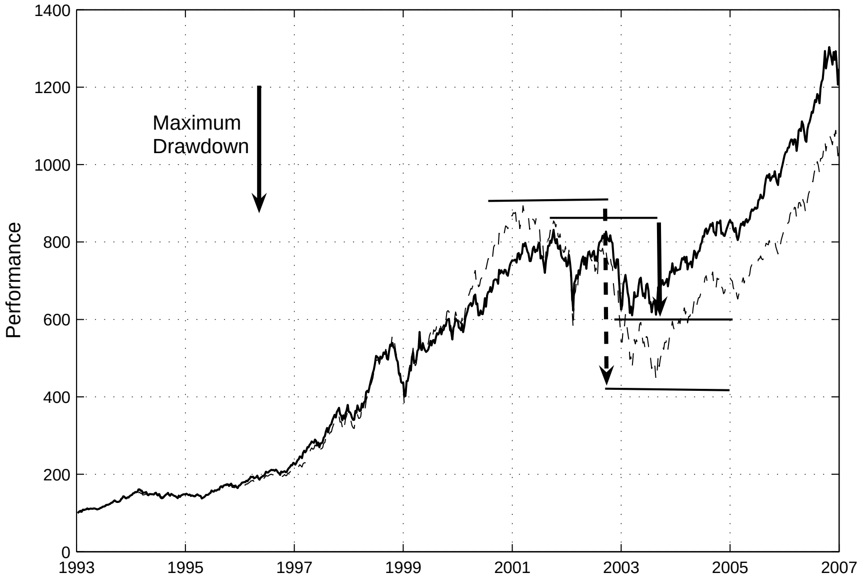

Figure 4: Downside Risk over Time with Perfect Foresight

The respective upper graphs give the evolution of downside risk over time. The light-shaded indicates the spread between the optimal portfolio (given perfect foresight of risk and return) and the benchmark. The solid line represents the optimized portfolio's realized downside measure. The respective lower graphs quantify the degree of spread exploitation that is attained in the corresponding optimization.

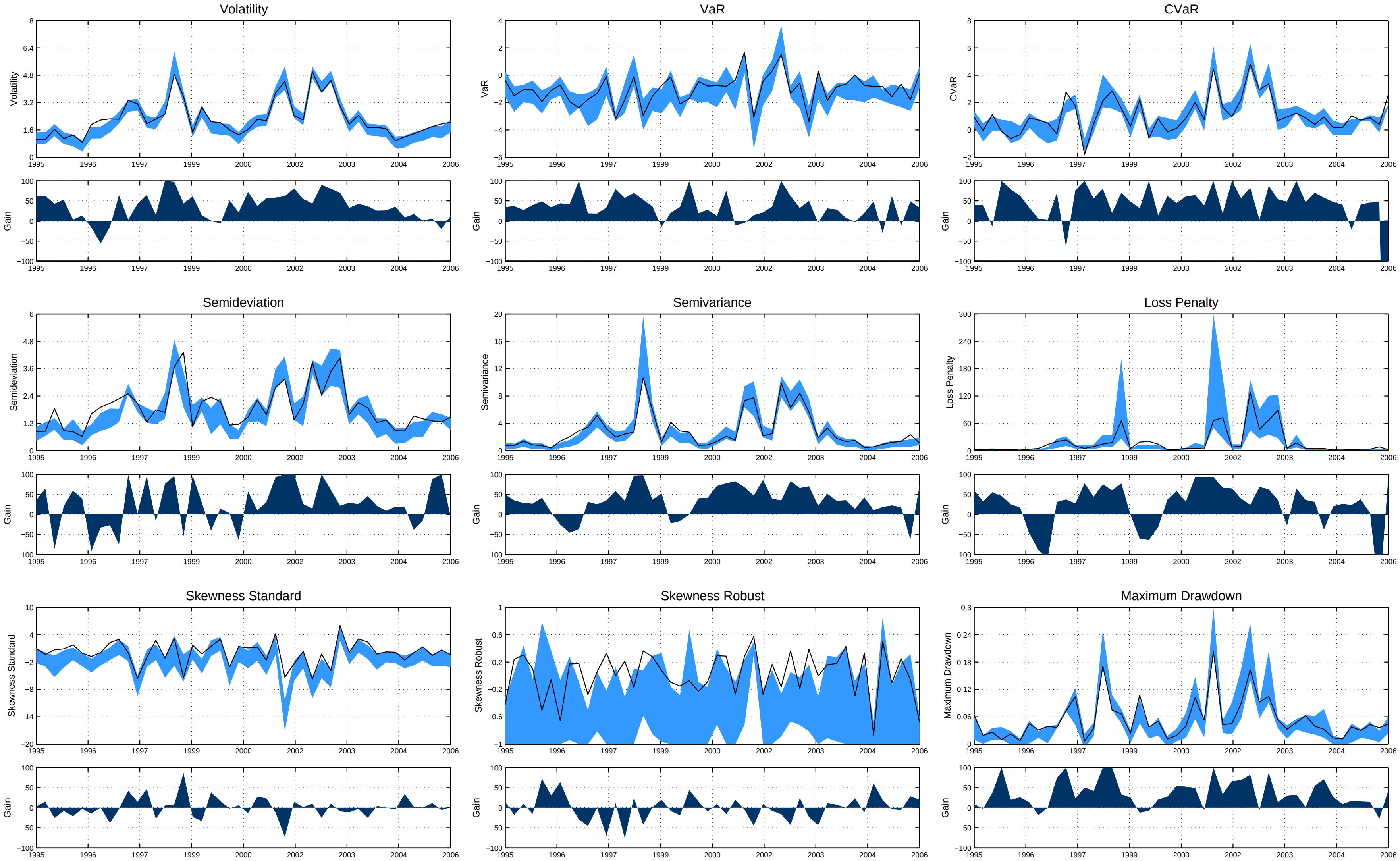

Figure 5: Downside Risk over Time with Forecasted Returns

The respective upper graphs give the evolution of downside risk over time. The light-shaded indicates the spread between the optimal portfolio (given perfect foresight of risk and return) and the benchmark. The solid line represents the optimized portfolio's realized downside measure. The respective lower graphs quantify the degree of spread exploitation that is attained in the corresponding optimization.

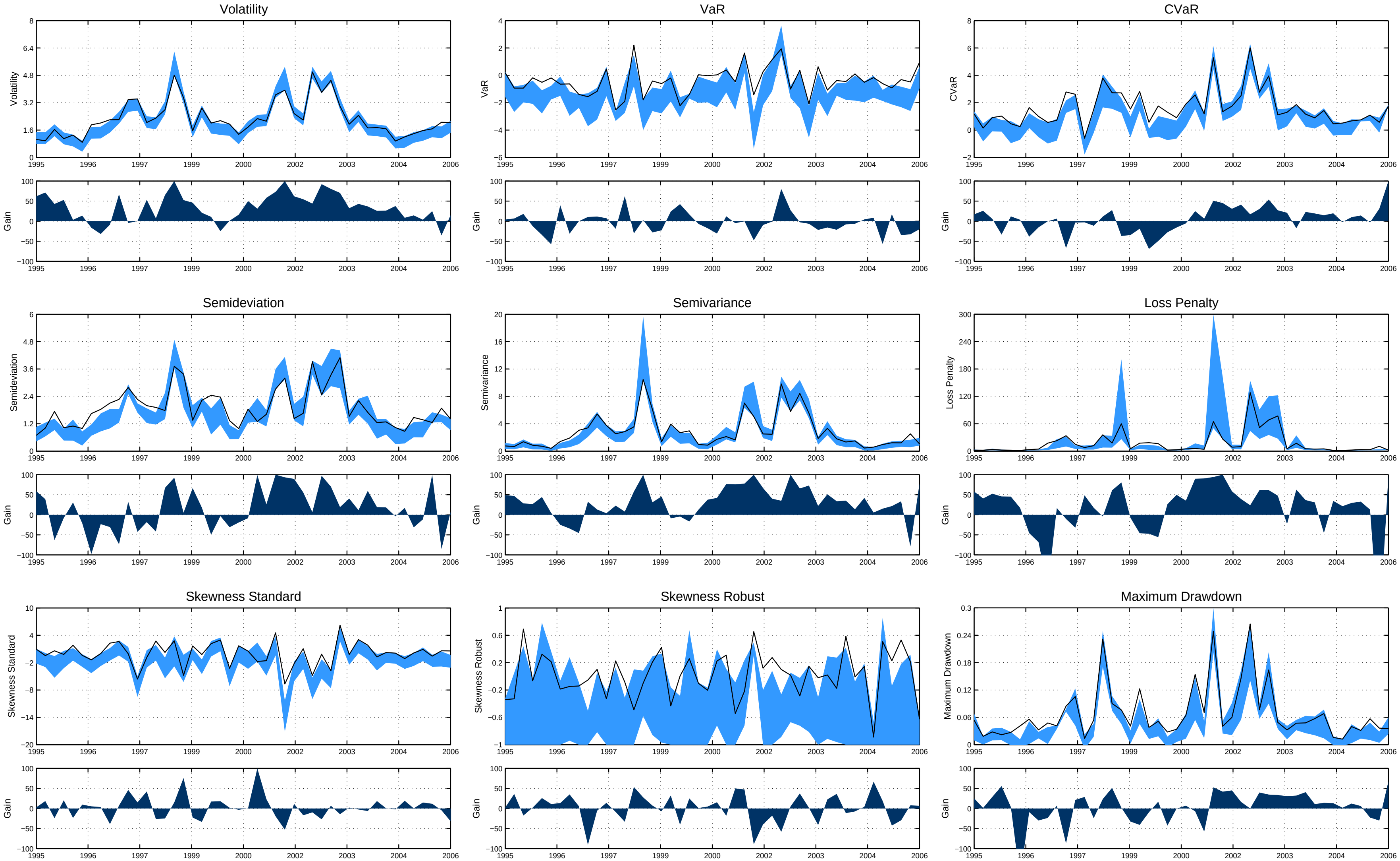

Figure 6: Performance of Downside Risk Strategies

The graphs give the cumulated return of several strategies. The light-shaded are indicates the spread between the highest cumulated return and the lowest cumulated return among the downside risk strategies. The solid line represents the cumulated return of one particular strategy as indicated by the graph's title.